

Quantum simulations of quantum electrodynamics in Coulomb gauge

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In recent years, the quantum computing method has been used to address the sign problem in traditional Monte Carlo lattice gauge theory (LGT) simulations. We propose that the Coulomb gauge (CG) should be used in quantum simulations of LGT. Since the redundant degrees of freedom of gauge fields can be eliminated in CG, the Hamiltonian in CG does not need to be gauge invariant, allowing the gauge field to be discretized naively. Then the discretized gauge fields and fermion fields should be placed on momentum and position lattices, respectively. Under this scheme, the CG condition and Gauss's law can be conveniently preserved by solving for the polarization vectors from algebraic equations. Furthermore, we discuss the mapping of gauge fields to qubits and evaluate the associated qubit and gate cost of this framework. We point out that this formalism is efficient for simulating hadron scattering processes on future fault-tolerant quantum computers. Finally, we calculate the vacuum expectation value of the U(1) plaquette operator and the Wilson loop on a classical device to test the performance of our discretization scheme.

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I. INTRODUCTION

Gauge field theory is an essential cornerstone of the standard model (SM) as it describes the fundamental laws governing the interactions between elementary particles [1–3]. There are nonperturbative problems within the SM, such as the long-distance physics in quantum chromodynamics (QCD), including the QCD phase diagram [4], hadron structure [5], etc. Nonperturbative approaches, such as lattice QCD, have been proposed to address these problems [6–8]. However, due to the sign problem of the Monte Carlo algorithms [9], lattice QCD faces challenges when simulating time evolutions and finite-density systems. These challenges prompt the exploration of alternative nonperturbative methods for simulating gauge field theories. The quantum computing method is one of the choices [10].

The quantum simulations of gauge fields involve a complete set of procedures [11–14]. The first step is the

discretization of gauge and fermion fields, with the Kogut-Susskind (K-S) formalism being the most widely employed discretization scheme [15]. This formalism is based on choosing a temporal gauge, $A^0 = 0$, where not all residual degrees of freedom are eliminated. Consequently, the K-S Hamiltonian must remain invariant under residual gauge transformations. Digitizing the continuous gauge group is the second step of simulating lattice gauge theory (LGT) on quantum computers. A critical aspect of this step is the preservation of Gauss's law, which ensures gauge invariance. In $(1 + 1)$ -dimensional cases, this can typically be achieved, as the gauge field can be eliminated by solving Gauss's law [16–30]. However, maintaining Gauss's law in higher dimensions is more complex. Several methods have been proposed to address this challenge, such as the truncation of the electric field (group representation) basis [11,31–39], discrete subgroup method [40–52], further gauge fixing [28,29,53–58], loop-string-hadron formalism [59–63], and the quantum link model [64–66].

Though the K-S formalism has achieved great success, it has challenges when using this formalism to simulate the hadron scattering process in more than $(1 + 1)$ -dimensional spacetime on quantum computers. According to the factorization theorem [5], the cross section of high energy

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hadron scattering σ can be factorized into $\sigma = f \tilde{\otimes} \hat{\sigma} \tilde{\otimes} D$, where $\tilde{\otimes}$ denotes a convolution operator. In this expression, f represents parton distribution functions, $\hat{\sigma}$ is the hard cross section for quarks and gluons, and D denotes non-perturbative functions that characterize the hadronization of partons. The calculation of those quantities has been explored in [67–72]. Ideally, we aim to calculate f , $\hat{\sigma}$, and D using the same formalism to avoid the complications associated with matching results obtained from different methods. This desired formalism should meet the following requirements: (1) It must efficiently reach the continuum limit. (2) The Wilson line must be unitary, as the operator definitions of f and D involve Wilson lines. (3) The final state of $\hat{\sigma}$ may include polarized gluons or photons, necessitating straightforward handling of these states within the formalism. (4) When simulating inclusive processes, it is crucial to sum over all physical asymptotic “out” states, which are the eigenstates of the Hamiltonian H . Thus, it is better to preserve Gauss’s law $G|\text{Phys}\rangle = 0$ as well as $[G, H] = 0$ strictly after digitizing the gauge fields. This guarantees the gauge invariance of all “out” states. However, those requirements may be sacrificed if we do not digitize the gauge fields carefully. For example, truncating the electric basis to the maximum eigenvalue Λ will break the commutation relation $[G, H] = 0$ at order $O(1/\Lambda)$. This implies that Gauss’s law operator does not commute with the Hamiltonian, so the Hamiltonian and Gauss’s law operators do not have common eigenstates. The quantum link model preserves Gauss’s law while truncating the electric basis, but this method sacrifices the unitarity of the gauge link. The discrete subgroup method can preserve Gauss’s law after digitizing the gauge group, and it can also reach the continuum limit of some model [73]. However, the largest subgroup of $SU(3)$ is $S(1080)$, so this method might sacrifice requirement (1) in the $SU(3)$ gauge theory. We can do further gauge fixing by solving Gauss’s law on the lattice. However, this method has a challenge when considering the theory with dynamical quarks, which is necessary to define the hadron.

In this work, we propose a formalism for simulating LGT on a quantum computer that aims to satisfy all four requirements, at least in the case of quantum electrodynamics (QED). At a cost, we need to introduce nonlocal interaction terms in the Hamiltonian, so more quantum gates are needed when simulating time evolution. Different from K-S formalism, our formalism is based on fixing the Coulomb gauge (CG). The choice of CG is motivated by its preservation of rotational symmetries, unlike the axial and light-cone gauges. Compared with the maximum tree gauge, CG can handle dynamical fermions naturally. Once the CG is fixed, there are no nontrivial gauge transformations that can simultaneously preserve both the Hamiltonian and the constraints. From this perspective, we can utilize the discrete version of

$A_\mu(x)$ to construct the Hamiltonian together with constraints $\chi(A) = 0$ (such as Gauss’s law and the CG condition) on the lattice (see Supplemental Material [74]). The lattice setup will be discussed in Sec. II. After that, the mapping of gauge fields a_μ^r and the fermion fields $\psi_\alpha(\mathbf{x})$ to qubits will be given in Sec. III. Sections IV and V address the complexity of our framework and present our results, respectively.

II. DISCRETIZATIONS OF COULOMB GAUGE QED

Both fermion fields and gauge fields need to be discretized. Here, we give important results of CG QED on the lattice, while the details can be found in the Supplemental Material [74]. As shown in Fig. 1, we put the fermion fields $\psi_\alpha(\mathbf{n})$ and gauge fields a_μ^r in the lattice of position and momentum space, respectively, where a_μ^r is the annihilation operator of photons with given momentum $\mathbf{p}$ and polarization $r = \pm$. On the lattice, the gauge potential $\hat{A}_0(\mathbf{n})$, $\hat{A}_i(\mathbf{n})$, and its conjugate momentum $\hat{\Pi}_{\perp,i}$ can be represented by

$$\begin{aligned}\hat{A}^0(\mathbf{n}) &= \sum_{\mathbf{m}} \sum_{\mathbf{p} \neq 0} \frac{J^0(\mathbf{m})}{M^3 \hat{E}_{\mathbf{p}}^2} e^{-i\mathbf{p} \cdot (\mathbf{n} - \mathbf{m})}, \\ \hat{A}_i(\mathbf{n}) &= \sum_{\mathbf{p} \neq 0} \frac{1}{\sqrt{2\hat{E}_{\mathbf{p}} M^3}} \sum_r [\hat{e}_i^r(\mathbf{p}) a_{\mathbf{p}}^r e^{i\mathbf{p} \cdot \mathbf{n}} + \text{H.c.}], \\ \hat{\Pi}_{\perp,i}(\mathbf{n}) &= \sum_{\mathbf{p} \neq 0} \sqrt{\frac{\hat{E}_{\mathbf{p}}}{2M^3}} \sum_r [-i\hat{e}_i^r(\mathbf{p}) a_{\mathbf{p}}^r e^{i\mathbf{p} \cdot \mathbf{n}} + \text{H.c.}],\end{aligned}\quad (1)$$

where $J^\mu(\mathbf{n}) = g\bar{\psi}(\mathbf{n})\gamma^\mu\psi(\mathbf{n})$ is the electric current, $\hat{E}_{\mathbf{p}} \equiv \sqrt{\frac{4}{a^2} \sum_i \sin^2(\frac{p_i a}{2})}$ is the dispersion relation of the photon on the lattice with lattice spacing a , and M is the number of lattice sites in each special dimension. On the lattice of momentum space, the momentum $\mathbf{p}$ can be evaluated as $\mathbf{p} = (2\pi k_x/Ma, 2\pi k_y/Ma, 2\pi k_z/Ma)$, with $k_x, k_y, k_z = 0, \pm 1, \dots, \pm[M/2]$. The unitary gauge link, given by $U_i(\mathbf{n}) = \exp(-igA_i(\mathbf{n}))$, fulfills requirement (2) in the Introduction. $\hat{e}_i^r(\mathbf{p})$ in Eq. (1) are polarization vectors of the photon, which need to be determined by solving

$$\begin{aligned}\sum_i (e^{ip_i a} - 1) \hat{e}_i^r(\mathbf{p}) &= 0, \\ \sum_i \hat{e}_i^r(\mathbf{p}) \hat{e}_i^s(\mathbf{p}) &= \delta_{rs}, \\ \sum_r \hat{e}_i^r(\mathbf{p}) \hat{e}_j^r(\mathbf{p}) &= \delta_{ij} - \frac{1}{\hat{E}_{\mathbf{p}}^2} (e^{-ip_i a} - 1)(e^{ip_i a} - 1).\end{aligned}\quad (2)$$

Once the solution of $\hat{e}_i^r(\mathbf{p})$ is obtained, Gauss’s law and CG condition will be satisfied automatically. It is worth

noting that both $\hat{A}_0$ and $\hat{e}_i^r(\mathbf{p})$ do not depend on the truncation of photon Fock states. This means that all constraints, including Gauss's law and the CG condition, will be preserved accurately after the digitization of gauge fields, which is important for simulating gauge field theories on quantum computers, and fulfill requirement (4) in our Introduction.

After discretizations of fields, the Hamiltonian of CG QED is given by $\hat{H} = \hat{H}_E + \hat{H}_B + \hat{H}_I + \hat{H}_V + \hat{H}_M + \hat{H}_W$ (we set $a = 1$ here):

$$\begin{aligned}\hat{H}_E + \hat{H}_B &= \sum_{\mathbf{p} \neq 0} \sum_r \hat{E}_{\mathbf{p}} a_{\mathbf{p}}^{r\dagger} a_{\mathbf{p}}^r, \\ \hat{H}_I &= \sum_{\mathbf{n}, i} \sum_{\mathbf{p} \neq 0} \sum_r \frac{J^i(\mathbf{n})}{M^{\frac{3}{2}} \sqrt{2\hat{E}_{\mathbf{p}}}} [\hat{e}_i^r(\mathbf{p}) a_{\mathbf{p}}^r e^{i\mathbf{p} \cdot \mathbf{n}} + \text{H.c.}], \\ \hat{H}_V &= \frac{1}{2} \sum_{\mathbf{n}, \mathbf{m}} \sum_{\mathbf{p} \neq 0} \frac{J^0(\mathbf{n}) J^0(\mathbf{m})}{\hat{E}_{\mathbf{p}}} e^{-i\mathbf{p} \cdot (\mathbf{n} - \mathbf{m})}, \\ \hat{H}_M &= \sum_{\mathbf{n}, i} \bar{\psi}(\mathbf{x}) \left[-i\gamma^i \frac{\psi(\mathbf{n} + \hat{i}) - \psi(\mathbf{n} - \hat{i})}{2} + m\bar{\psi}\psi \right], \\ \hat{H}_W &= \sum_{\mathbf{n}} -\frac{w}{2} \bar{\psi}(\mathbf{n}) \hat{\Delta} \psi(\mathbf{n}),\end{aligned}\quad (3)$$

where $\hat{H}_W$ comes from discretizing the fermion field by using the Wilson fermion method, and the parameter w needs to satisfy $0 < w < 1$.

III. MAPPING FIELDS TO QUBITS

Both fermion fields $\hat{\psi}(\mathbf{n})$ and photon annihilation operators $a_{\mathbf{p}}^r$ need to be mapped to qubits. In Fig. 1, we assume K qubits are used to represent Hilbert space $\mathcal{H}_{\mathbf{p},r}^{\text{ph}}$ of photons with momentum $\mathbf{p}$ and helicity r . Then, the entire Hilbert space of photons can be written as $\mathcal{H}^{\text{ph}} = \bigotimes_{\mathbf{p},r} \mathcal{H}_{\mathbf{p},r}$, where $\bigotimes$ denotes the direct product. The computational basis of $\mathcal{H}_{\mathbf{p},r}^{\text{ph}}$ can be written as $|\mathcal{N}\rangle_{\mathbf{p},r} \equiv |i_0 i_1 \dots i_{K-1}\rangle_{\mathbf{p},r}$, where $\mathcal{N}$ is the decimal numeral corresponding to the binary number $i_0 i_1, \dots, i_{K-1}$, which also can be denoted as $\mathcal{N}(i_{K-1}, \dots, i_0)$. As shown in Fig. 1, three lower indices $(J, \mathbf{p}, r)$, $J = 0, 1, \dots, K-1$ are used to label the qubits in the state $|\mathcal{N}\rangle_{\mathbf{p},r}$. $a_{\mathbf{p}}^r$ can raise and lower the state $|\mathcal{N}\rangle_{\mathbf{p},r}$

$$\begin{aligned}a_{\mathbf{p}}^r |\mathcal{N}\rangle_{\mathbf{p},r} &= \sqrt{\mathcal{N}} |\mathcal{N} - 1\rangle_{\mathbf{p},r}, \\ a_{\mathbf{p}}^{r\dagger} |\mathcal{N}\rangle_{\mathbf{p},r} &= \sqrt{\mathcal{N} + 1} |\mathcal{N} + 1\rangle_{\mathbf{p},r}.\end{aligned}\quad (4)$$

To satisfy Eq. (4) the qubit representations of $a_{\mathbf{p}}^r$ and $a_{\mathbf{p}}^{r\dagger}$ need to be written as

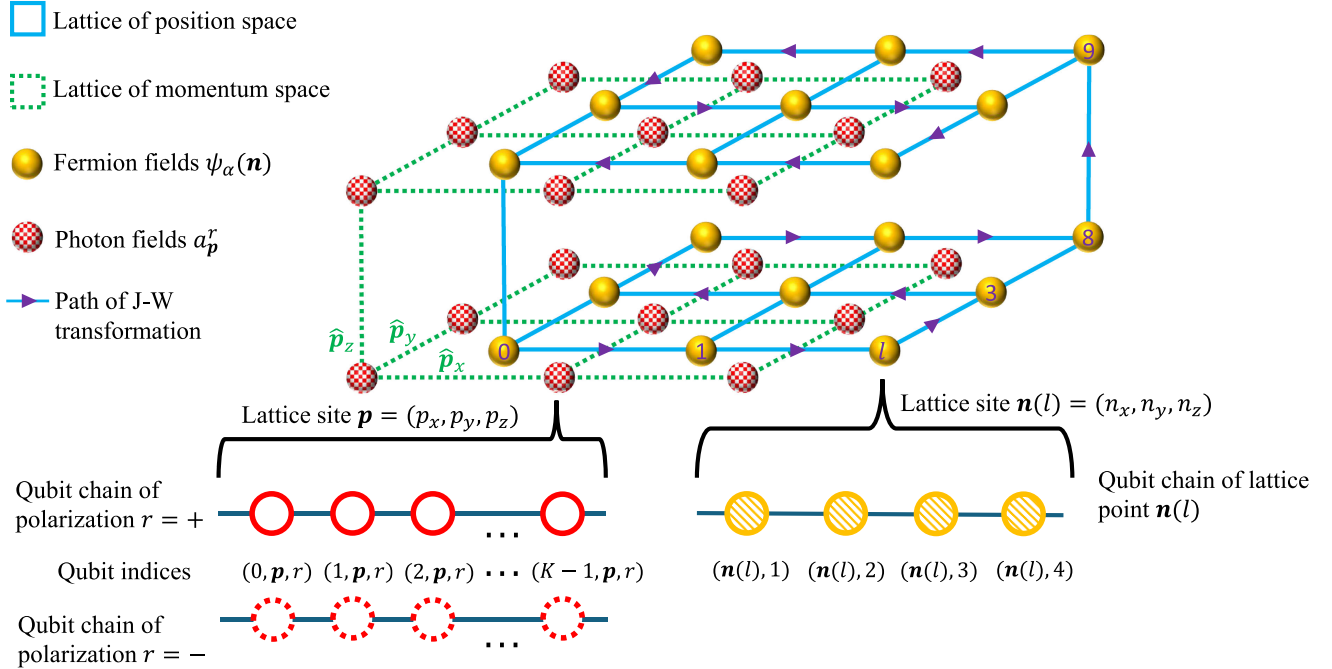


FIG. 1. The $(3 + 1)$ -dimensional lattice of position space (blue) and momentum space (green) is depicted. Fermion fields $\psi_{\alpha}(\mathbf{n})$ and photon fields $a_{\mathbf{p}}^r$ are placed on the position space lattice and the momentum space lattice, respectively. Under the choice of Fock state truncation $\Lambda = 2^K - 1$, $2K$ qubits need to be used to represent the photon fields at each point $\mathbf{p}$ on the momentum lattice. These $2K$ qubits are labeled by three indices $(J, \mathbf{p}, r)$, where $J = 0, 1, 2, \dots, K-1$. Four qubits, labeled by $(\mathbf{n}, \alpha)$, are used to represent the fermion fields $\psi_{\alpha}(\mathbf{n})$ at lattice point $\mathbf{n}$. The purple arrows show the path of the Jordan-Wigner transformation, while the purple numbers $0, 1, \dots, l, \dots$ are the parameters of the path.

$$a_{\mathbf{p}}^r = \left\{ \sum_{J=0}^{K-1} \left[\sigma_{J,\mathbf{p},r}^+ \left(\prod_{L=0}^{J-1} \sigma_{L,\mathbf{p},r}^- \right) \right] \right\} \sqrt{\hat{\mathcal{N}}_{\mathbf{p},r}},$$

$$a_{\mathbf{p}}^{r\dagger} = \sqrt{\hat{\mathcal{N}}_{\mathbf{p},r}} \sum_{J=0}^{K-1} \left[\sigma_{J,\mathbf{p},r}^- \left(\prod_{L=0}^{J-1} \sigma_{L,\mathbf{p},r}^+ \right) \right], \quad (5)$$

where $\sigma^+ = \frac{1}{2}(\sigma^1 + i\sigma^2)$ and $\hat{\mathcal{N}}_{\mathbf{p},r}$ is the particle number operator of the photon with quantum numbers $(\mathbf{p}, r)$:

$$\hat{\mathcal{N}}_{\mathbf{p},r} = \sum_{J=0}^{K-1} 2^J \left[\frac{1}{2} (I - \sigma_{J,\mathbf{p},r}^3) \right]. \quad (6)$$

Here, the Pauli operators I, σ^x, σ^y , and σ^z are denoted as $\sigma^0, \sigma^1, \sigma^2$, and σ^3 . In Eq. (5), the Fock states with photon occupation numbers greater than $\Lambda = 2^K - 1$ need to be truncated, as $a_{\mathbf{p}}^{r\dagger} |\Lambda\rangle = 0$ due to the finite-dimensional Hilbert space of a quantum computer. The square root of $\hat{\mathcal{N}}_{\mathbf{p},r}$ is given by

$$\sqrt{\hat{\mathcal{N}}_{\mathbf{p},r}} = \sum_{\mu_0, \dots, \mu_{K-1}=0,3} f_{\mu_{K-1}, \dots, \mu_0} \sigma_{K-1,\mathbf{p},r}^{\mu_{K-1}} \otimes \dots \otimes \sigma_{0,\mathbf{p},r}^{\mu_0}, \quad (7)$$

with

$$f_{\mu_{K-1}, \dots, \mu_0} = \frac{1}{2^K} \sum_{i_0, \dots, i_{K-1}=0}^1 \sqrt{\mathcal{N}(i_{K-1}, \dots, i_0)} \times (-1)^{i_{K-1}\mu_{K-1}} \times \dots \times (-1)^{i_0\mu_0}. \quad (8)$$

We point out that Eqs. (7) and (8) can be generalized to other functions $F(\hat{\mathcal{N}}_{\mathbf{p},r})$ of the particle-number operator. To achieve this, we only need to replace $\sqrt{\mathcal{N}(i_{K-1}, \dots, i_0)}$ in Eq. (8) by $F(\mathcal{N}(i_{K-1}, \dots, i_0))$. The successful mapping of $a_{\mathbf{p}}^{r\dagger}$ to qubits implies that we can effectively handle photons with specific polarization and momentum on quantum computers, thereby satisfying requirement (3) outlined in the Introduction.

The fermion fields $\hat{\psi}_\alpha(\mathbf{n})$ can be mapped to qubits by the Jordan-Wigner (J-W) transformation [75],

$$\psi_\alpha(\mathbf{n}) = \left[\prod_{l' < l} \left(\prod_{\beta=1}^4 \sigma_{\mathbf{m}(l'),\beta}^z \right) \right] \left(\prod_{\beta=1}^{\alpha-1} \sigma_{\mathbf{n}(l),\beta}^z \right) \times \sigma_{\mathbf{n}(l),\alpha}^+, \quad (9)$$

where l is the path parameter of the J-W transformation (see Fig. 1). We use the lower indices $(\mathbf{n}, \alpha)$ to label the qubits representing the fermion degrees of freedom. We can see that the J-W transformation is highly nonlocal. However, the fermion fields always appear as $\bar{\psi}\psi$ in the position space. Therefore, putting the fermion field on the lattice of position space instead of momentum space can significantly reduce the nonlocality of the qubit Hamiltonian. Now, both fermion and gauge fields are mapped to the qubits, and the whole Hilbert of QED can be written as $\mathcal{H} = H^{\text{ph}} \otimes H^{\text{f}}$, where H^{f} is the Hilbert of fermions.

IV. COMPLEXITY

Let us begin with the estimation of qubit scaling. Consider a d -dimensional spatial lattice system with M^d lattice sites on $(d+1)$ -dimensional spacetime. Fermion fields with N_s spinor components and N_f flavors are discretized on the lattice system. Then, $N_f N_s M^d$ qubits need to be used to encode the fermion fields. For example, the blue lattice in Fig. 1 shows the case with $N_f = 1$, $N_s = 4$, and $d = 3$. Photon fields on this lattice have $d-1$ different transverse directions. Therefore, $(d-1) \times \log_2 \Lambda \times M^d$ are needed to encode the photon fields. The case of $d-1 = 2$ and $\Lambda = 2^K - 1$ can be found in the green lattice of Fig. 1. The total qubit cost should sum over both the qubit cost of encoding fermion fields and photon fields:

$$N_{\text{CG}}^{\text{q,F}} + N_{\text{CG}}^{\text{q,G}} = [N_f N_s + (d-1) \log_2 \Lambda] M^d, \quad (10)$$

where $N_{\text{CG}}^{\text{q,F}}$ and $N_{\text{CG}}^{\text{q,G}}$ are the qubit cost of encoding the fermion fields and gauge fields in our formalism.

In Eq. (10), the cutoff Λ should be further estimated. Suppose the physics we are concerned about has an energy scale $E = \langle \Psi | H | \Psi \rangle$. If we want to control the Fock state truncation error of the arbitrary state $|\Psi\rangle$ under ε_s , the truncation Λ should be no more than Λ_{max} :

$$\Lambda \leq \frac{(d-1)M^d}{2\varepsilon_s} \left\{ \left[\frac{8dg}{(2\pi)^{\frac{3}{2}}} M^{\frac{d+3}{2}} + \left(\frac{8d^2g^2}{\pi^3} M^{d+3} + \frac{[3+4d^2(d-1)]g^2}{8\pi^3} M^{2d+3} + \frac{2}{\pi} \sqrt{3+m^2+12mw+36w^2} M^{d+1} + \frac{E}{\pi} M \right)^{\frac{1}{2}} \right]^2 - 1 \right\} \equiv \Lambda_{\text{max}}. \quad (11)$$

The proof of this inequality can be found in the Supplemental Material [74], and the procedure of proof is similar to [76,77]. From Eq. (11), we find that Λ depends on M, g, E, m, w , and ε_s polynomially, which shows our formalism can reach the continuum limit of QED efficiently as well as satisfy requirement (1). Equation (11) shows that the worst scaling behavior depends on ε_s because we consider an arbitrary state $|\Psi\rangle$ in the Hilbert space. If we consider a specific case, such as the ground state of the Hamiltonian, the truncated state will converge to the untruncated state exponentially (see Supplemental Material [74]). To make Eq. (11) look simpler, we can consider the $m = 0$ case and keep the leading term in the continuum limit $M \rightarrow \infty$:

$$\Lambda \sim O\left(\frac{g^2 M^{3d+3} + EM^{d+1}}{\varepsilon_s}\right). \quad (12)$$

From Eq. (12), we find that in the continuum limit, the $N_{\text{CG}}^{\text{q,G}}$ scales as

$$N_{\text{CG}}^{\text{q,G}} \sim (d-1)M^d \left\{ \log_2 \frac{1}{\varepsilon_s} + \log_2 [O(g^2 M^{3d+3} + EM^{d+1})] \right\}. \quad (13)$$

The complexity of simulating time evolution also needs to be estimated. For a given precision ε and a Hamiltonian H with a sum over $\mathcal{M}_G$ different terms, the time evolution operator $e^{-i\hat{H}t}$ can be approximated by the second-order Suzuki formula [78]:

$$e^{-i\hat{H}t} \approx \left[\prod_{\alpha=\mathcal{M}_G} \exp\left(-ih_\alpha \frac{\delta t}{2}\right) \prod_{\alpha=1}^{\mathcal{M}_G} \exp\left(-ih_\alpha \frac{\delta t}{2}\right) \right]^{N_T} + \mathcal{O}\left(\varepsilon = \frac{t^3}{N_T^2}\right), \quad (14)$$

where $\delta t = t/N_T$ and ε is the error of the trotter algorithm. The h_α is a Pauli string multiplied by a coefficient, so the evolution operator $\exp(-ih_\alpha \delta t)$ can be decomposed into one-qubit and CNOT gates (see Supplemental Material [74]). To estimate the gate cost of simulating $\exp(-i\hat{H}\delta t/2)$, we map the Hamiltonian of CG QED to qubits explicitly and give the qubit Hamiltonian in the Supplemental Material [74]. After some calculations (see Supplemental Material [74]), we obtain the one-qubit and CNOT gate cost of simulating $\exp[-i(\hat{H}_E + \hat{H}_B)\delta t/2]$:

$$\begin{aligned} N_{\text{OQ}}^{G,EB} &= (d-1)M^d \log_2 \Lambda, \\ N_{\text{CNOT}}^{G,EB} &= 0. \end{aligned} \quad (15)$$

The most complex evolution operator is $\exp(-i\hat{H}_I\delta t/2)$:

$$\begin{aligned} N_{\text{OQ}}^{G,I} &= 4(d-1)M^{2d} [d\Lambda(\log_2 \Lambda)(\log_2 \Lambda + 5) \\ &\quad - 4\Lambda \log_2 \Lambda], \\ N_{\text{CNOT}}^{G,I} &= 2(d-1)M^{2d} [d\Lambda(\log_2 \Lambda)(3\log_2 \Lambda + 5) \\ &\quad - 4\Lambda \log_2 \Lambda]. \end{aligned} \quad (16)$$

Then, we need to estimate the gate cost to simulate $\exp(-i\hat{H}_V\delta t/2)$:

$$\begin{aligned} N_{\text{OQ}}^{G,V} &= N_s M^{2d}, \\ N_{\text{CNOT}}^{G,V} &= 2N_s M^d (N_s M^d - 1). \end{aligned} \quad (17)$$

The gate cost of simulating $\exp(-i\hat{H}_M\delta t/2)$ is

$$\begin{aligned} N_{\text{OQ}}^{G,M}(d=2) &= 72M^2, \\ N_{\text{OQ}}^{G,M}(d=3) &= 104M^3, \\ N_{\text{CNOT}}^{G,M}(d=2) &= 16N_s M^2(1+M) + 8M^2, \\ N_{\text{CNOT}}^{G,M}(d=3) &= 16N_s M^3(1+M+M^2) + 8M^3, \end{aligned} \quad (18)$$

where $N_s = 4$ in both $d=2$ and $d=3$ and we only consider the $N_f = 1$ case here. The gate cost of simulating $\exp(-i\hat{H}_W\delta t/2)$ is the same as $N_{\text{OQ}}^{G,M}$ and $N_{\text{CNOT}}^{G,M}$. Finally, we can obtain $e^{-i\hat{H}t}$ by executing $N_T = t^{\frac{3}{2}}/\varepsilon^{\frac{1}{2}}$ times of $e^{-i\hat{H}\delta t}$, so the complexity of simulating $e^{-i\hat{H}t}$ is

$$\begin{aligned} N_{\text{CG,OQ}}^G &= \frac{t^{3/2}}{\varepsilon^{1/2}} (N_{\text{OQ}}^{G,EB} + N_{\text{OQ}}^{G,I} + N_{\text{OQ}}^{G,V} + N_{\text{OQ}}^{G,M} + N_{\text{OQ}}^{G,W}), \\ N_{\text{CG,CNOT}}^G &= \frac{t^{3/2}}{\varepsilon^{1/2}} (N_{\text{CNOT}}^{G,EB} + N_{\text{CNOT}}^{G,I} \\ &\quad + N_{\text{CNOT}}^{G,V} + N_{\text{CNOT}}^{G,M} + N_{\text{CNOT}}^{G,W}). \end{aligned} \quad (19)$$

The leading term of N_{CG}^G in the $M \rightarrow \infty$ and $\Lambda \rightarrow \infty$ limit is

$$N_{\text{CG}}^G \sim O\left(\frac{t^{3/2}}{\varepsilon^{1/2}} M^{2d} \Lambda (\log_2 \Lambda)^2\right). \quad (20)$$

In the K-S formalism, the leading term of the gate cost for simulating $e^{-iH_{\text{KS}}t}$ of U(1) gauge theory is [79]:

$$N_{\text{KS}}^G \sim O\left(\frac{t^{3/2}}{\varepsilon^{1/2}} M^{3d/2} \Lambda_{\text{KS}} (\log_2 \Lambda_{\text{KS}})^2\right), \quad (21)$$

where Λ_{KS} is the Hilbert space truncation of K-S formalism in some basis, which is different from our truncation Λ . Compared with K-S formalism, our formalism needs an additional gate cost that is proportional to $M^{d/2}$ to simulate time evolution. This additional gate cost comes from the nonlocal interactions $\hat{H}_I$ in our formalism.

V. RESULTS

To test the performance of our formalism, we simulate $(2+1)$ -dimensional pure U(1) gauge theory on a classical simulator. We first calculate the vacuum expectation value (VEV) of $\langle U_p(\mathbf{n}) \rangle$, where U_p is the plaquette operator of U(1) gauge theory:

$$U_p(\mathbf{n}) = U_i(\mathbf{n})U_j(\mathbf{n}+\hat{i})U_i^\dagger(\mathbf{n}+\hat{j})U_j^\dagger(\mathbf{n}), \quad (22)$$

with $U_i(\mathbf{n}) = \exp(-ig\hat{A}_i(\mathbf{n}))$. Because of the translation invariant of the vacuum state, the VEV $\langle U_p(\mathbf{n}) \rangle$ is independent of $\mathbf{n}$. Therefore, the $\langle U_p(\mathbf{n}) \rangle$ can be written as $\langle U_p \rangle$. In Fig. 2(a), we show the g dependence of $\langle U_p \rangle$, with different lattice volume $M_x \times M_y$ and a given Fock state cutoff $\Lambda = 1$. The points are our results, the black

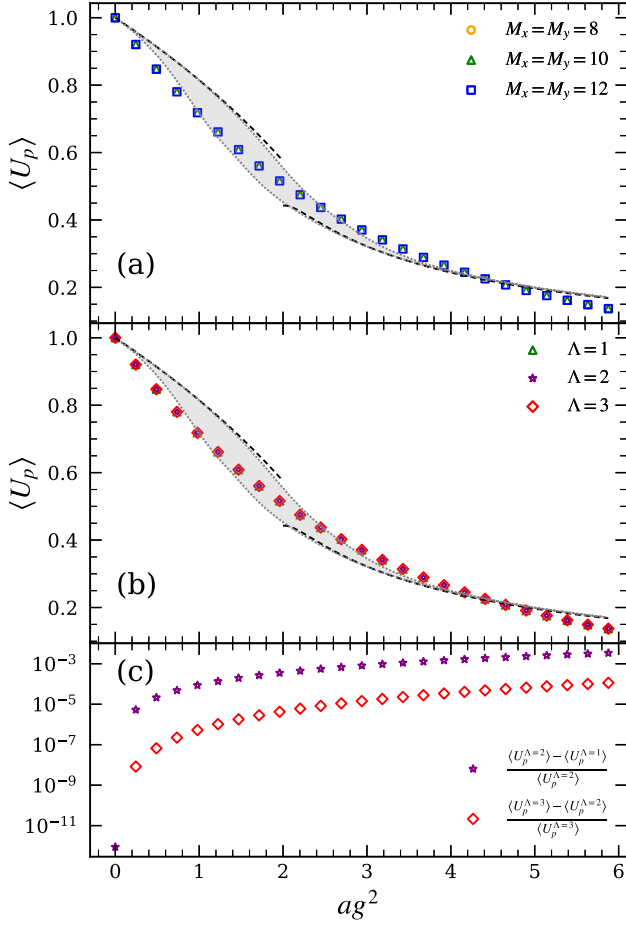


FIG. 2. (a) $\langle U_p \rangle$ dependence on coupling constant g with different lattice volumes $M_x \times M_y$ and a given Fock state cutoff $\Lambda = 1$. The points are our results, and the black dotted lines are the weak and strong coupling expansion results in [80,81]. The gray region is the bootstrap allowed region that was presented in [82]. (b) $\langle U_p \rangle$ dependence on coupling constant g with different Fock state cutoffs Λ and a given lattice volume $M_x = M_y = 10$. (c) Relative errors of $\langle U_p \rangle$ for different cutoffs. We set $a = 1$ in all of those subfigures.

dotted lines are the analytic weak and strong coupling expansion results in [80,81], and the gray region is the bootstrap allowed region of $\langle U_p \rangle$ that is presented in [82]. We find that our result agrees with those results qualitatively, but there are also some minor deviations between them. This is because our result is calculated by non-compact QED, while their result is obtained by compact QED. We also test the Fock state cutoff Λ dependence of $\langle U_p \rangle$ in Figs. 2(b) and 2(c), with a given lattice volume $M_x = M_y = 10$. It is shown in Fig. 2(c) that the precision of the $\Lambda = 2$ result, which only uses three states per lattice site, can reach more than 10^{-6} in the $g^2 a < 1$ region.

We also calculated the long-time behavior of the Wilson loop. The Wilson loop in the $(2 + 1)$ -dimensional CG QED can be written as

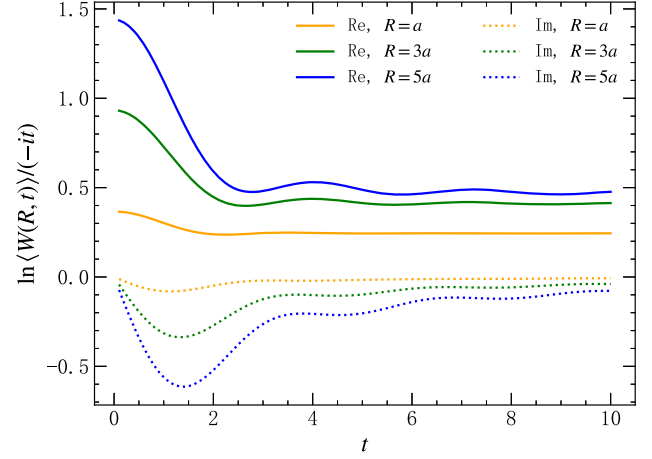


FIG. 3. The real part (solid line) and the imaginary part (dotted line) of $\frac{1}{-it} \ln \langle W(R, t) \rangle$, with $a = 1, M_x = M_y = 16, \Lambda = 4$, and $g = 0.7$.

$$W(R, t) = [e^{iga_t \hat{A}_0(0)} e^{i\hat{H}a_t}]^{M_t} \left\{ \prod_{s=1}^R U_i^\dagger((R-s)\hat{i}) \right\} \times [e^{-i\hat{H}a_t} e^{-iga_t \hat{A}_0(R\hat{i})}]^{M_t} \left[\prod_{s=0}^{R-1} U_i(s\hat{i}) \right]. \quad (23)$$

$i = 1, 2$ will give the same result because of the rotational symmetry of the CG. Considering a pair of static positron and electron that stay on the positions $\mathbf{n}_+$ and $\mathbf{n}_-$, then the gauge potential $\hat{A}_0(\mathbf{n})$ is given by

$$\hat{A}_0(\mathbf{n}; \mathbf{n}_+, \mathbf{n}_-) = \sum_{\mathbf{p} \neq 0} \frac{g}{\hat{E}_{\mathbf{p}}^2} (e^{-i\mathbf{p} \cdot (\mathbf{n} - \mathbf{n}_+)} - e^{-i\mathbf{p} \cdot (\mathbf{n} - \mathbf{n}_-)}). \quad (24)$$

Then, we can obtain the Wilson loop related to a pair of static electric positron sources by placing $\hat{A}_0(0) = \hat{A}_0(0; R\hat{i}, 0)$

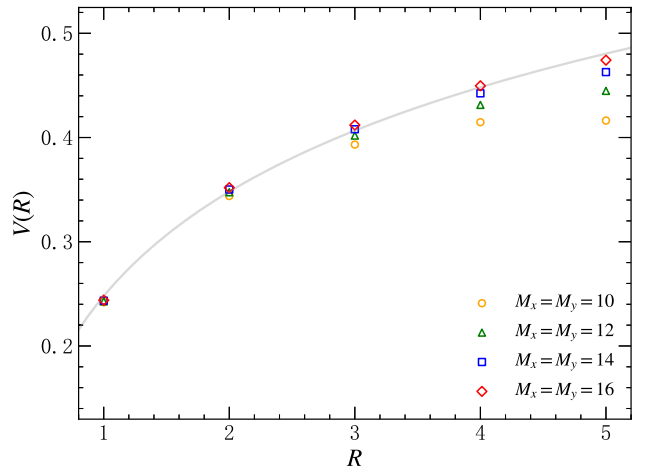


FIG. 4. The static potential extracted from Eq. (25) in different lattice volume $M_x \times M_y$, with fixing $g = 0.7, a = 1$, and $\Lambda = 3$. The point data come from the lattice calculation, while the gray line comes from fitting the $M_x = M_y = 16$ data using $V(R) = a_V \log R + b_V$. The fitting result is $a_V = 0.1445$ and $b_V = 0.2479$.

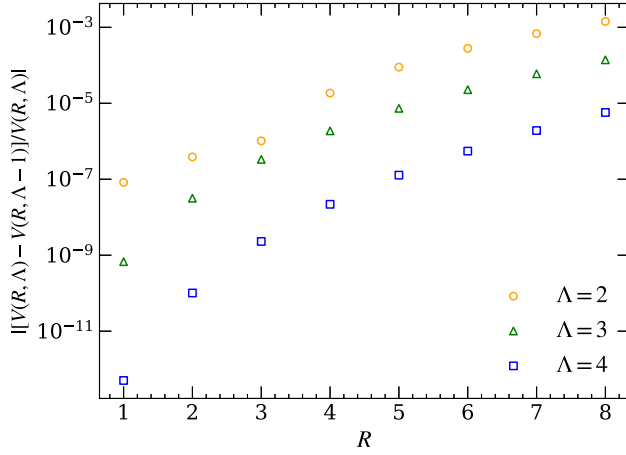


FIG. 5. The Fock state truncation Λ dependence of the static potential, with fixing $M_x = M_y = 16$, $g = 0.7$, and $a = 1$.

and $\hat{A}_0(R\hat{i}) = \hat{A}_0(R\hat{i}; R\hat{i}, \mathbf{0})$ in the Eq. (23). According to potential NRQCD [83], the static potential $V(r)$ can be extracted by calculating the logarithm of the VEV of the Wilson loop $\langle W(R, t) \rangle$:

$$\frac{1}{-it} \ln \langle W(R, t) \rangle = V(R) + O\left(\frac{1}{t}\right). \quad (25)$$

It is well known that the electron-positron pair has a logarithmic potential in $(2+1)$ -dimensional noncompact QED. So for a given R , the real part of $\frac{1}{-it} \ln \langle W(R, t) \rangle$ will converge to a nonzero real number, and the imaginary part will tend to zero in the long-time limit. That is what we have seen in Fig. 3. In Fig. 4, we show the static potential extracted from the long-time Wilson loop in different lattice volumes. We find that our result can be fitted by a logarithmic function, which is consistent with the potential in $(2+1)$ -dimensional QED. Finally, we test the Λ dependence of $V(R, \Lambda)$ in Fig. 5. We find that when $\Lambda = 3$, the relative error of $V(R < 5)$ is less than 10^{-5} , which shows the rationality of choosing $\Lambda = 3$ in Fig. 4.

VI. SUMMARY AND OUTLOOK

In this study, based on the CG, we propose a new formalism for simulating LGT on a quantum computer,

encompassing the discretization of fields and the mapping of these fields to qubits. As a preliminary exploration, we discuss the quantum simulations of QED. We discretize the gauge fields directly in our formalism. This formalism preserves Gauss's law and the CG condition by solving the lattice photon polarization vectors either numerically or analytically. One significant advantage of this approach is its efficiency in simulating hadron scattering processes on future fault-tolerant quantum computers. We also discuss the complexity of our formalism, including the qubit cost of encoding fields and the gate cost of simulating time evolution. We also investigate the convergence behavior in the limit $\Lambda \rightarrow \infty$ of our formalism. Compared with K-S formalism, our formalism costs more gates to simulate time evolution. Finally, the results for $\langle U_p \rangle$, the long-time behavior of the Wilson loop, and the static potential $V(R)$ validate the rationality of our formalism.

Extending this formalism to non-Abelian cases presents another intriguing area of investigation. The Hamiltonian for lattice $SU(N)$ gauge theory in the CG has been discussed in [84]. However, unlike the Abelian case, the CG in non-Abelian theories cannot eliminate all residual degrees of freedom due to the presence of Gribov ambiguity [85]. Consequently, addressing Gribov ambiguity on quantum computers constitutes a primary challenge for extending our formalism to non-Abelian gauge theories. Another challenge involves solving Gauss's law for non-Abelian gauge theories in the CG framework. Strategies for tackling these two challenges will be explored in our future work.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [86].

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- Material. We show the calculations of the truncation error for the photon Fock states in Sec. S3. Finally, in Sec. S4, we present the qubit Hamiltonian of CG QED and the derivation process of the gate cost for simulating the time evolution operator generated by the QED Hamiltonian in CG.
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