

# Revisiting the electroweak supersymmetry from the generalized minimal supergravity

Imtiaz Khan<sup>1,2,3,\*</sup>, Ali Muhammad<sup>4,5,†</sup>, Tianjun Li<sup>6,4,5,‡</sup> and Shabbar Raza<sup>7,§</sup>

<sup>1</sup>*Department of Physics, Zhejiang Normal University, Jinhua, Zhejiang 321004, China*

<sup>2</sup>*Research Center of Astrophysics and Cosmology, Khazar University, Baku, AZ1096, 41 Mehseti Street, Azerbaijan*

<sup>3</sup>*Zhejiang Institute of Photoelectronics, Jinhua, Zhejiang 321004, China*

<sup>4</sup>*CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China*

<sup>5</sup>*School of Physical Sciences, University of Chinese Academy of Sciences, No. 19A Yuquan Road, Beijing 100049, China*

<sup>6</sup>*School of Physics, Henan Normal University, Xinxiang 453007, People's Republic of China*

<sup>7</sup>*Department of Physics, Federal Urdu University of Arts, Science and Technology, Karachi 75300, Pakistan*



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We explore the electroweak supersymmetry (EWSUSY) scenario within the minimal supersymmetric Standard Model under the generalized minimal supergravity (GmSUGRA) framework, given that the anomalous magnetic moment of the muon may now be consistent with the Standard Model (SM) prediction, we consider both signs of the Higgsino mass parameter,  $\mu < 0$  and  $\mu > 0$ . A comprehensive scan of the parameter space is performed, subject to the experimental constraints from the LHC SUSY searches, Planck 2018 relic density, and LUX-ZEPLIN (LZ) direct detection limits. We identify the viable regions featuring neutralino dark matter production via coannihilation with stau, chargino, stop, sbottom, and gluino, as well as through  $A$ -funnel, Higgs-resonance, and  $Z$ -resonance mechanisms. Notably, the  $\mu < 0$  scenario yields a broader allowed parameter space, including for the first time sbottom-neutralino coannihilation solutions in GmSUGRA, which are absent for  $\mu > 0$ . While the Higgs-pole and  $Z$ -pole regions for  $\mu > 0$  are largely excluded by the current LZ bounds, substantial viable regions remain for  $\mu < 0$ . Gluino coannihilation scenarios are strongly constrained by the current LHC data. The characteristic mass ranges of interest include sbottoms (0.7–1.3 TeV), stops (up to 1.0 TeV for  $\mu > 0$  and 1.3 TeV for  $\mu < 0$ ), staus and charginos (up to 1.5 TeV), and pseudoscalar Higgs bosons in the  $A$  funnel (0.4–1.4 TeV). Moreover, the supersymmetric contributions to the muon anomalous magnetic moment remain within a  $2\sigma$  deviation from the SM prediction. And our findings suggest that significant portions of the parameter space can be probed at the future LHC SUSY searches and upcoming dark matter direct detection experiments.

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## I. INTRODUCTION

The supersymmetric Standard Models remain among the most attractive frameworks for physics beyond the Standard Model (BSM). They offer elegant solutions to several theoretical challenges, such as the gauge hierarchy

problem, and predict gauge coupling unification [1–6]. Moreover, if  $R$  parity is conserved, the lightest supersymmetric particle (LSP), often the neutralino, constitutes a viable dark matter (DM) candidate [7,8]. The framework also naturally accommodates radiative electroweak symmetry breaking via the large top Yukawa coupling. Within the minimal supersymmetric Standard Model (MSSM), the predicted mass for the lightest  $CP$ -even Higgs boson lies in the 100–135 GeV range [9], in agreement with experimental observations. As a result, supersymmetry (SUSY) remains a central focus at the Large Hadron Collider (LHC), linking high-scale theory with low-energy phenomenology. Following the LHC Run-2, no definitive SUSY signals have emerged, leading to stringent mass bounds on colored sparticles:  $\sim 1.25$  TeV for stops,

\*Contact author: [ikhanphys1993@gmail.com](mailto:ikhanphys1993@gmail.com)

†Contact author: [alimuhammad@phys.qau.edu.pk](mailto:alimuhammad@phys.qau.edu.pk)

‡Contact author: [tli@mail.itp.ac.cn](mailto:tli@mail.itp.ac.cn)

§Contact author: [shabbar.raza@fuuast.edu.pk](mailto:shabbar.raza@fuuast.edu.pk)

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$\sim 1.5$  TeV for sbottoms,  $\sim 2$  TeV for first- and second-generation squarks, and  $\sim 2.2$  TeV for gluinos [10–14]. These results suggest that colored superpartners are considerably heavy, typically above the TeV scale. However, viable models with an electroweak-scale binolike LSP and relatively light sleptons remain consistent with current constraints. Motivated by BSM theories, such scenarios allow slepton masses ranging from a few hundred GeV up to the TeV scale [15–17].

Recent advances in collider and DM experiments have significantly refined our understanding of physics BSM. The ATLAS and CMS collaborations have provided important constraints through searches for heavy Higgs bosons [18], detailed studies of charginos and neutralinos [19–22], and investigations of invisible Higgs decays [23]. In parallel, direct detection (DD) experiments such as PICO-60, LUX-ZEPLIN (LZ), XENON-1T, and PandaX-4T have set stringent limits on spin-independent (SI) and spin-dependent (SD) DM-nucleon scattering cross sections [24–30]. With the LHC entering Run-3, a timely reassessment of the MSSM parameter space is essential. In light of the most recent and stringent SI cross-section bounds reported by the LZ experiment [28,29], identifying phenomenologically viable regions that remain accessible to upcoming LHC searches is of critical importance.

We present a detailed phenomenological study of the generalized minimal supergravity (GmSUGRA) framework, investigating both signs of the Higgsino mass parameter,  $\mu > 0$  and  $\mu < 0$ , without relying on any fine-tuning assumptions. The sign of  $\mu$  remains phenomenologically important in SUSY grand unified theories (GUTs), since it determines the sign and magnitude of the dominant SUSY contribution to the anomalous magnetic moment of the muon,  $(g_\mu - 2)$ , arising primarily from neutralino-smuon and chargino-sneutrino loop diagrams [31,32]. Recent lattice-QCD-based evaluations of the hadronic vacuum polarization contribution have significantly reduced the discrepancy between the Standard Model prediction and the experimental measurement of  $(g_\mu - 2)$  [33,34]. Consequently, this observable no longer provides a strong indication of physics beyond the Standard Model. Nevertheless, because the SUSY contribution depends sensitively on the sign of  $\mu$ , it remains well motivated to investigate both  $\mu > 0$  and  $\mu < 0$  scenarios. Furthermore, recent studies have shown that the  $\mu < 0$  scenario can accommodate viable dark matter solutions through Higgs- and Z-boson resonance channels while remaining consistent with direct-detection and LHC constraints [35,36]. These developments motivate a comprehensive phenomenological investigation of SUSY GUT models with both signs of  $\mu$  in light of current LHC searches and dark matter experiments.

Focusing on dark matter phenomenology, we conduct extensive scans over the GmSUGRA parameter space and identify regions compatible with current limits from direct

detection experiments such as the LUX-ZEPLIN (LZ) Collaboration. Both  $\mu > 0$ , and  $\mu < 0$  scenarios yield solutions featuring resonant annihilation via the Higgs and Z bosons. However, direct detection bounds impose significantly tighter constraints on the  $\mu > 0$  case, largely excluding light Higgsinos in the Z- and H-pole regions. In contrast, the  $\mu < 0$  case allows for such light Higgsinos to remain viable under current LZ and electroweakino search limits. Our analysis uncovers a wide range of DM mechanisms compatible with collider and cosmological data, including: A-resonance annihilation, Higgs- and Z-boson resonance channels, as well as coannihilation scenarios involving stop, stau, gluino, sbottom, and tau with the lightest neutralino. All identified solutions respect constraints from LHC Run 2 data, projected sensitivities of Run 3, and future high-energy collider prospects. Moreover, these scenarios are consistent with the relic density bounds reported by Planck 2018 and adhere to current and forthcoming limits from direct and indirect DM detection efforts. Significantly, for the first time within the GmSUGRA framework, we identify sbottom-neutralino coannihilation as a viable mechanism exclusively in the  $\mu < 0$  scenario, whereas this channel is absent for  $\mu > 0$ . This further supports the case for investigating  $\mu < 0$  more thoroughly, given its greater flexibility under existing experimental bounds. Our results also demonstrate that the supersymmetric contribution to the muon’s anomalous magnetic moment,  $(g - 2)_\mu$ , remains compatible with the current experimental measurement, yielding deviations within the  $1\sigma$  to  $2\sigma$  deviation around the SM expectation for the  $\mu < 0$  scenario. Thus, our findings underscore the phenomenological richness of the GmSUGRA model and motivate focused studies on the  $\mu < 0$  regime to draw more conclusive insights in light of ongoing and future experimental results.

## II. THE GMSUGRA IN THE MSSM

Electroweak supersymmetry (EWSUSY) can naturally emerge within the framework of the GmSUGRA model [37,38], where the sleptons, bino, wino, and/or Higgsinos lie within the TeV scale, while squarks and gluinos may reside at multi-TeV scales [39]. The GmSUGRA is based on the  $SU(5)$  GUT, with the GUT symmetry broken by a Higgs field in the adjoint representation. Couplings of this field to gauge kinetic terms via higher-dimensional operators induce modifications in the gauge coupling and gaugino mass unification conditions once the adjoint acquires a vacuum expectation value.

The modified gauge coupling and gaugino mass relations at the GUT scale are given by,

$$\frac{1}{\alpha_2} - \frac{1}{\alpha_3} = k \left( \frac{1}{\alpha_1} - \frac{1}{\alpha_3} \right), \quad (1)$$

$$\frac{M_2}{\alpha_2} - \frac{M_3}{\alpha_3} = k \left( \frac{M_1}{\alpha_1} - \frac{M_3}{\alpha_3} \right), \quad (2)$$

where  $k = 5/3$  in the minimal GmSUGRA case. Here ( $k=5/3$ ) is fixed by the normalization of the hypercharge generator in  $SU(5)$ , where  $(\text{Tr}(Y^2) = 5/3)$  in the fundamental representation. In the GmSUGRA framework considered here, the  $SU(5)$  symmetry is broken by an adjoint Higgs field ( $\langle \Sigma \rangle \propto Y$ ), leading to the gaugino mass relation with ( $k = 5/3$ ) [37,38]. This corresponds to the commonly studied minimal realization based on the (24) representation of  $SU(5)$ , yielding a predictive nonuniversal gaugino mass pattern with only one additional parameter beyond universal mSUGRA. The value  $k = 5/3$  is analogous to the familiar  $SU(5)$  hypercharge normalization factor appearing in gauge coupling unification and therefore does not represent an additional arbitrary parameter. Assuming gauge coupling unification,  $\alpha_1 = \alpha_2 = \alpha_3$ , leads to a simplified gaugino mass relation,

$$M_2 - M_3 = \frac{5}{3}(M_1 - M_3), \quad (3)$$

which generalizes the universal gaugino mass condition  $M_1 = M_2 = M_3$  of minimal supergravity (mSUGRA). Solving Eq. (3) for  $M_3$ , we obtain,

$$M_3 = \frac{5}{2}M_1 - \frac{3}{2}M_2, \quad (4)$$

implying that  $M_3$  can vary from several hundred GeV to multiple TeV depending on the values of  $M_1$  and  $M_2$ , which are treated as free input parameters in EWSUSY. The soft supersymmetry-breaking (SSB) scalar mass relations at the GUT scale are derived under the assumption of an adjoint Higgs in  $SU(5)$  [38]. Taking the slepton masses as free parameters, the squark masses are,

$$m_{\tilde{Q}_i}^2 = \frac{5}{6}(m_0^U)^2 + \frac{1}{6}m_{\tilde{E}_i^c}^2, \quad (5)$$

$$m_{\tilde{U}_i^c}^2 = \frac{5}{3}(m_0^U)^2 - \frac{2}{3}m_{\tilde{E}_i^c}^2, \quad (6)$$

$$m_{\tilde{D}_i^c}^2 = \frac{5}{3}(m_0^U)^2 - \frac{2}{3}m_{\tilde{L}_i}^2, \quad (7)$$

where  $m_0^U$  is the universal scalar mass, and the tilde fields represent soft scalar masses of the corresponding sfermions. In EWSUSY, since  $m_{\tilde{L}}, m_{\tilde{E}^c} \lesssim 1$  TeV, the sleptons remain light. In the limit  $m_0^U \gg m_{\tilde{L}}, m_{\tilde{E}^c}$ , the approximate relations  $2m_{\tilde{Q}}^2 \approx m_{\tilde{U}^c}^2 \approx m_{\tilde{D}^c}^2$  hold.... Additionally, the Higgs soft masses  $m_{\tilde{H}_u}, m_{\tilde{H}_d}$ , and trilinear couplings  $A_U, A_D, A_E$  are all considered free parameters in the GmSUGRA framework [38,39].

### III. PHENOMENOLOGICAL CONSTRAINTS AND SCANNING PROCEDURE

We perform a random scan over the parameter space using the ISAJET 7.85 package [40]. The scan is carried out over the following ranges:

$$\begin{aligned} 0 \text{ GeV} &\leq m_0^U \leq 9000 \text{ GeV}, \\ 80 \text{ GeV} &\leq M_1 \leq 3000 \text{ GeV}, \\ 100 \text{ GeV} &\leq M_2 \leq 3100 \text{ GeV}, \\ 100 \text{ GeV} &\leq m_{\tilde{L}} \leq 1200 \text{ GeV}, \\ 100 \text{ GeV} &\leq m_{\tilde{E}^c} \leq 1200 \text{ GeV}, \\ 100 \text{ GeV} &\leq m_{\tilde{H}_{u,d}} \leq 5000 \text{ GeV}, \\ -16000 \text{ GeV} &\leq A_U = A_D \leq 16000 \text{ GeV}, \\ -6000 \text{ GeV} &\leq A_E \leq 6000 \text{ GeV}, \\ 2 &\leq \tan \beta \leq 60. \end{aligned} \quad (8)$$

The scan assumes  $\mu > 0$ ,  $\mu < 0$ , and a top quark mass of  $m_t = 173.3$  GeV [41], with  $m_b^{\overline{\text{DR}}}(M_Z) = 2.83$  GeV as implemented in ISAJET. Parameter space exploration is guided by the Metropolis-Hastings algorithm [42]. The scalar-mass relations in Eqs. (6)–(8) are derived in the GmSUGRA framework under the assumption that the  $SU(5)$ -breaking  $F$  term is aligned along the hypercharge direction ( $(F_T = 0)$ ) [38,39]. Under this assumption, the sfermion masses satisfy the relations given in Eqs. (6)–(8), while the Higgs-sector soft masses are not uniquely determined by the same  $SU(5)$ -breaking pattern. Consequently,  $(m_{\tilde{H}_u}^2)$  and  $(m_{\tilde{H}_d}^2)$  are treated as independent inputs, following the standard approach adopted in non-universal Higgs mass scenarios [39]. This additional freedom is phenomenologically important for achieving successful electroweak symmetry breaking and accommodating the observed Higgs-boson mass. Similarly, the trilinear couplings  $(A_U)$ ,  $(A_D)$ , and  $(A_E)$  are treated as free parameters. As shown in Ref. [38], the corresponding higher-dimensional operators involve the Yukawa matrices  $(y_{U,D,E})$ , which are themselves independent parameters. Therefore, the resulting trilinear soft terms are not uniquely fixed by the  $SU(5)$ -breaking sector alone and may be regarded as independent inputs in the low-energy phenomenological analysis [38,39]. Throughout the analysis, we adopt the notations  $A_t \equiv A_U$ ,  $A_b \equiv A_D$ , and  $A_\tau \equiv A_E$ .

It is instructive to compare the GmSUGRA framework with the phenomenological MSSM (pMSSM) [43,44]. While the pMSSM is formulated in terms of independent weak-scale soft SUSY-breaking parameters, GmSUGRA is defined by GUT-scale boundary conditions with nontrivial relations among gaugino and scalar masses arising from the underlying  $SU(5)$  structure [37–39]. As a result, both frameworks can accommodate similar low-energy

phenomenology, including neutralino dark matter realized through coannihilation or Higgs-funnel mechanisms. The key distinction is that, in GmSUGRA, these spectra emerge from a constrained grand-unified framework that predicts correlations among sparticle masses, such as the gaugino mass relations in Eqs. (3) and (4), which are absent in the generic pMSSM. Consequently, GmSUGRA provides a more predictive connection between low-energy observables and physics at the unification scale.

The collected data is subjected to a set of experimental and theoretical constraints discussed in the subsequent Section.

- (1) Basic constraints:
  - (a) Radiative electroweak symmetry breaking (REWSB).
  - (b) The LSP is a neutralino.
  - (c) Sparticle mass bounds:

$$\begin{aligned} m_{\tilde{t}_1}, m_{\tilde{b}_1} &\geq 100 \text{ GeV}, \\ m_{\tilde{\tau}_1} &\geq 105 \text{ GeV}, \\ m_{\tilde{\chi}_1^\pm} &\geq 103 \text{ GeV}, \end{aligned} \quad (9)$$

from LEP2 [45], and from LHC,

$$\begin{aligned} m_{\tilde{g}} &\geq 2.2 \text{ TeV}, \\ m_{\tilde{q}} &\geq 2 \text{ TeV}, \\ m_{\tilde{t}_1} &\gtrsim 1.25 \text{ TeV}, \\ m_{\tilde{b}_1} &\gtrsim 1.5 \text{ TeV} \end{aligned} \quad (10)$$

with Refs. [10–14].

- (d) Higgs boson mass,

$$122 \text{ GeV} \leq m_h \leq 128 \text{ GeV}, \quad (11)$$

to account for the uncertainty of  $m_h$  calculation in the MSSM with Refs. [46,47].

- (e)  $B$ -physics observables (evaluated using IsaTools [48–51]),

$$0.15 \leq \frac{\text{BR}(B_u \rightarrow \tau \nu_\tau)_{\text{MSSM}}}{\text{BR}(B_u \rightarrow \tau \nu_\tau)_{\text{SM}}} \leq 2.41(3\sigma), \quad (12)$$

$$2.99 \times 10^{-4} \leq \text{BR}(b \rightarrow s \gamma) \leq 3.87 \times 10^{-4}(2\sigma), \quad (13)$$

$$0.8 \times 10^{-9} \leq \text{BR}(B_s \rightarrow \mu^+ \mu^-) \leq 6.2 \times 10^{-9}(2\sigma). \quad (14)$$

within  $3\sigma$  and  $2\sigma$  experimental limits [52–54].

- (f) Planck 2018 relic density constraint [55],

$$0.114 \leq \Omega_{\text{CDM}} h^2 (\text{Planck2018}) \leq 0.126(5\sigma). \quad (15)$$

## IV. RESULTS AND DISCUSSION

We have systematically studied the GmSUGRA parameter space under the constraints described above.

Before discussing the phenomenological implications of the  $\tilde{b}_1 - \tilde{\chi}_1^0$  coannihilation region in detail, we comment on the theoretical uncertainties associated with the calculation of the sbottom mass spectrum. It is well known that for large  $\tan\beta$ , sizable trilinear couplings, and heavy scalar masses, different public spectrum generators can yield non-negligible differences in the predicted  $\tilde{b}_1$  mass [56–58]. These differences arise primarily from the treatment of two-loop renormalization-group running, threshold corrections, and the implementation of  $\tan\beta$ -enhanced bottom Yukawa effects, as well as from the choice of input bottom-quark mass. In particular, the right-handed sbottom soft mass is strongly influenced by the gluino mass  $M_3$  through renormalization-group evolution, and by the running bottom Yukawa coupling  $y_b$ , making the  $\tilde{b}_1$  mass more sensitive to theoretical assumptions than other sfermions. Consequently, the precise extent of the pure  $\tilde{b}_1 - \tilde{\chi}_1^0$  coannihilation strip may vary between spectrum generators. Nevertheless, our analysis demonstrates that in the  $\mu < 0$  GmSUGRA framework a viable region exists in which sbottom coannihilation contributes significantly to the effective annihilation cross section. While this region is narrower than suggested by a naïve random scan, it remains robust once local parameter variations are allowed that preserve the Higgs mass constraint and the Planck relic-density bound. We thus emphasize the existence of a phenomenologically viable *sbottom-assisted* coannihilation regime, rather than claiming a broad or generic sbottom-dominated region.<sup>1</sup> The upper panels of Fig. 1 display the parameter space in the  $m_{\tilde{t}_1} - m_{\tilde{\chi}_1^0}$  plane, separated into two scenarios:  $\mu > 0$  (left) and  $\mu < 0$  (right). The gray points satisfy REWSB and assume a neutralino LSP. Orange points satisfy all experimental bounds, including LEP constraints, rare  $B$ -meson decays, Higgs mass measurements, and LHC sparticle searches, while also being consistent with relic density exceeding the Planck upper bound. A subset of these, marked in green, correspond to relic density values below the Planck limit. Red points represent the most viable solutions consistent with

<sup>1</sup>The sbottom mass is subject to larger theoretical uncertainties than most other sparticles, particularly at large  $\tan\beta$ . Small differences between spectrum generators may therefore shift the precise location of the  $\tilde{b}_1 - \tilde{\chi}_1^0$  coannihilation point without altering the qualitative phenomenology.

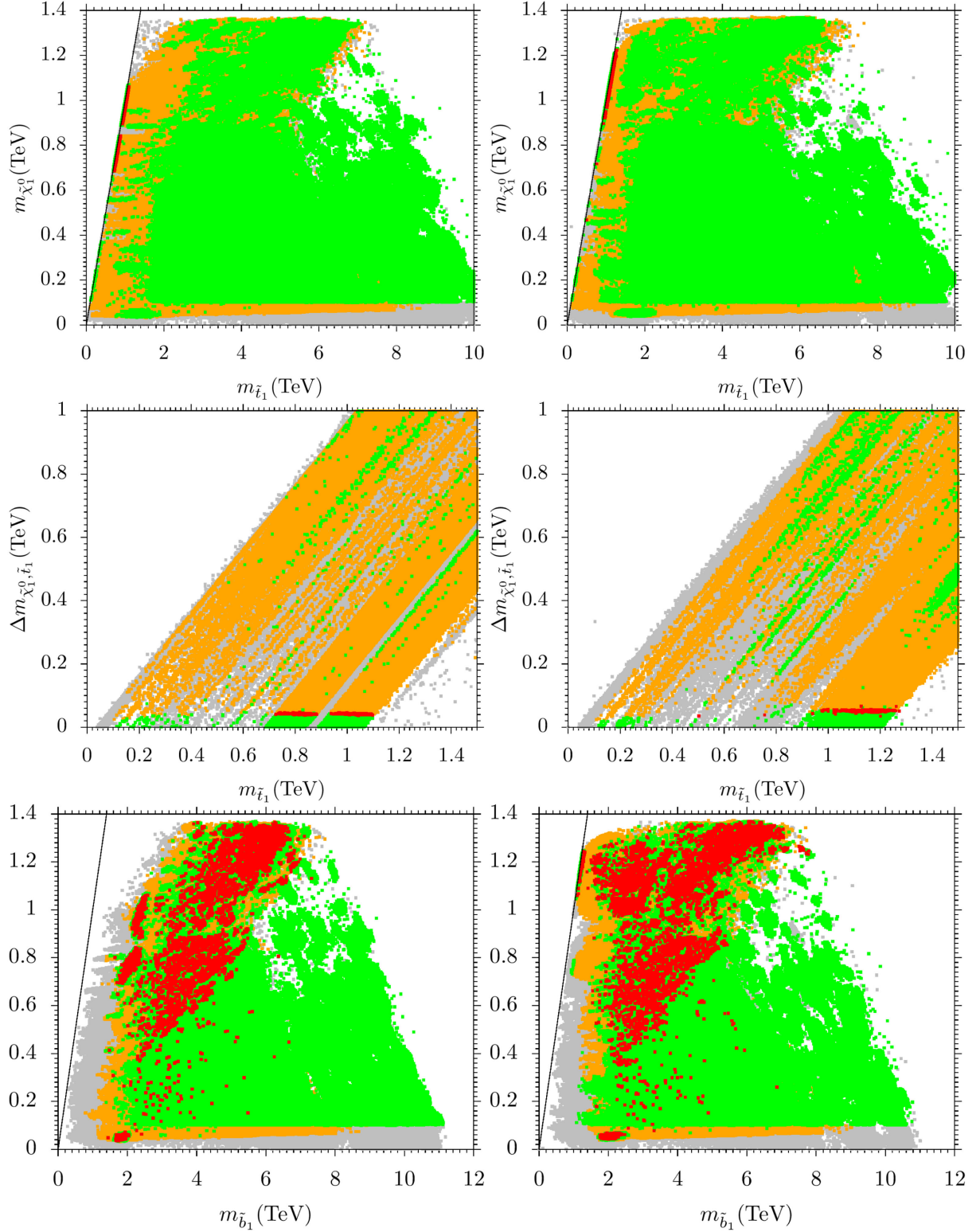


FIG. 1.  $m_{\tilde{t}_1}$  vs  $m_{\tilde{\chi}_1^0}$ ,  $m_{\tilde{t}_1}$  vs  $|\Delta m_{\tilde{\chi}_1^0, \tilde{t}_1}|$ , and  $m_{\tilde{b}_1}$  vs  $m_{\tilde{\chi}_1^0}$  planes with color coding as follows: The left and right panels correspond to the  $\mu > 0$  and  $\mu < 0$  scenarios, respectively. Grey points satisfy the REWSB and yield LSP neutralino. Orange points are a subset of the grey points and additionally satisfy the LEP mass limits, constraints from  $B$  physics observables, the Higgs boson mass requirement, and current LHC bounds on  $S$  particles, but satisfy an overabundant neutralino relic density. Green points form a further subset and satisfy all the above constraints except the relic-density upper bound, yielding an underabundant dark matter density. Points satisfying all constraints, including consistency with the Planck-measured relic density, are shown in red. The solid, black lines serve as visual cues for coannihilation as well as resonance solutions' sensitivity.

the observed relic density range within  $5\sigma$  uncertainty. Black solid curves indicate regions where neutralino-stop coannihilation or resonance effects are expected to dominate. Notably, red points aligned along these curves indicate viable next-to-lightest supersymmetric particle (NLSP) stop scenarios. These solutions occur for  $m_{\tilde{\tau}_1}$  values between approximately 0.15 TeV and 1.3 TeV. The apparent sparsity of red, green, and orange points in this region is due to the limited scan density rather than a true exclusion; denser scans would likely populate these regions further. The middle panel presents the mass gap  $\Delta m_{\tilde{\tau}_1, \tilde{\chi}_1^0}$  versus  $m_{\tilde{\chi}_1^0}$ . For red points, the stop-neutralino mass difference can reach up to 50 GeV. Despite this, the ratio  $\Delta m/m_{\tilde{\chi}_1^0}$  remains below 10%, consistent with coannihilation requirements. Large mass differences allow kinematically favorable decays such as  $\tilde{\tau}_1 \rightarrow t\tilde{\chi}_1^0$ , or three- and four-body decays like  $\tilde{\tau}_1 \rightarrow W + b + \tilde{\chi}_1^0$  and  $\tilde{\tau}_1 \rightarrow f + f' + b + \tilde{\chi}_1^0$ . In the case of small mass gaps, these decay channels are closed, and the loop-suppressed decay  $\tilde{\tau}_1 \rightarrow c\tilde{\chi}_1^0$  becomes dominant [59,60]. For recent LHC searches see [61–69]. In all of these studies, the maximum stop mass considered is 1.2 TeV, which is consistent with the maximum stop mass allowed by all constraints (red points) in our findings, indicating regions potentially accessible in future LHC runs. Even scenarios with small mass splittings where  $\tilde{\tau}_1 \rightarrow t\tilde{\chi}_1^0$  the dominant remains viable above 550 GeV, beyond current experimental constraints [66]. The lower panels of Fig. 1 show  $m_{\tilde{b}_1}$  as a function of  $m_{\tilde{\chi}_1^0}$  for  $\mu > 0$  and  $\mu < 0$ . The same color scheme applies. The diagonal line corresponds to mass-degenerate regions indicative of sbottom-neutralino coannihilation. In our current analysis, no sbottom-coannihilation is observed for  $\mu > 0$ . However, in the  $\mu < 0$  case, viable red points near 1–1.3 TeV hint at possible sbottom coannihilation. We note that the sbottom-coannihilation region visible for  $\mu < 0$  is more sparsely populated in a random scan, reflecting the enhanced sensitivity of the sbottom mass to high-scale parameters. Dedicated local scans confirm that viable solutions persist, albeit in a restricted region of parameter space. For  $\Delta m_{\tilde{b}_1, \tilde{\chi}_1^0} > m_b$ , dominant decays include  $pp \rightarrow \tilde{b}_1 \tilde{b}_1^* X \rightarrow b\tilde{b} + E_T$ , with  $\tilde{b}_1 \rightarrow b\tilde{\chi}_1^0$  being the primary decay channel. Additional signatures may arise from same-sign sbottom pair production:  $\tilde{b}_1 \tilde{b}_1$  or  $\tilde{b}_1^* \tilde{b}_1^*$ . Recent ATLAS results have excluded sbottom masses up to 1.5 TeV for decay chains involving heavier neutralinos or charginos [70–72]. For compressed spectra with  $\Delta m \sim 10$  GeV, searches employing secondary vertex tagging techniques exclude sbottoms up to 660 GeV [73]. Monojet searches have ruled out masses up to 600 GeV. Importantly, for  $m_{\tilde{b}_1} > 800$  GeV and small mass gaps, there are no stringent bounds [73], leaving our high-mass

sbottom solutions unconstrained and promising for discovery in Run-3 and future colliders.

Figure 2 presents our findings in the  $m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0}$  (top panel) and  $m_{\tilde{\tau}_1} - |\Delta m_{\tilde{\tau}_1, \tilde{\chi}_1^0}|$  (middle panel) parameter planes. The color coding is consistent with that used in Fig. 1. The top panel illustrates the stau-neutralino coannihilation region, while the middle panel highlights the corresponding mass splitting between the lightest stau ( $\tilde{\tau}_1$ ) and the lightest neutralino ( $\tilde{\chi}_1^0$ ). From our parameter scan, we observe that viable coannihilation solutions—characterized by near mass degeneracy between  $\tilde{\tau}_1$  and  $\tilde{\chi}_1^0$ —are located within the range  $0.15 \text{ TeV} \lesssim m_{\tilde{\tau}_1} \lesssim 1.4 \text{ TeV}$ . These findings are consistent with the experimental results reported by the CMS Collaboration based on  $137 \text{ fb}^{-1}$  data collected at  $\sqrt{s} = 13 \text{ TeV}$  [74]. We anticipate that future data from LHC Run 3 and high-luminosity phases will be capable of probing a significant portion of this parameter space. In addition to coannihilation scenarios, our scan also reveals the presence of Higgs funnel solutions. In these regions, the relic abundance of neutralino dark matter is effectively reduced through  $s$ -channel resonant annihilation via  $CP$ -even or  $CP$ -odd Higgs bosons, specifically  $h$ ,  $H$ , and  $A$ , with the condition  $m_A \simeq 2m_{\tilde{\chi}_1^0}$ . We also note that in this scenario  $m_A \simeq m_H$ . Our results indicate that such funnel solutions are present for  $m_A$  values ranging from approximately 1.2 TeV to 2.9 TeV. Recent constraints from the CMS experiment [75] demonstrate that for the decay mode  $A \rightarrow \tau^+ \tau^-$ , pseudoscalar Higgs masses below 1.7 TeV are excluded for  $\tan\beta \lesssim 30$ . Furthermore, projections from Refs. [76,77] indicate that for  $\tan\beta \lesssim 10$ , values of  $m_A \sim 1.0, 1.1$ , and 1.4 TeV may be testable at Run 2, Run 3, and the high-luminosity large hadron collider, respectively. Our parameter space includes scenarios that lie beyond these current exclusion limits, thereby motivating continued experimental exploration in this regime.

In addition to the previously discussed annihilation channels, our parameter space scan reveals viable chargino-neutralino coannihilation regions. These are illustrated in Fig. 3, where we present the distributions in the  $m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0}$  (top panel) and  $m_{\tilde{\chi}_1^\pm} - |\Delta m_{\tilde{\chi}_1^\pm, \tilde{\chi}_1^0}|$  (middle panel) planes. The red data points correspond to scenarios where the lightest chargino ( $\tilde{\chi}_1^\pm$ ) is nearly degenerate in mass with the lightest neutralino ( $\tilde{\chi}_1^0$ ), lying within the mass interval  $0.3 \text{ TeV} \lesssim m_{\tilde{\chi}_1^\pm} \lesssim 1.5 \text{ TeV}$ . Experimental bounds from direct searches for electroweakinos are provided in Ref. [78], which investigates final states arising from slepton- and  $W/Z/h$ -mediated decays of chargino and neutralino pairs. According to these constraints, degenerate scenarios with  $m_{\tilde{\chi}_1^\pm} \gtrsim 300 \text{ GeV}$  are not excluded at the 95% confidence level, thereby validating the heavier solutions in our scan. Furthermore, in regions of the parameter space where sleptons are heavier than charginos, decay channels mediated by virtual sleptons are

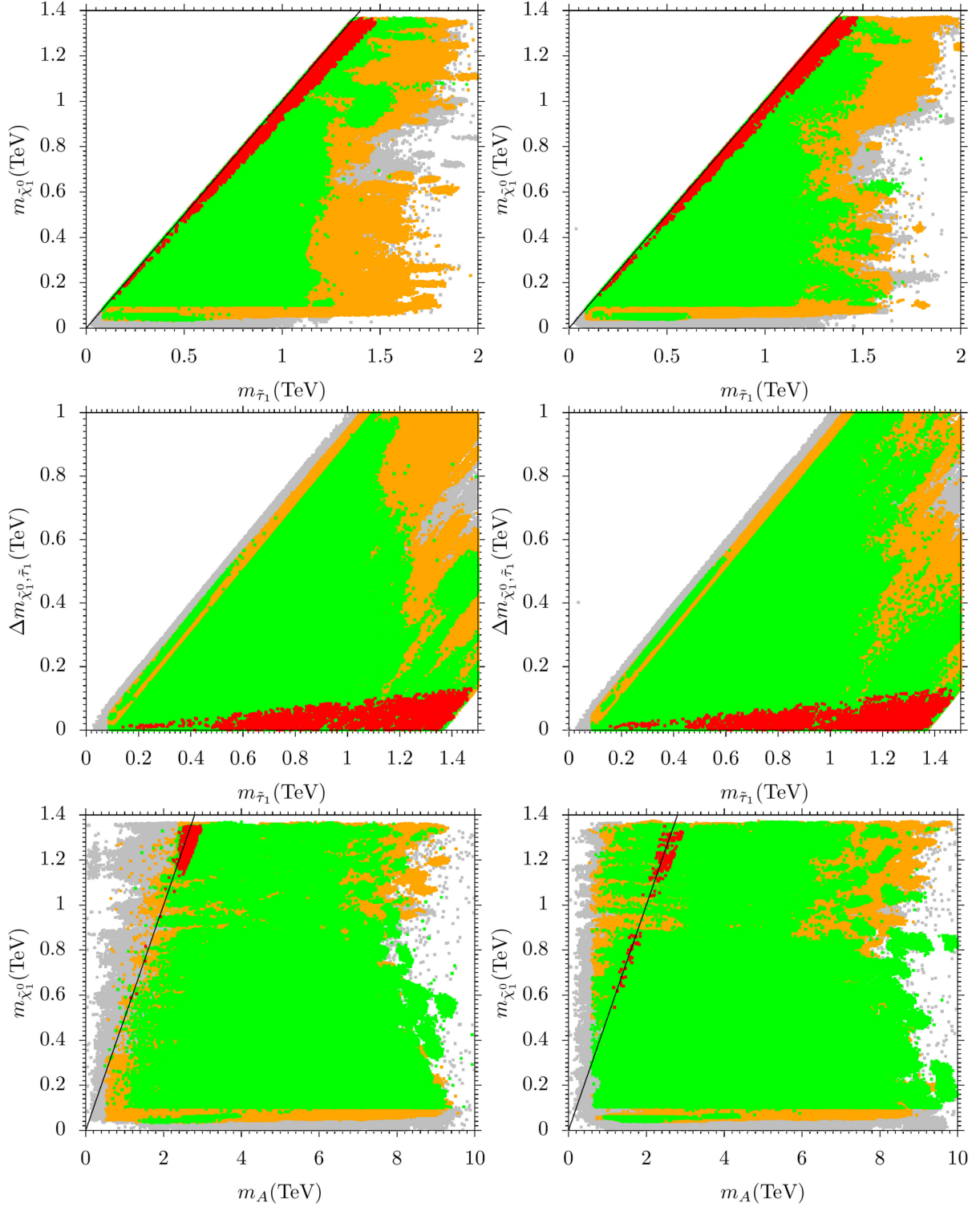


FIG. 2.  $m_{\tilde{\tau}_1}$  vs  $m_{\tilde{\chi}_1^0}$ ,  $m_{\tilde{\tau}_1}$  vs  $|\Delta m_{\tilde{\chi}_1^0, \tilde{\tau}_1}|$ , and  $m_A$  vs  $m_{\tilde{\chi}_1^0}$  planes with color coding as follows: The left and right panels correspond to the  $\mu > 0$  and  $\mu < 0$  scenarios, respectively. Gray points satisfy the REWSB and yield LSP neutralino. Orange points are a subset of the gray points and additionally satisfy the LEP mass limits, constraints from  $B$  physics observables, the Higgs boson mass requirement, and current LHC bounds on  $S$  particles, but satisfy an overabundant neutralino relic density. Green points form a further subset and satisfy all the above constraints except the relic-density upper bound, yielding an underabundant dark matter density. Points satisfying all constraints, including consistency with the Planck-measured relic density, are shown in red. The solid, black lines serve as visual cues for coannihilation as well as resonance solutions' sensitivity.

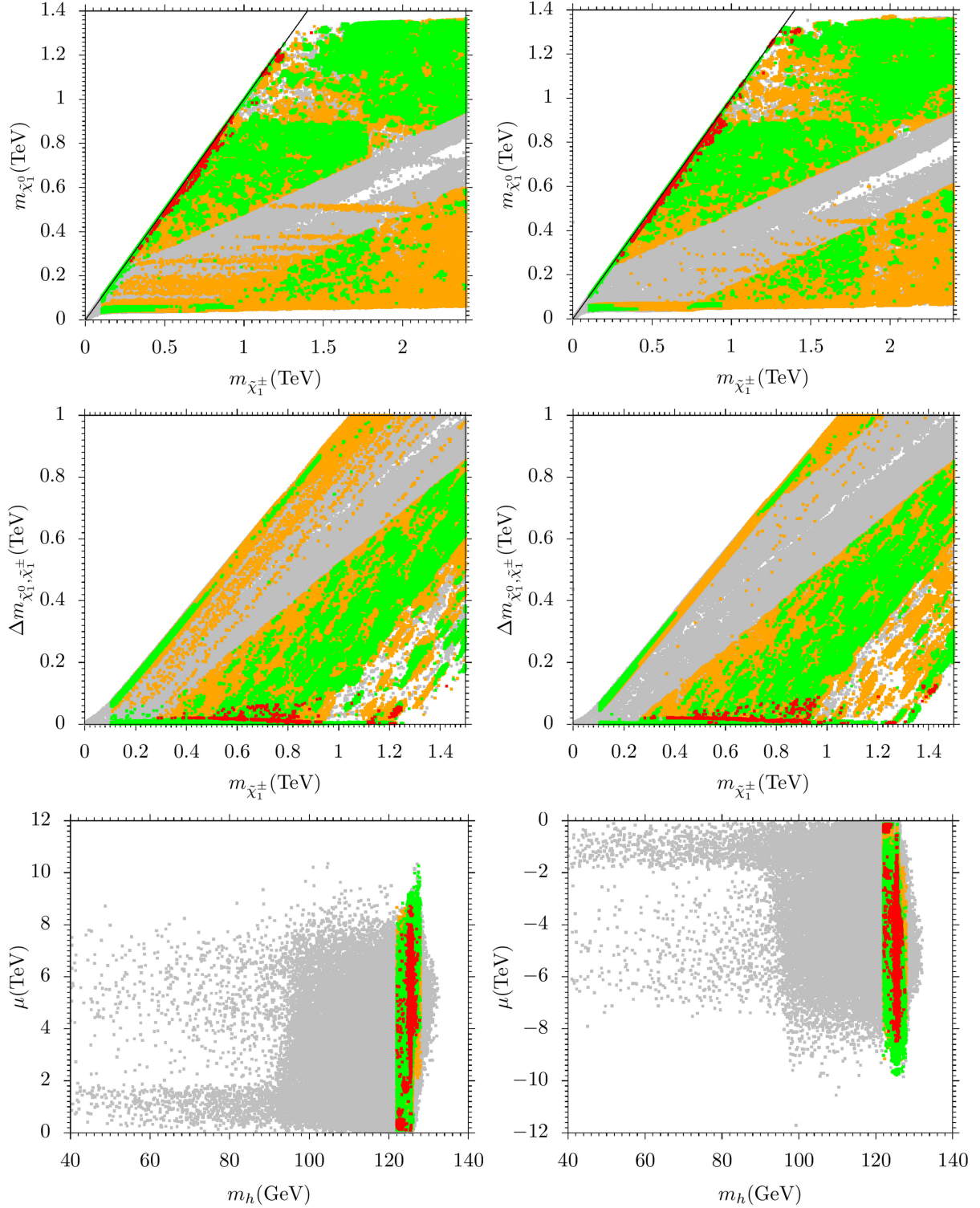


FIG. 3.  $m_{\tilde{\chi}_1^\pm}$  vs  $m_{\tilde{\chi}_1^0}$ ,  $m_{\tilde{\chi}_1^\pm}$  vs  $|\Delta m_{\tilde{\chi}_1^0, \tilde{\chi}_1^\pm}|$ , and  $\mu$  vs  $m_h$  planes with color coding as follows: The left and right panels correspond to the  $\mu > 0$  and  $\mu < 0$  scenarios, respectively. Grey points satisfy the REWSB and yield LSP neutralino. Orange points are a subset of the grey points and additionally satisfy the LEP mass limits, constraints from  $B$  physics observables, the Higgs boson mass requirement, and current LHC bounds on  $S$  particles, but satisfy an overabundant neutralino relic density. Green points form a further subset and satisfy all the above constraints except the relic-density upper bound, yielding an underabundant dark matter density. Points satisfying all constraints, including consistency with the Planck-measured relic density, are shown in red. The solid, black lines serve as visual cues for coannihilation as well as resonance solutions' sensitivity.

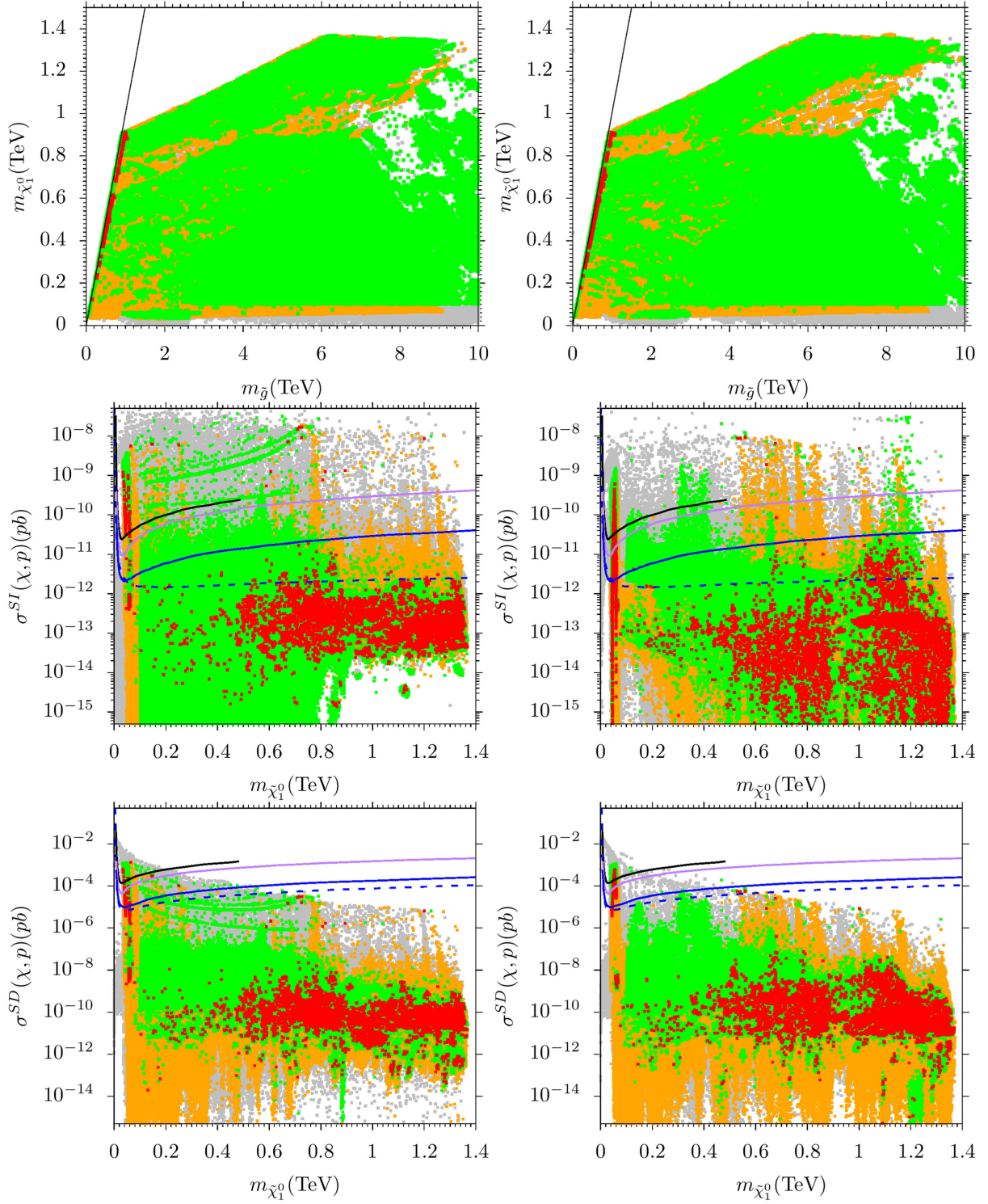


FIG. 4.  $m_{\tilde{\chi}_1^0}$  vs  $m_{\tilde{g}}$ ,  $\sigma^{SI}(\chi, p)(pb)$  and  $\sigma^{SD}(\chi, p)(pb)$ : The left and right panels correspond to the  $\mu > 0$  and  $\mu < 0$  scenarios, respectively. Grey points satisfy the REWSB and yield LSP neutralino. Orange points are a subset of the grey points and additionally satisfy the LEP mass limits, constraints from  $B$  physics observables, the Higgs boson mass requirement, and current LHC bounds on S particles, but satisfy an overabundant neutralino relic density. Green points form a further subset and satisfy all the above constraints except the relic-density upper bound, yielding an underabundant dark matter density. Points satisfying all constraints, including consistency with the Planck-measured relic density, are shown in red. The solid black line represents the XENONnT [80] and for the LUX-ZEPLIN, it is as follows: the solid purple line represents LZ (2022), the solid blue line represents LZ (2024), and the dotted blue line represents the LZ-1000 day sensitivity [28,29,81].

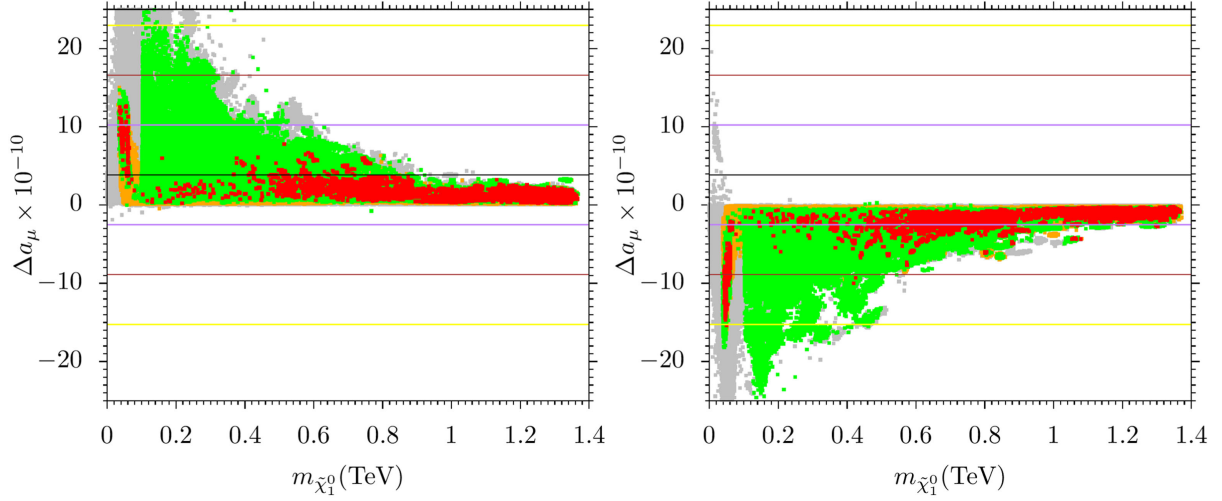


FIG. 5. Lightest neutralino DM candidate and the muon anomalous magnetic moment,  $a_\mu$ , is illustrated. Grey points satisfy the REWSB and yield LSP neutralino. Orange points are a subset of the grey points and additionally satisfy the LEP mass limits, constraints from  $B$  physics observables, the Higgs boson mass requirement, and current LHC bounds on S particles, but satisfy an overabundant neutralino relic density. Green points form a further subset and satisfy all the above constraints except the relic-density upper bound, yielding an underabundant dark matter density. Points satisfying all constraints, including consistency with the Planck-measured relic density, are shown in red. The black curve denotes the central value of  $\Delta a_\mu$ , while the purple, brown, and yellow curves correspond to the  $1\sigma$ ,  $2\sigma$ , and  $3\sigma$  confidence intervals, respectively, around this central value.

kinematically suppressed, favoring decays through Standard Model bosons. This supports the viability of our heavy chargino-neutralino coannihilation solutions in models where the slepton sector is decoupled. The bottom panel of Fig. 3 illustrates the light Higgs boson mass ( $m_h$ ) and the Higgsino mass parameter ( $\mu$ ). Our viable solutions cluster around the observed Higgs mass  $m_h \approx 125$  GeV, consistent with current experimental measurements. Notably, the parameter  $\mu$  spans a broad range, reaching up to approximately 10 TeV, with both moderate and large  $\mu$  solutions contributing to the viable regions. The presence of such solutions offers promising targets for future high-energy collider experiments.

The top panel of Fig. 4 illustrates the correlation between the mass of the lightest neutralino ( $m_{\tilde{\chi}_1^0}$ ) and that of the NLSP gluino ( $m_{\tilde{g}}$ ). The color scheme follows that of Fig. 3, with the exception that here we omit any imposed lower bounds on the gluino mass, as discussed in Sec. III. The diagonal clustering of red points in the mass range  $0.2 \text{ TeV} \lesssim m_{\tilde{g}} \lesssim 1.2 \text{ TeV}$  reflects a highly compressed spectrum, wherein the gluino is nearly degenerate with the neutralino. In such scenarios, the decay mode  $\tilde{g} \rightarrow b\bar{b}\tilde{\chi}_1^0$  tends to dominate. Although other decay modes are kinematically viable, they suffer from overwhelming Standard Model background contamination, particularly at low jet transverse momentum ( $p_T$ ). The advantage of the  $b$ -quark final state lies in its experimental signature: secondary vertices and displaced tracks from  $b$ -hadron decays can be reconstructed even when the energy is modest, enabling identification of soft  $b$  jets. In contrast, decay channels involving lighter quarks yield jets

that often fail to pass track or vertex reconstruction thresholds, reducing their utility in compressed searches. Experimental analyses from the ATLAS Collaboration [78,79] have placed stringent exclusion limits on this compressed topology, specifically, scenarios in which  $m_{\tilde{g}} \approx m_{\tilde{\chi}_1^0}$  are excluded for gluino masses up to approximately 1.2 TeV. Consequently, a significant portion of the red solutions presented in this region are already ruled out by LHC Run 2 data, emphasizing the experimental pressure on such compressed supersymmetric spectra. Finally, in Fig. 4, we present the predicted neutralino-nucleon scattering cross sections from our parameter scan, where the middle panel corresponds to the SI cross section  $\sigma_{\text{SI}}$  and the bottom panel to the SD cross section  $\sigma_{\text{SD}}$ , both as functions of the lightest neutralino mass  $m_{\tilde{\chi}_1^0}$ . Experimental exclusion bounds are overlaid for comparison: the solid black line denotes current XENONnT limits [80], while the solid purple and blue lines represent the 2022 and projected 1000-day sensitivity limits from the LZ experiment, respectively [28,81]. As evident in the  $m_{\tilde{\chi}_1^0}$ - $\sigma_{\text{SI}}$  plane, the majority of solutions (marked in red) lie safely below the exclusion thresholds set by XENONnT and the LZ 2022 results. Furthermore, a large fraction of these points fall within the future sensitivity range of the LZ 1000-day dataset, highlighting their experimental relevance. Apart from a few isolated points around  $m_{\tilde{\chi}_1^0} \sim 0.8 \text{ TeV}$ , nearly all red points evade current exclusion limits. Our scan includes resonance solutions where the LSP mass is close to half the mass of the SM Higgs boson ( $m_h/2$ ) or the Z boson ( $m_Z/2$ ), corresponding to the so-called Higgs and Z-funnel

TABLE I. Input model parameters and the resulting supersymmetric particle (sparticle) mass spectrum are summarized. All masses are provided in units of GeV, corresponding to the scenario with a negative Higgsino mass parameter ( $\mu < 0$ ).

|                                           | Point 1                 | Point 2                | Point 3                 | Point 4                | Point 5                | Point 6                | Point 7                | Point 8                |
|-------------------------------------------|-------------------------|------------------------|-------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| $m_0^U$                                   | 6588                    | 112.1                  | 82.2                    | 867.5                  | 4212                   | 103.4                  | 1145                   | 1845                   |
| $M_1, M_2, M_3$                           | 2009, 3095, 380         | 2498, 2883, 1920.5     | 2296, 2387, 2159.5      | 2616, 3039, 1981.5     | 1939, 1044, 3281.5     | 2749, 3074, 2261.5     | 89.51, 875.3, -1089.2  | 123.8, 1004, -1196.5   |
| $m_{E^c}, m_L$                            | 811, 827.8              | 1033, 692              | 988.2, 641.9            | 973.4, 934.9           | 1173, 1146             | 1142, 629.9            | 875.1, 125.9           | 543.4, 163.6           |
| $m_{H_u}, m_{H_d}$                        | 1021, 4285              | 113.7, 2829            | 111.9, 3448             | 2513, 4430             | 3013, 865.3            | 126, 3321              | 2706, 1880             | 3296, 1437             |
| $m_{\tilde{Q}}, m_{\tilde{D}^c}, m_{U^c}$ | 6023.1, 8505.1, 8479.1  | 433.96, 144.7, 0       | 410.35, 106.10          | 886, 1119.9, 789       | 3874.7, 5437.7, 5352.7 | 475.7, 133.5, 0        | 1104.6, 1478.2, 1294   | 1698.8, 2381.9, 2340.2 |
| $A_t = A_b, A_\tau$                       | 15240, 2946             | -6218, -1907           | -6772, -2076            | -5269, 5988            | -9039, -1298           | -7517, -1846           | 3966, -521.7           | 4326, -386.2           |
| $\tan \beta$                              | 37.7                    | 36.5                   | 31.9                    | 15                     | 30.3                   | 34.6                   | 16.6                   | 17.9                   |
| $\mu$                                     | -6789.4                 | -3146.4                | -3767.2                 | -2521                  | -5866.8                | -3788.3                | -417.63                | -218.12                |
| $m_h$                                     | 125.4                   | 124.7                  | 125.7                   | 125.5                  | 125.5                  | 124.4                  | 124                    | 123                    |
| $m_H$                                     | 5474                    | 2261                   | 4218                    | 5317                   | 4510                   | 2706                   | 1876                   | 1263                   |
| $m_A$                                     | 5439                    | <b>2247</b>            | 4191                    | 5282                   | 4480                   | 2689                   | 1864                   | 1255                   |
| $m_{H^\pm}$                               | 5475                    | 2264                   | 4219                    | 5318                   | 4511                   | 2708                   | 1878                   | 1266                   |
| $m_{\tilde{\chi}_{1,2}^0}$                | <b>902</b> , 2582       | <b>1122</b> , 2392     | <b>1024</b> , 1978      | <b>1171</b> , 2465     | <b>859</b> , 866       | <b>1240</b> , 2558     | <b>45</b> , 419        | <b>59</b> , 225        |
| $m_{\tilde{\chi}_{3,4}^0}$                | 6694, 6694              | 3098, 3101             | 3740, 3741              | 2525, 2572             | 5775, 5776             | 3731, 3733             | 431, 761               | 231, 866               |
| $m_{\tilde{\chi}_{1,2}^\pm}$              | 2586, 6662              | 2394, 4100             | 1985, 3743              | 2460, 2562             | <b>866</b> , 5739      | 2562, 3731             | 404, 750               | 211, 850               |
| $m_{\tilde{g}}$                           | <b>988</b>              | 4025                   | 4471                    | 4147                   | 6846                   | 4686                   | 2428                   | 2685                   |
| $m_{\tilde{u}_{L,R}}$                     | 6357, 8291              | 3929, 3494             | 4153, 3856              | 4110, 3733             | 6915, 7782             | 4486, 4057             | 2402, 2491             | 2878, 3258             |
| $m_{\tilde{t}_{1,2}}$                     | 3073, 5545              | 1540, 2415             | 1741, 2842              | <b>1224</b> , 3262     | 5216, 5617             | 1861, 2627             | 943, 1740              | 1701, 2096             |
| $m_{\tilde{d}_{L,R}}$                     | 6358, 8662              | 3930, 3500             | 4152, 3898              | 4111, 3761             | 6916, 7926             | 4487, 4068             | 2403, 2535             | 2879, 3273             |
| $m_{\tilde{b}_{1,2}}$                     | 3112, 7114              | 1177, 2422             | 2763, 2909              | 3309, 3568             | 5301, 7205             | <b>1321</b> , 2642     | 1734, 2416             | 2059, 3144             |
| $m_{\tilde{\nu}_1}$                       | 1237                    | 1967                   | 1614                    | 2114                   | 1001                   | 2054                   | 689                    | 690                    |
| $m_{\tilde{\nu}_3}$                       | 1102                    | 1881                   | 1439                    | 2037                   | 1010                   | 1971                   | 655                    | 664                    |
| $m_{\tilde{e}_{L,R}}$                     | 1149, 2639              | 1969, 1479             | 1620, 1469              | 2117, 1542             | 983, 1817              | 2056, 1642             | 701, 670               | 7001, 412              |
| $m_{\tilde{\tau}_{1,2}}$                  | 1028, 2507              | 1226, 1885             | <b>1048</b> , 1458      | 1330, 2043             | 957, 1782              | 1421, 1976             | 612, 669               | 349, 677               |
| $\Delta a_\mu$                            | $-1.08 \times 10^{-10}$ | $-9.9 \times 10^{-11}$ | $-1.11 \times 10^{-10}$ | $-3.2 \times 10^{-11}$ | $-2 \times 10^{-10}$   | $-8.3 \times 10^{-11}$ | $-5.2 \times 10^{-10}$ | $-5.4 \times 10^{-10}$ |
| $\sigma_{\text{SI}}(pb)$                  | $5.5 \times 10^{-18}$   | $5.1 \times 10^{-12}$  | $1.6 \times 10^{-13}$   | $2 \times 10^{-12}$    | $2 \times 10^{-13}$    | $8.8 \times 10^{-13}$  | $1.9 \times 10^{-12}$  | $1.3 \times 10^{-11}$  |
| $\sigma_{\text{SD}}(pb)$                  | $1.6 \times 10^{-11}$   | $2.9 \times 10^{-10}$  | $3.3 \times 10^{-11}$   | $2.1 \times 10^{-9}$   | $6.9 \times 10^{-10}$  | $9.3 \times 10^{-11}$  | $4.1 \times 10^{-6}$   | $6 \times 10^{-5}$     |
| $\Omega_{\text{CDM}} h^2$                 | 0.121                   | 0.124                  | 0.121                   | 0.125                  | 0.116                  | 0.120                  | 0.121                  | 0.115                  |

(pole) regions. In such cases, the LSP may contribute to the invisible decay widths of these bosons. For  $\mu > 0$ , we observe that the current LZ constraints already rule out Z-pole solutions and exclude portions of the Higgs-pole region (top left panel). In contrast, for  $\mu < 0$ , both the Z- and H-pole scenarios, they remain largely viable (top right panel), indicating that the negative  $\mu$  region retains significant phenomenological interest and merits focused study in light of upcoming experimental results. The  $m_{\tilde{\chi}_1^0} - \sigma_{\text{SD}}$  plot (bottom panel) reveals that all viable solutions remain consistent with existing limits and projected future sensitivities for SD scattering. These findings collectively underscore the complementarity of SI and SD detection modes in constraining supersymmetric DM scenarios.

Figure 5 presents the results from a comprehensive scan over the GmSUGRA parameter space. Our analysis reveals that in the  $\mu < 0$  regime, the supersymmetric contribution to  $(g-2)_\mu$  remains consistent with the experimental measurement within  $1\sigma$  to  $2\sigma$  uncertainty. Notably, viable solutions are found with neutralino masses extending up to 1.3 TeV that contribute within the  $1\sigma$  range. Additionally, the Higgs/Z-poles in this scenario yield contributions consistent within  $2\sigma$  while satisfying the Planck 2018 relic density constraints, as represented by the red points. On the other hand, in the  $\mu > 0$  scenario, although the Higgs and Z-funnel regions also produce contributions within  $1\sigma$ ,

these parameter spaces are already ruled out by the latest LZ direct detection limits. These results highlight the phenomenological viability of the  $\mu < 0$  scenario under current experimental bounds. These findings underscore the importance of further phenomenological studies of SUSY GUT models under the  $\mu < 0$  regime, as it remains consistent with both the updated  $(g-2)_\mu$  measurements in the SM and existing experimental bounds.

We conclude by presenting two benchmark tables. Table I summarizes representative scenarios for the case with a negative Higgsino mass parameter ( $\mu < 0$ ), while Table II corresponds to the case with  $\mu > 0$ . In Table I, Point 1 illustrates a scenario with a gluino as the NLSP, which has already been ruled out by current LHC SUSY searches. Point 2 represents a Higgs funnel (resonance) solution, where the pseudoscalar Higgs mass  $m_A = 2.247$  TeV and the heavy scalar Higgs mass  $m_H = 2.264$  TeV are nearly degenerate. The LSP in this case is a binolike neutralino with mass approximately 0.902 TeV, making it a valid A/H-resonance solution. Point 3 corresponds to an NLSP stau scenario in which the stau mass is approximately 1.048 TeV, and the LSP, a binolike neutralino, has a mass around 1.024 TeV. Point 4 exemplifies stop-neutralino coannihilation, with the NLSP stop mass around 1.224 TeV and the LSP neutralino (binolike) mass close to 1.171 TeV. Point 5 features chargino-neutralino coannihilation, where the LSP

TABLE II. Input model parameters and the resulting supersymmetric particle (sparticle) mass spectrum are summarized. All masses are provided in units of GeV, corresponding to the scenario with a positive Higgsino mass parameter ( $\mu > 0$ ).

|                                                   | Point 1               | Point 2               | Point 3                | Point 4                | Point 5                | point 6                | Point 7               |
|---------------------------------------------------|-----------------------|-----------------------|------------------------|------------------------|------------------------|------------------------|-----------------------|
| $m_0^U$                                           | 6710                  | 4837                  | 3664                   | 5414                   | 2648                   | 854.8                  | 1543                  |
| $M_1, M_2, M_3$                                   | 1749, 2722, 289.5     | 2963, 2391, 3821      | 2175, 2900, 1087.5     | 2555, 2232, 3039.5     | 2707, 1463, 4569       | 109.9, 886.9, -1055.6  | 122.4, 955.8, -1127.7 |
| $m_{\tilde{E}^c}, m_L$                            | 224, 1107             | 730.9, 830.5          | 186.2, 546.2           | 664.2, 954.2           | 328.9, 1044            | 868.1, 133             | 567.3, 111.5          |
| $m_{\tilde{H}_u}, m_{\tilde{H}_d}$                | 260, 1593             | 4899, 1378            | 3185, 3131             | 4321, 1427             | 670.1, 295.6           | 2575, 2027             | 2730, 2812            |
| $m_{\tilde{Q}}, m_{\tilde{D}^c}, m_{\tilde{U}^c}$ | 6126, 8662.6, 8660.6  | 4425.6, 6244.5, 6216  | 3345.6, 4730.2, 4727.8 | 4949.7, 6989.4, 6968.4 | 2358.2, 3328.2, 3314.6 | 857.03, 1103.5, 845.82 | 1427.5, 1992, 1937.4  |
| $A_t = A_b, A_\tau$                               | -11770, 2881          | -7490, 1703           | 15170, 2973            | -8342, 1580            | -10640, -2859          | 3944, -535.7           | 3380, -383.9          |
| $\tan \beta$                                      | 19                    | 46.7                  | 12.8                   | 56.2                   | 8.37                   | 17.6                   | 18.3                  |
| $\mu$                                             | 7037.7                | 5298                  | 4701.1                 | 5567.4                 | 7299.7                 | 180.52                 | 407.65                |
| $m_h$                                             | 126.5                 | 125.5                 | 125.5                  | 125.5                  | 125.4                  | 124                    | 122                   |
| $m_H$                                             | 6748                  | 3042                  | 5741                   | 2201                   | 7374                   | 1984                   | 2726                  |
| $m_A$                                             | 6704                  | 3022                  | 5704                   | <b>2186</b>            | 7326                   | 1971                   | 2708                  |
| $m_{H^\pm}$                                       | 6748                  | 3043                  | 5742                   | 2203                   | 7374                   | 1986                   | 2727                  |
| $m_{\tilde{\chi}_{1,2}^0}$                        | <b>805</b> , 2326     | <b>1334</b> , 1993    | <b>965</b> , 2390      | <b>1152</b> , 1872     | <b>1200</b> , 1207     | <b>49</b> , 185        | <b>59</b> , 410       |
| $m_{\tilde{\chi}_{3,4}^0}$                        | 7003, 7003            | 5300, 5301            | 4685, 5686             | 5547, 5547             | 7284, 7284             | 191, 766               | 420, 827              |
| $m_{\tilde{\chi}_{1,2}^\pm}$                      | 2329, 6984            | 1997, 5297            | 2411, 4714             | 1876, 5538             | <b>1201</b> , 7299     | 186, 762               | 419, 816              |
| $m_{\tilde{g}}$                                   | <b>888</b>            | 7863                  | 2467                   | 6446                   | 9137                   | 2346                   | 2531                  |
| $m_{\tilde{u}_{L,R}}$                             | 6373, 8467            | 8004, 8991            | 4260, 5025             | 7351, 8648             | 8131, 8454             | 2252, 2240             | 2637, 2860            |
| $m_{\tilde{t}_{1,2}}$                             | 4389, 5912            | 6104, 6677            | <b>1007</b> , 2389     | 5099, 6342             | 5812, 6943             | 486, 1595              | 1627, 2026            |
| $m_{\tilde{d}_{L,R}}$                             | 6374, 8802            | 8004, 9089            | 4261, 5175             | 7352, 8811             | 8131, 8504             | 2254, 2285             | 2639, 2935            |
| $m_{\tilde{b}_{1,2}}$                             | 4465, 8500            | 6186, 8145            | 2340, 5003             | 5190, 7529             | 6940, 8398             | 1600, 2147             | 2003, 2806            |
| $m_{\tilde{\nu}_1}$                               | 1130                  | 1486                  | 1699                   | 1262                   | 1227                   | 702                    | 515                   |
| $m_{\tilde{\nu}_3}$                               | 1065                  | 1496                  | 1673                   | 1497                   | 1213                   | 661                    | 404                   |
| $m_{\tilde{e}_{L,R}}$                             | 1049, 2473            | 1465, 1833            | 1701, 1511             | 1201, 2005             | 1288, 1405             | 713, 658               | 531, 721              |
| $m_{\tilde{\tau}_{1,2}}$                          | 1037, 2414            | <b>1413</b> , 1798    | 1449, 1678             | 1400, 2184             | 1222, 1413             | 577, 672               | 425, 585              |
| $\Delta a_\mu$                                    | $7 \times 10^{-11}$   | $1.5 \times 10^{-10}$ | $4 \times 10^{-11}$    | $2.2 \times 10^{-10}$  | $5.6 \times 10^{-11}$  | $6.2 \times 10^{-10}$  | $6.7 \times 10^{-10}$ |
| $\sigma_{\text{SI}}(pb)$                          | $3.3 \times 10^{-14}$ | $2.2 \times 10^{-13}$ | $5.2 \times 10^{-13}$  | $1.8 \times 10^{-13}$  | $1.1 \times 10^{-12}$  | $6.1 \times 10^{-10}$  | $5 \times 10^{-11}$   |
| $\sigma_{\text{SD}}(pb)$                          | $1 \times 10^{-11}$   | $9.9 \times 10^{-11}$ | $2.6 \times 10^{-11}$  | $6.9 \times 10^{-11}$  | $5 \times 10^{-10}$    | $1.3 \times 10^{-5}$   | $4.8 \times 10^{-6}$  |
| $\Omega_{\text{CDM}} h^2$                         | 0.117                 | 0.125                 | 0.115                  | 0.125                  | 0.125                  | 0.117                  | 0.118                 |

neutralino, dominantly bino with some wino admixture, has a mass of approximately 0.859 TeV, while the lightest chargino mass is around 0.866 TeV. Finally, Point 6 showcases sbottom-neutralino coannihilation, with the NLSP sbottom at 1.321 TeV and the binolike LSP neutralino at 1.240 TeV. Points 7 and 8 illustrate scenarios corresponding to the  $Z$ - and  $H$ -resonance (pole) solutions, respectively. In the  $Z$ -pole case (Point 7), LSP is a binolike neutralino with a mass of approximately 45 GeV, facilitating efficient annihilation via  $Z$ -boson resonance. For the  $H$ -pole case (Point 8), the LSP is a bino-dominated neutralino with a significant Higgsino admixture, having a mass near 59 GeV, enabling resonant annihilation through the light  $CP$ -even Higgs boson. These solutions are consistent with relic density constraints and remain viable under current collider and DM search limits.

Table II presents representative benchmark points corresponding to the  $\mu > 0$  scenario. Point 1 features a case with the gluino as the NLSP, which is already excluded by current LHC SUSY search limits. Point 2 represents an NLSP stau scenario, where the stau has a mass of approximately 1.413 TeV, and the LSP, a binolike neutralino, has a mass of about 1.133 TeV. Point 3 illustrates stop-neutralino coannihilation, with the NLSP stop mass around 1.007 TeV and the LSP neutralino (bino-dominated) at approximately 0.965 TeV. Point 4 corresponds to a Higgs

funnel (resonance) scenario, characterized by nearly degenerate heavy Higgs bosons with  $m_A = 2.186$  TeV and  $m_H = 2.202$  TeV, while the LSP neutralino, predominantly bino, has a mass close to 1.152 TeV. This near degeneracy allows us to interpret it as an  $A/H$ -resonance solution. Point 5 depicts a chargino-neutralino coannihilation scenario, where the lightest chargino has a mass of 1.202 TeV, and the LSP neutralino, which is primarily bino with admixtures of wino and Higgsino, has a mass around 1.200 TeV. Points 6 and 7 correspond to the  $Z$ - and  $H$ -resonance (pole) annihilation mechanisms, respectively. In the  $Z$ -pole scenario (Point 6), the LSP is a neutralino predominantly binolike with a non-negligible Higgsino component, and its mass is approximately 49 GeV, allowing resonant annihilation via the  $Z$  boson. In the  $H$ -pole scenario (Point 8), the LSP neutralino has a mass around 59 GeV and similarly exhibits a bino-dominated composition with significant higgsino admixture, enabling efficient annihilation through the light Higgs boson. Both scenarios yield viable DM relic densities and are consistent with current collider and DD constraints.

All benchmark points comply with existing bounds from LHC SUSY searches and DD DM experiments, except for Point 1 in both cases. The remaining scenarios could be probed in the LHC Run-3, future SUSY searches, and DD DM experiments.

## V. CONCLUSION

In this study, we have performed a comprehensive investigation of the parameter space of the GmSUGRA model within the framework of the MSSM, considering positive as well as negative signs of the Higgsino mass parameter  $\mu$ , as the anomalous magnetic moment of the muon may now be consistent with the SM prediction. Our findings indicate that, in the  $\mu < 0$  scenario, the supersymmetric contributions to the muon's anomalous magnetic moment remain within a  $2\sigma$  deviation from the central experimental value, taking into account the current Standard Model prediction. Moreover, the  $\mu < 0$  GmSUGRA scenario allows phenomenologically viable solutions that are absent or strongly suppressed for  $\mu > 0$ , including regions where sbottom coannihilation plays an important role. The DM production mechanisms explored encompass coannihilation with gluinos, staus, stops, sbottoms, and charginos, as well as annihilation through the  $A$ -,  $H$ -, and  $Z$ -boson resonance channels. While the  $\mu > 0$  scenario is significantly constrained by current limits from LZ DD data in the  $H/Z$  pole regions while the  $\mu < 0$  region remains largely viable and phenomenologically appealing. Notably, several benchmark points identified in our scan are within the projected discovery reach of LHC Run-3 and

the upcoming 1000-day exposure of the LZ experiment. These findings strongly motivate further experimental efforts focused on probing the electroweak sector of SUSY, especially in the context of the negative  $\mu$  scenario.

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## DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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