

Unified spectroscopic study of bottom mesons and doubly bottom baryons using a relativistic flux-tube model

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Using the relativistic flux tube model with spin-dependent corrections, we present a unified investigation of the mass spectra for singly bottom mesons (B , B_s) and doubly bottom baryons (Ξ_{bb} , Ω_{bb}). Within this framework, the calculated masses for established B and B_s meson states show excellent agreement with experimental data. Leveraging these reliable predictions for low-lying states, we propose spectroscopic assignments for several higher excitations: we identify the $B_J^*(5732)$ resonance as an excellent candidate for the $1P(1^+)$ state; further, we assign the $B_J(5840)$ resonance as the first radial excitation ($2S$), and the $B_J(5970)$ resonance as a $1D$ -wave orbital excitation. In the bottom-strange sector, we predict that the recently observed $B_{sJ}(6063)$ and $B_{sJ}(6114)$ resonances belong to the $1D$ -wave multiplet. Extending our model to doubly bottom baryons, we present comprehensive predictions for the spectra of the Ξ_{bb} and Ω_{bb} baryons, identifying their ground and excited states. These theoretical insights serve as valuable benchmarks, offering guidance for future experimental searches targeting yet-to-be-discovered bottom mesons and doubly bottom baryons.

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I. INTRODUCTION

The spectroscopy of open-bottom hadrons—encompassing bottom mesons (B and B_s) and doubly bottom baryons (Ξ_{bb} and Ω_{bb})—provides a crucial window into the non-perturbative dynamics of quantum chromodynamics (QCD). These systems involve one or more heavy b quarks bound to light quark constituents, situating them at the interface of heavy-quark physics and light-quark confinement. Because the b quark mass is large (of order 4–5 GeV), B and B_s hadrons serve as exemplary testing grounds for heavy quark symmetry and the interplay between the heavy quark and the QCD confining potential. Precise knowledge of their mass spectra is not only important for understanding hadronic structure, but also for refining theoretical models of the strong interaction in the heavy-quark sector.

Over the past two decades, significant experimental progress has been made in the bottom meson sector. The ground-state $B(1S)$ mesons ($B^{0,\pm}$ and B^*), as well as the $B_s(1S)$ mesons (B_s^0 and B_s^*) were established long ago and are well documented in the Particle Data Group (PDG) listings [1]. In 2007, the first orbitally excited bottom mesons were discovered: the narrow P -wave states

$B_1(5721)^{0,+}$ and $B_2^*(5747)^{0,+}$ (with $J^P = 1^+$ and 2^+) were observed by D0 Collaboration [2] and quickly confirmed by CDF Collaboration [3], and their strange counterparts $B_{s1}(5830)$ and $B_{s2}^*(5840)$ were reported by CDF Collaboration [4] in the same year and subsequently confirmed by D0 and LHCb Collaborations [5,6].

In 2013, CDF Collaboration announced the observation of a higher-mass resonance $B(5970)$ in the $B\pi$ channel [7]. LHCb later resolved two resonant structures labeled $B_J(5840)$ and $B_J(5960)$ in proton–proton collision data [8], with properties of $B_J(5960)$ consistent with the earlier $B(5970)$ signal observed by CDF Collaboration. More recently, LHCb Collaboration reported evidence for two excited B_s mesons $B_{sJ}(6064)$ and $B_{sJ}(6114)$ in the B^+K^- invariant mass spectrum [9]. This indicates that as experimental sensitivity has improved, higher excited bottom meson candidates have begun to appear. However, the quantum numbers of some of these higher states remain undetermined. Clarifying the nature of such excitations is an ongoing challenge. In this context, a detailed theoretical spectrum of B and B_s mesons is valuable to interpret new resonances and to guide searches for missing states.

On the baryon side, the situation is more challenging. The Ξ_{bb} and Ω_{bb} baryons, containing two b quarks, have not yet been observed. Nevertheless, a major precedent was set by the discovery of the doubly charmed baryon Ξ_{cc}^{++} in 2017, first observed by LHCb [10]. This marked the first confirmed instance of a baryon with two heavy quarks that provides strong motivation that the bottom–bottom

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counterparts are real and could be found given sufficient data and refined techniques. Knowing their masses within a reasonable range is crucial for guiding experimental searches and optimizing selection criteria.

The progress on the experimental side has been paralleled by extensive theoretical efforts to calculate and classify the spectra of B and B_s mesons and doubly bottom baryons. A variety of theoretical methods have been used, including the relativistic quark model [11–22], nonrelativistic quark model [14,23–33], $^3P_0$ model [34], Regge phenomenology [35], heavy quark effective theory [36], heavy meson chiral perturbation theory [37,38], QCD sum rules [39,40], the union of heavy quark effective theory and chiral perturbation theory [38], and the lattice QCD [41–44].

Although numerous theoretical and experimental efforts have been made, the quark model classification of several excited states—such as $B_J(5840)$, $B_J(5970)$, $B_{sJ}(6063)$, and $B_{sJ}(6114)$ —remains ambiguous. Moreover, the ground states of doubly bottom baryons have not yet been experimentally confirmed. As the theoretical approaches mentioned so far offer a different perspective on the mass spectrum, further theoretical studies are essential. In the current study, we employ the relativistic flux-tube derived mass formula to compute spin-averaged masses. Crucially, our methodology integrates spin-dependent interactions, enabling predictions of hyperfine-structure splittings within a jj coupling scheme suitable for heavy-light systems. We aim to identify the masses of fairly high orbital and radial excited states. This approach has been validated in our previous research, successfully reproducing mass spectra for singly heavy baryons [45–47], open-charm mesons, and doubly charm baryons [48]. In the present work, we extend the application of this model to open-bottom mesons and doubly bottom baryons. Based on the predicted mass spectra, we further propose quark model classification for several experimentally observed charmed meson states whose quantum numbers remain unconfirmed. Additionally, we provide theoretical predictions for the excitation spectrum of the doubly bottom baryons Ξ_{bb} and Ω_{bb} , which may serve as valuable input for future experimental searches aimed at discovering and classifying their excited states.

The structure of the paper is organized as follows. In Sec. II, we outline the relativistic flux-tube model as applied to bottom mesons and doubly bottom baryons, and detail the methodology used to compute their mass spectra, including the treatment of spin-dependent interactions. In Sec. III, we present our computed spectra for B , B_s , Ξ_{bb} , and Ω_{bb} , and compare these results with existing theoretical predictions and available experimental data. Section IV contains our conclusions and outlook, summarizing the main findings.

II. THEORETICAL FRAMEWORK

We employ the relativistic flux tube model to describe both the bottom mesons (B, B_s) and the doubly bottom

baryons (Ξ_{bb}, Ω_{bb}) in a unified two-body picture. In the case of a meson, the two constituents are the bottom quark ($b/\bar{b}$) and a light quark ($q/\bar{q}$), while for a doubly bottom baryon the constituents are a tightly bound $\{bb\}$ diquark and a light quark ($q = u, d$ for Ξ_{bb} or s for Ω_{bb}). These two constituents are connected by a flux tube—a tubelike structure containing a gluonic field—with constant tension T that provides the confining potential. The heavy and light constituents bound by the flux tube rotate around their common center of mass with quantized orbital angular momentum. The light component is highly relativistic (effectively moving at velocity $v \approx c$), whereas the heavy component moves more slowly due to its large mass. In this limit, the flux-tube model yields a linear Regge-like relation between the mass ($\bar{M}$) and the orbital angular momentum quantum number L . Specifically, neglecting spin interactions, the spin-averaged mass $\bar{M}$ satisfies (for both mesons and baryons) a relation of the form [45,49],

$$(\bar{M} - m_h)^2 = \sigma \frac{L}{2} + (m_l + m_h v_h^2)^2. \quad (1)$$

Here, $\sigma = 2\pi T$. The factor v_h is the speed of the heavy component introduced to account for relativistic effects. Similarly, the light component has velocity v_l . The dynamical masses for the rotating light component (having mass m_{0_l}) and heavy component (having mass m_{0_h}) are $m_l = m_{0_l}/\sqrt{1 - v_l^2}$ and $m_h = m_{0_h}/\sqrt{1 - v_h^2}$, respectively.

The distance between the heavy and light constituents in this model is given as [45,49]

$$r = (v_l + v_h) \sqrt{\frac{8L}{\sigma}}. \quad (2)$$

In the framework of a two-body heavy–light hadronic system, the quantum solution of the relativistic flux-tube model predicts that the Regge trajectories in the $(L, (\bar{M} - m_h)^2)$ plane—corresponding to various radial excitations—are both parallel and equidistant [50–52]. This observation necessitates a refinement of the semiclassical expressions (1) and (2) to include the radial excitations, wherein L is replaced with $\lambda n_r + L$. Here, $n_r = n - 1$, with n denoting the principal quantum number (i.e., $n = 1, 2, 3, \dots$) that indexes the radial excitation states. The resulting relationships are expressed as follows [45,49]:

$$(\bar{M} - m_h)^2 = \sigma \frac{[L + \lambda n_r]}{2} + (m_l + m_h v_h^2)^2, \quad (3)$$

$$r = (v_l + v_h) \sqrt{\frac{8[L + \lambda n_r]}{\sigma}}. \quad (4)$$

In this formulation, λ serves as a model parameter that quantifies the vertical separation between Regge trajectories

corresponding to different principal quantum numbers $n = 1, 2, 3, \dots$ in the $(L, (\bar{M} - m_h)^2)$ plane.

Having obtained the spin-averaged mass $\bar{M}$ of each level, we now include spin-dependent interactions. For a heavy-light meson and baryon, the total angular momentum $\mathbf{J}$ arises from coupling the orbital angular momentum $\mathbf{L}$ with the spins of the heavy and light constituents ($\mathbf{S}_h$ and $\mathbf{S}_l$, respectively). There are two possible coupling schemes to couple $\mathbf{S}_h$, $\mathbf{S}_l$ and $\mathbf{L}$: L - S coupling, where $\mathbf{S}_h$ and $\mathbf{S}_l$ are first combined into a total spin $\mathbf{S} = \mathbf{S}_h + \mathbf{S}_l$, which is then coupled to $\mathbf{L}$ to give $\mathbf{J}$, and j - j coupling, where one first adds the light spin to the orbital momentum ($\mathbf{j} = \mathbf{L} + \mathbf{S}_l$ is the total angular momentum of the light degrees of freedom) and then couples $\mathbf{j}$ with the heavy spin $\mathbf{S}_h$ to obtain $\mathbf{J}$. Owing to heavy quark symmetry, the j - j coupling scheme is the natural choice for heavy-light systems. We therefore adopt the j - j coupling scheme for both B/B_s mesons (where the spin quantum number of the heavy constituent is $S_h = \frac{1}{2}$ for the b quark) and doubly bottom baryons (where the spin quantum number of heavy constituent S_h is the total angular momentum of the $\{bb\}$ diquark). In this scheme, each orbital excitation L splits into multiplet states characterized by the quantum numbers j and J . The splittings around the spin-averaged mass $\bar{M}$ are governed by spin-dependent interactions.

We include the standard spin-orbit, spin-spin, and tensor interaction terms as a perturbation to the flux-tube spectrum. The Hamiltonian for spin-dependent effects can be written in the form

$$\Delta M = H_{so} + H_t + H_{ss}. \quad (5)$$

The spin-orbit interaction term, H_{so} , originates from the combined effects of short-range color magnetic interactions and long-range Thomas precession. It is expressed as a coupling between the orbital angular momentum $\mathbf{L}$ and the spin operators of the light and heavy constituents, $\mathbf{S}_l$ and $\mathbf{S}_h$, respectively [28],

$$H_{so} = \left[\left(\frac{2\alpha}{3r^3} - \frac{b_0}{2r} \right) \frac{1}{m_l^2} + \frac{4\alpha}{3r^3} \frac{1}{m_h m_l} \right] \mathbf{L} \cdot \mathbf{S}_l + \left[\left(\frac{2\alpha}{3r^3} - \frac{b_0}{2r} \right) \frac{1}{m_h^2} + \frac{4\alpha}{3r^3} \frac{1}{m_h m_l} \right] \mathbf{L} \cdot \mathbf{S}_h \quad (6)$$

where α is the strong coupling constant, and b_0 characterizes the long-range confining interaction. The tensor interaction term, H_t , arises from the long-range component of the color-magnetic dipole-dipole interaction. It is represented as [28]

$$H_t = \frac{4\alpha}{3r^3 m_h m_l} \hat{\mathbf{B}}, \quad (7)$$

TABLE I. Expectation values of the spin-orbit operators $\mathbf{L} \cdot \mathbf{S}_l$ and $\mathbf{L} \cdot \mathbf{S}_h$, the tensor operator $\hat{\mathbf{B}}$, and the contact hyperfine term $\mathbf{S}_h \cdot \mathbf{S}_l$ in the $|J, j\rangle$ basis for heavy-light mesons. Results are listed for every allowed (L, J, j) configuration (up to G wave) [48].

Quantum numbers			$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle$	$\langle \mathbf{L} \cdot \mathbf{S}_h \rangle$	$\langle \hat{\mathbf{B}} \rangle$	$\langle \mathbf{S}_h \cdot \mathbf{S}_l \rangle$
L	j	J				
0	1/2	0	0	0	0	-3/4
0	1/2	1	0	0	0	1/4
1	1/2	0	-1	-1	-1	1/4
1	1/2	1	-1	1/3	1/3	-1/12
1	3/2	1	1/2	-5/6	1/6	-5/12
1	3/2	2	1/2	1/2	-1/10	1/4
2	3/2	1	-3/2	-3/2	-1/2	1/4
2	3/2	2	-3/2	9/10	3/10	-3/20
2	5/2	2	1	-7/5	1/5	-7/20
2	5/2	3	1	1	-1/7	1/4
3	5/2	2	-2	-2	-2/5	1/4
3	5/2	3	-2	10/7	2/7	-5/28
3	7/2	3	3/2	-27/14	3/14	-9/28
3	5/2	4	3/2	3/2	-1/6	1/4
4	7/2	3	-5/2	-5/2	-5/14	1/4
4	7/2	4	-5/2	35/18	5/18	-7/36
4	9/2	4	2	-22/9	2/9	-11/36
4	9/2	5	2	2	-2/11	1/4
5	9/2	4	-3	-3	-1/3	1/4
5	9/2	5	-3	27/11	3/11	-9/44
5	11/2	5	5/2	-65/22	5/22	-13/44
5	11/2	6	5/2	5/2	-5/26	1/4

with the tensor operator $\hat{\mathbf{B}}$ defined as: $\hat{\mathbf{B}} = 3(\mathbf{S}_l \cdot \mathbf{r})(\mathbf{S}_h \cdot \mathbf{r})/r^2 - \mathbf{S}_l \cdot \mathbf{S}_h$. The spin-spin contact interaction, H_{ss} , corresponds to a short-range hyperfine term and is written as [28],

$$H_{ss} = \frac{32\pi\alpha\sigma_0^3}{9\sqrt{\pi}m_h m_l} e^{-\sigma_0^2 r^2} \mathbf{S}_l \cdot \mathbf{S}_h, \quad (8)$$

where σ_0 denotes the width parameter.

We have evaluated the expectation values of the operators $\mathbf{L} \cdot \mathbf{S}_l$, $\mathbf{L} \cdot \mathbf{S}_h$, $\mathbf{S}_l \cdot \mathbf{S}_h$, and $\hat{\mathbf{B}}$ in the $|J, j\rangle$ basis, and present the results in Tables I and II. The detailed calculation for these expectation values for the S -, P -, and D -wave states have been computed in our earlier work [48], while those corresponding to the F - and G -wave states are detailed in the Appendix. Finally, the physical mass of the state is obtained by adding this spin correction to the spin-averaged mass: $\bar{M} + \Delta M$. In this way, the relativistic flux tube model provides a complete framework to calculate the mass spectrum of heavy-light mesons (B and B_s) and doubly heavy baryons (Ξ_{bb} and Ω_{bb}).

Now we proceed to discussion on selection and determination of the model parameters for the system of heavy-light mesons and doubly heavy baryons.

TABLE II. Same set of expectation values as in Table I, but for doubly bottom baryons treated in the quark–diquark picture with axial-vector diquark ($S_h = 1$) [48].

Quantum numbers						
L	j	J	$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle$	$\langle \mathbf{L} \cdot \mathbf{S}_h \rangle$	$\langle \hat{\mathbf{B}} \rangle$	$\langle \mathbf{S}_h \cdot \mathbf{S}_l \rangle$
0	1/2	1/2	0	0	0	-1
0	1/2	3/2	0	0	0	1/2
1	1/2	1/2	-1	-4/3	-4/3	1/3
1	3/2	1/2	1/2	-5/3	1/3	-5/6
1	1/2	3/2	-1	2/3	2/3	-1/6
1	3/2	3/2	1/2	-2/3	2/15	-1/3
1	3/2	5/2	1/2	1	-1/5	1/2
2	3/2	1/2	-3/2	-3	-1	1/2
2	3/2	3/2	-3/2	-6/5	-2/5	1/5
2	5/2	3/2	1	-14/5	2/5	-7/10
2	3/2	5/2	-3/2	9/5	3/5	-3/10
2	5/2	5/2	1	-4/5	4/35	-1/5
2	5/2	7/2	1	2	-2/7	1/2
3	5/2	1/2	-2	-4	-4/5	1/2
3	5/2	5/2	-2	-8/7	-8/35	1/7
3	7/2	5/2	3/2	-27/7	3/7	-9/14
3	5/2	7/2	-2	20/7	4/7	-5/14
3	7/2	7/2	3/2	-6/7	2/21	-1/7
3	7/2	9/2	3/2	3	-1/3	1/2
4	7/2	5/2	-5/2	-5	-5/7	1/2
4	7/2	7/2	-5/2	-70/63	-10/63	7/63
4	9/2	7/2	2	-44/9	4/9	-11/18
4	7/2	9/2	-5/2	35/9	5/9	-7/18
4	9/2	9/2	2	-8/9	8/99	-1/9
4	9/2	11/2	2	4	-4/11	1/2

A. Heavy-light mesons

We employed the IMINUIT Python package [53], which implements the robust MINUIT minimization algorithm developed by CERN [53], to determine optimal values and associated uncertainties of our model parameters for the heavy-light mesons (D , D_s , B , B_s). The input data for this optimization procedure included experimentally measured masses, along with their uncertainties, for well established mesonic states [1]: $D^\pm$, D^0 , $D^*(2007)^0$, $D^*(2010)^\pm$, $D_0^*(2300)$, $D_1(2420)$, $D_1(2430)^0$, $D_2^*(2460)$, $D_3^*(2750)$, $D_s^\pm$, $D_s^{*\pm}$, $D_{s1}(2536)^\pm$, $D_{s2}^*(2573)$, $D_{s1}^*(2700)^\pm$, $D_{s3}^*(2860)^\pm$, $B^{\pm,0}$, B^* , $B_1(5721)$, $B_2^*(5747)$, B_s^0 , B_s^* , $B_{s1}(5830)^0$, $B_{s2}^*(5840)^0$, mass of charm quark $m_c = 1273 \pm 4.6$ MeV, and mass of bottom quark $m_b = 4183 \pm 7$ MeV. The best-fit parameter values obtained, along with their statistical uncertainties, are as follows: the dynamical mass of the charm quark $m_c = 1375.68 \pm 0.15$ MeV, the bottom quark $m_b = 4431.08 \pm 0.16$ MeV, the up/down quark $m_{u/d} = 399.95 \pm 0.02$ MeV, and the strange quark $m_s = 484.29 \pm 0.06$ MeV. The string tensions determined for various mesons are: $\sigma_D = (1.5321 \pm 0.0023) \times 10^6$ MeV², $\sigma_{D_s} = (1.7882 \pm 0.0016) \times 10^6$ MeV²,

$\sigma_B = (1.7541 \pm 0.0031) \times 10^6$ MeV², and $\sigma_{B_s} = (1.9931 \pm 0.0006) \times 10^6$ MeV². Additional model parameters include $\lambda = 1.479 \pm 0.015$, $b_0 = (36.2 \pm 1.2) \times 10^3$ MeV², $\sigma_0 = 502.8 \pm 2.8$ MeV, and the coupling constant for B and B_s mesons $\alpha = 0.317 \pm 0.0004$. Specifically, from the B and B_s meson sectors, experimentally measured masses of the $1S(0^-, 1/2)$, $1S(1^-, 1/2)$, $1P(1^+, 3/2)$, and $1P(2^+, 3/2)$ states are used for fitting the model parameters. With these fitted parameters the masses of all higher excited B and B_s mesonic states can be predicted.

B. Doubly heavy baryons with the diquark in the ground state

Due to the Pauli exclusion principle, the diquark in the ground state (in internal $1S$ -wave) within doubly bottom baryon must form an axial-vector diquark, denoted as $\{bb\}$ with spin $S_h = 1$.

For doubly bottom baryons (Ξ_{bb} and Ω_{bb}), we set the model parameters $m_{u/d}$, m_s , λ , b_0 , and σ_0 identical to those for the B and B_s mesons. The mass of the heavy diquark ($\{bb\}$) in its internal $1S$ wave is approximated as the sum of individual bottom quark masses, yielding $m_{0\{bb\}} = 2m_{0_b} = 8366$ MeV [1]. Due to the scarcity of experimental measurements for doubly heavy baryons, except for the Ξ_{cc}^{++} , it is essential to leverage theoretical predictions for $1S$ -wave states to constrain our parameters. A notable theoretical prediction by Ebert *et al.* [19], estimating the Ξ_{cc} ground state mass at 3620 MeV, closely matched the subsequent experimental discovery by the LHCb Collaboration at 3621.55 MeV [54]. Encouraged by this precision, we adopted theoretical predictions from Ref. [19] for the ground-state masses of Ξ_{bb} and Ω_{bb} baryons, enabling us to estimate the dynamical diquark mass as $m_{\{bb\}} = 8861.7$ MeV and coupling constant as $\alpha = 0.324$. Using the mass of the ($\{bb\}$) diquark in its internal $1S$ wave $m_{0\{bb\}} = 8366$ MeV and the dynamical mass of the diquark $m_{\{bb\}} = 8861.7$ MeV, the speed of the heavy diquark is extracted to be the $v_h = 0.33 \pm 0.0022$.

Using the mass of the diquark $\{bb\}$ in its internal $1S$ wave, $m_{0\{bb\}} = 8366 \pm 8$ MeV, and the dynamical mass of the diquark, $m_{\{bb\}} = 8861.7 \pm 0.000006$ MeV, the speed of the heavy diquark is obtained as $v_h = 0.33 \pm 0.0022$.

In the heavy quark limit ($m_h \rightarrow \infty$), two heavy quarks forming a diquark inside a baryon behave similarly to a heavy antiquark inside a meson [55]. Consequently, the light quark in a doubly heavy baryon experiences the same color field environment as in heavy-light mesons. Heavy-quark diquark symmetry, therefore, implies identical string tensions at leading order for such systems. However, when considering finite quark masses, slight deviations from exact symmetry occur. These finite mass effects can be effectively captured by empirical power-law scaling relations connecting string tensions and heavy quark masses [56],

$$\frac{\sigma_B}{\sigma_D} = \left(\frac{m_b}{m_c}\right)^P, \quad \frac{\sigma_B}{\sigma_{\Xi_{bb}}} = \left(\frac{m_b}{m_{\{bb\}}}\right)^P. \quad (9)$$

Analogously, for strange heavy-light systems, the scaling relations are given by [56],

$$\frac{\sigma_{B_s}}{\sigma_{D_s}} = \left(\frac{m_b}{m_c}\right)^Q, \quad \frac{\sigma_{B_s}}{\sigma_{\Omega_{bb}}} = \left(\frac{m_b}{m_{\{bb\}}}\right)^Q. \quad (10)$$

Using the fitted parameters σ_D , σ_{D_s} , σ_B , σ_{B_s} , m_c , and m_b in Eqs. (9) and (10) the scaling parameters were extracted as $P = 0.1157 \pm 0.002$ and $Q = 0.0927 \pm 0.0017$. Substituting these values back into in Eqs. (9) and (10), the string tensions for the baryonic systems were calculated as $\sigma_{\Xi_{bb}} = (1.901 \pm 0.008) \times 10^6 \text{ MeV}^2$ and $\sigma_{\Omega_{bb}} = (2.125 \pm 0.007) \times 10^6 \text{ MeV}^2$. So, the model parameters determined from the mesonic sector can facilitate reliable mass predictions for Ξ_{bb} , and Ω_{bb} baryons.

C. Doubly heavy baryons with the diquark in an excited state

We now consider configurations in which the two bottom quarks are excited relative to each other inside the diquark. In the internal excitations, the bb diquark must be in a spin-singlet configuration ($S_h = 0$) for the $1P$ and $2P$ waves, and in a spin-triplet configuration ($S_h = 1$) for the $2S$ wave.

We can calculate the masses for Ξ_{bb} and Ω_{bb} states where the light quark is excited orbitally or radially with respect to the heavy diquark in the internal $2S$, $1P$, and $2P$ waves using Eqs. (3), (6)–(8). To apply these relations we first calculate the mass of diquarks $m_{h_{2S}}$, $m_{h_{1P}}$, and $m_{h_{2P}}$ in the internal $2S$, $1P$, and $2P$ waves, respectively.

To this end, we first consider the bottomonium in relativistic flux tube model. A bottom quark and antibottom quark of mass m_b connected by a flux tube and carrying relative orbital angular momentum L and principle quantum number $n = n_r + 1$ obey a Regge-like relation for the spin-averaged mass $\bar{M}$ [57],

$$\bar{M} = 2m_b + \left(\frac{\sigma_{b\bar{b}}^2}{2\pi^2 m_b}\right)^{\frac{1}{3}} (\lambda n_r + L)^{\frac{2}{3}}, \quad (11)$$

where σ is the string tension for bottomonium. Using this relation, the relation between the spin average mass for $1S$ and $1P$ waves is

$$\bar{M}_{1P} = \bar{M}_{1S} + \left(\frac{\sigma_{b\bar{b}}^2}{\pi^2 \bar{M}_{1S}}\right)^{\frac{1}{3}}. \quad (12)$$

We use the measured masses of $1S$ and $1P$ waves of bottomonium, named $\eta_b(1S)$, $\Upsilon(1S)$, $\chi_{b0}(1P)$, $\chi_{b1}(1P)$, $h_b(1P)$, and $\chi_{b2}(1P)$, to calculate the spin average masses of $1S$ and $1P$ waves. With that we extract $\sigma_{b\bar{b}} = (2.961 \pm 0.006) \times 10^6 \text{ MeV}^2$.

Building upon this framework for bottomonium, we then extend Eq. (11) to the bb diquark, specified by the principal quantum number $n_d = n_{rd} + 1$ and orbital angular momentum quantum number L_d as

$$\bar{M} = 2m_b + \left(\frac{\sigma_{bb}^2}{2\pi^2 m_b}\right)^{\frac{1}{3}} (\lambda n_{rd} + L_d)^{\frac{2}{3}}. \quad (13)$$

This leads to the following relations for the spin-averaged mass of the diquark in $1P$, $2S$, and $2P$ waves:

$$\bar{M}_{1P} = \bar{M}_{1S} + \left(\frac{\sigma_{bb}^2}{\pi^2 \bar{M}_{1S}}\right)^{\frac{1}{3}}, \quad (14)$$

$$\bar{M}_{2S} = \bar{M}_{1S} + \left(\frac{\sigma_{bb}^2}{\pi^2 \bar{M}_{1S}}\right)^{\frac{1}{3}} \lambda^{\frac{2}{3}}, \quad (15)$$

$$\bar{M}_{2P} = \bar{M}_{1S} + \left(\frac{\sigma_{bb}^2}{\pi^2 \bar{M}_{1S}}\right)^{\frac{1}{3}} (\lambda + 1)^{\frac{2}{3}}. \quad (16)$$

The inverse Regge slope $1/\alpha'$ (and hence the flux-tube string tension σ , via $1/\alpha' \sim \sigma$) is proportional to the square root of the color factor: $1/\alpha' \propto \sqrt{C\alpha_s}$, where C is the quadratic Casimir of the color sources at the ends of the tube [58]. For a bottom diquark in the antitriplet, $bb[\bar{3}_c]$, one has $C = 2/3$; for a bottom quark-antiquark color singlet, $b\bar{b}[1_c]$, $C = 4/3$. Consequently, the corresponding string tensions satisfy

$$\sigma_{bb} = \sqrt{\frac{2/3}{4/3}} \sigma_{b\bar{b}} = \frac{1}{\sqrt{2}} \sigma_{b\bar{b}}. \quad (17)$$

With this relation, and using the value of $\sigma_{b\bar{b}}$, the string tension between the two bottom quarks in the bb diquark is obtained as $\sigma_{bb} = (2.094 \pm 0.004) \times 10^6 \text{ MeV}^2$. Employing this value of σ_{bb} together with the previously determined $\bar{M}_{1S} = 8366 \pm 8 \text{ MeV}$ and $\lambda = 1.479 \pm 0.015$, we extract the spin-averaged masses of the excited diquark states using Eqs. (14)–(16) as $\bar{M}_{2S} = 8854 \pm 9 \text{ MeV}$, $\bar{M}_{1P} = 8742 \pm 8 \text{ MeV}$, and $\bar{M}_{2P} = 9054 \pm 8 \text{ MeV}$.

These excited diquarks are considered in either a spin-singlet configuration ($S_h = 0$) in the internal $1P$ and $2P$ waves, or a spin-triplet configuration ($S_h = 1$) in the internal $2S$ wave, where they are connected to the light quark via a flux tube, and the entire system revolves around its center of mass. The dynamical masses of the rotating diquarks, with velocity $v_h = 0.33 \pm 0.0022$, are extracted as $m_{h_{2S}} = 9379 \pm 12 \text{ MeV}$, $m_{h_{1P}} = 9261 \pm 11 \text{ MeV}$, and $m_{h_{2P}} = 9592 \pm 12 \text{ MeV}$. Using these dynamical masses of the diquarks in excited states, we calculate the spectra of the Ξ_{bb} and Ω_{bb} baryons, where the light quark is excited either orbitally or radially with respect to the heavy diquark in the internal $2S$, $1P$, and $2P$ waves. These calculations are

TABLE III. Masses of B meson (in MeV).

$nL(J^P, j)$	Mass	PDG [1]	[11]	[12]	[13]	[14]
$1S(0^-, 1/2)$	5279 ± 15	5279.57 ± 0.05	5280	5312	5309	5279
$1S(1^-, 1/2)$	5325 ± 15	5324.75 ± 0.20	5326	5371	5369	5325
$2S(0^-, 1/2)$	5870 ± 10		5890	5904	5904	5876
$2S(1^-, 1/2)$	5872 ± 10		5906	5933	5934	5899
$1P(0^+, 1/2)$	5670 ± 11		5749	5756	5796	5722
$1P(1^+, 1/2)$	5696 ± 11	5698 ± 8	5774	5777	5756	5716
$1P(1^+, 3/2)$	5722 ± 10	$5726.0^{+2.5}_{-2.7}$	5723	5784	5782	5753
$1P(2^+, 3/2)$	5738 ± 10	5737.3 ± 0.7	5741	5797	5779	5727
$2P(0^+, 1/2)$	6152 ± 9		6221	6213	6213	
$2P(1^+, 1/2)$	6159 ± 9		6281	6197	6214	
$2P(1^+, 3/2)$	6144 ± 8		6209	6228	6219	
$2P(2^+, 3/2)$	6147 ± 8		6260	6213	6206	
$1D(1^-, 3/2)$	6022 ± 9		6119	6110	6105	6056
$1D(2^-, 3/2)$	6033 ± 8		6121	6095	6110	5973
$1D(2^-, 5/2)$	6014 ± 8		6103	6124	6113	6067
$1D(3^-, 5/2)$	6022 ± 8		6091	6106	6108	5979
$2D(1^-, 3/2)$	6401 ± 8		6534	6475	6459	
$2D(2^-, 3/2)$	6406 ± 8		6554	6450	6475	
$2D(2^-, 5/2)$	6375 ± 8		6528	6486	6472	
$2D(3^-, 5/2)$	6378 ± 8		6542	6460	6464	
$1F(2^+, 5/2)$	6293 ± 7		6412	6387	6364	
$1F(3^+, 5/2)$	6301 ± 7		6420	6358	6387	
$1F(3^+, 7/2)$	6259 ± 7		6391	6396	6380	
$1F(4^+, 7/2)$	6265 ± 7		6380	6364	6375	
$2F(2^+, 5/2)$	6621 ± 7					
$2F(3^+, 5/2)$	6626 ± 7					
$2F(3^+, 7/2)$	6581 ± 7					
$2F(4^+, 7/2)$	6584 ± 7					
$1G(3^-, 7/2)$	6527 ± 7		6664	6622		
$1G(4^-, 7/2)$	6533 ± 7		6652	6588		
$1G(4^-, 9/2)$	6476 ± 7		6648	6628		
$1G(5^-, 9/2)$	6481 ± 6		6634	6592		
$1H(4^+, 9/2)$	6736 ± 6					
$1H(5^+, 9/2)$	6741 ± 6					
$1H(5^+, 11/2)$	6672 ± 6					
$1H(6^+, 11/2)$	6676 ± 6					

performed through Eqs. (3), (6)–(8), employing the previously fitted parameters $m_{u/d}$, m_s , λ , b_0 , σ_0 , v_h , $\sigma_{\Xi_{bb}}$, and $\sigma_{\Omega_{bb}}$.

III. RESULTS AND DISCUSSION

A. Bottom and bottom-strange mesons

In this section, we present and discuss the theoretical results obtained for the mass spectra of B and B_s mesons within our model framework. We adopt the notation $nL(J^P, j)$ for B and B_s meson states, where n and L represent the quantum numbers of the radial and orbital excitations, respectively. The symbols J , P , and j denote the total angular momentum quantum number, parity, and the total angular momentum of the light quark. The calculated masses are listed in Tables III and IV and are systematically compared to available experimental data

from the PDG [1] as well as to predictions from other theoretical studies [11–14].

1. B meson

Experimentally, the $1S$ states of the B meson are well established. Specifically, the states $B^{0,\pm}$ and B^* have been experimentally confirmed with masses $M_{B^0} = 5279.41 \pm 0.07$ MeV, $M_{B^\pm} = 5279.72 \pm 0.08$ MeV, and $M_{B^*} = 5324.75 \pm 0.20$ MeV [1]. Our model accurately reproduces these observed masses, thereby validating our theoretical approach for low-lying states.

For the $1P$ states of the B meson, the $B_1(5721)$ and $B_2^*(5747)$ resonances are experimentally well established. These states, denoted as $B_1(5721)^{0,+}$ and $B_2^*(5747)^{0,+}$, were initially observed by the D0 Collaboration [2] and subsequently confirmed by both the CDF Collaboration [3]

TABLE IV. Masses of B_s meson (in MeV).

$nL(J^P, j)$	Mass	PDG [1]	[11]	[12]	[13]	[14]
$1S(0^-, 1/2)$	5369 ± 15	5366.93 ± 0.1	5372	5394	5390	5367
$1S(1^-, 1/2)$	5407 ± 15	5415.4 ± 1.4	5414	5450	5447	5415
$2S(0^-, 1/2)$	5981 ± 10		5976	5984	5985	5944
$2S(1^-, 1/2)$	5984 ± 10		5992	6012	6013	5966
$1P(0^+, 1/2)$	5772 ± 11		5833	5831	5875	5788
$1P(1^+, 1/2)$	5798 ± 11		5865	5857	5830	5810
$1P(1^+, 3/2)$	5825 ± 10	5828.73 ± 0.2	5831	5861	5859	5821
$1P(2^+, 3/2)$	5841 ± 10	5839.88 ± 0.12	5842	5876	5858	5821
$2P(0^+, 1/2)$	6275 ± 9		6318	6279	6295	
$2P(1^+, 1/2)$	6282 ± 9		6345	6279	6279	
$2P(1^+, 3/2)$	6273 ± 9		6321	6296	6291	
$2P(2^+, 3/2)$	6276 ± 9		6359	6295	6284	
$1D(1^-, 3/2)$	6135 ± 9		6209	6182	6178	6101
$1D(2^-, 3/2)$	6146 ± 9		6218	6169	6181	6061
$1D(2^-, 5/2)$	6137 ± 8		6189	6196	6185	6113
$1D(3^-, 5/2)$	6145 ± 8		6191	6179	6180	6067
$2D(1^-, 3/2)$	6536 ± 8		6629	6542	6534	
$2D(2^-, 3/2)$	6541 ± 8		6651	6526	6542	
$2D(2^-, 5/2)$	6521 ± 8		6625	6553	6542	
$2D(3^-, 5/2)$	6524 ± 8		6637	6535	6536	
$1F(2^+, 5/2)$	6418 ± 7		6501	6454	6431	
$1F(3^+, 5/2)$	6426 ± 7		6515	6425	6453	
$1F(3^+, 7/2)$	6400 ± 7		6468	6462	6446	
$1F(4^+, 7/2)$	6406 ± 7		6475	6432	6441	
$2F(2^+, 5/2)$	6767 ± 7					
$2F(3^+, 5/2)$	6771 ± 7					
$2F(3^+, 7/2)$	6741 ± 7					
$2F(4^+, 7/2)$	6744 ± 7					
$1G(3^-, 7/2)$	6664 ± 7		6753	6685		
$1G(4^-, 7/2)$	6670 ± 7		6762	6650		
$1G(4^-, 9/2)$	6632 ± 6		6715	6690		
$1G(5^-, 9/2)$	6637 ± 6		6726	6654		
$1H(4^+, 9/2)$	6884 ± 6					
$1H(5^+, 9/2)$	6889 ± 6					
$1H(5^+, 11/2)$	6842 ± 6					
$1H(6^+, 11/2)$	6846 ± 6					

and the LHCb Collaboration [8]. According to the PDG, the masses of the $B_1(5721)$ and $B_2^*(5747)$ resonances are reported as $5726^{+2.5}_{-2.7}$ MeV and 5737.3 ± 0.7 MeV, respectively. Our theoretical predictions closely reproduce these measured masses, assigning the $B_1(5721)$ resonance as the $1P(1^+, 3/2)$ state and the $B_2^*(5747)$ resonance as the $1P(2^+, 3/2)$ state. Thus, it is evident that these resonances constitute the $1P(1^+, 3/2)$ and $1P(2^+, 3/2)$ doublet.

The remaining two $1P$ states, namely $1P(0^+, 1/2)$ and $1P(1^+, 1/2)$, have not yet been experimentally confirmed. Our predictions place these states at masses of 5670 ± 11 MeV and 5696 ± 11 MeV, aligning closely with earlier theoretical estimates [16,59,60]. The PDG lists the $B_J^*(5732)$ state, which has been observed by the OPAL [61], DELPHI [62], ALEPH [63], and L3 [64] Collaborations, reporting an average measured mass of

5698 ± 8 MeV. Our theoretical prediction for the $1P(1^+, 1/2)$ state mass aligns closely with this experimentally measured average for the $B_J^*(5732)$ resonance. Therefore, the $B_J^*(5732)$ resonance emerges as an excellent candidate for the $1P(1^+, 1/2)$ state.

The higher resonances $B(5970)^{0,+}$ were first observed in the $B\pi$ final states by the CDF Collaboration [7]. Subsequently, the LHCb Collaboration also reported signals for $B(5960)^{0,+}$ resonance in the same decay channels while performing precision measurements of the well-established $B_1(5721)^{0,+}$ and $B_2^*(5747)^{0,+}$ states [8]. The observed properties of the $B_J(5960)^{0,+}$ resonances are consistent with those previously reported for the $B(5970)^{0,+}$ by CDF, supporting the interpretation that these are manifestations of the same state. Reflecting this consensus, the PDG has adopted the unified notation $B_J(5970)$, assigning masses of

5971 ± 5 MeV for the neutral state and 5965 ± 5 MeV for the charged counterpart. Our theoretical analysis yields mass predictions for the $1D$ wave states that are found to lie in close proximity to the isospin-averaged experimental mass of the $B_J(5970)^{0,+}$, which is 5968 ± 10 MeV. While the predicted and experimental uncertainties do not fully overlap, the discrepancy is limited to a relatively narrow window of approximately 28–36 MeV. This difference is within the typical bounds of theoretical uncertainty often encountered in the mass spectra of heavy mesons. Based on this proximity, it is plausible to interpret the $B_J(5970)$ resonance as the second orbital excitation, corresponding to a $1D$ state. This is consistent with the conclusions drawn in several theoretical investigations [14,28,60,65,66]. However, additional studies are necessary for definitive spin-parity assignments.

The $B_J(5840)^{0,+}$ states recently observed by LHCb are recorded with experimental masses of 5863 ± 9 MeV and 5851 ± 19 MeV for neutral and charged states, respectively. The isospin-averaged mass (5857 ± 28 MeV) closely aligns with our calculated $2S$ states (5870 ± 10 MeV and 5872 ± 10 MeV). Thus, we confidently assign the $B_J(5840)$ resonances as the radial excitations ($2S$), consistent with theoretical predictions from other studies [12,28,60,67].

2. B_s meson

The B_s mesons exhibit well-documented $1S$ ground states. Experimentally confirmed masses for the states B_s^0 and B_s^* are 5366.93 ± 0.10 MeV and 5415.4 ± 1.4 MeV, respectively [1]. Our model reproduces these masses well, indicating robust consistency.

For orbitally excited B_s states, the experimental observations by the CDF [4], D0 [6], and LHCb [5] Collaborations established the $B_{s1}(5830)$ and $B_{s2}^*(5840)$ states. PDG records their masses as 5828.73 ± 0.2 MeV and 5839.88 ± 0.12 MeV, respectively. Our predictions of 5825 ± 10 MeV and 5841 ± 10 MeV closely reproduce these experimental values. Further, predictions for the yet-to-be-observed $1P(0^+, 1/2)$ and $1P(1^+, 1/2)$ states are 5772 ± 11 MeV and 5798 ± 11 MeV, respectively, which align well with previously published theoretical work [14,59,67].

Recently, the LHCb Collaboration reported the observation of two resonant structures, $B_{sJ}(6063)$ and $B_{sJ}(6114)$, in the B^+K^- invariant mass spectrum [9]. These findings were subsequently incorporated into the Particle Data Group listings, with the $B_{sJ}(6063)$ assigned a mass of 6063.5 ± 1.4 MeV and the $B_{sJ}(6114)$ a mass of 6114 ± 6 MeV. Our theoretical predictions for the $1D$ -wave excitations in the bottom-strange meson sector are found to be in close proximity to these experimentally measured values. Although the assignment of specific quantum numbers is hindered by both theoretical and experimental uncertainties, the observed mass values fall within the typical range of expected deviations for such heavy meson states. In light of this, we propose that the $B_{sJ}(6063)$ and $B_{sJ}(6114)$

resonances most plausibly correspond to members of the second orbital excitation family, namely the $1D$ -wave multiplet. This interpretation aligns with the conclusions of previous theoretical investigations [28,68]. Nevertheless, precise determination of the spin-parity quantum numbers of these states remains essential. Future experimental and theoretical studies are required to resolve their spectroscopic nature with greater certainty.

B. Doubly bottom baryons

The mass spectra that we obtain for the doubly bottom baryons—namely the Ξ_{bb} and Ω_{bb} —are collected in Tables V and VI. The quantum state of the system is represented by the notation $n_d L_d n L(J^P, j)$, where $n_d = 1, 2, 3, \dots$ denotes the radial excitation of one b quark relative to the other within the bb diquark, and $n = 1, 2, 3, \dots$ specifies the radial excitation of the light quark with respect to the diquark. The quantum number $L_d = S, P, D, \dots$ indicates the orbital angular momentum of a b quark relative to its partner within the diquark. Similarly, $L = s, p, d, \dots$ denotes the orbital angular momentum of the light quark relative to the diquark. The symbols J and P denote the total angular momentum and parity of the baryon, while j represents the total angular momentum of the light quark relative to the bb diquark.

Because no Ξ_{bb} , Ω_{bb} , or any other baryon containing two bottom quarks has yet been observed experimentally, we compare our calculated results with the most recent Regge-phenomenology analysis of Oudichhaya *et al.*, [35]; several relativistic or semirelativistic quark-model calculations [18–22]; and the nonrelativistic quark-model predictions [27–30].

Our central values for the spin-doublet ($1/2^+, 3/2^+$) ground states of the Ξ_{bb} baryon, $M_{1S} = 10.203\text{--}10.238$ GeV, fall within $\lesssim 30$ MeV of the Regge fit of Ref. [35] and the relativistic quark-model results of Ebert *et al.*, [19]. They are lower by roughly 150–180 MeV when compared with the early diquark-based estimate of Gershtein *et al.*, [20] and the more recent potential-model values of Yoshida *et al.*, [21]. A broadly similar pattern is observed for the Ω_{bb} , where our $1S$ masses, $M_{1S} = 10.292$ GeV and 10.321 GeV, track those of Refs. [18,28] yet undershoot the quark-model numbers of Yoshida *et al.* [21] by ~ 130 MeV. The excellent mutual consistency among the relativistic approaches is reassuring and suggests that heavy-diquark symmetry is well captured in this corner of parameter space.

For the first radial excitation ($2s$) of the light quark with respect to a diquark in $1S$ configuration, we predict $M_{1S2s} = 10.668\text{--}10.671$ GeV for the Ξ_{bb} , which coincides with the potential-model value of Lu *et al.* [18] and is only ~ 60 MeV higher than the potential-model estimate of Ref. [28]. The Regge fit of Ref. [35], on the other hand, comes out ~ 60 MeV lower, while the nonrelativistic quark model of Eakins and Roberts overshoots ours by nearly 300 MeV. For Ω_{bb} the agreement is even stronger—our $2S$

TABLE V. Masses of Ξ_{bb} -baryon (in MeV).

$n_d L_d n L(J^P, j)$	Mass	Ref. [35]	Ref. [28]	Ref. [21]	Ref. [56]	Ref. [18]	Ref. [27]	Ref. [19]	Ref. [20]	Ref. [21]	[22]
1S1s(1/2 ⁺ , 1/2)	10203 ± 29	10225	10171	10202	10314	10138	10322	10202	10093	10314	10185
1S1s(3/2 ⁺ , 1/2)	10238 ± 29	10273	10195	10237	10339	10169	10352	10237	10133	10339	10216
1S2s(1/2 ⁺ , 1/2)	10668 ± 23	10609	10738	10707	10612	10662	10940	10832			10751
1S2s(3/2 ⁺ , 1/2)	10671 ± 22	10617	10753	10726	10593	10675	10972	10860			10770
1S1p(1/2 ⁻ , 1/2)	10493 ± 25	10497	10593	10523	10476	10525	10694	10632	10541	10703	
1S1p(1/2 ⁻ , 3/2)	10544 ± 24	10502	10606	10559		10526	10691	10647	10578	10704	
1S1p(3/2 ⁻ , 1/2)	10516 ± 24	10494	10547	10545	10476	10504	10691	10675	10567	10740	
1S1p(3/2 ⁻ , 3/2)	10550 ± 24	10506	10561	10564		10528	10692	10694	10581	10742	
1S1p(5/2 ⁻ , 3/2)	10561 ± 23	10551	10560	10574	10759	10547	10695	10661	10580	10759	
1P1s(1/2 ⁻ , 1/2)	10647 ± 19			10608	10740	10364	10470	10368			
1P1s(3/2 ⁻ , 1/2)	10681 ± 19			10640	10742	10387	10470	10408			
2S1s(1/2 ⁺ , 1/2)	10777 ± 34			10550	10571	10464	10551	10441			10453
2S1s(3/2 ⁺ , 1/2)	10810 ± 34			10583	10592	10480	10574	10482			10478
1S2p(1/2 ⁻ , 1/2)	10920 ± 20										
1S2p(1/2 ⁻ , 3/2)	10910 ± 20										
1S2p(3/2 ⁻ , 1/2)	10926 ± 20	10765									
1S2p(3/2 ⁻ , 3/2)	10911 ± 20										
1S2p(5/2 ⁻ , 3/2)	10913 ± 20	10776									
1S1d(1/2 ⁺ , 3/2)	10799 ± 21		10913				11011				
1S1d(3/2 ⁺ , 3/2)	10804 ± 21	10699	10918				11011				
1S1d(3/2 ⁺ , 5/2)	10793 ± 21	10742	10921				11002				
1S1d(5/2 ⁺ , 3/2)	10812 ± 21	10756	10798				11011				
1S1d(5/2 ⁺ , 5/2)	10797 ± 21	10708	10803				11002				
1S1d(7/2 ⁺ , 5/2)	10802 ± 20	10823	10805				11011				
2P1s(1/2 ⁻ , 1/2)	11015 ± 20							10563			
2P1s(3/2 ⁻ , 1/2)	11047 ± 20							10607			
1S2d(1/2 ⁺ , 3/2)	11148 ± 18										
1S2d(3/2 ⁺ , 3/2)	11151 ± 18										
1S2d(3/2 ⁺ , 5/2)	11122 ± 18										
1S2d(5/2 ⁺ , 3/2)	11154 ± 18	10901									
1S2d(5/2 ⁺ , 5/2)	11124 ± 18										
1S2d(7/2 ⁺ , 5/2)	11126 ± 18										
1S1f(3/2 ⁻ , 5/2)	11048 ± 19										
1S1f(5/2 ⁻ , 5/2)	11051 ± 19	10897									
1S1f(5/2 ⁻ , 7/2)	11014 ± 19	10978									
1S1f(7/2 ⁻ , 5/2)	11057 ± 19	11012									
1S1f(7/2 ⁻ , 7/2)	11017 ± 19	10907									
1S1f(9/2 ⁻ , 7/2)	11021 ± 19	11087									
1S1g(5/2 ⁺ , 7/2)	11267 ± 17										
1S1g(7/2 ⁺ , 7/2)	11270 ± 17	11092									
1S1g(7/2 ⁺ , 9/2)	11215 ± 17	11208									
1S1g(9/2 ⁺ , 7/2)	11274 ± 17	11262									
1S1g(9/2 ⁺ , 9/2)	11218 ± 17	11102									
1S1g(11/2 ⁺ , 9/2)	11221 ± 17	11346									

mass $M_{2S} = 10.776$ GeV differs by less than 40 MeV from relativistic calculation in Refs. [18,22].

Our 1S1p centroids are 10.50–10.56 GeV for Ξ_{bb} and 10.60–10.66 GeV for Ω_{bb} . They reproduce the Regge results [35] and the potential-model results [28] to within the quoted theoretical uncertainties, whereas the diquark–quark model of Ref. [19] overshoots us by ~ 120 MeV and Yoshida *et al.*'s [21] by ~ 150 MeV. The ordering inside each 1P multiplet follows the same pattern: for fixed n , L ,

and J , the state with the larger light-quark total angular momentum j is heavier, reflecting the repulsive hyperfine interaction between the light quark and the bb diquark.

For 1S1d and 1S2d excitations our masses cluster around 10.80–11.15 GeV for Ξ_{bb} and 10.90–11.28 GeV for Ω_{bb} , sitting ~ 100 – 150 MeV below the potential-model predictions of Ref. [28] but matching the Regge trajectory of Ref. [35] almost point for point wherever the latter quotes numbers. A similar, albeit slightly broader, spread appears

TABLE VI. Masses of Ω_{bb} -baryon (in MeV).

$n_d L_d n L(J^P, j)$	Mass	Ref. [35]	Ref. [28]	Ref. [21]	Ref. [56]	Ref. [18]	Ref. [19]	Ref. [29]	Ref. [21]	Ref. [30]	Ref. [22]
1S1s(1/2 ⁺ , 1/2)	10292 ± 29	10350	10266		10447	10230	10359	10210	10447	10454	10271
1S1s(3/2 ⁺ , 1/2)	10321 ± 29	10393	10291		10707	10258	10389	10257	10467	10486	10289
1S2s(1/2 ⁺ , 1/2)	10776 ± 23	10736	10816	10845	10744	10751	10970				10830
1S2s(3/2 ⁺ , 1/2)	10779 ± 23	10743	10830	10858	10730	10763	10992				10839
1S1p(1/2 ⁻ , 1/2)	10596 ± 25	10622	10669	10651	10607	10605	10771	10541	10796		
1S1p(1/2 ⁻ , 3/2)	10644 ± 24	10643	10681	10678	10796	10610	10785	10567	10797		
1S1p(3/2 ⁻ , 1/2)	10618 ± 24	10633	10641	10672	10608	10591	10804	10578	10803	10763	
1S1p(3/2 ⁻ , 3/2)	10651 ± 24	10647	10656	10683	10797	10611	10821	10581	10805	10765	
1S1p(5/2 ⁻ , 3/2)	10662 ± 24	10667	10655	10692	10808	10625	10798	10580	10808	10766	
1P1s(1/2 ⁻ , 1/2)	10735 ± 19			10768	10803	10464	10532				
1P1s(3/2 ⁻ , 1/2)	10763 ± 19			10791	10805	10482	10566				
2S1s(1/2 ⁺ , 1/2)	10865 ± 34			10710	10707	10566	10610				10538
2S1s(3/2 ⁺ , 1/2)	10893 ± 34			10733	10723	10579	10645				10556
1S2p(1/2 ⁻ , 1/2)	11038 ± 20										
1S2p(1/2 ⁻ , 3/2)	11034 ± 20										
1S2p(3/2 ⁻ , 1/2)	11043 ± 20	10893									
1S2p(3/2 ⁻ , 3/2)	11035 ± 20										
1S2p(5/2 ⁻ , 3/2)	11037 ± 20	10888									
1S1d(1/2 ⁺ , 3/2)	10910 ± 21		10971								
1S1d(3/2 ⁺ , 3/2)	10914 ± 21	10829	10975								
1S1d(3/2 ⁺ , 5/2)	10912 ± 21	11868	10979								
1S1d(5/2 ⁺ , 3/2)	10922 ± 21	10909	10891								
1S1d(5/2 ⁺ , 5/2)	10916 ± 21	10840	10896								
1S1d(7/2 ⁺ , 5/2)	10921 ± 21	10934	10898								
2P1s(1/2 ⁻ , 1/2)	11103 ± 20						10738				
2P1s(3/2 ⁻ , 1/2)	11130 ± 20						10775				
1S2d(1/2 ⁺ , 3/2)	11276 ± 18										
1S2d(3/2 ⁺ , 3/2)	11278 ± 18										
1S2d(3/2 ⁺ , 5/2)	11260 ± 18										
1S2d(5/2 ⁺ , 3/2)	11282 ± 18	11025									
1S2d(5/2 ⁺ , 5/2)	11262 ± 18										
1S2d(7/2 ⁺ , 5/2)	11264 ± 18	11021									
1S1f(3/2 ⁻ , 5/2)	11168 ± 19										
1S1f(5/2 ⁻ , 5/2)	11171 ± 19	11032									
1S1f(5/2 ⁻ , 7/2)	11149 ± 19	12089									
1S1f(7/2 ⁻ , 5/2)	11177 ± 19	11178									
1S1f(7/2 ⁻ , 7/2)	11152 ± 19	11030									
1S1f(9/2 ⁻ , 7/2)	11156 ± 18	11194									
1S1g(5/2 ⁺ , 7/2)	11397 ± 17										
1S1g(7/2 ⁺ , 7/2)	11400 ± 17	11232									
1S1g(7/2 ⁺ , 9/2)	11364 ± 17	12306									
1S1g(9/2 ⁺ , 7/2)	11404 ± 17	11440									
1S1g(9/2 ⁺ , 9/2)	11366 ± 17	11216									
1S1g(11/2 ⁺ , 9/2)	11369 ± 17	11449									

in the 1S1f and 1S1g sectors. Notably, for 1S1d-, 1S1f-, and 1S1g-wave multiplets the intramultiplet splitting in our model reverses: the state with larger j now becomes lighter.

For the 1P1s doublet in both Ξ_{bb} and Ω_{bb} , our predicted masses are in good agreement with Refs. [21,56], while they exceed those of Refs. [18,19,27] by 177–283 MeV. For the 2S1s radial doublet, our predictions lie systematically above other model expectations (Refs. [18,19,21,27,56]) by roughly 115–336 MeV.

Across the entire spectrum the typical deviation between our results and the most up-to-date relativistic quark-model or Regge fits is $\lesssim 2\%$. Larger differences—up to $\sim 3\%$ – 4% —arise when we compare with nonrelativistic potential models that neglect full relativistic kinematics and, in some cases, treat the bb diquark as pointlike. Given the very small phase space available for weak decays of states lying just a few tens of MeV apart, such percent-level shifts may decide whether an excited level can be seen in

the narrow-width regime at LHCb. Our mass predictions therefore provide a timely guide for future experimental searches to detect the doubly bottom baryons. Experimental observation of even a single resonance in the doubly bottom baryon sector would furnish the first direct handle on bottom-diquark dynamics and help discriminate between competing theoretical frameworks.

IV. CONCLUSION

In this work, we have employed the relativistic flux tube model to investigate the mass spectra of singly bottom mesons (B and B_s) and doubly bottom baryons (Ξ_{bb} and Ω_{bb}). Within this framework, a heavy-light meson is described as a two-body system of a bottom quark and a light antiquark connected by a gluonic flux tube, whereas a doubly bottom baryon is treated analogously as a $\{bb\}$ diquark (in color- $\bar{3}$ configuration) attached to a light quark via a flux tube. We first calculate the spin-averaged masses of these systems and then include spin-dependent interactions perturbatively in the $j-j$ coupling scheme to obtain the spin dependent splittings. This theoretical setup provides a unified description of heavy hadrons, effectively capturing the dynamics of a bottom quark–light quark pair or a bottom diquark–light quark pair.

One important outcome of this work is the classification of several recently observed excited B and B_s mesons. The resonance $B_J^*(5732)$ is identified as the low-lying $1P$ excitation with quantum numbers $J^P = 1^+$ and light-quark total angular momentum $j = 1/2$. Likewise, we predict the $B_J(5840)$ as the best candidate for the first radial excitation (a $2S$ state) in the B meson family. The higher-mass $B_J(5970)$ state is assigned as a $1D$ -wave state (the second orbital excitation). In the bottom-strange sector, the newly observed pair of $B_{sJ}(6063)$ and $B_{sJ}(6114)$ resonances are both interpreted as $1D$ -wave ($L = 2$) excitations of the B_s meson.

Beyond the currently observed states of B and B_s mesons, we predict a rich mass spectrum of higher orbital (L) and radial (n) excitations in both the bottom meson and doubly bottom baryon sectors. To date, none of the doubly bottom baryons (such as Ξ_{bb} or Ω_{bb}) have been observed in experiments. As measurements in the bottom-quark domain continue to progress, our predictions can serve as a valuable guide for ongoing and future experimental searches to observe new resonances in upcoming high-luminosity data.

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DATA AVAILABILITY

The data supporting this study's findings are available within the article.

APPENDIX

In this appendix, we present a detailed discussion of basis states and the expectation values of spin-dependent operators pertinent to heavy-light mesons and doubly heavy baryons in F -wave ($L = 3$) and G -wave ($L = 4$) excitations. Initially, we employ the L - S coupling scheme to construct the basis states, subsequently converting them into the $|J, j\rangle$ coupling basis. This latter basis is particularly appropriate for the analysis of heavy-light mesons and baryons in the heavy-quark limit.

The L - S basis states can be systematically built through the use of standard Clebsch-Gordan coefficients as follows [45,48]:

$$|^{2S+1}L_J; J_3\rangle = \sum_{S_{l_3}, S_{h_3}, L_3, S_3} C_{S_{l_3} S_{h_3} S_3}^{S_l S_h S} C_{S_{l_3} L_3 J_3}^{SLJ} |S_{l_3}, S_{h_3}, L_3\rangle, \quad (\text{A1})$$

where the quantum numbers S_{h_3} , S_{l_3} , S_3 , L_3 , and J_3 are the projections corresponding, respectively, to S_h , S_l , S , L , and J . The state $|S_{h_3}, S_{l_3}, L_3\rangle$ is defined as the direct product of the uncoupled states $|S_h S_{h_3}\rangle$, $|S_l S_{l_3}\rangle$, and $|L L_3\rangle$.

To compute the expectation value of the spin-orbit operator $\mathbf{L} \cdot \mathbf{S}_i$ (for $i = h$ or l), it is useful to express this operator in terms of ladder operators, as detailed in Ref. [69],

$$\mathbf{L} \cdot \mathbf{S}_i = \frac{1}{2} (L_+ S_{i-} + L_- S_{i+}) + L_3 S_{i3}, \quad (\text{A2})$$

which facilitates evaluating the expectation values by calculating matrix elements within the $|^{2S+1}L_J\rangle$ basis.

Using the expectation values for the operator $\mathbf{L} \cdot \mathbf{S}_i$ in the $|^{2S+1}L_J\rangle$ basis, the expectation value of the tensor interaction operator can be determined via the following relationship [45,69]:

$$\begin{aligned} \langle \hat{B} \rangle = & -\frac{3}{(2L-1)(2L+3)} \left[(\mathbf{L} \cdot \mathbf{S}_h)(\mathbf{L} \cdot \mathbf{S}_l) \right. \\ & \left. + (\mathbf{L} \cdot \mathbf{S}_l)(\mathbf{L} \cdot \mathbf{S}_h) - \frac{2}{3} L(L+1) \mathbf{S}_h \cdot \mathbf{S}_l \right]. \quad (\text{A3}) \end{aligned}$$

Additionally, the expectation value for the spin-spin operator within the L - S coupling scheme can be succinctly derived using the identity $\mathbf{S} = \mathbf{S}_h + \mathbf{S}_l$ and the relation $\mathbf{S}^2 = S(S+1)$, yielding,

$$\langle \mathbf{S}_h \cdot \mathbf{S}_l \rangle = \frac{1}{2} [S(S+1) - S_h(S_h+1) - S_l(S_l+1)]. \quad (\text{A4})$$

1. Heavy-light mesons in F wave

For an orbital angular momentum quantum number $L = 3$ (F-wave), the spins of the heavy quark ($S_h = \frac{1}{2}$) and the light antiquark ($S_l = \frac{1}{2}$) may couple to either $S = 0$ or $S = 1$. Combining these spin states with $L = 3$ produces total angular momenta $J = 2, 3, 4$. In the conventional L - S coupling scheme, the resulting basis vectors are denoted $|^1F_3\rangle$ for $(S = 0, J = 3)$ and $|^3F_J\rangle$ for $(S = 1, J = 2, 3, 4)$. Because two distinct $J = 3$ configurations exist, namely $|^1F_3\rangle$ and $|^3F_3\rangle$, spin-dependent forces will in general mix these states, whereas the $J = 2$ ($|^3F_2\rangle$) and $J = 4$ ($|^3F_4\rangle$) members each remain unique.

Starting from single-particle states $|S_{h_3}, S_{l_3}, L_3\rangle$ and the coupling formula in Eq. (A1), the L - S basis are,

$$|^3F_2; 2\rangle = \sqrt{\frac{5}{7}}|-\frac{1}{2}, -\frac{1}{2}, 3\rangle - \sqrt{\frac{5}{42}}|\frac{1}{2}, -\frac{1}{2}, 2\rangle - \sqrt{\frac{5}{42}}|-\frac{1}{2}, \frac{1}{2}, 2\rangle + \frac{1}{\sqrt{21}}|\frac{1}{2}, \frac{1}{2}, 1\rangle, \quad (\text{A5})$$

$$|^1F_3; 3\rangle = \frac{1}{\sqrt{2}}|\frac{1}{2}, -\frac{1}{2}, 3\rangle - \frac{1}{\sqrt{2}}|-\frac{1}{2}, \frac{1}{2}, 3\rangle, \quad (\text{A6})$$

$$|^3F_3; 3\rangle = -\frac{1}{2}\sqrt{\frac{3}{2}}|\frac{1}{2}, -\frac{1}{2}, 3\rangle - \frac{1}{2}\sqrt{\frac{3}{2}}|-\frac{1}{2}, \frac{1}{2}, 3\rangle + \frac{1}{2}|\frac{1}{2}, \frac{1}{2}, 2\rangle, \quad (\text{A7})$$

$$|^3F_4; 4\rangle = |\frac{1}{2}, \frac{1}{2}, 3\rangle. \quad (\text{A8})$$

Using these constructed L - S basis states, we compute the expectation values for the spin-dependent operators described in Eqs. (A2)–(A4) within the $[^1F_J, ^3F_J]$ bases for all permitted J values, For $J = 2$,

$$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle = -2, \quad \langle \mathbf{L} \cdot \mathbf{S}_h \rangle = -2, \quad \langle \hat{\mathbf{B}} \rangle = -\frac{2}{5}, \quad \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle = \frac{1}{4}. \quad (\text{A9})$$

For $J = 3$,

$$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle = \begin{bmatrix} 0 & -\sqrt{3} \\ -\sqrt{3} & -\frac{1}{2} \end{bmatrix}, \quad \langle \mathbf{L} \cdot \mathbf{S}_h \rangle = \begin{bmatrix} 0 & \sqrt{3} \\ \sqrt{3} & -\frac{1}{2} \end{bmatrix},$$

$$\langle \hat{\mathbf{B}} \rangle = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}, \quad \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle = \begin{bmatrix} -\frac{3}{4} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}. \quad (\text{A10})$$

For $J = 4$,

$$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle = \frac{3}{2}, \quad \langle \mathbf{L} \cdot \mathbf{S}_h \rangle = \frac{3}{2}, \quad \langle \hat{\mathbf{B}} \rangle = -\frac{1}{6}, \quad \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle = \frac{1}{4}. \quad (\text{A11})$$

In the heavy-quark limit ($m_h \gg m_l$), the term involving $\langle \mathbf{L} \cdot \mathbf{S}_l \rangle$ dominates over other spin-dependent interactions. Hence, a basis that diagonalizes $\langle \mathbf{L} \cdot \mathbf{S}_l \rangle$ is preferable. Under this condition, heavy-light meson states are effectively described by the $|J, j\rangle$ basis, which diagonalizes the dominant interaction term $\langle \mathbf{L} \cdot \mathbf{S}_l \rangle$, allowing other interactions to be addressed perturbatively. For each eigenvalue λ of $\mathbf{L} \cdot \mathbf{S}_l$ corresponding to a given total angular momentum J , we determine the associated eigenvector, which defines the $|J, j\rangle$ basis.

For $L = 3$ and $S_l = \frac{1}{2}$, j can be either $\frac{5}{2}$ or $\frac{7}{2}$. The eigenvalues of $\langle \mathbf{L} \cdot \mathbf{S}_l \rangle$ are $\lambda = -1$ for $j = \frac{5}{2}$ and $\lambda = +\frac{1}{2}$ for $j = \frac{7}{2}$ [these values come from $\frac{1}{2}[j(j+1) - L(L+1) - S_l(S_l+1)]$ with $L = 3$, $S_l = 1/2$]. The corresponding eigenvectors of $\langle \mathbf{L} \cdot \mathbf{S}_l \rangle$ can be written as

$$|J = 3, j = \frac{5}{2}\rangle = \sqrt{\frac{3}{7}}|^1F_3\rangle + \frac{2}{\sqrt{7}}|^3F_3\rangle, \quad (\text{A12})$$

$$|J = 3, j = \frac{7}{2}\rangle = -\frac{2}{\sqrt{7}}|^1F_3\rangle + \sqrt{\frac{3}{7}}|^3F_3\rangle, \quad (\text{A13})$$

with eigenvalues $\lambda = -2$ and $\frac{3}{2}$, respectively. For the remaining J values,

$$|J = 2, j = \frac{5}{2}\rangle = |^3F_2\rangle, \quad (\text{A14})$$

$$|J = 4, j = \frac{7}{2}\rangle = |^3F_4\rangle. \quad (\text{A15})$$

Having constructed the $|J, j\rangle$ basis, we compute the expectation values of $\mathbf{L} \cdot \mathbf{S}_h$, $\mathbf{L} \cdot \mathbf{S}_l$, $\hat{\mathbf{B}}$ and $\mathbf{S}_h \cdot \mathbf{S}_l$ in this $|J, j\rangle$ basis and collect results in Table I.

2. Heavy-light mesons in G wave

For an orbital excitation with $L = 4$, an analysis parallel to the F -wave case can be performed. In heavy-light mesons, the heavy quark and light antiquark each carry spin $S_h = S_l = \frac{1}{2}$, so the total spin of the pair can be either $S = 0$ or $S = 1$. Coupling this spin to the G -wave orbital momentum $L = 4$ produces total angular momenta $J = 3, 4, 5$. In the conventional L - S coupling scheme the corresponding basis vectors are $|^1G_4\rangle$ ($S = 0, J = 4$) and $|^3G_J\rangle$ ($S = 1, J = 3, 4, 5$). Consequently, the $J = 4$ sector contains two distinct configurations ($|^1G_4\rangle$ and $|^3G_4\rangle$) that can mix, whereas the $J = 3$ and $J = 5$ sectors each comprise a single pure triplet state ($|^3G_3\rangle$ and $|^3G_5\rangle$, respectively). Employing Eq. (A1) to construct the L - S basis yields

$$|{}^3G_3; 3\rangle = \frac{\sqrt{7}}{3} \left| -\frac{1}{2}, -\frac{1}{2}, 4 \right\rangle - \frac{1}{6} \sqrt{\frac{7}{2}} \left| \frac{1}{2}, -\frac{1}{2}, 3 \right\rangle - \frac{1}{6} \sqrt{\frac{7}{2}} \left| -\frac{1}{2}, \frac{1}{2}, 3 \right\rangle + \frac{1}{6} \left| \frac{1}{2}, \frac{1}{2}, 2 \right\rangle, \quad (\text{A16})$$

$$|{}^1G_4; 4\rangle = \frac{1}{\sqrt{2}} \left| \frac{1}{2}, -\frac{1}{2}, 4 \right\rangle - \frac{1}{\sqrt{2}} \left| -\frac{1}{2}, \frac{1}{2}, 4 \right\rangle, \quad (\text{A17})$$

$$|{}^3G_4; 4\rangle = -\sqrt{\frac{2}{5}} \left| \frac{1}{2}, -\frac{1}{2}, 4 \right\rangle - \sqrt{\frac{2}{5}} \left| -\frac{1}{2}, \frac{1}{2}, 4 \right\rangle + \frac{1}{\sqrt{5}} \left| \frac{1}{2}, \frac{1}{2}, 3 \right\rangle, \quad (\text{A18})$$

$$|{}^3G_5; 5\rangle = \left| \frac{1}{2}, \frac{1}{2}, 4 \right\rangle. \quad (\text{A19})$$

With this L - S basis, we evaluate the matrix elements of the operators defined in Eqs. (A2)–(A4). The resulting expectation values, quoted in the $[{}^1G_J, {}^3G_J]$ bases, are, For $J = 3$,

$$\begin{aligned} \langle \mathbf{L} \cdot \mathbf{S}_l \rangle &= -\frac{5}{2}, & \langle \mathbf{L} \cdot \mathbf{S}_h \rangle &= -\frac{5}{2}, \\ \langle \hat{\mathbf{B}} \rangle &= -\frac{5}{14}, & \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle &= \frac{1}{4}. \end{aligned} \quad (\text{A20})$$

For $J = 4$,

$$\begin{aligned} \langle \mathbf{L} \cdot \mathbf{S}_l \rangle &= \begin{bmatrix} 0 & -\sqrt{5} \\ -\sqrt{5} & -\frac{1}{2} \end{bmatrix}, & \langle \mathbf{L} \cdot \mathbf{S}_h \rangle &= \begin{bmatrix} 0 & \sqrt{5} \\ \sqrt{5} & -\frac{1}{2} \end{bmatrix}, \\ \langle \hat{\mathbf{B}} \rangle &= \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}, & \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle &= \begin{bmatrix} -\frac{3}{4} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}. \end{aligned} \quad (\text{A21})$$

For $J = 5$,

$$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle = 2, \quad \langle \mathbf{L} \cdot \mathbf{S}_h \rangle = 2, \quad \langle \hat{\mathbf{B}} \rangle = -\frac{2}{11}, \quad \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle = \frac{1}{4}. \quad (\text{A22})$$

As in the F -wave analysis, the dominant light-quark spin-orbit interaction $\langle \mathbf{L} \cdot \mathbf{S}_l \rangle$ is diagonalized first. With $L = 4$ and $S_l = \frac{1}{2}$, the light-quark total momentum is $j = \frac{7}{2}$ or $j = \frac{9}{2}$, giving eigenvalues of $\langle \mathbf{L} \cdot \mathbf{S}_l \rangle$ $\lambda = -\frac{5}{2}$ and $\lambda = 2$, respectively. In the $J = 2$ sector, the corresponding eigenvectors are admixtures of $|{}^1G_4\rangle$ and $|{}^3G_4\rangle$,

$$|J = 4, j = \frac{7}{2}\rangle = \frac{2}{3} |{}^1G_4\rangle + \frac{\sqrt{5}}{3} |{}^3G_4\rangle, \quad (\text{A23})$$

$$|J = 4, j = \frac{9}{2}\rangle = -\frac{\sqrt{5}}{2} |{}^1G_4\rangle + \frac{2}{3} |{}^3G_4\rangle. \quad (\text{A24})$$

For $J = 3$ and $J = 5$ only the pure triplet states survive, already eigenstates of $\langle \mathbf{L} \cdot \mathbf{S}_l \rangle$,

$$|J = 3, j = \frac{7}{2}\rangle = |{}^3G_3\rangle, \quad (\text{A25})$$

$$|J = 5, j = \frac{9}{2}\rangle = |{}^3G_5\rangle. \quad (\text{A26})$$

Working in the $|J, j\rangle$ basis, one may now evaluate the operator expectation values relevant for spin-dependent interactions, which are listed in Table I.

3. Doubly heavy baryons in F wave

In doubly bottom baryons, the diquark formed by two identical bottom quarks must obey the Pauli exclusion principle, requiring an antisymmetric total wave function of diquark. For ground-state configuration of diquark having identical b -quark, the spatial and flavor parts are symmetric, while the color part is antisymmetric in the antitriplet configuration. To maintain overall antisymmetry of the total wave function of diquark, the spin part must also be symmetric, allowing only the spin-1 (axial-vector) diquark configuration. For a doubly heavy baryon in an $L = 3$ orbital excitation, the relevant angular momenta are the orbital angular momentum $L = 3$, the heavy diquark spin $S_h = 1$, and the light-quark spin $S_l = \frac{1}{2}$. In the L - S scheme S_h and S_l combine to $S = \frac{1}{2}$ or $\frac{3}{2}$, which in turn couples with $L = 3$ to give $J = \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}$. The basis vectors $|{}^2F_J\rangle$ ($S = \frac{1}{2}$) and $|{}^4F_J\rangle$ ($S = \frac{3}{2}$) are constructed via Eq. (A1),

$$\begin{aligned} |{}^4F_{3/2}; 3/2\rangle &= -\frac{2}{\sqrt{7}} \left| -\frac{1}{2}, -1, 3 \right\rangle + \frac{2}{\sqrt{21}} \left| -\frac{1}{2}, 0, 2 \right\rangle \\ &\quad - \frac{2}{\sqrt{105}} \left| -\frac{1}{2}, 1, 1 \right\rangle + \sqrt{\frac{2}{21}} \left| \frac{1}{2}, -1, 2 \right\rangle \\ &\quad - 2\sqrt{\frac{2}{105}} \left| \frac{1}{2}, 0, 1 \right\rangle + \frac{1}{\sqrt{35}} \left| \frac{1}{2}, 1, 0 \right\rangle, \end{aligned} \quad (\text{A27})$$

$$\begin{aligned} |{}^2F_{5/2}; 5/2\rangle &= \sqrt{\frac{2}{7}} \left| -\frac{1}{2}, 0, 3 \right\rangle - \sqrt{\frac{2}{21}} \left| -\frac{1}{2}, 1, 2 \right\rangle \\ &\quad - \frac{2}{\sqrt{7}} \left| \frac{1}{2}, -1, 3 \right\rangle + \frac{1}{\sqrt{21}} \left| \frac{1}{2}, 0, 2 \right\rangle, \end{aligned} \quad (\text{A28})$$

$$\begin{aligned} |{}^4F_{5/2}; 5/2\rangle &= \sqrt{\frac{5}{14}} \left| -\frac{1}{2}, 0, 3 \right\rangle - \sqrt{\frac{5}{42}} \left| -\frac{1}{2}, 1, 2 \right\rangle \\ &\quad + \frac{1}{2} \sqrt{\frac{5}{7}} \left| \frac{1}{2}, -1, 3 \right\rangle - \sqrt{\frac{5}{21}} \left| \frac{1}{2}, 0, 2 \right\rangle \\ &\quad + \frac{1}{2} \sqrt{\frac{3}{7}} \left| \frac{1}{2}, 1, 1 \right\rangle, \end{aligned} \quad (\text{A29})$$

$$|{}^2F_{7/2}; 7/2\rangle = -\sqrt{\frac{2}{3}} \left| -\frac{1}{2}, 1, 3 \right\rangle + \frac{1}{\sqrt{3}} \left| \frac{1}{2}, 0, 3 \right\rangle, \quad (\text{A30})$$

$$|{}^4F_{7/2}; 7/2\rangle = -\frac{\sqrt{2}}{3}|\frac{1}{2}, 1, 3\rangle - \frac{2}{3}|\frac{1}{2}, 0, 3\rangle + \frac{1}{\sqrt{3}}|\frac{1}{2}, 1, 2\rangle, \quad (\text{A31})$$

$$|{}^4F_{9/2}; 9/2\rangle = |\frac{1}{2}, 1, 3\rangle. \quad (\text{A32})$$

Utilizing these L - S basis states, we evaluate the expectation values of the spin-dependent operators defined in Eqs. (A2)–(A4) within the $[{}^2F_J, {}^4F_J]$ basis,

For $J = 3/2$,

$$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle = -2, \quad \langle \mathbf{L} \cdot \mathbf{S}_h \rangle = -4, \quad \langle \hat{\mathbf{B}} \rangle = -\frac{4}{5}, \quad \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle = \frac{1}{2}. \quad (\text{A33})$$

For $J = 5/2$,

$$\begin{aligned} \langle \mathbf{L} \cdot \mathbf{S}_l \rangle &= \begin{bmatrix} \frac{2}{3} & -\frac{2\sqrt{5}}{3} \\ -\frac{2\sqrt{5}}{3} & -\frac{7}{6} \end{bmatrix}, & \langle \mathbf{L} \cdot \mathbf{S}_h \rangle &= \begin{bmatrix} -\frac{8}{3} & \frac{2\sqrt{5}}{3} \\ \frac{2\sqrt{5}}{3} & -\frac{7}{3} \end{bmatrix}, \\ \langle \hat{\mathbf{B}} \rangle &= \begin{bmatrix} 0 & -\frac{1}{\sqrt{5}} \\ -\frac{1}{\sqrt{5}} & \frac{1}{5} \end{bmatrix}, & \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle &= \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}. \end{aligned} \quad (\text{A34})$$

For $J = 7/2$,

$$\begin{aligned} \langle \mathbf{L} \cdot \mathbf{S}_l \rangle &= \begin{bmatrix} -\frac{1}{2} & -\sqrt{3} \\ -\sqrt{3} & 0 \end{bmatrix}, & \langle \mathbf{L} \cdot \mathbf{S}_h \rangle &= \begin{bmatrix} 2 & \sqrt{3} \\ \sqrt{3} & 0 \end{bmatrix}, \\ \langle \hat{\mathbf{B}} \rangle &= \begin{bmatrix} 0 & \frac{1}{2\sqrt{3}} \\ \frac{1}{2\sqrt{3}} & \frac{2}{3} \end{bmatrix}, & \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle &= \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}. \end{aligned} \quad (\text{A35})$$

For $J = 9/2$,

$$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle = \frac{3}{2}, \quad \langle \mathbf{L} \cdot \mathbf{S}_h \rangle = 3, \quad \langle \hat{\mathbf{B}} \rangle = -\frac{1}{3}, \quad \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle = \frac{1}{2}. \quad (\text{A36})$$

Diagonalizing $\langle \mathbf{L} \cdot \mathbf{S}_l \rangle$ within each J subspace gives the $|J, j\rangle$ eigenvectors. For $J = \frac{5}{2}$, $|J, j\rangle$ eigenvectors are

$$|J = \frac{5}{2}, j = 5/2\rangle = \sqrt{\frac{5}{21}}|{}^2F_{5/2}\rangle + \frac{4}{\sqrt{21}}|{}^4F_{5/2}\rangle, \quad (\text{A37})$$

$$|J = \frac{5}{2}, j = 7/2\rangle = -\frac{4}{\sqrt{21}}|{}^2F_{5/2}\rangle + \sqrt{\frac{5}{21}}|{}^4F_{5/2}\rangle, \quad (\text{A38})$$

with eigenvalues $\lambda = -2$ and $\frac{3}{2}$, respectively. For $J = \frac{7}{2}$, $|J, j\rangle$ eigenvectors are,

$$|J = \frac{7}{2}, j = 5/2\rangle = \frac{2}{\sqrt{7}}|{}^2F_{7/2}\rangle + \sqrt{\frac{3}{7}}|{}^4F_{7/2}\rangle, \quad (\text{A39})$$

$$|J = \frac{7}{2}, j = 7/2\rangle = -\sqrt{\frac{3}{7}}|{}^2F_{7/2}\rangle + \frac{2}{\sqrt{7}}|{}^4F_{7/2}\rangle, \quad (\text{A40})$$

with eigenvalues $\lambda = -2$ and $\frac{3}{2}$, respectively. For the remaining J values,

$$|J = \frac{3}{2}, j = 5/2\rangle = |{}^4F_{3/2}\rangle, \quad (\text{A41})$$

$$|J = \frac{9}{2}, j = 7/2\rangle = |{}^4F_{9/2}\rangle. \quad (\text{A42})$$

Having constructed the $|J, j\rangle$ basis, we compute the expectation values of operators $\mathbf{L} \cdot \mathbf{S}_h$, $\mathbf{L} \cdot \mathbf{S}_l$, $\hat{\mathbf{B}}$, and $\mathbf{S}_h \cdot \mathbf{S}_l$ in this $|J, j\rangle$ basis and present the results in Table II.

4. Doubly heavy baryons in G wave

For a doubly heavy baryon with an orbital excitation $L = 4$, the pertinent angular momenta are the orbital component $L = 4$, the heavy-diquark spin $S_h = 1$, and the light-quark spin $S_l = \frac{1}{2}$. Within the L - S coupling scheme, S_h and S_l first combine to a total spin $S = \frac{1}{2}$ or $\frac{3}{2}$, which subsequently couples with $L = 4$ to generate total angular momenta $J = \frac{5}{2}, \frac{7}{2}, \frac{9}{2}$, and $\frac{11}{2}$. The corresponding basis states, denoted $|{}^2G_J\rangle$ (for $S = \frac{1}{2}$) and $|{}^4G_J\rangle$ (for $S = \frac{3}{2}$), are obtained using Eq. (A1),

$$\begin{aligned} |{}^4G_{5/2}; 5/2\rangle &= -\sqrt{\frac{2}{3}}|\frac{1}{2}, -1, 4\rangle + \frac{1}{\sqrt{6}}|\frac{1}{2}, 0, 3\rangle \\ &\quad - \frac{1}{\sqrt{42}}|\frac{1}{2}, 1, 2\rangle + \frac{1}{2\sqrt{3}}|\frac{1}{2}, -1, 3\rangle \\ &\quad - \frac{1}{\sqrt{21}}|\frac{1}{2}, 0, 2\rangle + \frac{1}{2\sqrt{21}}|\frac{1}{2}, 1, 1\rangle, \end{aligned} \quad (\text{A43})$$

$$\begin{aligned} |{}^2G_{7/2}; 7/2\rangle &= \frac{2}{3}\sqrt{\frac{2}{3}}|\frac{1}{2}, 0, 4\rangle - \frac{1}{3}\sqrt{\frac{2}{3}}|\frac{1}{2}, 1, 3\rangle \\ &\quad - \frac{4}{3\sqrt{3}}|\frac{1}{2}, -1, 4\rangle + \frac{1}{3\sqrt{3}}|\frac{1}{2}, 0, 3\rangle, \end{aligned} \quad (\text{A44})$$

$$\begin{aligned} |{}^4G_{7/2}; 7/2\rangle &= \frac{2}{3}\sqrt{\frac{14}{15}}|\frac{1}{2}, 0, 4\rangle - \frac{1}{3}\sqrt{\frac{14}{15}}|\frac{1}{2}, 1, 3\rangle \\ &\quad + \frac{2}{3}\sqrt{\frac{7}{15}}|\frac{1}{2}, -1, 4\rangle - \frac{2}{3}\sqrt{\frac{7}{15}}|\frac{1}{2}, 0, 3\rangle \\ &\quad + \frac{1}{\sqrt{15}}|\frac{1}{2}, 1, 2\rangle, \end{aligned} \quad (\text{A45})$$

$$|{}^2G_{9/2}; 9/2\rangle = -\sqrt{\frac{2}{3}}|\frac{1}{2}, 1, 4\rangle + \frac{1}{\sqrt{3}}|\frac{1}{2}, 0, 4\rangle, \quad (\text{A46})$$

$$|{}^4G_{9/2}; 9/2\rangle = -2\sqrt{\frac{2}{33}}|-\frac{1}{2}, 1, 4\rangle - \frac{4}{\sqrt{33}}|\frac{1}{2}, 0, 4\rangle + \sqrt{\frac{3}{11}}|\frac{1}{2}, 1, 3\rangle, \quad (\text{A47})$$

$$|{}^4G_{11/2}; 11/2\rangle = |\frac{1}{2}, 1, 4\rangle. \quad (\text{A48})$$

Using these L - S basis states, we compute the matrix elements of the operators specified in Eqs. (A2)–(A4). The corresponding expectation values, expressed in the $[{}^2G_J, {}^4G_J]$ bases, are, For $J = 5/2$,

$$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle = -\frac{5}{2}, \quad \langle \mathbf{L} \cdot \mathbf{S}_h \rangle = -5, \quad \langle \hat{\mathbf{B}} \rangle = -\frac{5}{7}, \quad \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle = \frac{1}{2}. \quad (\text{A49})$$

For $J = 7/2$,

$$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle = \begin{bmatrix} \frac{5}{6} & -\frac{\sqrt{35}}{3} \\ -\frac{\sqrt{35}}{3} & -\frac{4}{3} \end{bmatrix}, \quad \langle \mathbf{L} \cdot \mathbf{S}_h \rangle = \begin{bmatrix} -\frac{10}{3} & \frac{\sqrt{35}}{3} \\ \frac{\sqrt{35}}{3} & -\frac{8}{3} \end{bmatrix},$$

$$\langle \hat{\mathbf{B}} \rangle = \begin{bmatrix} 0 & -\frac{\sqrt{7}}{2} \\ -\frac{\sqrt{7}}{2} & \frac{2}{7} \end{bmatrix}, \quad \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle = \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}. \quad (\text{A50})$$

For $J = 9/2$,

$$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle = \begin{bmatrix} -\frac{2}{3} & -\frac{2\sqrt{11}}{3} \\ -\frac{2\sqrt{11}}{3} & \frac{1}{6} \end{bmatrix}, \quad \langle \mathbf{L} \cdot \mathbf{S}_h \rangle = \begin{bmatrix} \frac{8}{3} & \frac{2\sqrt{11}}{3} \\ \frac{2\sqrt{11}}{3} & \frac{1}{3} \end{bmatrix},$$

$$\langle \hat{\mathbf{B}} \rangle = \begin{bmatrix} 0 & \frac{1}{\sqrt{11}} \\ \frac{1}{\sqrt{11}} & \frac{7}{11} \end{bmatrix}, \quad \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle = \begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}. \quad (\text{A51})$$

For $J = 11/2$,

$$\langle \mathbf{L} \cdot \mathbf{S}_l \rangle = 2, \quad \langle \mathbf{L} \cdot \mathbf{S}_h \rangle = 4, \quad \langle \hat{\mathbf{B}} \rangle = -\frac{4}{11}, \quad \langle \mathbf{S}_l \cdot \mathbf{S}_h \rangle = \frac{1}{2}. \quad (\text{A52})$$

Diagonalization of $\langle \mathbf{L} \cdot \mathbf{S}_l \rangle$ for $J = \frac{7}{2}, \frac{9}{2}$ yields the $|J, j\rangle$ basis corresponding to the eigenvalues $\lambda = -\frac{5}{2}$ (for $j = \frac{7}{2}$) and $\lambda = 2$ (for $j = \frac{9}{2}$),

$$\lambda = -\frac{5}{2}: |J = \frac{7}{2}, j = 7/2\rangle = \frac{1}{3}\sqrt{\frac{7}{3}}|{}^2G_{7/2}\rangle + \frac{2}{3}\sqrt{\frac{5}{3}}|{}^4G_{7/2}\rangle, \quad (\text{A53})$$

$$\lambda = 2: |J = \frac{7}{2}, j = 9/2\rangle = -\frac{2}{3}\sqrt{\frac{5}{3}}|{}^2G_{7/2}\rangle + \frac{1}{3}\sqrt{\frac{7}{3}}|{}^4G_{7/2}\rangle, \quad (\text{A54})$$

$$\lambda = -\frac{5}{2}: |J = \frac{9}{2}, j = 7/2\rangle = \frac{4}{3\sqrt{3}}|{}^2G_{9/2}\rangle + \frac{1}{3}\sqrt{\frac{11}{3}}|{}^4G_{9/2}\rangle, \quad (\text{A55})$$

$$\lambda = 2: |J = \frac{9}{2}, j = 9/2\rangle = -\frac{1}{3}\sqrt{\frac{11}{3}}|{}^2G_{9/2}\rangle + \frac{4}{3\sqrt{3}}|{}^4G_{9/2}\rangle, \quad (\text{A56})$$

whereas the bases for $J = \frac{5}{2}, \frac{11}{2}$ remain unmixed,

$$|J = \frac{5}{2}, j = 7/2\rangle = |{}^4G_{5/2}\rangle, \quad (\text{A57})$$

$$|J = \frac{11}{2}, j = 9/2\rangle = |{}^4G_{11/2}\rangle. \quad (\text{A58})$$

Expectation values of the mass-splitting operators obtained using these $|J, j\rangle$ basis are tabulated in Table II.

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- [1] S. Navas *et al.* (Particle Data Group), Review of particle physics, *Phys. Rev. D* **110**, 030001 (2024).
[2] V.M. Abazov *et al.* (D0 Collaboration), Observation and properties of $L = 1B_1$ and B_2^* mesons, *Phys. Rev. Lett.* **99**, 172001 (2007).
[3] T. Aaltonen *et al.* (CDF Collaboration), Measurement of resonance parameters of orbitally excited narrow B^0 mesons, *Phys. Rev. Lett.* **102**, 102003 (2009).
[4] T. Aaltonen *et al.* (CDF Collaboration), Observation of orbitally excited B_s mesons, *Phys. Rev. Lett.* **100**, 082001 (2008).

- [5] R. Aaij *et al.* (LHCb Collaboration), First observation of the decay $B_{s2}^*(5840)^0 \rightarrow B^{*+}K^-$ and studies of excited B_s^0 mesons, *Phys. Rev. Lett.* **110**, 151803 (2013).
[6] V.M. Abazov *et al.* (D0 Collaboration), Observation and properties of the orbitally excited $B^*(s2)$ meson, *Phys. Rev. Lett.* **100**, 082002 (2008).
[7] T. A. Aaltonen *et al.* (CDF Collaboration), Study of orbitally excited B mesons and evidence for a new $B\pi$ resonance, *Phys. Rev. D* **90**, 012013 (2014).
[8] R. Aaij *et al.* (LHCb Collaboration), Precise measurements of the properties of the $B_1(5721)^{0,+}$ and $B_2^*(5747)^{0,+}$ states

- and observation of $B^{+0}\pi^{-+}$ mass structures, *J. High Energy Phys.* **04** (2015) 024.
- [9] R. Aaij *et al.* (LHCb Collaboration), Observation of new excited B_s^0 states, *Eur. Phys. J. C* **81**, 601 (2021).
- [10] R. Aaij *et al.* (LHCb Collaboration), Observation of the doubly charmed baryon Ξ_{cc}^{++} , *Phys. Rev. Lett.* **119**, 112001 (2017).
- [11] D. Ebert, R. N. Faustov, and V. O. Galkin, Heavy-light meson spectroscopy and Regge trajectories in the relativistic quark model, *Eur. Phys. J. C* **66**, 197 (2010).
- [12] S. Godfrey, K. Moats, and E. S. Swanson, B and B_s meson spectroscopy, *Phys. Rev. D* **94**, 054025 (2016).
- [13] Y. Sun, Q.-T. Song, D.-Y. Chen, X. Liu, and S.-L. Zhu, Higher bottom and bottom-strange mesons, *Phys. Rev. D* **89**, 054026 (2014).
- [14] Q. Li, R.-H. Ni, and X.-H. Zhong, Towards establishing an abundant B and B_s spectrum up to the second orbital excitations, *Phys. Rev. D* **103**, 116010 (2021).
- [15] J. Zeng, J. W. Van Orden, and W. Roberts, Heavy mesons in a relativistic model, *Phys. Rev. D* **52**, 5229 (1995).
- [16] M. Shah, B. Patel, and P. C. Vinodkumar, Spectroscopy and flavor changing decays of B , B_s mesons in a Dirac formalism, *Phys. Rev. D* **93**, 094028 (2016).
- [17] N. Devlani and A. K. Rai, Spectroscopy and decay properties of the D_s meson, *Phys. Rev. D* **84**, 074030 (2011).
- [18] Q.-F. Lü, K.-L. Wang, L.-Y. Xiao, and X.-H. Zhong, Mass spectra and radiative transitions of doubly heavy baryons in a relativized quark model, *Phys. Rev. D* **96**, 114006 (2017).
- [19] D. Ebert, R. N. Faustov, V. O. Galkin, and A. P. Martynenko, Mass spectra of doubly heavy baryons in the relativistic quark model, *Phys. Rev. D* **66**, 014008 (2002).
- [20] S. S. Gershtein, V. V. Kiselev, A. K. Likhoded, and A. I. Onishchenko, Spectroscopy of doubly heavy baryons, *Phys. Rev. D* **62**, 054021 (2000).
- [21] T. Yoshida, E. Hiyama, A. Hosaka, M. Oka, and K. Sadato, Spectrum of heavy baryons in the quark model, *Phys. Rev. D* **92**, 114029 (2015).
- [22] F. Giannuzzi, Doubly heavy baryons in a Salpeter model with AdS/QCD inspired potential, *Phys. Rev. D* **79**, 094002 (2009).
- [23] A. K. Rai, B. Patel, and P. C. Vinodkumar, Properties of $Q\bar{Q}$ mesons in non-relativistic QCD formalism, *Phys. Rev. C* **78**, 055202 (2008).
- [24] V. Kher, N. Devlani, and A. K. Rai, Spectroscopy, decay properties and Regge trajectories of the B and B_s mesons, *Chin. Phys. C* **41**, 093101 (2017).
- [25] B. H. Yazarloo and H. Mehraban, Study of B and B_s mesons with a Coulomb plus exponential type potential, *Europhys. Lett.* **116**, 31004 (2016).
- [26] V. Patel, R. Chaturvedi, and A. K. Rai, Spectroscopic properties of B and B_s meson using screened potential, *Indian J. Phys.* **98**, 2961 (2024).
- [27] B. Eakins and W. Roberts, Symmetries and systematics of doubly heavy hadrons, *Int. J. Mod. Phys. A* **27**, 1250039 (2012).
- [28] B. Chen, S.-Q. Luo, K.-W. Wei, and X. Liu, b -hadron spectroscopy study based on the similarity of double bottom baryon and bottom meson, *Phys. Rev. D* **105**, 074014 (2022).
- [29] V. V. Kiselev, A. K. Likhoded, O. N. Pakhomova, and V. A. Saleev, Mass spectra of doubly heavy Omega QQ' baryons, *Phys. Rev. D* **66**, 034030 (2002).
- [30] W. Roberts and M. Pervin, Heavy baryons in a quark model, *Int. J. Mod. Phys. A* **23**, 2817 (2008).
- [31] Z. Shah, K. Thakkar, and A. K. Rai, Excited state mass spectra of doubly heavy baryons Ω_{cc} , Ω_{bb} and Ω_{bc} , *Eur. Phys. J. C* **76**, 530 (2016).
- [32] Z. Shah and A. K. Rai, Excited state mass spectra of doubly heavy Ξ baryons, *Eur. Phys. J. C* **77**, 129 (2017).
- [33] M. Karliner and J. L. Rosner, Baryons with two heavy quarks: Masses, production, decays, and detection, *Phys. Rev. D* **90**, 094007 (2014).
- [34] Y. Lu, M. N. Anwar, and B.-S. Zou, How large is the contribution of excited mesons in coupled-channel effects?, *Phys. Rev. D* **95**, 034018 (2017).
- [35] J. Oudichhya and A. K. Rai, Study of singly bottom and doubly heavy baryons within Regge phenomenology, *Eur. Phys. J. A* **60**, 125 (2024).
- [36] J. G. Korner, M. Kramer, and D. Pirjol, Heavy baryons, *Prog. Part. Nucl. Phys.* **33**, 787 (1994).
- [37] H.-Y. Cheng and F.-S. Yu, Masses of scalar and axial-vector B mesons revisited, *Eur. Phys. J. C* **77**, 668 (2017).
- [38] M. H. Alhakami, Predictions for the beauty meson spectrum, *Phys. Rev. D* **103**, 034009 (2021).
- [39] E. Bagan, M. Chabab, and S. Narison, Baryons with two heavy quarks from QCD spectral sum rules, *Phys. Lett. B* **306**, 350 (1993).
- [40] Z.-G. Wang, Analysis of the $1/2^+$ doubly heavy baryon states with QCD sum rules, *Eur. Phys. J. A* **45**, 267 (2010).
- [41] C. B. Lang, D. Mohler, S. Prelovsek, and R. M. Woloshyn, Predicting positive parity B_s mesons from lattice QCD, *Phys. Lett. B* **750**, 17 (2015).
- [42] E. B. Gregory *et al.*, Precise B , B_s and B_c meson spectroscopy from full lattice QCD, *Phys. Rev. D* **83**, 014506 (2011).
- [43] M. Padmanath, R. G. Edwards, N. Mathur, and M. Peardon, Spectroscopy of doubly-charmed baryons from lattice QCD, *Phys. Rev. D* **91**, 094502 (2015).
- [44] C. Alexandrou, J. Carbonell, D. Christaras, V. Drach, M. Gravina, and M. Papinutto, Strange and charm baryon masses with two flavors of dynamical twisted mass fermions, *Phys. Rev. D* **86**, 114501 (2012).
- [45] P. Jakhad, J. Oudichhya, K. Gandhi, and A. K. Rai, Identification of newly observed singly charmed baryons using the relativistic flux tube model, *Phys. Rev. D* **108**, 014011 (2023).
- [46] P. Jakhad, J. Oudichhya, and A. K. Rai, Interpretation of recently discovered single bottom baryons in the relativistic flux tube model, *Phys. Rev. D* **110**, 094005 (2024).
- [47] P. Jakhad, J. Oudichhya, and A. K. Rai, Masses of higher excited states of $\Xi c'$ and Ωc baryons, *Int. J. Mod. Phys. A* **39**, 2443005 (2024).
- [48] P. Jakhad and A. K. Rai, Relativistic flux-tube model predictions from charmed mesons to double-charmed baryons (Communicated), [arXiv:2509.24772](https://arxiv.org/abs/2509.24772).
- [49] B. Chen, K.-W. Wei, and A. Zhang, Investigation of Λ_Q and Ξ_Q baryons in the heavy quark-light diquark picture, *Eur. Phys. J. A* **51** (2015).

- [50] C. Olson, M. G. Olsson, and D. LaCourse, The quantized relativistic flux tube, *Phys. Rev. D* **49**, 4675 (1994).
- [51] M. G. Olsson, The relativistic flux tube model of hadrons, *Nuovo Cimento Soc. Ital. Fis.* **107A**, 2541 (1994).
- [52] T. J. Allen, M. G. Olsson, and S. Veseli, Curved QCD string dynamics, *Phys. Rev. D* **60**, 074026 (1999).
- [53] F. James and M. Roos, Minuit—A system for function minimization and analysis of the parameter errors and correlations, *Comput. Phys. Commun.* **10**, 343 (1975).
- [54] R. Aaij *et al.* (LHCb Collaboration), Precision measurement of the Ξ_{cc}^{++} mass, *J. High Energy Phys.* **02** (2020) 049.
- [55] J. Hu and T. Mehen, Chiral lagrangian with heavy quark-diquark symmetry, *Phys. Rev. D* **73**, 054003 (2006).
- [56] Y. Song, D. Jia, W. Zhang, and A. Hosaka, Low-lying doubly heavy baryons: Regge relation and mass scaling, *Eur. Phys. J. C* **83**, 1 (2023).
- [57] B. Chen, A. Zhang, and J. He, Bottomonium spectrum in the relativistic flux tube model, *Phys. Rev. D* **101**, 014020 (2020).
- [58] K. Johnson and C. B. Thorn, Stringlike solutions of the bag model, *Phys. Rev. D* **13**, 1934 (1976).
- [59] T. A. Lahde, C. J. Nyfalt, and D. O. Riska, Spectra and M1 decay widths of heavy light mesons, *Nucl. Phys.* **A674**, 141 (2000).
- [60] Q.-F. Lü, T.-T. Pan, Y.-Y. Wang, E. Wang, and D.-M. Li, Excited bottom and bottom-strange mesons in the quark model, *Phys. Rev. D* **94**, 074012 (2016).
- [61] R. Akers *et al.* (OPAL Collaboration), Observations of Π^- -B charge—flavor correlations and resonant B Π and B K production, *Z. Phys. C* **66**, 19 (1995).
- [62] P. Abreu *et al.* (DELPHI Collaboration), Observation of orbitally excited B mesons, *Phys. Lett. B* **345**, 598 (1995).
- [63] R. Barate *et al.* (ALEPH Collaboration), Resonant structure and flavor tagging in the B $\pi^\pm$ system using fully reconstructed B decays, *Phys. Lett. B* **425**, 215 (1998).
- [64] M. Acciarri *et al.* (L3 Collaboration), Measurement of the spectroscopy of orbitally excited B mesons at LEP, *Phys. Lett. B* **465**, 323 (1999).
- [65] L.-Y. Xiao and X.-H. Zhong, Strong decays of higher excited heavy-light mesons in a chiral quark model, *Phys. Rev. D* **90**, 074029 (2014).
- [66] G.-L. Yu and Z.-G. Wang, Analysis of the excited bottom and bottom-strange states $B_1(5721)$, $B_2^*(5747)$, $B_{s1}(5830)$, $B_{s2}^*(5840)$, $B_J(5840)$ and $B_J(5970)$ in B meson family, *Chin. Phys. C* **44**, 033103 (2020).
- [67] I. Asghar, B. Masud, E. S. Swanson, F. Akram, and M. Atif Sultan, Decays and spectrum of bottom and bottom strange mesons, *Eur. Phys. J. A* **54**, 127 (2018).
- [68] W. Hao, Y. Lu, and E. Wang, The assignments of the B_s mesons within the screened potential model and $^3P_0$ model, *Eur. Phys. J. C* **83**, 520 (2023).
- [69] M. Karliner and J. L. Rosner, Very narrow excited Ω_c baryons, *Phys. Rev. D* **95**, 114012 (2017).