

ϕNN , $J/\psi NN$, $\eta_c NN$ systems based on HAL QCD interactionsLiang-Zhen Wen¹, Yao Ma^{2,*}, Lu Meng^{3,†} and Shi-Lin Zhu^{2,‡}¹*School of Physics, Peking University, Beijing 100871, China*²*School of Physics and Center of High Energy Physics, Peking University, Beijing 100871, China*³*Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany*

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We investigate the existence of bound states and resonances in the ϕNN , $J/\psi NN$, $\eta_c NN$ systems using HAL QCD interactions for ϕN , $J/\psi N$, and $\eta_c N$. We employ the Gaussian expansion method to solve the complex-scaled Schrödinger equation and find no resonances or bound states in the $J/\psi NN$ and $\eta_c NN$ systems. We estimate the interaction between charmonium and nuclei, concluding that the J/ψ or η_c is likely to bind with ^3H , ^3He , ^4He , and heavier nuclei. For the ϕNN system, the lattice QCD $\phi N(^2S_{1/2})$ interaction is absent. We combine the ϕp correlation function analysis and HAL QCD results in model A. We assume the spin-spin interactions for $J/\psi N$ and ϕN systems are inversely proportional to their masses in model B. Model A predicts a stronger $\phi N(^2S_{1/2})$ interaction and permits a two-body bound state, whereas model B suggests the interaction is attractive but too weak to form a bound state. Both models predict bound states for the $I(J^P) = 0(0^-)$ and $0(1^-)$ ϕNN systems. In model A, these states are deeply bound with binding energies exceeding 15 MeV and remain existent when considering parameter uncertainties. In contrast, these states are very loosely bound in model B, with binding energies below 1 MeV and an existent probability of about 60% when parameter uncertainties are considered. In both models, there exist very loosely bound $I(J^P) = 0(2^-)$ three-body states which resemble a ϕ -d atom with the ϕ meson surrounding the deuteron, but their existence is sensitive to parameter uncertainties. No bound states or resonances are found in the isovector $I(J^P) = 1(1^-)$ ϕNN system.

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I. INTRODUCTION

The fundamental theory giving rise to strong interaction among hadrons is quantum chromodynamics (QCD). However, due to the nonperturbative properties of QCD at the low-energy scale, many issues about hadronic interaction remain unresolved to this day. Over the past several decades, the nuclear force has been the most extensively studied hadronic interaction [1–3], primarily driven by the light meson exchange. However, this mechanism is inapplicable to interactions involving heavy quarkonium, since heavy quarkonium and nucleons do not share the same valence light quarks. Therefore, exploring the interactions between charmonium and nucleons could provide insights into different aspects of QCD that cannot be revealed through the study of nucleon-nucleon (NN)

systems. Recently, the discovery of pentaquark states in the $J/\psi p$ final states [4,5] has renewed interest in the $J/\psi N$ interaction, as it plays a key role in the coupled-channel dynamics underlying the formation and production of pentaquarks [6–8]; see Refs. [8–16] for more comprehensive reviews. Another related system is the ϕN , where the ϕ meson is an almost pure $s\bar{s}$ state. The significant mass hierarchy between the strange quark and the u/d quarks occasionally makes it resemble a heavy quark [17]. Understanding the ϕN interaction is also crucial for exploring hidden strange pentaquarks [18].

Several approaches have been used to describe the interactions between quarkonium and nucleons: QCD van der Waals forces [19,20], and interactions allowed by the Okubo-Zweig-Iizuka rule, which arise from the nonzero components of $c\bar{c}$, $s\bar{s}$ in the nucleon [21], as well as meson-baryon coupled-channel dynamics [22–24]. The possibility of quarkonium-nucleus-bound states has also been investigated. In the 1980s, Brodsky *et al.*, proposed that nuclear matter could bind quarkonium through the QCD van der Waals force, which becomes dominant when two interacting color-singlet hadrons share no common quarks [25]. Building on this idea, Gao *et al.* [26] suggested the potential existence of a ϕN bound state, as the interaction is expected to be enhanced by a factor of

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$(\frac{m_c}{m_s})^3$ when transitioning from $c\bar{c}$ to $s\bar{s}$. This proposition sparked significant theoretical interest, leading to extensive studies of nucleus-bound quarkonium [27–61]. However, no experimental evidence has been found for a ϕN bound state in photon-production measurements. The LEPS Collaboration measured the differential cross section of photoproduction of ϕ mesons on protons [62]. Based on these data, Titov *et al.* [63] utilized the vector meson dominance (VMD) assumption and obtained $a_{\phi p} \approx -0.15$ fm, which is consistent with the scattering length derived with the QCD sum rule analysis [64]. The CLAS Collaboration measured the near-threshold differential cross section for the $\gamma p \rightarrow \phi p$ process, covering center-of-mass energies from 1.97 to 2.84 GeV [65]. Strakovsky *et al.* applied the VMD approach to the CLAS measurements, yielding $a_{\phi p} \approx -0.063 \pm 0.010$ fm [66].

Until 2021, the ALICE Collaboration analyzed the $p\phi$ correlation function in proton-proton collisions, revealing for the first time a strong attraction between ϕ and nucleons, with a real part of the scattering length $\text{Re}[a_{\phi p}] = -0.85 \pm 0.34(\text{stat}) \pm 0.14(\text{syst})$ fm [67]. Furthermore, ALICE proposed that elastic contributions dominate in ϕN interactions in vacuum, which contrasts with earlier measurements of ϕ absorption off various nuclear targets in different reactions [68–70].

For the charmonium sector, a scattering length $a_{J/\psi p} = -(3.08 \pm 0.55(\text{stat}) \pm 0.45(\text{syst})) \times 10^{-3}$ fm is extracted from the near-threshold photon-production total cross section using GlueX data [71] as reported by Strakovsky *et al.* [72]. Another value of $a_{J/\psi p} = -0.046 \pm 0.005$ fm is derived from a global fit to both differential and total cross sections of earlier photon-production data [73]. The above two results in Refs. [72,73] were both extracted within the VMD framework. Recently, JPAC combined the latest GlueX and $J/\psi - 007$ measurements of near-threshold J/ψ photoproduction cross sections with constraints from the low-energy unitarity, yielding a scattering length $a_{J/\psi p} \approx O(1)$ fm [74], where the VMD was not adopted.

In summary, no experimental evidence has been found for the existence of ϕp or $J/\psi p$ bound states. Moreover, significant inconsistencies persist among the reported scattering lengths. The discrepancies in ϕN scattering lengths between photon-production processes and proton-proton collisions may arise from differences in the measurement methods. Alternatively, the smaller scattering lengths observed in the photoproduction of ϕp [63,66] and $J/\psi p$ [72,73] within the VMD framework, compared to those obtained without the VMD assumption [67,74], could be attributed to the off-shell effects of the vector meson [75] inherent in the VMD framework. Another limitation is that all the above experimental measurements are restricted to spin-averaged results. Therefore, novel inputs are essential for a more comprehensive understanding of these issues.

Recently, the lattice QCD started from the first principle and made progress on the $J/\psi N$, $\eta_c N$, and ϕN

interactions. In 2022, the HAL QCD Collaboration extracted the interaction potential for ϕN in the $^4S_{3/2}$ channel for the first time near the physical pion mass [76]. Their results showed that the $\phi N(^4S_{3/2})$ interaction is attractive at all ranges, with scattering length $a_{\phi p} = -1.43(23)_{\text{stat}}^{(+36)}_{(-06)\text{syst}}$ fm. Recently, HAL QCD also presented their new results on nucleon-charmonium interactions near the physical pion mass [77]. They found attraction in both the $^4S_{3/2}$ and $^2S_{1/2}$ nucleon-charmonium channels, with the scattering lengths given by $a_{J/\psi N}^{^4S_{3/2}} = -0.30(2)(^{+2}_{-0})$ fm and $a_{J/\psi N}^{^2S_{1/2}} = -0.38(4)(^{+3}_{-0})$ fm, which are consistent with some theoretical calculations [44,46,78] but are explicitly larger than those obtained from photoproduction experiments. In addition, several previous lattice QCD studies have investigated charmonium-nucleon interactions using unphysical pion masses [79–82].

Because of its strong coupling with the ΛK and ΣK channels, HAL QCD is unable to effectively extract the interaction of the $\phi N(^2S_{1/2})$ channel. To address this issue, one approach involves using the potential to fit the correlation function [83], which claims the existence of a deep bound state in the $\phi N(^2S_{1/2})$ channel. In this work, we refer to this interaction as model A. However, it is important to note that the same correlation function can also be fitted well by coupled-channel dynamics [84,85], without requiring a bound state predominantly composed of ϕN . To investigate other possibilities, in this work, we propose a new potential for the $\phi N(^2S_{1/2})$ interaction that connects it with the interactions of $\phi N(^4S_{3/2})$, $J/\psi N(^4S_{3/2})$, and $J/\psi N(^2S_{1/2})$, referred to as model B.

In addition to two-body systems, few-body systems also hold significant implications for understanding hadronic interactions, as reviewed in Ref. [15]. For instance, the existence of a DDD^* bound state would impose strong constraints on the DD^* and $D\bar{D}^*$ interactions, providing insights into the pole position of the $Z_c(3900)$ [86]. In the ϕNN , $J/\psi NN$, and $\eta_c NN$ three-body systems, several pioneering studies have been conducted. Notably, Belyaev *et al.* used Faddeev equations in differential form [58,59], while Sofianos *et al.* [60] employed a two-variable integro-differential equation based on the Faddeev expansion of the wave function to conduct advanced research on ϕNN . Based on the ϕN interaction proposed by Gao *et al.* [26], both Belyaev *et al.* and Sofianos *et al.* found ϕNN bound states. Yokota *et al.* applied the Gaussian expansion method (GEM) to study $J/\psi NN$ and $\eta_c NN$ [61], concluding that the small scattering length obtained from quenched lattice QCD [80] cannot support a three-body charmonium nucleus-bound state. Additionally, Belyaev *et al.* used the Alt-Grassberger-Sandhas (AGS) equation to investigate $\eta_c NN$ [57], predicting a shallow bound state based on the potential proposed by Brodsky *et al.* [25]. Etminan and Aalimi solved the Faddeev equation with hyperspherical

harmonics [87]. However, the interactions between heavy quarkonium and nucleons used in these studies were relatively crude, precluding definitive conclusions based on these potentials.

The HAL QCD potential for ϕN near the physical pion mass revealed that neither the Yukawa-type nor power-type potentials used in previous studies accurately described the hadronic interaction. Instead, the two-pion exchange potential dominates at long range. Although none of the extracted potentials identified any two-body bound states in ϕN , $J/\psi N$, or $\eta_c N$ systems, the potential with a scattering length smaller than -1 fm has attracted attention for investigating three-body bound states in ϕNN systems. Recently, Filikhin *et al.* conducted a comprehensive analysis of $I(J^P) = 0(0^-)$, $0(1^-)$, $1(1^-)$, and $0(2^-)$ ϕNN systems using Faddeev equations in configuration space [88]. They identified bound states in the $0(0^-)$, $0(1^-)$, and $1(1^-)$ systems using the HAL QCD and model A potentials. A key issue is that Filikhin *et al.* used a single-channel spin-averaged potential for the $0(1^-)$ and $1(1^-)$ ϕNN channels. However, in model A, the interactions in the $\phi N(^4S_{3/2})$ and $\phi N(^2S_{1/2})$ channels differ significantly. It is more reasonable to account for the complex coupling effects between spin channels. Additionally, the $\phi N(^2S_{1/2})$ potential, determined using correlation functions, neglects the coupled-channel effects, making its uncertainty uncontrollable. Exploring alternative models for the $\phi N(^2S_{1/2})$ interaction, as well as investigating the ϕNN system within new interactions, would be of great significance. Meanwhile, it is also crucial to investigate the bound states and resonances of the $J/\psi NN$, $\eta_c NN$, and charmonium-nucleus systems.

In this work, we perform complete dynamical calculations on the ϕNN , $J/\psi NN$, and $\eta_c NN$ systems using the complex scaling method (CSM) [89–95] and GEM [29]. Beyond the bound state, the CSM enables the study of resonances if they exist in these systems. The theoretical frameworks accurately describe the properties of multi-quark systems, few-hadron systems, and few-lepton systems [86,96–110].

In this study, we employ the HAL QCD potential to investigate the ϕNN system with the highest spin of $0(2^-)$, the $J/\psi NN$ system with $0(0^-)$, $0(1^-)$, $1(1^-)$, and $0(2^-)$, and the $\eta_c NN$ system with $0(1^-)$ and $1(0^-)$. For the $S = 1$ and $S = 0$ ϕNN channels, we utilize the two aforementioned models for comparison. Furthermore, we provide an approximate estimation of the minimal mass number of a nucleus required for the formation of bound charmonium-nucleus states.

This paper is organized as follows: In Sec. II, we outline the theoretical framework, including the Hamiltonian, wave function construction, complex scaling method, and the approach for analyzing spatial structures. In Sec. III, we explore the possible two-body bound states and the scattering lengths and effective ranges. In Sec. IV, we

present the numerical results for the ϕNN system. In Sec. V, we analyze the results for the $J/\psi NN$ and $\eta_c NN$ systems. Finally, in Sec. VI, we provide a brief summary and discussion.

II. THEORETICAL FRAMEWORK

A. Hamiltonian

The nonrelativistic Hamiltonian of a three-body hadron system can be expressed as

$$H = \sum_i \left(m_i + \frac{\mathbf{p}_i^2}{2m_i} \right) + \sum_{i < j=1}^3 V_{ij}, \quad (1)$$

where m_i and $\mathbf{p}_i$ are the mass and momentum of hadron i . In Sec. IV, the typical binding energy of bound states is 2–50 MeV corresponding to a characteristic momentum of 40–150 MeV. For ϕNN systems, the mass scale is 1 GeV, giving $p/m \approx 0.1$. In charmonium-nucleus systems, p/m is much smaller than that in ϕNN . Thus, nonrelativistic kinematics in the Hamiltonian remain appropriate.

For nucleon-nucleon interactions, we utilize the modified Malfliet-Tjon (MT) potential [111,112]. It only incorporates S-wave Yukawa-type potentials characterized by a short-range repulsion and a long-range attraction. The NN potential reads

$$V_{NN}^{1S_0}(r) = \sum_{i=1}^2 C_i^{1S_0} \frac{e^{-\mu_i r}}{r}, \quad (2)$$

$$V_{NN}^{3S_1}(r) = \sum_{i=1}^2 C_i^{3S_1} \frac{e^{-\mu_i r}}{r}. \quad (3)$$

The parameters are given in Table I.

For $J/\psi N$ and $\eta_c N$ interactions, we adopt the potential from HAL QCD in Ref. [77], which employed three Gaussian functions for fitting. The potentials are expressed as follows:

$$V_{J/\psi N}^{4S_{3/2}}(r) = \sum_{i=1}^3 a_{i1} e^{-(r/b_{i1})^2}, \quad (4)$$

$$V_{J/\psi N}^{2S_{1/2}}(r) = \sum_{i=1}^3 a_{i2} e^{-(r/b_{i2})^2}, \quad (5)$$

$$V_{\eta_c N}^{2S_{1/2}}(r) = \sum_{i=1}^3 a_{i3} e^{-(r/b_{i3})^2}. \quad (6)$$

The parameters are collected in Table II.

TABLE I. The parameters of the modified MT model.

	$C_1(\text{MeV} \cdot \text{fm})$	$\mu_1(\text{fm}^{-1})$	$C_2(\text{MeV} \cdot \text{fm})$	$\mu_2(\text{fm}^{-1})$
$^1S_0$	−514	1.55	1439	3.11
$^3S_1$	−627	1.55	1439	3.11

TABLE II. The parameters of $c\bar{c}N$ potential a_1, a_2, a_3 (in MeV), and b_1, b_2, b_3 (in fm) are taken from Ref. [77].

	a_1	b_1	a_2	b_2	a_3	b_3
$J/\psi N(^4S_{3/2})$	-51(1)	0.09(1)	-13(6)	0.49(7)	-22(5)	0.82(6)
$J/\psi N(^2S_{1/2})$	-101(1)	0.13(1)	-33(6)	0.44(5)	-23(8)	0.83(9)
$\eta_c N(^2S_{1/2})$	-264(14)	0.11(1)	-28(13)	0.24(6)	-22(2)	0.77(3)

For the ϕN spin quartet interaction, we utilize the HAL QCD potential [76], which incorporates a combination of two Gaussian potentials and the two-pion exchange potential. The potential reads

$$V_{\phi N}^{4S_{3/2}}(r) = \sum_{i=1}^2 \alpha_i e^{-(r/\beta_i)^2} + \alpha_3 m_\pi^4 f(r; \beta_3) \left(\frac{e^{-m_\pi r}}{r} \right)^2, \quad (7)$$

$$f(r; \beta_3) = (1 - e^{-(r/\beta_3)^2})^2. \quad (8)$$

The parameters are shown in Table III.

For the spin doublet potential, no direct lattice QCD results are available due to the complexity of the coupled-channel effect. As noted in Sec. I, the potential extracted from the combined analysis of the correlation function and lattice QCD spin-3/2 data also neglects the coupled-channel effect, which may compromise the reliability of the potential. Therefore, in spin-1/2 systems, we explore two different models to investigate alternative possibilities. In model A, a spin-1/2 potential function is constructed by modifying Eq. (7) with an additional factor applied to the Gaussian part [83]

$$V_{\phi N}^{2S_{1/2},A}(r) = \beta \sum_{i=1}^2 \alpha_i e^{-(r/\beta_i)^2} + \alpha_3 m_\pi^4 f(r; \beta_3) \left(\frac{e^{-m_\pi r}}{r} \right)^2. \quad (9)$$

The parameter β is determined by fitting the $p - \phi$ correlation functions, while other parameters adopt the values from the HAL QCD spin-3/2 potential [76,83]. The value of β is shown in Table III. It is worth noting that the lattice QCD calculations are not performed at the physical pion mass but rather at a value close to it. Model A is constructed by directly varying the explicit pion mass in Eq. (9) without accounting for the pion-mass dependence of the parameters.

In model B, we construct the spin-1/2 ϕN potential by relating it to that of the $J/\psi N$ system without using $p - \phi$

TABLE III. The parameters of ϕN potential α_1, α_2 (in MeV), and $\beta_1, \beta_2, \beta_3$, as well as $\alpha_3 m_\pi^4$ (in fm) are taken from Refs. [76,83].

α_1	β_1	α_2	β_2	$\alpha_3 m_\pi^4$	β_3	β
-371(27)	0.13(1)	-119(39)	0.30(5)	-97(14)	0.63(4)	6.9

correlation function data. We assume that the spin-spin part in the $J/\psi N$ and ϕN interaction is inversely proportional to the hadron mass. Using this assumption, we can derive the ϕN doublet potential as follows:

$$V_{J/\psi N}^{\text{Spin}}(r) = \frac{V_{J/\psi N}^{4S_{3/2}} - V_{J/\psi N}^{2S_{1/2}}}{3}, \quad (10)$$

$$V_{\phi N}^{\text{Spin},B}(r) = \frac{m_{J/\psi}}{m_\phi} V_{J/\psi N}^{\text{Spin}}(r), \quad (11)$$

$$V_{\phi N}^{2S_{1/2},B}(r) = V_{\phi N}^{4S_{3/2}}(r) - 3V_{\phi N}^{\text{Spin},B}(r), \quad (12)$$

where the spin-spin interaction is labeled by the superscript “Spin.”

Most of the above potentials are derived from HAL QCD results. Since the lattice QCD simulations are performed at an unphysical pion mass, our calculations will also use the corresponding unphysical hadron masses, as listed in Table IV. In the charm sector, lattice QCD employs two sets of parameters to interpolate the physical mass of the charm quark and reproduce the dispersion relation for the spin-averaged 1S charmonium state [77]. Consequently, we adopt the experimental masses of J/ψ and η_c . It is worth emphasizing that these masses closely approximate their physical counterparts.

B. Wave function construction

The single hadron wave function space is the direct product of spatial space χ_r , spin space χ_s , and isospin space χ_f ,

$$\psi = \mathcal{A}(\chi_r \otimes \chi_s \otimes \chi_f), \quad (13)$$

where $\mathcal{A}$ is the antisymmetrization operator representing the exchange of two nucleons. For the ϕNN system, $\mathcal{A} = (1 - P_{23})$, where P_{ij} permutes the i th and j th hadron.

For the spatial wave function, the GEM [29] is employed. Namely, the spatial wave function is expanded using the following basis:

$$\chi_{nlm}(\mathbf{r}) = \sqrt{\frac{2^{l+5/2}}{\Gamma(l + \frac{3}{2}) r_n^3}} \left(\frac{r}{r_n} \right)^l e^{-\frac{r^2}{r_n^2}} Y_{lm}(\hat{r}), \quad (14)$$

where the r_n is taken in geometric progression, $r_n = r_0 a^{n-1}$. Y_{lm} is the spherical harmonics.

TABLE IV. The masses of ϕ, N, π (in GeV) used in HAL QCD are taken from Refs. [76,77], while the masses of $J/\psi, \eta_c$ (in GeV) are taken from experiments.

m_N	m_π	m_ϕ	$m_{J/\psi}$	m_{η_c}
0.954	0.146	1.048	3.096	2.984

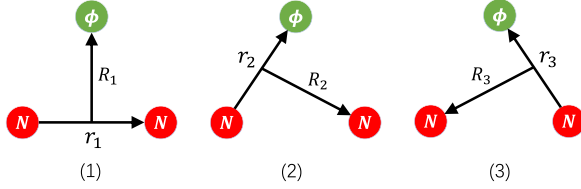


FIG. 1. The Jacobi coordinate configurations of the ϕNN system.

The total orbital angular momentum serves as a good quantum number due to the spatial rotational invariance of each two-body potential. We focus on $L = 0$ states as they correspond to the low-energy levels. Since the interactions we employ are purely S-wave, the S-wave Gaussian bases corresponding to the three distinct Jacobi configurations illustrated in Fig. 1 form a complete set for expanding the spatial wave function. We employ subscripts 1, 2, and 3 to distinguish the three distinct Jacobi coordinate configurations.

In our calculations, we utilize 20 basis functions per coordinate, with $n_{\max} = 20$. The configuration of the basis parameters is as follows:

$$r_0 = 0.02 \text{ fm}, \quad r_{n_{\max}} = 40.0 \text{ fm}, \quad (15)$$

where we use the same parameters of Gaussian basis $r_0, r_{n_{\max}}$ for different Jacobi coordinates.

It is worth noting that the GEM was often employed for the local potential, contributing to all partial waves, as illustrated in Refs. [86,98–110]. However, in this work, the interaction described in Sec. III is restricted to the S-wave only. We lack sufficient experimental data to constrain the potential for higher partial waves. Since our focus is exclusively on the ground state, the influence of higher partial waves is neglected. Utilizing the S-wave-projected interaction, however, introduces some computational complexities when applied to the GEM framework. For any given potential $\hat{V}$, it can be expanded in the angular momentum basis $|rLM\rangle$ as follows:

$$\hat{V} = \sum_{L,M,L',M'} \int r^2 dr r'^2 dr' |rLM\rangle \langle rLM| \hat{V} |r'L'M'\rangle \langle r'L'M'|. \quad (16)$$

When employing an S-wave potential, we obtain

$$\begin{aligned} \langle rLM| \hat{V} |r'L'M'\rangle &= \delta_{L,0} \delta_{L',0} V(r) \frac{\delta(r-r')}{r^2}, \\ \hat{V} &= \int r^2 dr |r00\rangle V(r) \langle r00|. \end{aligned} \quad (17)$$

When working with the Jacobi configurations of $|\chi_3\rangle$, which are identical to the configuration of the potential

$V_3(r_3)$, the form of the matrix element of the S-wave potential is the same as that of a typical local potential,

$$\begin{aligned} \langle \chi_3 | \hat{V}_3 | \chi_2 \rangle &= \int r_3^2 dr_3 \langle \chi_3 | r_3 00 \rangle \langle r_3 00 | \chi_2 \rangle V(r_3) \\ &= \sum_{L,M} \int r_3^2 dr_3 \langle \chi_3 | r_3 LM \rangle V_3(r_3) \langle r_3 LM | \chi_2 \rangle \\ &= \int d\vec{r}_3 \langle \chi_3 | \vec{r}_3 \rangle V_3(r_3) \langle \vec{r}_3 | \chi_2 \rangle. \end{aligned} \quad (18)$$

The second equality arises because the wave function $\langle \chi_3 | r_3 LM \rangle$ is purely S-wave, and only $\langle \chi_3 | r_3 00 \rangle$ survives. When working with the Jacobi configurations of $|\chi_1\rangle$ and $|\chi_2\rangle$, which differ from the configuration of the potential $V_3(r_3)$, it is necessary to transform the Gaussian basis functions to align with the potential's configuration $|r_3 LM\rangle$ to calculate the corresponding matrix elements. However, transforming an S-wave basis from its original configuration to a new one via Jacobian transformations introduces higher partial wave components. Given that the extracted potential is purely S-wave, we must project the wave function onto the S-wave configuration of the potential $|r_3 00\rangle$ as follows:

$$\begin{aligned} \langle \chi_1 | r_3 00 \rangle &= \int d\vec{r}_1 d\vec{r}_3 \langle \chi_1 | \vec{r}_1 \rangle \langle \vec{r}_1 | \vec{r}_3 \rangle \langle \vec{r}_3 | r_3 00 \rangle \\ &= \int d\vec{r}_3 \langle \chi_1 | \vec{r}_3 \rangle Y_{00}(\hat{r}_3). \end{aligned} \quad (19)$$

The second step involves obtaining $\langle \chi_1 | \vec{r}_3 \rangle$ through a coordinate transformation, while the final step integrates out the angular dependence of the newly transformed wave function to achieve the S-wave projection. Finally, the matrix element can be expressed as

$$\begin{aligned} \langle \chi_1 | \hat{V}_3 | \chi_2 \rangle &= \int r_3^2 dr_3 \langle \chi_1 | r_3 00 \rangle \langle r_3 00 | \chi_2 \rangle V_3(r_3) \\ &= \int r_3^2 dr_3 \langle \chi_1 | r_3 00 \rangle V_3(r_3) \langle r_3 00 | \chi_2 \rangle. \end{aligned} \quad (20)$$

For the spin-flavor wave functions $[(NN)_{S_{NN}, I_{NN}} \phi]_{S,I}$ and $[(\phi N)_{S_{\phi N}, I_{\phi N}} N]_{S,I}$, we have included all possible basis functions that align with the corresponding S-wave Jacobi configurations. The inner products of these discrete wave functions for total spin $S = 0, 1, 2$ and total isospin $I = 0, 1$ are detailed in Appendix.

C. Complex scaling method

Few-body systems encompass subchannels such as breakup and rearrangement channels. Near-threshold bound states or resonant states can be determined using the complex scaling method (CSM). The CSM provides a direct approach for calculating the energies and decay

widths of resonant states in many-body systems by performing an analytical continuation of the Schrödinger equation [89,90,113]. This is accomplished by applying a complex rotation to the coordinate $\mathbf{r}$ and momentum $\mathbf{p}$ given by

$$U(\theta)\mathbf{r} = \mathbf{r}e^{i\theta}, \quad U(\theta)\mathbf{p} = \mathbf{p}e^{-i\theta}. \quad (21)$$

Under the rotation, the Hamiltonian in Eq. (1) becomes

$$H(\theta) = \sum_{i=1}^3 \left(m_i + \frac{p_i^2 e^{-2i\theta}}{2m_i} \right) + \sum_{i<j=1}^3 V_{ij}(r_{ij}e^{i\theta}). \quad (22)$$

Meanwhile, for the resonant states with pole positions within the range of the rotated angle, their wave functions become normalizable by integration, thereby solvable through localized Gaussian bases in the same way as bound states. As a result, solving the complex-scaled Schrödinger equation will simultaneously yield the eigenenergies of bound states and resonant states within the rotated angle.

A typical pattern of the solved eigenenergies in the complex energy plane is as follows: The bound states lie on the negative real axis of the energy plane. The continuum states align along beams originating from thresholds with $\text{Arg}(E) = -2\theta$. The resonant states with mass M_R and width Γ_R are located at $E_R = M_R - i\Gamma_R/2$, and only those within $|\text{Arg}(E_R)| < 2\theta$ can be solved. The positions of the bound and resonant states remain unchanged with the variation of the rotation angle. The CSM is therefore an effective tool for distinguishing between scattering states and near-threshold bound states. One can find more details in Refs. [96,99,100].

D. Spatial structure

The root-mean-square (rms) radius is a good physical quantity for reflecting the spatial structure of the hadron states. For near-threshold states, this is also an auxiliary criterion for determining whether it is a bound state or a scattering state.

The definition of the rms radius under CSM is

$$r_{ij}^{\text{rms},C} \equiv \text{Re} \left[\sqrt{\frac{(\Psi(\theta)|r_{ij}^2 e^{2i\theta}|\Psi(\theta))}{(\Psi(\theta)|\Psi(\theta))}} \right], \quad (23)$$

where the $\Psi(\theta)$ is the obtained complex wave function of the state. The round bra-ket represents the so-called c-product [114] defined as

$$(\phi_n|\phi_m) \equiv \int \phi_n(\mathbf{r})\phi_m(\mathbf{r})d^3\mathbf{r}, \quad (24)$$

without taking the complex conjugate of the bra-state. This procedure ensures the function inside the integral is

analytic, thereby the expectation value of the physical quantity remains stable as the rotation angle changes. The rms radius calculated from the c-product is generally not real for resonance. However, it is real for the bound state, as discussed in Ref. [115].

The one-body density distribution is also a physical quantity that reflects the radial distributions of hadrons, where $\hat{r}_i$ represents the radius operator of the i th particle relative to the center of mass [116]

$$\rho(r) = \langle \Psi | \sum_i \delta(r - \hat{r}_i) | \Psi \rangle. \quad (25)$$

III. TWO-BODY SYSTEMS

For each two-body channel, we solve the Schrödinger equation to determine the bound state, including its theoretical binding energy and rms radii. The scattering parameters are obtained by solving the Lippmann-Schwinger equation. The corresponding values are provided in Table V.

The $NN(^3S_1)$ channel features a well-known shallow bound state, the deuteron (d), with a binding energy of 2.41 MeV. The relatively deeper binding energy observed in our calculations, compared to the experimental value, arises from our use of lattice hadron mass results to ensure consistency with the HAL QCD quarkonium-nucleon interactions. The $^1S_0$ channel exhibits an extremely large absolute scattering length, which is attributed to the presence of a nearby virtual state.

In the $\phi N(^4S_{3/2})$ channel, no bound state is formed. However, the $^2S_{1/2}$ channel exhibits significantly different outcomes between the two models. Model A predicts a deeply bound state $\tilde{N}$ with a binding energy of approximately 18 MeV, whereas model B shows no bound state but suggests strong attraction, as evidenced by a large absolute scattering length.

In the charmonium sector, no bound states are observed, and the absolute scattering lengths are relatively small. The negative scattering lengths indicate attractive interactions. Our calculated scattering parameters agree with those reported in Refs. [76,77]. Notably, the spin-dependent

TABLE V. The bound states (BS), including their binding energies, rms radii, scattering lengths, and effective ranges for two-body systems, are presented below.

	BS	E_{Theo} (MeV)	$r_{\text{Theo}}^{\text{rms}}$ (fm)	a_0 (fm)	r_{eff} (fm)
$NN(^1S_0)$	...	...	...	-34.14	3.09
$NN(^3S_1)$	d	2.41	3.84	5.28	1.94
$\phi N(^4S_{3/2})$	...	...	...	-1.44	2.36
$\phi N(^2S_{1/2}), A$	$\tilde{N}$	18.4	1.27	1.76	0.45
$\phi N(^2S_{1/2}), B$	...	...	...	-2.19	1.88
$J/\psi N(^4S_{3/2})$	...	...	...	-0.30	3.28
$J/\psi N(^2S_{1/2})$	...	...	...	-0.38	2.67
$\eta_c N(^2S_{1/2})$	...	...	...	-0.22	3.61

interaction in charmonium-nucleon systems is weak, resulting in nearly identical scattering lengths and effective ranges for $J/\psi N(^4S_{3/2})$ and $J/\psi N(^2S_{1/2})$. This suggests that the spin interaction is significantly weaker than the central force in both charmonium-nucleon and ϕN interactions. In model B, the spin interaction is inversely proportional to mass, which aligns well with these findings.

IV. ϕNN SYSTEM

A. ϕNN states in model A

The eigenstates of the model A complex-scaled Hamiltonian are shown in Fig. 2. We use blue, orange, and green markers to represent the results from three different complex scaling angles: 0° , 9° , 12° . We obtain discrete scattering states due to the finite basis employed in our calculations. The dashed lines represent the theoretical continuum scattering states for the respective angles. Each bound state remains at the same position regardless of the complex scaling angle. The binding energies and rms radii of bound states are provided in Table VI.

In the $0(0^-)$ ϕNN system, there are two two-body thresholds, $\tilde{N}N$ and $d\phi$, that lie below the three-body threshold. A deep bound state is identified in the top-left panel of Fig. 2, with a binding energy of 52.36 MeV relative to the three-body threshold and 34.0 MeV with

respect to the $\tilde{N}N$ threshold. Among the three bound states listed in Table VI, the $0(0^-)$ system is the most deeply bound because it involves only the strong $NN(^3S_1)$ and $\phi N(^2S_{1/2})$ interactions. Because of the tight binding, the $r_{NN}^{\text{rms}} = 1.55$ fm is significantly smaller than that of the deuteron, and the $r_{\phi N}^{\text{rms}}$ is the smallest among the three bound states in Table VI. The $r_{\phi N}^{\text{rms}}$ increases monotonically with binding energy, making it useful for assessing the tightness of few-body bound states. Notably, the $r_{\phi N}^{\text{rms}}$ is comparable to that of the two-body ϕN bound state $\tilde{N}$. Since the NN pairs are identical particles, the $r_{\phi N}^{\text{rms}}$ inherently includes contributions from both nucleons. This can be addressed by considering the one-body density distributions of ϕ . As shown in Fig. 3, we find that the distribution of ϕ is peaked at approximately 0.5 fm, which is significantly smaller than the rms radius of the $\phi N(^2S_{1/2})$ two-body bound states, 1.27 fm. This suggests a tighter configuration of the three-body bound states compared to the ϕN two-body bound states.

The $0(1^-)$ channel shares the same two-body thresholds, $\tilde{N}N$ and $d\phi$, with the $0(0^-)$ channel. The pair interaction between two nucleons involves only the $NN(^3S_1)$ channel, similar to the $0(0^-)$ ϕNN system. The $0(1^-)$ channel differs by incorporating the mixing of the $\phi N(^4S_{3/2})$ and

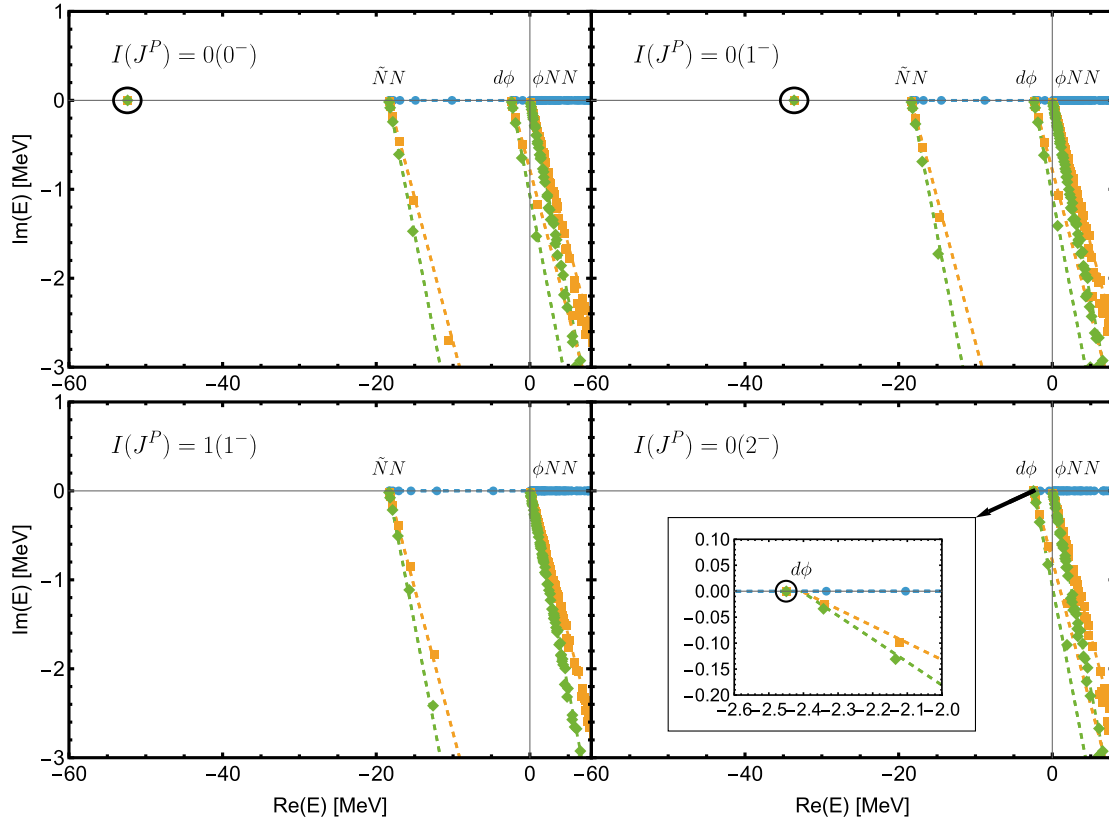


FIG. 2. The complex energy eigenvalues of the ϕNN states in model A with varying θ in the CSM. The dashed lines represent the continuum lines rotating along $\text{Arg}(E) = -2\theta$. The bound states do not shift as θ changes and are marked out by black circles.

TABLE VI. The binding energies and rms radii of the ϕNN bound states in two different models. The E_B represent the binding energies relative to the corresponding lowest two-body thresholds, with the values in parentheses referring to the binding energies relative to the three-body threshold. In the column labeled “2-body thresh.,” the two-body thresholds are listed in ascending order. For comparison, the binding energies and the lowest two-body thresholds of ϕNN states obtained from Ref. [88] are presented.

$I(J^P)$	Model A				Model B				Ref. [88]	
	2-body thresh.	E_B (MeV)	r_{NN}^{rms} (fm)	$r_{\phi N}^{\text{rms}}$ (fm)	2-body thresh.	E_B (MeV)	r_{NN}^{rms} (fm)	$r_{\phi N}^{\text{rms}}$ (fm)	2-body thresh.	E_B (MeV)
$0(0^-)$	$\tilde{N}N, d\phi$	34.0(52.4)	1.55	1.26	$d\phi$	0.95(3.35)	3.10	4.87	$d\phi$	64.1(66.4)
$0(1^-)$	$\tilde{N}N, d\phi$	15.2(33.6)	1.81	1.58	$d\phi$	0.52(2.93)	3.28	6.21	$d\phi$	14.9(17.1)
$1(1^-)$	$\tilde{N}N$	...	...	...	...	...	...	...	...	...(5.47)
$0(2^-)$	$d\phi$	0.04(2.45)	3.68	17.9	$d\phi$	0.04(2.45)	3.68	17.9	$d\phi$	...(...

$\phi N(^2S_{1/2})$ spin components. The overlap coefficients for these channels are provided in Eq. (A2), where the strongly attractive component of $\phi N(^2S_{1/2})$ exhibits dominant overlap with $NN(^3S_1)$. We find a bound state in the $0(1^-)$ channel with a binding energy of 33.6 MeV relative to ϕNN threshold and 15.2 MeV relative to the lowest two-body threshold $\tilde{N}N$. The $\phi N(^4S_{3/2})$ component disfavors a bound state, resulting in a weaker interaction for the $S = 1$, $I = 0$ ϕNN system compared to the $S = 0$, $I = 0$ case. The corresponding r^{rms} are shown in Table VI, where each r^{rms} is slightly larger than its $0(0^-)$ partner. Additionally, the

distributions of N and ϕ in the $0(1^-)$ channel are more dispersed compared to those in the $0(0^-)$ channel, as shown in Fig. 3.

For the $1(1^-)$ channel, we do not observe any bound states or resonances, as illustrated in Fig. 2. This absence is attributed to the smaller overlap matrix element between the strongly attractive component $\phi N(^2S_{1/2})$ and $NN(^1S_0)$, compared to that in the $0(1^-)$ channel. From the perspective of discrete wave functions, the interaction between the $\phi N(^2S_{1/2})$ pair and the spectator N in the $1(1^-)$ channel is weaker by a factor of $1/\sqrt{2}$ compared to that in the $0(1^-)$

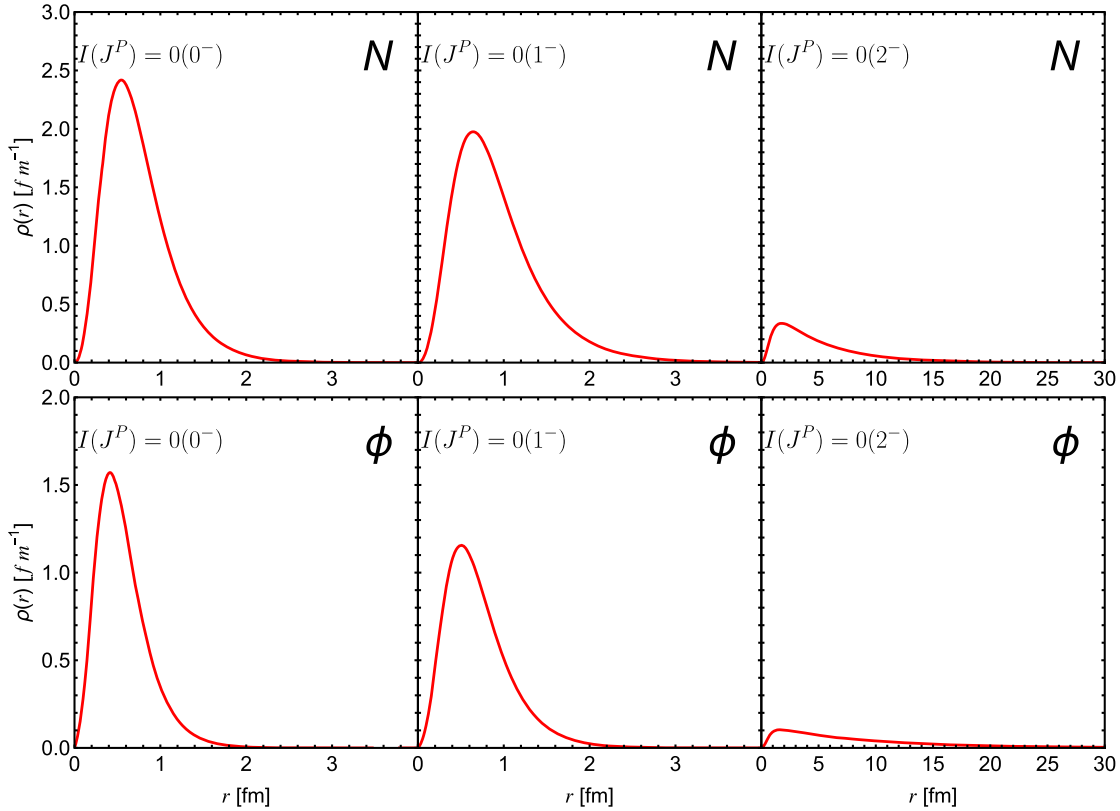


FIG. 3. The one-body density distributions $\rho(r)$ (in units of fm^{-1}) for nucleons and the ϕ meson in the ϕNN states within model A. The top subfigures show the one-body density distributions of nucleons, while the bottom subfigures show the one-body density distributions of the ϕ meson.

sector [see Eqs. (A2) and (A4)], which disfavors the formation of three-body bound states. Additionally, the NN interaction in the $1(1^-)$ ϕNN system corresponds to the $NN(^1S_0)$ virtual state, which is weaker than the $NN(^3S_1)$ interaction associated with the deuteron. By adjusting the interaction parameters, we find that the $1(1^-)$ state behaves as a virtual state relative to the $\tilde{N}N$ threshold, while being a bound state relative to the three-body threshold. Thus, the existence of few-body bound states requires not only a strong interaction but also a competition with subchannels. It is worth noting that the quantum numbers of $\tilde{N}$ align with those of nucleon excited states N^* , such as $N^*(1535)$. Therefore, when the conversion potential is taken into account, the three-body bound states of ϕNN become quasibound states that can decay into the N^*N channel. However, given that the dominant Fock state of $\tilde{N}$ contains an $s\bar{s}$ component, it is expected that the decay width would be relatively small compared to the rearrangement width, if the latter exists.

B. ϕNN states in model B

We note that Ref. [88] employs an average potential to treat the $\phi N(^2S_{1/2})$ and $\phi N(^4S_{3/2})$ channels, rather than performing a complete coupled-channel calculation. The

average ϕN potentials for $0(1^-)$ and $1(1^-)$ three-body systems are given by

$$\begin{aligned}\bar{V}_{\phi N}^{0(1^-)} &= \frac{2}{3}V_{\phi N(^2S_{1/2})} + \frac{1}{3}V_{\phi N(^4S_{3/2})}, \\ \bar{V}_{\phi N}^{1(1^-)} &= \frac{1}{3}V_{\phi N(^2S_{1/2})} + \frac{2}{3}V_{\phi N(^4S_{3/2})}.\end{aligned}\quad (26)$$

Apparently, this average potential approach is merely an assumption and lacks dynamical detail. What is more important, the averaged ϕN interaction would prevent the existence of the $\tilde{N}$ two-body $\phi N(^2S_{1/2})$ bound states and consequently make the two-body scattering states $\tilde{N}N$ absent in the three-body system, which would essentially affect the analyticity of the S-matrix. They obtained bound states in both the $0(1^-)$ and $1(1^-)$ channels, as shown in Table VI. The binding energy of the $0(1^-)$ state is 14.90 MeV relative to the $d\phi$ two-body threshold and 17.1 MeV relative to the three-body threshold, whereas the binding energy of the $1(1^-)$ state is 5.47 MeV relative to the three-body threshold. Notably, the binding energy of the $0(1^-)$ state is significantly underestimated when compared to complete dynamical calculations. In the case of the $1(1^-)$ channel, the average potential approach results in a spurious bound state.

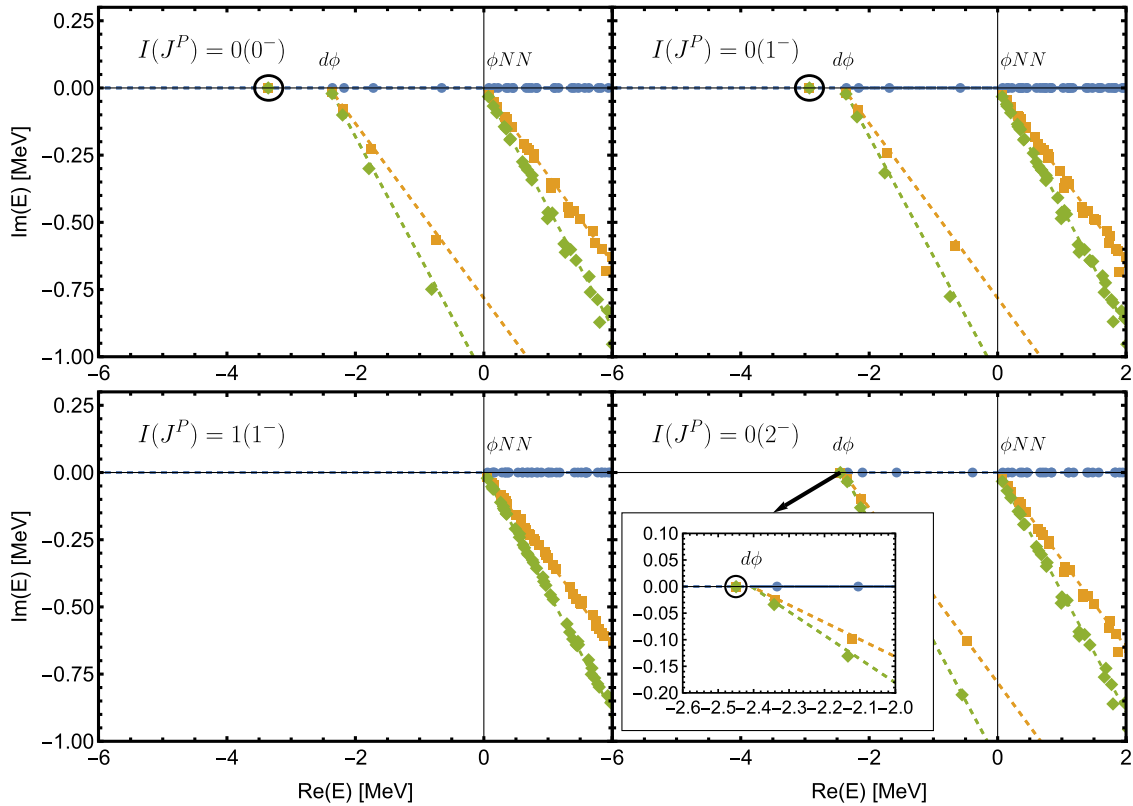


FIG. 4. The complex energy eigenvalues of the ϕNN states in model B with varying θ in the CSM. The dashed lines represent the continuum lines rotating along $\text{Arg}(E) = -2\theta$. The bound states do not shift as θ changes and are marked out by the black circles.

The spin-2 channel features a very shallow bound state with a binding energy of only 0.04 MeV relative to the $d\phi$ threshold, as shown in the last column of Table VI. This channel comprises only the $\phi N(^4S_{3/2})$ component, depending exclusively on HAL QCD $\phi N(^4S_{3/2})$ potential. The r_{NN}^{rms} of this state is similar to that of the deuteron, while the $r_{\phi N}^{\text{rms}}$ is approximately 17.9 fm. The structure of this state resembles a $\phi - d$ atom, with the ϕ meson surrounding the deuteron. It is important to note that our results are based on the original parameters at $m_\pi = 146.4$ MeV from HAL QCD [76]. In contrast, Ref. [88] reports an inability to find a bound state in this channel. This discrepancy likely arises from their substitution of the pion mass in (7) with the physical pion mass. The two-pion exchange term includes an m_π^4 factor, which amplifies the effects of small changes in the pion mass. The present parametrized two-pion exchange term, $e^{-2m_\pi r}/r^2$, contributes significantly to the short-range interaction, particularly in the range $0.5 \text{ fm} < r < 1 \text{ fm}$. However, one could construct different mechanisms (with different pion-mass dependence) to model the short-range interaction. Consequently, it is difficult to believe that a straightforward substitution of m_π exactly captures the true pion-mass dependence in the short-range regime.

In model A, relatively deep bound states exist for the $0(1^-)$ and $1(1^-)$ systems. Even when considering the uncertainties in the parameters of the $\phi N(^4S_{3/2})$ potential listed in Table III, their existence is unlikely to be affected, only with the potential shifting of their binding energies. Since this work focuses primarily on qualitative results, we will not delve further into the uncertainties of their binding energies, as their existence is established. However, for the $0(2^-)$ ϕNN system, its existence could be affected by uncertainties in the potential parameters. As the interactions for the $0(2^-)$ system are identical in both model A and model B, we will investigate its uncertainties in the following section.

The eigenstates of the complex-scaled Hamiltonian for model B are illustrated in Fig. 4. The three-body bound states in the nominal values of the parameters in Tables III and II are summarized in Table IV. In model B, there is no ϕN two-body bound state. The difference between the $\phi N(^4S_{3/2})$ and $\phi N(^2S_{1/2})$ interactions is relatively minor. The $\phi N(^2S_{1/2})$ interaction exhibits slightly stronger attraction than the $\phi N(^4S_{3/2})$. The binding energy order of the three isoscalar ϕNN depends on the components of the $(\phi N)_{\frac{1}{2},\frac{1}{2}}N$, as detailed in Appendix, which represents the most attractive spin channel. Specifically, the greater

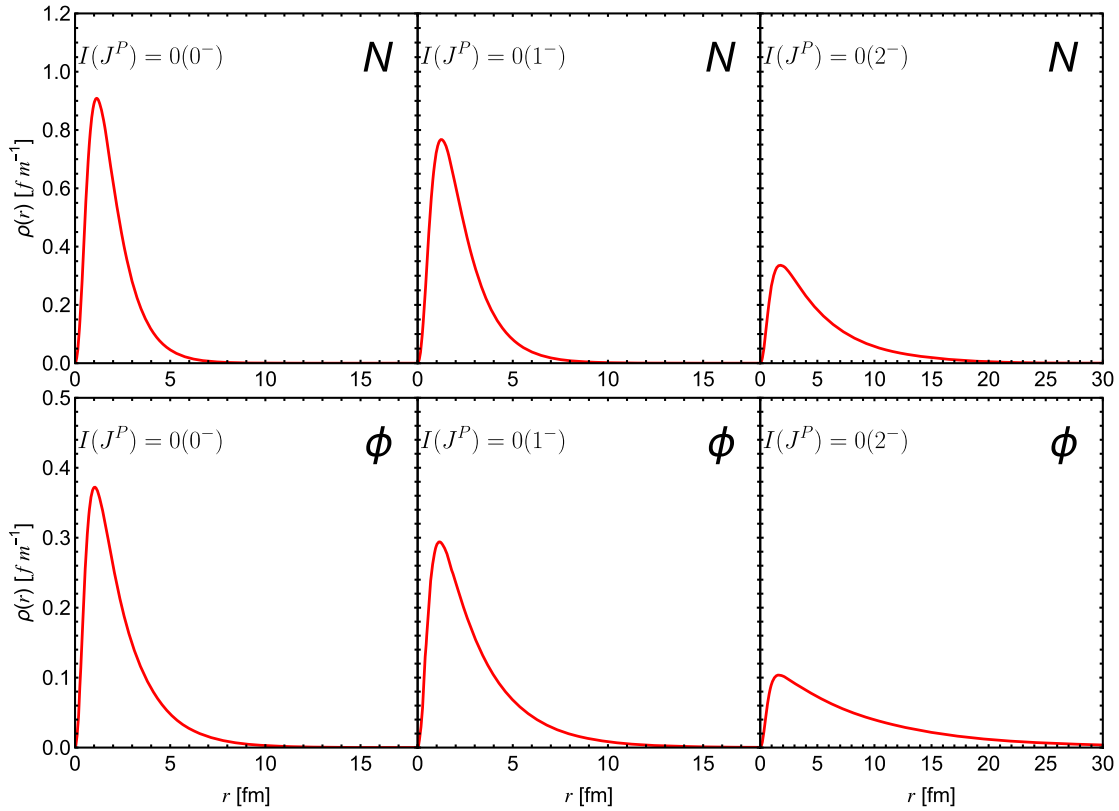


FIG. 5. The one-body density distributions $\rho(r)$ (in units of fm^{-1}) for nucleons and the ϕ meson in the ϕNN states within model B. The top subfigures show the one-body density distributions of nucleons, while the bottom subfigures show the one-body density distributions of the ϕ meson.

the contribution of $(\phi N)_{\frac{1}{2}^+}N$ there is, the deeper the bound states become. Thus, we have the following order:

$$E_b^{0(2^-)} < E_b^{0(1^-)} < E_b^{0(0^-)}. \quad (27)$$

The presence of a shallow $0(2^-)$ ϕNN bound state suggests that similar shallow bound states should also exist for the $0(0^-)$ and $0(1^-)$ channels. Specifically, the $0(0^-)$ and $0(1^-)$ bound states are situated near the $d\phi$ threshold, with binding energies of 0.95 and 0.52 MeV, respectively. The radii of N and ϕ in these bound states are significantly more extended compared to the range of the two-body force within 1 fm, consistent with the shallow binding energy in each channel, as shown in Fig. 5. Consequently, the bound states in each channel can be characterized as a ϕ -d two-body molecular state. This characterization is further supported by the observation that $r_{NN}^{\text{rms}} \approx r_d^{\text{rms}}$.

Because of the relatively weak binding energies in ϕNN systems, uncertainties cannot be ignored in the analysis. Therefore, we extracted the uncertainties from the HAL QCD potential function for both the ϕN and $J/\psi N$ interactions. For each parameter α_1 , α_2 , α_3 , β_1 , β_2 , β_3 in $\phi N(^4S_{3/2})$ interactions, we selected three points: the central value, the central value plus 1 standard deviation (σ), and the central value minus 1 standard deviation (σ), resulting in a total of 3^6 parameter sets. For the spin-dependent interaction of ϕN determined from $J/\psi N$ interactions, we only account for the uncertainties in the linear coefficients and use perturbation theory to propagate these uncertainties. This approach is justified as the spin-dependent interaction is relatively weak compared to the central interaction. Specifically, for the $J/\psi N(^4S_{3/2})$ and $J/\psi N(^2S_{1/2})$ interactions, we varied each parameter α_1 , α_2 , α_3 individually within the ϕN interaction parameter space, generating six parameters across two channels. By comparing the changes in binding energy of bound states with and without altering the $J/\psi N$ interaction parameters, we assessed the impact of $J/\psi N$ interactions on the binding energies and existence of bound states in the ϕN parameter space. This process ultimately resulted in a dataset of binding energies comprising 3^{12} parameter sets. We introduced a binding probability of three-body bound state P_{bound} across the parameter space. Its distribution is shown in Fig. 6. We find that, due to the uncertainties in the HAL QCD extraction of the ϕN and $J/\psi N$ interactions, the existence of a three-body bound state is not certain. Only approximately 60% of the parameter sets support the formation of bound states in ϕNN systems. We also use the 68% probability (1σ significance) to estimate the uncertainty of the binding energies, which is summarized in Fig. 6. Under the 1σ significance, the $0(0^-)$ ϕNN state varies from an unbound state to the bound state with a binding energy 10.87 MeV. For the $0(1^-)$ ϕNN state, it varies from an unbound state to the bound state with a

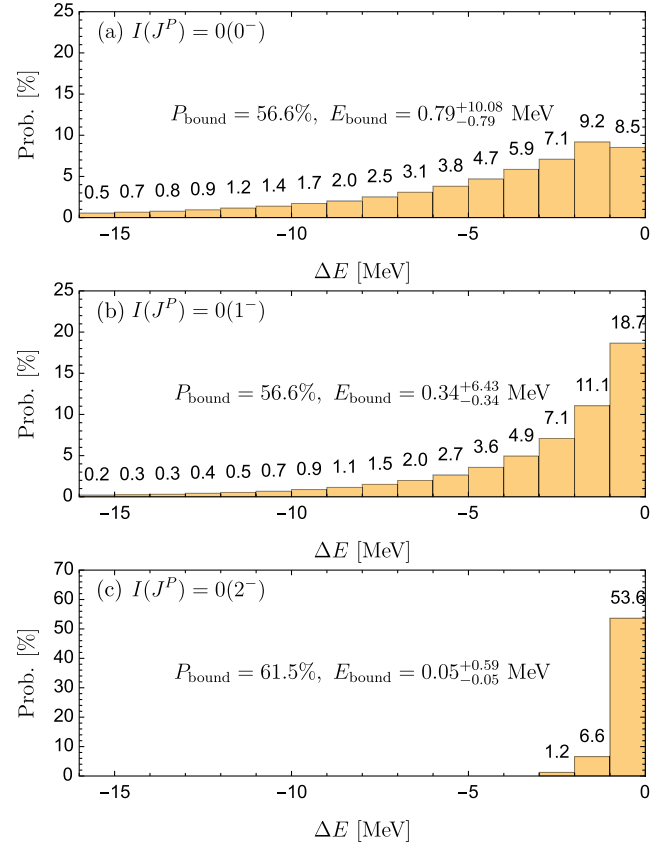


FIG. 6. The distribution of the ϕNN binding energies in the $0(0^-)$, $0(1^-)$, and $0(2^-)$ channels accounting for parameter uncertainties in model B. The numbers on each bar represent the percentage of bound states within 1 MeV energy intervals.

binding energy 6.77 MeV. The $0(2^-)$ ϕNN state has the smallest binding energy range, varying from an unbound state to the bound state with a binding energy 0.64 MeV.

V. $J/\psi NN$ AND $\eta_c NN$ STATES

Our results for $J/\psi NN$ and $\eta_c NN$ based on the GEM are shown in Figs. 7 and 8, respectively. Although both $J/\psi N$ and $\eta_c N$ interactions are attractive, we find that neither bound states nor resonances exist in $J/\psi NN$ and $\eta_c NN$ systems. This conclusion is consistent with previous GEM calculations [61], which were based on earlier quenched lattice QCD results [117].

We also estimate the requirement of the two-body scattering lengths of $J/\psi N$ or $\eta_c N$ in order to form the three-body bound states. Using HAL QCD Gaussian-type potentials in Eq. (4) with an extra adjustable factor, we determine that for a three-body bound state to exist, the two-body scattering length must be less than approximately -1 fm. This finding is consistent with that using the Gaussian-type and Yukawa-type potential in Ref. [61]. Therefore, the two-body scattering length serves as a crucial criterion for identifying three-body bound states.

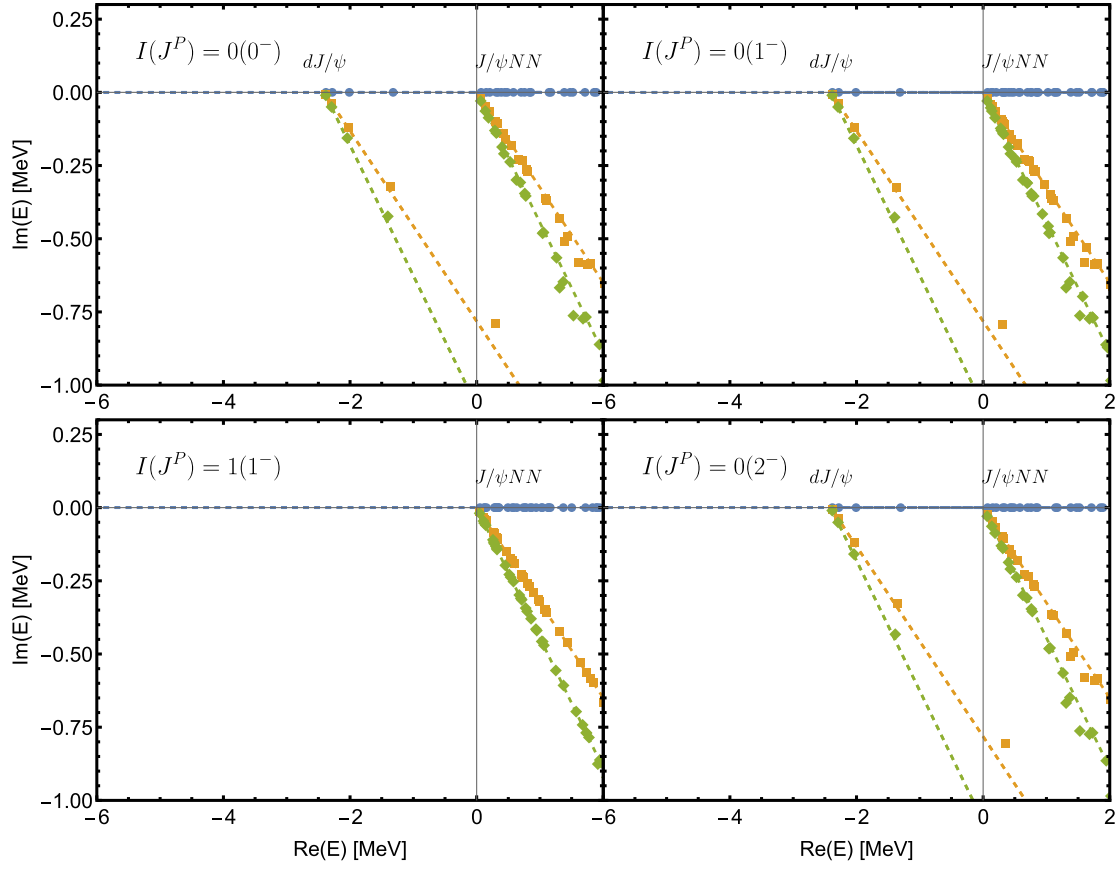


FIG. 7. The complex energy eigenvalues of the $J/\psi NN$ states with varying θ in the CSM. The dashed lines represent the continuum lines rotating along $\text{Arg}(E) = -2\theta$. No bound state or resonance exists.

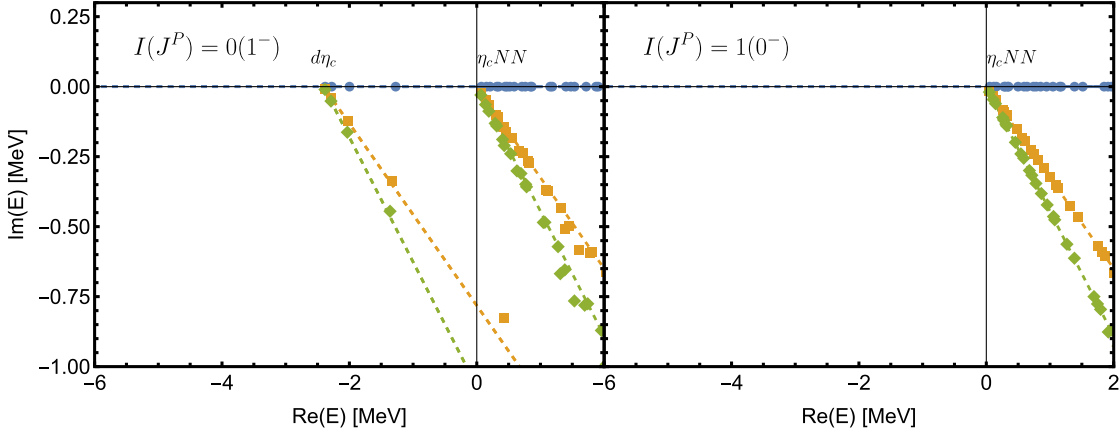


FIG. 8. The complex energy eigenvalues of the $\eta_c NN$ states with varying θ in the CSM. The dashed lines represent the continuum lines rotating along $\text{Arg}(E) = -2\theta$. No bound state or resonance exists.

With the suppression of the spin-dependent $J/\psi N$ interaction and the spin independence of the $\eta_c N$ interaction, we can roughly estimate the interaction between charmonium and a nucleus with more than two nucleons. Since both the $J/\psi N$ and $\eta_c N$ interactions are expected to be isospin independent and largely insensitive to spin, we

can use the potential $V_{(c\bar{c})N}$ (taking the spin-averaged value for the $J/\psi N$ system) to approximate the interaction $V_{(c\bar{c})^A X}$:

$$V_{(c\bar{c})^A X} = A \times V_{(c\bar{c})N},$$

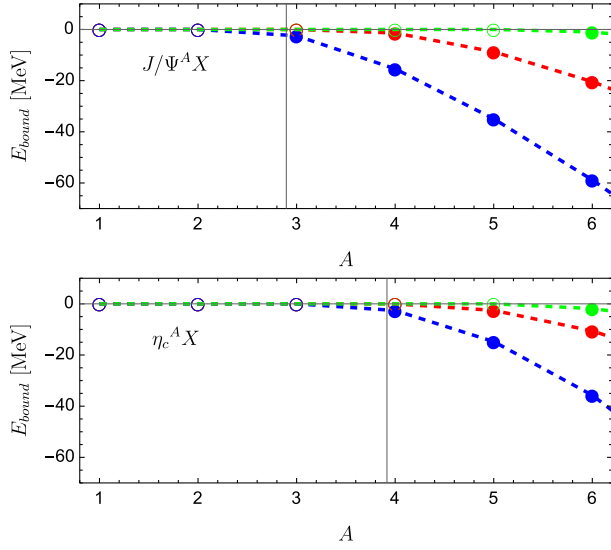


FIG. 9. The binding energies (in MeV) of the $(c\bar{c})$ -nuclei systems vary with nuclear mass number A . Solid circles represent existing bound states, while hollow circles denote the absence of bound states. The red, green, and blue lines (or markers) correspond to results obtained using the central values, upper limits, and lower limits of the parameters, respectively, to account for uncertainties.

where A is the mass number of the nucleus. Using this approach, we can estimate the minimum A required for the $(c\bar{c})$ to bind to a nucleus $^A X$. The binding energy is shown in Fig. 9. It is evident that, with nominal parameters, the lightest nucleus capable of binding $(c\bar{c})$ has a mass number $A = 4$, such as ^4He . Considering uncertainties, there is still a significant probability that nuclei with $A = 3$, such as ^3H or ^3He , could also bind J/ψ . For η_c , a nucleus with mass number $A = 5$ can bind η_c with nominal parameters, and there are substantial possibilities to form bound states with $A = 4$.

VI. SUMMARY AND DISCUSSION

We investigate the ground states of the ϕNN and $J/\psi NN$ systems with isospin-spin-parity combinations $I(J^P) = 0(0^-), 0(1^-), 1(1^-), 0(2^-)$, as well as the $\eta_c NN$ system with $I(J^P) = 1(0^-), 0(1^-)$, based on HAL QCD interactions. The Gaussian expansion method is employed to solve the complex-scaled Schrödinger equation.

For the ϕNN system, the lattice QCD $\phi N(^2S_{1/2})$ interaction is not available. In this work, we explore the interaction using two models: model A, which combines the ϕp correlation function analysis and HAL QCD results, and model B, which assumes that the spin-spin interactions for $J/\psi N$ and ϕN systems are inversely proportional to their masses. Model A predicts a stronger $\phi N(^2S_{1/2})$ interaction, permitting a two-body bound state, while model B indicates that the interaction is attractive but not strong enough to support a bound state. Both models,

however, predict bound states for the $I(J^P) = 0(0^-)$ and $0(1^-)$ ϕNN systems. In model A, these states are deeply bound with binding energies exceeding 15 MeV and remain existent when considering parameter uncertainties. In contrast, these states are very loosely bound in model B, with binding energies below 1 MeV and existent probability of about 60% when parameter uncertainties are considered. In both models, there exist very loosely bound $I(J^P) = 0(2^-)$ three-body states. However, its existence is sensitive to parameter uncertainties. Additionally, we find no bound states or resonances in the isovector $I(J^P) = 1(1^-)$ ϕNN system in either model.

We also calculate the rms radii, r^{rms} , as well as the one-body density distributions, $\rho(r)$, of these states to further elucidate their spatial configurations. In model A, the bound states with $I(J^P) = 0(0^-)$ and $0(1^-)$ are tightly bound, leading to significantly smaller r_{NN}^{rms} compared to r_d^{rms} . In contrast, in model B, the bound states with $I(J^P) = 0(0^-)$ and $0(1^-)$ are shallow, resembling a ϕ -d atom similar to the $0(2^-)$ bound state. However, due to the indistinguishability of nucleons, the spatial structure of the tightly bound states cannot be fully determined using $r_{\phi N}^{\text{rms}}$ alone. We utilize the one-body density distributions to further determine the tightness of the three-body bound states.

We also investigate the ground state in $J/\psi NN$ and $\eta_c NN$ three-body systems using the newly proposed HAL QCD potential. For $J/\psi NN$, we consider $I(J^P) = 0(0^-), 0(1^-), 1(1^-), 0(2^-)$, and for $\eta_c NN$, we consider $0(1^-), 1(0^-)$. No bound states or resonances are found in these systems, which can be attributed to the small absolute value of the two-body scattering length in the $(c\bar{c})N$ interactions.

The criterion for the existence of a three-body bound state is that the two-body $(c\bar{c})N$ scattering length is approximately less than -1 fm, which lies outside the range of most previous experimental and theoretical approaches. Furthermore, we also estimate the interaction between charmonium and nuclei, concluding that the J/ψ or η_c is likely to form bound states with ^3H , ^3He , ^4He , and heavier nuclei. Therefore, we suggest exploring the possibility of these bound states in the future.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data supporting this study's findings are available within the article.

APPENDIX: OVERLAP OF SPIN-FLAVOR WAVE FUNCTIONS

The overlap matrix of the spin-flavor wave functions is presented below with the bases $[(NN)_{1,0}\phi]$ for the $I = 0$ system ($[(NN)_{0,1}\phi]$ for the $I = 1$ system), $[(\phi N)_{\frac{1}{2},\frac{1}{2}}N]$, $[(\phi N)_{\frac{3}{2},\frac{1}{2}}N]$, $[N(\phi N)_{\frac{1}{2},\frac{1}{2}}]$, and $[N(\phi N)_{\frac{3}{2},\frac{1}{2}}]$:

$$\begin{pmatrix} 1 & \frac{1}{\sqrt{3}} & -\sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} \\ \frac{1}{\sqrt{3}} & 1 & 0 & \frac{1}{3} & \frac{2\sqrt{2}}{3} \\ -\sqrt{\frac{2}{3}} & 0 & 1 & \frac{2\sqrt{2}}{3} & -\frac{1}{3} \\ -\frac{1}{\sqrt{3}} & \frac{1}{3} & \frac{2\sqrt{2}}{3} & 1 & 0 \\ \sqrt{\frac{2}{3}} & \frac{2\sqrt{2}}{3} & -\frac{1}{3} & 0 & 1 \end{pmatrix} \quad (\text{A1})$$

for the $S = 0, I = 0$ system;

$$\begin{pmatrix} 1 & -\sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \\ -\sqrt{\frac{2}{3}} & 1 & 0 & -\frac{1}{3} & -\frac{2\sqrt{2}}{3} \\ -\frac{1}{\sqrt{3}} & 0 & 1 & -\frac{2\sqrt{2}}{3} & \frac{1}{3} \\ \sqrt{\frac{2}{3}} & -\frac{1}{3} & -\frac{2\sqrt{2}}{3} & 1 & 0 \\ \frac{1}{\sqrt{3}} & -\frac{2\sqrt{2}}{3} & \frac{1}{3} & 0 & 1 \end{pmatrix} \quad (\text{A2})$$

for the $S = 1, I = 0$ system;

$$\begin{pmatrix} 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 \end{pmatrix} \quad (\text{A3})$$

for the $S = 2, I = 0$ system;

$$\begin{pmatrix} 1 & \frac{1}{\sqrt{3}} & -\sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} \\ \frac{1}{\sqrt{3}} & 1 & 0 & \frac{1}{3} & \frac{2\sqrt{2}}{3} \\ -\sqrt{\frac{2}{3}} & 0 & 1 & \frac{2\sqrt{2}}{3} & -\frac{1}{3} \\ -\frac{1}{\sqrt{3}} & \frac{1}{3} & \frac{2\sqrt{2}}{3} & 1 & 0 \\ \sqrt{\frac{2}{3}} & \frac{2\sqrt{2}}{3} & -\frac{1}{3} & 0 & 1 \end{pmatrix} \quad (\text{A4})$$

for the $S = 1, I = 1$ system.

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