

Energy-energy correlator from the AdS Virasoro-Shapiro amplitude

Lecheng Ren (任乐成)^{1,*} Bo Wang (王波)^{2,†} and Congkao Wen (温从烤)^{1,‡}

¹*Centre for Theoretical Physics, Department of Physics and Astronomy,
Queen Mary University of London, London E1 4NS, United Kingdom*

²*Zhejiang Institute of Modern Physics, School of Physics, Zhejiang University,
Hangzhou, Zhejiang 310058, China*



(Received 22 January 2026; accepted 27 March 2026; published 12 May 2026)

We establish a precise formula relating the world sheet integral of the anti-de Sitter (AdS) Virasoro-Shapiro amplitude to the energy-energy correlator (EEC) in $\mathcal{N} = 4$ super Yang-Mills theory at strong coupling. This mapping allows us to evaluate the coefficients of the AdS curvature expansion of the EEC in terms of the world sheet integral over a unit disk. To illustrate this idea, we explicitly compute the flat-space contribution and the first curvature correction to the EEC. Our results provide a rigorous description of the stringy energy flow, demonstrating how world sheet correlators imprint themselves on collider observables and offering a potential template for effective string descriptions of energy correlators in general gauge theories.

DOI: [10.1103/m9q6-47z2](https://doi.org/10.1103/m9q6-47z2)

I. INTRODUCTION

Understanding the flow of energy in quantum field theory is a fundamental endeavor that bridges the gap between formal correlation functions and physical detectors. In the context of QCD, energy-energy correlators (EECs) have emerged as powerful observables for probing the dynamics of the strong interaction [1,2] and the internal structure of jets [3,4] (see [5] for a recent comprehensive review). By weighting the cross section with the energy of detected particles, the EEC offers an infrared-safe probe of the theory's asymptotic states. Formally, it is defined in terms of the correlation of light-ray operators [6,7]. However, the complicated operator structure, running coupling effects and confinement inherent to QCD make its nonperturbative structure difficult to analyze.

$\mathcal{N} = 4$ super Yang-Mills theory (SYM) serves as the ideal laboratory for this endeavor. Despite being conformal, it captures the physical evolution of energy flow analogous to QCD. By tuning the 't Hooft coupling, one observes a transition from collimated “jets” to a uniform distribution. At weak coupling, the EEC mirrors perturbative QCD [8–11] with power-law singularities from light-ray operator product expansions [7,12–18]. Conversely, at strong coupling, due to

AdS/CFT [19–21] it is described by the supergravity approximation and becomes flat [22].

Despite our control over the weak- and infinite-coupling limits, the intermediate regime remains elusive (see [23] for recent work in this direction). This is the domain where the “stringy” nature of the theory is manifest, which is dual to the finite-size effects of strings. Understanding how these stringy features imprint themselves on the angular distribution of energy is a crucial step toward connecting the pointlike descriptions of perturbative scattering with the extended objects of holographic models. Recent breakthroughs in bootstrapping the AdS Virasoro-Shapiro amplitude [24,25] permit a precise analysis of the stringy dynamics.

In this paper, we present a rigorous derivation of the stringy energy flow from a world sheet point of view. We establish a direct mapping between the world sheet dynamics and the statistical energy correlations on the boundary. By transforming the AdS Virasoro-Shapiro amplitude from Mellin space to the detector sphere, we express the coefficients of the EEC in the AdS curvature expansion as integrals over the string moduli space. This result provides an explicit dictionary for translating world sheet resonances into observable angular patterns. As a concrete application, we compute the flat-space limit and the first curvature correction, offering a prototype for constructing effective string descriptions of energy flow applicable to QCD-like theories.

II. ENERGY-ENERGY CORRELATOR FROM LOCAL CORRELATOR

Consider a scattering process created by some source operator $\mathcal{O}$ with total momentum q^μ , the EEC measures the energy flux correlation between two detectors, defined as

* Contact author: lecheng.ren@qmul.ac.uk

† Contact author: b_w@zju.edu.cn

‡ Contact author: c.wen@qmul.ac.uk

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

$$\text{EEC}(\xi) = \frac{8\pi^2}{q^2 \sigma_0} \int d^4 x_{13} e^{iq \cdot x_{13}} \langle \mathcal{O}(x_1) \mathcal{E}(n_2) \mathcal{E}(n_4) \mathcal{O}(x_3) \rangle,$$

where $x_{ij} = x_i - x_j$ and σ_0 is the total cross section. The energy flow operator is given by the integral of the stress tensor along null infinity,

$$\mathcal{E}(\vec{n}) = \lim_{r \rightarrow \infty} r^2 \int_0^\infty dt n_i T_{0i}(t, r\hat{n}). \quad (1)$$

Here, $\mathcal{E}$ represents the energy-flow operator placed at null infinity in the direction specified by the unit vector $n = (1, \hat{n})$ and $T_{\mu\nu}$ is the stress tensor. The correlator depends on a single cross ratio ξ related to the angle θ between two detectors, in the rest frame of the source we have $\xi = \sin^2(\theta/2)$.

For $\mathcal{N} = 4$ SYM, the EEC can be related to a four-point function of superconformal primary operators of dimension two (denoted $\mathcal{O}_{20'}$ in the literature). Crucially, these operators belong to the same supermultiplet as a stress tensor, which makes it possible to relate their correlation function to the EEC. A key step is to Wick rotate the Euclidean correlation function—where most explicit computations in the literature have been performed—to the corresponding Wightman function. This procedure leads to the Mellin representation of the energy-energy correlator established in [9,10],

$$\text{EEC}(\xi) = \int_{-i\infty}^{+i\infty} \frac{ds}{2\pi i} \frac{dt}{2\pi i} M(s, t) K(t, \xi), \quad (2)$$

where the kernel is given by

$$K(t, \xi) = -\frac{\pi t^2 (t-2)^2}{128 \xi^3 \sin(\pi t/2)} \left(\frac{\xi}{1-\xi} \right)^{1+\frac{t}{2}}. \quad (3)$$

We can choose the contour between $1 < \text{Re}(s), \text{Re}(t) < 2$. $M(s, t)$ denotes the Mellin amplitude of the four-point function $\mathcal{H}(u, v)$ of $\mathcal{O}_{20'}$'s,

$$\mathcal{H}(u, v) = \int_{-i\infty}^{+i\infty} \frac{ds dt}{(2\pi i)^2} u^s v^t M(s, t) \times \Gamma(2-s/2)^2 \Gamma(2-t/2)^2 \Gamma(2-\tilde{u}/2)^2, \quad (4)$$

where u, v are cross-ratios and $s+t+\tilde{u}=4$. We will focus on the planar limit. In the strong coupling expansion, $M(s, t)$ comprises the supergravity contribution and an infinite series of stringy corrections (see e.g., [26–32]),

$$M(s, t) = \frac{8}{(s-2)(t-2)(\tilde{u}-2)} + \frac{120\zeta_3}{\lambda^{3/2}} + \dots \quad (5)$$

Higher-order corrections consist of symmetric polynomials in the Mellin variables s, t , and $\tilde{u}$. While substituting the leading supergravity term into Eq. (2) yields precisely 1 [22], a naïve term-by-term integration of the higher-order

polynomials leads to divergences in the s -integration. To obtain a well-defined result, we must reorganize the summation and integration. The physical EEC, which is expected to be free of divergence, can be organized in the strongly coupled regime as

$$\text{EEC}(\xi) = 1 + \sum_{k=0}^{\infty} \lambda^{-\frac{k}{2}-1} \sum_{n=0}^k c_{k,n} Q_{k-n}(\xi). \quad (6)$$

Here, the functions $Q_n(\xi)$ carry the dependence on the variable t in the kernel, defined as [33]

$$Q_n(\xi) = \int_{-i\infty}^{+i\infty} \frac{dt}{2\pi i} t^n K(t, \xi). \quad (7)$$

It is easy to note $Q_0(\xi) = 1 - 6\xi + 6\xi^2$, and more general $Q_n(\xi)$ can be determined through a recurrence relation $Q_n(\xi) = \partial_\xi(2(1-\xi)\xi Q_{n-1}(\xi)) - 2(\xi-1)Q_{n-1}(\xi)$.

The validity of the organization in Eq. (6) is rooted in the structure of stringy corrections. The generic form of the stringy corrections to the Mellin amplitude scales as $M(s, t) \sim \lambda^{-\frac{3+k}{2}} \mathcal{P}_k(s, t)$, where $\mathcal{P}_k(s, t)$ is a polynomial of degree k . After integrating out s , the polynomials $\mathcal{P}_k(s, t)$ reduce to polynomials in t . This naturally organizes the expansion; terms with the highest power t^k correspond to the index $n=0$ (i.e., $c_{k,0}$) and are governed strictly by the flat-space limit. Subleading powers t^{k-1} correspond to $n=1$ (i.e., $c_{k,1}$) and receive contributions from the first curvature correction, and so forth. Thus, the index n effectively counts the order in the curvature expansion away from the flat-space limit. Previous studies have determined that $c_{0,0} = 4\pi^2$ [22,26], as well as results for $c_{k,0}$ and specific coefficients such as $c_{1,1}$ obtained recently in [23]. The objective of this paper is to provide a universal algorithm for computing the general coefficients $c_{k,n}$.

A rigorous derivation of these coefficients, which circumvents the ambiguities of the naïve expansion, is achieved by establishing a connection between the EEC and the AdS Virasoro-Shapiro amplitude $A(S, T)$ [24,25]. Since $A(S, T)$ is related to the Mellin amplitude via a Borel transform [34], we obtain

$$\text{EEC}(\xi) = \frac{1}{2\lambda^{3/2}} \int_0^\infty d\beta \int_{-i\infty}^{+i\infty} \frac{ds dt}{(2\pi i)^2} \frac{\beta^5}{e^\beta} A(\beta\hat{s}, \beta\hat{t}) K(t, \xi), \quad (8)$$

where

$$\hat{s} = \frac{s-4/3}{2\sqrt{\lambda}}, \quad \hat{t} = \frac{t-4/3}{2\sqrt{\lambda}}. \quad (9)$$

Recently, it was shown that the AdS Virasoro-Shapiro amplitude is expressed as a simple integral of a world sheet correlator over the Riemann sphere [35],

$$A(S, T) = \int d^2z |z|^{-2S-2} |1-z|^{-2T-2} G^{\text{tot}}(z, \bar{z}; S, T), \quad (10)$$

where the measure reads $d^2z = \frac{dz \wedge d\bar{z}}{-2\pi i}$. The world sheet correlator admits a perturbation series in the curvature expansion,

$$G^{\text{tot}}(z, \bar{z}; S, T) = \sum_{k=0}^{\infty} \lambda^{-\frac{k}{2}} G^{\text{tot},(k)}(z, \bar{z}; S, T), \quad (11)$$

which is bootstrapped up to second order curvature corrections [25,44]. For a general integrand $G^{\text{tot},(k)}$, it is convenient to write down the following decomposition:

$$G^{\text{tot},(k)} = \frac{1}{U^2} G^{(k)}(z; S, T) + \frac{|z|^2}{S^2} G^{(k)}\left(\frac{1}{z}; U, T\right) + \frac{|1-z|^2}{T^2} G^{(k)}\left(\frac{z}{z-1}; S, U\right), \quad (12)$$

where $U = -S - T$. This decomposition renders the crossing symmetry of $A(S, T)$ manifest.

Altogether, Eq. (8) provides a world sheet formulation for the EEC at strong coupling. Taking this expression as our starting point, we will demonstrate that by interchanging the order of integrals, one can systematically derive the expression for the general coefficients $c_{k,n}$.

III. DEALING WITH INTEGRALS

While Eq. (8) involves multiple layers of integration, each component admits a clear physical interpretation. We first recast the expression into the following form:

$$\text{EEC}(\xi) = \frac{1}{\lambda} \int_{-i\infty}^{+i\infty} \frac{dt}{2\pi i} \int_0^\infty d\beta e^{-\beta} \beta^4 I(\beta \hat{t}) K(t, \xi). \quad (13)$$

Here, we have introduced the function $I(T)$, defined as

$$I(T) = \int_{-i\infty}^{+i\infty} \frac{ds}{2\pi i} \frac{\beta}{2\sqrt{\lambda}} A(\beta \hat{s}, \beta \hat{t}) = \int_{-i\infty}^{+i\infty} \frac{dS}{2\pi i} A(S, T), \quad (14)$$

where we have identified $S = \beta \hat{s}$ and $T = \beta \hat{t}$, absorbing the scaling factors appropriately. This reformulation disentangles the specific contributions of each integral. Since $I(T)$ is the central ingredient in computing the EEC, the relation Eq. (14) establishes a direct connection between superstring amplitudes in AdS and the EEC in the dual CFT. For stringy corrections, $I(T)$ can be expanded as a series in $\beta^i t^j$ in the large λ expansion. The β -integral in Eq. (13) then effectively acts as a rescaling, mapping $\beta^i \rightarrow \Gamma(i+5)$ and t^j to the functions $Q_j(\xi)$. This mapping renders the relationship between the EEC and the strong coupling

expansion of $I(T)$ transparent. The amplitude $A(S, T)$ admits a curvature expansion analogous to that of the world-sheet correlator; accordingly, we define $I^{(k)}(T)$ as the integral of the k th order amplitude $A^{(k)}(S, T)$ over S . Finally, the determination of the coefficients $c_{k,n}$ is reduced to the small- T expansion of $I(T)$.

It is worth commenting on the phenomenon of “enhancement” characteristic of the EEC at strong coupling. Previous studies on the leading coefficient $c_{0,0}$ revealed that the result is enhanced to $4\pi^2/\lambda$, exceeding the naïve expectation from leading stringy corrections. Similar enhancements have been observed in higher-order large- N corrections [45]. We identify that this enhancement arises from the S -integration within $I(T)$. The Jacobian factor $\sqrt{\lambda}$, emerging from the transformation between Mellin variables and flat-space parameters, increases the stringy correction from order $\lambda^{-3/2}$ to λ^{-1} . This mechanism is universal across the entire stringy correction sector.

We now turn to the evaluation of the world-sheet and S -integrals. A significant simplification is achieved by interchanging the order of integration. The generic integral of interest takes the following form:

$$\mathcal{J}_j(T) = \int \frac{dS}{2\pi i} \int d^2z S^j |z|^{-2S-2} J(z, \bar{z}; T), \quad (15)$$

where $J(z, \bar{z}; T)$ collects the remaining parts of the integrand of the AdS Virasoro-Shapiro amplitude. Note that terms involving a U pole can be mapped to the same form via the shift $S \rightarrow S - T$. We evaluate the integral depending on whether the integer j is negative or non-negative.

Case 1: $j < 0$. We can employ the Schwinger parametrization,

$$S^j = \frac{1}{\Gamma(-j)} \int_0^\infty d\alpha \alpha^{-j-1} e^{-\alpha S}.$$

Substituting this into $\mathcal{J}_j(T)$ and performing the S -integral yields a delta function constraint,

$$\int_{-i\infty}^{+i\infty} \frac{dS}{2\pi i} e^{-S(\ln|z|^2 + \alpha)} = \delta(\alpha + \ln|z|^2).$$

Since the integration range for the Schwinger parameter is $\alpha \in [0, \infty)$, the support of delta function imposes the constraint $\ln|z|^2 < 0$, thereby restricting the spatial integral to the unit disk $|z| < 1$ [46]. This allows us to derive explicit results for specific cases, such as $j = -1, -2$,

$$\mathcal{J}_{-1}(T) = \int_{|z|<1} d^2z |z|^{-2} J(z, \bar{z}; T), \quad (16)$$

$$\mathcal{J}_{-2}(T) = \int_{|z|<1} d^2z |z|^{-2} (-\ln|z|^2) J(z, \bar{z}; T). \quad (17)$$

Case 2: $j \geq 0$. Let us first rewrite the integrand using $|z|^{-2S-2} = |z|^{-2} e^{-S \ln |z|^2}$ and utilize the integral representation of delta function,

$$\int_{-\infty}^{+\infty} \frac{dS}{2\pi i} S^j e^{-SX} = (-1)^j \delta^{(j)}(X).$$

Setting $X = \ln |z|^2$, the integral is localized on the unit circle $|z| = 1$. To evaluate this, we switch to polar coordinates $z = r e^{i\theta}$. Using the property $\int \delta^{(j)}(X) f(X) dX = (-1)^j f^{(j)}(0)$, the integral reduces to a boundary integral over the unit circle,

$$\mathcal{J}_j(T) = \oint_{|z|=1} \frac{d\theta}{2\pi} \hat{D}^j J(z, \bar{z}; T), \quad \text{with } \hat{D} = \frac{r}{2} \partial_r. \quad (18)$$

We stress that the integral representations derived above remain valid for the general world sheet correlation function $G^{(k)}(z, \bar{z}; S, T)$. This establishes a systematic strategy for computing energy-energy correlations at higher orders of curvature corrections at strong coupling. Furthermore, when the contributions are combined, we observe that the calculation undergoes significant simplification. We will apply this approach to the flat-space contribution and the first curvature correction as concrete examples.

IV. FLAT SPACE CONTRIBUTION

We begin by examining the flat space contributions $I^{(0)}(T)$, which governs the coefficients $c_{k,0}$. In this case, $G^{\text{tot},(0)} = |z|^2/S^2$ in Eq. (12) corresponds to $J(z, \bar{z}; T) = |z|^2 |1 - z|^{-2T-2}$ with $j = -2$ in Eq. (15) [47]. Identifying this with Eq. (17), we evaluate the integral $I^{(0)}(T)$ as follows:

$$\begin{aligned} I^{(0)}(T) &= \int_0^1 \frac{r dr}{\pi} (-2 \ln r) \int_0^{2\pi} d\theta |1 - z|^{-2T-2} \\ &= \frac{1}{T^2} \left(\frac{\Gamma(1-2T)}{\Gamma(1-T)^2} - 1 \right). \end{aligned} \quad (19)$$

The Taylor expansion of $I^{(0)}(T)$ for small T reads

$$I^{(0)}(T) = \zeta_2 + 2\zeta_3 T + \frac{19}{4} \zeta_4 T^2 + \dots \quad (20)$$

We observe that these coefficients align precisely with the leading contributions from the highest-spin sector [23]. Indeed, these coefficients are uniquely fixed by the flat-space limit. Crucially, due to the relation $T = \beta \hat{t}$, the flat-space terms affect the subleading coefficients $c_{k,n}$ via a shift $t \rightarrow 4/3$, which should not be neglected.

Alternatively, exploiting the symmetry allows us to consider an equivalent integrand $J(z, \bar{z}; T) = |1 - z|^{-2T}/T^2$, with $j = 0$ in Eq. (15). This corresponds to *case 2*, and the resulting integral is localized on the boundary of a disk,

$$I^{(0)}(T) = \frac{1}{T^2} \int_0^{2\pi} \frac{d\theta}{2\pi} \left[4 \sin^2 \left(\frac{\theta}{2} \right) \right]^{-T}. \quad (21)$$

Note that the $1/T^2$ term in small- T expansion should be removed, since it corresponds to the supergravity contribution, which we have excluded by focusing on stringy effects. It is worth noting that this alternative form can also be obtained by interpreting the energy detector as a shockwave perturbation of the dual AdS geometry [22], and the two-point correlator of the corresponding shockwave vertex operators then reproduces the expression in Eq. (21) (see also [23]).

V. FIRST CURVATURE CORRECTION

We now turn to the first curvature correction. This contribution can be decomposed into bulk and boundary integrals. For completeness, the full expression for the first curvature correction to the AdS Virasoro-Shapiro amplitude Eq. (A2), is provided in Appendix A [48], where details of the calculation are presented. We observe that restricting the integration domain precipitates dramatic cancellations among complicated polylogarithmic structures; the resulting concise integral representation hints at the intrinsic simplicity of the EEC. In summary, the final integrated result is organized into four distinct functions,

$$I^{(1)}(T) = \sum_{i=1}^4 I_i^{(1)}(T), \quad (22)$$

where $I_1^{(1)}(T)$ and $I_2^{(1)}(T)$ originate from the bulk integration, while $I_3^{(1)}(T)$ and $I_4^{(1)}(T)$ arise from the boundary contributions, and their explicit world-sheet integral representations are given in Appendix A. We summarize the results as follows.

The bulk contributions, supported on the disk $|z| < 1$, are given by

$$I_1^{(1)}(T) + I_2^{(1)}(T) = -\frac{T}{9} \frac{\partial^3}{\partial T^3} \frac{2\Gamma(-2T)}{\Gamma(1-T)^2}. \quad (23)$$

The small- T expansion yields a well-behaved series where the coefficients are in terms of zeta values. The remaining contributions are localized on the boundary $|z| = 1$ and involve polylogarithms arising from the world sheet correlator expansion,

$$I_3^{(1)} = 2\zeta_3 \frac{\Gamma(-2T-1)}{\Gamma(-T)^2} + F(T), \quad (24)$$

$$I_4^{(1)} = -\frac{\partial^3}{\partial T^3} \left(\frac{5\Gamma(1-2T)}{18\Gamma(1-T)^2} \right) - F(T-1), \quad (25)$$

where $F(T)$ is defined by an infinite sum

$$F(T) = \sum_{n=1}^{\infty} \frac{2 \sin(\pi T) \Gamma(-2T-1) \Gamma(n+T+1)}{\pi n^3 \Gamma(n-T)}. \quad (26)$$

Although $F(T)$ can be resummed into a generalized hypergeometric function, the infinite sum representation is more advantageous for the Taylor expansion in T .

Collecting all contributions, we find that the complete first curvature correction is captured by the following compact form:

$$I^{(1)}(T) = \frac{1}{3T^4} \frac{\partial}{\partial T} T^5 \frac{\partial^2}{\partial T^2} \frac{\Gamma(-2T)}{\Gamma(1-T)^2} + \frac{2\zeta_3 \Gamma(-2T-1)}{\Gamma(-T)^2} + F(T) - F(T-1). \quad (27)$$

We stress again that $I^{(1)}(T)$ appears to be much simpler than the corresponding AdS Virasoro-Shapiro amplitude.

Evaluating Eq. (6) requires the small- T expansion of $I^{(1)}(T)$, see Appendix B [48] for technical details. Remarkably, all contributions involving the Euler-Mascheroni constant cancel out, revealing a structure governed by multiple zeta values. The first few terms are given by

$$I^{(1)}(T) = \zeta_2 - \frac{10}{3} \zeta_3 + \left(-2\zeta_2 + 4\zeta_3 - \frac{53}{2} \zeta_4 \right) T + \dots,$$

where we have dropped the singular term in the small- T expansion, which is associated with the supergravity contribution that we should exclude. Finally, combining the contributions from the flat-space results and first curvature corrections, we can determine all the coefficients $c_{k,0}$ and $c_{k,1}$ in Eq. (6). In particular, $c_{k,0}$ can be straightforwardly read off from Eq. (19), yielding

$$c_{0,0} = 24\zeta_2, \quad c_{1,0} = 120\zeta_3, \quad c_{2,0} = 855\zeta_4, \dots, \quad (28)$$

and for $c_{k,1}$, we derive the following results:

$$c_{1,1} = 24\zeta_2 - 240\zeta_3, \quad (29)$$

$$c_{2,1} = -120\zeta_2 + 240\zeta_3 - 3870\zeta_4, \dots, \quad (30)$$

and similar results for higher-order terms. This gives the strong coupling expansion of the EEC,

$$\begin{aligned} \text{EEC}(\xi) = 1 + \frac{24\zeta_2}{\lambda} Q_0(\xi) \\ + \frac{(24\zeta_2 - 240\zeta_3) Q_0(\xi) + 120\zeta_3 Q_1(\xi)}{\lambda^{3/2}} + \dots \end{aligned} \quad (31)$$

As a consistency check, we performed an independent calculation for $c_{0,0}$, $c_{1,0}$ and $c_{1,1}$ using the low-energy

expansion method in Appendix C [48], which yields exactly identical results [54].

VI. DISCUSSION

In this paper, we have established a rigorous connection between the EEC in strongly coupled $\mathcal{N} = 4$ SYM and the world sheet description of the AdS Virasoro-Shapiro amplitude. By reformulating the problem from Mellin space into the integration over the string moduli space, we demonstrated that the computation of stringy corrections to the EEC reduces to evaluating integrals over a unit disk and its boundary on the Riemann sphere. In practice, this powerful construction leads to an extremely efficient way of computing the EEC in $\mathcal{N} = 4$ SYM at strong coupling. We demonstrated this by explicitly computing the flat space and first curvature correction to the EEC and obtained finite results.

The results of this paper open up a variety of interesting directions for future work. A natural extension is to consider the second order of the higher-curvature correction [25]. Such higher-order results would also allow for a more precise comparison with the bootstrap bounds [23]. Another direction is to study the EECs in $\mathcal{N} = 4$ SYM involving operators with higher dimensions [55] for the relevant correlators [36]. Perhaps an even more ambitious goal is to bootstrap the EEC directly. The observed structural simplicity of the EEC suggests it may be feasible to determine these corrections without referencing the AdS Virasoro-Shapiro amplitude.

Furthermore, $\mathcal{N} = 4$ SYM is known to enjoy S -duality [56–58]. It has been shown that the first few orders of stringy corrections to the correlator can be determined exactly in terms of known nonholomorphic modular functions, which incorporate all perturbative and instanton contributions and manifest S -duality [59,60]. It would be interesting to study the modular properties of the EEC and its nonperturbative instanton effects.

Finally, our world sheet formulation may offer new insights into the back-to-back limit ($\xi \rightarrow 1$) of EEC. Its singular behavior is governed by Sudakov logarithms [18,61–63], which are effectively described by the geometry of Wilson lines [64,65]. Intriguingly, a parallel geometric description exists at strong coupling, where correlation functions in the light-like limit are known to map directly to null polygonal Wilson loops [66]. The prominence of Wilson loops at both weak and strong coupling suggests a universal physical picture: the infrared dynamics of energy flow may be effectively described by specific Wilson loop configurations, even in the intermediate stringy regime. It is tantalizing to speculate that our world sheet results could be reorganized into such an effective “stringy Wilson loop” description. Identifying these dominant operators that control this regime would be a crucial step toward constructing an effective string theory for jet physics.

ACKNOWLEDGMENTS

The authors would like to thank Hao Chen, Francesco Galvagno, Alessandro Georgoudis, Max Jackson, Murat Kologlu, Rodolfo Russo, Joao Vilas Boas, Alexander Zhiboedov, and Huaxing Zhu for valuable discussions on related topics. L.R. is supported by the Royal Society via a Newton International Fellowship. B.W. is supported by the National Natural Science Foundation of China under Grants No. 124B2095 and No. 12175197.

C.W. is supported by a Royal Society University Research Fellowship, No. URF\R\221015 and a STFC Consolidated Grant No. ST/T000686/1 “Amplitudes, strings and duality.”

DATA AVAILABILITY

The data that support the findings of this article are available within this article.

-
- [1] C.L. Basham, L.S. Brown, S.D. Ellis, and S.T. Love, Energy correlations in electron—positron annihilation: Testing QCD, *Phys. Rev. Lett.* **41**, 1585 (1978).
 - [2] C.L. Basham, L.S. Brown, S.D. Ellis, and S.T. Love, Energy correlations in electron-positron annihilation in quantum chromodynamics: Asymptotically free perturbation theory, *Phys. Rev. D* **19**, 2018 (1979).
 - [3] H. Chen, I. Moulton, X. Zhang, and H. X. Zhu, Rethinking jets with energy correlators: Tracks, resummation, and analytic continuation, *Phys. Rev. D* **102**, 054012 (2020).
 - [4] P. T. Komiske, I. Moulton, J. Thaler, and H. X. Zhu, Analyzing N-point energy correlators inside jets with CMS open data, *Phys. Rev. Lett.* **130**, 051901 (2023).
 - [5] I. Moulton and H. X. Zhu, Energy correlators: A journey from theory to experiment, [arXiv:2506.09119](#).
 - [6] P. Kravchuk and D. Simmons-Duffin, Light-ray operators in conformal field theory, *J. High Energy Phys.* **11** (2018) 102.
 - [7] M. Kologlu, P. Kravchuk, D. Simmons-Duffin, and A. Zhiboedov, The light-ray OPE and conformal colliders, *J. High Energy Phys.* **01** (2019) 128.
 - [8] A. V. Belitsky, S. Hohenegger, G. P. Korchemsky, E. Sokatchev, and A. Zhiboedov, Energy-energy correlations in $N = 4$ supersymmetric Yang-Mills theory, *Phys. Rev. Lett.* **112**, 071601 (2014).
 - [9] A. V. Belitsky, S. Hohenegger, G. P. Korchemsky, E. Sokatchev, and A. Zhiboedov, From correlation functions to event shapes, *Nucl. Phys.* **B884**, 305 (2014).
 - [10] A. V. Belitsky, S. Hohenegger, G. P. Korchemsky, E. Sokatchev, and A. Zhiboedov, Event shapes in $\mathcal{N} = 4$ super-Yang-Mills theory, *Nucl. Phys.* **B884**, 206 (2014).
 - [11] J. M. Henn, E. Sokatchev, K. Yan, and A. Zhiboedov, Energy-energy correlation in $N = 4$ super Yang-Mills theory at next-to-next-to-leading order, *Phys. Rev. D* **100**, 036010 (2019).
 - [12] C.-H. Chang, M. Kologlu, P. Kravchuk, D. Simmons-Duffin, and A. Zhiboedov, Transverse spin in the light-ray OPE, *J. High Energy Phys.* **05** (2020) 059.
 - [13] H. Chen, I. Moulton, and H. X. Zhu, Quantum interference in jet substructure from spinning gluons, *Phys. Rev. Lett.* **126**, 112003 (2021).
 - [14] H. Chen, I. Moulton, and H. X. Zhu, Spinning gluons from the QCD light-ray OPE, *J. High Energy Phys.* **08** (2021) 233.
 - [15] H. Chen, QCD factorization from light-ray OPE, *J. High Energy Phys.* **01** (2023) 035.
 - [16] H. Chen, P. F. Monni, Z. Xu, and H. X. Zhu, Scaling violation in power corrections to energy correlators from the light-ray operator product expansion, *Phys. Rev. Lett.* **133**, 231901 (2024).
 - [17] L. J. Dixon, I. Moulton, and H. X. Zhu, Collinear limit of the energy-energy correlator, *Phys. Rev. D* **100**, 014009 (2019).
 - [18] G. P. Korchemsky, Energy correlations in the end-point region, *J. High Energy Phys.* **01** (2019) 008.
 - [19] J. M. Maldacena, The Large N limit of superconformal field theories and supergravity, *Adv. Theor. Math. Phys.* **2**, 231 (1998).
 - [20] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Gauge theory correlators from noncritical string theory, *Phys. Lett. B* **428**, 105 (1998).
 - [21] E. Witten, Anti de Sitter space and holography, *Adv. Theor. Math. Phys.* **2**, 253 (1998).
 - [22] D. M. Hofman and J. Maldacena, Conformal collider physics: Energy and charge correlations, *J. High Energy Phys.* **05** (2008) 012.
 - [23] R. Dempsey, R. Karlsson, S. S. Pufu, Z. Zahraee, and A. Zhiboedov, Conformal collider bootstrap in $\mathcal{N} = 4$ SYM, [arXiv:2512.10796](#).
 - [24] L. F. Alday, T. Hansen, and J. A. Silva, Emergent worldsheet for the AdS Virasoro-Shapiro amplitude, *Phys. Rev. Lett.* **131**, 161603 (2023).
 - [25] L. F. Alday and T. Hansen, The AdS Virasoro-Shapiro amplitude, *J. High Energy Phys.* **10** (2023) 023.
 - [26] V. Gonçalves, Four point function of $\mathcal{N} = 4$ stress-tensor multiplet at strong coupling, *J. High Energy Phys.* **04** (2014) 150.
 - [27] L. Rastelli and X. Zhou, Mellin amplitudes for $\text{AdS}_5 \times S^5$, *Phys. Rev. Lett.* **118**, 091602 (2017).
 - [28] D. J. Binder, S. M. Chester, S. S. Pufu, and Y. Wang, $\mathcal{N} = 4$ Super-Yang-Mills correlators at strong coupling from string theory and localization, *J. High Energy Phys.* **12** (2019) 119.
 - [29] J. M. Drummond, H. Paul, and M. Santagata, Bootstrapping string theory on $\text{AdS}_5 \times S^5$, *Phys. Rev. D* **108**, 026020 (2023).
 - [30] S. M. Chester and S. S. Pufu, Far beyond the planar limit in strongly-coupled $\mathcal{N} = 4$ SYM, *J. High Energy Phys.* **01** (2020) 103.

- [31] T. Abl, P. Heslop, and A. E. Lipstein, Towards the Virasoro-Shapiro amplitude in $\text{AdS}_5 \times S^5$, *J. High Energy Phys.* **04** (2020) 237.
- [32] F. Aprile, J. M. Drummond, H. Paul, and M. Santagata, The Virasoro-Shapiro amplitude in $\text{AdS}_5 \times S^5$ and level splitting of 10d conformal symmetry, *J. High Energy Phys.* **11** (2020) 109.
- [33] Note that we use a slightly different definition for $Q_n(\xi)$ compared to [26]. In particular, $Q_n(\xi)$ defined here is equal to $Q_{n+1}(\xi)$ in [26].
- [34] J. Penedones, Writing CFT correlation functions as AdS scattering amplitudes, *J. High Energy Phys.* **03** (2010) 025.
- [35] The similar construction is also valid for more holographic backgrounds such as [36–43].
- [36] G. Fardelli, T. Hansen, and J. A. Silva, AdS Virasoro-Shapiro amplitude with KK modes, *J. High Energy Phys.* **11** (2023) 064.
- [37] B. Wang, D. Wu, and E. Y. Yuan, Kaluza-Klein AdS Virasoro-Shapiro amplitude near flat space, *Phys. Rev. Lett.* **135**, 041603 (2025).
- [38] L. F. Alday, S. M. Chester, T. Hansen, and D.-l. Zhong, The AdS veneziano amplitude at small curvature, *J. High Energy Phys.* **05** (2024) 322.
- [39] L. F. Alday and T. Hansen, Single-valuedness of the AdS Veneziano amplitude, *J. High Energy Phys.* **08** (2024) 108.
- [40] B. Wang, Bootstrap AdS veneziano amplitude with arbitrary Kaluza-Klein modes, *J. High Energy Phys.* **02** (2026) 201.
- [41] S. M. Chester and D.-l. Zhong, $\text{AdS}_3 \times S^3$ Virasoro-Shapiro amplitude with Ramond-Ramond flux, *Phys. Rev. Lett.* **134**, 151602 (2025).
- [42] H. Jiang and D.-l. Zhong, $\text{AdS}_3 \times S^3$ Virasoro-Shapiro amplitude with KK modes, *J. High Energy Phys.* **02** (2026) 071.
- [43] S. M. Chester, T. Hansen, and D.-l. Zhong, The type IIA Virasoro-Shapiro amplitude in $\text{AdS}_4 \times \text{CP}^3$ from ABJM theory, *J. High Energy Phys.* **05** (2024) 040.
- [44] We have used the AdS/CFT dictionary, $R^4/\ell_s^4 = \lambda$, where R is the AdS curvature and ℓ_s is the string length, to express the large AdS curvature expansion in terms of the large- λ expansion.
- [45] H. Chen, R. Karlsson, and A. Zhiboedov, Energy correlations and Planckian collisions, *J. High Energy Phys.* **03** (2026) 078.
- [46] We have independently verified this conclusion using the residue theorem. The convergence condition for the contour integral depends on the sign of $\ln|z|^2$. Assuming $\text{Re}(S) > 0$ requires closing the contour in the left half-plane, which selects the region $|z| < 1$. Conversely, assuming $\text{Re}(S) < 0$ dictates closing the contour to the right, corresponding to $|z| > 1$. While these domains are complementary, the sign reversal from the change in contour orientation cancels the sign difference between the radial integrals using the formal integral relation $\int_0^1 dr r^k = -\int_1^\infty dr r^k$. Consequently, the final result is independent of the assumption regarding $\text{Re}(S)$.
- [47] Note we should remove the supergravity amplitude $1/(STU)$ for focusing the stringy effects.
- [48] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/m9q6-47z2> for details on first curvature correction to the EEC, which includes Refs. [24,25,49–52]; for series expansion of $I(T)$; and for low-energy expansion, which includes Refs. [53].
- [49] A. B. Goncharov, Multiple polylogarithms, cyclotomy and modular complexes, *Math. Res. Lett.* **5**, 497 (1998).
- [50] A. B. Goncharov, Multiple polylogarithms and mixed tate motives, [arXiv:math/0103059](https://arxiv.org/abs/math/0103059).
- [51] F. C. S. Brown, Polylogarithmes multiples uniformes en une variable, *C. R. Math.* **338**, 527 (2004).
- [52] C. Duhr and F. Dulat, PolyLogTools—polylogs for the masses, *J. High Energy Phys.* **08** (2019) 135.
- [53] L. F. Alday, T. Hansen, and J. A. Silva, AdS Virasoro-Shapiro from single-valued periods, *J. High Energy Phys.* **12** (2022) 010.
- [54] We note our result for $c_{1,1}$ disagrees with that in the Ref. [23], corresponding to the $\lambda^{-3/2}$ term in Eq. (4.6) of that reference. Converting to our basis, their expression implies $c_{1,1} = 48\zeta_2 - 240\zeta_3$. We find that this discrepancy arises from performing the s -integration prior to the δ -summation in [23], which omits a certain subtle contribution (more precisely, a term with a 0/0 structure). We have explicitly verified that including such contribution restores precise agreement. We thank Alexander Zhiboedov for very helpful discussions on this point.
- [55] D. Chicherin, G. P. Korchemsky, E. Sokatchev, and A. Zhiboedov, Energy correlations in heavy states, *J. High Energy Phys.* **11** (2023) 134.
- [56] C. Montonen and D. I. Olive, Magnetic monopoles as gauge particles?, *Phys. Lett.* **72B**, 117 (1977).
- [57] E. Witten and D. I. Olive, Supersymmetry algebras that include topological charges, *Phys. Lett.* **78B**, 97 (1978).
- [58] H. Osborn, Topological charges for $N = 4$ supersymmetric gauge theories and monopoles of spin 1, *Phys. Lett.* **83B**, 321 (1979).
- [59] S. M. Chester, M. B. Green, S. S. Pufu, Y. Wang, and C. Wen, Modular invariance in superstring theory from $\mathcal{N} = 4$ super-Yang-Mills, *J. High Energy Phys.* **11** (2019) 016.
- [60] S. M. Chester, M. B. Green, S. S. Pufu, Y. Wang, and C. Wen, New modular invariants in $\mathcal{N} = 4$ Super-Yang-Mills theory, *J. High Energy Phys.* **04** (2020) 212.
- [61] J. C. Collins and D. E. Soper, Back-to-back jets in QCD, *Nucl. Phys.* **B193**, 381 (1981); **B213**, 545(E) (1983).
- [62] I. Moulton and H. X. Zhu, Simplicity from recoil: The three-loop soft function and factorization for the energy-energy correlation, *J. High Energy Phys.* **08** (2018) 160.
- [63] H. Chen, X. Zhou, and H. X. Zhu, Power corrections to energy flow correlations from large spin perturbation, *J. High Energy Phys.* **10** (2023) 132.
- [64] G. P. Korchemsky and A. V. Radyushkin, Renormalization of the Wilson loops beyond the leading order, *Nucl. Phys.* **B283**, 342 (1987).
- [65] H. Chen, P. F. Monni, Z. Pang, G. Vita, and H. X. Zhu, Correlation function/Wilson loop duality in gauge theory from EFT, [arXiv:2510.07377](https://arxiv.org/abs/2510.07377).
- [66] L. F. Alday, B. Eden, G. P. Korchemsky, J. Maldacena, and E. Sokatchev, From correlation functions to Wilson loops, *J. High Energy Phys.* **09** (2010) 123.