

Resolved photoproduction of the B_c meson in electron-proton collisions

Na Cai^{*} and Xi-Jie Zhan[†]

*Department of Physics, Hebei University, Baoding 071002, People's Republic of China;
Hebei Key Laboratory of High-precision Computation and Application of Quantum Field Theory,
Baoding 071002, People's Republic of China,
and Hebei Research Center of the Basic Discipline for Computational Physics,
Baoding 071002, People's Republic of China*

Tai-Fu Feng[‡]

*Department of Physics, Hebei University, Baoding 071002, People's Republic of China;
Department of Physics, Guangxi University, Nanning 530004, People's Republic of China;
Hebei Key Laboratory of High-precision Computation and Application of Quantum Field Theory,
Baoding 071002, People's Republic of China,
and Hebei Research Center of the Basic Discipline for Computational Physics,
Baoding 071002, People's Republic of China*



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We present a systematic study of prompt B_c meson photoproduction at electron-proton colliders within the framework of nonrelativistic QCD factorization. In addition to the dominant direct channel $\gamma + g \rightarrow B_c + X$, we include resolved contributions initiated by $g + g$ and $q + \bar{q}$ ($q = u, d, s$) subprocesses. The production of the $1S$, $2S$, and $1P$ color-singlet B_c states is calculated, and the feeddown contributions from the excited states are included in the prompt- B_c yield. Total cross sections and transverse-momentum distributions are presented for several collider configurations, including HERA, LHeC, FCC- ep , and EIC. The numerical results show that the direct $\gamma + g$ channel provides the leading contribution over the entire kinematic range. The resolved $g + g$ channel gives a non-negligible correction, reaching the level of $\mathcal{O}(10\%)$ in the low- p_T region where most events are produced, and it becomes increasingly important at higher collision energies.

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I. INTRODUCTION

The B_c meson is a distinctive heavy-flavored hadron in the Standard Model, consisting of a bottom quark and a charm antiquark (or charge-conjugated state). As the only known meson composed of two heavy quarks of different flavors, the B_c system occupies a unique position between charmonium and bottomonium. Its spectroscopy, decay properties, and production mechanisms provide sensitive probes of heavy-quark dynamics, nonrelativistic effective theories, and nonperturbative aspects of QCD.

At high-energy colliders, the production of B_c mesons is governed predominantly by strong interactions. In contrast

to quarkonium production, the formation of a B_c meson requires the simultaneous production of both a $b\bar{b}$ and a $c\bar{c}$ pair within a single hard scattering. Consequently, its production rate is suppressed relative to conventional heavy quarkonia, while remaining fully calculable within perturbative QCD. The hard subprocesses involve energy scales set by the heavy-quark masses, where perturbation theory is applicable, whereas the subsequent hadronization into a bound $b\bar{c}$ state probes the transition to the nonperturbative regime. This interplay makes B_c production a sensitive probe of QCD factorization and heavy-quark binding dynamics [1,2].

Experimentally, the existence of the B_c meson was first firmly established by the CDF Collaboration in 1998 through analyses of proton-antiproton collision data at the Tevatron [3,4]. In 2014, the ATLAS Collaboration reported the observation of the radially excited $B_c(2S)$ states [5]. These results were subsequently confirmed and refined by the CMS and LHCb Collaborations in 2019 with higher statistical significance and improved mass resolution [6,7]. More recently, further experimental advances have led to the observation of orbitally excited B_c states [8].

^{*}Contact author: yoloitw@outlook.com

[†]Contact author: zhanxj@hbu.edu.cn

[‡]Contact author: fengtf@hbu.edu.cn

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To date, all observations of B_c mesons and their excited states have been obtained exclusively at hadron colliders.

Theoretical investigations of B_c meson production have been pursued across a wide range of collision environments and mechanisms. At hadron colliders such as the Tevatron and the LHC, dominant production channels arise from gluon–gluon fusion and heavy-quark fragmentation, calculable within the nonrelativistic QCD (NRQCD) factorization framework at leading and higher orders in α_s [1,2,9–27]. Beyond hadronic collisions, B_c production has also been studied in the context of high-luminosity electron–positron colliders [28–39]. Indirect sources of B_c mesons through decays of heavy particles have been explored [40–49], such as the top quark, W boson and Higgs boson, highlighting the role of heavy-particle decay as a complementary production channel at high energies. An important class of processes at e^+e^- or electron-proton machines involves photoproduction, where initial-state radiation or Weizsäcker-Williams photons interact to produce a B_c meson accompanied by heavy quarks [32,48,50–53]. The quasireal photon can interact as a pointlike particle, and at high energies, the photon can also fluctuate into a hadronic state and develop a nontrivial partonic structure. This gives rise to resolved photoproduction processes, in which partons inside the photon participate in the hard scattering. Such resolved contributions are known to play an important role in various quarkonium production processes, particularly in kinematic regions characterized by low transverse momentum and high center-of-mass energy. Detailed studies at e^+e^- colliders show that both direct $\gamma\gamma \rightarrow B_c + b + \bar{c}$ and resolved photon channels can yield substantial production rates under suitable collider configurations [54]. For ep collisions, earlier calculations of $B_c^{(*)}$ photoproduction mainly considered the direct $\gamma + g$ channel and, in some cases, extrinsic heavy-quark initiated mechanisms such as $\gamma + c$ and $\gamma + \bar{b}$ [25,50]. The production of orbitally excited P -wave B_c^{*} states at the LHeC was studied in Ref. [51]. The resolved-photon contributions to B_c photoproduction, however, have received less attention, especially in the kinematic regime relevant for future high-luminosity ep colliders.

In this work, we present a systematic study of B_c photoproduction at electron-proton colliders, including both direct and resolved photon contributions. In addition to the conventional $\gamma + g$ channel, we incorporate resolved processes initiated by $g + g$ and $q + \bar{q}$ ($q = u, d, s$) partonic subprocesses. We calculate the production of the $1S$, $2S$, and $1P$ color-singlet B_c states and include the feeddown of excited states into the prompt B_c yield. We perform numerical analyses for several representative collider configurations, including HERA, LHeC, FCC- ep , and EIC, and investigate both total cross sections and differential distributions. The inclusion of resolved photon contributions enables us to assess the role of the photon's

partonic structure in B_c production. In particular, we analyze the energy and transverse-momentum dependence of different production channels and identify the kinematic regions where resolved processes become non-negligible. Such studies are relevant for future experimental programs aiming to explore QCD dynamics in novel regimes and may provide complementary constraints on parton distribution functions in both the proton and the photon.

II. FORMULATION

Within the framework of the Weizsäcker-Williams approximation (WWA), the energy distribution of photons originating from bremsstrahlung emission can be expressed as [55]

$$f_{\gamma/e}(x) = \frac{\alpha}{2\pi} \left[\frac{1 + (1-x)^2}{x} \log \frac{Q_{\max}^2}{Q_{\min}^2} + 2m_e^2 x \left(\frac{1}{Q_{\max}^2} - \frac{1}{Q_{\min}^2} \right) \right], \quad (1)$$

where $x = E_\gamma/E_e$ denotes the fraction of the longitudinal momentum carried by the emitted photon, α is the electromagnetic fine-structure constant, and the kinematic limits are given by $Q_{\min}^2 = m_e^2 x^2/(1-x)$ and $Q_{\max}^2 = (E_e \theta_c)^2(1-x) + Q_{\min}^2$. The parameter $\theta_c = 32$ mrad corresponds to the maximum allowed scattering angle of the outgoing electron, imposed to ensure that the emitted photon remains quasireal.

Within the NRQCD factorization formalism, following the approach in Ref. [54], the cross section for producing a color-singlet B_c state H at an electron-proton collider can be factorized into contributions from the photon flux, the parton distributions in the proton and in the photon, and the partonic hard-scattering subprocesses,

$$\begin{aligned} d\sigma(e^- + P \rightarrow H + X) \\ = \sum_{i,j,n} \int dx_1 dx_2 dx_i f_{\gamma/e}(x_1) f_{j/P}(x_2, \mu_{F,P}) \\ \times f_{i/\gamma}(x_i, \mu_{F,\gamma}) d\hat{\sigma}_{ij,n}(\mu_R) \langle \mathcal{O}^H[n] \rangle. \end{aligned} \quad (2)$$

Here $f_{j/P}(x, \mu_{F,P})$ denotes the parton distribution function (PDF) of parton j inside the proton, while $f_{i/\gamma}(x, \mu_{F,\gamma})$ ($i = \gamma, g, u, d, s$) represents the Glück-Reya-Schienbein (GRS) distribution function of parton i in the photon [56]. The special case $f_{\gamma/\gamma}(x) = \delta(1-x)$ corresponds to the direct photoproduction contribution. The scale $\mu_{F,\gamma}$ is the factorization scale at which the partonic structure of the photon is probed. The resolved photoproduction channels effectively probe the partonic structure of the quasireal photon. The quantity $d\hat{\sigma}_{ij,n}(\mu_R)$ denotes the short-distance partonic cross section for $ij \rightarrow c\bar{b}[n] + b + \bar{c}$, which can be calculated perturbatively within QCD. The intermediate

heavy-quark pair $c\bar{b}[n]$ is characterized by the quantum numbers n , and the associated long-distance matrix element (LDME) $\langle\mathcal{O}^H[n]\rangle$ encodes the nonperturbative transition probability for the $c\bar{b}$ pair to hadronize into a physical B_c state. In contrast to charmonium and bottomonium systems, where color-octet channels can play an essential role in describing experimental data, the situation for the B_c meson is qualitatively different. The production of a B_c state requires the simultaneous creation of two heavy-quark pairs, which already suppresses the overall production rate. As a consequence, fragmentation-type mechanisms that typically enhance color-octet contributions in quarkonium production are less effective in the B_c case. From the perspective of NRQCD power counting, color-octet contributions are suppressed by powers of the relative velocity v of the heavy quarks inside the bound state. For the B_c system, one expects $v^2 \sim \mathcal{O}(0.1)$, leading to an additional suppression factor of $\mathcal{O}(v^4)$ for the dominant octet channels. In the present analysis, we restrict ourselves to the leading contribution in the NRQCD velocity expansion, namely the color-singlet channel. Under this approximation, the corresponding LDME can be evaluated using potential models.

For definiteness, we consider the following partonic subprocesses contributing to three distinct production channels,

$$\gamma + g \rightarrow H + b + \bar{c}, \quad (3)$$

$$g + g \rightarrow H + b + \bar{c}, \quad (4)$$

$$q + \bar{q} \rightarrow H + b + \bar{c}, \quad (5)$$

where $q = u, d, s$ and H denotes $B_c(1^1S_0)$, $B_c^*(1^3S_1)$, $B_c(2^1S_0)$, $B_c^*(2^3S_1)$, $B_c^{**}(1^1P_1)$, or $B_c^{**}(1^3P_J)$ with $J = 0, 1, 2$. When prompt B_c production is quoted, we use

$$\sigma_{\text{prompt}}^{S+P} = \sigma[B_c(1^1S_0)] + \sum_{H' \in \mathcal{S}_{\text{fd}}} \mathcal{B}(H' \rightarrow B_c + X) \sigma(H'), \quad (6)$$

where $\mathcal{S}_{\text{fd}} = \{B_c^*, B_c(2^1S_0), B_c^*(2^3S_1), B_c^{**}(1P)\}$. We take $\mathcal{B}(H' \rightarrow B_c + X) = 1$ for the inclusive feeddown estimate. Representative Feynman diagrams for the subprocesses in Eqs. (3)–(5) are shown in Fig. 1. Both the analytical derivation and numerical evaluation of the partonic amplitudes are performed using the well-established Feynman

Diagram Calculation (FDC) package [57]. Within this framework, the standard NRQCD projection method [58] is employed to extract contributions from specific heavy-quark configurations.

III. NUMERICAL RESULTS AND DISCUSSIONS

The input parameters in the calculation are taken as follows. The fine structure constant is fixed as $\alpha = 1/137$. We take $m_b = 4.8$ GeV, $m_c = 1.5$ GeV, and $M_{B_c} = m_b + m_c$. The proton PDFs are evaluated with the CT14ll0 set through an external PDF interface, and the one-loop running strong coupling $\alpha_s(\mu_R)$ associated with the same PDF set is used consistently [60,61]. The GRS distributions are used for the photon PDFs [56]. We distinguish the renormalization scale μ_R , the proton factorization scale $\mu_{F,p}$, and the photon factorization scale $\mu_{F,\gamma}$. Unless otherwise stated, their central values are chosen as

$$\mu_R = \mu_{F,p} = \mu_{F,\gamma} = \mu_0 = m_T(B_c) = \sqrt{M_{B_c}^2 + p_T^2}, \quad (7)$$

with p_T being the transverse momentum of the produced B_c meson. The LDMEs $\langle\mathcal{O}^H[n]\rangle$ are related to the wave function at the origin. For the S -wave states we use $\langle\mathcal{O}^H[n]\rangle \simeq N_c |R_S(0)|^2 / (2\pi)$ with $|R_{1S}(0)|^2 = 1.642 \text{ GeV}^3$ and $|R_{2S}(0)|^2 = 0.983 \text{ GeV}^3$ [62,63]. For the $1P$ states, whose LDMEs are proportional to the derivative of the radial wave function at the origin, we take $|R'_{1P}(0)|^2 = 0.201 \text{ GeV}^5$ [62,63]. Several representative electron-proton colliders [64–67] were selected, and their corresponding center-of-mass energies are listed in Table I.

The integrated cross sections for the photoproduction of B_c states at various electron-proton colliders are summarized in Table II. The results are presented for the $1S$ and $2S$ states, B_c , B_c^* , $B_c(2^1S_0)$, and $B_c^*(2^3S_1)$, as well as for the orbitally excited states $B_c^{**}(1^1P_1)$ and $B_c^{**}(1^3P_J)$ ($J = 0, 1, 2$). For each state, the three entries in parentheses give the direct γg contribution and the resolved gg and $q\bar{q}$ contributions.

Table II shows that the direct γg channel remains dominant for all collider configurations. For the full prompt yield defined in Eq. (6), the gg resolved fraction increases with the collider energy: it is about 1.25% at HERA, about 3.70% at LHeC-2, and about 7.28% at FCC-ep-2. The

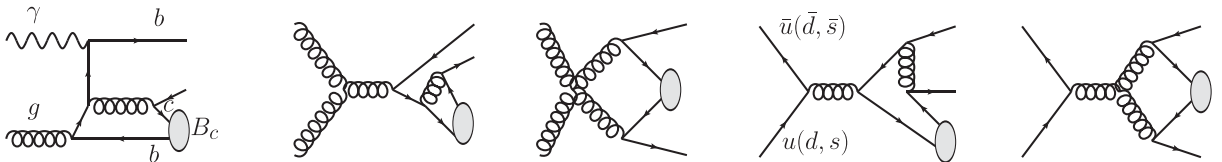


FIG. 1. Some typical Feynman diagrams for calculating the partonic cross section $\hat{\sigma}$ of B_c photoproduction at ep collider. The diagrams are drawn by JaxoDraw [59].

TABLE I. The center-of-mass energies ($\sqrt{S}$) of representative electron-proton colliders, along with the corresponding electron energies (E_e) and proton energies (E_p).

Collider	$\sqrt{S}$	E_e	E_p
HERA	319 GeV	27.5 GeV	920 GeV
LHeC-1	1.30 TeV	60 GeV	7 TeV
LHeC-2	1.98 TeV	140 GeV	7 TeV
FCC- ep -1	7.07 TeV	250 GeV	50 TeV
FCC- ep -2	10.0 TeV	500 GeV	50 TeV
EIC-1	45 GeV	5 GeV	100 GeV
EIC-2	140 GeV	18 GeV	275 GeV

corresponding $q\bar{q}$ contribution is much smaller at high energies, below the percent level for the LHeC and FCC- ep configurations. The larger relative importance of the gg channel at higher energies is caused by the enhancement of the gluon densities in both the proton and the photon at smaller momentum fractions.

The excited-state contributions are important for the prompt- B_c yield. The spin-triplet B_c^* states have larger cross sections than the corresponding spin-singlet states, mainly because of the spin multiplicity. The summed $1P$ -wave feeddown contribution is more modest in the channel set considered here: it increases the integrated prompt- B_c cross section by about 3.5%–4%. This is smaller than the enhancement found in the dedicated P -wave photoproduction study of Ref. [51], because that work also included extrinsic heavy-quark initiated channels such as $\gamma + c$ and $\gamma + \bar{b}$. Those channels are outside the present resolved-photon study and require a separate treatment of

TABLE III. Representative integrated luminosities and the corresponding numbers of produced prompt B_c mesons. The prompt cross section $\sigma_{\text{prompt}}^{S+P}$ includes the $1S$, $2S$, and $1P$ contributions defined in Eq. (6). The event numbers do not include decay branching fractions, detector acceptance, trigger efficiency, or reconstruction efficiency.

Collider	$\sigma_{\text{prompt}}^{S+P}$	Assumed $\mathcal{L}_{\text{int}}$	N_{prompt}^{S+P}
HERA	3.99 pb	0.5 fb $^{-1}$ [68]	2.0×10^3
LHeC-1	30.91 pb	1 ab $^{-1}$ [69]	3.1×10^7
LHeC-2	54.31 pb	1 ab $^{-1}$	5.4×10^7
FCC- ep -1	196.47 pb	1 ab $^{-1}$ [70]	2.0×10^8
FCC- ep -2	281.85 pb	1 ab $^{-1}$	2.8×10^8
EIC-1	14.68 fb	100 fb $^{-1}$ [71]	1.5×10^3
EIC-2	768.34 fb	100 fb $^{-1}$	7.7×10^4

heavy-quark PDFs and possible double counting with gluon-splitting contributions.

Table III lists representative integrated luminosities and the corresponding numbers of produced prompt B_c mesons. At FCC- ep -2, the central prompt cross section is 281.85 pb, which gives about 2.8×10^8 produced prompt B_c mesons for an integrated luminosity of 1 ab $^{-1}$. The corresponding estimate for LHeC-2 is about 5.4×10^7 . In contrast, the EIC configurations lead to much smaller event samples, about 1.5×10^3 and 7.7×10^4 produced prompt B_c mesons for EIC-1 and EIC-2, respectively, if 100 fb $^{-1}$ is accumulated. These numbers refer to produced prompt B_c mesons and should be further multiplied by decay branching fractions, detector acceptance, trigger efficiencies, and reconstruction efficiencies when estimating observable event yields.

TABLE II. The integrated cross sections for the photoproduction of the B_c states at representative electron-proton colliders. Each parenthesis contains the three channel contributions ($\sigma_{\gamma g}, \sigma_{gg}, \sigma_{q\bar{q}}$) corresponding to Eqs. (3)–(5). The units are indicated in the first column of each block; the smaller HERA and EIC- $1P$ -wave entries are displayed in fb and ab, respectively.

Collider	σ_{B_c} ($\sigma_{\gamma g}, \sigma_{gg}, \sigma_{q\bar{q}}$)	$\sigma_{B_c(2^1S_0)}$ ($\sigma_{\gamma g}, \sigma_{gg}, \sigma_{q\bar{q}}$)	$\sigma_{B_c^*}$ ($\sigma_{\gamma g}, \sigma_{gg}, \sigma_{q\bar{q}}$)	$\sigma_{B_c^*(2^3S_1)}$ ($\sigma_{\gamma g}, \sigma_{gg}, \sigma_{q\bar{q}}$)
HERA(pb)	0.41(0.40, 0.0089, 0.0025)	0.24(0.24, 0.0053, 0.0015)	2.00(1.96, 0.021, 0.017)	1.20(1.17, 0.013, 0.010)
LHeC-1(pb)	3.33(3.16, 0.16, 0.0095)	1.99(1.89, 0.095, 0.0057)	15.32(14.88, 0.39, 0.061)	9.17(8.91, 0.23, 0.037)
LHeC-2(pb)	5.94(5.57, 0.35, 0.015)	3.55(3.33, 0.21, 0.0087)	26.85(25.89, 0.86, 0.094)	16.07(15.50, 0.52, 0.056)
FCC- ep -1(pb)	22.25(20.04, 2.17, 0.036)	13.32(12.00, 1.30, 0.022)	96.36(90.78, 5.35, 0.23)	57.69(54.35, 3.20, 0.14)
FCC- ep -2(pb)	32.29(28.66, 3.57, 0.049)	19.33(17.16, 2.14, 0.029)	137.88(128.74, 8.83, 0.30)	82.54(77.07, 5.29, 0.18)
EIC-1(fb)	1.32(1.29, 0.0032, 0.029)	0.79(0.77, 0.0019, 0.018)	7.50(7.25, 0.0066, 0.24)	4.49(4.34, 0.0039, 0.15)
EIC-2(fb)	75.14(73.48, 0.87, 0.79)	44.98(43.99, 0.52, 0.47)	387.95(380.17, 2.02, 5.75)	232.25(227.60, 1.21, 3.44)
Collider	$\sigma_{B_c^{**}(1^1P_1)}$ ($\sigma_{\gamma g}, \sigma_{gg}, \sigma_{q\bar{q}}$)	$\sigma_{B_c^{**}(1^3P_0)}$ ($\sigma_{\gamma g}, \sigma_{gg}, \sigma_{q\bar{q}}$)	$\sigma_{B_c^{**}(1^3P_1)}$ ($\sigma_{\gamma g}, \sigma_{gg}, \sigma_{q\bar{q}}$)	$\sigma_{B_c^{**}(1^3P_2)}$ ($\sigma_{\gamma g}, \sigma_{gg}, \sigma_{q\bar{q}}$)
HERA(fb)	33.49(32.93, 0.37, 0.19)	8.59(8.36, 0.16, 0.071)	19.84(19.34, 0.32, 0.18)	80.60(79.34, 0.76, 0.51)
LHeC-1(pb)	0.26(0.25, 0.0070, 0.00072)	0.065(0.062, 0.0027, 0.00027)	0.16(0.15, 0.0060, 0.00066)	0.61(0.59, 0.014, 0.0019)
LHeC-2(pb)	0.45(0.44, 0.016, 0.0011)	0.11(0.11, 0.0059, 0.00041)	0.28(0.26, 0.013, 0.0010)	1.06(1.02, 0.032, 0.0029)
FCC- ep -1(pb)	1.64(1.54, 0.098, 0.0028)	0.41(0.37, 0.035, 0.0010)	1.03(0.94, 0.083, 0.0025)	3.77(3.56, 0.20, 0.0069)
FCC- ep -2(pb)	2.35(2.19, 0.16, 0.0037)	0.59(0.53, 0.057, 0.0014)	1.48(1.34, 0.14, 0.0034)	5.39(5.05, 0.33, 0.0092)
EIC-1(ab)	131.25(129.05, 0.11, 2.09)	38.95(38.00, 0.080, 0.87)	71.72(69.36, 0.099, 2.26)	340.32(333.42, 0.21, 6.69)
EIC-2(fb)	6.52(6.42, 0.035, 0.062)	1.72(1.68, 0.017, 0.023)	3.78(3.69, 0.031, 0.059)	16.00(15.75, 0.070, 0.17)

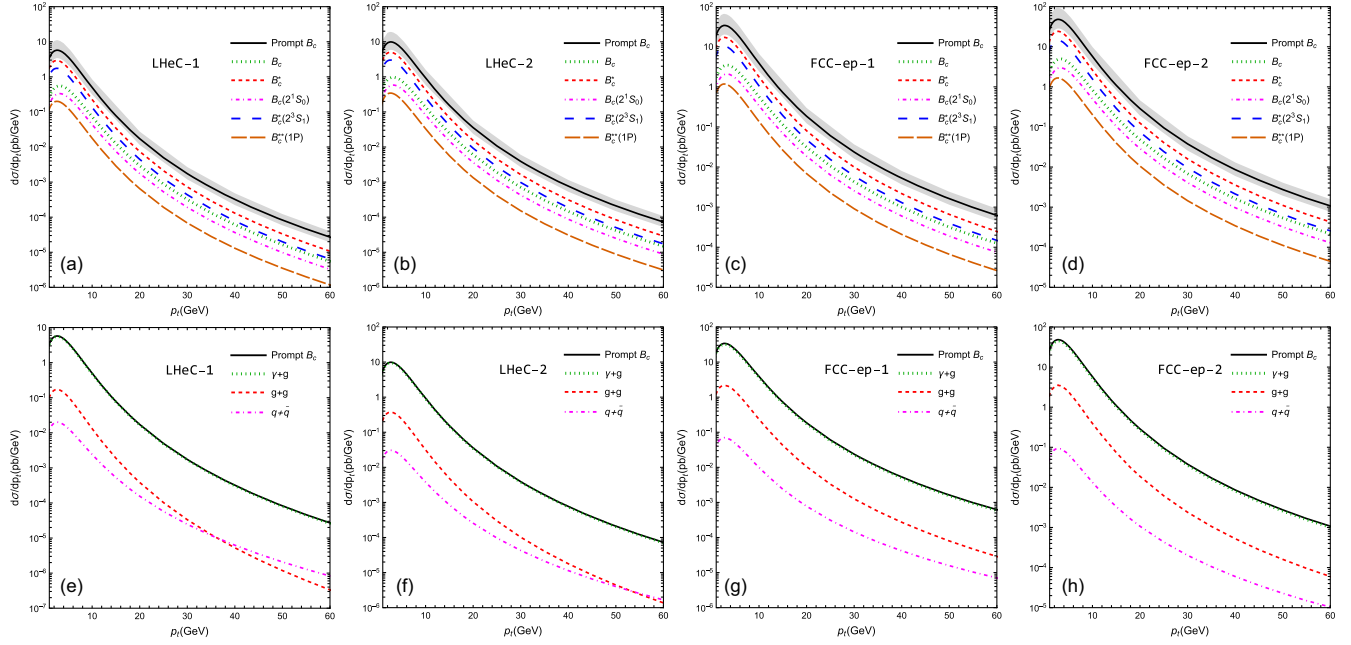


FIG. 2. The p_T distributions for B_c photoproduction. Panels (a)–(d) show the distributions of the dominant S -wave states, the summed $1P$ -wave feeddown contribution denoted by $B_c^{**}(1P)$, and the prompt B_c yield. The prompt yield includes the feeddown contributions from the excited S - and P -wave states with $\mathcal{B}(H \rightarrow B_c + X) = 1$. The shaded bands around the prompt- B_c curves in panels (a)–(d) show the bin-by-bin envelope obtained by varying m_c , m_b , μ_R , $\mu_{F,P}$, and $\mu_{F,Y}$ according to the same prescription as in Table IV and V. Panels (e)–(h) show the decomposition of the prompt- B_c yield into the three production channels in Eqs. (3)–(5).

The transverse momentum (p_T) distributions for B_c photoproduction provide deeper insights into the kinematic features of different production mechanisms. Figure 2 presents these distributions for the LHeC and FCC- ep configurations. From Figure 2(a)–(d), the distributions for the S -wave states exhibit similar shapes, peaking at low p_T and decreasing rapidly with increasing transverse momentum. The summed $B_c^{**}(1P)$ feeddown contribution is much smaller than the dominant B_c contribution, but it is included in the prompt- B_c curve. As a result, the prompt B_c distribution is enhanced by the excited S - and P -wave states while retaining the same overall low- p_T dominated behavior.

Figure 2(e)–(h) shows the production-channel decomposition of the prompt- B_c yield. The $\gamma + g$ channel dominates over the whole p_T range. The resolved $g + g$ contribution is most visible at small p_T and becomes relatively less important at larger p_T , whereas the $q\bar{q}$ channel has a harder shape but remains numerically small. This pattern is consistent with the integrated cross sections in Table II. As a representative example, at FCC- ep -2 the $g + g$ channel contributes about 15% of the total cross section at $p_T = 1$ GeV, whereas this fraction is reduced to 6% at $p_T = 50$ GeV. It reflects the fact that low- p_T production probes smaller momentum fractions, where the gluon distributions in both the proton and the photon are enhanced. Since realistic event samples are dominated by the low- p_T region, the $g + g$ resolved photoproduction channel should be retained in phenomenological predictions.

We now turn to the theoretical uncertainties of our predictions. Instead of listing the variations for each individual intermediate state, we focus on the experimentally relevant prompt- B_c yield $\sigma_{\text{prompt}}^{S+P}$, with all partonic channels summed. Table IV shows the dependence on the heavy-quark masses m_c and m_b . A variation of m_c by ± 0.1 GeV changes the prompt cross section by about +25% and –19% at both LHeC-2 and FCC- ep -2. The dependence on m_b is milder, about +9% and –8% for the same mass variation. This stronger sensitivity to m_c reflects the proximity of the $c\bar{b}$ system to threshold.

TABLE IV. Dependence of the prompt- B_c production cross section on the heavy-quark masses. The prompt yield includes the direct production of $B_c(1^1S_0)$ and the feeddown contributions from the excited S - and P -wave states. All partonic channels, γg , gg , and $q\bar{q}$, are summed. The central input is $m_c = 1.5$ GeV and $m_b = 4.8$ GeV. The relative change is defined as $\Delta_\sigma = (\sigma - \sigma_0)/\sigma_0$.

Variation	Value	LHeC-2		FCC- ep -2	
		$\sigma_{\text{prompt}}^{S+P}$ [pb]	Δ_σ [%]	$\sigma_{\text{prompt}}^{S+P}$ [pb]	Δ_σ [%]
Central	$m_c = 1.5,$ $m_b = 4.8$	54.31	0.0	281.85	0.0
m_c	1.4 GeV	68.26	+25.7	351.99	+24.9
m_c	1.6 GeV	43.82	–19.3	228.70	–18.9
m_b	4.7 GeV	59.40	+9.4	305.99	+8.6
m_b	4.9 GeV	49.71	–8.5	259.85	–7.8

TABLE V. Independent scale dependence of the prompt- B_c production cross section. The central scale is $\mu_0 = m_T(B_c) = \sqrt{M_{B_c}^2 + p_T^2}$, and $\mu_R = \xi_R \mu_0$, $\mu_{F,p} = \xi_p \mu_0$, $\mu_{F,\gamma} = \xi_\gamma \mu_0$. In each row only one of the three scales is varied, while the other two are fixed at their central values. The CT14llo proton PDFs and the associated running α_s are used. The relative change is defined as $\Delta_\sigma = (\sigma - \sigma_0)/\sigma_0$.

Scale setting	$(\xi_R, \xi_p, \xi_\gamma)$	LHeC-2		FCC- ep -2	
		$\sigma_{\text{prompt}}^{S+P}$ [pb]	Δ_σ [%]	$\sigma_{\text{prompt}}^{S+P}$ [pb]	Δ_σ [%]
Central	(1, 1, 1)	54.31	0.0	281.85	0.0
$\mu_R/2$	(1/2, 1, 1)	101.82	+87.5	530.18	+88.1
$2\mu_R$	(2, 1, 1)	32.79	-39.6	169.53	-39.9
$\mu_{F,p}/2$	(1, 1/2, 1)	42.45	-21.8	189.72	-32.7
$2\mu_{F,p}$	(1, 2, 1)	64.03	+17.9	370.86	+31.6
$\mu_{F,\gamma}/2$	(1, 1, 1/2)	53.58	-1.3	274.31	-2.7
$2\mu_{F,\gamma}$	(1, 1, 2)	55.08	+1.4	290.04	+2.9

The dependence on the three unphysical scales is presented in Table V. We vary one scale at a time by a factor of two around $\mu_0 = m_T(B_c)$, keeping the other two scales fixed at their central values. The renormalization-scale dependence is the largest, as expected for a leading-order calculation: varying μ_R from $\mu_0/2$ to $2\mu_0$ changes the prompt cross section by approximately +88% and -40%. The proton factorization scale also has a visible effect, especially at FCC- ep -2 where the variation is about -33% to +32%. By contrast, the photon-PDF factorization scale changes the total prompt cross section only by a few percent, because at the present leading-order accuracy $\mu_{F,\gamma}$ affects only the resolved-photon part. A systematic cancellation of the photon-factorization-scale dependence between the direct and resolved components would require the corresponding next-to-leading-order direct contribution, which is beyond the scope of the present work.

IV. SUMMARY

In this work, we have performed a systematic analysis of prompt B_c meson photoproduction in electron-proton collisions, incorporating both direct and resolved photon contributions. The production of the $1S$, $2S$, and $1P$ color-singlet B_c states has been investigated. Assuming inclusive feeddown into the ground-state B_c , the excited states significantly enhance the prompt B_c yield and should be included in realistic phenomenological studies. In the direct and resolved channel set considered here, the $1P$ -wave feeddown increases the integrated prompt cross section by about 4%. The direct $\gamma + g$ channel dominates the B_c production rate over a wide range of collider energies and kinematic regions. The resolved $g + g$ contribution is nevertheless non-negligible, especially in the low transverse-momentum region and at high center-of-mass

energies. For the integrated prompt yield it contributes about 7% at FCC- ep -2, and its relative contribution can reach the level of $\mathcal{O}(10\%)$ in the low- p_T region. In contrast, the $q + \bar{q}$ channel remains strongly suppressed in the high-energy ep configurations. With representative integrated luminosities, the numbers of produced prompt B_c mesons can reach 5.4×10^7 at LHeC-2 and 2.8×10^8 at FCC- ep -2, before decay branching fractions and detector efficiencies are included.

We have also examined the dependences on the heavy-quark masses and on the three unphysical scales μ_R , $\mu_{F,p}$, and $\mu_{F,\gamma}$. The leading-order prediction is most sensitive to the renormalization scale, while the photon-PDF factorization scale produces only a small change in the total prompt cross section because it affects only the resolved-photon contribution at this order. These findings indicate that B_c photoproduction can serve as a complementary probe of the gluon content of the photon, especially in the small- x regime accessible at future high-energy ep colliders.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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