

Resurgent structure of the 't Hooft-Polyakov monopole

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(Received 20 February 2026; accepted 15 June 2026; published 14 July 2026)

In this paper, we present a comprehensive analysis of the differential equations governing the spatial profile of the 't Hooft-Polyakov monopole from the viewpoint of resurgence theory. We note that the universality of the gauge-component asymptotics, together with the relative simplicity of its Borel transform and the associated Volterra equations' kernels, gives rise to a perturbative expansion featuring a good control over the proliferation of the Borel-plane singularities to all orders, along with useful information about their structure. Moreover, its partial resummation reveals remarkably simple universal analytic "dressed" background, around which one may develop a global perturbative expansion of the exact solutions for any $\lambda/e^2 > 0$ and, at least in principle, get a grip on the numerical parameters governing the local expansions of the profile functions.

DOI: 10.1103/mm8d-1qd4

Introduction. It is well known that nonlinear ordinary differential equations (ODEs) with domains stretching all the way to infinity typically do not admit convergent large- x transseries expansions and, with the notable exception of the Bogomol'nyi-Prasad-Sommerfield (BPS) case [1,2], the ODEs governing the spatial profile of the 't Hooft-Polyakov monopole [3,4],

$$y'' = yz^2 + y(y^2 - 1)/x^2, \quad (1)$$

$$z'' + 2z'/x = 2zy^2/x^2 + \beta z(z^2 - 1) \quad (2)$$

(with $\beta \equiv \lambda/e^2$, x corresponding to r , y to K , and zx to H of Ref. [1]), are no different in this respect. Hence, the usual approach to their solution exploits various types of numerical accounts based on controlled small-parameter or local expansions, or even vanilla ODE integrators like Runge-Kutta (cf. [5–10], and references therein).

What we claim in this paper is that even the basic methods of resurgence theory (cf. [11–21]) make it possible to do much better in this respect. The point is the full exploitation of the analytic form of the lowest exponential level gauge-sector (1) asymptotics' Borel resum and a careful bookkeeping of the proliferation of singularities

throughout higher levels of the relevant Volterra equations. The key to this is the fact that, due to the exponentially fast saturation of the $z|_{x \rightarrow \infty} \rightarrow 1$ boundary condition in the $\beta > 0$ regime, the asymptotics of solutions of the gauge profile ODE (1) is universal:

$$y(x \rightarrow +\infty) \rightarrow A_\beta \sqrt{x} K_\nu(x) \quad \forall \beta > 0, \quad (3)$$

up to a β -dependent real constant A_β ; here, K_ν denotes the modified Bessel K function of imaginary order $\nu = i\sqrt{3}/2$. At the same time, the $\sqrt{x}K_\nu(x)$ piece of Eq. (3) can be cast in terms of a Laplace transform of an associated ${}_2F_1$ hypergeometric function (see, e.g., [21]):

$$\sqrt{x}K_\nu(x) = \sqrt{\frac{\pi}{2}} x e^{-x} \int_0^\infty e^{-xt} {}_2F_1\left(\frac{1}{2} - \nu, \frac{1}{2} + \nu; 1; -\frac{t}{2}\right), \quad (4)$$

where the preintegral $x e^{-x}$ factor is fully compatible with the inverse-power-exponential shape of the relevant asymptotic transseries, as well as with that of the asymptotically fast decaying y^3/x^2 nonlinearity of Eq. (1). Hence, the hypergeometric integrand of Eq. (4) can be viewed as a seed of the Borel-plane structure of solutions of Eqs. (1) and (2) regardless of the value of $\beta > 0$.

Interestingly, the discrete singularity structure of $\hat{f}_0(t) = A_\beta \sqrt{\frac{\pi}{2}} {}_2F_1(\frac{1}{2} - \nu, \frac{1}{2} + \nu; 1; -\frac{t}{2})$ in Eq. (4), limited to a single branch cut starting at $t = -2$, along with the triangular (i.e., iterative) nature of the Borel-plane Volterra equations equivalent to Eqs. (1) and (2), make it possible to track the resurgent structure of the y and z functions to any

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desired level—higher-order singularities are generated from the lower-order ones at specific points along the real- t axis, inheriting all their structure from the fundamental $\hat{f}_0$ seed. This suggests that one may even attempt to approximate the full forms of y and z in the corresponding perturbative sector in terms of Laplace transforms of thus constructed collection of Borel-plane germs; the singularities developed along the positive real semiaxis would then have to be dealt with by suitable deformations of the relevant integration contours (lateral transforms) underpinning the median resummation method; see, e.g., [17]. Note also that a detailed information about the Borel-plane singularity structure may facilitate a reconstruction of the solutions of Eqs. (1) and (2) in terms of Padé approximants; cf. [21].

Furthermore, we shall see that thus obtained Borel-plane perturbation series can even be rearranged and partially resummed in such a way to provide a *global expansion scheme developed around analytic nonperturbative “dressed” background profiles* governed by simple vector- and scalar-sector “seed functions” $\hat{f}_0^R$ and $\hat{g}_0^R$, that yield x -plane *backgrounds automatically obeying all boundary conditions*. These, in turn, may also provide information on the *a priori* unknown parameters governing the local expansions of both y and z .

In what follows, we shall first demonstrate these principles on the simplified case of the “maximally non-BPS” (MNBPS) monopole corresponding to $\beta \rightarrow \infty$, where only the gauge ODE (1) retains a nontrivial form (Sec. II). In Sec. III, we shall comment on how the vector sector feeds into the structure of its scalar counterpart in case of general $\beta > 0$ configurations. The partial resummation of the naïve expansion around $\hat{f}_0$ and the explicit construction of the dressed global background profile $\hat{f}_0^R$, along with a sample calculation of the lowest-order contribution to the B_∞ parameter governing the local expansion of y_{MNBPS} at $x = 0$, are discussed in Sec. IV. Most of the technical details concerning, namely, the finite $\beta > 0$ (NBPS) case are deferred to a set of Appendixes and an extended study [22].

The MNBPS monopole ($\beta \rightarrow \infty$). The variant of Eq. (1) governing the gauge-field profile of the MNBPS monopole, subject to boundary conditions $y|_{x \rightarrow 0} \rightarrow 1$ and $y|_{x \rightarrow \infty} \rightarrow 0$, reads

$$y'' = y + (y^3 - y)/x^2. \quad (5)$$

Despite its apparent simplicity [note the triviality of the crosstalk with the scalar sector assuming $z(x) = 1 \ \forall \ x > 0$], Eq. (5) is still notoriously hard (second order, nonlinear, nonconservative, with boundary conditions as limits in the two singular points of its domain).

The Borel-plane representation of the solution (4) of its linearized version (cf. Appendix A) suggests a substitution

$y(x) = xe^{-x}f(x)$, for which Eq. (5) assumes a simple symbolic form $L_\infty[f](x) = e^{-2x}f(x)^3$. Here,

$$L_\infty[f](x) \equiv f''(x) - 2f'(x) + \frac{2f'(x)}{x} - \frac{2f(x)}{x} + \frac{f(x)}{x^2} \quad (6)$$

is the differential operator of the linear problem, and the rhs is a fast-decaying nonlinear forcing term. In the Borel plane, Eq. (5) is, thus, equivalent to

$$t(t+2)\hat{f}(t) + \int_0^t ds K(t,s)\hat{f}(s) = [\hat{f} * \hat{f} * \hat{f}](t-2), \quad (7)$$

where $K(t,s) = t - 3s - 2$ is a regular kernel corresponding to the nonderivative part of the differential operator (6) and the star symbols in the square bracket stand for (associative and commutative) Borel convolutions

$$[\hat{h}_1 * \hat{h}_2](t) \equiv \int_0^t ds \hat{h}_1(s)\hat{h}_2(t-s).$$

Proliferation of singularities of $\hat{f}$: The Borel-plane picture of the solution of the linearized version of Eq. (5) written in terms of $\hat{f}_0(t)$ (see Sec. I) trivially obeys Eq. (7) with zero rhs, and, as such, it represents the initial level of the ladder of approximations to the full $\hat{f} = \hat{f}_0 + \hat{f}_1 + \dots$. Higher orders are then obtained by iterations exploiting the triangular structure of Eq. (7), for instance,

$$t(t+2)\hat{f}_1(t) + \int_0^t ds K(t,s)\hat{f}_1(s) = [\hat{f}_0 * \hat{f}_0 * \hat{f}_0](t-2),$$

and so on. Throughout these, proliferation of the primary $t = -2$ singularity of $\hat{f}_0$ is governed by two simple rules: (i) Convolutions generate new singularities at points obtained by shifts corresponding to position vectors of singularities of individual components. (ii) The $(t-2)$ argument on the rhs of Eq. (7) then shifts all such structures by two units (in t) to the right. Hence, the singularities of $\hat{f}$ will eventually populate the real axis of the Borel plane at discrete $t_{\text{sing}} = 2m$ ($m \in \mathbb{Z}$) points.

To this end, it is worth noting that the BPS monopole profiles $y_{\text{BPS}}(x) = x/\sinh x$ and $z_{\text{BPS}}(x) = \coth x - 1/x$ exhibit a very similar singularity pattern like that encountered in the MNBPS case above; for more details, the reader is kindly deferred to the extended study [22].

The general NBPS setting ($\beta \in \mathbb{R}_+$). Interestingly, a similar singularity pattern may emerge even for finite $\beta > 0$. Indeed, with $y(x) = xe^{-x}f(x)$ and $z(x) = 1 + g(x)$, Eqs. (1) and (2) cast in the f and g coordinates read

$$\begin{aligned} L_f[f] &= e^{-2x}f^3 + g(g+2)f, \\ L_g[g] &= 2e^{-2x}(g+1)f^2 + \beta g^2(g+3), \end{aligned} \quad (8)$$

with $L_f[f] = L_\infty[f]$ of Eq. (6) and $L_g[g](x) = g''(x) + 2\frac{g'(x)}{x} - 2\beta g(x)$ defining the linear sectors. Note also that the MNBPS limit in Sec. II is achieved for $g(x) \rightarrow 0$ which corresponds to $z(x) \rightarrow 1$. With this at hand, the relevant Volterra equations can be written readily:

$$\begin{aligned}\hat{L}_f[\hat{f}](t) &= [\hat{f} * \hat{f} * \hat{f}](t-2) + 2[\hat{f} * \hat{g}](t) + [\hat{f} * \hat{g} * \hat{g}](t), \\ \hat{L}_g[\hat{g}](t) &= 2[\hat{f} * \hat{f}](t-2) + 2[\hat{f} * \hat{f} * \hat{g}](t-2) \\ &\quad + \beta[3\hat{g} * \hat{g} + \hat{g} * \hat{g} * \hat{g}](t),\end{aligned}\quad (9)$$

with $\hat{L}_f[\hat{f}](t)$ corresponding to the lhs of Eq (7) and $\hat{L}_g[\hat{g}](t) = (t^2 - 2\beta)\hat{g}(t) + \int_0^t ds \tilde{K}(t, s)\hat{g}(s)$, with $\tilde{K}(t, s) = -2s$. The system (9) is quite remarkable for several reasons: (i) Because of $g(x)|_{x \rightarrow \infty} \rightarrow 0$ [cf. Eq. (8)], the Borel-plane asymptotic scalar profile is trivial $[\hat{g}_0(t) = 0]$, and the asymptotic gauge profile $\hat{f}_0(t)$ is still proportional to ${}_2F_1(\frac{1}{2} - \nu, \frac{1}{2} + \nu; 1; -\frac{t}{2})$ like in the MNBPS case of Sec. II (albeit with different normalization factors $A_\beta \neq A_\infty$; cf. Sec. I); (ii) the proliferation of singularities due to (9) follows a similar pattern as before—the primary branch point of $\hat{f}_0(t)$ at $t = -2$ is complemented by the potential discontinuities of $\hat{g}_0$ at $\pm\sqrt{2\beta}$, the rhs convolutions combine these and, due to the $t \rightarrow t-2$ shifts, spread them accordingly along the entire real axis (to $t = 2k + \sqrt{2\beta}l$; $k, l \in \mathbb{Z}$); (iii) *the scalar sector is “enslaved” by the gauge one*—the first nontrivial contribution to $\hat{g}$ is generated by $\hat{f}_0$ through the $2[\hat{f} * \hat{f}](t-2)$ convolution on the rhs of Eq. (9). It is interesting to observe that β values of the form $\beta = m^2/2$ ($m \in \mathbb{N}$) may be special because of a resonance among the two sectors placing all germs to integers, thus restoring the full periodicity like that of the MNBPS and BPS cases above.

Partial resummation of $\hat{f}$. Concerning the lowest-order calculation of the B_∞ parameter of the local power-log expansion of the MNBPS gauge profile around $x = 0$ promised in Sec. I, unfortunately, the asymptotic background profiles $\hat{f}_0$ and $\hat{g}_0$ of Secs. II and III are not particularly suitable for this task. The reason is that $\hat{f}_0$ yields an expansion of y around a “wild” background corresponding to its Laplace transform $e^x K_{i\sqrt{3}/2}(x)/\sqrt{x}$ [cf. Eq. (4)], divergent and infinitely fast oscillating at the origin. Hence, the regular pattern of the local expansion around $x = 0$ required by $y|_{x \rightarrow 0} \rightarrow 1$ is unlikely to emerge perturbatively in such a scheme; see also [23].

Dressing the asymptotic $\hat{f}_0$ propagator: Remarkably enough, there is a trick (corresponding, in Dyson’s language, to a dressing of the hypergeometric “propagator” $\hat{f}_0$) that makes it possible to reformulate the problem as a perturbative expansion around another, global analytic nonperturbative background, that is even universal for all

gauge profiles of 't Hooft–Polyakov monopoles with arbitrary $\beta > 0$; cf. [22]. The key is a simple deformation of the $L_\infty[f]$ operator in Eq. (6):

$$L_\infty \rightarrow L_\infty - \frac{3}{x^2} \equiv L_\infty^R, \quad (10)$$

which makes it possible to transform the original MNBPS-case ODE for f , i.e., $L_\infty[f](x) = e^{-2x}f(x)^3$ [equivalent, upon $y \rightarrow xe^{-x}f$, to Eq. (5)], to the form

$$L_\infty^R[f^R] = \frac{e^x}{x} + e^{-2x}f^R(x)^3 - 3\frac{e^{-x}}{x}f^R(x)^2, \quad (11)$$

where $f^R \equiv f - \frac{e^x}{x}$. The power of this rearrangement stems from the fact that the Volterra equation equivalent to Eq. (11) then assumes a rather special form

$$t(t+2)\hat{f}^R(t) - 2(t+1)\int_0^t ds \hat{f}^R(s) = \hat{R}(t), \quad (12)$$

where $\hat{R}$ is the Borel-plane equivalent of the rhs of Eq. (11),

$$\begin{aligned}\hat{R}(t) &= 1_{-1}(t) + (\hat{f}^R * \hat{f}^R * \hat{f}^R)(t-2) \\ &\quad - 3(\hat{f}^R * \hat{f}^R * 1)(t-1).\end{aligned}\quad (13)$$

Note, in particular, the absence of an s -dependent kernel from Eq. (12) and the fact that its homogeneous solution defining the fundamental mode of this expansion is very simple, namely,

$$\hat{h}(t) = C(t+1). \quad (14)$$

Hence, the general solution to the full Eq. (12) can be formally written as

$$\hat{f}^R(t) = \frac{\hat{R}(t)}{t(t+2)} + 2(t+1)\int_0^t ds \frac{\hat{R}(s)}{s^2(s+2)^2}. \quad (15)$$

The expansion $\hat{f}^R = \hat{f}_0^R + \hat{f}_1^R + \dots$ with each $\hat{f}_n^R$ given by Eq. (15) with a suitable part $\hat{R}_n$ of the total $\hat{R} = \hat{R}_0 + \hat{R}_1 + \dots$ can then be viewed as an expansion around a resummed background profile $\hat{f}_0^R$ corresponding to $\hat{R}_0 = 1_{-1}$ (unity anchored at $t = -1$) with the perturbative modes $\hat{f}_n^R$ ($n \in \mathbb{N}$) defined by the triangular partition of the nonlinear part of $\hat{R}$ of Eq. (13) corresponding to the expansion of $\hat{f}^R$. The key point is that the solution (15) of Eq. (12) with $\hat{R}_0$ on the rhs is very simple (in $u = t+1$, the local coordinate of the germ at -1), namely,

$$\hat{f}_0^R(u)_{-1} = \frac{1 + u_0(u - u_0)}{u_0^2 - 1} + u(\operatorname{arctanh} u - \operatorname{arctanh} u_0), \quad (16)$$

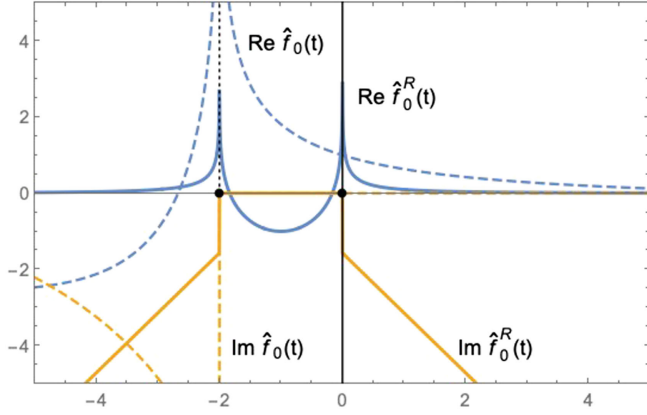


FIG. 1. Borel-plane structure of the “naked” propagator $\hat{f}_0$ (dashed lines) of Secs. II and III and its “dressed” counterpart $\hat{f}_0^R$ of Sec. IV (solid lines); real parts in blue, imaginary in yellow. One can see the emergence of the second branch cut of $\hat{f}_0^R$ at $t = 0$ due to the partial resummation of $\hat{f}$. Note also the fast diminishing of the asymptotic tails of $\hat{f}_0^R$, as compared to $\hat{f}_0$.

where u_0 is an integration constant to be fixed by one of the boundary conditions. Interestingly, $u_0 = 0$ yields

$$\hat{f}_0^R(u)_{-1} = -1 + u \operatorname{arctanh} u, \quad (17)$$

which, upon Laplace transform (with u as a local coordinate), gives

$$f_0^R(x) = \frac{1}{2x^2} [e^{2x}(x-1)\operatorname{Ei}(-x) + (x+1)\operatorname{Ei}(x)] - \frac{e^x}{x}.$$

Crucially, y_0 derived from this structure *obeys both boundary conditions* $y_0|_{x \rightarrow 0} \rightarrow 1$ and $y_0|_{x \rightarrow \infty} \rightarrow 0$ *simultaneously*. At first glance, this looks like pure serendipity, but it is actually the intended consequence of the very specific reparametrization of $\hat{f}$ in terms of the resummed (global) quantities.

On a similar footing (cf. [22]), one obtains $g_0^R(x) = [-1 + \exp(-\sqrt{2\beta}x)(1 + \sqrt{2\beta}x)]/\beta x^2$, that corresponds to the Borel-plane structure $\hat{g}_0^R(t) = \theta(t - \sqrt{2\beta})t/\beta - t/\beta$, localized at $t = \sqrt{2\beta}$ (θ -shifted, exponential sector germ) and $t = 0$ (regular germ), respectively.

Structure of the resummed expansion: Interestingly, switching from the naïve hypergeometric asymptotic background $\hat{f}_0(t) \propto {}_2F_1(e^{-i\pi/3}, e^{i\pi/3}; 1; -\frac{t}{2})$ to the global resummed background $\hat{f}_0^R$ in Eq. (17), the Borel-plane structure of the perturbative expansion of $\hat{f}^R$ (and $\hat{g}^R$) also changes with respect to that of $\hat{f}$ and $\hat{g}$ discussed in Secs. II and III. Note, in particular, that the resummed (or dressed) vector “seed profile” $\hat{f}_0^R$, which shares the $t = -2$ branch cut with its naked asymptotic variant $\hat{f}_0$, develops a new singularity at $t = 0$ where $\hat{f}_0$ was regular; cf. Fig. 1. Second, both real and imaginary parts of $\hat{f}_0^R$ are very nicely behaved in the

large- t region where the real part of $\hat{f}_0$ suffers from oscillatory behavior with roots at exponentially distant points. Third, unlike in $\hat{f}_0$, no free constant remains in $\hat{f}_0^R$. This is a clear indication of a significant qualitative difference of the two types of perturbative expansions.

Moreover, since $\hat{f}_0^R$ on its own generates an x -plane profile y_0 that obeys *both* boundary conditions, higher-order corrections do not need to produce any nonperturbative jump at $x = 0$ like was required from the expansion around $\hat{f}_0$. Hence, they may not only vanish at the $x = 0$ singularity and naturally decay at $x \rightarrow \infty$, but also gradually diminish on the entire $\mathbb{R}_+$; thus, the resummed expansion may even converge uniformly to the exact solution. This is demonstrated explicitly in [22].

Sample lowest-order B_∞ calculation: Finally, y_0 as the nonperturbative background of the full perturbative expansion of y behaves around $x = 0$ as

$$y_0(x) = 1 + \frac{1}{3}x^2 \left(\gamma_E - \frac{4}{3} + \log x \right) + \dots, \quad (18)$$

where γ_E is the Euler-Mascheroni constant. Thus, one verifies the structure of the leading $x^2 \log x$ term of the expansion (C1) and obtains the lowest-order part of B_∞ :

$$B_{\infty,0} = \frac{1}{3} \left(\gamma_E - \frac{4}{3} \right) \approx -0.252. \quad (19)$$

This is in the right ballpark of the “true” value $B_\infty \approx -0.484$ obtained from the numerics; cf. Appendix C. Higher-order corrections, along with an exact analytic formula for B_∞ , are given in the extended study [22].

Conclusions. In this paper, we have elucidated the remarkable Borel-plane structure of the solutions of the ‘t Hooft–Polyakov monopole profile equations for all positive values of the $\beta = \lambda/e^2$ parameter therein. It was shown that, within the canonical perturbative scheme suggested by the universal asymptotics of the NBPS monopole gauge profiles, the emerging discrete singularity pattern is fully driven by the ${}_2F_1(e^{-i\pi/3}, e^{i\pi/3}; 1; -\frac{t}{2})$ hypergeometric function featuring a logarithmic branch cut at $t = -2$. We have argued that an even more interesting global expansion scheme for the profile functions can be devised around a specific resummed global background featuring a surprisingly simple analytic structure (explicitly constructed for the vector sector) which, in turn, has been used as a basis for a sample calculation of the numerical parameter B_∞ governing the local $x = 0$ expansion of the gauge-sector profile in the MNBPS $\beta \rightarrow \infty$ setting.

Hence, the Borel-plane perspective looks like a very natural viewpoint for contemplating topological defects in spontaneously broken gauge theories.

Acknowledgments. The author is grateful to Petr Beneš and Filip Blaschke for valuable feedback on the manuscript. The work has been performed with the support from the Charles University Research Center of Excellence UNCE/24/SCI/016 grant and from the FORTE Project No. CZ.02.01.01/00/22_008/0004632 cofunded by the EU and the Ministry of Education, Youth and Sports of the Czech Republic.

Data availability. There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the author.

Appendix A: Universal $\beta > 0$ gauge asymptotics. The asymptotic profiles of the solutions of Eq. (1) correspond to a mere replacement of z by its asymptotic limit $z(x) = 1$ which, for $\beta > 0$, is saturated exponentially fast, i.e., $1 - z(x) \sim e^{-2x}$ for large x [unlike for the BPS case in which the convergence is only hyperbolic, $1 - z(x) \sim x^{-1}$]. With the fast-decaying y^3/x^2 piece neglected, the profile equation (1) assumes a simple linearized form

$$y''(x) = \left(1 - \frac{1}{x^2}\right)y(x). \quad (\text{A1})$$

For $y(x) \equiv \sqrt{x}u(x)$ this structure can be readily mapped onto the defining equation of Bessel functions $x^2 u''(x) + xu'(x) - (x^2 + \nu^2)u(x) = 0$, with $\nu^2 = -3/4$, and, as such, the physically interesting (i.e., decaying) universal asymptotic solution of Eq. (1) can be written as $A\sqrt{x}K_\nu(x)$, where K_ν is the modified Bessel K function of imaginary order $\nu = i\sqrt{3}/2$ and A is a real constant.

Appendix B: The MNBPS $y(x)$ transseries at $x \rightarrow \infty$. The large- x transseries for y in the $\beta \rightarrow \infty$ limit obtained from an ansatz

$$y(x) = \sum_{m=0}^{\infty} \sum_{n=0}^m a_{m,n} x^{-m} e^{-(2n+1)x} \quad (\text{B1})$$

by a mere order-by-order comparison of the coefficients at the lhs and rhs of Eq. (5) clearly reveals its

asymptotic nature in the numerical behavior of $a_{m,n}$; for instance, the $n = 0$ tower (stripped from an *a priori* unknown overall factor A_∞) reads

$$a_{m,0} \propto \left\{1, -\frac{1}{2}, \frac{3}{8}, -\frac{7}{16}, \frac{91}{128}, -\frac{1911}{1280}, \frac{19747}{5120}, \dots\right\}.$$

As expected, these are (up to the standard Borel-transform factorials) exactly the coefficients of the power expansion of ${}_2F_1(e^{-i\pi/3}, e^{i\pi/3}; 1; -\frac{t}{2})$ around $t = 0$.

Appendix C: The MNBPS $y(x)$ transseries at $x = 0$. Similarly, the behavior of the gauge profile of the MNBPS ($\beta \rightarrow \infty$) monopole around $x = 0$ can be written in terms of a locally convergent power-log transseries

$$y(x) = \sum_{m=0}^{\infty} \sum_{n=0}^m b_{m,n} x^{2m} (\log x)^n, \quad (\text{C1})$$

whose $b_{m,n}$ coefficients can be fully determined recursively from the $y|_{x \rightarrow 0} \rightarrow 1$ initial condition, up to one *a priori* unknown constant B_∞ . The first few terms of this expansion read (see also [10])

$$\begin{aligned} y(x) = & 1 + x^2 \left(\frac{1}{3} \log x + B_\infty \right) \\ & + x^4 \left[\frac{1}{30} (\log x)^2 + \left(\frac{1}{5} B_\infty - \frac{1}{75} \right) \log x \right. \\ & \left. + \frac{1}{750} (225 B_\infty^2 - 30 B_\infty + 2) \right] + \dots, \end{aligned} \quad (\text{C2})$$

with a numerical fit (see Ref. [22]) revealing $B_\infty \approx -0.484$. This series has a finite convergence radius R which can be estimated, for instance, from the progression of its leading-log coefficients $b_{n,n}$ that, asymptotically, is purely geometrical; for large n , $b_{n,n} \approx \sqrt{8} q^n$ with $q \approx 0.109374$. Hence, $R \lesssim \exp[\frac{1}{2} W(2/q)] \sim 2.92$, where W is the Lambert function.

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