Letter

Effective description of ajar systems with a $U(1)$ symmetry

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We introduce the concept of "ajar systems" as an intermediate case between closed and open systems, where the time scale for charge exchange with the environment is parametrically larger than all other characteristic time scales. The Schwinger-Keldysh effective action for such finite-temperature systems exhibits a symmetry group $G_1 \times G_2$ weakly broken to its diagonal subgroup G_{diag}, which we systematically describe using spurion techniques. For $G = U(1)$, we calculate leading-order corrections to correlation functions in both diffusive and spontaneously broken phases. Unlike systems with approximate symmetries where both real and imaginary parts of dispersion relations receive corrections, ajar systems modify only the imaginary part—preserving gapless modes while adding finite damping.

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Introduction. The application of effective field theory techniques to hydrodynamic systems and, more in general, systems out of equilibrium has recently received significant attention (see, e.g., [1–3]). The relevant framework to discuss these issues is the Schwinger-Keldysh (or "in-in") formalism [4] and generalizations thereof [5].

Previous work [6,7] showed that a local Schwinger-Keldysh effective action at low energies ($E \ll T$) for a *closed* system at finite temperature T must possess two copies of all global internal symmetries, forming a group $G_1 \times G_2$, partially realized nonlinearly. The individual symmetries G_1, G_2 do not have a direct physical interpretation, however the Noether current of the off-diagonal symmetry corresponds to the physically observed currents, and their (non)conservation can be used to diagnose the openness of the system [6]. More precisely, an *open* system, which exchanges conserved charges with an environment, has an effective action invariant only under the diagonal subgroup G_{diag} [6,8], reflecting the fact that charge conservation is now only satisfied on average in equilibrium, and not as an exact operator statement.

This distinction suggests an intermediate scenario, termed *ajar* systems, where interactions with the environment occur over time scales much longer than internal dynamics. Such systems experience weak explicit symmetry breaking from $G_1 \times G_2$ to G_{diag}, that can be systematically modeled via a spurion technique [9]. Introducing a spurion field transforming bilinearly under $G_1 \times G_2$,

$$O \to U_1 O U_2^\dagger, \tag{1}$$

and assigning it an expectation value $\langle O \rangle = \gamma$ invariant under G_{diag} allows systematic accounting of symmetry-breaking effects. The parameter γ characterizes the interaction strength with the environment; small γ relative to the EFT cutoff ensures approximate symmetry, yielding predictions for corrections to observables.

Before proceeding, we emphasize the difference between our approach and that of [10]. The latter studied the hydrodynamic behavior of closed systems exhibiting approximate symmetries (see also [11]), introducing two spurion fields O_1 and O_2 transforming as

$$O_1 \to U_1 O_1, \qquad O_2 \to U_2 O_2. \tag{2}$$

When these fields acquire nonzero expectation values, the full symmetry group $G_1 \times G_2$ is weakly broken. In contrast, our setup preserves the diagonal subgroup G_{diag}. Thus, an open system can be viewed as a particular case of this general scenario in which spurions appear only through combinations preserving G_{diag}, specifically $O_1^\dagger O_1$, $O_2^\dagger O_2$, and $O = O_1 O_2^\dagger$ (and its conjugate).[1] The first two combinations, scalars under $G_1 \times G_2$, modify exactly invariant terms by an amount proportional to the spurion expectation value $\langle O \rangle$. As we will show, these invariant contributions

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[1]We do not need to consider additional structures involving derivatives, because they would drop out once the spurions are replaced with their expectation value.

are crucial for ensuring unitarity in the underlying microscopic theory. Consequently, our effective action will depend not only on O but also explicitly on $\langle O\rangle$, slightly departing from the usual spurion formalism. In the Supplemental Material [12], we consider a simple example of a concrete UV completion that motivates our proposal and elucidates the origin of the dependence on $\langle O\rangle$.[2]

In the remainder of this letter, we will illustrate our approach in the simplest case of Abelian symmetries, $G = U(1)$. In this case, our spurion field will be a complex field that transforms under $U(1)_1 \times U(1)_2$ like

$$O \to e^{i(\alpha_1-\alpha_2)}O, \tag{3}$$

ensuring that the spurion expectation value preserves $U(1)_{\text{diag}}$. Gauged $U(1)$ symmetries in open systems—where the off-diagonal symmetry is badly broken—were recently discussed in [14].

$U(1)$ diffusion.

Closed systems: We start by reviewing the effective field theory (EFT) of $U(1)$ diffusion for closed systems in a thermal state. In this case, the Schwinger-Keldysh effective action must be invariant under $U(1)_1 \times U(1)_2$ *spontaneously* broken down to $U(1)_{\text{diag}}$ [7]. The corresponding Goldstone mode, φ_a, shifts under the "axial" $U(1)$, is a singlet under $U(1)_{\text{diag}}$, and is paired with a "matter field" ρ_r.[3] Thermality implies that all correlation functions satisfy the KMS condition. This can be enforced at the level of the effective action by imposing invariance under the dynamical KMS (DKMS) symmetry [15,16][4]

$$\varphi_a'(x) = -\varphi_a(-x) - i\beta\rho_r(-x) + \mathcal{O}(\beta^2), \tag{4a}$$

$$\rho_r'(x) = \rho_r(-x) - \frac{i\beta}{4}\partial_i^2\varphi_a(-x) + \mathcal{O}(\beta^2). \tag{4b}$$

We have expanded these transformation rules in powers of $\beta = 1/T$, anticipating the fact that the effective action will be organized in powers of E/T. A discussion of these transformations at all orders in β and of the systematics of power counting in E/T can be found in [17].

The Schwinger-Keldysh effective action must also satisfy some constraints that follow from unitarity considerations [2]

$$S[\varphi_a = 0, \rho_r] = 0, \tag{5a}$$

$$S^*[\varphi_a, \rho_r] = -S[-\varphi_a, \rho_r], \tag{5b}$$

$$\text{Im}\, S \geq 0. \tag{5c}$$

At lowest order in derivatives and quadratic order in fields, the effective action that satisfies these constraints and is invariant under all the symmetries mentioned above is [18][5]

$$S = \int d^4x \left[n(\rho_r)\partial_t\varphi_a + \sigma(\rho_r)\partial_i\varphi_a\left(\frac{i}{\beta}\partial_i\varphi_a - \partial_i\rho_r\right)\right], \tag{6}$$

where $d^4x \equiv dtd^3r$ and n and σ are analytic around $\rho_r = 0$, and $\sigma \geq 0$ to satisfy (5c). Imposing invariance under charge conjugation constrains n (σ) to contain only odd (even) powers of ρ_r. Demanding that the terms in (6) are all of the same order implies the power counting relations $\rho_r \sim T\varphi_a$ and $\partial_t \sim \partial_i^2$. Interestingly, the first of these relations also implies that the second term on the right-hand side of (4b) is negligible compared to the first one—an approximation usually justified by invoking an expansion in powers of $\hbar$.

The equation of motion obtained by varying S with respect to ρ_r always admits the solution $\varphi_a = 0$, which is the background we will be interested in.[6] On this background, the equation for φ_a reduces to a diffusion equation for $n(\rho_r)$

$$\partial_t n(\rho_r) = \partial_i[\sigma(\rho_r)\partial^i\rho_r], \tag{7}$$

In fact, $n(\rho_r)$ is also the Noether charge density associated with invariance under shifts of φ_a. These results suggest identifying ρ_r with fluctuations of the chemical potential around equilibrium.

Leading corrections in ajar systems: We will now supplement the action S for a closed system with the leading terms that depend on the spurion field O. We will need to ensure that these additional terms are invariant under the DKMS symmetry and satisfy the appropriate unitarity constraints. To this end, we will proceed in two steps.

[2]We emphasize that $\langle O\rangle$ is chosen by hand to parametrize the explicit symmetry breaking, and is not an order parameter of the system whose value is to be determined from the dynamics. Its appearance in the EFT is an artifact of the fact that the spurion we introduce does not come in two copies.

[3]The a and r subscript are introduced to match standard notation in the literature—see, e.g., [2].

[4]The DKMS symmetry relies on a discrete symmetry of the action involving time reversal. While CPT is a natural candidate for Lorentz-invariant actions, C is spontaneously broken at finite charge density. Thus, implementing DKMS via PT, with additional C symmetry imposed through $\varphi_a'(x) = -\varphi_a(x)$ and $\rho_r'(x) = -\rho_r(x)$, becomes more convenient. The transformation rules (4) assume invariance under PT, and differ from the ones in [7] which are built using CT.

[5]In the incompressible limit, boosts are explicitly broken by neglecting thermal bath phonons. The DKMS symmetry is realized perturbatively up to corrections of $\mathcal{O}(E/T)$.

[6]This is more than just a convenient choice, it is also the physical one since it ensures that φ_a does not propagate, which would lead to an instability otherwise [14]. This conclusion can be avoided for purely imaginary saddle points of φ_a [4].

First, it will be convenient to think again of our spurion field as being a composite object of the form $O = O_1 O_2^\dagger$. This is because the transformation properties of the operators $O_{1,2}$ under the DKMS symmetry are quite simple [2]

$$O'_{1,2}(x) = O^\dagger_{1,2}(-x) + \mathcal{O}(\beta\partial_t O^\dagger_{1,2}), \tag{8}$$

where the explicit form of the second term on the righthand side will not be needed because it will vanish when the spurions are replaced with their expectation value. The first two unitarity constraints can also be easily amended to account for the fields $O_{1,2}$ [2]

$$S[\varphi_a = 0, \rho_r, O_1, O_2 = O_1] = 0, \tag{9a}$$

$$S^*[\varphi_a, \rho_r, O_1, O_2] = -S[-\varphi_a, \rho_r, O_2, O_1]. \tag{9b}$$

These conditions stem from the unitarity of the microscopic physics describing the combined system and bath, and therefore must be imposed even on the effective action of an open system [16].

Second, following the usual template for coupling Goldstone modes to additional "matter fields" [19], we will find it convenient to work directly with the combination $\tilde{O} = Oe^{-i\varphi_a}$, since it is invariant under $U(1)_1 \times U(1)_2$. By combining our definitions, we can easily derive the transformation rule for $\tilde{O}$ and its complex conjugate under DKMS

$$\tilde{O}'(x) = \tilde{O}^\dagger(-x) + \beta\rho_r(-x)\tilde{O}^\dagger(-x) + \mathcal{O}(\beta\partial_t), \tag{10a}$$

$$\tilde{O}^{\dagger\prime}(x) = \tilde{O}(-x) - \beta\rho_r(-x)\tilde{O}(-x) + \mathcal{O}(\beta\partial_t), \tag{10b}$$

while the first two unitarity constraints reduce to

$$S[\varphi_a = 0, \rho_r, \tilde{O} \geq 0] = 0, \tag{11a}$$

$$S^*[\varphi_a, \rho_r, \tilde{O}] = -S[-\varphi_a, \rho_r, \tilde{O}^\dagger]. \tag{11b}$$

At linear order in O and $\langle O\rangle$, there is only one term that is invariant under the DKMS symmetry, and satisfies *all three* unitarity constraints.

$$\Delta S = \frac{i}{\beta}\int d^4x F(\rho_r)\left[2\langle O\rangle - (\tilde{O}^\dagger + \tilde{O}) + \frac{\beta}{2}\rho_r(\tilde{O}^\dagger - \tilde{O})\right], \tag{12}$$

where $F(\rho_r)$ is analytic around $\rho_r = 0$ and, without loss of generality, such that $F(0) = 1$. Note that the first term proportional to $\langle O\rangle$ is crucial to ensure that the unitarity condition (11a) is satisfied. Replacing $O \to \langle O\rangle = \gamma$, this expression reduces to

$$\Delta S = \int d^4x 4\gamma(\rho_r)\sin\frac{\varphi_a}{2}\left(\frac{i}{\beta}\sin\frac{\varphi_a}{2} - \frac{\rho_r}{2}\cos\frac{\varphi_a}{2}\right), \tag{13}$$

where we have defined $\gamma(\rho_r) \equiv \gamma F(\rho_r)$.

It is easy to check that this expression is invariant under the DKMS transformations (4) up to corrections of $\mathcal{O}(E/T)$ and satisfies the unitarity constraints (5) provided $\gamma(\rho_r) \geq 0$. In the presence of charge conjugation symmetry, $\gamma(\rho_r)$ only contains even powers of ρ_r [under C, $O'(x) = O^\dagger(x)$]. The nonlinear structure of (13) implies that, in the limit of weak interaction with the environment, any n-point function will introduce only one new free parameter at leading order, i.e. successive measurements of each n-point function will fix the coefficient of the $(n-1)$th parameter in the expansion of $\gamma(\rho_r)$. We will now present the corrections to the 2- and 3-point functions for the charge density in ajar systems due to the terms in (13).

Density correlation functions: The quadratic action for diffusive ajar systems follows from expanding $S + \Delta S$ to quadratic order in ρ_r and ϕ_a. Inverting these terms yields the 2-point functions for φ_a and ρ_r, or equivalently φ_a and the charge density n

$$\begin{aligned}\langle n(p)n(-p)\rangle &= \frac{2}{\beta}\frac{(\gamma + \sigma k^2)}{\omega^2 + (\gamma/\chi + \mathcal{D}k^2)^2},\\ \langle n(p)\varphi_a(-p)\rangle &= \frac{1}{\omega + i(\gamma/\chi + \mathcal{D}k^2)},\end{aligned} \tag{14}$$

where $p = (\omega, \vec{k})$, $\chi = dn/d\rho_r|_{\rho_r=0}$, $\sigma = \sigma(\rho_r = 0)$, and $\mathcal{D} = \sigma/\chi$ is the diffusion coefficient, and delta functions enforcing energy-momentum conservation have been omitted. Notably, environmental interactions gap only the imaginary part of the pole, unlike an approximate $U(1)$ symmetry, where the real part is also gapped.

The leading 3-point function of charge density arises from expanding $S + \Delta S$ to cubic order in ρ_r and φ_a. The closed-system result was recently derived in [18]. Expressing interactions in terms of $n \simeq \chi(\rho_r + \frac{1}{2}\chi'\rho_r^2)$ yields

$$\begin{aligned}S_3 + \Delta S_3 = \int d^4x \Bigg[&\frac{\sigma\chi'}{\chi^2} n\partial_i\varphi_a n + \sigma' n\partial_i\varphi_a\partial^i\left(\frac{i}{\beta}\varphi_a - \frac{n}{\chi}\right)\\ &+ \frac{\gamma\chi'}{2\chi^2}n^2\varphi_a + \gamma' n\varphi_a\left(\frac{i}{\beta}\varphi_a - \frac{n}{\chi}\right)\varphi_a\Bigg],\end{aligned} \tag{15}$$

where primes denote derivatives with respect to n. The resulting three-point function of n is (with $q_i \equiv \sqrt{\mathcal{D}}k_i$ and $\tilde{\gamma} \equiv \gamma/\chi$)

$$\langle n(p_1)n(p_2)n(p_3)\rangle = (\chi T)^2\Big\{12\tilde{\gamma}\frac{\chi'}{\chi}\frac{(\tilde{\gamma}+q_1^2)(\tilde{\gamma}+q_2^2)(\tilde{\gamma}+q_3^2)}{\left[\omega_1^2+(\tilde{\gamma}+q_1^2)^2\right]\left[\omega_2^2+(\tilde{\gamma}+q_2^2)^2\right]\left[\omega_3^2+(\tilde{\gamma}+q_3^2)^2\right]}$$
$$+\left[\frac{4\chi'}{\chi}-\frac{2\sigma'}{\sigma}\right]\frac{(\tilde{\gamma}+q_1^2)(\tilde{\gamma}+q_2^2)(\tilde{\gamma}+q_3^2)(q_1^2+q_2^2+q_3^2)}{\left[\omega_1^2+(\tilde{\gamma}+q_1^2)^2\right]\left[\omega_2^2+(\tilde{\gamma}+q_2^2)^2\right]\left[\omega_3^2+(\tilde{\gamma}+q_3^2)^2\right]}$$
$$+\frac{4\sigma'}{\sigma}\frac{\omega_2\omega_3(q_2\cdot q_3)(\tilde{\gamma}+q_1^2)+\omega_1\omega_3(q_1\cdot q_3)(\tilde{\gamma}+q_2^2)+\omega_1\omega_2(q_1\cdot q_2)(\tilde{\gamma}+q_3^2)}{\left[\omega_1^2+(\tilde{\gamma}+q_1^2)^2\right]\left[\omega_2^2+(\tilde{\gamma}+q_2^2)^2\right]\left[\omega_3^2+(\tilde{\gamma}+q_3^2)^2\right]}$$
$$-\frac{4\gamma'}{\chi^2}\frac{3\left[(\tilde{\gamma}+q_1^2)(\tilde{\gamma}+q_2^2)(\tilde{\gamma}+q_3^2)\right]+\omega_2\omega_3(\tilde{\gamma}+q_1^2)+\omega_1\omega_3(\tilde{\gamma}+q_2^2)+\omega_1\omega_2(\tilde{\gamma}+q_3^2)}{\left[\omega_1^2+(\tilde{\gamma}+q_1^2)^2\right]\left[\omega_2^2+(\tilde{\gamma}+q_2^2)^2\right]\left[\omega_3^2+(\tilde{\gamma}+q_3^2)^2\right]}\Big\}. \quad (16)$$

$U(1)$ goldstone mode.

Effective action for ajar systems: The approach discussed in the previous section applies also when the $U(1)$ symmetry is spontaneously broken. In this case, the corresponding Schwinger-Keldysh effective action for a closed system realizes the entire group $U(1)_1 \times U(1)_2$ nonlinearly. The relevant degrees of freedom are the two Goldstone modes π_r and π_a, which transform under the DKMS symmetry as follows [7]:

$$\pi_a'(x) = -\pi_a(-x) + i\beta\partial_t\pi_r(-x) + \mathcal{O}(\beta^2), \quad (17a)$$

$$\pi_r'(x) = -\pi_r(-x) - \frac{i\beta}{4}\partial_t\pi_a(-x) + \mathcal{O}(\beta^2). \quad (17b)$$

The effective action must be invariant under (17) as well as shifts of $\pi_{r,a}$, and must satisfy unitarity constraints of the form (5) and (11) with $\rho_r \to \partial_t\pi_r, \varphi_a \to \pi_a$. The shift symmetries imply that our fields must enter the effective Lagrangian with at least one derivative. If our system was closed, the effective action would be [20]

$$S = \frac{f^2}{c_s^3}\int d^4x\Big\{\partial_t\pi_a\partial_t\pi_r - c_s^2\partial_i\pi_a\partial_i\pi_r + \frac{\Sigma_\pi}{f}\partial_t\pi_a\left(\frac{i}{\beta}\partial_t\pi_a - \partial_t^2\pi_r\right) + \frac{\sigma_\pi c_s^2}{f}\partial_i\pi_a\left(\frac{i}{\beta}\partial_i\pi_a - \partial_t\partial_i\pi_r\right) + \cdots\Big\}, \quad (18)$$

where we have shown explicitly only the terms quadratic in the fields. From the quadratic action, we deduce the power-counting rules $\pi_a \sim (E/T)\pi_r$ and $\partial_t \sim \partial_i$.

The exchange of charge with an environment explicitly breaks the shift symmetry of π_a. In ajar systems, where this breaking is soft, it can again be modeled by a spurion O, or equivalently $\tilde{O} = Oe^{-i\pi_a}$, that now transforms under DKMS as follows:

$$\tilde{O}'(x) = \tilde{O}^\dagger(-x) - \beta\partial_t\pi_r(-x)\tilde{O}^\dagger(-x) + \mathcal{O}(\beta^2), \quad (19a)$$

$$\tilde{O}^{\dagger\prime}(x) = \tilde{O}(-x) + \beta\partial_t\pi_r(-x)\tilde{O}(-x) + \mathcal{O}(\beta^2), \quad (19b)$$

At leading order in the spurion field, the effective action receives the following correction

$$\Delta S = \frac{i}{\beta}\int d^4x F(\partial_t\pi_r, \partial_i\pi_r\partial^i\pi_r) \times\left[2\langle O\rangle - (\tilde{O}^\dagger + \tilde{O}) + \frac{\beta}{2}\partial_t\pi_r(\tilde{O}^\dagger - \tilde{O})\right], \quad (20)$$

where, once again, we can assume $F(0,0) = 1$ without loss of generality.

Goldstone correlation functions: The two-point functions that result from inverting the quadratic part of $S + \Delta S$ are

$$\langle\pi_r\pi_r\rangle = \frac{2c_s^3}{\beta f^2}\frac{\tilde{\gamma} + \frac{1}{f}\left(\Sigma_\pi\omega^2 + \sigma_\pi c_s^2k^2\right)}{\left(\omega^2 - c_s^2k^2\right)^2 + \omega^2\left[\tilde{\gamma} + \frac{1}{f}\left(\Sigma_\pi\omega^2 + \sigma_\pi c_s^2k^2\right)\right]^2},$$
$$\langle\pi_r\pi_a\rangle = \frac{c_s^3}{f^2}\frac{i}{\omega^2 - c_s^2k^2 + i\tilde{\gamma}\omega + \frac{i}{f}\left(\Sigma_\pi\omega^3 + \sigma_\pi c_s^2\omega k^2\right)}, \quad (21)$$

where $\tilde{\gamma} \equiv \gamma c_s^3/f^2$. The second correlator is simply the retarded Green's function for the Goldstone mode.

Once again, we see that "openness" adds a finite contribution to the imaginary part of the Goldstone dispersion relation, rather than turning this mode into a pseudo-Goldstone boson by gapping the real part, as would be the case for an approximate symmetry. Physically, it means that the Goldstone mode can only propagate for wave numbers $k \gtrsim \tilde{\gamma}/c_s$. Higher-point functions for the Goldstone mode can be calculated in a straightforward way by expanding $S + \Delta S$ up to the desired order.

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[1] F. M. Haehl, R. Loganayagam, and M. Rangamani, J. High Energy Phys. 06 (2017) 069.
[2] H. Liu and P. Glorioso, Proc. Sci. TASI2017 (**2018**) 008 [arXiv:1805.09331].
[3] S. A. Salcedo, T. Colas, and E. Pajer, J. High Energy Phys. 10 (2024) 248.
[4] A. Kamenev, *Field Theory of Non-Equilibrium Systems*, 2nd ed. (Cambridge University Press, Cambridge, England, 2023).
[5] F. M. Haehl and M. Rangamani, arXiv:2410.10602.
[6] L. M. Sieberer, M. Buchhold, and S. Diehl, Rep. Prog. Phys. **79**, 096001 (2016).
[7] C. O. Akyuz, G. Goon, and R. Penco, J. High Energy Phys. 06 (2024) 004.
[8] M. Hongo, S. Kim, T. Noumi, and A. Ota, Phys. Rev. D **103**, 056020 (2021).
[9] R. Penco, arXiv:2006.16285.
[10] M. Hongo, N. Sogabe, M. A. Stephanov, and H.-U. Yee, arXiv:2411.08016.
[11] M. Baggioli, Y. Bu, and V. Ziogas, J. High Energy Phys. 09 (2023) 019.
[12] See Supplemental Material at http://link.aps.org/supplemental/10.1103/mrzp-zh4v, which includes Ref. [13], for an example of a UV model that motivates our spurion construction.
[13] A. O. Caldeira and A. J. Leggett, Physica A (Amsterdam) **121A**, 587 (1983).
[14] S. A. Salcedo, T. Colas, and E. Pajer, J. High Energy Phys. 03 (2025) 138.
[15] L. M. Sieberer, A. Chiocchetta, A. Gambassi, U. C. Täuber, and S. Diehl, Phys. Rev. B **92**, 134307 (2015).
[16] P. Glorioso, M. Crossley, and H. Liu, J. High Energy Phys. 09 (2017) 096.
[17] E. Firat, A. Gomes, F. Nardi, R. Penco, and R. Rattazzi (to be published).
[18] L. V. Delacretaz and R. Mishra, SciPost Phys. **16**, 047 (2024).
[19] S. Weinberg, *The Quantum Theory of Fields. Vol. 2: Modern Applications* (Cambridge University Press, Cambridge, England, 2013).
[20] A. Donos and P. Kailidis, J. High Energy Phys. 01 (2024) 110.