

Challenging the $\omega_0\omega_a$ CDM parametrization through rational expansions in view of the DESI DR2

Youri Carloni^{1,2,3,*}, Orlando Luongo^{1,2,3,4,5,†} and Marek Biesiada^{6,‡}

¹*Università di Camerino, Via Madonna delle Carceri, Camerino, 62032, Italy*

²*INAF—Osservatorio Astronomico di Brera, Milano, Italy*

³*Istituto Nazionale di Fisica Nucleare, Sezione di Perugia, Perugia, 06123, Italy*

⁴*SUNY Polytechnic Institute, 13502 Utica, New York, USA*

⁵*Al-Farabi Kazakh National University, Al-Farabi avenue 71, 050040 Almaty, Kazakhstan*

⁶*National Centre for Nuclear Research, Pasteura 7, 02-093 Warsaw, Poland*



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In view of the new Dark Energy Spectroscopic Instrument 2025 results, we analyze three types of *Padé* cosmology, based on rational series making use of Padé approximants over the equations of state, namely Padé ^{ω} (0, 1) and Padé (1, 1), plus a Padé ^{q} (0, 1), i.e., a rational expansion on the dark energy deceleration parameter, in which the numerator and denominator orders are incorporated into the above brackets. These scenarios appear as alternative dark energy parametrizations with respect to the well-known $\omega_0\omega_a$ CDM model. Accordingly, we perform Monte Carlo Markov chain analyses with the publicly available CLASS Boltzmann code, including the three Padé parametrizations, along with the $\omega_0\omega_a$ CDM and Λ CDM standard pictures. To this end, we combine independent probes from high to low redshifts to obtain reliable constraints on the cosmological parameters of these models and compare them using statistical selection criteria. Our results show that $\omega_0\omega_a$ CDM parametrization is not significantly preferred over the Padé cosmology. On the contrary, the deviance information criterion identifies Padé (1, 1) as *the best-fit model* along with the $\omega_0\omega_a$ CDM parametrization. Based on our bounds, we further investigate the evolution of the squared sound speed, revealing that the Padé ^{q} (0, 1) and Padé ^{ω} (0, 1) parametrizations exhibit enhanced stability compared with the other cases here considered and, therefore, describe robust alternatives for the cosmological background.

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I. INTRODUCTION

Observations over the past few decades suggest that the Universe is undergoing an accelerated expansion driven by an exotic component, known as dark energy, which constitutes approximately 70% of the total energy budget of the Universe [1–3]. The remaining components are dark matter, accounting for about 25%, and baryonic matter, which contributes only $\sim 5\%$ [4–9].

The standard background that accounts for both the energy composition and the observed accelerated expansion is the Λ CDM paradigm, in which Λ represents the

cosmological constant, while CDM denotes *cold dark matter*.

Despite its success in passing a wide range of observational tests, this model has always faced significant challenges, e.g. the cosmological constant problem, the persistent cosmological tensions [10–19], etc. that culminated in the recent results, from the Dark Energy Spectroscopic Instrument (DESI) collaboration, favoring the $\omega_0\omega_a$ CDM parametrization [20–26].

Recent developments, however, showed that this may be only partially true [27–35] since the preferred model may depend on the choice of the combined datasets and/or on the functional structure of dark energy evolution, instead. In addition, it appears still unclear whether the $\omega_0\omega_a$ CDM scenario is the most suitable one or not.

In light of these considerations, we here perform a further analysis employing *rational expansions of dark energy*, modeled by means of *Padé approximants* as an alternative dark energy equation of state [36–42]. Precisely, in analogy to the w CDM model [43], we employ a Padé (0, 1) approach first, then adding one more parameter, we consider a Padé (1, 1) scenario, expanding in both the cases

* Contact author: youri.carloni@unicam.it

† Contact author: orlando.luongo@unicam.it

‡ Contact author: marek.biesiada@ncbj.gov.pl

the equation of state. Afterward, motivating our analysis from a genuine thermodynamics, we move on expanding the dark energy deceleration parameter through a Padé (0, 1) as a further scheme to analyze, extending the findings found in Ref. [41]. In all the cases, we thus compare our outcomes with those obtained for the $\omega_0\omega_a$ CDM, to check if the so-constructed *Padé cosmology* provides a consistent way to avoid the high redshift divergences that affect Taylor expansions and thereby offer an effective replacement for the $\omega_0\omega_a$ CDM scenario. Our Padé cosmology is thus based on alternative parametrizations, namely Padé^ω (0, 1), Padé (1, 1), and Padé^q (0, 1), from low to high redshift observations. In particular, we consider the cosmic microwave background (CMB), the second data release (DR2) of baryon acoustic oscillation (BAO) measurements from the DESI mission, the *Pantheon+* type Ia supernova (SN Ia), and the observational Hubble data (OHD). By performing a Markov chain Monte Carlo (MCMC) analysis using the above cosmic probes, we conclude that Padé approximants provide a viable description of dynamical dark energy, *as they are not statistically excluded*, as certified by exploring Akaike's information criterion (AIC) and deviance information criterion (DIC) statistical criteria [44–48].

These quantities, extensively used in regression model assessment [49], are defined as

$$\text{AIC} \equiv -2 \ln \mathcal{L}_m + 2d, \quad (1a)$$

$$\text{DIC} \equiv -2 \ln \mathcal{L}_m + 2p, \quad (1b)$$

where $\ln \mathcal{L}_m$ denotes the maximum log-likelihood, see Table III, d is the number of free parameters in a given model, and $p = \langle -2 \ln \mathcal{L} \rangle + 2 \ln \mathcal{L}_m$, where brackets denote the average over the posterior distribution of parameters. The quantity p , called Bayesian complexity, has the meaning of an effective number of parameters of a given model constrained by the data [45]. One can note formal similarity of mathematical expressions for AIC and DIC.

The conceptual foundation of these estimators relies on two probability distributions: $h(x)$, the true but unknown distribution, and $g(x|\theta)$, a parametric model characterized by the parameter set θ . An optimal parameter set $\theta_{\min}$ exists that minimizes the discrepancy between $g(x|\theta)$ and $h(x)$ [50]. The AIC is derived from an approximate minimization of the Kullback-Leibler information entropy, which is one of the ways to quantify the discrepancy between the true distribution of the data and the model distribution. By using the effective number of parameters, the DIC overcomes the problem of the AIC that it does not discount parameters which are unconstrained by the data. In other words, in contrast to AIC, DIC penalizes only those parameters that are effectively constrained by the data [45].

Since $h(x)$ is unknown, the absolute values of AIC, or DIC for a single model are not directly informative.

Instead, model comparison is performed through relative differences,

$$\Delta X = X_i - X_0, \quad X_i \in \{\text{AIC}, \text{DIC}\}, \quad (2)$$

where X_0 denotes the minimum value among the competing models. We adopt, as the baseline, the model that minimizes the information criterion, i.e., the one for which $\Delta X = 0$. The magnitude of ΔX quantifies how strongly a model is disfavored with respect to the best-fitting model: $\Delta X \geq 2$ (weak), $\Delta X \geq 6$ (moderate), and $\Delta X \geq 10$ (strong) [51].

In our work, model comparison shows that the $\omega_0\omega_a$ CDM model yields the lowest AIC value; however, Padé (1, 1) and Padé^q (0, 1) are not excluded. Instead, according to the DIC criterion, both $\omega_0\omega_a$ CDM and Padé (1, 1) are identified as the best-fit models. Further, based on the results of our analysis, we also compute the squared sound speed and find that Padé^q (0, 1) and Padé^ω (0, 1) exhibit a more stable behavior at late times compared to the other dark energy parametrizations. The overall theoretical and numerical analysis indicates that alternative expansions can be employed to characterize dark energy, and suggests that the $\omega_0\omega_a$ CDM model appears less crucial than previously claimed [20,23].

The paper is organized as follows. In Sec. II, we introduce the theoretical framework of Padé polynomials used as a parametrization for dark energy. In Sec. III, we outline the experimental setup and detail the cosmic probes employed in our analysis. In Sec. IV, we report our findings, comparing the Padé approximants equation of state with the $\omega_0\omega_a$ CDM parametrization using the AIC and DIC statistical criteria, and computing the squared sound speed for each model. Finally, in Sec. V, we summarize our results and provide an outlook for future work.

II. DARK ENERGY PARAMETRIZATION

Characterizing dark energy in terms of barotropic fluids can be complicated by the lack of direct information toward the nature of the constituent associated with dark energy itself [52–55]. On one hand, barotropic fluid is the only one physical description of constituents of the Universe compatible with the homogeneity and isotropy. On the other hand, the detailed equation of state relating fluid's pressure P and density ρ would always be based on a specific physical model of that constituent. Hence, approaches to go through this issue includes *a priori* and/or *a posteriori* methods that try to determine the dark energy equation of state, see e.g. [56]. One possibility is to use a reasonable assumption that an evolving dark energy equation of state should be an analytical function of the cosmic time [57–59]. Respecting this constraint, one can in principle use Taylor expansion in terms of the redshift, scale factor

or the Hubble parameter, obtaining usually quite different results, also due to the fact that the expansions are typically limited to very small cosmographic quantities, implying a convergence problem, see e.g. [36,60,61]. Anyway, this heuristic, phenomenological approach is a promising one as it provides a common point at which observations can meet the theory. Namely, the cosmographic observational tests are able to constrain the coefficients of such expansions, while any potential specific theoretical model of dark energy should be able to predict the values of these coefficients. The best known example of such an approach is the scalar field model of quintessence [57,62].

In this work, we analyze alternative ways to parametrize dynamical dark energy without relying on specific model properties, instead constructing the background using fundamental series. We then compare these parametrizations with the well-known $\omega_0\omega_a$ CDM and Λ CDM models. Precisely, we proceed as follows:

- (i) We introduce the $\omega_0\omega_a$ CDM scenario, which is the most commonly employed parametrization derived from a Taylor expansion in the scale factor, and we recast it in terms of the redshift;
- (ii) We propose two methods to *stabilize* the expansions at *high redshift* by invoking rational series constructed via Padé reconstruction, which was first introduced in cosmographic analyses in Ref. [36];
- (iii) We derive a thermodynamically motivated model, as third expansion, constructed from the simplest viable expansion and expressed through the dark energy deceleration parameter, originally introduced in Ref. [41].

These proposals are thus compared among them first and with respect to the Λ CDM and $\omega_0\omega_a$ CDM standard scenarios, by employing the most recent late-time datasets. In so doing, we will demonstrate that the so-constructed *Padé cosmology* is able to better perform than the Λ CDM and $\omega_0\omega_a$ CDM approaches, opening new avenues toward how dark energy evolves.

A. Standard Taylor expansions: From z expansions to the $\omega_0\omega_a$ CDM model

Any dark energy model can be parametrized by

$$E(z)^2 = \Omega_m(1+z)^3 + \Omega_{de}f(z), \quad (3)$$

in a spatially flat Universe. Throughout the paper, we assume spatial flatness of the Universe as well supported by observations [9,63]. Here, Ω_m and Ω_{de} are the present-day dimensionless densities of matter and dark energy, respectively, $E(z) = H(z)/H_0$ is the reduced Hubble parameter, and $f(z)$ describes the redshift evolution of the dark energy component, i.e.,

$$f(z) = \exp \left[\int_0^z \frac{3(1+\omega(z'))}{1+z'} dz' \right], \quad (4)$$

with $\omega(z)$ denoting the equation of state coefficient.

A naive Taylor expansion of $\omega(z)$ coefficient around the present epoch $z = 0$

$$\omega(z) = \sum_{n=0}^{\infty} \omega_n z^n, \quad (5)$$

has a finite radius of convergence and, when truncated at finite order, induces an exponential contribution at high redshift. As a consequence, its applicability is effectively limited to $z < 1$, leading to systematic biases and reduced numerical accuracy, see e.g. [60,64,65].

In fact, truncating the expansion at first order, i.e., $\omega(z) = \omega_0 + \omega_1 z$, the corresponding reduced Hubble parameter would read

$$E(z)^2 = \Omega_m(1+z)^3 + \Omega_{de}(1+z)^{3(\omega_0-\omega_1+1)} e^{3\omega_1 z}, \quad (6)$$

explicitly showing a divergence for high redshifts.

To solve this issue, a widely adopted approach is reformulating the expansion in terms of the scale factor, $a = (1+z)^{-1}$ (which is a true physical degree of freedom), as

$$\omega(a) = \sum_{n=0}^{\infty} \omega_n (1-a)^n. \quad (7)$$

Then, truncating this series at the first order, yields the well-known $\omega_0\omega_a$ CDM parametrization,¹

$$\omega(a) = \omega_0 + \omega_a(1-a), \quad (8)$$

which, in light of DESI results obtaining $\omega_0 - \omega_a$ contours disjoint from Λ CDM at 2σ level, implies that the $\omega_0\omega_a$ CDM parametrization turns out to be taken as a possible viable cosmological model.

In this case, the corresponding reduced Hubble parameter reads

$$E(a)^2 = \Omega_m a^{-3} + \Omega_{de} a^{-3(\omega_0+\omega_a+1)} e^{-3\omega_a(1-a)}, \quad (9)$$

turning out to be stable at high redshift as

$$E(z)^2 = \Omega_m(1+z)^3 + \Omega_{de}(1+z)^{3(\omega_0+\omega_a+1)} e^{-\frac{3\omega_a z}{1+z}}. \quad (10)$$

¹The model was first proposed by Chevallier and Polarski [66] and later by Linder [67], being usually denoted as CPL parametrization. The phantom nature of the equation of state, however, was not predicted by the three authors, suggesting a novel nomenclature under the form of $\omega_0\omega_a$ CDM parametrization.

B. Going beyond Taylor series: Rational expansions and stabilization of Taylor series

It appears natural to go beyond the standard Taylor series and to investigate alternative formulations, expressed in function of the redshift, in which the polynomial series is replaced by a *rational polynomial parametrization*, also known as Padé approximant. This prerogative is motivated by the fact that one may invoke the stability of ω directly on it, without passing through the trick of reformulating everything in function of a .

To do so, we can work out the Padé parametrization or Padé reconstructions, defined on ω by

$$\omega(z) = \frac{\sum_{i=0}^N a_i z^i}{1 + \sum_{j=1}^M b_j z^j}, \quad (11)$$

which, for $N \leq M$, avoids the divergence problems of the standard Taylor expansion and extends the redshift domain of convergence.

In particular, for Padé^ω (0, 1), the equation of state is given by $\omega(z) = \frac{a_0}{1+b_1 z}$, while for Padé (1, 1) is provided by $\omega(z) = \frac{a_0+a_1 z}{1+b_1 z}$, leading to

$$E(z)^2 = \Omega_m(1+z)^3 + \Omega_{de}(1+z)^{\frac{3(-a_0+b_1-1)}{b_1-1}}(1+b_1 z)^{\frac{3a_0}{b_1-1}}, \quad (12)$$

and

$$E(z)^2 = \Omega_m(1+z)^3 + \Omega_{de}(1+z)^{\frac{3(-a_0+a_1+b_1-1)}{b_1-1}} \times (1+b_1 z)^{\frac{3a_1-3a_0b_1}{b_1-b_1^2}}, \quad (13)$$

respectively. Here, for physical consistency, we impose $a_0 < -1/3$ to reproduce the expected dark energy behavior, and require $b_1 > 0$ to prevent poles in the denominator.

C. Thermodynamic reconstructions from a rational deceleration parameter

At this stage, we extend our analysis further to explore an alternative use of Padé approximants to characterize the dark energy behavior in a model-independent way, as shown in Ref. [41].

Indeed, we can reconstruct the expansion history of the Universe by considering the evolution of kinematic quantities such as the deceleration parameter. We adopt this approach since any cosmological model should remain compatible with the observational constraints arising from the formation of the cosmic structure.

Accordingly, the evolution of cosmic expansion can be described in terms of the deceleration parameter $q = -1 - \frac{\ddot{H}}{H^2}$, which, up to the epoch of structure formation, assumes the form

$$q(z) = -1 + \frac{3\Omega_m(1+z)^3\Omega_{de}(1+z)\frac{df(z)}{dz}}{2(\Omega_m(1+z)^3 + \Omega_{de}f(z))}. \quad (14)$$

Consequently, $q(z)$ is expected to satisfy a set of asymptotic conditions, which are commonly adopted as

$$q(z) \rightarrow -1 \left(\frac{dq}{dz} > 0 \right) \quad \text{as } z \rightarrow -1, \quad (15a)$$

$$q(z) \geq -1 \quad \forall z, \quad (15b)$$

$$q(z) \rightarrow \frac{1}{2} \quad \text{for } z \gg 1. \quad (15c)$$

These conditions summarize the physically motivated asymptotic behavior of a viable cosmological expansion history. In particular, Eq. (15a) corresponds to the asymptotic limit $z \rightarrow -1$, where a dark energy-dominated Universe is expected to approach a de Sitter attractor with $\dot{H} \rightarrow 0$. The additional requirement $dq/dz > 0$ implies a monotonic approach to this asymptotic state: as $z \rightarrow -1$, the deceleration parameter smoothly relaxes to -1 . Equation (15b), $q(z) \geq -1$, excludes phantom regimes, which are associated with pathological behavior. Therefore, this constraint guarantees a nonphantom and thermodynamically consistent expansion history. Finally, Eq. (15c) recovers the standard high redshift evolution of a matter-dominated Universe.

At this stage, we consider the dark energy deceleration parameter

$$q_{de}(z) = -1 + \frac{(1+z)}{H_{de}} \frac{dH_{de}}{dz}, \quad (16)$$

and to describe it we apply the Padé^q (0, 1) approximant

$$q_{de}(z) = \frac{q_{de}}{1 + q_1 z}, \quad (17)$$

to have a total $q(z)$ that fulfills all the above requirements.

Thus, with this choice, Eq. (14) becomes

$$q(z) = \frac{2q_{de}(1-\Omega_m)(1+z+q_{de}z) + \Omega_m(1+z)^3}{2[(1-\Omega_m)(1+z+q_{de}z^2) + \Omega_m(1+z)^3]}, \quad (18)$$

where we imposed $q_1 = 1 + q_{de}$ to respect Eq. (15c). This expression for the deceleration parameter also respects Eq. (15a), while Eq. (15b) requires constraining the value of q_{de} through observational data.

Moreover, this model-independent Padé formulation leads to the following equation of state coefficient

$$\omega(z) = \frac{1}{3} \left[\frac{2q_{de}}{1+z(1+q_{de})} - 1 \right] \quad (19)$$

and to the corresponding reduced Hubble parameter

$$E(z)^2 = \Omega_m(1+z)^3 + \Omega_{de}(1+z+q_{de}z)^2. \quad (20)$$

Then, we also examine whether the model-dependent dark energy parametrizations, i.e., Padé ^{ω} (0, 1), Padé (1, 1), and $\omega_0\omega_a$ CDM, satisfy the three conditions mentioned above. In particular, Eq. (15c) is automatically fulfilled for all dark energy parametrizations, whereas we have to check the validity of Eqs. (15a)–(15b) after obtaining the MCMC results.

D. The role of sound speed

The standard theory of structure formation implies that the linear perturbations in Fourier space are given by

$$\ddot{\delta} + 2H\dot{\delta} + c_s^2 \left(\frac{k^2}{a^2} - k_J^2 \right) \delta = 0, \quad (21)$$

where δ indicates the density perturbations and k_J is the physical Jeans wave number.

Here, we can clearly see that the sound speed plays a crucial role in determining the formation of cosmic structures. In particular, comparing the length scales of perturbations to the Jeans length permits one to characterize at which stage of the Universe's evolution structures may form. This is particularly useful especially in two cases, (i) to compare fluids with matter, where the latter clusterizes at all scales, (ii) in view of the DESI results that unequivocally showed that dark energy might exhibit a negative squared sound speed, at least for what concerns the $\omega_0\omega_a$ CDM parametrization.

Nevertheless, considering λ as a characteristic length scale and the Jeans length defined as

$$\lambda_J = c_s \sqrt{\frac{\pi}{\rho}}, \quad (22)$$

assuming dust matter, i.e., a fluid with equation of state $\omega \equiv \frac{P}{\rho} = 0$, implies that within the linear cosmological regime the Jeans length vanishes, $\lambda_J = 0$. This follows from the fact that, for a generic fluid with $P = \omega(z)\rho$, the squared sound speed is given by

$$c_s^2(z) = \frac{dP}{d\rho} = \omega + \rho \frac{d\omega}{d\rho}. \quad (23)$$

Consequently, within the framework of the dynamical dark energy parametrizations investigated in this work, we obtain the following analytical expressions for Eq. (23), computed at $z = 0$:

(i) Padé ^{ω} (0, 1)

$$c_s^2(z) = \frac{a_0(\Omega_m - 1)(3a_0 - b_1 + 3)}{3(a_0(\Omega_m - 1) - 1)}; \quad (24)$$

(ii) Padé (1, 1)

$$c_s^2(z) = -\frac{(\Omega_m - 1)(a_0(3a_0 - b_1 + 3) + a_1)}{3\Omega_m - 3(a_0 + 1)(\Omega_m - 1)}; \quad (25)$$

(iii) $\omega_0\omega_a$ CDM

$$c_s^2(z) = \frac{(\Omega_m - 1)(3\omega_0(\omega_0 + 1) + \omega_a)}{3((\Omega_m - 1)(\omega_0 + 1) - \Omega_m)}; \quad (26)$$

(iv) Padé ^{q} (0, 1)

$$c_s^2(z) = -\frac{2(\Omega_m - 1)(1 - q_{de})(q_{de} + 1)}{3\Omega_m(2q_{de} - 1) - 6(q_{de} + 1)}. \quad (27)$$

III. DATA USED

To constrain the three types of Padé dark energy parametrizations, we employ the following independent cosmological probes: *Pantheon+* SN Ia, DESI DR2, Planck 2018 CMB, and OHD data.

The characteristics of these probes are summarized as follows:

- (i) *Pantheon+ and SH0ES*. The *Pantheon+* catalog comprises 1701 SNe Ia over the redshift range $z \in [0, 2.3]$, collected from 18 independent subsamples [68]. This dataset also incorporates the SH0ES Cepheid-calibrated host galaxy distances, which yield an estimate of H_0 that is in tension with CMB-based determinations [69].

When including the SH0ES calibration, the residuals in the SN distance modulus are defined as

$$\Delta\mu_i = \begin{cases} \mu_i - \mu_{C,i}, & i \in \text{Cepheid hosts}, \\ \mu_i - \mu_{th}(z_i), & \text{otherwise}, \end{cases} \quad (28)$$

where μ_i denotes the measured SN distance modulus, $\mu_{C,i}$ is the SH0ES Cepheid-based modulus for the host galaxy, and $\mu_{th}(z_i)$ is the theoretical model prediction,

$$\mu_{th}(z_i) = m_i - M = 5 \log \left[\frac{D_L(z_i)}{10 \text{ pc}} \right], \quad (29)$$

with m_i being the rest-frame B -band apparent magnitude and M the absolute B -band magnitude, treated as a nuisance parameter and marginalized over [70]. The SN log-likelihood is then

TABLE I. The DESI DR2 dataset provides measurements, along with their corresponding uncertainties, for the bright galaxy survey (BGS), luminous red galaxies (LRG), emission line galaxies (ELG), quasars (QSO), Lyman- α forest quasars (Ly α QSO), and the combined LRG + ELG sample [23].

Tracer	z_{eff}	D_M/r_d	D_H/r_d	D_V/r_d
BGS	0.295	...	...	7.942 ± 0.075
LRG1	0.510	13.588 ± 0.167	21.863 ± 0.425	...
LRG2	0.706	17.351 ± 0.177	19.455 ± 0.330	...
LRG3 + ELG1	0.934	21.576 ± 0.152	17.641 ± 0.193	...
ELG2	1.321	27.601 ± 0.318	14.176 ± 0.221	...
QSO	1.484	30.512 ± 0.760	12.817 ± 0.516	...
Ly α QSO	2.330	38.988 ± 0.531	8.632 ± 0.101	...

$$\ln \mathcal{L}_{\text{SN}} = -\frac{1}{2} \Delta \vec{\mu}^T C^{-1} \Delta \vec{\mu}, \quad (30)$$

where the total covariance matrix is $C = C_{\text{SN}} + C_C$, incorporating both statistical and systematic uncertainties.²

- (ii) *DESI-BAO DR2*. For BAO constraints, we utilize the DESI DR2 compilation given in Tab. I, consisting of $N_D = 13$ measurements: 6 transverse comoving distances D_M , 6 line-of-sight distances D_H , and 1 angle-averaged distance D_V . All quantities are expressed in units of the comoving sound horizon at the baryon drag epoch [23].

The BAO observables are provided by

$$\frac{D_M(z)}{r_d} = \frac{c}{r_d} \int_0^z \frac{dz'}{H(z')}, \quad (31a)$$

$$\frac{D_H(z)}{r_d} = \frac{c}{r_d H(z)}, \quad (31b)$$

$$\frac{D_V(z)}{r_d} = \frac{[z D_H(z) D_M^2(z)]^{1/3}}{r_d}, \quad (31c)$$

where r_d is the comoving size of the sound horizon at baryon drag epoch.

At this point, we can write the BAO log-likelihood as

$$\ln \mathcal{L}_{\text{BAO}} \propto -\frac{1}{2} \sum_{i=1}^{N_D} \Delta Y_i^T C^{-1} \Delta Y_i, \quad (32)$$

where $\Delta Y_i = Y_i - Y(z_i)$. Here, Y_i are the DR2 measurements, $Y(z_i) \in \{D_M(z_i)/r_d, D_H(z_i)/r_d, D_V(z_i)/r_d\}$, and C is the covariance matrix.³

- (iii) *Planck 2018 CMB*. We further include the Planck 2018 CMB data, consisting of high- ℓ TTTEE, low- ℓ TTEE, and lensing likelihoods. We adopt

the `Planck` high- ℓ likelihood [71], which includes TT data in the multipole range $30 \leq \ell \leq 2508$ and TE and EE data over $30 \leq \ell \leq 1996$. At low multipoles, the TT likelihood covering $2 \leq \ell \leq 29$ is derived using the `Commander` algorithm, while the EE likelihood over the same range is based on the `SimAll` likelihood [9,72]. In addition, we incorporate the CMB lensing likelihood constructed with the `Planck` pipeline, following the methodology described in Ref. [73].

- (iv) *OHD*. Finally, we employ the compilation of OHD, consisting of $N_O = 32$ measurements of $H(z)$ over the redshift interval $z \in [0.07, 1.965]$, shown in Table II. Although these measurements exhibit relatively large uncertainties, they are obtained in a fully model-independent manner using the *cosmic chronometer* approach [74]. This method relies on passively evolving red galaxies, for which $H(z)$ is inferred via the differential age technique by evaluating the age difference between pairs of galaxies formed simultaneously but observed at slightly different redshifts, according to

$$H(z) = -\frac{1}{1+z} \frac{\Delta z}{\Delta t}. \quad (33)$$

Here, the best-fit parameters are determined by maximizing the following log-likelihood function

$$\ln \mathcal{L}_{\text{OHD}} \propto -\frac{1}{2} \sum_{i=1}^{N_O} \left[\frac{H_i - H(z_i)}{\sigma_{H_i}} \right]^2, \quad (34)$$

where σ_{H_i} denotes the uncertainties in the $H(z)$ measurements.

Therefore, by combining all these cosmic probes, the final constraints on the cosmological model parameters are obtained by maximizing the total log-likelihood provided as follows

$$\ln \mathcal{L} = \ln \mathcal{L}_{\text{SN}} + \ln \mathcal{L}_{\text{BAO}} + \ln \mathcal{L}_{\text{CMB}} + \ln \mathcal{L}_{\text{OHD}}, \quad (35)$$

²<https://github.com/PantheonPlusSH0ES/DataRelease>.

³https://github.com/LauraHerold/MontePython_desilike.

TABLE II. OHD catalog with redshifts (first column), measurements with uncertainties (second column), and references (third column).

z	$H(z)$ (km/s/Mpc)	References
0.07	69.0 ± 19.6	[75]
0.09	69.0 ± 12.0	[74]
0.12	68.6 ± 26.2	[75]
0.17	83.0 ± 8.0	[76]
0.179	75.0 ± 4.0	[77]
0.199	75.0 ± 5.0	[77]
0.20	72.9 ± 29.6	[75]
0.27	77.0 ± 14.0	[76]
0.28	88.8 ± 36.6	[75]
0.352	83.0 ± 14.0	[78]
0.38	83.0 ± 13.5	[78]
0.4	95.0 ± 17.0	[76]
0.4004	77.0 ± 10.2	[78]
0.425	87.1 ± 11.2	[78]
0.445	92.8 ± 12.9	[78]
0.47	89.0 ± 23.0	[79]
0.4783	80.9 ± 9.0	[78]
0.48	97.0 ± 62.0	[80]
0.593	104.0 ± 13.0	[77]
0.68	92.0 ± 8.0	[77]
0.75	98.8 ± 33.6	[81]
0.781	105.0 ± 12.0	[77]
0.875	125.0 ± 17.0	[77]
0.88	90.0 ± 40.0	[80]
0.9	117.0 ± 23.0	[76]
1.037	154.0 ± 20.0	[77]
1.3	168.0 ± 17.0	[76]
1.363	160.0 ± 33.6	[82]
1.43	177.0 ± 18.0	[76]
1.53	140.0 ± 14.0	[76]
1.75	202.0 ± 40.0	[76]
1.965	186.5 ± 50.4	[82]

and the uncertainties are assessed through the MCMC analysis.

IV. NUMERICAL ANALYSIS

In this section, we perform a numerical analysis by applying the cosmic probes introduced in Sec. IV to four dynamical dark energy parametrizations, i.e., Padé^{*ω*} (0, 1), Padé (1, 1), $\omega_0\omega_a$ CDM, Padé^{*q*} (0, 1) and to the Λ CDM model.

To do so, we carry out the MCMC analysis with CLASS, a modular Boltzmann solver widely used in cosmology for computing CMB anisotropies and related observables. Its structure allows straightforward implementation of non-standard physics. In our setup, it evolves perturbations in the early Universe, including photons, baryons, cold dark matter, neutrinos (with $m_\nu = 0.06$ eV), and our dark energy models.

Then, we adopt information-theoretic criteria to ensure a fair and consistent model comparison. In this way, we verify whether the Padé parametrizations, as an alternative phenomenological description of dark energy are favored over the $\omega_0\omega_a$ CDM model, which emerged as a viable, if not preferred description of late-time cosmology according to the DESI collaboration.

A. Results and models comparison

The outcomes of the MCMC analysis are summarized in Table III, which reports the mean values of the cosmological parameters along with their corresponding 1σ and 2σ confidence intervals.

The analysis employs physically motivated priors on the cosmological parameters.

The priors used for our analysis and the corresponding contour plots, representing the parameter constraints, are given in Appendix A.

The behavior of the deceleration parameter is determined from the results obtained by MCMC analysis, with the corresponding plots shown in Appendix B. Among the considered parametrizations, Padé^{*q*} (0, 1) satisfies all

TABLE III. Mean of cosmological parameters with 1σ (2σ) uncertainties and best-fit log-likelihood values of Padé^{*ω*} (0, 1), Padé (1, 1), $\omega_0\omega_a$ CDM, Padé^{*q*} (0, 1), and Λ CDM models, obtained by fitting SNe Ia, BAO, CMB, and OHD datasets.

Parameter	Padé ^{<i>ω</i>} (0, 1)	Padé (1, 1)	$\omega_0\omega_a$ CDM	Padé ^{<i>q</i>} (0, 1)	Λ CDM
H_0 (km/s/Mpc)	$69.61^{+0.54(1.08)}_{-0.52(1.07)}$	$69.25^{+0.55(1.10)}_{-0.57(1.10)}$	$69.21^{+0.56(1.12)}_{-0.55(1.10)}$	$69.93^{+0.51(1.06)}_{-0.54(1.05)}$	$68.82^{+0.28(0.57)}_{-0.29(0.56)}$
$100\omega_b$	$2.258^{+0.014(0.028)}_{-0.014(0.027)}$	$2.249^{+0.014(0.028)}_{-0.014(0.028)}$	$2.249^{+0.013(0.028)}_{-0.014(0.027)}$	$2.255^{+0.014(0.027)}_{-0.014(0.028)}$	$2.266^{+0.012(0.026)}_{-0.013(0.025)}$
ω_{cdm}	$0.1179^{+0.0008(0.0016)}_{-0.0008(0.0016)}$	$0.1191^{+0.0009(0.0018)}_{-0.0009(0.0019)}$	$0.1192^{+0.0009(0.0018)}_{-0.0009(0.0018)}$	$0.1185^{+0.0009(0.0017)}_{-0.0009(0.0018)}$	$0.1171^{+0.0007(0.0013)}_{-0.0006(0.0013)}$
$10^9 A_s$	$2.12^{+0.03(0.07)}_{-0.04(0.07)}$	$2.10^{+0.03(0.07)}_{-0.03(0.06)}$	$2.10^{+0.03(0.06)}_{-0.03(0.06)}$	$2.11^{+0.03(0.07)}_{-0.03(0.06)}$	$2.13^{+0.03(0.07)}_{-0.04(0.07)}$

(Table continued)

TABLE III. (Continued)

Parameter	Padé ^ω (0, 1)	Padé (1, 1)	$\omega_0\omega_a$ CDM	Padé ^q (0, 1)	Λ CDM
n_s	$0.9709^{+0.0036(0.0072)}_{-0.0036(0.0071)}$	$0.9680^{+0.0036(0.0074)}_{-0.0037(0.0073)}$	$0.9679^{+0.0037(0.0074)}_{-0.0037(0.0074)}$	$0.9695^{+0.0037(0.0072)}_{-0.0036(0.0073)}$	$0.9731^{+0.0035(0.0069)}_{-0.0034(0.0069)}$
τ_{reio}	$0.0601^{+0.0072(0.0161)}_{-0.0083(0.0153)}$	$0.0552^{+0.0074(0.0155)}_{-0.0079(0.0159)}$	$0.0548^{+0.0071(0.0154)}_{-0.0076(0.0149)}$	$0.0578^{+0.0073(0.0158)}_{-0.0080(0.0154)}$	$0.0633^{+0.0076(0.0164)}_{-0.0087(0.0157)}$
a_0	$-1.039^{+0.023(0.044)}_{-0.022(0.045)}$	$-0.939^{+0.043(0.087)}_{-0.044(0.087)}$	...	...	...
b_1	$1.50(\times 10^{-4})^{+0.44(0.88)}_{-0.44(0.87)}$	$0.339^{+0.104(0.226)}_{-0.128(0.220)}$	...	...	...
a_1	...	$-0.619^{+0.157(0.299)}_{-0.143(0.289)}$	...	...	...
ω_0	...	...	$-0.905^{+0.055(0.110)}_{-0.055(0.110)}$	...	...
ω_a	...	...	$-0.55^{+0.22(0.43)}_{-0.21(0.43)}$	...	...
q_{de}	$-0.92^{+0.08(0.16)}_{-0.08(0.15)}$	$-0.95^{+0.06(0.11)}_{-0.06(0.11)}$	$-0.86^{+0.08(0.16)}_{-0.08(0.15)}$	$-1.05^{+0.02(0.05)}_{-0.02(0.05)}$	-1
Ω_m	$0.2913^{+0.0043(0.0089)}_{-0.0046(0.0089)}$	$0.2965^{+0.0050(0.0100)}_{-0.0051(0.0100)}$	$0.2971^{+0.0050(0.0100)}_{-0.0050(0.0100)}$	$0.2909^{+0.0043(0.0085)}_{-0.0044(0.0087)}$	$0.2964^{+0.0036(0.0071)}_{-0.0035(0.0072)}$
Ω_{de}	$0.7086^{+0.0046(0.0089)}_{-0.0043(0.0089)}$	$0.7033^{+0.0053(0.0100)}_{-0.0050(0.0100)}$	$0.7028^{+0.0050(0.0100)}_{-0.0050(0.0100)}$	$0.7102^{+0.0044(0.0085)}_{-0.0043(0.0087)}$	$0.7035^{+0.0035(0.0072)}_{-0.0036(0.0071)}$
$\ln \mathcal{L}_m$	-2063.53	-2059.95	-2059.50	-2061.35	-2064.70

the conditions introduced in Sec. II, whereas the others exhibit a different behavior at future times, i.e., for $z \rightarrow -1$.

In addition, the squared sound speed, represented in Fig. 1, shows that $\omega_0\omega_a$ CDM and Padé (1, 1) lead to negative values at late times, indicating unphysical behavior. In contrast, such behavior is avoided in the Padé^q (0, 1) and Padé^ω (0, 1) frameworks, which remain stable also at late times.

As mentioned above, we performed a quantitative comparison of DESI supported $\omega_0\omega_a$ CDM parametrization with the Padé approximants.

For this purpose, we applied the well-established AIC and DIC model selection criteria. The results, shown in Table IV, indicate that the Padé (1, 1) parametrization, together with the $\omega_0\omega_a$ CDM model, emerges as the best-fit model according to the DIC test. The AIC criterion favors the $\omega_0\omega_a$ CDM model, showing only a weak preference over Padé (1, 1) and Padé^q. In contrast, the Λ CDM paradigm remains moderately disfavored according to AIC and strongly disfavored according to DIC, in agreement with the findings reported by the DESI collaboration.

To better quantify the preference for dynamical dark energy in our analysis, we follow the methodology adopted by the DESI collaboration [23], based on the $\Delta\chi^2$ between the dynamical dark energy models and the concordance paradigm, with $\chi^2 = -2 \ln \mathcal{L}_m$.

Indeed, since the Λ CDM model is nested with both Padé cosmology and $\omega_0\omega_a$ CDM parametrization, we can apply the Wilks' theorem [83].

The Wilks' theorem states that, under the null hypothesis, $\Delta\chi^2$ is asymptotically distributed as a χ^2 with ν degrees of freedom, where ν equals the number of additional parameters in the extended model. The

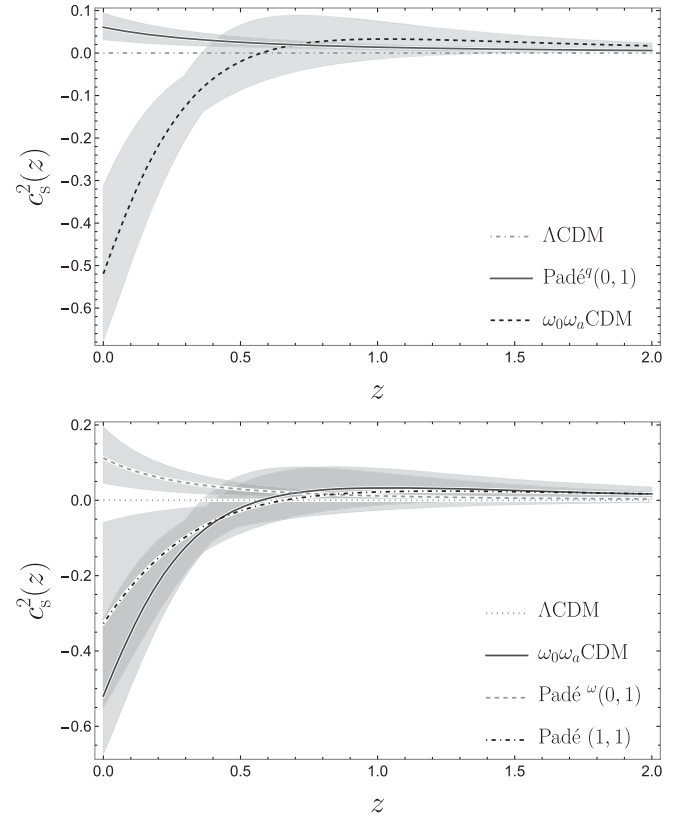


FIG. 1. Sound speed as a function of redshift, obtained from the MCMC analysis of the proposed Padé dark energy parametrization, compared with the Λ CDM and $\omega_0\omega_a$ CDM models. The shaded regions denote the 1σ confidence intervals.

TABLE IV. Model selection criteria and relative differences with respect to the baseline for the considered parametrizations.

$X(\Delta X)$	Padé ^ω (0, 1)	Padé (1, 1)	$\omega_0\omega_a$ CDM	Padé ^q (0, 1)	Λ CDM
AIC (Δ AIC)	4143(8)	4138(3)	4135(0)	4137(2)	4141(6)
DIC (Δ DIC)	4175(7)	4168(0)	4168(0)	4171(3)	4182(14)

corresponding Gaussian significance $N\sigma$ is obtained by equating cumulative probabilities,

$$\text{CDF}_{\chi^2}(\Delta\chi^2|\nu) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\Delta\chi^2} e^{-t^2/2} dt, \quad (36)$$

thus providing a direct conversion between $\Delta\chi^2$ and the equivalent number of Gaussian standard deviations.

Accordingly, we assess the statistical preference for the Padé cosmology over Λ CDM using the corresponding $\Delta\chi^2$ values reported in Table V.

Therefore, the results obtained suggest that the Padé approximants provide a viable alternative to the $\omega_0\omega_a$ CDM parametrization for describing the behavior of dark energy.

Specifically, only the Padé^ω (0, 1) parametrization appears to be statistically disfavored, as it is the closest to Λ CDM due to the small value of b_1 .

From a stability perspective, however, the Padé^ω (0, 1) and Padé^q (0, 1) parametrizations exhibit a more stable behavior compared to the other dynamical dark energy models, showing a positive squared sound speed at late times.

V. FINAL OUTLOOKS

The DESI mission has shown that dynamical dark energy is favored over the cosmological constant. This fact was a *unicum* in the recent developments in cosmology, where experimental tests always preferred the non-evolving cosmological constant and the corresponding Λ CDM paradigm, especially while combining low with high cosmic probes.

Since several exceptions and criticisms against DESI have been raised immediately after the DESI release, it

appears natural to wonder whether dark energy, under the form of a parametrization, is or is not the final framework to describe dark energy at all stages of Universe evolution.

In this respect, as Taylor series in z or a simply suggest that the dark energy equation of state is analytical, it appears natural to work with rational approximations that, instead, are constructed to be more stable as the redshift or scale factor increases.

The main purpose of this work was, in fact, to test rational approximations, constructed starting from the well-known Taylor series, namely coincident with these expansions, up to a given order and by means of the well-established technique offered by Padé polynomials. More precisely, we here adopted Padé^ω (0, 1), (1, 1), and Padé^q (0, 1) parametrizations of dark energy as alternatives to the $\omega_0\omega_a$ CDM model, using DESI DR2 data, i.e., the second DESI release, recently released. Padé polynomials provide a consistent way to regulate the high redshift divergence of Taylor expansions expressed in redshift, whereas the $\omega_0\omega_a$ CDM model corresponds to a Taylor expansion around the scale factor normalized to unity today. In other words, we employed some sort of *Padé cosmology* at the background, invoking expansions on the equations of state and the dark energy deceleration parameter.

Our analysis showed that $\omega_0\omega_a$ CDM is supported by the data; however, Padé (1, 1) also emerges as the best-fit model according to the DIC. Nevertheless, the AIC suggests $\omega_0\omega_a$ CDM as the preferred model, although Padé (1, 1) and Padé^q (0, 1) remain consistent with the data. Concluding, DESI preference for the $\omega_0\omega_a$ CDM is confirmed as claimed by the collaboration and recent findings [20,22,23,84]. At the same time, in line with recent criticisms that showed that there is no clear indication toward the model [28–31,85], we find that alternative parametrizations, such as Padé cosmology, can also provide a valid description of dark energy. In addition, our results certified that the Λ CDM model can be either preferred or discarded in favor of more complicated models than the $\omega_0\omega_a$ CDM parametrization.

The values of the best-fitted cosmological parameters H_0 , Ω_m , Ω_{de} are almost identical for the different parametrizations of the cosmic equation of state. It was shown, by employing MCMC analysis based on the free available CLASS code, modifying the background, and adopting

TABLE V. Difference in the effective $\chi^2 = -2 \ln \mathcal{L}_m$ value for the considered dynamical dark energy model relative to Λ CDM, i.e., $\Delta\chi^2 = \chi^2_{\text{model}} - \chi^2_{\Lambda\text{CDM}}$. The significance is computed assuming k extra free parameters and Wilks' theorem.

Model	$\Delta\chi^2$	Significance
Padé ^ω (0, 1)	−2.36	1.01 σ
Padé (1, 1)	−9.50	2.26 σ
$\omega_0\omega_a$ CDM	−10.40	2.78 σ
Padé ^q (0, 1)	−6.70	2.59 σ

SH0ES data points, together with the OHD data and Planck 2018 CMB data. Obviously, every analysis has been carried out including also the DESI DR2 data, consisting of eight points with refined uncertainties, as compared with the DR1 release.

Then, the best-fitted models have been compared using AIC and DIC information criteria. The overall analysis, as claimed above, suggested that (i) *the best benchmark paradigm for DIC is not only the simple parametrization in terms of a* , as claimed by DESI, (ii) if we consider both AIC and DIC, there is no compelling statistical evidence disfavoring the Padé^q (0, 1), which allows for a positive squared sound speed.

Future works, combining DESI DR2 with other independent cosmic probes, will be essential to further test Padé parametrizations and to clarify whether $\omega_0\omega_a$ CDM can still be regarded as the best dark energy model, as claimed by the DESI collaboration. The most important output of our work was the need to extend the standard parametrizations, claimed as definitive by the collaboration and previous literature. Deeper understanding of significance that the negative sound speed squared of dark energy fluid may have for both structure formation and cosmic evolution would shed light on the nature of dark energy. Remarkably, in Λ CDM scenario dark energy was supposed to be the cosmological constant, with an identically vanishing sound speed. Departing from this scenario toward models exhibiting phantomlike behavior demands carefulness and rigorous physical justification.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: CONTOUR PLOTS

In this Appendix, we report the contour plots obtained with GETDIST, see Fig. 2, together with the details of the MCMC analyses carried out for the different parametrizations. We assess chain convergence using the Gelman-Rubin criterion, requiring $R - 1 < 0.01$ [86].

Moreover, in Table VI, we specify the priors adopted for each model.

APPENDIX B: DECELERATION PARAMETER EVOLUTION

Here, we present the evolution of the deceleration parameter $q(z)$, shown in Fig. 3, evaluated using the mean values obtained from the MCMC analysis. The results indicate that all parametrizations behave consistently at high redshift, i.e., for $z \gg 1$, as expected by construction.

However, the $\omega_0\omega_a$ CDM model exhibits positive values of $q(z)$ as $z \rightarrow -1$, implying a future time behavior that differs from our initial expectations. Furthermore, model-dependent Padé parametrizations also show a slightly nonstandard evolution in this regime as $z \rightarrow -1$, we obtain for Padé^ω (0, 1) and Padé (1, 1), $q \sim -1.06$ and $q \sim -0.23$, respectively.

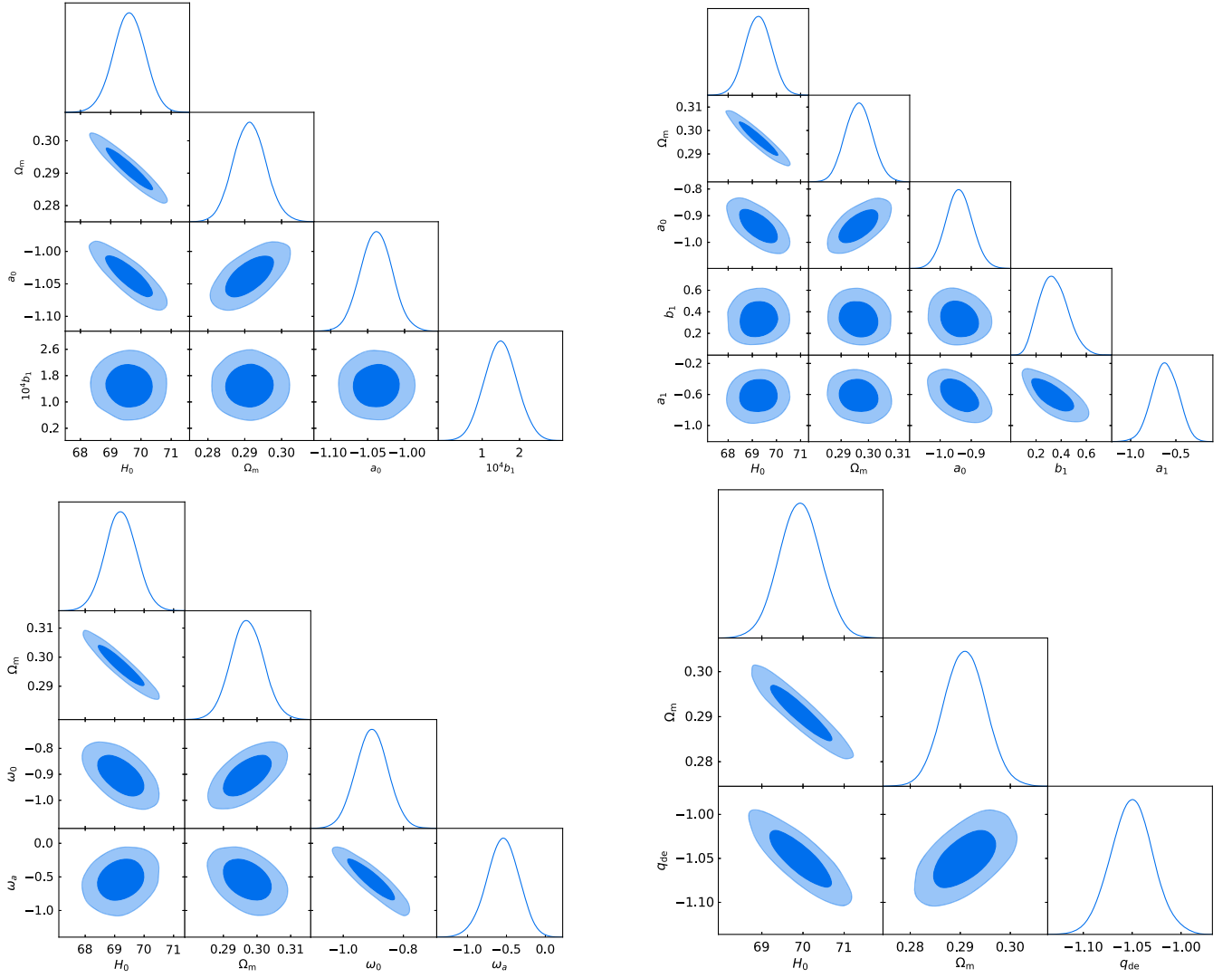


FIG. 2. Contour plots of the cosmological parameters for the three dark energy parametrizations considered in this work, obtained with GETDIST. The top row corresponds to $\text{Padé}^\omega(0, 1)$ and $\text{Padé}(1, 1)$, while the bottom panel shows $\omega_0\omega_a$ CDM and $\text{Padé}^q(0, 1)$.

TABLE VI. Priors adopted for each cosmological model.

Parameter	$\text{Padé}^\omega(0, 1)$	$\text{Padé}(1, 1)$	$\omega_0\omega_a$ CDM	$\text{Padé}^q(0, 1)$	Λ CDM
H_0 (km/s/Mpc)	[50, 80]	[50, 80]	[50, 80]	[50, 80]	[50, 80]
$100\omega_b$	[1.8, 3]	[1.8, 3]	[1.8, 3]	[1.8, 3]	[1.8, 3]
ω_{cdm}	[0.1, 0.2]	[0.1, 0.2]	[0.1, 0.2]	[0.1, 0.2]	[0.1, 0.2]
n_s	[0.9, 1.1]	[0.9, 1.1]	[0.9, 1.1]	[0.9, 1.1]	[0.9, 1.1]
$10^9 A_s$	[1.8, 3]	[1.8, 3]	[1.8, 3]	[1.8, 3]	[1.8, 3]
τ_{reio}	[0.004, 0.12]	[0.004, 0.12]	[0.004, 0.12]	[0.004, 0.12]	[0.004, 0.12]
a_0	$[-3, -0.5]$	$[-3, -0.5]$	...	...	...
b_1	$[10^{-7}, 10^{-2}]$	[0, 3]	...	...	...
a_1	...	$[-3, 1]$	...	...	...
q_{de}	...	...	...	$[-1.5, -0.5]$	...
ω_0	...	...	$[-3, 1]$	...	...
ω_a	...	...	$[-3, 0.75]$	...	...

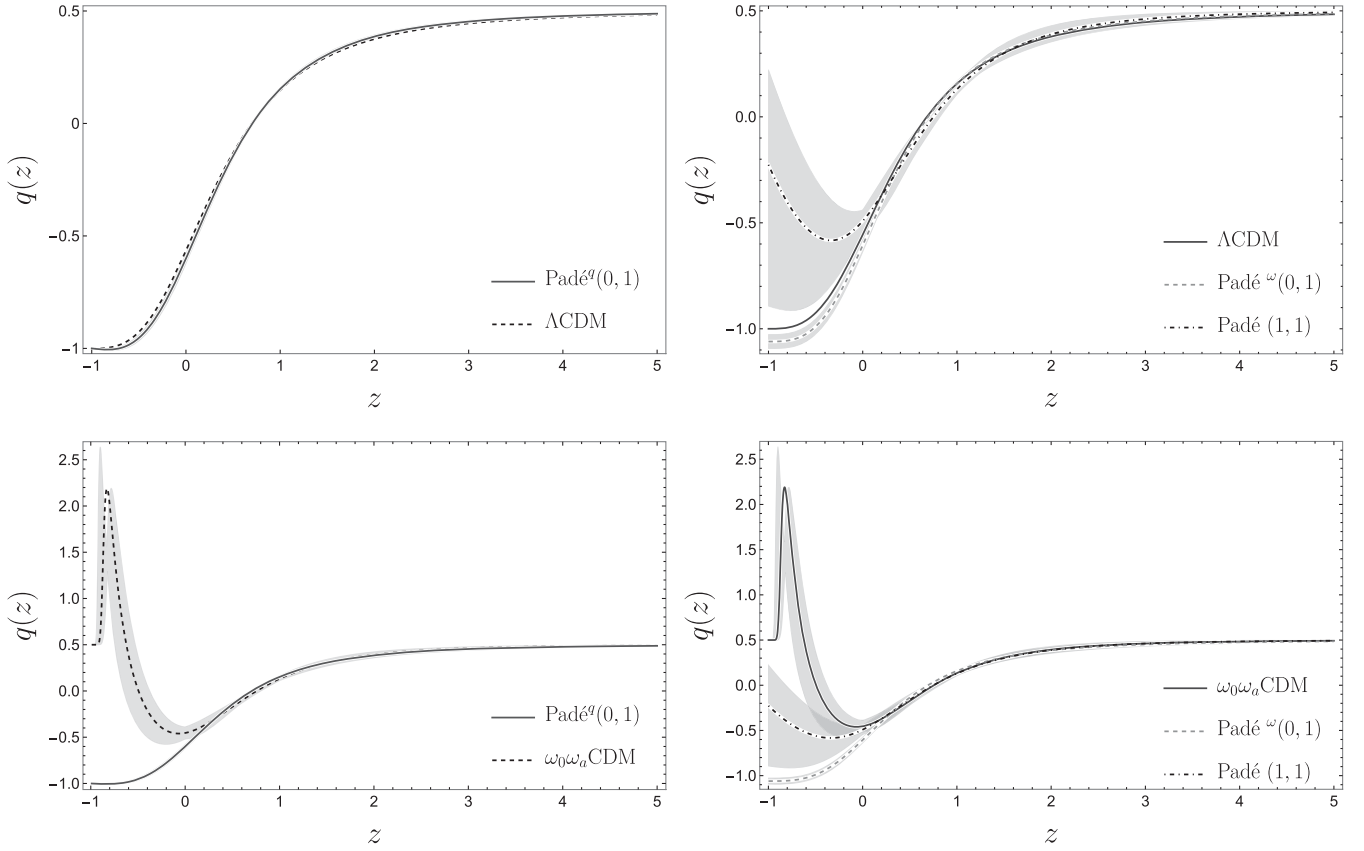


FIG. 3. Reconstructed evolution of the deceleration parameter $q(z)$ based on the MCMC constraints for the proposed Padé dark energy parametrizations, compared to the Λ CDM and $\omega_0\omega_a$ CDM models. The shaded bands indicate the 1σ uncertainty ranges around the mean.

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