

# Implications of the evidence for direct $CP$ violation in $D \rightarrow \pi^+ \pi^-$ decays

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The observation of  $CP$  violation in the difference of  $CP$  asymmetries between  $D \rightarrow K^+ K^-$  and  $D \rightarrow \pi^+ \pi^-$  has raised a debate whether the observed asymmetries can be regarded as a signal of physics beyond the standard model (SM). In this paper we obtain all the topological amplitudes and isospin amplitudes directly from measured observables for  $D \rightarrow \pi\pi$ . These results unambiguously imply a very large penguin contribution, having a central value 4.74 times the magnitude of the amplitude for  $D^0 \rightarrow \pi^+ \pi^-$ . This fitted central value differs from a reasonable SM estimate of 10% with a significance greater than  $3.3\sigma$ . In contrast to previous studies, we present model-independent arguments based only on unitarity of rescattering amplitudes to show that large penguins cannot arise from rescattering alone and likely indicate physics beyond the SM. In a model-independent approach we show how a very small contribution from physics beyond the SM with a large weak phase alleviates the problem.

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## I. INTRODUCTION

The observation of  $CP$  violation in  $D$  meson decays has long been regarded as a smoking gun signal for new physics (NP) or physics beyond the standard model (SM). However, the observation of  $CP$  violation in singly Cabibbo suppressed  $D$  meson decays [1,2] has raised a debate on whether the observed asymmetries can be regarded as a signal of NP. The confusion stems from the difficulty in reliably estimating long distance contributions and also convincingly establishing that the penguin contribution is indeed too large to be acceptable within the SM. Several studies [3–15] have examined the issues indicating the likelihood that the observed asymmetries may arise from NP. In this paper we estimate the size of the penguin and tree contributions as well as isospin contributions directly from experimental data. We find that the penguin contributions are unexpectedly large, having a central value 4.74 times the magnitude of the amplitude for

$D^0 \rightarrow \pi^+ \pi^-$ . This fitted central value differs from a reasonable SM estimate of 10% with a significance greater than  $3.3\sigma$ . We argue that the evidence for  $CP$  violation in  $D \rightarrow \pi^+ \pi^-$  is a signal of NP at greater than  $3.3\sigma$ . We also examine final state interactions in a completely model-independent way based only on unitarity in contrast to previous studies. Our conclusion depends largely on the measured value of the direct  $CP$  asymmetry in  $D \rightarrow \pi^+ \pi^-$ . In a model-independent approach we show how a very small contribution from NP with a large weak phase alleviates the problem. If this evidence is indeed a signal of NP, large  $CP$  violation must show up in other modes and experiments. Searches for other signals of  $CP$  violation in  $D$  decays are essential to substantiate the evidence of NP.

## II. ANALYSIS OF $D \rightarrow \pi\pi$ AMPLITUDES

The amplitudes for  $D \rightarrow \pi\pi$  decay modes can be written assuming isospin as

$$\begin{aligned} A(D^0 \rightarrow \pi^+ \pi^-) &= \sqrt{2}(\lambda_d(T + E^d) + \lambda_i(P^i + PE^i + PA^i)), \\ A(D^0 \rightarrow \pi^0 \pi^0) &= (\lambda_d(C - E^d) - \lambda_i(P^i + PE^i + PA^i)), \\ A(D^+ \rightarrow \pi^+ \pi^0) &= \lambda_d(T + C), \end{aligned} \quad (1)$$

where  $\lambda_i = V_{ci}^* V_{ui}$  and it is assumed that  $i$  is summed over  $d, s$ , and  $b$  quarks.  $T, C, E, P, PE$ , and  $PA$  are color-allowed external  $W$ -emission, color-suppressed internal  $W$ -emission,  $W$ -exchange, penguin, penguin-exchange, and penguin-annihilation topological amplitudes, respectively.

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Unitarity of the Cabibbo-Kobayashi-Maskawa matrix requires that  $\lambda_d + \lambda_s + \lambda_b = 0$ , which can be used to eliminate  $\lambda_b$  in Eq. (1). We choose this parametrization, resulting in more accurate estimates. Within the SM,  $\lambda_{d,s}$  can be defined within the Wolfenstein parametrization up to  $\mathcal{O}(\lambda^6)$  as

$$\begin{aligned}\lambda_d &= \left(-\lambda + \frac{1}{2}A^2\lambda^5(1 - 2(\rho - i\eta))\right) \left(1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4\right) \\ &= (0.219113 \pm 0.000625)e^{i(-0.000633 \pm 0.000026)}, \\ \lambda_s &= \lambda \left(1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2)\right) \\ &= 0.219045 \pm 0.000624,\end{aligned}\quad (2)$$

$$\begin{aligned}\lambda &= 0.22501 \pm 0.00068, \quad A = 0.826_{-0.015}^{+0.016}, \quad (\bar{\rho}, \bar{\eta}) = \\ &(\rho, \eta) \times (1 - \lambda^2/2 + \dots), \quad \bar{\rho} = 0.1591 \pm 0.0094, \quad \bar{\eta} = \\ &0.3523_{-0.0071}^{+0.0073} \quad [16].\end{aligned}$$

The decay amplitudes in Eq. (1) can be recast as,

$$\begin{aligned}A(D^0 \rightarrow \pi^+\pi^-) &\equiv A^{+-} = \sqrt{2}(t + e^{i\phi}p), \\ A(D^0 \rightarrow \pi^0\pi^0) &\equiv A^{00} = c - e^{i\phi}p, \\ A(D^+ \rightarrow \pi^+\pi^0) &\equiv A^{+0} = (t + c),\end{aligned}\quad (3)$$

where within the SM the weak phase  $\phi \equiv \arg(-\lambda_s/\lambda_d)$  is  $\phi = (0.000633 \pm 0.000026) = (0.0363 \pm 0.0015)^\circ$ , and

$$\begin{aligned}t &= |\lambda_d|(T + E^d + P^{db} + PE^{db} + PA^{db}), \\ c &= |\lambda_d|(C - E^d - P^{db} - PE^{db} - PA^{db}), \\ p &= |\lambda_s|(P^{sb} + PE^{sb} + PA^{sb}),\end{aligned}\quad (4)$$

with  $P^{ij} \equiv P^i - P^j$ ,  $PE^{ij} \equiv PE^i - PE^j$ , and  $PA^{ij} \equiv PA^i - PA^j$ . It may be noted that  $t$ ,  $c$ , and  $p$  are defined to include both the magnitude and any relevant strong phase. Amplitudes for the conjugate modes  $A(\bar{D}^0 \rightarrow \pi^+\pi^-) \equiv \bar{A}^{+-}$ ,  $A(\bar{D}^0 \rightarrow \pi^0\pi^0) \equiv \bar{A}^{00}$ , and  $A(D^- \rightarrow \pi^-\pi^0) \equiv \bar{A}^{+0}$  can be defined analogously with  $\lambda_{d,s}$  being replaced by  $\lambda_{d,s}^*$ .

Note that the isospin triangle relation

$$\frac{1}{\sqrt{2}}A^{+-} + A^{00} = A^{+0}, \quad (5)$$

has been implicitly imposed by the expressions in Eq. (3), with a similar expression for the conjugate modes. The amplitudes for  $D \rightarrow \pi\pi$  decay modes can also be decomposed [17] in terms of the  $I = 0$  and  $I = 2$  amplitudes as follows:

$$\begin{aligned}A^{+-} &= \sqrt{2}(A_2 - A_0), \\ A^{00} &= (2A_2 + A_0), \\ A^{+0} &= 3A_2,\end{aligned}\quad (6)$$

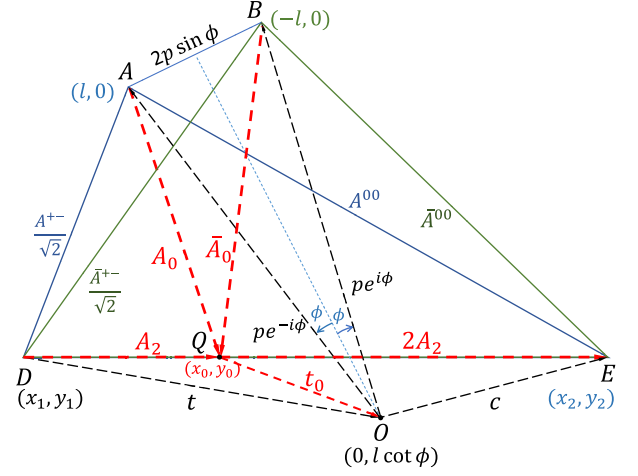


FIG. 1. Isospin triangles for the  $D \rightarrow \pi\pi$  and  $\bar{D} \rightarrow \pi\pi$  modes. The triangles are illustrative and do not correspond to measured observables.

with analogous expressions for the conjugate mode amplitudes. It may be noted that the decay amplitudes of the conjugate modes can be recast after rotating each of the amplitudes by the weak phase of  $\lambda_d$ , which results in  $A^{+0}$  and  $\bar{A}^{+0}$  having the same phase.

In Fig. 1 we show the triangles for  $D \rightarrow \pi\pi$  modes and their respective rotated conjugate modes. The relevant information regarding the amplitudes  $t$ ,  $c$ ,  $p$ , and weak phase  $\phi$  is also encoded in the figure. Note that the triangles are determined purely from experimental data. Given the two triangles, the largest value of  $p$  is obtained for a parametrization where  $\phi$  is the smallest. Since our aim is to determine the size of the penguin contribution to the  $D \rightarrow \pi\pi$  mode we choose the parametrization defined in Eq. (3). It also turns out that the error in  $\phi$  is the smallest among all possible choices of parametrization phases that can be used to describe our modes. This results in a more accurate measurement of  $\phi$ . In Fig. 1 we also show the isospin amplitudes  $A_0$  and  $A_2$  and another interesting parameter  $t_0$  corresponding to the isospin zero component of  $t$ . The amplitude  $p$  contributes only to  $A_0$ , whereas,  $t$ ,  $c$  contribute to both  $A_2$  and  $A_0$ . One can express  $t = t_2 + t_0$  and  $c = c_2 + c_0$ , where the subscripts denote the isospin decomposition of  $t$  and  $c$ . It is then possible to relate the isospin and topological amplitudes as follows:

$$\begin{aligned}A^{+-} &= \sqrt{2}(A_2 - A_0) = \sqrt{2}(t_2 + t_0 + e^{i\phi}p), \\ A^{00} &= (2A_2 + A_0) = (c_2 + c_0 - e^{i\phi}p), \\ A^{+0} &= 3A_2 = (t_2 + c_2 + t_0 + c_0).\end{aligned}\quad (7)$$

Using the above equations one can solve for  $A_2$  and  $A_0$  and easily see that  $c_0 = -t_0$  and  $c_2 = 2t_2$  must hold so that  $A_0$  and  $A_2$  receive contributions only from the relevant isospin amplitudes. This implies that  $A_2 = t_2$  and  $A_0 = -t_0 - e^{i\phi}p$ , such that

$$\begin{aligned}
A^{+-} &= \sqrt{2}(A_2 - A_0) = \sqrt{2}(t_2 + t_0 + e^{i\phi}p), \\
A^{00} &= (2A_2 + A_0) = (2t_2 - t_0 - e^{i\phi}p), \\
A^{+0} &= 3A_2 = 3t_2.
\end{aligned} \tag{8}$$

### III. CONSTRAINING THE AMPLITUDES FROM OBSERVABLES

To determine the amplitudes in terms of the observables we assign a coordinate system such that points  $A$  and  $B$  correspond to  $(l, 0)$  and  $(-l, 0)$  and the points  $D$  and  $E$  are  $(x_1, y_1)$  and  $(x_2, y_2)$ , respectively. The coordinate of  $O$  is then determined to be  $(0, l \cot \phi) \equiv (0, p \cos \phi)$ . The coordinate of  $Q$ , is determined to be

$$(x_0, y_0) = \left( \frac{2x_1 + x_2}{3}, \frac{2y_1 + y_2}{3} \right). \tag{9}$$

The branching fractions ( $\mathcal{B}$ ) and direct  $CP$  asymmetries are defined as,

$$\begin{aligned}
\mathcal{B}(D^0 \rightarrow \pi^+ \pi^-) &\equiv B_{+-} = \frac{1}{2}(|A^{+-}|^2 + |\bar{A}^{+-}|^2), \\
\mathcal{B}(D^0 \rightarrow \pi^0 \pi^0) &\equiv B_{00} = \frac{1}{2}(|A^{00}|^2 + |\bar{A}^{00}|^2), \\
\mathcal{B}(D^+ \rightarrow \pi^+ \pi^0) &\equiv B_{+0} = |A^{+0}|^2 \frac{\tau_{D^+}}{\tau_{D^0}},
\end{aligned} \tag{10}$$

$$a_{+-} \equiv \frac{(|A^{+-}|^2 - |\bar{A}^{+-}|^2)}{(|A^{+-}|^2 + |\bar{A}^{+-}|^2)}, \quad a_{00} \equiv \frac{(|A^{00}|^2 - |\bar{A}^{00}|^2)}{(|A^{00}|^2 + |\bar{A}^{00}|^2)},$$

where the amplitudes are defined by including the lifetime and phase space corrections. Note that we have normalized the amplitude  $A^{+0}$  by lifetime ratios, so that all amplitudes have the same scale. We have used PDG [16] and HFLAV [18] values  $B_{+-} = (1.454 \pm 0.024) \times 10^{-3}$ ,  $B_{00} = (8.26 \pm 0.25) \times 10^{-4}$  and  $B_{+0} = (1.247 \pm 0.033) \times 10^{-3}$ . The Belle collaboration has measured the  $CP$  asymmetry  $a_{00} = (-3 \pm 64) \times 10^{-4}$  [19] and recently LHCb measured the value  $a_{+-} = (23.2 \pm 6.1) \times 10^{-4}$  [2]. The conclusions we will obtain are insensitive to the value of  $a_{00}$ . We have verified that our conclusions remain unaltered even if we use the new less restrictive value of  $a_{00} = (0.30 \pm 0.72 \pm 0.20)\%$  as reported by the recent Belle II paper [20] instead. Note that both the Belle and Belle II measurements of  $a_{00}$  are consistent with  $CP$  symmetry in  $D^0 \rightarrow \pi^0 \pi^0$  decays.

In terms of the coordinates the observables are derived to be

$$\begin{aligned}
B_{+-} &= 2(x_1^2 + y_1^2) + 2l^2, & B_{00} &= (x_2^2 + y_2^2) + l^2, \\
B_{+0} &= (x_1^2 + y_1^2) + (x_2^2 + y_2^2) - 2x_1x_2 - 2y_1y_2, \\
a_{+-} &= -\frac{4x_1p \sin \phi}{B_{+-}}, & a_{00} &= -\frac{2x_2p \sin \phi}{B_{00}}.
\end{aligned} \tag{11}$$

The five parameters  $x_1$ ,  $y_1$ ,  $x_2$ ,  $y_2$  and  $l$  that define the coordinates can be solved in terms of the five observables as follows:

$$\begin{aligned}
x_1 &= -\frac{a_{+-}B_{+-}}{4l}, & x_2 &= -\frac{a_{00}B_{00}}{2l}, \\
y_1 &= \pm \frac{\sqrt{a_{+-}^2 B_{+-}^2 + 8B_{+-}l^2 + 16l^4}}{4l}, \\
y_2 &= \pm \frac{\sqrt{a_{00}^2 B_{00}^2 + 4B_{00}l^2 + 4l^4}}{2l},
\end{aligned} \tag{12}$$

and  $l^2$  can now be solved up to a quadratic ambiguity using the expression for  $B_{+0}$  in Eq. (11). The amplitudes and strong phases of  $t$ ,  $c$ ,  $p$ ,  $A_2$ ,  $A_0$ ,  $\bar{A}_0$ , and  $t_0$  can thus be determined purely in terms of experimental data. Note that the solutions have four ambiguities that correspond to the relative orientation of the triangles, a fact that is well known. We show in Fig. 2 the relative sizes of  $t$ ,  $c$ , and  $p$  contributions (normalized to  $A^{+0}$ ) using central values from data. The two panels depicted in Fig. 2 correspond to those two orientations of isospin triangles (out of four possible orientations) that have smaller penguin contributions. The penguin values corresponding to the other two orientations are an order of magnitude larger than tree contribution and are hence not considered. Note that very large penguin contributions are indicated by data.

We write the amplitudes in terms of complex coordinate system as follows:

$$t = -x_1 + i(l \cot \phi - y_1), \tag{13}$$

$$c = x_2 + i(y_2 - l \cot \phi), \tag{14}$$

$$p = -il \csc \phi, \tag{15}$$

$$A_0 = (2x_1 + x_2)/3 - l + i(2y_1 + y_2)/3, \tag{16}$$

$$\bar{A}_0 = (2x_1 + x_2)/3 + l + i(2y_1 + y_2)/3, \tag{17}$$

$$A_2 = (-x_1 + x_2)/3 + i(-y_1 + y_2)/3, \tag{18}$$

$$t_0 = (2x_1 + x_2)/3 + i(l \cot \phi - (2y_1 + y_2)/3). \tag{19}$$

Using complex coordinates to estimate amplitudes enables evaluation of both magnitudes and relative phase with ease. The amplitudes of  $t$ ,  $c$ ,  $p$ , and  $t_0$  as well as their phases are evaluated using a simulated dataset of  $B_{+-}$ ,  $B_{00}$ ,  $B_{+0}$ ,  $a_{+-}$ ,  $a_{00}$ , and  $\phi$  with a million points.

In Fig. 3 the obtained normalized amplitudes  $p_{+-} \equiv p/\sqrt{B_{+-}}$  versus  $t_{+-} \equiv t/\sqrt{B_{+-}}$  are shown. The central values of the fit for  $(p_{+-}, t_{+-})$  are (4.74, 4.19) and (4.80, 5.43), indicating that the observed  $p_{+-}$  is indeed large. If we make the choice to ignore the small  $CP$  violating asymmetry  $a_{+-}$  relative to  $A^{+-}$  and  $\bar{A}^{+-}$ , we can directly compare with the estimate  $p_{+-} = (0.09 \pm 0.01)$  made in Ref. [21], using light cone sum rule techniques. Similar

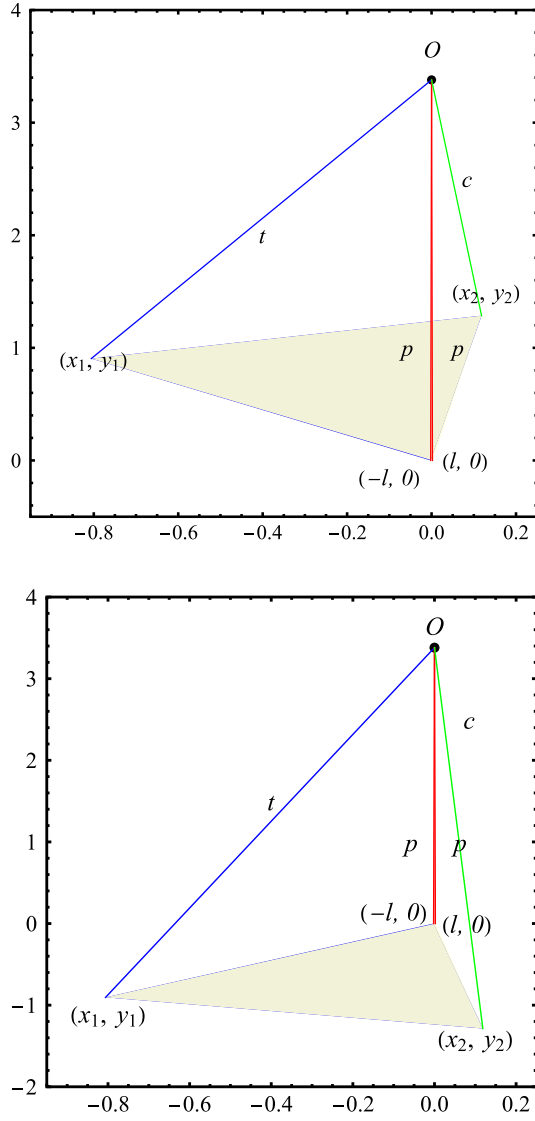


FIG. 2. Two of the four isospin triangle orientations for the  $D \rightarrow \pi\pi$  and  $\bar{D} \rightarrow \pi\pi$  modes representing the central values of the measured observables. Note that the triangles corresponding to the mode and conjugate mode almost overlap. The triangle for  $\bar{D} \rightarrow \pi\pi$  in dark blue is barely visible under the translucent yellow triangle corresponding to  $D \rightarrow \pi\pi$ . The triangles shown are chosen to be the ones with smaller penguin contributions. All amplitudes are normalized to  $A^{+0}$ .

estimates for  $p_{+-}$  are obtained in Ref. [22]. Since the momentum flowing through the penguin loop is of order 1 GeV or larger, a perturbative calculation of this loop is not unreasonable. Convincing arguments on the reliability of these estimates have been made in Ref. [23]. Note, that these are estimates of penguin  $p$  compared to  $\sqrt{B^{+-}}$ , and it would be unreasonable to expect a much larger value. It is seen from Fig. 3 that a 10% value of  $p_{+-}$  is  $\sim 3.4\sigma$  away from the estimated value indicating an unacceptably large penguin contribution. We note that  $a_{+-}$  is nonzero at  $3.8\sigma$  and our results are consistent with this measurement.

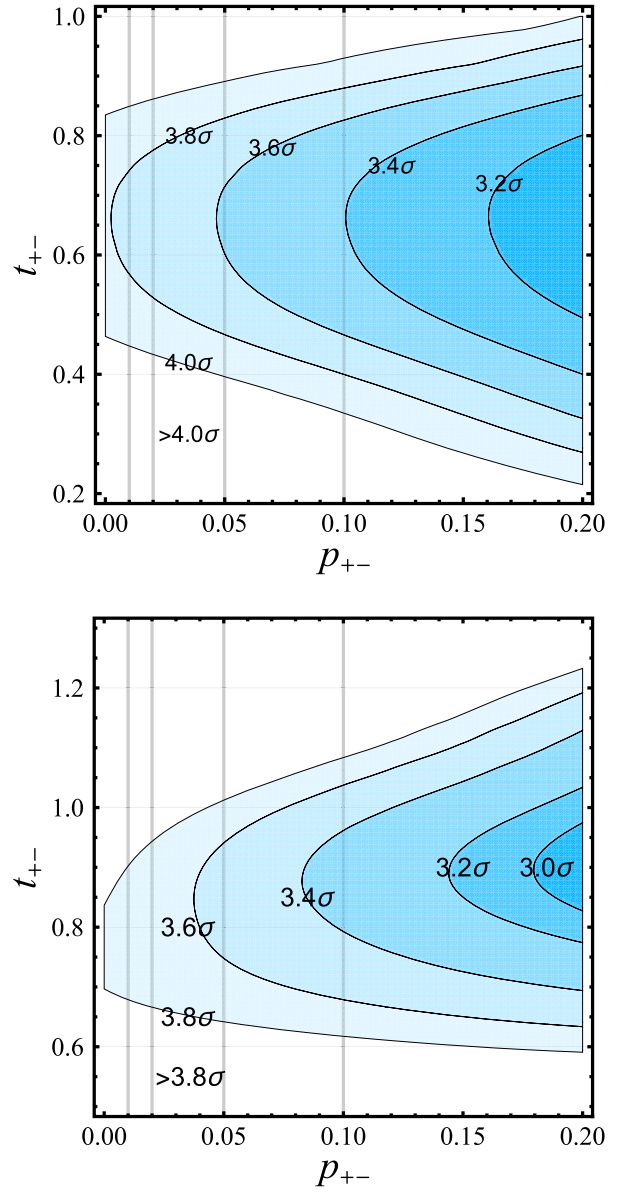


FIG. 3. Amplitudes  $p_{+-}$  versus  $t_{+-}$  defined by normalizing  $p$  and  $t$  by  $\sqrt{B_{+-}}$ , corresponding to the two triangle orientations with smaller values of  $p$ . The central values of the fit for  $(p_{+-}, t_{+-})$  for the top and bottom panels are (4.74, 4.19) and (4.80, 5.43), respectively. It is seen that a 10% value of  $p_{+-}$  is at least  $3\sigma$  away from the estimated value, indicating an unacceptably large penguin contribution.

Even with a simulation of a million points no point below  $p_{+-} < 0.15$  was found limiting our accuracy at vanishing  $p_{+-}$  by interpolation errors. It is worth noting that  $p_{+-} \sim 1$  at  $1\sigma$ ,  $p_{+-} \sim 0.6$  at  $2\sigma$ ,  $p_{+-} \sim 0.2$  at  $3\sigma$ . Thus the data indicates a very large penguin contribution. It may be noted that since the parametrization of Ref. [15] and our paper differ, the expressions and estimates of the penguin also differ. However, our estimates for  $p_{+-}$  are in broad agreement with those of Ref. [15]. For completeness we have also plotted the  $p/t$  values as a polar plot in Fig. 4.

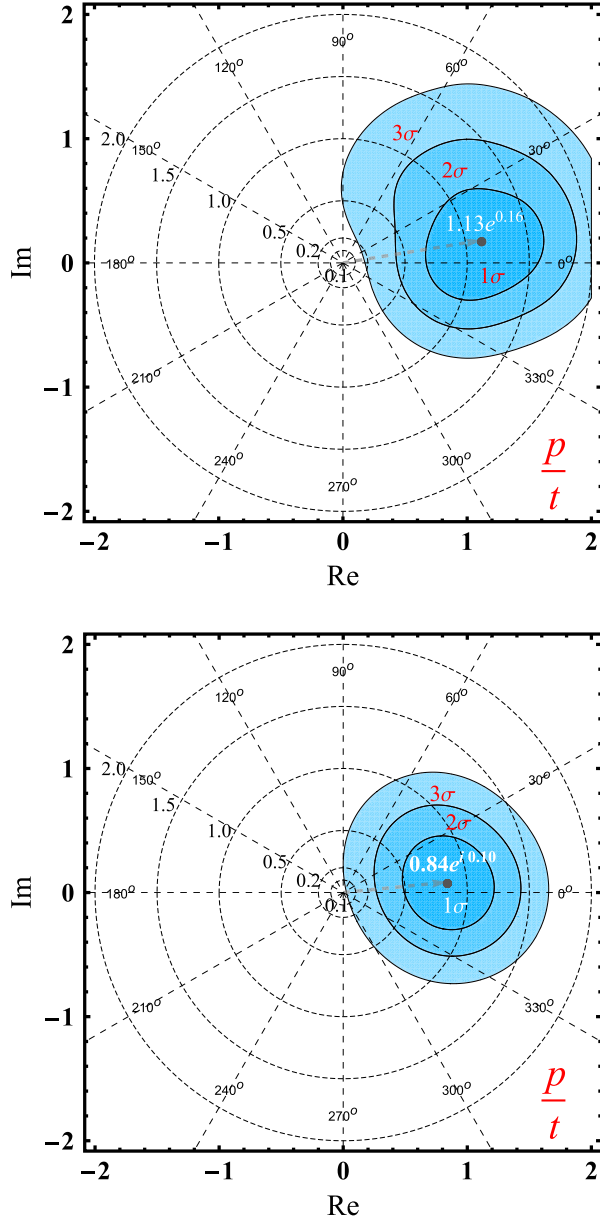


FIG. 4.  $p/t$  corresponding to the two triangle orientations smaller  $p$  are shown corresponding to the two orientations. Note that small values of  $p/t \sim 0.1$  are possible only at  $\sim 3\sigma$ .

The  $p/t$  values once again indicate that  $p$  is indeed large and small values of  $p/t \sim 0.1$  are possible only at  $\sim 3\sigma$ .

In Fig. 5, the values of the ratio of isospin amplitudes  $A_0/A_2$  are shown. The orange contours referred to as SM assume that  $a_{+-} = 0$  at  $\mathcal{O}(\lambda^4)$  as expected in the SM. This ensures that  $A_0$  and  $\bar{A}_0$  amplitudes are identical. In the limit  $a_{+-} = 0$  the isospin ratio is the same for mode and conjugate mode. The blue contours correspond to the observed  $a_{+-}$ . There is another solution with the same magnitude but a phase around  $3\pi/2$ , which is not shown, since it is not a true solution. The observed  $a_{+-}$  is indeed quite small, and does not result in a visible difference in the

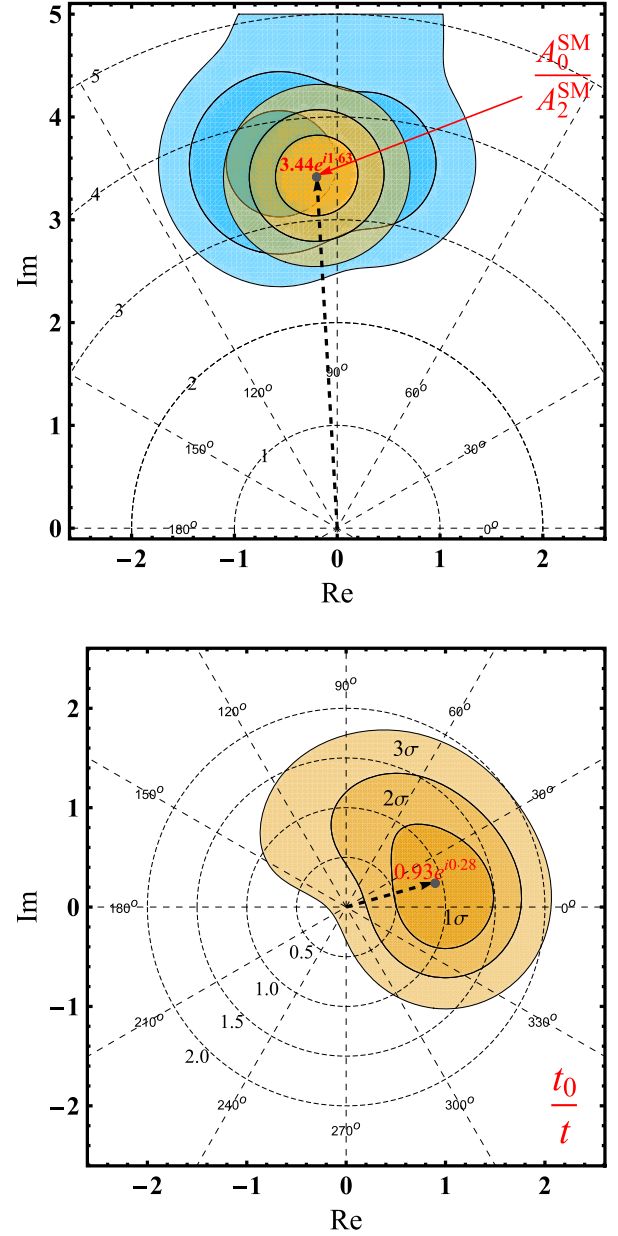


FIG. 5. Values of  $A_0/A_2$  are shown in the top panel. The orange contours correspond to the SM and assume  $a_{+-} = 0$ . The blue contours are for the observed  $a_{+-}$ .  $|A_0/A_2| \gtrsim 2$  at  $3\sigma$  indicates an inherent  $\Delta I = 1/2$  enhancement in  $D \rightarrow \pi\pi$  mode. The panel below shows contours for the ratio of  $t_0/t$  indicating that  $t_0$  can be almost as large as  $t$ .

blue contours between  $D \rightarrow \pi\pi$  and  $\bar{D} \rightarrow \pi\pi$ . Hence, true solutions cannot have significant deviation from SM isospin values. The value  $|A_0/A_2| \gtrsim 2$  at  $3\sigma$  irrespective of the value of the observed  $CP$  violation, indicates an inherent  $\Delta I = 1/2$  enhancement in  $D \rightarrow \pi\pi$  modes in agreement with Ref. [8]. The bottom panel of Fig. 5 shows contours for the ratio of  $t_0/t$ , which indicates that  $t_0$  can be almost as large as  $t$ . This indicates that the tree topology also plays a role in  $\Delta I = 1/2$  enhancement.

#### IV. COMMENTS ON CONTRIBUTIONS FROM FINAL STATE INTERACTIONS

While arguments [23] have been made on the broad reliability of the estimates for  $p_{+-}$ , it may be noted that explicit final state interactions might alter the size of the penguin contributions. We therefore examine in a model-independent way effects of final state interactions and draw general conclusions. In two-body scattering of  $n$ -coupled channels a convenient parametrization of a unitary S matrix is in terms of the K matrix,

$$S(s) = (1 - iK(s))^{-1}(1 + iK(s)), \quad (20)$$

where  $K(s)$  is a  $n \times n$  Hermitian matrix and  $s$  is set equal to  $m_D^2$  for  $D$  decays. A set of  $n$  ununitarized weak decay amplitudes  $A^O(s)$  are unitarized into a set  $A^U(s)$  through multichannel final state interactions via a matrix equation,

$$A^U(s) = (1 - iK(s))^{-1}A^O(s). \quad (21)$$

Since strong interactions conserve isospin and  $G$  parity, Eq. (21) has to be written for each isospin and  $G$  parity (where it can be defined). Let  $U$  be a unitary matrix that diagonalizes the Hermitian matrix  $K$  with real eigenvalues  $\lambda_i (i = 1, \dots, n)$ . Then,

$$\sum_i^n A_i^{U\dagger} A_i^U = \sum_i^n (UA^O)^\dagger_i (1 + \lambda_i^2)^{-1} (UA^O)_i. \quad (22)$$

Since  $\lambda_i$  are real this results in the inequality

$$\sum_i^n |A_i^U|^2 \leq \sum_i^n |A_i^O|^2. \quad (23)$$

This inequality implies that the sum of transition probabilities after interchannel mixing cannot exceed the sum before mixing. Clearly all channels contributing to any specific isospin channel can rescatter among each other. The channels  $D^0 \rightarrow \pi\pi$ ,  $D^0 \rightarrow KK$  as well as other channels can rescatter in the isospin zero channel resulting in the huge enhancement in the  $I = 0$ ,  $D \rightarrow \pi\pi$  amplitudes. It has to be kept in mind however, that since the penguin amplitude  $p$  has a weak phase it cannot receive contributions from any other channel that does not have the same weak phase. Equation (21) must therefore apply separately to the  $p$  amplitudes, implying the constraint,

$$\sum_{i=\pi\pi, KK, \dots} |p_i^U|^2 \leq \sum_{i=\pi\pi, KK, \dots} |p_i^O|^2, \quad (24)$$

where,  $\dots$  represent any other channel contributing to  $I = 0$ . If the anomalously large penguin contributions arises from within the SM, one must generate large penguin contributions in at least one of the channels that rescatters to produce the observed asymmetries. This is difficult to accommodate within the SM. If  $D \rightarrow KK$  penguin amplitude rescatters to  $D \rightarrow \pi\pi$  such that  $D \rightarrow \pi\pi$  penguin contributions become

very large it would result in  $D \rightarrow KK$  penguin amplitude becoming very small and consequently a vanishing  $CP$  asymmetry. For any other channel [8] contributing to  $I = 0$ , the amplitude of the penguin must be established to be large theoretically. We must therefore conclude that attempts [12] to explain large penguin contributions and large  $CP$  asymmetry, may inherently violate unitarity in rescattering and should be viewed with skepticism.

In our analysis we have ignored electroweak penguins ( $p_{EW}$ ), which is justifiable within SM, but they can be sizable in the presence of NP. In the absence of  $p_{EW}$  the measurements of branching ratios and the two  $CP$  asymmetries provide a complete determination of all parameters that describe the decay. However, this is not true in the presence of  $p_{EW}$  where one parameter remains unsolved [24] without an additional input. The direct  $CP$  asymmetry measured in the  $D^+ \rightarrow \pi^+ \pi^0$  mode is  $a_{+0} = (4.0 \pm 8.0) \times 10^{-3}$ . Hence, the two triangles will not in principle share a common base, since  $A^{+0}$  and  $\bar{A}^{+0}$  can differ by as much as 0.012 at  $1\sigma$  confidence.<sup>1</sup> However, this difference is small given the estimate of penguin contributions shown in Fig. 3, hence to a very good approximation we can assume that  $p$  includes  $p_{EW}$  contributions in all our estimations. We note that even a small  $p_{EW} = 0.005$  and new physics weak phase of  $\pi/6$  can produce an angle between  $A^{+0}$  and  $\bar{A}^{+0}$  of 8.2 deg. However, this would not affect our analysis.

#### V. PROBING THE SIZE OF GENERIC NP CONTRIBUTION

We next examine if NP can resolve the puzzle of large penguin and the observed direct  $CP$  asymmetry. This has been a subject of intense study and it is generally believed that a variety [25–30] of NP scenarios can contribute to the observed anomaly. Let us assume that NP also contributes to the decay mode via the  $p + p_{EW}$  topologies. We parametrize such NP by  $Ne^{i\phi_{NP}}$ , where  $\phi_{NP}$  is the weak phase of NP and  $N$  includes the magnitude and any relevant strong phase. This NP amplitude can easily be rewritten such that  $Ne^{i\phi_{NP}} \equiv N_1 + N_2 e^{i\phi}$ . NP affects the amplitudes such that

$$\begin{aligned} A^{+-} &\rightarrow \sqrt{2}(t + e^{i\phi}p + Ne^{i\phi_{NP}}) \\ &= \sqrt{2}((t + N_1) + (p + N_2)e^{i\phi}), \\ A^{00} &\rightarrow c - (e^{i\phi}p + Ne^{i\phi_{NP}}) \\ &= (c - N_1) - (p + N_2)e^{i\phi}, \\ A^{+0} &= t + c. \end{aligned} \quad (25)$$

Note that the amplitudes with NP contributions still obey the isospin triangle relation of Eq. (5).

<sup>1</sup>This is the precision with which the presence of NP should explain the observation.

It is easy to see that  $N_1$  and  $N_2$  can be expressed in terms of  $N$ ,  $\phi_{NP}$  and  $\phi$  to be

$$\begin{aligned} N_1 &= N \cos \phi_{NP} - N \cot \phi \sin \phi_{NP}, \\ N_2 &= N \csc \phi \sin \phi_{NP}. \end{aligned} \quad (26)$$

Clearly taking into account NP results in  $t$  being modified by  $N_1$  and  $p$  by  $N_2$ , so that the weak phases are aligned. This implies that both  $t$  and  $p$  are altered by NP. The observed value of the penguin would include contributions from both SM penguin and NP. Hence,

$$p_{+-}^{\text{obs}} = (p_{+-}^{\text{SM}} + N_{+-} e^{i\delta_{NP}} \csc \phi \sin \phi_{NP}) e^{i\phi}, \quad (27)$$

where all amplitudes are normalized by  $|A_{+-}|$ ;  $p_{+-}^{\text{obs}} \equiv p_{+-}^{\text{obs}}/|A_{+-}|$ ,  $p_{+-}^{\text{SM}} \equiv p_{+-}^{\text{SM}}/|A_{+-}|$ ,  $N_{+-} = N/|A_{+-}|$ , and  $\delta_{NP}$  is the difference between the strong phase of  $N$  and  $p$ . Without loss of generality, the domain of  $\delta_{NP}$  is chosen such that the amplitudes  $p_{+-}^{\text{SM}}$  and  $N_{+-}$  are always positive. We hence have

$$\begin{aligned} (p_{+-}^{\text{obs}})^2 &= (p_{+-}^{\text{SM}})^2 + 2N_{+-} p_{+-}^{\text{SM}} \cos \delta_{NP} \csc \phi \sin \phi_{NP} \\ &\quad + N_{+-}^2 \csc^2 \phi \sin^2 \phi_{NP}. \end{aligned} \quad (28)$$

Note that  $p_{+-}^{\text{obs}}$  has already been obtained from data and  $p_{+-}^{\text{SM}}$  has been estimated in Ref. [21]. We can use Eq. (28) to solve for  $N_{+-}$  in terms of  $\phi_{NP}$  and  $\delta_{NP}$ , since  $\phi$  is known with errors. The simulated dataset used to obtain  $p_{+-}^{\text{obs}}$  is extended to include randomly generated Gaussian values of  $p_{+-}^{\text{SM}}$ , whereas  $\delta_{NP}$  and  $\phi_{NP}$  are randomly varied in the range  $(0, \pi)$ . The range for  $N_{+-}$  as function of  $\phi_{NP}$  is shown in Fig. 6. We find that large  $CP$  violation observed and resulting very large penguin can arise due to a  $N_{+-}$  with a larger weak phase, even with  $N_{+-}$  smaller than  $p_{+-}^{\text{SM}}$ , indicating that NP could result in the larger than expected  $CP$  asymmetry. The reader may wonder how a small NP amplitude can generate a large  $CP$  asymmetry. We emphasize that even a small  $\phi_{NP} = \pi/6$  results in an enhancement of  $\sim 800$  in  $a_{+-}$  compared to that from SM phase  $\phi$ , resulting in a much larger asymmetry even with a smaller  $N_{+-}$  compared to SM penguin  $p_{+-}$ .

While the case for NP looks robust, it may be noted that it depends largely on the measured value of  $a_{+-}^{KK} = (7.7 \pm 5.7) \times 10^{-4}$  [2]. Let us consider a scenario, in future measurements, where  $a_{+-}^{KK} = (-7.6 \pm 2.3) \times 10^{-4}$  indicating a shift by  $2.5\sigma$ . The robust measurement of the difference between direct  $CP$  asymmetries,  $\Delta a_{CP} = a_{+-}^{KK} - a_{+-}^{\pi\pi} = (-15.4 \pm 2.9) \times 10^{-4}$  [1], would then imply  $a_{+-}^{\pi\pi} \equiv a_{+-} = (7.8 \pm 2.6) \times 10^{-4}$ , a downward shift of  $2.5\sigma$ . The extracted value of  $p_{+-}$  then reduces to within  $\sim 1\sigma$  of the estimate  $p_{+-} = 0.09$  [21], indicating consistency with

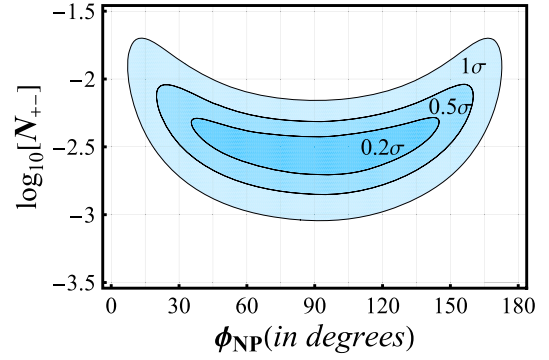


FIG. 6. Contours for  $\log_{10}[N_{+-}]$  versus  $\phi_{NP}$  (in degrees). It is easy to see that reasonable value of  $N_{+-}$  smaller than  $p_{+-}^{\text{SM}}$  can result in large observed penguin and  $CP$  violation.

SM and no evidence for enhanced penguins. Interestingly, in such a case one would also have an almost exact  $U$ -spin symmetry.

## VI. CONCLUSION

The evidence of larger than expected  $CP$  violation in singly Cabibbo suppressed D meson decay  $D \rightarrow \pi^+ \pi^-$  has been keenly debated in literature with the view of ascertaining if the observed  $CP$  violating asymmetry is indeed a signal of NP. We avoid the difficulties in arriving at a convincing conclusion by providing an estimate of the size of the penguin contributions directly from experimental data. Second, we examine final state interactions in a model-independent way using only unitarity, in contrast to previous approaches. The model-independent constraints derived indicate that attempts [12] to explain large penguin contributions and large  $CP$  asymmetry, may inherently violate unitarity in rescattering and should be viewed with skepticism. We conclude that the measurement of  $CP$  violation in  $D \rightarrow \pi^+ \pi^-$ ,  $a_{+-} = (23.2 \pm 6.1) \times 10^{-4}$ , provides evidence of NP at about  $3\sigma$  significance. In a model-independent approach we show how a very small contribution from NP with a large weak phase alleviates the problem. If this evidence is indeed a signal of NP at the level observed in  $D \rightarrow \pi^+ \pi^-$ , large  $CP$  violation must show up in other modes and experiments. Searches for other signals of  $CP$  violation in D decays are needed to substantiate the evidence of NP.

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## DATA AVAILABILITY

No data were created or analyzed in this study.

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