

Determination of the pion generalized parton distributions at zero skewness

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We perform a global QCD analysis of the pion electromagnetic form factor (FF) data from pion electroproduction and elastic pion scattering to extract the valence pion generalized parton distributions (GPDs) at zero skewness. The analysis uses three different sets of pion parton distribution functions (PDFs), namely, xFitter, JAM21, and MAP23, to construct the GPD ansatz. Through a χ^2 minimization and a careful parametrization scan, we determine the profile function parameters and find that only two parameters are sufficient to describe the data. The extracted valence pion GPDs from different analyses have similar x -dependence, with minor differences at small momentum transfer. The resulting theoretical predictions for the pion electromagnetic FF and its squared magnitude show good agreement with experimental measurements. Among the three analyses, the one using the MAP23 PDFs provides the best overall fit and is adopted as the final GPD set. Our results offer a consistent determination of the valence pion GPDs, indicating a minor impact of the choice of pion PDFs. The present study provides a solid foundation for future investigations of pion structure, including its charge radius, tomography, and mechanical properties.

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I. INTRODUCTION

The pion plays an important role in our understanding of the strong interaction as described by quantum chromodynamics (QCD). It is the simplest quark-antiquark ($q\bar{q}$) bound state in the quark-parton model of hadrons. The pion mediates nucleon-nucleon forces and is one of the Nambu-Goldstone bosons associated with spontaneous chiral symmetry breaking in QCD, making it essential for explaining the origin of mass in hadronic matter [see, e.g., Refs. [1–4] for detailed discussions of pion physics]. Despite being theoretically simpler than the proton, the

internal structure of the pion remains less well understood, especially in comparison to that of the proton [5].

The concept of parton distribution functions (PDFs), which describe the probability densities of partons within a hadron as a function of the fraction of the hadron's momentum that they carry, is fundamental to high-energy particle physics, particularly in the description of scattering processes and collisions [6,7]. The extraction of PDFs is often formulated as a nonlinear regression problem, where the goal is to determine functional forms from experimental data [see, e.g., Refs. [8–15] for unpolarized PDFs and Refs. [16–25] for polarized PDFs of the proton]. These distributions are universal objects, meaning they can be applied across a wide range of processes involving parton dynamics. In this context, the determination of pion PDFs, either through global analyses of experimental data [26–37] or via theoretical approaches [38–49], has attracted considerable interest in recent years. One can point to the efforts by the xFitter [30], JAM [31–33], and MAP [34] collaborations for the case of global analysis of pion PDFs.

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Although PDFs provide valuable information about the internal structure of hadrons in the longitudinal (beam) direction, they cannot offer insight into the transverse spatial distribution of partons. In this regard, generalized parton distributions (GPDs) have emerged as a powerful framework for obtaining a more comprehensive picture of hadron structure [50–62]. GPDs correlate the transverse position and longitudinal momentum of partons, thus enabling a three-dimensional characterization of hadrons. They are also an essential ingredient in the theoretical description of hard exclusive and elastic scattering processes, as various moments of GPDs are related to different hadronic form factors (FFs) [63–66]. While significant progress has been made in the study of proton GPDs, the determination of pion GPDs has also attracted growing interest in recent years [67–81].

The electromagnetic FF of the pion is a fundamental observable that encodes essential information about its internal structure and spatial charge distribution. The pion is the simplest mesonic system composed of a quark and an antiquark. So, it offers a clean environment to study the dynamics of the strong interaction in both perturbative and nonperturbative regimes of QCD. Study of the pion FF provides a critical testing ground for a variety of theoretical approaches [82–102], including perturbative QCD [82–85], lattice QCD [86–89], the Dyson–Schwinger equation (DSE) [90,91], light-front holography (LFH) [92], the dispersive matrix approach [93], the light-cone quark model (LCQM) [102], and QCD sum rules [94,95]. On the experimental side, the spacelike pion FF has been measured via pion electroproduction processes such as $ep \rightarrow e'\pi^+n$ [103–108] and elastic pion scattering [109–112]. The timelike region has also been explored in $e^+e^- \rightarrow \pi^+\pi^-$ annihilation [113,114]. A very important point in this regard is that the pion FF is closely related to the first moment of the pion GPD, making it a vital input and constraint in the modeling and extraction of the pion's GPDs. Thus, understanding the pion FF not only sheds light on the nature of confinement and hadronization in QCD but also provides key input for the determination of the pion's GPDs. In addition to the electromagnetic form factor, the gravitational FFs (GFFs) of the pion, which are related to moments of the energy-momentum tensor, have also attracted significant interest in recent years [115–119].

Following the MMGPDs studies on the extraction of proton GPDs in the zero-skewness limit [120,121], in the present work, we aim to determine the pion GPDs through a global analysis of all available pion FF data from both pion electroproduction and elastic pion scattering. This is the first application of the MMGPDs framework to the pion, extending its utility beyond the proton sector. To this end, we adopt the same functional ansatz used in the proton analysis and investigate its sensitivity to the choice of input pion PDFs. In Sec. II, we introduce the pion PDF sets

used in our analysis, highlighting their key features and differences. In Sec. III, we introduce the phenomenological framework used to extract the pion GPDs from the FF data. Section IV is devoted to the experimental data selection and treatment. A detailed discussion of the extracted GPDs and a comparison with the input data is presented in section Sec. V. Finally, a summary and our conclusions are given in Sec. VI.

II. PION PDFS

Pion PDFs play an important role in our understanding of hadron structure and QCD dynamics. As the lightest meson and the simplest quark-antiquark bound state, the pion provides a unique framework for testing theoretical models of parton dynamics, confinement, and chiral symmetry breaking. In particular, pion PDFs are essential inputs in the theoretical description of high-energy processes involving pions, such as Drell-Yan (DY) production [122–124], leading neutron (LN) electroproduction [125,126], and prompt photon production [127]. They can also play a crucial role in the extraction of pion GPDs, where the forward limit of the GPD is directly related to the PDF. Unlike proton PDFs, which are constrained by a wide range of precise experimental data, pion PDFs are less well known due to the absence of stable pion targets. However, several recent efforts have provided improved extractions of pion PDFs through both data analysis [29–37] and theoretical calculations [40–49]. The recent determinations of the pion PDFs have adopted distinct frameworks, methodologies, and theoretical inputs, reflecting complementary approaches to constraining the pion's partonic structure. In this section, we outline the defining features of the xFitter [30], JAM21 [32], and MAP23 [34] pion PDF sets, highlighting their key features and differences.

The xFitter Collaboration has provided an open-source pion PDF fit based on a combination of pion-induced DY and prompt-photon production data for the first time. [30]. In this phenomenological approach, flexible functional forms are used to parametrize the PDFs at an input scale, and Dokshitzer-Gribov-Lipatov-Altarelli-Parisi evolution is applied to evolve them to experimental scales. An important property of the xFitter analysis is its comprehensive treatment of uncertainties which incorporates both experimental and theoretical uncertainties. The systematic uncertainties include also the scale variations and α_s dependence. Prompt-photon data included in the xFitter analysis play a crucial role considering the fact that they offer direct sensitivity to the gluon PDF. However, their impact is reduced by statistical limitations and theoretical uncertainties in the photon sector.

The JAM Collaboration has performed a global QCD analysis using pion-induced DY and LN electroproduction data [32] (we call it JAM21 in this paper). As a distinguishing feature of this study, one can point to the systematic

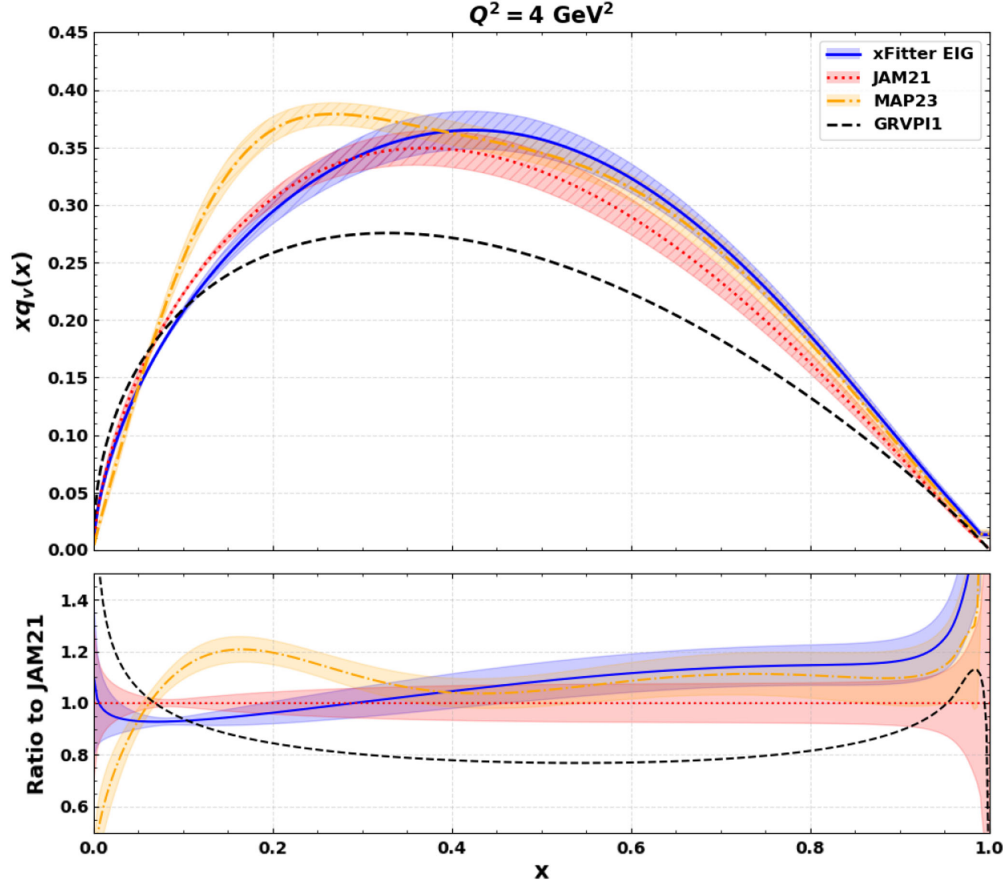


FIG. 1. Comparison of the valence pion PDF $q_v(x)$ at $Q^2 = 4 \text{ GeV}^2$ from the xFitter [30], JAM21 [32], MAP23 [34], and GRVPI1 [26] determinations. For xFitter, the EIG grid is used. The lower panel displays the ratio of each PDF to the JAM21 result.

implementation of threshold resummation at next-to-leading logarithmic accuracy. The aim was to investigate the sensitivity of high- x observables to soft-gluon emissions near the partonic threshold, where x denotes the longitudinal momentum fraction of the pion carried by a constituent parton. JAM21 explored multiple resummation schemes, including Mellin-Fourier and double-Mellin techniques, to test the universality of resummation corrections to the large- x behavior of the valence distribution. The analysis demonstrated that resummation can significantly affect the final results, particularly at larger values of x . It should be noted that JAM21 considers PDFs as parametric functions (just like xFitter) constrained by data, and uses Bayesian methods to explore uncertainties. Sea and gluon distributions are allowed to vary and are constrained by the combined DY and LN datasets. This provides a more data-driven evaluation of the pion's partonic content at various scales.

The MAP Collaboration (referred to as MAP23 in this paper) has utilized a light-front constituent quark model approach rooted in an explicit Fock-space expansion of the pion state up to four-parton configurations, including $q\bar{q}$, $q\bar{q}g$, $q\bar{q}gg$, and $q\bar{q}q\bar{q}$ sectors [34]. The model directly encodes valence, sea, and gluon contributions into the

structure of the light-front wave functions (LFWFs). An important feature of the MAP23 analysis is the factorized parameterization of the longitudinal and transverse momentum components of the LFWFs. In this approach, the information on the longitudinal profile comes from the pion distribution amplitudes and the transverse profile is constrained by fits to the electromagnetic FF. This model offers a complementary, theory-driven perspective on pion PDFs which link them explicitly to hadronic wave functions and form-factor phenomenology.

Figure 1 compares the valence pion PDF, $xq_v(x, Q^2)$, at $Q^2 = 4 \text{ GeV}^2$ from the xFitter [30], JAM21 [32], and MAP23 [34] analyses, along with the older Gluck-Reya-Vogt (GRV) determination [26] (denoted GRVPI1). Assuming charge symmetry for the valence quark PDF, we have $q_v \equiv \bar{u}_v^{\pi^-} = \bar{u}^{\pi^-} - u^{\pi^-} = d_v^{\pi^+}$. For xFitter, the EIG grid was employed in the comparison. To emphasize the differences, the lower panel of Fig. 1 shows the ratios of each result to JAM21. Distinct deviations appear mainly in the small- and large- x regions, while xFitter and JAM21 agree more closely within their respective uncertainty bands. Such differences can have a non-negligible impact on phenomenological applications involving pion PDFs, including the present analysis of pion GPDs. In this work,

we explicitly examine this sensitivity by repeating our extraction with different pion PDF inputs to assess the degree of PDF dependence in the final results.

III. PHENOMENOLOGICAL FRAMEWORK

As mentioned in the Introduction, in the present study, we are going to determine the pion GPDs through a global analysis of all available pion electromagnetic FF data from both pion electroproduction and elastic pion scattering (see the next section). In this section, we present the phenomenological framework used in our analysis. Note that the pion electromagnetic FF is evaluated in the spacelike region, $t < 0$, where t is the Mandelstam variable representing the squared four-momentum transfer. This kinematic domain is where GPDs are defined and can be interpreted in terms of PDFs within the pion. The t -dependence of GPDs encodes information about the spatial distribution of quarks inside the pion.

The contribution of a given quark flavor q to the pion electromagnetic FF, denoted by $F_\pi^q(t)$, can be expressed in terms of the valence unpolarized pion GPD $H_v^q(x, t, Q^2)$ as [65,120]

$$F_\pi^q(t) = \int_0^1 dx H_v^q(x, t, Q^2). \quad (1)$$

Here, x is the longitudinal momentum fraction carried by the quark, and Q^2 is the scale at which the internal structure of pion is probed. In principle, GPDs also depend on the skewness parameter ξ , the longitudinal momentum transfer. Equation (1) holds for any value of ξ , but the ξ -dependence drops out upon integration over x due to the polynomiality property [50,55]. This kind of GPDs which is related to the FFs is commonly referred to as the zero-skewness GPDs [61]. Considering the quark content of the charged pions ($u\bar{d}$ for π^+ and $\bar{d}u$ for π^-) and the corresponding quark electric charges e_q , the total pion FF can be written as

$$F_\pi(t) = \sum_q e_q F_\pi^q(t). \quad (2)$$

Equation (1) expresses the sum rule connecting the pion electromagnetic FF to the first moment of the GPD at fixed scale Q^2 . This relation ensures that the GPDs are properly normalized and that the charge conservation condition $F_\pi(0) = 1$ is automatically satisfied when the valence distributions are used. In the zero-skewness limit ($\xi = 0$), which corresponds to the case where no longitudinal momentum is transferred between the initial and final pion states, the GPDs reduce to the form relevant for describing elastic scattering processes.

To extract the pion GPDs from FF data, we adopt a functional form inspired by previous MMGPD studies on the proton [120,121]:

$$H_v^q(x, t, Q^2) = q_v(x, Q^2) \exp[tf_v^q(x)], \quad (3)$$

where $q_v(x, Q^2)$ is the valence PDF of flavor q and $f_v^q(x)$ is a profile function encoding the t -dependence of the GPD. The factorized ansatz in Eq. (3) is widely used in phenomenological studies because it cleanly separates the longitudinal-momentum structure, encoded in the PDF $q_v(x, Q^2)$, from the transverse spatial structure, governed by the exponential profile function $f_v^q(x)$. The exponential form ensures positivity and smoothly reproduces known Regge behavior at the respective kinematic limits. This approach has been successfully applied to nucleon and pion GPDs in previous analyses and provides a practical framework for fitting experimental data. In this work, we take $q_v(x, Q^2)$ from the xFitter [30], JAM21 [32], and MAP23 [34] analyses at $Q^2 = 4 \text{ GeV}^2$, as provided through the LHAPDF package [128]. This allows us to test the sensitivity of our results to the choice of pion PDFs and identify the set that best describes the data.

Following Refs. [65,120,121], we parametrize the profile function as

$$f_v^q(x) = \alpha'(1-x)^3 \ln \frac{1}{x} + B(1-x)^3 + Ax(1-x)^2, \quad (4)$$

where α' , A , and B are free parameters to be determined separately for each quark flavor. This functional form ensures Regge-like behavior at small x through the logarithmic term and a power-law falloff at large x . To be more precise, the logarithmic term $\ln(1/x)$ controls the small x rise of the GPD, consistent with the Regge trajectory slope α' , while the power terms in $(1-x)$ govern the falloff at large x , corresponding to the dominance of configurations where the struck quark carries most of the pion's momentum. Therefore, the parameters α' , A , and B have direct physical meaning: α' determines the effective Regge slope, while A and B adjust the transition between the non-perturbative and perturbative regimes.

The optimal values of these parameters are obtained by minimizing the χ^2 function:

$$\chi^2 = \sum_{i=1}^n \left(\frac{\mathcal{E}_i - \mathcal{T}_i}{\delta \mathcal{E}_i} \right)^2, \quad (5)$$

where $\mathcal{E}_i$ is the i th experimental data point, $\mathcal{T}_i$ is the corresponding theoretical prediction, and $\delta \mathcal{E}_i$ is the total experimental uncertainty which is obtained by adding statistical and systematic errors in quadrature. The sum runs over all data points from the datasets discussed in Sec. IV. In practice, we performed a systematic parameter-scan procedure [129] to identify the most stable minimum. This reduces the risk of local minima and ensures that the extracted parameters correspond to a genuine global optimum. The fit quality for each dataset and the resulting parameter uncertainties are reported in Sec. V.

The minimization is carried out using the Python interface of the CERN MINUIT library [130], iMINUIT [131]. The uncertainties on the extracted GPDs are estimated using the standard Hessian method [132] as implemented in iMINUIT, with $\Delta\chi^2 = 1$ corresponding to the 68% confidence level.

IV. DATA SELECTION

Our analysis relies on a comprehensive set of experimental measurements of the charged pion electromagnetic FF, $F_\pi(t)$, as well as its absolute square, $|F_\pi(t)|^2$, obtained from two main sources: pion electroproduction and elastic pion scattering. Pion electroproduction involves measurements of the pion FF extracted from the longitudinal cross section of the exclusive process $ep \rightarrow e'\pi^+n$, where a leading neutron is produced in the final state. Elastic pion scattering provides direct measurements of $|F_\pi(t)|^2$ from electron-pion ($e-\pi$) scattering experiments. In the present analysis, the momentum transfer t lies in the spacelike region ($t < 0$), consistent with the kinematics of pion electroproduction and elastic scattering experiments from which the data are taken.

The full list of datasets used in this study is summarized in Table I, together with the corresponding kinematic ranges and the number of data points, N_{pts} . The electroproduction category includes 40 data points covering the range $0.18 < |t| < 9.77 \text{ GeV}^2$. For the Brauel dataset [106], we use the reanalyzed value reported in the dataset of Volmer [107]. The elastic scattering category is dominated by classic low- $|t|$ data, including 101 data points covering the range $0.0138 < |t| < 0.092 \text{ GeV}^2$. These measurements provide crucial constraints on the slope of the form factor at the origin and, hence, on the pion charge radius. Since these data are free from the model assumptions inherent in electroproduction extractions, they play an important role in our global fit.

TABLE I. Summary of the experimental datasets used in this analysis. The kinematic coverage is given in terms of the squared momentum transfer t , and N_{pts} denotes the number of data points included from each source.

Experiment	Process	$-t$ range (GeV^2)	N_{pts}
Brown [103]	$ep \rightarrow e'\pi^+n$	0.176–1.188	5
Ackermann [104]	$ep \rightarrow e'\pi^+n$	0.35	1
Bebek [105]	$ep \rightarrow e'\pi^+n$	0.18–9.77	21
Brauel [106]	$ep \rightarrow e'\pi^+n$	0.7	1
Volmer [107]	$ep \rightarrow e'\pi^+n$	0.6–1.6	4
Huber [108]	$ep \rightarrow e'\pi^+n$	0.6–2.45	8
Adylov [109]	$e-\pi$ scattering	0.0138–0.0353	22
Dally [110]	$e-\pi$ scattering	0.0317–0.0705	20
Dally [111]	$e-\pi$ scattering	0.039–0.092	14
Amendolia [112]	$e-\pi$ scattering	0.015–0.253	45
Total	–	0.0138–9.77	141

The combined dataset, including 141 data points, spans a broad kinematic range in t , from the near-forward region $|t| = 0.0138 \text{ GeV}^2$ (from elastic scattering) up to $|t| = 9.77 \text{ GeV}^2$ (from electroproduction). This coverage ensures sensitivity to both the long-distance and short-distance structure of the pion. Before fitting, we examined all datasets for potential inconsistencies by comparing overlapping measurements in the same t region. We didn't find statistically significant tensions.

V. RESULTS

In this section, we present the results of our global QCD analyses of the pion electromagnetic FF data. The fits have been performed using the three different pion PDF sets introduced in Sec. II, namely, xFitter [30] (EIG grid), JAM21 [32], and MAP23 [34], to construct the GPD ansatz of Eq. (3). In addition to determining the pion GPDs, we also investigate the sensitivity of the results to the choice of input PDFs. The GPD sets extracted from these three analyses will be referred to as Set 1, Set 2, and Set 3, respectively. The pion PDFs are taken at $Q^2 = 4 \text{ GeV}^2$ using the LHAPDF package [128], in analogy with previous proton GPD analyses [61,120]. For each analysis, the optimum values of the free parameters in the profile function $f_v^q(x)$ [see Eq. (4)] are obtained by minimizing the global χ^2 function defined in Eq. (5).

Following the standard assumption adopted in pion PDF analyses, namely, $\bar{u}_v^- = d_v^-$, we also assume that the profile functions of the constituent quarks are identical, so that a single profile function $f_v(x)$ needs to be determined. This reduces the number of free parameters from six to three. Furthermore, based on the parametrization scan procedure used in previous analyses [61,120], originally proposed in the QCD analysis of PDFs by the H1 and ZEUS Collaborations [129], we find that it is sufficient to vary only the parameters α' and A in order to achieve a satisfactory description of the data. In fact, allowing parameter B to vary does not lead to a significant reduction in the total χ^2 . Physically, α' and A control the t -dependence of the GPDs at small and large $-t$, respectively, while B mainly affects the intermediate $-t$ region. Our investigations indicate that a full description of the data can be achieved by considering only α' and A .

Table II summarizes the best-fit values of the profile function parameters, α' and A , for each analysis. The extracted parameters exhibit a mild dependence on the choice of pion PDF set, while the overall trends remain consistent across the three fits. In particular, we observe an inverse correlation: as the value of α' increases, the corresponding value of A decreases. The quoted uncertainties represent the 1σ confidence intervals, obtained using the Hessian method described in Sec. III.

To quantify the quality of the fits, Table III presents the χ^2 values divided by the number of data points, χ^2/N_{pts} ,

TABLE II. Best-fit values of the GPD profile function parameters with their uncertainties, obtained from analyses using different pion PDF sets. Set 1, Set 2, and Set 3 correspond to the xFitter [30], JAM21 [32], and MAP23 [34] pion PDFs, respectively.

GPD set	α'	A
Set 1	1.008 ± 0.024	2.929 ± 0.146
Set 2	1.123 ± 0.023	2.724 ± 0.146
Set 3	1.196 ± 0.024	2.384 ± 0.148

TABLE III. χ^2 values per number of data points, χ^2/N_{pts} , obtained for each dataset in the three analyses. The last row shows the total χ^2 divided by the number of degrees of freedom, $\chi^2/\text{d.o.f.}$, for each fit.

Dataset	Set 1	Set 2	Set 3
Brown [103]	2.576/5	2.734/5	2.885/5
Ackermann [104]	0.036/1	0.055/1	0.068/1
Bebek [105]	31.372/21	31.282/21	30.564/21
Volmer [107]	3.971/5	4.058/5	4.205/5
Huber [108]	10.145/8	9.915/8	9.538/8
Adylov [109]	9.449/22	9.455/22	9.488/22
Dally [110]	43.419/20	43.202/20	43.032/20
Dally [111]	9.139/14	9.159/14	9.140/14
Amendolia [112]	51.474/45	52.656/45	52.224/45
Total	161.581/139	162.516/139	161.144/139

obtained for each experimental dataset included in the analysis. The last row shows the total χ^2 divided by the number of degrees of freedom, $\chi^2/\text{d.o.f.}$, for each fit. Note that in this table the dataset of Volmer [107] also includes the reanalyzed data point of Brauel [106]. Overall, all three analyses provide an acceptable description of the data, with no significant differences observed. This indicates that our ansatz for the GPDs in Eq. (3) is not strongly dependent on the choice of PDFs, which is encouraging. The use of the MAP23 PDFs (Set 3) leads to a slightly smaller $\chi^2/\text{d.o.f.}$, and we therefore adopt this set as our final GPD set. For this fit, the total χ^2 divided by the number of data points for the pion electroproduction and elastic pion scattering processes is 47.260/40 and 113.884/101, respectively.

In Fig. 2, we compare the extracted valence pion GPDs, $xH_v^q(x, t, Q^2, \xi = 0)$, from the three analyses at $Q^2 = 4 \text{ GeV}^2$ for representative values of t , namely, $t = -0.01$, -1 , -3 , and -6 GeV^2 . Note the uncertainties also include the corresponding ones from the pion PDFs. As expected, the GPDs are suppressed as the absolute value of t increases. The general x -dependence is similar across all fits, with some differences emerging primarily in the

small- $|t|$ region, where the underlying pion PDFs differ the most. These results provide further evidence that our GPD ansatz, and thus the extracted universal pion GPD, is only weakly sensitive to the choice of pion PDFs.

Figures 3 and 4 show the comparison between the theoretical predictions for the pion form factor, $F_\pi(t)$, and its absolute square, $|F_\pi(t)|^2$, obtained from the extracted GPDs, with the experimental measurements from pion electroproduction and elastic scattering. All three fits provide a satisfactory description of the data across the full t range. As can be seen, all GPD sets lead to essentially the same predictions. As mentioned earlier, since the analysis using MAP23 PDFs results in a slightly smaller χ^2 , we take it as our final result.

Figure 5 shows a comparison between the theoretical predictions for the pion electromagnetic form factor, $F_\pi(t)$, obtained from the present analysis (Set 3), and representative results from several established theoretical approaches, including the DSE framework [90], LFH [92], LCQM [102], and the thermal soft-wall model (TSWM) of holographic QCD [80], together with recent lattice QCD results labeled as Lattice 1 [87], Lattice 2 [88], and Lattice 3 [89]. Overall, our prediction agrees very well with these approaches in whole $|t|$ region, particularly with the DSE, LFH, and lattice QCD calculations at small $|t|$ region. At larger $|t|$, our result lies within the spread of existing model predictions and follows the average trend of the different calculations. This comparison demonstrates that the GPD-based extraction obtained in the present work is fully consistent with a broad range of modern theoretical frameworks.

In addition to the pion electromagnetic FF itself, it is instructive to compare the results with the scaled quantity $|t|F_\pi(t)$. This observable highlights the asymptotic behavior of the FF and provides a direct connection to perturbative QCD (pQCD) predictions [83]. Experimentally, measurements of $|t|F_\pi(t)$ from Jefferson Lab (JLab) [107,108] have provided valuable insights into the transition region between the nonperturbative and perturbative regimes of QCD. In addition, results from lattice QCD and other theoretical and phenomenological approaches are available for this quantity, allowing for a direct comparison between theory, phenomenology, and experiment.

Figure 6 presents our theoretical predictions for $|t|F_\pi(t)$ obtained from the final GPD set (Set 3), together with the experimental measurements from JLab [108] as well as results from lattice QCD [133], LFH by the BLFQ Collaboration [67], and parameter-free prediction of Ref. [97] obtained using a symmetry-preserving truncation of the quantum field equations describing hadron properties (labeled as YBR24). The agreement between our predictions and the data is very good, except in the large- $|t|$

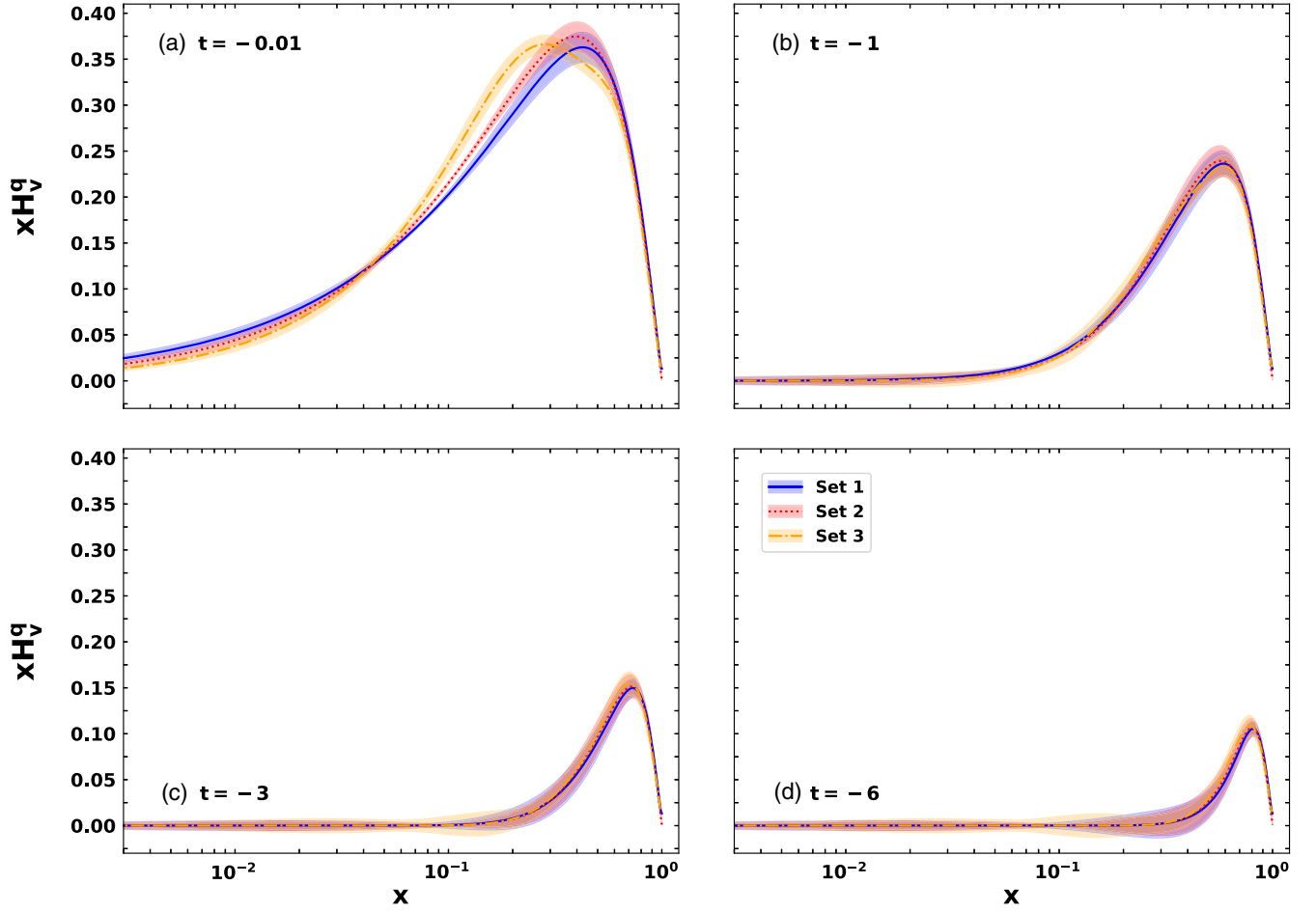


FIG. 2. Comparison of the extracted valence pion GPDs, $xH_v^q(x, t, Q^2)$, corresponding to Set 1, Set 2, and Set 3, obtained using the xFitter, JAM21, and MAP23 PDFs, respectively, shown at $Q^2 = 4 \text{ GeV}^2$ for representative values of t .

region. Note that the JLab $F_\pi(t)$ data were already included in our analyses, labeled as Huber 2008 in Tables I and III, with a corresponding χ^2 of 9.538 for 8 data points (for Set 3 in Table III), indicating excellent fit quality. The minor deviation observed in Fig. 6 at large $|t|$ reflects the sensitivity of the scaled quantity $|t|F_\pi(t)$, and hence,

highlights its role as a precision observable. From another point of view, it is also possible that the JLab data may require refinement with future precise measurements, especially since pQCD predicts that $|t|F_\pi(t)$ should approach a constant value at large momentum transfers. Such a behavior well reproduced by our results.

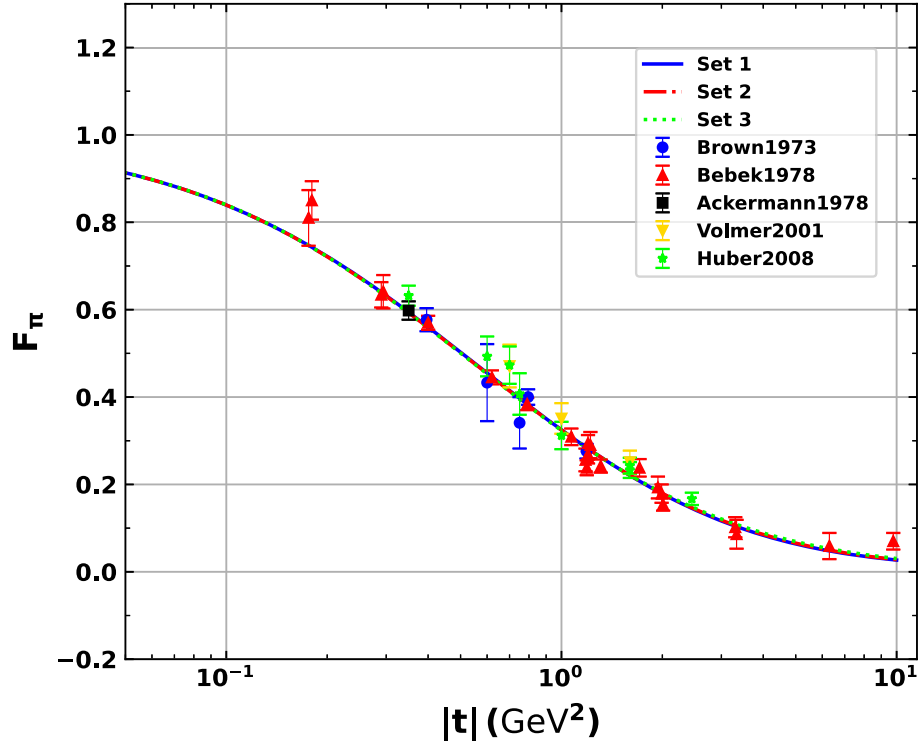


FIG. 3. Comparison of the experimental data for the pion electromagnetic form factor, $F_\pi(t)$, from pion electroproduction with the theoretical predictions obtained from the three fits.

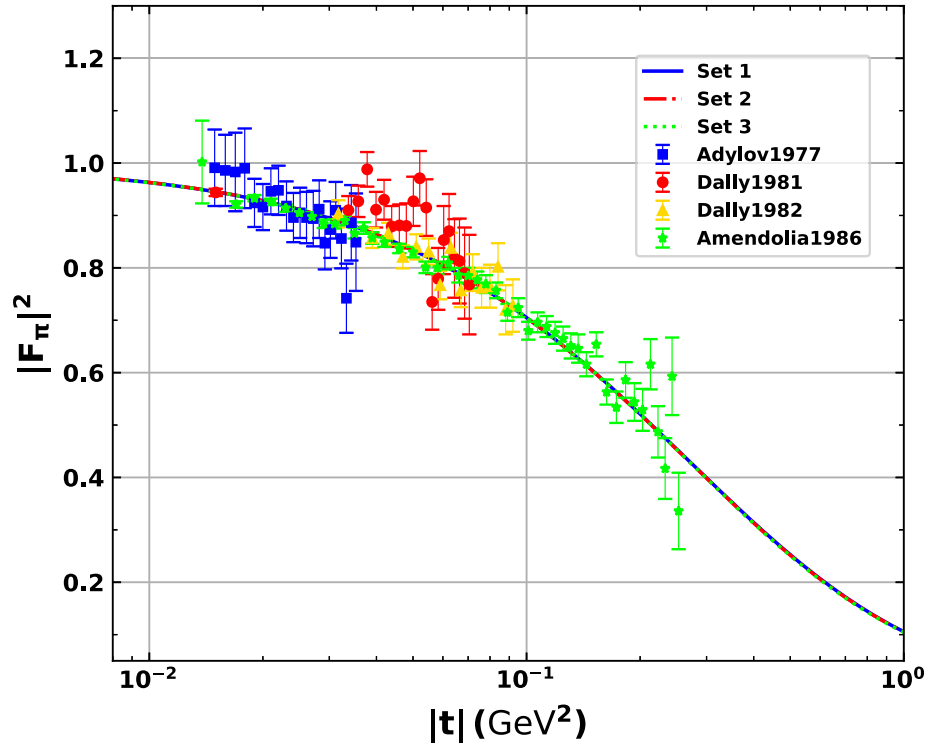


FIG. 4. Comparison of the experimental data for the squared pion electromagnetic form factor, $|F_\pi(t)|^2$, from elastic pion scattering with the theoretical predictions obtained from the three fits.

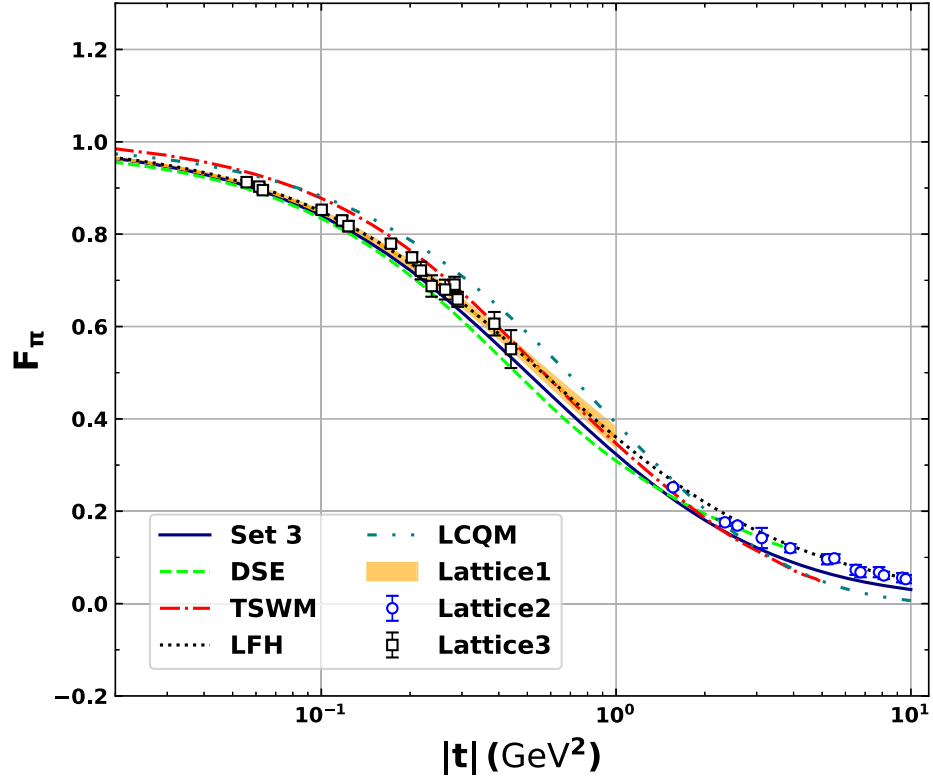


FIG. 5. Comparison of the pion electromagnetic form factor, $F_\pi(t)$, obtained from the present analysis (Set 3), with representative theoretical predictions from the DSE approach [90], LFH [92], LCQM [102], and TSWM of holographic QCD [80], together with recent lattice QCD results [87–89].

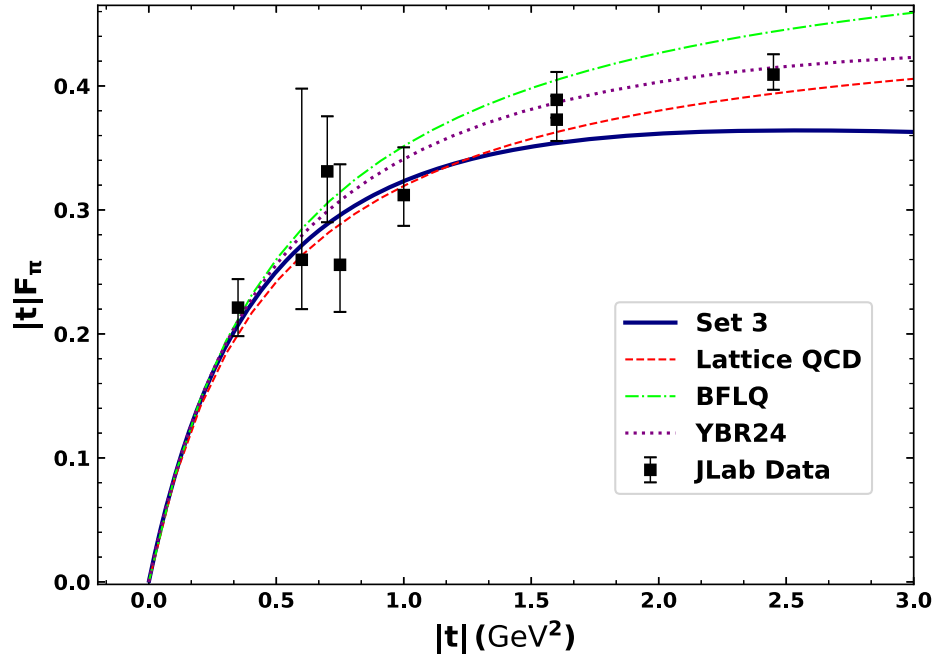


FIG. 6. Comparison of the experimental data for $|t|F_\pi(t)$ from JLab [108] with theoretical predictions from our analysis (Set 3), as well as results from lattice QCD [133], BLFQ [67], and YBR24 [97].

VI. SUMMARY AND CONCLUSION

In this work, we have performed a global QCD analysis of the pion electromagnetic FF data to extract the valence pion GPDs at zero skewness. The analysis was carried out using three different sets of pion PDFs, namely, xFitter [30], JAM21 [32], and MAP23 [34], which were used to construct the GPD ansatz. We carefully examined the dependence of the extracted GPDs on the choice of the underlying pion PDF and performed a parametrization scan procedure to determine the optimum values of the model parameters.

The profile function parameters of the GPDs were determined through a χ^2 minimization. The total number of experimental data points included in the analyses is 141, comprising 40 points from pion electroproduction and 101 points from elastic pion scattering. By following standard assumptions on the pion PDFs and their valence quark symmetry, and employing a parametrization scan procedure, we reduced the number of free parameters and found that only the parameters α' and A were sufficient to achieve a satisfactory description of the data. Our results indicate that the extracted parameters exhibit only a mild dependence on the choice of PDF set, while the overall trends remain consistent across the three analyses.

A detailed comparison of the extracted valence pion GPDs from analyses with different input PDFs shows that the x -dependence is very similar in all fits. Only minor differences are seen at small t which reflects variations in the input PDFs. The theoretical predictions for the pion

electromagnetic form factor, $F_\pi(t)$, and its absolute square, $|F_\pi(t)|^2$, are in good agreement with the experimental measurements. This good agreement demonstrates the reliability of our GPD ansatz. Among the three analyses, the one using the MAP23 PDFs (Set 3) leads to a slightly smaller $\chi^2/\text{d.o.f.}$ and is therefore adopted as our final GPD set.

In summary, our study provides a comprehensive determination of the valence pion GPDs at $Q^2 = 4 \text{ GeV}^2$ and $\xi = 0$, showing that the choice of pion PDFs has only a minor effect on the extracted distributions. These results represent a significant step forward in understanding the internal structure of the pion within QCD and provide a solid foundation for future phenomenological studies, such as investigations of the pion charge radius, pion tomography, and pion mechanical properties, similar to recent analyses performed for the proton [67,121,134,135].

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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