

Transition between Schwarzschild black hole and string black hole

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In this paper, we aim to study the quantum transition between a Schwarzschild black hole and a string black hole in the large D limit. Classically, such a transition between these two distinct black hole geometries is forbidden. The only feasible discussion is centered on how a black hole evaporates, loses mass, and transitions into highly excited fundamental strings. Building upon our previous work on T-duality between the Schwarzschild and string black holes, we reduce the problem to two dimensions, where the corresponding Wheeler–de Witt equation can be derived. Using this equation, we identify the two black hole geometries as distinct wave function states. This allows us to easily compute the transition probability between these two geometries, driven by the string coupling.

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I. INTRODUCTION

The evolution of a Schwarzschild black hole into highly excited fundamental strings through mass loss during evaporation is an intriguing problem [1–4], as it bridges two distinct regimes: the black hole interior and string microstates [5]. When the black hole’s mass is sufficiently large, the highly excited fundamental strings form a single long string, whose self-gravitation gives rise to a string star. The string star continues to evaporate, eventually transitioning into free strings. Traditionally, this process has been studied using the Horowitz-Polchinski effective field theory [4]. Relevant discussions are provided in Refs. [6–8]. In this paper, we approach the problem from a new perspective. We do not focus on how the Schwarzschild black hole directly evolves into fundamental strings; instead, we investigate its intermediate step, specifically how the Schwarzschild black hole transitions into the string black hole.¹

Classically, the transition from a Schwarzschild black hole to a string black hole is forbidden, as they represent solutions to different actions—the Einstein-Hilbert action and the low-energy effective action of string theory, respectively. However, quantum dynamics allows transitions between these two distinct geometric configurations.

Our goal is to describe this evolution using a quantum framework. Although the Schwarzschild and string black holes arise from different actions, their connection can be established through key observations:

- (1) The low-energy effective action admits two kinds of black hole solutions. The first is the string black hole, with a nontrivial dilaton and a naked singularity but no event horizon. The second is the Schwarzschild black hole, with a constant or vanishing dilaton source. These two solutions are T-dual to each other [9–12]; the dilaton vanishes in the T-dual transformation, linking the two black hole solutions.
- (2) In the framework of large D gravity [13], near-horizon geometries of both solutions reduce to two-dimensional black strings, whose relationship is described by the well-known scale-factor duality of two-dimensional low-energy effective theory [14]. This simplification reduces the complex problem to a two-dimensional framework.

Crucially, the two-dimensional reduced solutions from Schwarzschild and string black holes share the same low-energy effective action and cover different regions of the two-dimensional spacetime, as illustrated in Fig. 1. This result closely parallels developments in string cosmology [15–18]. In that context, the low-energy effective action of the bosonic string in a cosmological background reveals a scale-factor duality $a(t) \leftrightarrow a(t)^{-1}$. When combined with time-reversal symmetry, this duality leads to the so-called pre-big-bang scenario, which predicts a long evolutionary phase preceding the big bang. The pre- and post-big-bang branches are classically disconnected by the big bang singularity, rendering transitions between them classically forbidden. Nevertheless, the quantum string cosmological dynamics admits nonvanishing transition probabilities between the pre- and post-big-bang phases,

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¹Here, a string black hole refers to a black hole solution of the low-energy effective action in string theory, featuring a nontrivial dilaton field.

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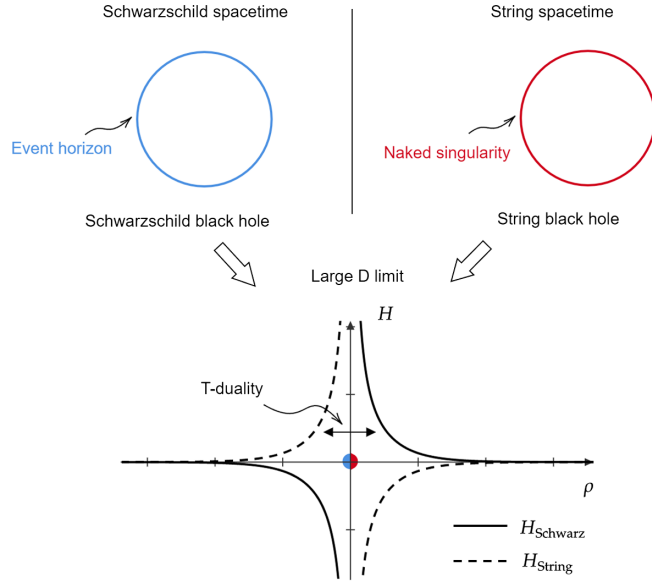


FIG. 1. This figure depicts our strategy for connecting two distinct black holes—one from Einstein’s gravity and the other from string theory. In the large D limit, the near-horizon geometries of Schwarzschild and string black holes reduce to two T-dual, two-dimensional black strings that cover different regions of the two-dimensional spacetime. In this figure, H represents the Hubble-like parameter for the background, and ρ is a spacelike coordinate. Detailed definitions and further explanations can be found in the rest of the paper.

despite their classical separation by a singularity [19–23]. More specifically, in the string cosmology framework, the classical low-energy effective action is quantized via the Wheeler–de Witt (WDW) approach. This yields a wave function defined on superspace, where each point corresponds to a particular geometric configuration. By solving the WDW equation under appropriate tunneling boundary conditions, one can determine the wave function and thereby compute quantum transition probabilities between classically disconnected spacetime regions, such as those separated by a big bang singularity. These transition probabilities are often extracted from the reflection coefficient of the wave function.

In this paper, we adopt the well-established methodology of quantum string cosmology [19] to explore the possibility of quantum transitions between distinct black hole geometries. Our starting point is a pair of T-dual black hole solutions derived from the low-energy effective action of string theory, corresponding respectively to the Schwarzschild and string black holes. In the large D limit, these solutions reduce to two T-dual black string configurations, together covering the entire two-dimensional spacetime [14], as illustrated in Fig. 1. We then derive the Hamiltonian constraint and the associated WDW equation for the system. Consistent with results from quantum string cosmology [19], our analysis shows that the classically forbidden transition between

Schwarzschild and string black holes becomes allowed at the quantum level. This establishes a potential quantum connection between general relativity and string theory and suggests the intriguing possibility that a black hole with an event horizon could quantum mechanically transition into a configuration featuring a naked singularity.

This paper is organized as follows. In Sec. II, we provide a review of the higher-dimensional black hole solutions and their T-dual counterparts in string theory. We then demonstrate the scale-factor duality between these two black hole solutions in the large D limit. In Sec. III, we derive the WDW equation for the two-dimensional low-energy effective action and compute the transition probability between the Schwarzschild black hole and the string black hole. Finally, in Sec. IV, we present a discussion and conclude the paper.

II. BRIEF REVIEW OF T-DUAL LARGE D BLACK HOLES

In our previous work [14], we obtained the D -dimensional T-dual black hole solutions from the low-energy effective string theory. We begin with the low-energy effective action of string theory in D dimensions, assuming a vanishing Kalb–Ramond field and cosmological constant,

$$I_{\text{String}} = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g} e^{-2\phi} (R + 4(\partial\phi)^2), \quad (2.1)$$

where ϕ represents the D -dimensional dilaton field. To extend the four-dimensional black hole solution [11] to higher dimensions and its T-dual, we proceed as follows.

A. Ordinary solution

The ordinary higher-dimensional black hole solution is given by

$$ds_{\text{Schwarz}}^2 = - \left(1 - \left(\frac{2\eta}{r} \right)^n \right)^{\frac{m+\sigma}{\eta}} dt^2 + \left(1 - \left(\frac{2\eta}{r} \right)^n \right)^{\frac{\sigma-m}{\eta}} dr^2 + \left(1 - \left(\frac{2\eta}{r} \right)^n \right)^{1+\frac{\sigma-m}{\eta}} r^2 d\Omega_{n+1}^2, \quad (2.2)$$

where $d\Omega_{n+1}^2$ denotes the metric of an $(n+1)$ -dimensional sphere, and the dilaton solution is

$$\phi_{\text{Schwarz}}(r) = \frac{1}{4\eta} ((n-1)(\eta-m) + (n+1)\sigma) \ln \left(1 - \left(\frac{2\eta}{r} \right)^n \right), \quad (2.3)$$

subject to the constraint

$$\begin{aligned} &-(n+1)m^2 - (n-3)\eta^2 - (n+1)\sigma^2 + 2(n-1)m(\eta + \sigma) \\ &- 2(n-1)\eta\sigma = 0. \end{aligned} \quad (2.4)$$

B. T-dual solution

Applying the Buscher rules with a vanishing Kalb-Ramond field along the g_{00} direction,

$$\tilde{g}_{00} = \frac{1}{g_{00}}, \quad \phi_{\text{String}} = \phi_{\text{Schwarz}} - \frac{1}{2} \ln(-g_{00}), \quad (2.5)$$

the T-dual black hole solution is

$$\begin{aligned} ds_{\text{String}}^2 = & - \left(1 - \left(\frac{2\eta}{r} \right)^n \right)^{\frac{-m-\sigma}{\eta}} dt^2 + \left(1 - \left(\frac{2\eta}{r} \right)^n \right)^{\frac{\sigma-m}{\eta}} dr^2 \\ & + \left(1 - \left(\frac{2\eta}{r} \right)^n \right)^{1+\frac{\sigma-m}{\eta}} r^2 d\Omega_{n+1}^2, \end{aligned} \quad (2.6)$$

with the shifted dilaton solution:

$$\phi_{\text{String}}(r) = \frac{1}{4\eta} ((n-1)(\eta + \sigma) - (n+1)m) \ln \left(1 - \left(\frac{2\eta}{r} \right)^n \right). \quad (2.7)$$

The constraint for the parameters is same as (2.4).

To recover the Schwarzschild black hole, we choose identical values for m , η , and σ , such that $\sigma = 0$ and $m = \eta$. Consequently, the metrics (2.2) and (2.6) are given by

$$\begin{aligned} ds_{\text{Schwarz}}^2 = & - \left(1 - \left(\frac{r_0}{r} \right)^n \right) dt^2 + \frac{dr^2}{(1 - (\frac{r_0}{r})^n)} + r^2 d\Omega_{n+1}^2, \\ \phi_{\text{Schwarz}}(r) = & 0, \end{aligned} \quad (2.8)$$

$$\begin{aligned} ds_{\text{String}}^2 = & - \left(1 - \left(\frac{r_0}{r} \right)^n \right)^{-1} dt^2 + \frac{dr^2}{(1 - (\frac{r_0}{r})^n)} + r^2 d\Omega_{n+1}^2, \\ \phi_{\text{String}}(r) = & -\frac{1}{2} \ln \left(1 - \left(\frac{r_0}{r} \right)^n \right), \end{aligned} \quad (2.9)$$

where $r_0 = 2m$. Here, ds_{Schwarz}^2 represents the Schwarzschild-Tangherlini black hole solution in Einstein's gravity with a vanishing dilaton source, where r_0 denotes the event horizon. On the other hand, ds_{String}^2 corresponds to the string black hole solution with a non-vanishing dilaton source, where r_0 indicates the curvature singularity.

To analyze the large D limit of the metric, we introduce the coordinate transformation $R = (\frac{r}{r_0})^n$. The metric for the Schwarzschild black hole (2.8) then becomes

$$ds_{\text{Schwarz}}^2 = -\frac{R-1}{R} dt^2 + \frac{r_0^2}{n^2} R^{\frac{2}{n}} \frac{1}{R(R-1)} dR^2 + r_0^2 R^{\frac{2}{n}} d\Omega_{n+1}^2, \quad (2.10)$$

which is equivalent to performing a dimensional reduction on a sphere in the Einstein-Hilbert action:

$$ds^2 = \underbrace{\mathbb{G}_{\mu\nu}(x^\mu) dx^\mu dx^\nu}_{2 \text{ dimensions}} + \underbrace{r_0^2 e^{-4\hat{\phi}(x^\mu)/(n+1)} d\Omega_{n+1}^2}_{n+1 \text{ dimensional sphere}}. \quad (2.11)$$

This transformation introduces the effective dilaton $\hat{\phi}(x^\mu)$ into the Einstein-Hilbert action, yielding the low-energy effective action of string theory. The solution for the effective dilaton is obtained by comparing the two metrics (2.10) and (2.11):

$$\hat{\phi}(R) = -\frac{n+1}{2n} \ln R. \quad (2.12)$$

To study the near-horizon geometry as $r \rightarrow r_0$ [13], we use the coordinate transformation

$$R = \cosh^2(\lambda\rho), \quad (2.13)$$

where $\lambda = \frac{n}{2r_0}$. Applying the large n limit to both the ordinary (2.8) and T-dual (2.9) metrics, we obtain the expressions

$$\begin{aligned} ds_{\text{Schwarz}}^2 = & d\rho^2 - \tanh^2(\lambda\rho) dt^2, \quad \hat{\phi}(\rho) = -\frac{1}{2} \ln \cosh^2(\lambda\rho), \\ ds_{\text{String}}^2 = & d\rho^2 - \coth^2(\lambda\rho) dt^2, \quad \tilde{\phi}(\rho) = -\frac{1}{2} \ln \sinh^2(\lambda\rho), \end{aligned} \quad (2.14)$$

where $\tilde{\phi}(\rho)$ arises from the dimensional reduction (2.11), which incorporates the effective dilaton $\hat{\phi}(\rho)$ into the standard one $\phi_{\text{String}}(\rho)$ from the solution (2.9):

$$\begin{aligned} \tilde{\phi}(\rho) = & \hat{\phi}(\rho) + \phi_{\text{String}}(\rho) \\ = & -\frac{1}{2} \ln \cosh^2(\lambda\rho) - \frac{1}{2} \ln \tanh^2(\lambda\rho) \\ = & -\frac{1}{2} \ln \sinh^2(\lambda\rho). \end{aligned} \quad (2.15)$$

Thus, the near-horizon geometry $r \rightarrow r_0$ can be effectively described by a two-dimensional low-energy effective action that incorporates the effective dilaton [13].

Before further discussion, let us recall the coordinate transformation (2.13). When $r \geq r_0$, $R \geq 1$. Since $\cosh(\rho) \geq 1$, it implies that the coordinate ρ can range from $-\infty$ to $+\infty$. To make this clearer, let us introduce the Hubble-like parameter,

$$H(\rho) \equiv \frac{\partial_\rho a(\rho)}{a(\rho)}, \quad (2.16)$$

for the metric

$$ds^2 = d\rho^2 - a(\rho)^2 dt^2. \quad (2.17)$$

The Hubble-like parameter determines the scalar curvature for this specific form of the metric. Considering the dual metrics (2.14), the corresponding Hubble-like parameters are given by

$$H_{\text{Schwarz}}(\rho) = 2\lambda \text{csch}(2\lambda\rho), \quad H_{\text{String}}(\rho) = -2\lambda \text{csch}(2\lambda\rho). \quad (2.18)$$

Note that $H_{\text{Schwarz}}(\rho) \leftrightarrow -H_{\text{String}}(\rho)$ and $\hat{\phi}(\rho) \leftrightarrow \tilde{\phi}(\rho)$ represent a well-known scale-factor duality [24,25]. The Hubble-like parameters are plotted in the Fig. 1.

III. WHEELER-DE WITT EQUATION IN TWO SPACETIMES

We have noted in the previous section that the Schwarzschild-Tangherlini black hole (with a constant or vanishing dilaton source ϕ) and its T-dual, namely, the string black hole, are both solutions to the low-energy effective action of closed string theory. Since these two black hole solutions can reduce to the two T-dual string black holes (2.14) in the large D limit, the corresponding D -dimensional action must also reduce to the relevant two-dimensional action simultaneously. To clarify this, we will consider these two cases separately. Our method and analysis closely follow the approach of Ref. [19], which may be viewed as a black hole extension of this well-established framework.

We first examine the Schwarzschild-Tangherlini black hole with a vanishing dilaton source. The action is simply the Einstein-Hilbert action:

$$I_{\text{EH}} = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g} R. \quad (3.1)$$

By performing dimensional reduction on a sphere and introducing the effective dilaton field $\hat{\phi}(x^\mu)$, we write the metric as

$$ds^2 = \underbrace{\mathbb{G}_{\mu\nu}(x^\mu) dx^\mu dx^\nu}_{2 \text{ dimensions}} + \underbrace{r_0^2 e^{-4\hat{\phi}(x^\mu)/(n+1)} d\Omega_{n+1}^2}_{n+1 \text{ dimensional sphere}}, \quad (3.2)$$

where the Einstein-Hilbert action becomes

$$I_{\text{EH}} = \frac{\Omega_{n+1} r_0^{n+1}}{16\pi G_D} \int d^2 x \sqrt{-\mathbb{G}} e^{-2\hat{\phi}} \times \left(\mathbb{R} + \frac{4n}{n+1} (\partial\hat{\phi})^2 + \frac{n(n+1)}{r_0^2} e^{-4\hat{\phi}/(n+1)} \right), \quad (3.3)$$

where $\mathbb{R}$ is the Ricci scalar of the two-dimensional metric $\mathbb{G}_{\mu\nu}$ and the volume of the unit sphere is given by $\Omega_{n+1} = 2\pi^{\frac{n+2}{2}}/\Gamma(\frac{n+2}{2})$. In the large n limit ($n \rightarrow \infty$), the action reduces to the two-dimensional string effective action. This action possesses an $SU(2)_k/U(1)$ symmetry,

$$I_{\text{EH}}^{2D} = \frac{1}{16\pi G_2} \int d^2 x \sqrt{-\mathbb{G}} e^{-2\hat{\phi}} (\mathbb{R} + 4(\partial\hat{\phi})^2 + 4\lambda^2), \quad (3.4)$$

where $G_2 = \lim_{n \rightarrow \infty} \frac{G_D}{\Omega_{n+1} r_0^{n+1}}$ and $\lambda = \frac{n}{2r_0}$.

On the other hand, recall the T-dual action for the Einstein-Hilbert action, which is the low-energy effective action with a nonvanishing dilaton source ϕ_{String} :

$$I_{\text{String}} = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g} e^{-2\phi_{\text{String}}} (R + 4(\partial\phi_{\text{String}})^2). \quad (3.5)$$

Using the dimensional reduction on the sphere (3.2), the action (3.5) becomes

$$I_{\text{String}} = \frac{\Omega_{n+1} r_0^{n+1}}{16\pi G_D} \int d^2 x \sqrt{-\mathbb{G}} e^{-2(\phi_{\text{String}} + \hat{\phi})} \left[\mathbb{R} + 4(\partial\phi_{\text{String}})^2 + 8\partial\hat{\phi}\partial\phi_{\text{String}} + \frac{4n}{n+1} (\partial\hat{\phi})^2 + \frac{n(n+1)}{r_0^2} e^{\frac{4\hat{\phi}}{n+1}} \right], \quad (3.6)$$

where $\mathbb{R}$ is the Ricci scalar of the two-dimensional metric $\mathbb{G}_{\mu\nu}$. In the limit $n \rightarrow \infty$, the action reduces to the two-dimensional low-energy effective action,

$$I_{\text{String}}^{2D} = \frac{1}{16\pi G_2} \int d^2 x \sqrt{-\mathbb{G}} e^{-2\tilde{\phi}} (\mathbb{R} + 4(\partial\tilde{\phi})^2 + 4\lambda^2), \quad (3.7)$$

where $\tilde{\phi}(\rho) = \hat{\phi}(\rho) + \phi_{\text{String}}(\rho)$. We also use the relations $G_2 = \lim_{n \rightarrow \infty} \frac{G_D}{\Omega_{n+1} r_0^{n+1}}$ and $\lambda = \frac{n}{2r_0}$. As expected, the Einstein-Hilbert action (3.1) and low-energy effective action (3.5) both reduce to the same two-dimensional low-energy effective action (3.7) in the large D limit. Now, considering the combination of the solutions (2.14), we focus on the following configuration:

$$\begin{aligned} \rho < 0, \quad a(\rho) &= \coth(\lambda\rho), \quad \tilde{\phi}(\rho) = -\frac{1}{2}\ln \sinh^2(\lambda\rho), \quad \text{string spacetime,} \\ \rho > 0, \quad a(\rho) &= \tanh(\lambda\rho), \quad \hat{\phi}(\rho) = -\frac{1}{2}\ln \cosh^2(\lambda\rho), \quad \text{Schwarzschild spacetime.} \end{aligned} \quad (3.8)$$

Therefore, the two geometries can be stitched together in the (t, ρ) plane, allowing us to study their quantum properties. Next, it is easy to check that the action (3.7) is invariant under the following T-dual transformations, also known as scale-factor duality:

$$a(\rho) \leftrightarrow a(\rho)^{-1}, \quad H(\rho) \leftrightarrow -H(\rho), \quad \hat{\phi}(\rho) \leftrightarrow \tilde{\phi}(\rho). \quad (3.9)$$

Now, our goal is to obtain the WDW equation from the action (3.7). In quantum mechanics, a wave function controlled by the Schrödinger equation is used to describe a particle in Hilbert space. Similarly, if we treat the whole spacetime as a particle, we can mimic quantum mechanics to use a wave function to study the quantum dynamics of the spacetime manifold. These quantum effects allow for transitions between different geometries that are forbidden in classical theory. In this analogy, the Hilbert space is replaced by the superspace, and the wave function is referred to the WDW equation. In other words, spacetime can be viewed as a particle state in superspace. For our case, this two-dimensional superspace $\beta(\rho)$, $\Phi(\rho)$, called minisuperspace, is defined by

$$\beta(\rho) = \ln a(\rho), \quad \Phi(\rho) = 2\tilde{\phi}(\rho) - \beta(\rho). \quad (3.10)$$

To derive the WDW equation using the variables $\beta(\rho)$ and $\Phi(\rho)$ in the minisuperspace, we must recover the temporal-like gauge $n(\rho)$. Thus, the ansatz for the metric is given by

$$ds^2 = n(\rho)^2 d\rho^2 - a(\rho)^2 dt^2. \quad (3.11)$$

The action (3.7) takes the form

$$\begin{aligned} \rho < 0, \quad \Pi_\beta &= -\beta'(-\rho)e^{-\Phi(-\rho)} = -\lambda \equiv -k, \quad \text{string spacetime,} \\ \rho > 0, \quad \Pi_\beta &= -\beta'(\rho)e^{-\Phi(\rho)} = -\lambda \equiv -k, \quad \text{Schwarzschild spacetime,} \end{aligned} \quad (3.16)$$

B. Classical solution for Π_Φ

Similarly, we can derive the solution for Π_Φ :

$$\begin{aligned} \rho < 0, \quad \Pi_\Phi &= \Phi'(-\rho)e^{-\Phi(-\rho)} = \lambda \cosh(2\lambda\rho), \quad \text{string spacetime,} \\ \rho > 0, \quad \Pi_\Phi &= \Phi'(\rho)e^{-\Phi(\rho)} = -\lambda \cosh(2\lambda\rho), \quad \text{Schwarzschild spacetime.} \end{aligned} \quad (3.17)$$

The relation between $\Phi(\rho)$ and ρ is illustrated in Fig. 2.

$$I_{\text{String}}^{2D} = \int d\rho e^{-\Phi} \frac{1}{2n} [\Phi'^2 - \beta'^2 + n^2(4\lambda^2)], \quad (3.12)$$

where the prime denotes the derivative with respect to ρ . Note that Φ is also referred to as the $O(d, d)$ dilaton, and $\beta' = H$ is the Hubble-like parameter. The action is invariant under the transformations $\Phi \leftrightarrow \Phi$ and $H \leftrightarrow -H$. From this action, the Hamiltonian-like constraint is derived:

$$\left. \frac{\delta I_{\text{String}}^{2D}}{\delta n} \right|_{n=1} = 0 \Rightarrow \Phi'^2 - \beta'^2 - 4\lambda^2 = 0. \quad (3.13)$$

To introduce the canonical momenta, we define

$$\Pi_\beta = \left. \frac{\delta I_{\text{String}}^{2D}}{\delta \beta'} \right|_{n=1} = -\beta' e^{-\Phi}, \quad \Pi_\Phi = \left. \frac{\delta I_{\text{String}}^{2D}}{\delta \Phi'} \right|_{n=1} = \Phi' e^{-\Phi}. \quad (3.14)$$

The Hamiltonian is then

$$\mathcal{H} = \Pi_\beta^2 - \Pi_\Phi^2 + 4\lambda^2 e^{-2\Phi}. \quad (3.15)$$

This Hamiltonian satisfies the momentum conservation condition: $[\Pi_\beta, \mathcal{H}] = 0$.

To relate the wave functions to Schwarzschild spacetime and string spacetime, we present the following classical solutions for Π_β and Π_Φ :

A. Classical solution for Π_β

Using the solutions from Eq. (3.8), we can verify the following:

At the strong coupling regime, where $\Phi \rightarrow +\infty$ corresponds to $\rho \rightarrow 0$, we have the following limits:

$$\begin{aligned} \rho < 0, \quad \lim_{\Phi \rightarrow +\infty} \Pi_\Phi = \lambda = k, \quad & \text{string spacetime,} \\ \rho > 0, \quad \lim_{\Phi \rightarrow +\infty} \Pi_\Phi = -\lambda = -k, \quad & \text{Schwarzschild spacetime.} \end{aligned} \quad (3.18)$$

For $\Phi \rightarrow -\infty$ ($\rho \rightarrow \pm\infty$), we have

$$\begin{aligned} \rho \rightarrow -\infty, \quad \lim_{\Phi \rightarrow -\infty} \Pi_\Phi = 2\lambda e^{-\Phi} \equiv z, \quad & \text{string spacetime,} \\ \rho \rightarrow +\infty, \quad \lim_{\Phi \rightarrow -\infty} \Pi_\Phi = -2\lambda e^{-\Phi} \equiv -z, \quad & \text{Schwarzschild spacetime.} \end{aligned} \quad (3.19)$$

The corresponding Hamiltonian-like constraint from Eq. (3.13) is

$$\Pi_\beta^2 - \Pi_\Phi^2 + 4\lambda^2 e^{-2\Phi} = 0. \quad (3.20)$$

Thus, we obtain the WDW equation for the action (3.7) through $\Pi_\beta \rightarrow i\partial_\beta$ and $\Pi_\Phi \rightarrow i\partial_\Phi$:

$$[\partial_\Phi^2 - \partial_\beta^2 + 4\lambda^2 e^{-2\Phi}] \Psi(\beta, \Phi) = 0. \quad (3.21)$$

To solve this equation, we employ the method of separation of variables:

$$\Psi(\beta, \Phi) = \psi_\beta \psi_\Phi. \quad (3.22)$$

Since

$$\Pi_\beta \Psi(\beta, \Phi) = i\partial_\beta \Psi(\beta, \Phi), \quad (3.23)$$

and using Eq. (3.16), the wave function ψ_β is given by

$$\begin{aligned} \rho < 0, \quad \Pi_\beta \psi_\beta(\rho < 0) = -k\psi_\beta(\rho < 0) \rightarrow \psi_\beta(\rho < 0) = e^{ik\beta}, \quad & \text{string spacetime,} \\ \rho > 0, \quad \Pi_\beta \psi_\beta(\rho > 0) = -k\psi_\beta(\rho > 0) \rightarrow \psi_\beta(\rho > 0) = e^{ik\beta}, \quad & \text{Schwarzschild spacetime.} \end{aligned} \quad (3.24)$$

Therefore, we can fix the solution for ψ_β :

$$\psi_\beta = e^{ik\beta}. \quad (3.25)$$

Since β increases monotonically from $-\infty$ to $+\infty$, it can be identified as a timelike coordinate in the WDW equation. Therefore, the tunneling will be triggered by the increasing curvature. The complete solution thus becomes

$$\Psi(\beta, \Phi) = \psi_\Phi e^{ik\beta}. \quad (3.26)$$

Substituting this into the WDW equation (3.21), we obtain

$$[\partial_\Phi^2 + k^2 + 4\lambda^2 e^{-2\Phi}] \psi_\Phi = 0. \quad (3.27)$$

The general solution to this equation is

$$\psi_\Phi = c_1 \Gamma(1-\nu) J_{-\nu}(z) + c_2 \Gamma(1+\nu) J_{+\nu}(z), \quad (3.28)$$

where Γ is the Euler gamma function, $J_{\pm\nu}(z)$ are Bessel functions, $\nu = ik$ and $z = 2\lambda e^{-\Phi}$. Before proceeding further, it is essential to clarify the physical significance of this linear combination of Bessel functions. As is well known:

$$\lim_{\Phi \rightarrow +\infty} J_{\pm ik}(2\lambda e^{-\Phi}) \sim e^{\mp ik\Phi}. \quad (3.29)$$

Based on the relations (3.18) and $\Pi_\Phi \Psi(\beta, \Phi) = i\partial_\Phi \Psi(\beta, \Phi)$, we can identify the wave function regions:

$$\begin{aligned} \psi_\Phi(\rho < 0) &= c_2 \Gamma(1+\nu) J_{+\nu}(z), \\ \psi_\Phi(\rho > 0) &= c_1 \Gamma(1-\nu) J_{-\nu}(z). \end{aligned} \quad (3.30)$$

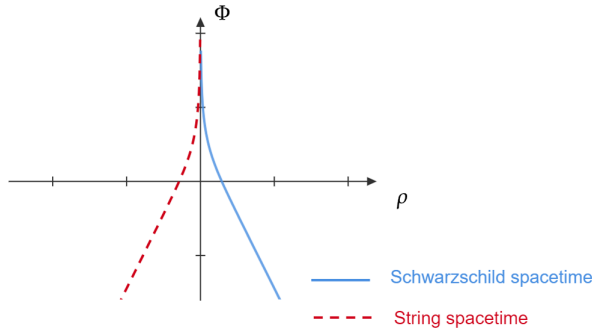


FIG. 2. The relations between $\Phi(\rho)$ and ρ for the classical solutions (3.8) describe how the dilaton field $\Phi(\rho)$ evolves as a function of the spacelike coordinate ρ in the reduced two-dimensional near-horizon spacetime.

As $\Phi \rightarrow +\infty$, the potential in the WDW equation (3.27) vanishes, and the plane-wave solution becomes

$$\psi_{\Phi \rightarrow +\infty} = \psi_{\Phi \rightarrow +\infty}(\rho < 0) + \psi_{\Phi \rightarrow +\infty}(\rho > 0) \sim e^{-ik\Phi} + e^{ik\Phi}. \quad (3.31)$$

Thus, the complete wave function in the strong coupling region can be written as a superposition of right-moving

($\rho < 0$) and left-moving ($\rho > 0$) waves:

$$\Psi(\beta, \Phi \rightarrow +\infty) = \psi_{\Phi \rightarrow +\infty}(\rho < 0)e^{ik\beta} + \psi_{\Phi \rightarrow +\infty}(\rho > 0)e^{ik\beta}. \quad (3.32)$$

Before imposing the boundary condition, we first clarify the value range of β . Referring to the classical solutions (3.8), we find

$$\begin{aligned} \rho < 0, \quad \beta(-\rho) &= \ln(\coth(-\lambda\rho)) > 0, & \text{string spacetime,} \\ \rho > 0, \quad \beta(\rho) &= \ln(\tanh(\lambda\rho)) < 0, & \text{Schwarzschild spacetime.} \end{aligned} \quad (3.33)$$

Now, consider the initial wave incoming from the low-energy regime, where $\beta < 0$ and $\Phi \rightarrow -\infty$. This wave corresponds to the Schwarzschild spacetime, as determined by (3.33). In this paper, we are only interested in a specific boundary condition—namely, the tunneling boundary condition—such that only the right-moving wave, $\psi_{\Phi}(\rho > 0)$, evolves toward the singularity at $\beta > 0$ and $\Phi \rightarrow +\infty$ (strong coupling regime) [26,27]. It is worth noting that, although the divergence of the dilaton indicates a breakdown of the classical description near the horizon, our methodology (taking a large D limit) considerably suppresses fluctuations in the transverse sphere. Hence, the

dominant dynamics remain effectively two-dimensional, allowing the semiclassical approximation to capture the essential tunneling physics. In future work, it is possible to study the higher-order corrections to this process [14]. Under this boundary condition, the specific solution is given by

$$\Psi(\beta, \Phi) = \psi_{\Phi}(\rho > 0)e^{ik\beta} = c_1 \Gamma(1-\nu) J_{-\nu}(z) e^{ik\beta}. \quad (3.34)$$

On the other hand, in the regime $\Phi \rightarrow -\infty$ (or equivalently $z \rightarrow +\infty$), the solution can be expanded as

$$\lim_{\Phi \rightarrow -\infty} \Psi(\beta, \Phi) = \frac{c_1 e^{ik\beta}}{\sqrt{2\pi z}} \left[\exp\left(i\left(z - \frac{\pi}{4}\right)\right) \exp\left(-\frac{k\pi}{2}\right) + \exp\left(-i\left(z - \frac{\pi}{4}\right)\right) \exp\left(\frac{k\pi}{2}\right) \right]. \quad (3.35)$$

Based on (3.19), we identify the wave functions in the following limits:

$$\begin{aligned} \rho \rightarrow +\infty, \quad \Psi_{-}(\beta, \Phi) &= \frac{c_1 e^{ik\beta}}{\sqrt{2\pi z}} \exp\left(i\left(z - \frac{\pi}{4}\right)\right) \exp\left(-\frac{k\pi}{2}\right), & \text{string spacetime} \\ \rho \rightarrow -\infty, \quad \Psi_{+}(\beta, \Phi) &= \frac{c_1 e^{ik\beta}}{\sqrt{2\pi z}} \exp\left(-i\left(z - \frac{\pi}{4}\right)\right) \exp\left(\frac{k\pi}{2}\right), & \text{Schwarzschild spacetime,} \end{aligned} \quad (3.36)$$

through

$$\lim_{\Phi \rightarrow -\infty} \Pi_{\Phi} \Psi_{\pm}(\beta, \Phi) = \lim_{\Phi \rightarrow -\infty} \mp z \Psi(\beta, \Phi). \quad (3.37)$$

Consequently, we consider the incoming wave $\Psi_{+}(\beta, \Phi \rightarrow -\infty)$, representing the Schwarzschild black hole, originating from the low-energy limit $\beta < 0$ and $\Phi \rightarrow -\infty$. This wave partially transmits to the singularity at $\beta > 0$ and $\Phi \rightarrow +\infty$, while the remaining portion, $\Psi_{-}(\beta, \Phi \rightarrow -\infty)$ reflects to the string black hole. Finally, the transition can be understood as a reflection of the wave function in the minisuperspace (β, Φ) . The transition probability is then determined by the reflection coefficient:

$$P = \frac{|\Psi_{-}(\beta, \Phi \rightarrow -\infty)|^2}{|\Psi_{+}(\beta, \Phi \rightarrow -\infty)|^2} = \exp(-2k\pi). \quad (3.38)$$

This result implies that, although the corresponding transition being classically forbidden, this quantum process has a nonzero probability. The similar result in string cosmology can be found in Refs. [19,22]. Remarkably, this result is consistent with predictions from loop quantum gravity. As the black hole approaches the final stages of its evaporation, the probability of tunneling into a white hole is no longer nonperturbatively suppressed [28]. Moreover, in both cases, the tunneling occurs near the horizon, $r \sim r_0$. Finally, this result not only provides the transition

probability between a Schwarzschild black hole and a string black hole but also serves as a counterexample to the weak cosmic censorship hypothesis, suggesting that a black hole with an event horizon may transition to a black hole with a naked singularity.

IV. CONCLUSION

In this paper, we investigated the large D limit of T-dual Schwarzschild and string black holes. In this limit, the near-horizon geometries reduced to two-dimensional black string solutions. These two geometries were T dual to each other, shared the same low-energy effective action, and described different regions of the two-dimensional target space. Because of this property, we derived the corresponding WDW equation for this two-dimensional background, which enabled us to study the transition between the two T-dual geometries. The transition probability was also computed. Our results demonstrated that the classically forbidden transition between Schwarzschild and string black holes can be realized in the large D limit through quantum dynamics.

In future work, we aim to explore the following topics:

- (i) After deriving the transition probability between the near-horizon geometries of large D Schwarzschild and string black holes, it will be valuable to investigate how this transition extends to fundamental strings.
- (ii) We are interested in exploring potential observable consequences of this result. Specifically, since Schwarzschild black holes and naked string black holes have different photon spheres, a transition between them should manifest in observable changes in the photon sphere.
- (iii) The weak cosmic censorship conjecture is also a topic worth reexamining in this new framework.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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