

Electroweak phase transition and bubble wall velocity in local thermal equilibrium

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The dynamics of the electroweak phase transition in the early Universe has profound implications for cosmology and particle physics. We systematically study the steady-state dynamics of bubble walls in scenarios where the transition is first order within three representative beyond the Standard Model frameworks, characterized by the presence of an additional scalar in different electroweak representations. Focusing on the local thermal equilibrium regime, we numerically solve the coupled scalar and hydrodynamic equations to extract key properties of the phase transition front: the wall velocity, width, plasma, and field profiles. We find a near-universal behavior across models when expressed in terms of thermodynamic quantities, which can be captured by simple fitting functions, useful for phenomenological applications. These results also provide an upper bound on the bubble velocity and represent the first necessary step for the full inclusion of out-of-equilibrium effects. As an application, we determine the gravitational wave signal and the amount of baryon asymmetry generated by the transition in local thermal equilibrium.

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I. INTRODUCTION

It has been known for a long time that interactions with a plasma at high temperature tend to stabilize scalar fields around the origin of field space [1–3], raising the question of how the electroweak (EW) vacuum was reached in the evolution of the Universe from the initial hot state to the present day. It was shown in the late 1990s that in the Standard Model (SM) the transition toward the EW vacuum is a smooth crossover [4–6], and does not generate any particularly interesting observational signature. This conclusion can be overturned in beyond the SM (BSM) scenarios with enlarged scalar sectors, where a first-order phase transition (PhT) can be easily accommodated.

If a first-order EW phase transition (EWPhT) truly occurred in the early Universe, it is expected to have produced

a variety of cosmological signatures, such as a matter-antimatter asymmetry, primordial black holes, topological defects, dark matter remnants, and, most notably, a stochastic background of gravitational waves. The recent observation of the latter in astrophysical systems [7,8] has fueled renewed interest in the subject. The possibility to confront theories of the EWPhT with cosmological observables would make BSM scenarios falsifiable, providing an extraordinarily rich arena to test new physics complementary to collider searches. Given a specific BSM model, an accurate description of the PhT dynamics is fundamental to give a quantitative assessment of its impact on cosmological observables. In particular, the expansion speed of the PhT front and its width are two fundamental quantities that characterize, for instance, the spectrum of emitted gravitational waves and the amount of matter asymmetry generated in scenarios of EW baryogenesis (see [9,10] for recent reviews of cosmological phase transitions).

A first-order PhT proceeds as follows: initially, scalar fields reside in a homogeneous configuration corresponding to the trivial vacuum, which is favored at high temperatures. As the Universe cools below the critical temperature T_c , where the minima of the potential become degenerate, new energetically deeper minima emerge. When the probability to decay per Hubble volume exceeds the Hubble expansion rate, the transition proceeds through the nucleation and

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growth of bubbles of true vacuum, which eventually collide, merge, and fill up the whole space, leaving as an imprint the various cosmological signatures mentioned above.

At a more fundamental level, bubbles correspond to nontrivial scalar field configurations that solve the scalar equations of motion with spherical symmetry. These solutions smoothly interpolate between the true vacuum (the deeper minimum of the potential) and the false vacuum (the higher minimum). When the potential difference between the two vacua is small, this transition occurs rapidly over a narrow spatial region. In this regime, the solution can be effectively interpreted as a spherical bubble of true vacuum, separated from the surrounding metastable phase by a domain wall (DW).

The potential difference between the phases generates an outward pressure on the wall that accelerates it. In turn, the presence of a moving front induces a backreaction of the false vacuum background giving rise to a friction that hinders the acceleration and can eventually lead to the onset of a stable stationary solution, where the wall reaches a terminal velocity v_w . Given the importance of this quantity in the determination of gravitational wave signals, a huge effort has been put forward in recent years to develop methods for its calculation [11–78]. This requires one to simultaneously solve the equations of motion for the scalar fields, the continuity equations for the plasma, and the Boltzmann equation for the distribution functions of the particle species, making it a highly nontrivial task.

The calculation of the bubble wall velocity, and more generally the determination of a stationary solution, has a long history, with the first full approach dating back to the seminal works of Moore and Prokopec [11,12], where the computation was performed in the SM. In these works, the distribution functions are determined using a fluid ansatz and solving the (integro-differential) Boltzmann equation taking suitable weighted moments that allow one to express it as a system of ordinary differential equations.

In [48,56,64] significant progress was made in the development of a fully quantitative numerical method that allows one to directly tackle the Boltzmann equation without resorting to any ansatz or arbitrary choice of decomposition. Technically, the collision integral represents the most challenging part of the computation. An iterative procedure was then devised, where, starting from the solution found in local thermal equilibrium (LTE), the out-of-equilibrium distribution functions are determined at each step inserting the solution found at the previous step in the source. At a more technical level, the great simplification allowed us to obtain for the first time a full solution to the Boltzmann equation, whereas previous works only dealt with a linearized (in the deviation from equilibrium of the distribution function) version of it. Important insight on the structure of the equation was also gained.

In this work, we begin a systematic analysis of the EWPhT dynamics in BSM models. In [48,56,64], the

numerical algorithm was tested on two benchmark points in the $\mathbb{Z}_2$ -symmetric real scalar extension of the Standard Model. As noted there, the efficiency reached with the various simplifications of the collision integral opened the door to the possibility of performing scans of the parameter space of particle physics models. As a first step toward this goal, we present here the results for the bubble wall velocity and all the parameters entering in the steady-state dynamics obtained in local thermal equilibrium of three BSM models, namely the $\mathbb{Z}_2$ -symmetric real singlet extension of the SM (SSM), the $\mathbb{Z}_2$ -symmetric real triplet extension of the SM (RTSM), involving a $SU(2)_L$ triplet with vanishing hypercharge, and the inert doublet model (IDM), featuring a second Higgs doublet that is odd under a $\mathbb{Z}_2$ symmetry. All three models share a common feature: they allow for a two-step EW symmetry-breaking pattern in which the Universe is first transitioning into a phase where a BSM scalar acquires a vacuum expectation value (VEV), followed by a first-order transition to the EW vacuum. Such scenarios generically give rise to tree-level potential barriers between vacua, circumventing the limitations of perturbation theory at finite temperature and allowing for a strong first-order EWPhT.

Our primary goal is to calculate the steady-state wall velocity v_w and the (field and fluid) profiles in LTE across the relevant parameter space of these models. This represents the first step in a broader program aimed at eventually incorporating out-of-equilibrium dynamics, using iterative techniques, and will be used in subsequent works as the starting point for the full evaluation of out-of-equilibrium effects in the parameter space of BSM models.

In the following analysis, we do not impose phenomenological constraints, such as those arising from collider searches, EW precision observables, or dark matter relic density and direct detection bounds, since our focus is on the accurate description of the wall dynamics. This choice allows us to explore the behavior of the solutions and the structure of the equations governing them as functions of the parameters of the models, independently of external constraints. A detailed phenomenological analysis, incorporating these constraints and assessing the interplay between cosmological and experimental signatures, is beyond the scope of this work and can be addressed in future studies.

It is also important to emphasize that the wall velocities computed within the LTE framework represent an upper bound on the actual wall velocities expected in a complete treatment. In realistic settings, deviations from equilibrium near the wall, captured by solving the Boltzmann equations, introduce additional sources of friction. These out-of-equilibrium effects tend to further slow down the motion of the wall, reducing the velocity relative to the LTE estimate. Therefore, the LTE results obtained here provide a crucial reference point, both for identifying the qualitative behavior of solutions and for initializing more complete iterative procedures that include effects beyond (local) equilibrium.

In this paper, we solve the coupled scalar equations of motion and hydrodynamic conservation laws numerically. The resulting wall velocities, widths, and field displacements are obtained by imposing consistency conditions on the pressures, as well as on their gradients across the wall. Special attention is given to the classification of the hydrodynamic regimes (deflagration, detonation, hybrid) and their realization within the LTE framework. We find that stationary solutions exist throughout the parameter space corresponding to two-step transitions, except in regions where the wall experiences runaway behavior. Actually, in the $v_w \rightarrow 1$ limit a friction is known to arise that stops the bubble wall acceleration [16,28,76,77], and these solutions become ultrarelativistic detonations. This effect can not be captured in LTE. Moreover, our results exhibit an approximate model-independence in the behavior of v_w when expressed in terms of thermodynamic quantities characterizing the transition. Our analysis shows the presence of common features across BSM scenarios. To make this result of practical use, we have captured this universality in the form of simple fitting functions, which can be readily employed in independent phenomenological studies, such as gravitational wave signal modeling, baryogenesis estimates, or model parameter scans, without going through the full computationally demanding numerical simulation, thus greatly facilitating phenomenological analyses. As a preliminary application, we determine the spectrum of gravitational waves and the baryon asymmetry generated by the EWPhT in the SSM augmented with a CP -violating wall-fermion interaction.

The structure of the paper is as follows: In Sec. II, we review the hydrodynamic description of the plasma, derive the continuity equations and the equations of motion for the scalar fields, and describe the numerical algorithm we used to solve them. Section III details the BSM models under consideration, including renormalization and thermal resummation schemes. Section IV provides analytic insight into the structure of the constraint equations, aiding in the interpretation of numerical results. Section V presents our numerical results for the wall velocity and associated parameters in the three models. We apply our results to the gravitational wave spectrum and EW baryogenesis in Sec. VI. We conclude in Sec. VII with a discussion of the implications and future directions. In the Appendix we provide the analytic expressions of the thermal masses employed for the considered BSM scenarios.

II. HYDRODYNAMICS

The determination of the wall dynamics relies, among other things, on the hydrodynamic description of the plasma interacting with the DW. The expansion of the bubble drives the plasma out of equilibrium, the deviations from it being larger in regions close to the wall and for particles (more strongly) coupled to it. The plasma can thus be conveniently described as a mixture of two components:

species directly coupled to the DW and out of equilibrium, as the top quark, and background particles, assumed to be in local equilibrium. The interaction between the plasma and the DW gives rise to position-dependent temperature and velocity plasma profiles, which must be accurately described to account for the friction exerted on the moving wall, and eventually determine the wall terminal velocity.

A. Hydrodynamic equations

The dynamics of a relativistic plasma can be described using the stress-energy tensor, which can be written in terms of the distribution function $f_i(k, x) = f_i^{(0)}(k, x) + \delta f_i(k, x)$ of the particle species as

$$T_{\mu\nu}^{\text{pl}} = \sum_i \int \frac{d^3k}{(2\pi)^3 2E_k} k_\mu k_\nu f_i(k, x) = T_{\mu\nu}^{\text{lte}} + T_{\mu\nu}^{\text{out}}, \quad (1)$$

where the sum extends to all the species in the plasma and $T_{\mu\nu}^{\text{lte}}$ describes the local thermal equilibrium contribution given by the position-dependent equilibrium distributions $f_i^{(0)}(k, x)$ while $T_{\mu\nu}^{\text{out}}$ is the out-of-equilibrium part determined by $\delta f_i(k, x)$, with the largest contribution coming from the species that are more strongly coupled to the DW, and in a region close to it.

In the following, for the plasma in local equilibrium $T_{\mu\nu}^{\text{lte}}$ simplifies to

$$T_{\mu\nu}^{\text{lte}} = w u_\mu u_\nu - g_{\mu\nu} p, \quad (2)$$

with $w = e + p = T \partial_T p$ the enthalpy, $u_\mu = \gamma(1, \vec{v}_p)$ the plasma four-velocity, p the pressure, and e the energy density.

The total stress-energy tensor of the system (the plasma and the DW) is thus given by the sum

$$T_{\mu\nu} = T_{\mu\nu}^{\text{pl}} + T_{\mu\nu}^{\phi}, \quad (3)$$

where $T_{\mu\nu}^{\phi}$ stands for the stress-energy tensor of the scalar fields driving the phase transition,

$$T_{\mu\nu}^{\phi} = \sum_j \left[\partial_\mu \phi_j \partial_\nu \phi_j - g_{\mu\nu} \frac{(\partial \phi_j)^2}{2} \right] + g_{\mu\nu} V_{T=0}, \quad (4)$$

with $V_{T=0} \equiv V(\phi_1, \dots, \phi_n, T=0)$ the zero-temperature effective potential.

The hydrodynamic equations are obtained from the conservation of $T_{\mu\nu}$. Neglecting the expansion of the Universe, and taking a planar approximation for the wall, the conservation equations can be simply written in the domain wall reference frame as

$$\partial_z T^{z0} = \partial_z T^{zz} = 0, \quad (5)$$

where the z axis is chosen to be aligned to the propagation direction of the wall. Integrating Eq. (5) we obtain

$$T^{30} \equiv w\gamma^2 v_p + T_{\text{out}}^{30} = c_1, \quad (6)$$

$$T^{33} \equiv \sum_{i=1}^n \frac{(\partial_z \phi_i)^2}{2} - V(\phi_1, \dots, \phi_n, T) + w\gamma^2 v_p^2 + T_{\text{out}}^{33} = c_2, \quad (7)$$

where $V(\phi_1, \dots, \phi_n, T) \equiv V_{T=0} - p$ is the finite temperature effective potential. The constants c_1 and c_2 are determined by the boundary conditions on the plasma velocity $v_{\pm}$, the plasma temperature $T_{\pm}$, and the scalar fields VEVs $\phi_i^{\pm}$, which can be imposed either inside or outside the bubble (to be consistent with the literature, we use the $+$ sign for quantities in front of the wall and the $-$ sign for quantities behind it). They are more easily determined far away from the wall at $z \rightarrow \pm\infty$ where $\delta f \rightarrow 0$, and depend on the class of the hydrodynamic solution, as we will briefly review below.

Integrating the conservation equations (5) across the DW, v_p and T in front and behind the wall are found to be related by the matching equations

$$v_+ v_- = \frac{p_+ - p_-}{e_+ - e_-}, \quad \frac{v_+}{v_-} = \frac{e_- + p_+}{e_+ + p_-}, \quad (8)$$

where p was redefined to include the contribution from the zero-temperature potential $p \rightarrow p - V_{T=0}$, and e is the total energy density associated to it, $e \rightarrow e + V_{T=0} = T \partial_T p - p$. The scalar VEVs $\phi_i^{\pm}$ are straightforwardly found by minimizing the potential $V(\phi, T) \equiv V(\phi_1, \dots, \phi_n, T)$ at T_+ and T_- . The dynamics of the system is then fully described by the equation of state (EoS) for the plasma, and the equations of motion (EoM) for the scalar fields.

Concerning the EoS, we take the generalized bag EoS (see [18]), where the quantities of interest are determined from the finite-temperature effective potential $V(\phi, T) = -p$ as

$$p_{\pm} = \frac{1}{3} a_{\pm} T_{\pm}^4 - \epsilon_{\pm}, \quad e_{\pm} = a_{\pm} T_{\pm}^4 + \epsilon_{\pm} \quad (9)$$

and

$$a_{\pm} = \frac{3}{4T_{\pm}^3} \left(\frac{\partial p}{\partial T} \right)_{\pm} = \frac{3\omega_{\pm}}{4T_{\pm}^4}, \quad \epsilon_{\pm} = \frac{1}{4} (e_{\pm} - 3p_{\pm}), \quad (10)$$

where $\epsilon_{\pm}$ stands for the potential energy inside and outside the bubble. Inserting (9) in (8), v_+ and v_- are expressed in terms of T_+ and T_- .

Far away from the wall, the scalar fields become constant and the solution must be matched to a self-similar dynamical one where the profiles of the plasma velocity and temperature are described by the conservation of $T_{le}^{\mu\nu}$. Projecting the equation $\partial_{\mu} T_{le}^{\mu\nu} = 0$ along the flow ($u_{\nu} \partial_{\mu} T_{le}^{\mu\nu} = 0$) and in the direction perpendicular to it ($\bar{u}_{\nu} \partial_{\mu} T_{le}^{\mu\nu} = 0$, with $\bar{u}_{\mu} u^{\mu} = 0$ and $\bar{u}^2 = 1$), one gets, in the bubble frame,

$$(\xi - v) \frac{\partial_{\xi} e}{w} = 2 \frac{v}{\xi} + (1 - \gamma^2 v (\xi - v)) \partial_{\xi} v, \quad (11)$$

$$(1 - v\xi) \frac{\partial_{\xi} p}{w} = \gamma^2 (\xi - v) \partial_{\xi} v, \quad (12)$$

where a spherical symmetry was assumed, and ξ is defined as $\xi \equiv r/t$, with r the distance from the center of the bubble and t the time from nucleation. Using $c_s^2 = dp/de$, with c_s the speed of sound, the equations above can be rearranged into

$$2 \frac{v_p}{\xi} = \gamma^2 (1 - v_p \xi) \left(\frac{\mu^2}{c_s^2} - 1 \right) \partial_{\xi} v_p, \quad (13)$$

$$\partial_{\xi} T = \gamma^2 \mu T \partial_{\xi} v_p, \quad (14)$$

where μ is the Lorentz-transformed fluid velocity

$$\mu(\xi, v_p) = \frac{\xi - v_p}{1 - \xi v_p}. \quad (15)$$

In these coordinates, the bubble wall is located at $\xi = v_w$.

B. Hydrodynamic regimes

The conservation equations give rise to different types of solutions that can be classified by the wall velocity v_w . In terms of v_+ and v_- , Eq. (8) determines two branches, one with $v_+ > v_-$ (detonations), and another with $v_+ < v_-$ (deflagrations). The lowest value v_+ can assume in the detonation regime is the so-called Jouguet velocity v_J and is obtained for a solution with $v_- = c_s$, where $c_s = (\partial_T V / (T \partial_T^2 V))_-$ is the speed of sound inside the bubble and the subscript indicates that the derivative must be evaluated inside the bubble. Below we briefly recall the most important features of the three types of solution (we refer to [18,79] for further details).

Detonation ($v_w > v_J$). The wall moves faster than sound, leaving a rarefaction wave behind it. The plasma in front of the bubble is at rest, with perturbations occurring only behind it, so that $v_w = v_+$, and $T_n = T_+$. The plasma temperature and velocity have opposite behavior,¹ so that $v_+ > v_-$ implies $T_+ < T_-$.

Deflagration ($v_w < c_s$). The wall velocity is lower than the speed of sound inside the bubble, generating a shock wave ahead of it. Perturbations are found in front of the wall, while the plasma is at rest behind it and in front of the shock wave. The wall velocity is then $v_w = v_-$, and $T_n = T_+^{SW}$, where T_+^{SW} is the temperature in front of the shock wave. Quantities at the shock front are related to one another as in a detonation solution, and one finds the following chain $T_+ > T_-^{SW} > T_+^{SW} > T_-$.

¹In local equilibrium, this is due to entropy conservation, which imposes $\gamma(z)T(z) = \text{constant}$ [44].

Hybrid ($c_s < v_w < v_J$). These solutions are given by a combination of deflagration and detonation ones. They have both a shock front and a rarefaction wave, with $T_+^{SW} = T_n$ and $v_- = c_s$. Computationally, they are similar to deflagration solutions, and we will generically refer to deflagrations for both.

As the wall velocity approaches the Jouguet velocity from below, the wall and the shock front get closer and closer until the latter disappears at $v_w = v_J$, and the type of solution changes from a hybrid to a detonation. The theoretical setup described above, and in particular Eqs. (6) and (7), ignores the presence of the shock front, and requires a sufficiently large distance between the latter and the bubble wall to give an accurate description of the region in between. As a consequence, the approach presented here is not valid when $v_w \rightarrow v_J$ from below, and, more generally, the limit $v_w \rightarrow v_J$ should be considered with care.

C. Scalar equations of motion

As our main interest is in multistep phase transitions driven by two scalar fields, in the following we consider a system of two scalars, h and s . In the applications presented below, the first one will be the Higgs field, and the second one a CP -even, neutral, additional scalar. Assuming a steady-state expansion and a mean free path of particles in the plasma much smaller than the scale of variation of the scalars, the equations of motion for h and s read [11,12]

$$E_h \equiv -\partial_z^2 h + \frac{\partial V(h, s, T)}{\partial h} + \frac{F_h^{\text{out}}(z)}{h'} = 0, \quad (16)$$

$$E_s \equiv -\partial_z^2 s + \frac{\partial V(h, s, T)}{\partial s} + \frac{F_s^{\text{out}}(z)}{s'} = 0, \quad (17)$$

where ($\phi_j = h, s$)

$$F_j^{\text{out}}(z) = \sum_i \frac{n_i}{2} \frac{\partial m_i^2}{\partial \phi_j} \int \frac{d^3 p}{(2\pi)^3 E_p} \delta f_i \quad (18)$$

accounts for the out-of-equilibrium contributions and n_i counts the number of degrees of freedom. An approximate solution to these equations can be obtained taking a *tanh* ansatz for the fields,²

$$h(z) = \frac{h_-}{2} \left(1 + \tanh \left(\frac{z}{L_h} \right) \right), \quad (19)$$

$$s(z) = \frac{s_+}{2} \left(1 - \tanh \left(\frac{z}{L_s} - \delta_s \right) \right), \quad (20)$$

where L_h and L_s are the thicknesses of the h and s walls [the z axis is chosen here so that positive values of z

²Having in mind the application to the EWPhT, we consider here a transition from the state $(h, s) = (0, s_+)$ to $(h, s) = (h_-, 0)$.

correspond to the interior of the (h -)bubble, while negative values correspond to the exterior].

Exploiting translation invariance, the first solution is centered in $z = 0$, and δ_s is the displacement of the s wall with respect to the h one. The coefficients h_- and s_+ are the VEVs of the h and s fields inside and outside the bubble, respectively

$$\frac{\partial V(h_-, 0, T_-)}{\partial h} = 0, \quad \frac{\partial V(s_+, 0, T_+)}{\partial s} = 0. \quad (21)$$

The EoM are then traded for four constraints, obtained taking suitable moments of them,

$$\begin{aligned} P_h &= \int dz E_h h' = 0, & G_h &= \int dz E_h \left(2 \frac{h}{h_-} - 1 \right) h' = 0, \\ P_s &= \int dz E_s s' = 0, & G_s &= \int dz E_s \left(2 \frac{s}{s_+} - 1 \right) s' = 0, \end{aligned} \quad (22)$$

where $h' = \partial_z h$ and $s' = \partial_z s$. Here, P_h and P_s determine the pressure acting on the h and s walls, respectively, while G_h and G_s are the corresponding pressure gradients (see [11,12]). Vanishing of P_h and P_s is necessary for a steady-state regime to be reached, where the outward pressure, which tends to accelerate the wall, is balanced by the friction acting on it. The equations $G_h = 0$ and $G_s = 0$ correspond to the requirement that the pressure gradient vanishes, enforcing that the solution has fixed widths.

When out-of-equilibrium effects are considered, the Boltzmann equation must be included to determine δf . An iterative procedure was recently proposed to get a complete (numerical) solution, and was tested for given benchmark points in a specific model [48,56,64]. In this approach, the equilibrium solution is the first, crucial, step, needed to proceed to the iterative evaluation of the out-of-equilibrium effects and of their impact on the physical quantities of interest. The present work is only concerned with the first part of this program, i.e., the determination of the equilibrium solution.

D. Numerical algorithm

In this section, we describe the numerical method used to solve the hydrodynamic equations and the EoMs. This approach allows us to determine the bubble wall velocity v_w , the parameters δ_s , L_h , L_s , and the profiles for the plasma, $T(z)$ and $v_p(z)$. The profiles of the fields $h(z)$ and $s(z)$ are then determined too.

For a given point in the parameter space of a model, the Jouguet velocity v_J and the speed of sound inside the bubble c_s are calculated once and for all, before the numerical loop starts, using a numerical technique for root-finding and function minimization. The algorithm then proceeds with the following steps.

1. Initial guess

The algorithm starts with an initial guess for v_w , δ_s , L_h , and L_s . When scanning the parameter space of a model, the region where the scan is performed is divided into a grid of points. For each point, the initial guess corresponds to the result that was obtained for one of its nearest neighbours.

2. Boundary conditions

The first step is to calculate the four constraints P_{tot} , ΔP , G_h and G_s using the initial guess, where $P_{\text{tot}} \equiv P_h + P_s$ and $\Delta P \equiv P_s - P_h$ (we will motivate the choice of P_{tot} and ΔP , rather than P_h and P_s , later). To compute these quantities, the plasma temperature profile $T(z)$ must be known in the appropriate combustion regime. This is determined by comparing the bubble wall velocity v_w from the initial guess to the Jouguet velocity v_J , and then classifying the tentative solution.

3a. Detonations ($v_w > v_J$)

For detonations, $T_+ = T_n$ ($s_+ = s_n$) and $v_+ = v_w$. The two unknown T_- and v_- are then easily found using the matching equations (8). Note that there is an intrinsic dependence on T_- in h_- , so T_- is found searching for the root of those equations once h_- is determined as the minimum of the potential in the h direction at the temperature $T = T_-$. The velocity v_- is then computed, and h_- is used to construct the profile $h(z)$ [together with $s(z)$, which was already available], from which the plasma profiles $T(z)$ and $v_p(z)$ are in turn determined, using (6), (7).

3b. Deflagrations/ hybrids ($v_w < v_J$)

The determination of the plasma profiles for deflagrations is more involved, as the boundary conditions $T_+^{SW} = T_n$ and $v_- = v_w$ (or $v_- = c_s$ for hybrids) are given at two different points. Before the matching equations can be used, the additional task is to find the value of T_+ . This is done in two sequential steps. The first step calculates T_+^{SW} for a given tentative value of T_+ with the following procedure. The tentative value of T_+ , along with v_w , is then used to determine v_+ and T_- from the matching conditions. Equations (13) and (14) are then integrated from the bubble wall to the shock front³ to find T_-^{SW} and v_-^{SW} , from which T_+^{SW} is obtained. The second step refines the value of T_+ until the condition $T_+^{SW} = T_n$ is satisfied. Once T_+ is found, the determination of the profiles proceeds as for detonations.

³A technical remark is in order here. To deal with single-valued functions, the equations are inverted and solved for $\xi(v_p)$ and $T(v_p)$ in the bubble frame. The integration is performed from $v_p = 0$ to $v_p = \mu(v_w, v_+)$.

4. Root finding

The constraints are finally evaluated on the initial guess using an adaptive quadrature integration technique. Starting from the initial guess, the values of v_w , δ_s , L_h , and L_s are progressively refined until the solution is found. As a check, the constraints are calculated on the final result, which is accepted only if each of them is below a predefined threshold.

III. EXTENSIONS OF THE HIGGS SECTOR

In this section we present the three BSM models used to test our numerical algorithm and calculate the bubble wall velocity, each characterized by an extra scalar field with different charge under the EW $SU(2)$ gauge group. We consider SSM, RTSM, and IDM. An enlarged scalar sector offers the possibility to realize a first-order EWPhT with a two-step pattern, where the EW minimum is reached from a configuration in which the additional neutral, CP -even scalar possesses a VEV. The potential barrier between the two minima receives sizeable contributions from the tree-level potential, so that the EWPhT is typically strong with observable GW signatures and more efficient mechanisms for EW baryogenesis. On the contrary, when the EWPhT proceeds directly from the completely symmetric vacuum, this cannot happen in general. Indeed, as the Higgs field quadratic term must be positive at the nucleation temperature for the origin to be a minimum itself, the barrier can only be generated by a combination of loop and thermal contributions, exposing it more to the (well-known) issues of the perturbative expansion at finite temperature.

A. SSM

We consider a $\mathbb{Z}_2$ -symmetric extension of the SM with a scalar field s . The tree-level Lagrangian of the scalar sector is

$$\mathcal{L} = (D_\mu H)^\dagger (D^\mu H) + \frac{1}{2} \partial_\mu s \partial^\mu s - V_0(H, s), \quad (23)$$

where H is the Higgs doublet

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_1 + i\chi_2 \\ h + i\chi_3 \end{pmatrix}, \quad (24)$$

h the Higgs field and V_0 the potential

$$V_0(H, s) = \mu_h^2 |H|^2 + \lambda_h |H|^4 + \frac{\mu_s^2}{2} s^2 + \frac{\lambda_s}{4} s^4 + \lambda_{hs} |H|^2 s^2. \quad (25)$$

In terms of h and s , it reads

$$V_0(h, s) = \frac{\mu_h^2}{2} h^2 + \frac{1}{4} \lambda_h h^4 + \frac{\mu_s^2}{2} s^2 + \frac{\lambda_s}{4} s^4 + \frac{\lambda_{hs}}{2} h^2 s^2. \quad (26)$$

The model consists of three free parameters, μ_s , λ_s , and λ_{hs} , with the former that can be traded for the mass of the singlet in the EW vacuum $(h, s) = (v, 0)$, $m_s^2 = \mu_s^2 + \lambda_{hs}v^2$, while μ_h^2 and λ_h are fixed by the usual relations $\lambda_h = m_h^2/2v^2$ and $\mu_h^2 = -\lambda_h v^2$. To match the parameters in the tree-level potential directly to the measured values of the Higgs VEV and mass, we will use an on-shell renormalization scheme. The singlet mass will also be fixed to its tree-level value by the renormalization conditions given below.

When $\mu_s^2 < 0$, a nontrivial minimum is found in the singlet direction. The viability of this scenario requires that, at $T = 0$, the EW minimum is deeper than the minimum in the s direction. A two-step phase transition (with a first-order transition toward the EW vacuum in the second step) can then occur if the nontrivial minimum in the s direction appears at temperatures higher than the one in the h direction, and conditions for nucleation toward the EW vacuum are met. The region of parameter space where this is verified is the one of interest for our analysis. A rough analytic estimate of this region can be obtained working with the Parwani-resummed tree-level potential [30], i.e., the tree-level potential supplemented by thermal masses.

We choose to adopt the Parwani resummation method [80], where daisy diagrams are resummed *ab initio*. In terms of the thermal eigenmasses $m_i^2(h, s, T)$, the one-loop effective potential in the $\overline{\text{MS}}$ scheme with renormalization scale $\mu = v$ reads

$$V_{CW}(h, s, T) = \frac{1}{64\pi^2} \sum_i n_i m_i^4(h, s, T) \left(\log \frac{m_i^2(h, s, T)}{v^2} - c_i \right), \quad (27)$$

where n_i is the number of degrees of freedom of the corresponding particle, $i \in \{h, s, \chi, t, W_{L/T}, Z_{L/T}, \gamma\}$, with $t, W_{L/T}, Z_{L/T}$ and γ the top quark, (longitudinal and transverse components of the) W bosons, (longitudinal and transverse components of the) Z boson, and the photon, respectively. For the scalar fields, longitudinal W/Z , and the top quark, $c_i = 3/2$, while for transverse gauge bosons, $c_i = 1/2$. The expressions for the field-dependent thermal masses $m_i^2(h, s, T)$ can be found in Appendix A 1.

As mentioned above, we compensate for radiative shifts of the Higgs VEV, Higgs mass, and singlet mass in the EW vacuum by introducing finite counterterms

$$V_{CT}(h, s) = \frac{1}{2} \delta m_h^2 h^2 + \frac{1}{2} \delta m_s^2 s^2 + \frac{1}{4} \delta \lambda_h h^4, \quad (28)$$

and imposing the conditions

$$\left. \frac{\partial(V_{CW} + V_{CT})}{\partial h} \right|_{h=v, s=0}^{T=0} = 0, \quad (29)$$

$$\begin{aligned} \left. \frac{\partial^2(V_{CW} + V_{CT})}{\partial h^2} \right|_{h=v, s=0}^{T=0} &= 0, \\ \left. \frac{\partial^2(V_{CW} + V_{CT})}{\partial s^2} \right|_{h=v, s=0}^{T=0} &= 0. \end{aligned} \quad (30)$$

The second derivative with respect to h of the Goldstone contribution is singular in the EW vacuum and needs to be regularized. We do so by shifting the field-dependent mass $m_\chi^2, m_\chi^2 \rightarrow m_\chi^2 + \mu_{\text{IR}}^2$, in the singular terms. Specifically, we make the following substitution (V_{CW}^χ indicates the Goldstone contribution to V_{CW})

$$\left. \frac{\partial^2 V_{CW}^\chi}{\partial h^2} \right|_{h=v, s=0}^{T=0} \rightarrow \frac{n_\chi}{8\pi^2} \lambda_h v^2 \log \left(\frac{\mu_{\text{IR}}^2}{v^2} \right), \quad (31)$$

and we choose $\mu_{\text{IR}} = m_h$. The counterterms are then unambiguously determined as

$$\delta m_h^2 = \left(\frac{1}{2} \frac{\partial^2 V_{CW}}{\partial h^2} - \frac{3}{2v} \frac{\partial V_{CW}}{\partial h} \right) \Big|_{h=v, s=0}^{T=0}, \quad (32)$$

$$\delta m_s^2 = - \frac{\partial^2 V_{CW}}{\partial s^2} \Big|_{h=v, s=0}^{T=0}, \quad (33)$$

$$\delta \lambda_h = \frac{1}{2v^2} \left(\frac{1}{v} \frac{\partial V_{CW}}{\partial h} - \frac{\partial^2 V_{CW}}{\partial h^2} \right) \Big|_{h=v, s=0}^{T=0}. \quad (34)$$

The thermal contribution to the one-loop potential is given by

$$V_T(h, s, T) = \frac{T^4}{2\pi} \left(\sum_i n_i J_B \left(\frac{m_i^2(h, s, T)}{T^2} \right) - n_t J_F \left(\frac{m_t^2(h)}{T^2} \right) \right), \quad (35)$$

where the thermal functions $J_{B/F}$ for the bosons and fermions are (the + sign is for bosons, the – for fermions)

$$J_{B/F}(y) = \int_0^\infty dx x^2 \log(1 \mp e^{-\sqrt{x^2+y}}). \quad (36)$$

B. RTSM

This model is a $\mathbb{Z}_2$ extension of the SM that includes a real scalar isotriplet field Σ with zero hypercharge. The scalar sector Lagrangian reads

$$\mathcal{L} = (D_\mu H)^\dagger (D^\mu H) + \frac{1}{2} (D_\mu \Sigma) (D^\mu \Sigma) - V_0(H, \Sigma), \quad (37)$$

where the covariant derivative of Σ is

$$(D_\mu \Sigma)^a = (\partial_\mu \delta^{ac} - g \epsilon^{abc} W_\mu^b) \sigma_c, \quad (38)$$

and σ_a are the three real components of the triplet Σ , with the Higgs doublet parametrized as in (24). The physical degrees of freedom are the two charged fields $\sigma^\pm = (\sigma_1 \mp i\sigma_2)/\sqrt{2}$, and the neutral σ_3 , with the $\mathbb{Z}_2$ symmetry ensuring stability of the latter.

The tree-level scalar potential $V_0(H, \Sigma)$ is

$$V_0(H, \Sigma) = \mu_h^2 |H|^2 + \lambda_h |H|^4 + \frac{\mu_\sigma^2}{2} \sigma^a \sigma^a + \frac{\lambda_\sigma}{4} (\sigma^a \sigma^a)^2 + \lambda_{h\sigma} |H|^2 \sigma^a \sigma^a. \quad (39)$$

In terms of the neutral CP -even scalars h and σ_3 , V_0 reduces to (26) (with the obvious substitutions $s \rightarrow \sigma_3$ and $\mu_s, \lambda_s, \lambda_{hs} \rightarrow \mu_\sigma, \lambda_\sigma, \lambda_{h\sigma}$). The $\mathbb{Z}_2$ symmetry of the model forbids gauge-invariant cubic operators such as $H^\dagger \sigma^a H$ or $\sigma^a \sigma^b \sigma^c$.

As for the singlet, we adopt the Parwani resummation scheme, and use the same renormalization conditions defined in (28)–(30). The field-dependent mass terms that appear in the Coleman-Weinberg and thermal one-loop contributions can be found in Appendix A 2.

C. IDM

In the inert two-Higgs doublet model, a discrete $\mathbb{Z}_2$ symmetry is imposed, under which the SM fields are even and the second doublet H_2 is odd. This prevents H_2 from decaying into lighter SM particles. The Lagrangian of the two doublets reads

$$\mathcal{L} = (D_\mu H_1^\dagger)(D^\mu H_1) + (D_\mu H_2^\dagger)(D^\mu H_2) - V_0(H_1, H_2), \quad (40)$$

where H_1 is the SM Higgs doublet and the tree-level potential V_0 is

$$V_0(H_1, H_2) = \mu_1^2 |H_1|^2 + \mu_2^2 |H_2|^2 + \lambda_1 |H_1|^4 + \lambda_2 |H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 |H_1^\dagger H_2|^2 + \frac{\lambda_5}{2} [(H_1^\dagger H_2)^2 + \text{h.c.}] \quad (41)$$

Similar to H_1 in (24), the second doublet can be expressed in terms of its components

$$H_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_4 + i\chi_5 \\ h' + i\chi_6 \end{pmatrix}. \quad (42)$$

Stability of the tree-level potential imposes the constraints

$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 + \lambda_4 - \lambda_5 + 2\sqrt{\lambda_1 \lambda_2} > 0 \quad (43)$$

on the quartic couplings.

As for the other cases, we only consider the vacua landscape for the neutral and CP -even fields h and h' , in terms of which the tree-level potential reduces to

$$V_0(h, h') = \frac{\mu_1^2}{2} h^2 + \frac{\lambda_1}{4} h^4 + \frac{\mu_2^2}{2} h'^2 + \frac{\lambda_2}{4} h'^4 + \frac{\lambda_{345}}{4} h^2 h'^2, \quad (44)$$

with $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$. In the EW vacuum $(h, h') = (v, 0)$, the physical masses of the various fields are

$$m_h^2 = 2\lambda_1 v^2, \quad m_{h'}^2 = \mu_2^2 + \frac{\lambda_{345}}{2} v^2, \\ m_{\chi_{4/5}}^2 = \mu_2^2 + \frac{\lambda_3}{2} v^2, \quad m_{\chi_6}^2 = \mu_2^2 + \frac{\tilde{\lambda}_{345}}{2} v^2, \quad (45)$$

where m_h^2 and $m_{h'}^2$ are the masses of h and h' , $m_{\chi_{4/5}}^2$ the mass of χ_4 and χ_5 , giving rise to the charged Higgs $H^\pm$, $m_{\chi_6}^2$ the mass of the CP -odd scalar χ_6 , and we defined $\tilde{\lambda}_{345} \equiv \lambda_3 + \lambda_4 - \lambda_5$.

The field and temperature dependent mass matrices that enter into the one-loop contributions to the potential are given in Appendix A 3. We introduce the counterterms $\delta m_1^2, \delta m_2^2$, and $\delta \lambda_1$ and adopt the same renormalization conditions introduced for the SSM to fix v , m_h , and $m_{h'}$ to their tree-level value. The only difference here stems from the Goldstone sector: the mass matrices in the (h, h') background at finite temperature mix χ_1 with χ_4 , χ_2 with χ_5 , and χ_3 with χ_6 .

IV. ANALYTIC CONSIDERATIONS

Before moving on to the numerical analysis, we make here some general considerations on the structure of the constraint equations, which serve as a guide for a better reading of the results presented in the next section.

The four constraints in (22), together with the matching equations (8), can be solved for the four unknown parameters, v_w, δ_s, L_h , and L_s . The wall velocity is found by first determining the temperature behind the wall T_- (we recall here that T_+ is given in terms of T_n for both detonations and deflagrations) that enters in (22) through h_- , and then solving (8) for v_+ and v_- , the plasma velocity in front and behind the wall.

In local thermal equilibrium, the total pressure $P_{\text{tot}} = P_h + P_s$ can be expressed as

$$P_{\text{tot}} = \int_{-\infty}^{+\infty} dz \left[\frac{\partial V}{\partial h} h' + \frac{\partial V}{\partial s} s' \right] = \Delta V - \int_{-\infty}^{+\infty} dz \frac{\partial V}{\partial T} T', \quad (46)$$

where $\Delta V \equiv V(h_-, 0, T_-) - V(0, s_+, T_+)$ is the difference between the potential of the two asymptotic states $(h_-, 0, T_-)$ and $(0, s_+, T_+)$. Field-independent terms do not enter in P_{tot} , and the latter can equivalently be written either in terms of V or in terms of $\tilde{V}(h, s, T) \equiv V(h, s, T) - V(0, 0, T)$.

Concerning the subtracted potential $\tilde{V}$, the difference $\Delta \tilde{V}$ must be negative, as the latter is the driving force of

the expansion, and a solution to the equation $P_{\text{tot}} = 0$ can only be found if $\int dz(\partial_T \tilde{V})T'(z)$, which represents the back-reaction of the plasma, is negative too. In this case, the nontrivial temperature profile of the plasma generates a friction term that allows for a steady-state expansion of the wall. As for the potential difference ΔV , its sign is not known *a priori*, but simple considerations can be made to estimate it. The potential V is dominated by the contribution from species that are light with respect to the temperature T . For these, the plasma behaves as a relativistic gas. Contributions from heavier species are Boltzmann suppressed, and the full potential V can thus be approximated as

$$V(h, s, T) \simeq -g_* \frac{\pi^2}{90} T^4 + \tilde{V}(h, s, T), \quad (47)$$

where g_* is the effective number of relativistic degrees of freedom. The potential difference ΔV is then given as

$$\Delta V \simeq \frac{\pi^2}{90} (g_*^+ T_+^4 - g_*^- T_-^4) + \Delta \tilde{V}, \quad (48)$$

where the superscript $\pm$ in g_* indicates the vacuum where it is calculated. Most of the particles are massless outside the bubble and massive inside, so that $g_*^+ > g_*^-$. Unless large deviations in the temperature are found across the wall, ΔV is expected to be positive, so with reversed sign as compared to $\Delta \tilde{V}$.

The requirement for the friction term to actually hinder the wall acceleration can then be better understood considering the conservation equations (6), (7). For our purposes, the latter can be conveniently rewritten in terms of the potential as

$$\frac{(h')^2 + (s')^2}{2} - V + c_1 v_p = c_2, \quad (49)$$

and, evaluating the resulting equation far away from the wall, we get

$$\Delta V = c_1 (v_- - v_+), \quad (50)$$

with constant c_1 obtained from (6) at, for instance, $z \rightarrow +\infty$,

$$c_1 = -(T \partial_T V)|_{(h_-, 0, T_-)} \frac{v_-}{1 - v_-^2}. \quad (51)$$

To determine the sign of $\partial_T V$, we write it as $\partial_T V = \partial_T V_T + \partial_T V_{CW}$ with

$$\partial_T V_T = \frac{2T^3}{\pi^2} \left(\sum_i n_i Y_B \left(\frac{m_i^2}{T^2} \right) - n_i Y_F \left(\frac{m_i^2}{T^2} \right) \right), \quad (52)$$

$$\partial_T V_{CW} = \sum_i \frac{n_i}{32\pi^2} m_i^2 \frac{\partial m_i^2}{\partial T} \left(\log \frac{m_i^2}{\mu^2} - 1 \right) \quad (53)$$

where the functions $Y_{B/F}$ are defined as

$$Y_{B/F}(y) = \int_0^\infty dx x^2 \left(\log(1 \mp e^{-\sqrt{x^2+y}}) \mp \frac{2y - \partial_T(T^2 y)/T}{8(e^{\sqrt{x^2+y}} - 1)\sqrt{x^2+y}} \right), \quad (54)$$

and the sum in (52) extends to all bosons, while the sum in (53) is only for degrees of freedom that acquire a thermal mass. The terms proportional to $\partial_T m^2$ in the equations above are a byproduct of the Parwani resummation. For bosons, we can write $m_i^2(h, s, T) = \bar{m}_i^2(h, s) + c_i T^2$, so that the numerator in the second term of (54) reduces to $\bar{m}_i^2(h, s)$. It is then easily understood that a bosonic contribution to $\partial_T V_T$ is negative for $\bar{m}_i^2(h, s) \geq 0$, which is clearly the case at $z \rightarrow +\infty$. Similar considerations lead us to conclude that the fermionic contribution is always negative, thus implying that $\partial_T V_T|_{(h_-, 0, T_-)} < 0$.

On the other hand, the sign of $\partial_T V_{CW}$ is not defined, as it depends on the choice of the renormalization scale μ (we took $\mu = v$ in this work). However, $\partial_T V_{CW}$ receives contributions only from scalars and longitudinal gauge bosons, and, in general, for reasonable choices of μ , it turns out to be subdominant with respect to $\partial_T V_T$. Thus, we expect $\partial_T V$ to be negative for $z \rightarrow +\infty$.

Having determined the constant c_1 in (51) to be positive, from (50) we deduce that the argument on the sign of ΔV suggests that in local thermal equilibrium a solution to $P_{\text{tot}} = 0$ can only be found if $v_- > v_+$. As the sign of $\partial_T V$ is the same for all values of z (51), a comparison with (46) also indicates that, on the solution, the temperature $T(z)$ must be a decreasing function of z . We then conclude that in local thermal equilibrium detonations are unlikely to be found.⁵ Anticipating the results, this will also be the outcome of our numerical analysis, and it is in agreement with the conclusions of [57,72], where an alternative derivation based on entropy conservation was used. The LTE detonation solutions found (in a narrow region of parameter space) in [57,72] are known to be unstable, as in LTE the pressure

⁴In addition to that, $\partial_T V_{CW}$ arises only from the specific choice of resummation.

⁵Detonations require $\Delta V < 0$ and can only arise if a cancellation takes place in (48) as to overthrow the argument on the positivity of ΔV . Using $\tilde{V}$, there is yet another way to see that detonations require some amount of tuning that makes them unlikely to appear in LTE. Besides the ambiguity related to the renormalization scale, the contribution of a field i to $\partial_T \tilde{V}$ can be shown to be positive for $m_i^2 \geq 0$. In the excursion from $-\infty$ to $+\infty$, only scalar masses can become negative, though this is not necessarily the case, as maxima of the potential typically are not reached in multifield transitions. All the other fields give positive contributions to $\partial_T \tilde{V}$. On the other hand, detonations require $\partial_T \tilde{V}$ to be negative in a large enough interval for the negative part to dominate over the positive one in the integral $\int dz \partial_T \tilde{V} T'(z)$.

decreases with v_w for $v_w > v_J$. We will come back to this point as we comment on the numerical results.

Coming back to the constraint equations (22), an important point both for the development of our code and for the interpretation of the numerical results was to understand the dependence of each one of the constraints on the parameters. Starting with P_{tot} in Eq. (46), we observe that the first term on the right-hand side (ΔV) is clearly independent of L_h , L_s , and δ_s . The second term, in contrast, has a dependence on these parameters that is governed by the profiles of the scalars. However, a few general and model-independent considerations can be made. For the sake of the present discussion, we here consider a sharp profile for the temperature, $T(z) = T_- \theta(z) + T_+ \theta(-z)$. The generalization to a smoother profile is straightforward, as we will briefly mention.

When the potential is such that $\partial_T V$ does not contain terms that mix the two fields h and s (take, for instance, the high-temperature potential in the quadratic truncation), changing the integration variable $z \rightarrow z/L_h$ or $z \rightarrow z/L_s$ in the various pieces shows that P_{tot} only depends on T_- and δ_s , with the δ_s dependence arising from the nontrivial spatial profile of the temperature $T(z)$. In this respect, we note that a smoother profile for $T(z)$ would introduce a dependence on the ratios L_h/L_T and L_s/L_T , with L_T the width of the region where the temperature has a significant variation. However, from the hydrodynamic equations (6), (7), we expect the field and temperature widths to be related, and so their ratio not to vary much. Similar considerations can be made in the more general case where $\partial_T V$ contains terms that mix h and s , and one is led to conclude that P_{tot} is a function of three variables, $P_{\text{tot}} = P_{\text{tot}}(T_-, \delta_s, L_h/L_s)$. Combining the observations above with the fact that a solution to $P_{\text{tot}} = 0$ with $\Delta V \neq 0$ necessarily requires a nontrivial profile for the temperature, we expect P_{tot} to have the largest dependence on T_- , and in turn on v_w .

Similar to P_{tot} , the pressure difference

$$\Delta P = P_s - P_h = \int dz \left(\frac{\partial V}{\partial s} s' - \frac{\partial V}{\partial h} h' \right) \quad (55)$$

is easily found to be a function of the same three variables, $\Delta P = \Delta P(T_-, \delta_s, L_h/L_s)$, and we intuitively envisage that the largest dependence is on δ_s , since the pressure difference controls the displacement between the two fields.⁶

⁶In terms of the original equations of motion, δ_s serves to realize the condition $\partial_h V = 0$ at $z = 0$ (center of the h bubble), while the ratio L_h/L_s allows one to realize the analogous condition $\partial_s V = 0$ at $z = L_s \delta_s$ (s -bubble center). When $\delta_s = 0$, $\partial_h V$ and $\partial_s V$ both vanish for the same field configuration $h = h_-/2$, $s = s_+/2$. When $\delta_s \neq 0$, $\partial_h V|_{z=0} = 0$ for $h = h_-/2$, $s = s_+(1 + \tanh \delta_s)/2 \equiv s_*/2$ and $\partial_s V|_{z=L_s \delta_s} = 0$ for $h = h_-(1 + \tanh(L_s \delta_s/L_h))/2 \equiv h_*/2$ and $s = s_+/2$. The (absolute value of the) offset δ_s increases as the difference between the configurations that make $\partial_h V$ vanish and $\partial_s V$ vanish increases, with the ratio L_s/L_h adjusting for the asymmetry in the deviations $h_* - h_-$ and $s_* - s_+$.

The considerations above are at the basis of our choice to use the equations $P_{\text{tot}} = 0$, $\Delta P = 0$ instead of $P_h = 0$, $P_s = 0$ in our numerical algorithm, as discussed in Sec. II D, and have been used to guide the root-finding procedure.

The values of L_h and L_s are fixed by the pressure gradients G_h and G_s , which can be thought of as averages of the pressure over the profiles ($\phi_i = h, s$)

$$G_i = \pm \int_{-\infty}^{+\infty} dz P_i(z) \tanh\left(\frac{z}{L_i} - \delta_i\right) = \langle P_i \rangle_{\phi_i} \quad (56)$$

where we defined $P_i(z) \equiv E_{\phi_i} \phi_i'$. In our case, the $+$ sign applies for the Higgs field h , and the minus sign for s . Within the $\tanh$ ansatz, G_i can be expressed as

$$G_i = \int_{-\infty}^{+\infty} dz \left(-\phi_i'' + \frac{\partial V}{\partial \phi_i} \right) \phi_i' \left(\frac{2\phi_i}{\bar{\phi}_i} - 1 \right) \\ = \pm \frac{2\bar{\phi}_i^2}{15L_i^2} \pm \frac{2}{\bar{\phi}_i} \int_{-\infty}^{+\infty} dz \left(\frac{\partial V}{\partial \phi_i} \right) \phi_i' \left(\frac{2\phi_i}{\bar{\phi}_i} - 1 \right), \quad (57)$$

where $\bar{\phi}_i = h_-, s_+$.

Following the discussions above, we can then write G_h and G_s as

$$G_h = \frac{2h_-^2}{15L_h^2} + g_1 \left(T_-, \delta_s, \frac{L_h}{L_s} \right), \\ G_s = -\frac{2s_+^2}{15L_s^2} - g_2 \left(T_-, \delta_s, \frac{L_h}{L_s} \right). \quad (58)$$

On the solution, one has

$$\frac{h_-^2}{s_+^2} = \frac{g_1(T_-, \delta_s, L_h/L_s) L_h^2}{g_2(T_-, \delta_s, L_h/L_s) L_s^2}. \quad (59)$$

As one can infer from (57), the ratio g_1/g_2 gives a measure (weighted by $(\phi_i')^2$) of the differences between the gradient of the potential encountered in the h and s directions. If no large hierarchy in the shape and height of V along h and s is present, it is reasonable to expect $g_1/g_2 \sim \mathcal{O}(1)$, and the ratio of the widths can be approximated to $L_h/L_s \sim h_-/s_+$. Such a relation can be taken as a rough estimate, to be used, for example, as an initial guess in a numerical analysis. As for the individual widths, according to the explicit calculation of [11, 12], the ratio between the square of the jump in the field $\bar{\phi}_i^2$ and the square of the width L_i^2 should be proportional to the potential difference ΔV . They are thus expected to be related to ΔV as $L_i^2 \propto \bar{\phi}_i^2/\Delta V$.

V. NUMERICAL RESULTS

In this section, we present the numerical results for the bubble wall velocity and, more generally, for the system of equations that describe the wall dynamics, obtained using our code that implements the algorithm described in

Sec. II D. The analysis was performed for the three models introduced in Sec. III. As already stressed, our main interest is in regions of the parameter space where the transition proceeds with a two-step pattern, schematically $(0, 0) \rightarrow (0, s) \rightarrow (h, 0)$, with the last one being first order. In this case the potential barrier between the two states involved in the EW transition is generated at tree-level in most of the parameter space. Our algorithm needs as input some specifics of the transition such as the nucleation temperature T_n and the values of the fields in the false and true vacuum at T_n . Therefore, for each model considered, we performed a preliminary analysis of their parameter space with the `Cosmotransitions` package [81] and with BSMP [82–84] for cross-checking, to determine the transition pattern and individuate the region of interest.

A. SSM

For the singlet extension of the Standard Model we present the results for two different choices of the singlet self-coupling, $\lambda_s = 1$ and $\lambda_s = 2$.

The region where the EW vacuum at $T = 0$ is reached through a two-step phase transition is reported in Fig. 1. As can be seen, for larger values of the singlet self-coupling λ_s , the two-step region is shifted to larger values of λ_{hs} . This can be understood taking a high temperature approximation for the effective potential (tree-level plus quadratic corrections)

$$V(h, s, T) = \frac{1}{2}\mu_h^2 h^2 + \frac{1}{2}\mu_s^2 s^2 + \frac{\lambda_h}{4}h^4 + \frac{\lambda_s}{4}s^4 + \frac{\lambda_{hs}}{2}h^2 s^2 + \frac{c_h T^2}{2}h^2 + \frac{c_s T^2}{2}s^2, \quad (60)$$

where all the model-dependence is encoded in the coefficients c_h and c_s of the thermal masses, and $m_s^2 = \mu_s^2 + \lambda_{hs}v^2$. For $\mu_s^2 + c_s T^2 < 0$, a nontrivial minimum is found in the s direction, $w^2(T) = -(\mu_s^2 + c_s T^2)/\lambda_s$. The Hessian

calculated on $(h, s) = (0, w(T))$ is diagonal,

$$\mathcal{H}(0, w(T)) = \begin{pmatrix} \mu_h^2 + c_h T^2 + \frac{\lambda_{hs}}{2}w^2(T) & 0 \\ 0 & \mu_s^2 + c_s T^2 + 3\lambda_s w^2(T) \end{pmatrix}, \quad (61)$$

and stability around it requires that its eigenvalues are positive. As $w^2(T)$ is inversely proportional to λ_s , it is immediately apparent that an increase in λ_s must be compensated by a simultaneous increase of the portal λ_{hs} for the state $(0, w(T))$ to be stable along the Higgs direction.

A comparison of the region in Fig. 1 with the one obtained through its analytic determination with the quadratic potential (60) (see, for instance, [30]) shows a substantial overlap between the two, and confirms the crucial role played by the tree-level barrier in strong two-step PhTs. Below the colored region of Fig. 1, the transition toward the EW vacuum consists of only one step, either first or second order, schematically $(0, 0) \rightarrow (h, 0)$, with the SM result recovered in the limit $\lambda_{hs} \rightarrow 0$. For large λ_{hs} above the colored region, the EW vacuum is not the global minimum of the potential at $T = 0$, as an increase in λ_{hs} with fixed value of m_s^2 determines an increase in the absolute value of μ_s^2 . Before this happens, for intermediate values of λ_{hs} above the upper boundary of the two-step region, one finds that, though the EW vacuum is the global minimum of the potential at zero temperature, there is no transition toward it. This is due to the absence of a sufficiently large difference in potential values between the two vacua at $T = 0$. In fact, for fixed m_s the critical temperature T_c and the nucleation temperature T_n decrease with λ_{hs} . Above a certain maximal value of λ_{hs} , T_c drops to such low values that the conditions for nucleation cannot be satisfied, so that the system remains trapped in the false vacuum all the way down to $T = 0$.

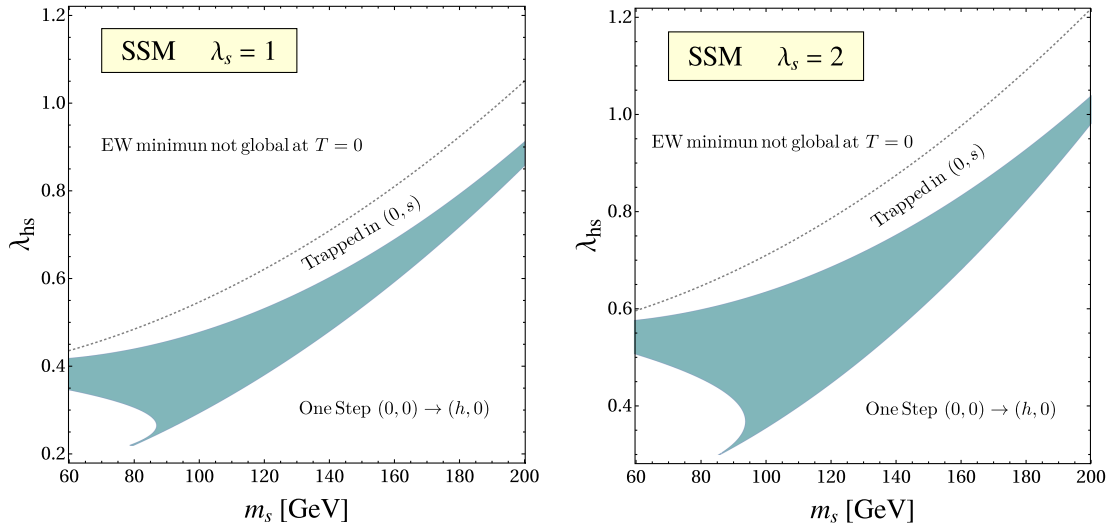


FIG. 1. Regions on the plane (m_s, λ_{hs}) with different patterns for the EW phase transition for the SSM with $\lambda_s = 1$ (left panel) and $\lambda_s = 2$ (right panel). In the colored region the EWPhT is two step.

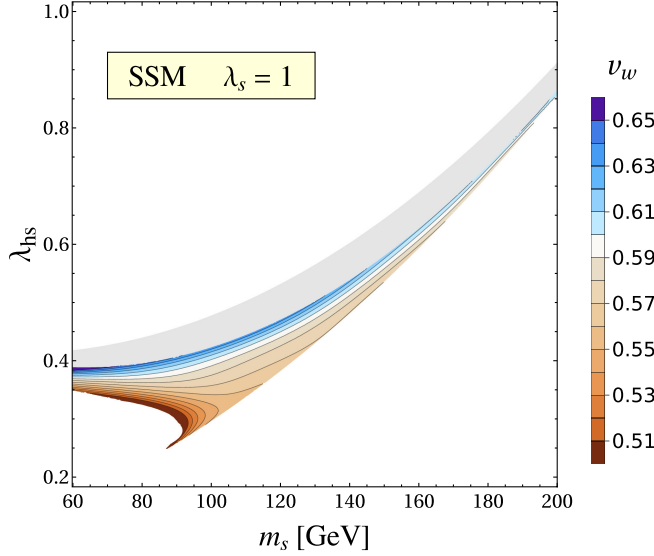


FIG. 2. Contour plot of the wall velocity v_w as a function of the parameters m_s and λ_{hs} for the SSM with $\lambda_s = 1$. The bubble wall velocity is constant on the black lines, with the color gradient describing its variation and the color bar serving as a legend. In the grey region no steady-state solutions are found in LTE.

Given the parameters describing the EWPhT obtained with these preliminary scans, the bubble wall velocity v_w , the wall thicknesses L_h , L_s , and the displacement δ_s are obtained with our algorithm. Stationary solutions were found in the whole two-step parameter space but a strip on the upper part of it. This is related to the nonappearance of detonation solutions in local thermal equilibrium, as discussed in Sec. IV.

1. SSM, $\lambda_s = 1$

The results for the wall velocity in the (m_s, λ_{hs}) plane are shown in Fig. 2. For constant m_s the velocity tends to

increase as λ_{hs} increases until v_w gets too close to the Jouguet velocity v_J and the framework described above does not provide an accurate description of the wall dynamics anymore. As already stressed, the limit $v_w \rightarrow v_J$ from below, in fact, corresponds to the case of an hybrid solution where the wall gets infinitely close to the shock front, and the description of the bubble as a planar domain wall breaks down before the limit is reached. The left panel in Fig. 3 confirms that the upper boundary of v_w is given by the condition $v_w \sim v_J$.

As seen in Fig. 3 (left), our analysis does not find any detonation solution, in agreement with the considerations of Sec. IV. Instead, the grey region above the upper boundary of the colored one should be interpreted as an ultrarelativistic detonation regime, if the appropriate friction was included. Conversely, smaller values of v_w are found in the lower part of the plot (especially in the left corner), where iso-lines tend to become denser and the transition is weaker, with the potential barrier between the vacua becoming flatter as one approaches it. For completeness, the results for the speed of sound behind the wall c_s are presented in the right panel of Fig. 3, showing an opposite behavior to that of v_w , i.e., it decreases while going from the lower to the upper boundary of the parameter space. It is worth noting that c_s turns out to be smaller (up to the percent level) than the speed of sound in the vacuum $c_0 = 1/\sqrt{3}$, as it should, with larger deviations for stronger transitions.

A similar analysis can be performed for the other parameters, L_h , L_s , and δ_s , whose contour plots are shown in Fig. 4. The wall widths have a behavior opposite to the velocity, i.e., they decrease as the strength of the phase transition increases, as one would intuitively guess. This reflects the physical relation between energy dissipation and wall dynamics, with lower velocities allowing for greater spreading of the profiles. As mentioned in Sec. II C,

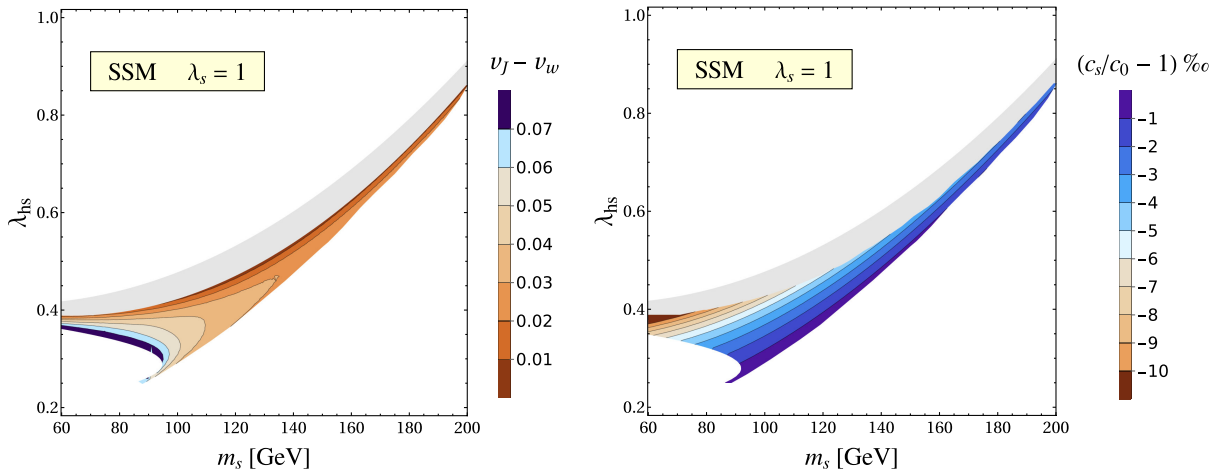


FIG. 3. Contour plots of the difference between the Jouguet and the bubble wall velocities $v_J - v_w$ (left panel), and of the deviation of the speed of sound c_s from $c_0 = 1/\sqrt{3}$ (right panel), in the plane (m_s, λ_{hs}) for the SSM with $\lambda_s = 1$. In the grey band no steady-state solution is found in LTE.

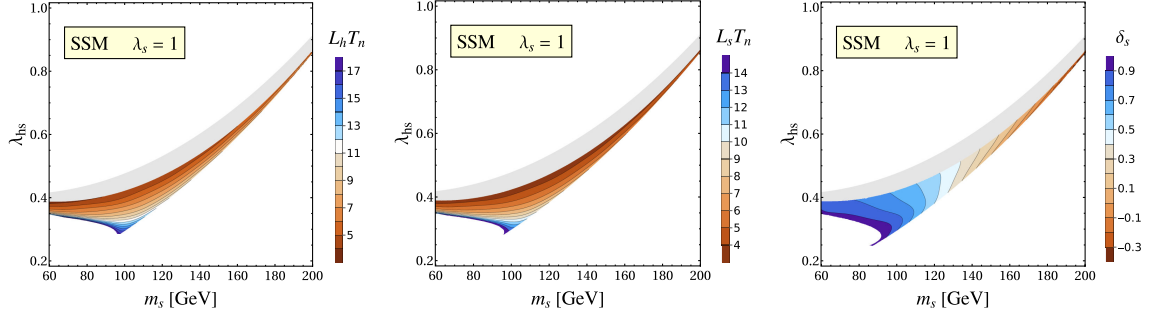


FIG. 4. Contour plots of the wall widths L_h , L_s , and of the displacement δ_s (left, central, and right panels, respectively) in the plane (m_s, λ_{hs}) for the SSM with $\lambda_s = 1$. In the grey band no steady-state solution is found in LTE.

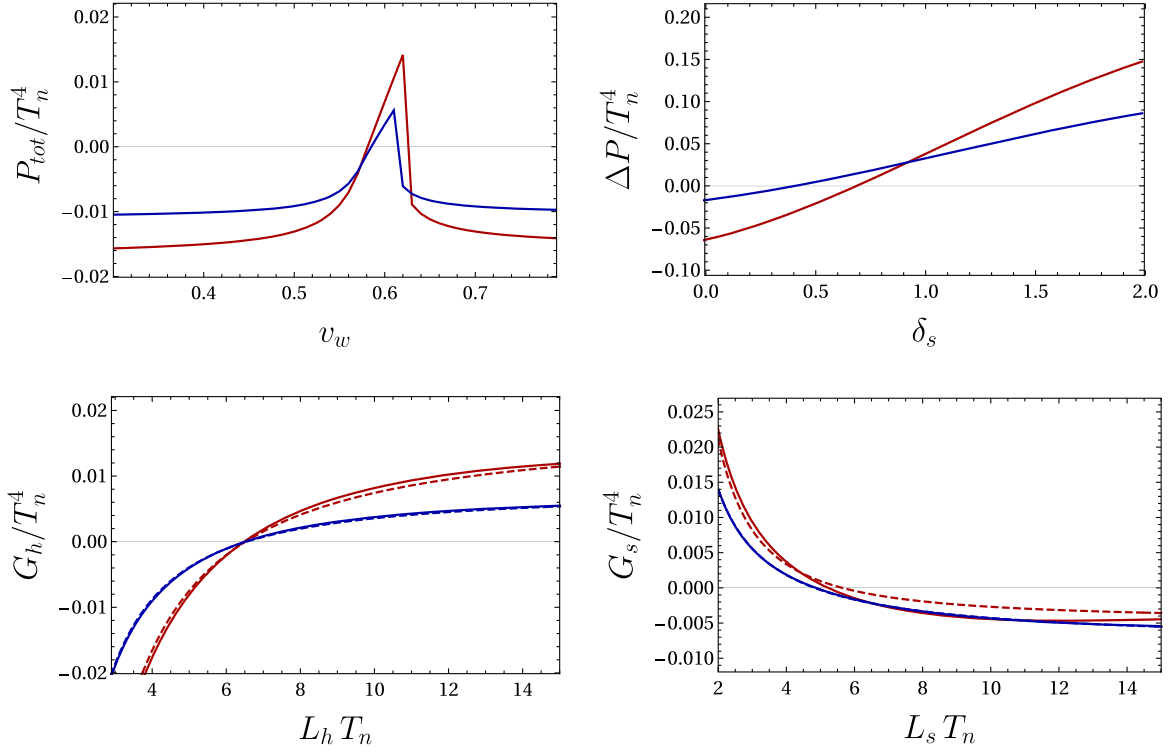


FIG. 5. *Upper panel:* Total pressure $P_{\text{tot}} = P_h + P_s$ as a function of the wall velocity v_w for two benchmark points in the SSM with $\lambda_s = 1$: BP1 ($m_s = 105$ GeV, $\lambda_{hs} = 0.39$, red curve) and BP2 ($m_s = 131$ GeV, $\lambda_{hs} = 0.47$, blue curve) (left). Pressure difference $\Delta P = P_s - P_h$ as a function of δ_s (right). *Lower panel:* Pressure gradient G_h as a function of the wall width L_h . The dashed lines are for the fitting function $G_h(L_h)$, whose form is given in the text (left). Pressure gradient G_s as a function of the wall width L_s . The dashed lines are for the fitting function $G_s(L_s)$ (right).

it is worth to recall here that, when $L_h T_n \rightarrow 1$ ($L_s T_n \rightarrow 1$), the approximations leading to the scalar equations of motion (16) break down, as they are obtained taking a WKB approximation to evaluate thermal averages [11,12]. We also observe that, for fixed m_s , the widths L_h and L_s decrease with λ_{hs} , that is with the PhT strength. As the latter is proportional to the potential difference in the two vacua, this also suggests that the widths tend to decrease with ΔV . This is in good agreement with expectations. Moreover, throughout the parameter space, the ratio $L_h/L_s \simeq h_-/s_+ \gtrsim 1$, and it decreases for decreasing λ_{hs} , as per the arguments given in Sec. IV.

As discussed in Sec. IV, analytic considerations suggest that the bubble wall velocity v_w mainly depends on the total pressure $P_{\text{tot}} = P_h + P_s$, the displacement δ_s on the pressure difference $\Delta P = P_s - P_h$, and the wall widths L_h , L_s on G_h and G_s , respectively, so that in the following we will typically refer to P_{tot} and ΔP rather than P_h and P_s when discussing constraints. Fixing the other three quantities to the solution, in Fig. 5 the dependence of each one of the four constraints above on their “primary” variable is shown for two different benchmark points.

In the upper left panel, we see that, starting from negative values, P_{tot} increases with v_w until it eventually crosses the

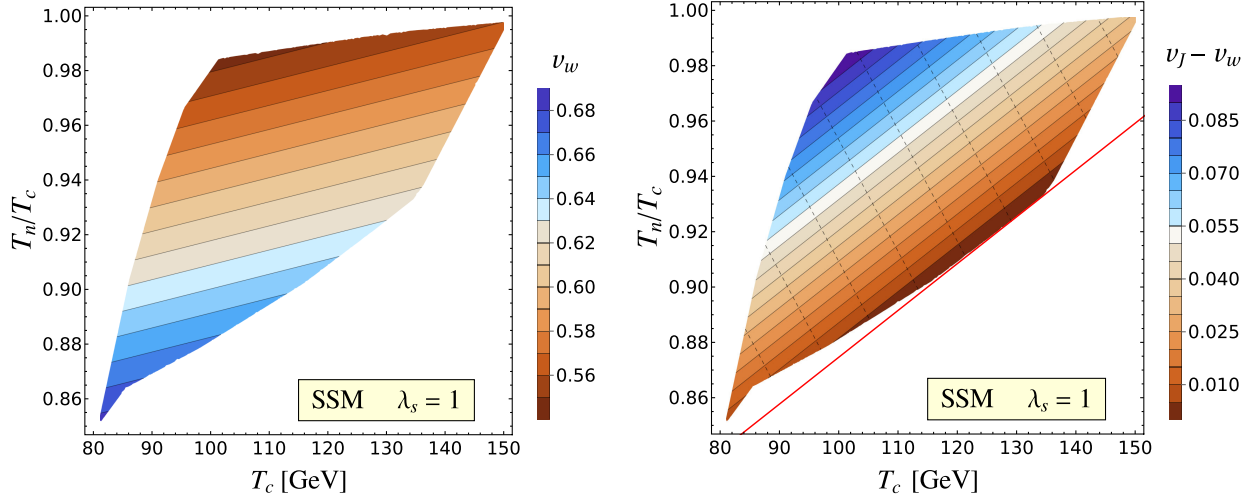


FIG. 6. *Left panel.* Contour plot of the wall velocity v_w in the plane $(T_c, T_n/T_c)$ for the SSM with $\lambda_s = 1$. The straight lines represent the curves of constant v_w for the fitting function in Table I. *Right panel.* Contour plot of $(v_J - v_w)$. The straight lines represent the curves of constant $v_J - v_w$ obtained using linear fitting functions for both v_w and v_J (see Table I). The dashed lines represent curves of constant v_J (v_J decreases going from the left to the right part of the colored region). On the red line, $v_w = v_J$, as explained in the text.

axis. A peak is then reached at the Jouguet velocity, beyond which P_{tot} decreases and relaxes to negative values again. Actually, as stressed several times, the hydrodynamic equations cannot be used for v_w too close to the Jouguet velocity. In particular, while the first zero of $P_{\text{tot}}(v_w)$ represents the deflagration solution found in our analysis, the second zero, located right beyond the Jouguet velocity, should not be taken for a detonation solution. The fact that $P_{\text{tot}}(v_w)$ is monotonically decreasing for $v_w > v_J$ means that an increase in the wall velocity does not induce an increase in the friction acting on the wall, and thus the second zero does not represent a stable stationary expansion. This is a further manifestation of the nonappearance of detonation solutions in local equilibrium. In this respect, it is important to remember that, as shown for instance in [49,56,64], out-of-equilibrium effects give a positive and increasing contribution to P_{tot} for $v_w > v_J$, opening the door to the possibility of finding detonations. Concerning ΔP (upper right panel) we see that, as expected, the offset δ_s adjusts to equate the two pressures P_h and P_s . When $P_h > P_s$ ($\Delta P < 0$), an increase in δ_s , corresponding to a shift of the s -profile in the direction opposite to propagation, is required. The converse is true for $P_h < P_s$.

As for the other constraints, the pressure gradients are shown in the second row of Fig. 5. The dashed lines represent a fit to the data of the form $G^{\text{fit}}(x) = c_1 + c_2 x^{-1} + c_3 x^{-2}$, that can be appreciated to reproduce the results to a good approximation, especially around the solution $G = 0$. It can be verified that the coefficient c_3 is $c_{3,h} \sim -2h_-^2/15$ for G_h^{fit} and $c_{3,s} \sim 2s_+^2/15$ for G_s^{fit} , in agreement with expectations from Sec. IV. As both the constraints are shown in terms of only one width, with the other one constant, a net dependence of G_h from L_h and of G_s from L_s comes from the functions $g_1(T_-, \delta_s, L_h/L_s)$ and $g_2(T_-, \delta_s, L_h/L_s)$, respectively. The

latter are responsible for the terms x^0 and x^{-1} in $G^{\text{fit}}(x)$. Since the fitting function above nicely reproduces the numerical results around $G = 0$ for both the pressure gradients G_h and G_s , the widths can be approximately determined as the roots of the functions $G_h^{\text{fit}}(x)$ and $G_s^{\text{fit}}(x)$. Noting that $c_2^2/4c_1^2$ is always subdominant with respect to c_3/c_1 (at least for our benchmark points), it is then understood that the widths determined in this way are approximately given as⁷ $L_h^2 \sim -c_{3,h}/c_{1,h}$ and $L_s^2 \sim -c_{3,s}/c_{1,s}$, so that $L_h^2/L_s^2 \sim (c_{3,h}/c_{3,s}) \times (c_{1,h}/c_{1,s}) \sim (h_-^2/s_+^2)(c_{1,h}/c_{1,s})$. For the two benchmark points in the figure, $\sqrt{c_{1,h}/c_{1,s}} \sim 1.6$ (red curve) and $\sqrt{c_{1,h}/c_{1,s}} \sim 1.9$ (blue curve).

Moving forward, we now study the dependence of the bubble wall velocity on the PhT parameters. For instance, in the left panel of Fig. 6 we show the wall velocity as a function of the critical temperature T_c and of the ratio T_n/T_c , that provides a measure of the amount of supercooling related to the transition. To better appreciate the outcome, we have excluded from the plot a small portion of space with weak transitions ($v_w \lesssim 0.54$), that, due to the near flatness of the potential, contains larger numerical errors and introduces small fluctuations in the figure. The points excluded all collapse on the upper left part of the plot.

The figure displays a particularly clear dependence of v_w on T_c and T_n/T_c , that is very well approximated by a linear function, which is given in Table I. The velocity shows a tendency to decrease as the supercooling decreases, that is for larger T_n/T_c , and to increase with the critical temperature. The first feature amounts to the wall being faster for stronger transitions, which is quite intuitive. The second one

⁷For G_h^{fit} , $c_{1,h} > 0$ and $c_{3,h} < 0$, while for G_s^{fit} one has $c_{1,s} < 0$ and $c_{3,s} > 0$.

TABLE I. Fitting functions for the wall velocity (*second column*) and the Jouguet velocity (*third column*) in terms of $T_c/v(\equiv x)$ and $T_n/T_c(\equiv y)$, and of the line of minimal T_n/T_c (*fourth column*) for the different models studied.

Model	$v_w(T_c/v, T_n/T_c)$	$v_J(T_c/v, T_n/T_c)$	$T_n/T_c _{\min}(T_c/v)$
SSM $\lambda_s = 1$	$1.60 + 0.15x - 1.14y$	$0.96 - 0.23x - 0.23y$	$0.71 + 0.42x$
SSM $\lambda_s = 2$	$1.60 + 0.15x - 1.14y$	$0.96 - 0.22x - 0.23y$	$0.71 + 0.42x$
RTSM $\lambda_\sigma = 1$	$1.60 + 0.13x - 1.12y$	$0.96 - 0.22x - 0.24y$	$0.73 + 0.39x$
RTSM $\lambda_\sigma = 2$	$1.59 + 0.13x - 1.12y$	$0.97 - 0.22x - 0.25y$	$0.72 + 0.40x$
IDM $\lambda_2 = 1/2$	$1.60 + 0.07x - 1.10y$	$0.97 - 0.21x - 0.25y$	$0.75 + 0.34x$
IDM $\lambda_2 = 1$	$1.60 + 0.05x - 1.08y$	$0.98 - 0.21x - 0.26y$	$0.76 + 0.32x$

can be understood on the same ground as follows [using the quadratic approximation for the potential, Eq. (60), as a guide]. On a line of constant T_n/T_c , an increase in the critical temperature corresponds to an increase in the difference $T_c - T_n$. At temperatures such that nontrivial minima exist in both field directions, the cancelation between μ_h^2 and $c_h T^2$ sets the scale of the minimum $\bar{h}^2(T)$ [similarly for $\bar{s}^2(T)$], with the potential in the vacuum being quadratically sensitive to this cancelation, $V(\bar{h}(T), 0, T) \sim -(\mu_h^2 + c_h T^2)^2/\lambda_h$, $V(0, \bar{s}(T), T) \sim -(\mu_s^2 + c_s T^2)^2/\lambda_s$. At T_c , both $\mu_h^2 + c_h T_c^2$ and $\mu_s^2 + c_s T_c^2$ are negative, so $-\mu_h^2 > c_h T_c^2$, $-\mu_s^2 > c_s T_c^2$. The larger the excursion from T_c to T_n , the more $\mu_h^2 + c_h T_n^2$ and $\mu_s^2 + c_s T_n^2$ tend to μ_h^2 and μ_s^2 , respectively, and the potential difference between the two vacua ΔV at $T = T_n$ grows and approaches its zero-temperature value. We then understand that an increase in T_c with fixed T_n/T_c corresponds to an increase in ΔV at $T = T_n$, and thus an increase in the strength of the transition.

In the right panel of Fig. 6, we show the difference $v_J - v_w$ in the $(T_c, T_n/T_c)$ plane, together with the lines of constant v_J (dashed lines). The isolines of v_w in the left panel of the figure and of v_J (right panel, dashed lines) are nearly orthogonal. The limit $v_w \rightarrow v_J$ from below is realized when two lines with same value (of v_w and v_J , respectively) cross each other. This gives a boundary for

deflagrations, and more generally for steady-state solutions in LTE, and is represented by the red line in the figure. With our fitting functions, the condition $v_w \lesssim v_J$ in terms of T_c and T_n/T_c is given in Table I, which is an upper boundary $T_n/T_c|_{\min}$ on the amount of supercooling.

Before closing this section, let us turn our attention to the plasma. Its temperature and velocity profiles $T(z)$ and $v_p(z)$, normalized by T_+ and v_w , respectively, are shown in Fig. 7 for two benchmark points, namely $(m_s/\text{GeV}, \lambda_{hs}) = (105, 0.39)$ (red curves) and $(m_s/\text{GeV}, \lambda_{hs}) = (131, 0.47)$ (blue curves). In the plots, the axis $z = 0$ represents the location of the h planar domain wall, with positive (negative) values of z indicating the region inside (outside) the h bubble. The coordinate is normalized by L_h , so that on the axis the h width is equal to 1. The center of the s bubble is indicated by dashed lines, with the colored bands showing the corresponding widths. Besides the single profiles $v_p(z)$ and $T(z)$, our analysis finds that the product $\gamma(z)T(z)$ is constant. It was recently argued in [44] that, in local thermal equilibrium, entropy conservation enforces this condition. The latter can be used as an additional hydrodynamic equation alongside (8).

Coming back to the profiles themselves, Fig. 7 shows a decreasing temperature, and an increasing velocity, as one moves toward the domain wall from the symmetric phase, as expected in the deflagration/hybrid regime. In particular,

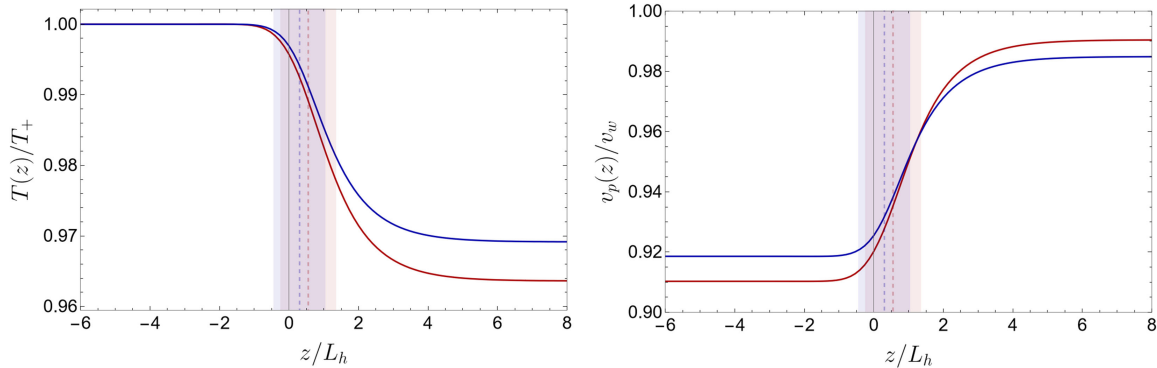


FIG. 7. *Left panel.* Plasma temperature profile normalized by T_+ , $T(z)/T_+$ as a function of z/L_h in the SSM with $\lambda_s = 1$ for two different benchmark points: BP1 ($m_s = 105$ GeV, $\lambda_{hs} = 0.39$, red curve) and BP2 ($m_s = 131$ GeV, $\lambda_{hs} = 0.47$, blue curve). The coordinate $z = 0$ indicate the h bubble wall, with the horizontal axis pointing toward the inside of the bubble. Dashed lines locate the center of the s profile, with the band indicating the width of the latter. *Right panel.* Plot of the plasma velocity normalized by the wall velocity, $v_p(z)/v_w$. The colors are for the same benchmark points as for the left panel.

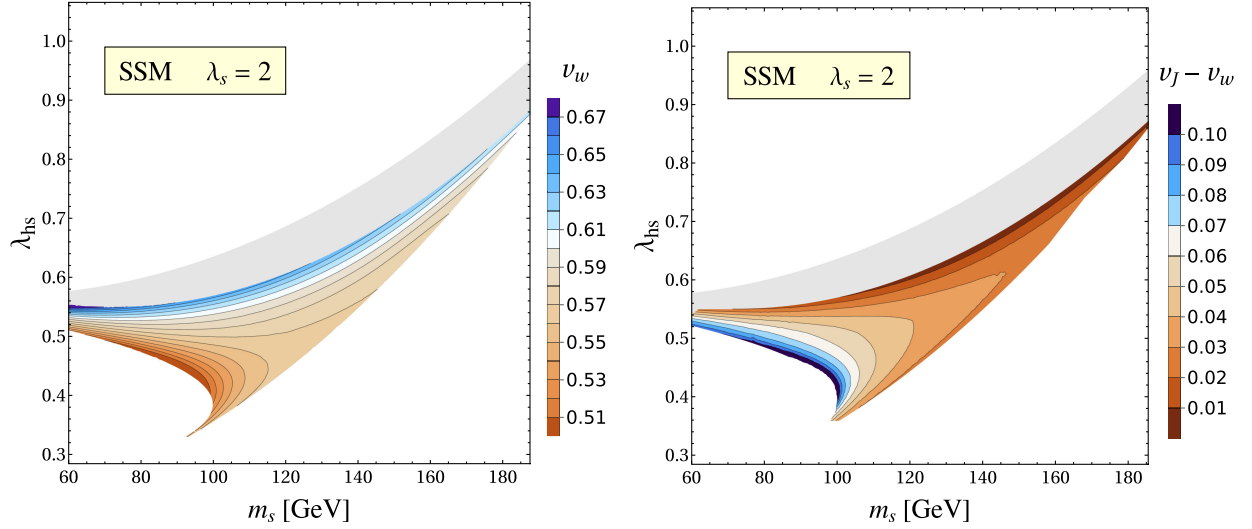


FIG. 8. Contour plots of the bubble wall velocity (*left panel*) and of the difference between the Jouguet and bubble wall velocities (*right panel*), in the (m_s, λ_{hs}) plane for the SSM with $\lambda_s = 2$.

the profiles are nearly flat far from the wall, and the plasma is approximately unperturbed up until the bubble front, with a significant variation only inside the bubble and in the vicinity of the wall. It is worth remembering here that the gradient of the temperature is fully responsible for the friction force exerted by the plasma on the expanding domain wall in local equilibrium, as per Eq. (46), and the latter can be easily determined from the knowledge of the profile $T(z)$. This also means that the contribution to the friction comes, to a large extent, from the region right behind the bubble wall.

2. SSM, $\lambda_s = 2$

To check the dependence on the singlet self coupling, we repeated the analysis for $\lambda_s = 2$. To keep the discussion

more concise, here and in the following we refrain from presenting the results for all the quantities, as we did in the previous section, but concentrate only on the main ones.

The results for the wall velocity and the difference $v_J - v_w$ in the $m_s - \lambda_{hs}$ plane are shown in Fig. 8. It can be immediately appreciated that the dependence of both v_w and $v_J - v_w$ on m_s and λ_{hs} is quite similar to the one in Figs. 2 and 3, respectively. The overall range of variation is also very similar, with the weakest transitions displaying a velocity $v_w \sim 0.5$, and the strongest ones, in the upper left corner of the plot, having $v_w \sim 0.67$. In agreement with the arguments of Sec. IV, only deflagration solutions were found in this case too, and only in a subset of the entire two-step parameter space. No LTE solution was found in the grey region of Fig. 8. The right panel of

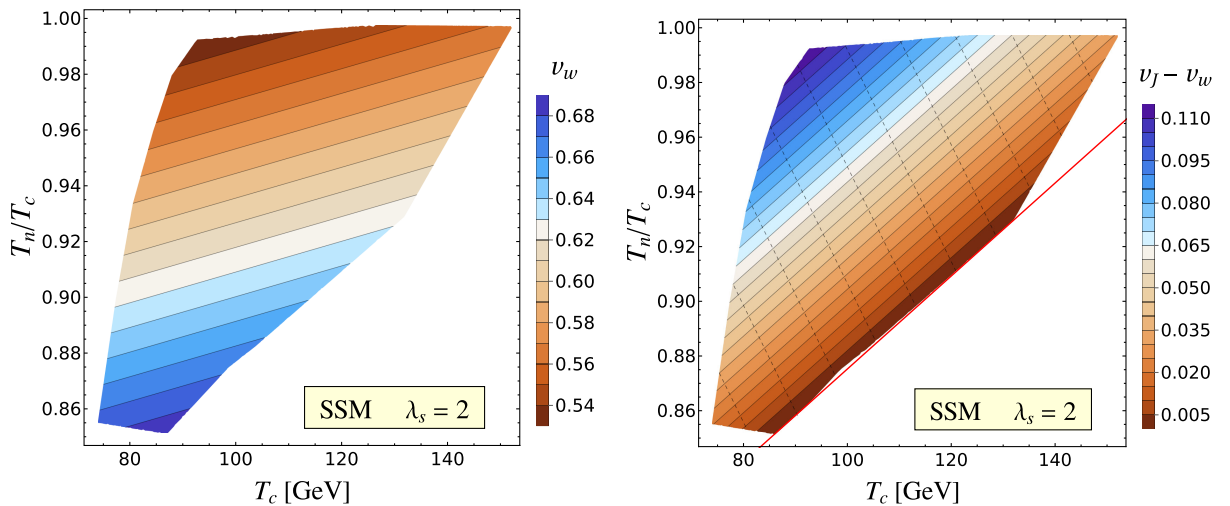


FIG. 9. *Left panel*. Contour plot of the wall velocity v_w in the $(T_c, T_n/T_c)$ plane for the SSM with $\lambda_s = 2$. *Right panel*. Contour plot of $v_J - v_w$, with dashed lines representing curves of constant v_J . On the red line, $v_w = v_J$.

Fig. 8 shows that the upper boundary of the region with a steady-state solution is reached as v_w approaches v_J from below.

The dependence of the velocity on the parameters of the transition T_c and T_n/T_c is shown in the left panel of Fig. 9. Again the results are reproduced to a very good approximation by a linear fit, the coefficients of which are reported in Table I. The agreement of the numerical fit to the one obtained for $\lambda_s = 1$ can hardly be missed. The largest difference between the coefficients is below the percent level. This can be taken as indicating a very mild dependence of the bubble wall velocity, and more generally of the dynamics of the phase transition, on the self-coupling of the additional scalar.

The behavior of the difference $v_J - v_w$ in the $(T_c, T_n/T_c)$ plane is shown in the right panel of Fig. 9. The similarity to the $\lambda_s = 1$ case is again evident: the iso-lines of the Jouguet velocity (dashed lines) are nearly orthogonal to those of v_w , and this again leads to an “upper” boundary for deflagrations, with the condition $v_w \lesssim v_J$ obtained with our linear fits reported in Table I. The agreement between the two realizations with different λ_s is again below the percent level.

The results for the other parameters not shown in this section are all very similar to those presented for $\lambda_s = 1$.

B. RTSM

We now proceed with our analysis and consider the real triplet extension of the Standard Model. Similar to the

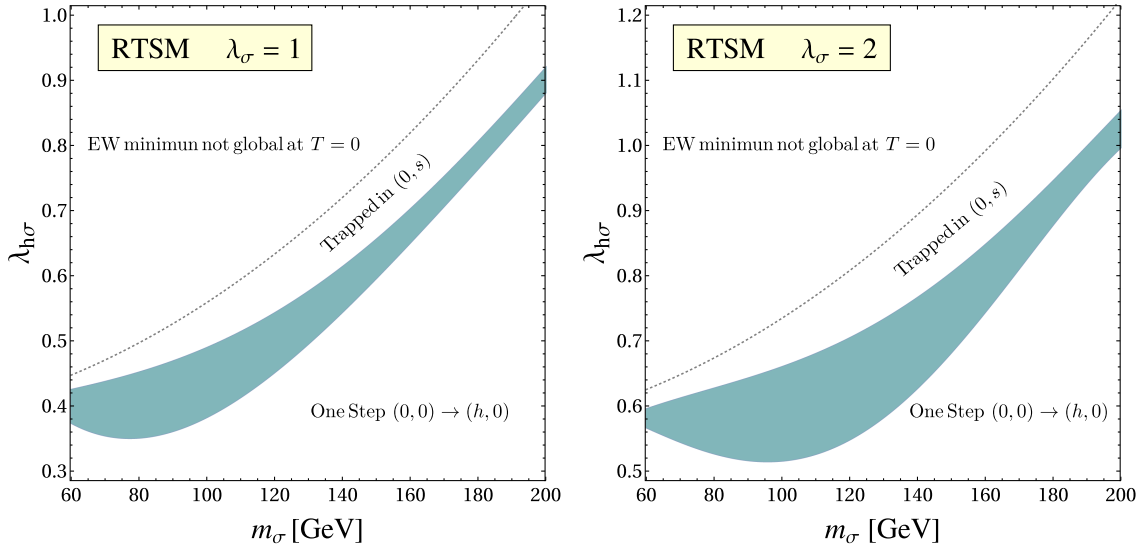


FIG. 10. Regions in the plane $(m_\sigma, \lambda_{h\sigma})$ with corresponding patterns for the EW phase transition in the RTSM for $\lambda_\sigma = 1$ (left panel) and $\lambda_\sigma = 2$ (right panel). In the colored region the EW phase transition is two step.

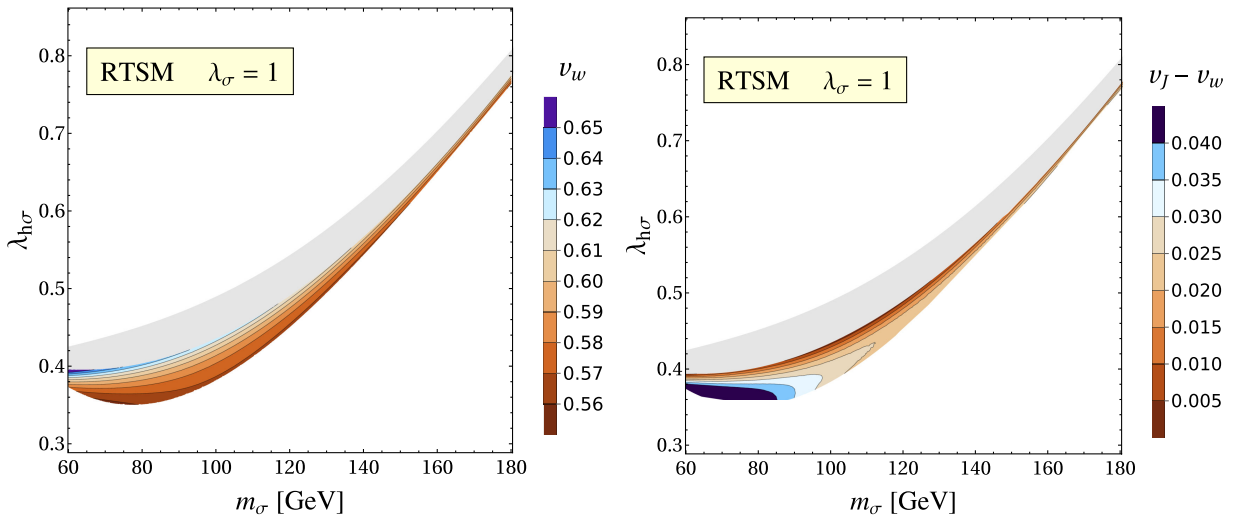


FIG. 11. Left panel. Contour plot of v_w in the $(m_\sigma, \lambda_{h\sigma})$ plane for the RTSM with $\lambda_\sigma = 1$. Right panel. Contour plot of the difference between the Jouguet and the bubble wall velocities ($v_J - v_w$).

previous case, we performed the computation for two benchmark values of the extra scalar self-coupling, $\lambda_\sigma = 1$ and $\lambda_\sigma = 2$.

The parameter space with a two-step PhT found with Cosmotransitions is given by the colored regions in Fig. 10 for $\lambda_\sigma = 1$ (left panel) and $\lambda_\sigma = 2$ (right panel). As for the SSM, the two-step region gets shifted to larger values of $\lambda_{h\sigma}$ for larger values of the self-coupling λ_σ , and only in a portion of these regions is it possible to find a deflagration solution in local thermal equilibrium.

The results turn out to be qualitatively model independent, so that, for both $\lambda_\sigma = 1$ and $\lambda_\sigma = 2$, we will only show the plots for the wall velocity and the difference between the Jouguet and the bubble wall velocities ($v_J - v_w$).

The bubble wall velocity in the $(m_\sigma, \lambda_{h\sigma})$ plane is shown in the left panel of Fig. 11 for $\lambda_\sigma = 1$, with the difference

between the Jouguet velocity and v_w in the right panel of the same figure. The analogous results for $\lambda_\sigma = 2$ are shown in Fig. 13. As can be seen, they have the same features as those found in the analysis of the SSM, with v_w and v_J increasing from the lower to the upper boundary of the deflagration parameter space, where v_w approaches v_J . The largest values of v_w and v_J are found in the upper left corner of the plot, with the smallest ones in the lower left corner. Again, no detonation solution with $v_w > v_J$ was found in the grey regions of Figs. 11 and 13.

A plot of v_w in the $T_c - T_n/T_c$ plane is given in Fig. 12 for $\lambda_\sigma = 1$ and in Fig. 14 for $\lambda_\sigma = 2$. To a good approximation, the velocity shows a linear dependence on the parameters of the transition, in all similar to the one found in the SSM for both $\lambda_s = 1$ and $\lambda_s = 2$. The fitting functions are given in Table I. The contour plots of the difference $v_J - v_w$ are in

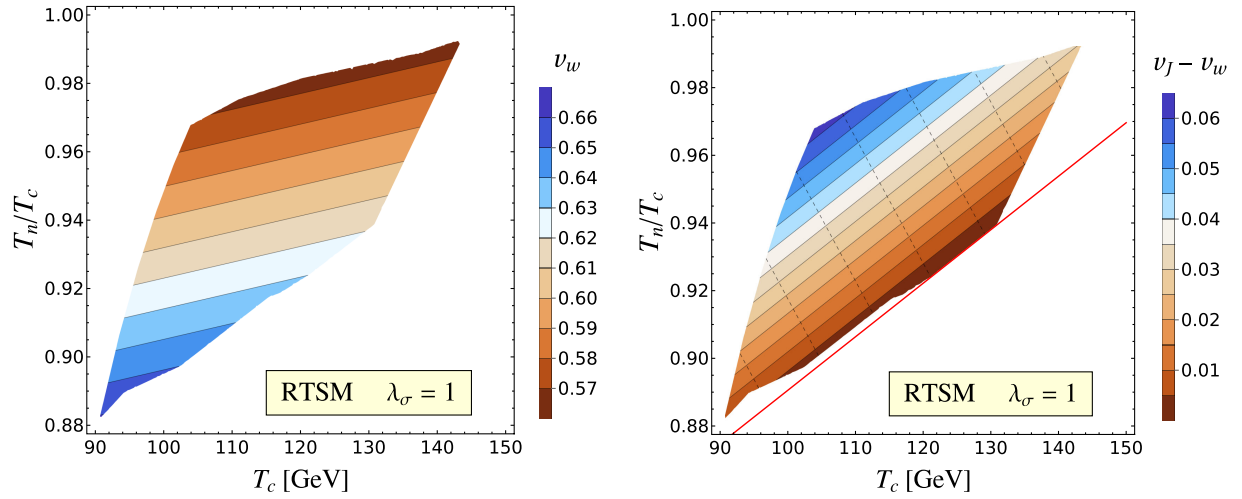


FIG. 12. *Left panel.* Contour plot of the wall velocity v_w in the $T_c - T_n/T_c$ space for the RTSM with $\lambda_\sigma = 1$. *Right panel.* Contour plot of $v_J - v_w$. Dashed lines are for constant v_J . On the red line, $v_w = v_J$.

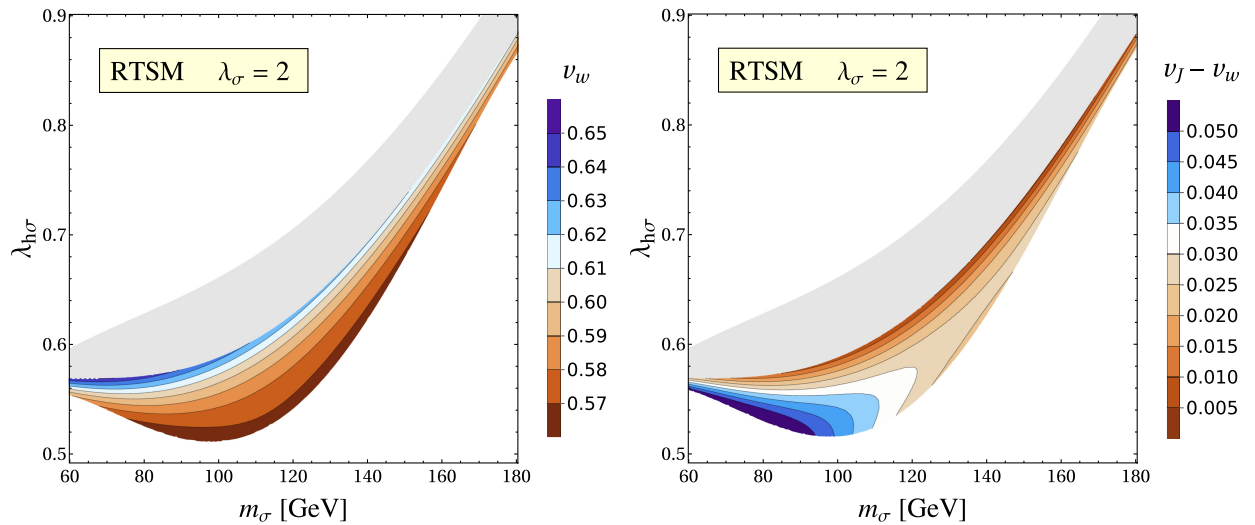


FIG. 13. *Left panel.* Contour plot of v_w in the $(m_\sigma, \lambda_{h\sigma})$ plane for the RTSM with $\lambda_\sigma = 2$. *Right panel.* Contour plot of the difference between the Jouguet and the bubble wall velocities ($v_J - v_w$).

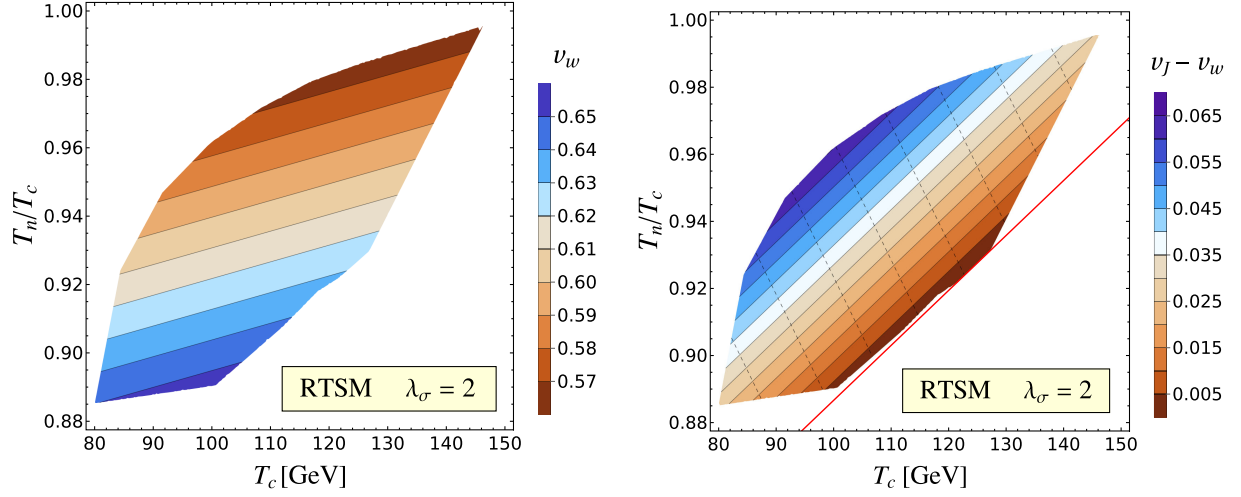


FIG. 14. *Left panel.* Contour plot of the wall velocity v_w in the $(T_c, T_n/T_c)$ plane for the RTSM with $\lambda_\sigma = 2$. *Right panel.* Contour plot of $v_J - v_w$. Dashed lines are for constant v_J . On the red line, $v_w = v_J$.

the right panel of Fig. 12 and 14, together with the lines of constant Jouguet velocity (dashed lines). The features discussed for the SSM are again found here. As in the other case, the lines of constant v_w and v_J lead to an upper bound on the amount of supercooling. The expression for the fitting functions is given in Table I.

C. IDM

Finally, we show in this section the results for the scan of the IDM. Similar to the cases above, we fixed the value of the H_2 self-coupling. Moreover, since the relevant coupling for the potential of the two neutral CP -even fields is λ_{345} ,

we chose to perform the analysis fixing the values of λ_4 and λ_5 to $\lambda_4 = 1/4$ and $\lambda_5 = -1/3$, and varying λ_3 .

The region of parameter space with a two-step phase transition is depicted in Fig. 15 for the two choices of the H_2 self-coupling $\lambda_2 = 1/2$ (left panel) and $\lambda_2 = 1$ (right panel), with the second one involving larger values of λ_3 , as expected.

We show once more the results for v_w and for $v_J - v_w$ in the parameter space and in the $T_c - T_n/T_c$ plane. Concerning the first ones, the plots of the wall velocity (left panel) and of the difference (right panel) are in Fig. 16 for $\lambda_2 = 1/2$ and in Fig. 18 for $\lambda_2 = 1$. They do not show any significant difference with respect to the corresponding plots in the

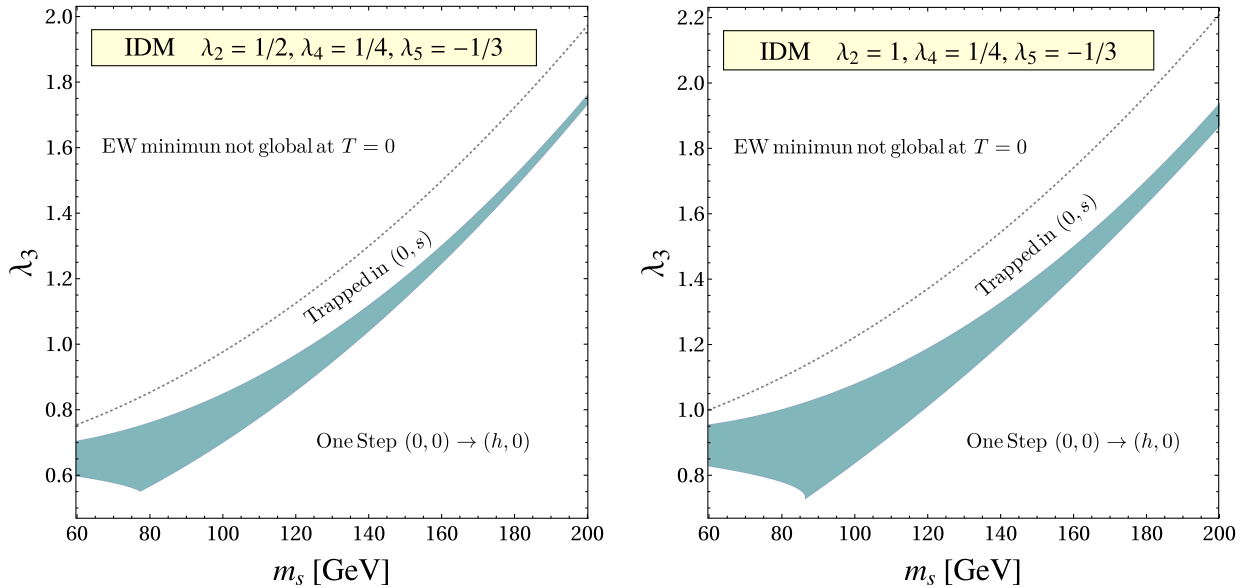


FIG. 15. Regions in the plane (m_s, λ_3) with corresponding patterns for the EWPhT in the IDM for $\lambda_4 = 1/4$, $\lambda_5 = -1/3$, $\lambda_2 = 1/2$ (left panel) and $\lambda_2 = 1$ (right panel). In the colored region the EWPhT is two step.

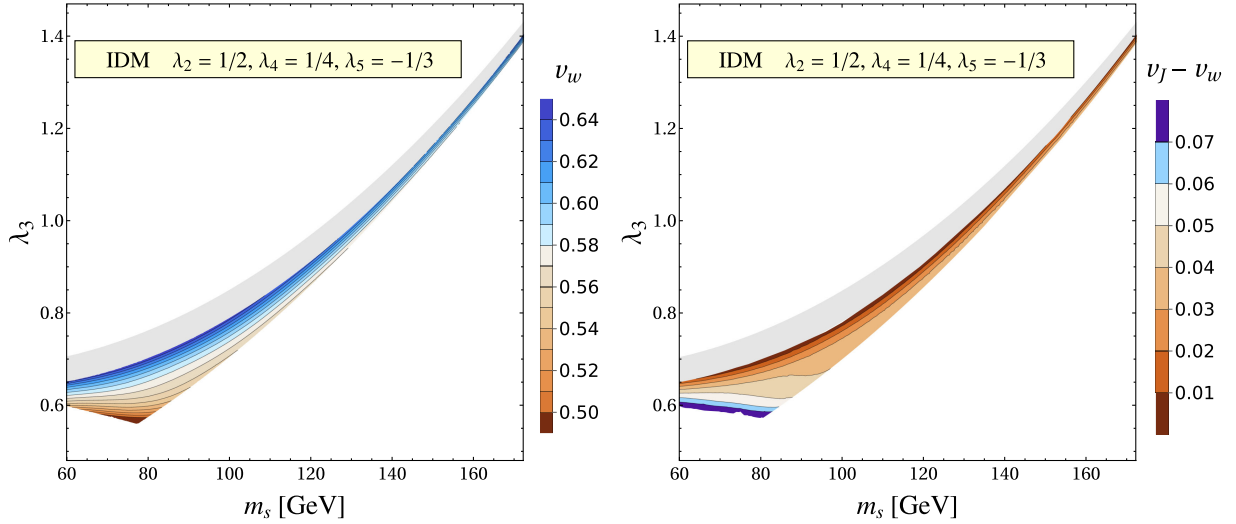


FIG. 16. *Left panel.* Contour plot of v_w in the (m_s, λ_3) plane for the IDM with $\lambda_2 = 1/2$, $\lambda_4 = 1/4$, $\lambda_5 = -1/3$. *Right panel.* Contour plot of the difference $v_J - v_w$.

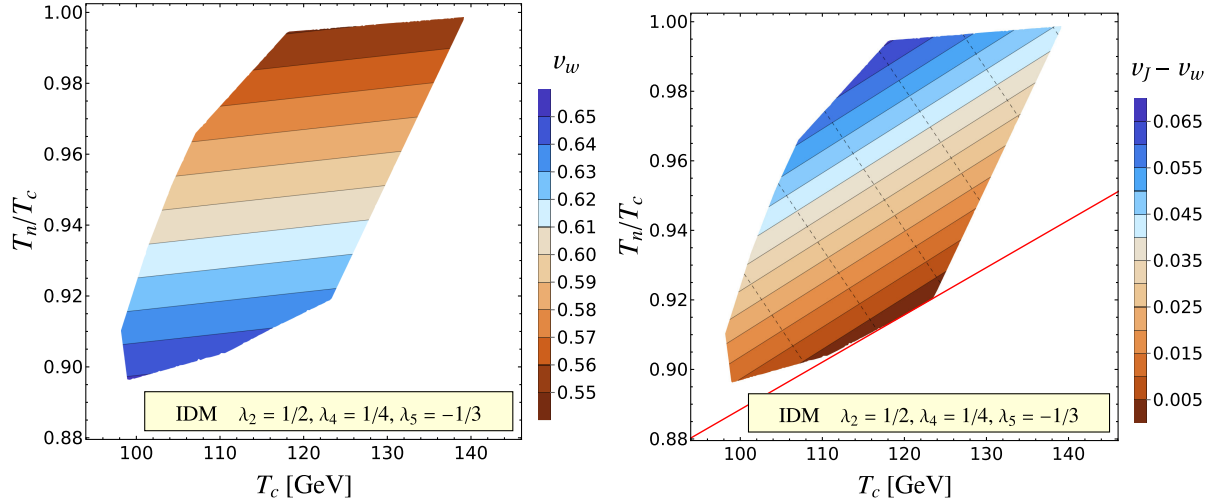


FIG. 17. *Left panel.* Contour plot of the wall velocity v_w in the $(T_c, T_n/T_c)$ plane for the IDM with $\lambda_2 = 1/2$, $\lambda_4 = 1/4$ and $\lambda_5 = -1/3$. *Right panel.* Contour plot of $v_J - v_w$. Dashed lines are for constant v_J . On the red line, $v_w = v_J$.

SSM and in the RTSM, with v_w getting closer to v_J as the upper boundary of the (colored) parameter space is approached. The bubble runs away in the grey region above it.

The wall velocity in the T_c - T_n/T_c plane is shown in the left panel of Fig. 17 for $\lambda_2 = 1$ and in the left panel of Fig. 19 for $\lambda_2 = 1$. The qualitative behavior is as observed in the previous cases, with the lines of constant v_w that appear somehow flatter. Numerical data are again fitted to a good approximation by a linear function, that can be found together with the others in Table I. The lines of constant Jouguet velocity still form a large angle with those of constant v_w , as can be seen in the right panel of the same figures. The portion of T_c - T_n/T_c plane available for deflagrations is then restricted to $T_n/T_c \geq (T_n/T_c)_{\min}$, with $(T_n/T_c)_{\min}$ given in Table I.

D. Comments and comparison between the models

We show the fitting functions for the wall velocity, the Jouguet velocity, and the equation determining the upper bound on the supercooling found in the T_c - T_n/T_c plane for the different models in Table I. Approximate independence of all of them from the self-coupling of the additional scalar is clearly suggested by the results, as its variation induces differences at the percent level at most.

A slight dependence emerges when comparing different models. In particular, the coefficient of T_c/v in the fitting function of v_w appears to have the largest variation, though the velocity mostly depends on T_n/T_c . The coefficient of T_n/T_c shows a slight tendency to decrease as the additional scalar becomes more strongly coupled. As for the Jouguet velocity v_J , the coefficient of T_c/v slightly decreases the

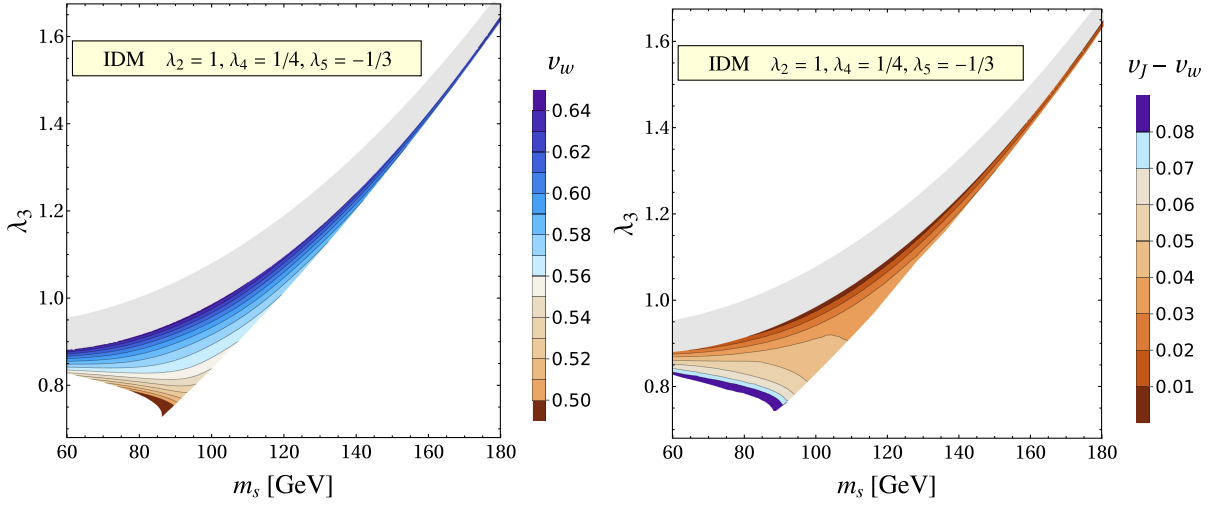


FIG. 18. *Left panel.* Contour plot of v_w in the (m_s, λ_3) plane for the IDM with $\lambda_2 = 1$, $\lambda_4 = 1/4$ and $\lambda_5 = -1/3$. *Right panel.* Contour plot of $(v_J - v_w)$.

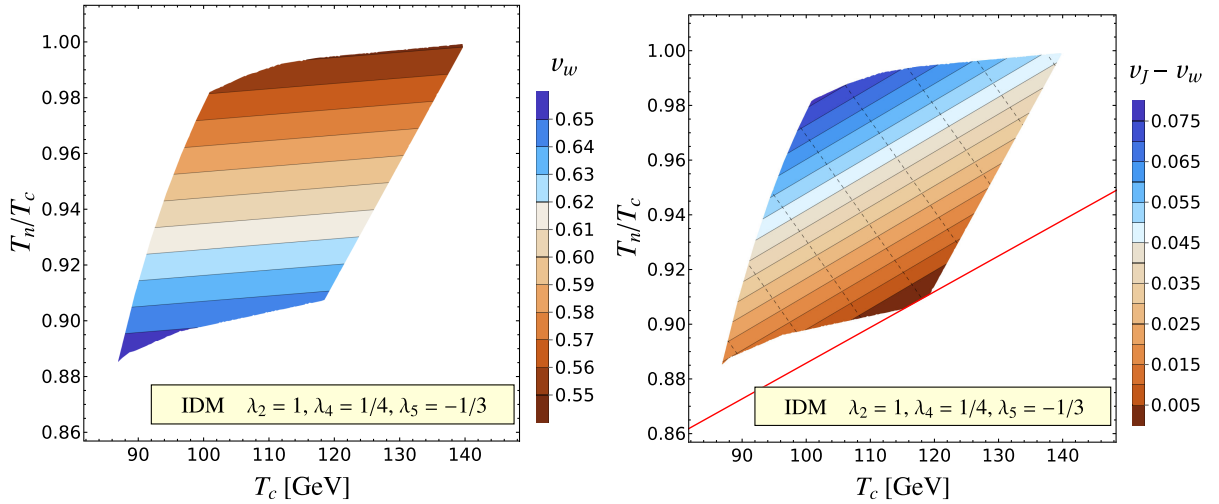


FIG. 19. *Left panel.* Contour plot of the wall velocity v_w in the $(T_c, T_n/T_c)$ plane for the IDM with $\lambda_2 = 1$, $\lambda_4 = 1/4$, and $\lambda_5 = -1/3$. *Right panel.* Contour plot of $(v_J - v_w)$. Dashed lines are for constant v_J . On the red line, $v_w = v_J$.

TABLE II. Fitting functions for the wall velocity (*second column*), the Jouguet velocity (*third column*) in terms of $T_c/v(\equiv x)$ and $T_n/T_c(\equiv y)$, and of the line of minimal T_n/T_c (*fourth column*) in the $\overline{\text{MS}}$ scheme with different values of the renormalization scale μ , and using the Arnold-Espinoza resummation method.

SSM $\lambda_s = 1$	$v_w(T_c/v, T_n/T_c)$	$v_J(T_c/v, T_n/T_c)$	$T_n/T_c _{\min}(T_c/v)$
$\overline{\text{MS}}, \mu = v/2$	$1.59 + 0.15x - 1.13y$	$0.97 - 0.22x - 0.24y$	$0.70 + 0.43x$
$\overline{\text{MS}}, \mu = v$	$1.59 + 0.14x - 1.12y$	$0.94 - 0.21x - 0.22y$	$0.72 + 0.39x$
$\overline{\text{MS}}, \mu = 2v$	$1.60 + 0.13x - 1.12y$	$0.93 - 0.19x - 0.21y$	$0.74 + 0.35x$
AE	$1.59 + 0.14x - 1.12y$	$0.96 - 0.22x - 0.23y$	$0.71 + 0.41x$

larger the $SU(2)$ representation of the scalar, while the coefficient of T_n/T_c slightly increases. The fitting functions for $T_n/T_c|_{\min}$ also show a mild model dependence. We observe a tendency to be shifted upward for scalars in

larger representations of $SU(2)$, as well as a tendency to become flatter.

We verified that the fitting functions are approximately independent of both the renormalization scheme and the

resummation method. For definiteness, we explored the impact of those in the SSM scenario with $\lambda_s = 1$. The results of the investigation are collected in Table II. The analysis presented above was performed using the on-shell renormalization scheme. Here we wish to investigate how the results change if we adopt the $\overline{\text{MS}}$ renormalization scheme. To this end, we first define the $\overline{\text{MS}}$ parameters appearing in the effective potential at $\mu = M_Z$ in terms of the pole masses and the Fermi constant using one-loop matching conditions. The parameters are then evolved to a chosen renormalization scale $\mu = \{v/2, v, 2v\}$ with one-loop β functions. Indicating the coefficients of the v_w fitting function with the symbol a_i , $v_w = a_1 + a_2(T_c/v) + a_3(T_n/T_c)$, we found variations at the percent level for a_1 and a_3 , and variations $\lesssim 10\%$ for a_2 . In this respect, it should be noted however that for typical values of $T_c \lesssim v$, the term a_2 gives a subdominant contribution to v_w , leaving the overall variation of v_w in the $\mu \in (v/2, 2v)$ interval at the percent level. As for resummation, changing the scheme from Parwani to Arnold-Espinosa [85], we found variations at or below the percent level for the three fitting coefficients. Slightly larger variations, with differences of order $\sim 10\%$ in some cases, are found for the coefficients of the v_J and $T_n/T_c|_{\min}$ fitting functions. We then conclude that, while the relations between input parameters and thermodynamic quantities are more affected by the choice of the schemes, the dependence of the velocity from the critical and nucleation temperature is instead less sensitive, suggesting an approximate model independence of v_w in the two-step phase transition scenario when expressed in terms of thermodynamic quantities.

VI. APPLICATIONS

As mentioned in the Introduction, the determination of the bubble wall dynamics is crucial to establish the cosmological signals related to a putative first order EWPhT. In this section we present results for the emitted spectrum of gravitational waves and for the matter asymmetry obtained

with the parameters calculated in the previous section. A similar analysis was recently presented in [78].

A. Gravitational waves

Gravitational waves are one of the most compelling cosmological relics of first order phase transitions. There are three main mechanisms of production: bubble collisions, sound waves, which are generated in the plasma after the collisions and before the expansion dissipates the kinetic energy, and turbulence effects that take longer to dissipate. For the thermal transitions of interest to our analysis, numerical simulations have shown that sound waves give the dominant contribution to the gravitational wave spectrum [86]. The frequency f^{peak} and the amplitude $\Omega_{\text{SW}}^{\text{peak}}$ at the peak of the spectrum for the sound wave contribution can be found in [18,86–95].

We show here the results for the peak amplitude and frequency for the three BSM scenarios considered. As showcases we limit ourselves to $\lambda_s = 1$ for the SSM, $\lambda_\sigma = 1$ for the RTSM, and $\lambda_2 = 1$ (with $\lambda_4 = 1/4$ and $\lambda_5 = -1/3$) for the IDM. As can be seen in Fig. 20 the three models show common predictions. Most of the deflagrations are below the sensitivity of the experimental proposals. Within the LTE approximation, we find that a small fraction of the two-step parameter space with a deflagration solution can lead to gravitational wave signals observable at BBO and very marginally at DECIGO. On the other hand, ultrarelativistic detonations tend to be more in reach of the proposed experiments, with LISA probing a fraction of the parameter space, which, as we verified, is larger for values of the self-coupling. This is expected since the detonation region opens up as the latter grows, allowing larger values of the portal coupling and, correspondingly, stronger transitions.

We expect these conclusions to get somehow worse with the inclusion of out-of-equilibrium contributions, as the latter decrease the value of the wall velocity with respect to

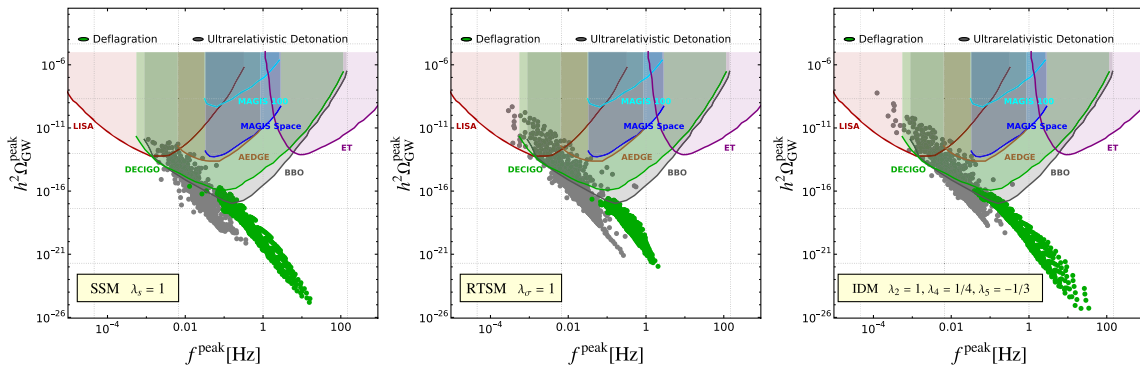


FIG. 20. Peak of the gravitational wave signal $h^2 \Omega_{\text{SW}}^{\text{peak}}$ as function of the peak frequency f_{peak} for the SSM with $\lambda_s = 1$ (left), for the RTSM with $\lambda_\sigma = 1$ (center), and for the IDM with $\lambda_2 = 1$, $\lambda_4 = 1/4$, $\lambda_5 = -1/3$ (right). Green (gray) points correspond to deflagration (ultrarelativistic detonation) profiles. Also shown are the sensitivity curves of some future interferometers.

the corresponding LTE solution for deflagrations and possibly convert some LTE detonations into deflagrations.

B. Electroweak baryogenesis

A crucial ingredient for EW baryogenesis is the presence of CP violation in the interactions between the wall and matter. This effect can be described, in the simplest scenarios, through an effective dimension-5 operator, such as $s\bar{Q}_L H t_R$, with complex coefficient, where Q_L and t_R are the left- and right-handed top-quark fields, H is the Higgs doublet, and s an additional scalar with a nontrivial spatial profile. The CP violation becomes efficient during the EWPhT, when both scalar fields acquire a space dependent VEV. As such, the phase in the top mass cannot be reabsorbed by a redefinition of the fermionic field. This operator emerges, for instance, from the nonlinear dynamics of composite Higgs models [30,96]. The relevant term in the Lagrangian is

$$\mathcal{L} \supset \frac{y_t}{\sqrt{2}} h(z) \bar{t}_L \left(1 + ic_5 \frac{s(z)}{\Lambda} \right) t_R + \text{h.c.}, \quad (62)$$

where Λ is the scale of the effective interaction and c_5 its Wilson coefficient. The latter gives rise to a field-dependent top-quark mass and phase

$$\begin{aligned} \tilde{m}_t(z) &= \frac{y_t h(z)}{\sqrt{2}} \sqrt{1 + c_5^2 \frac{s^2(z)}{\Lambda^2}}; \\ \theta(z) &= \arctan \left(c_5 \frac{s(z)}{\Lambda} \right). \end{aligned} \quad (63)$$

In the following, we will focus on the SSM scenario.

We have verified that the addition of this dimension-5 operator has a negligible impact on the PhT dynamics, so we can safely use all the results found in the previous section. With the mass term (63), the Boltzmann equation for the perturbations δf_i can be projected into its CP -even and its CP -odd parts. The former determines the out-of-equilibrium dynamics of the bubble wall, more precisely the (out-of-equilibrium) friction acting on the expanding front, while the CP -odd equation is the relevant one for the calculation of the baryon asymmetry. The latter is eventually determined by solving the equations for the CP -odd component of the chemical potential of the species in the plasma.

For details we refer to Ref. [31], whose method we closely follow here for the calculation. The CP -odd equations are solved by exploiting a moment expansion, truncated to second order. This strategy has been recently generalized to an arbitrary number n of moments in [97].

Compared to the LTE calculation of the wall velocity through the entropy conservation method, the upshot of our procedure is that we fully determine the plasma and field profiles (through the parameters δ_s , L_h and L_s). These enter in both the CP -even and the CP -odd components of the Boltzmann equation, and the final result for the asymmetry crucially depends on them. Our results thus provide a complete determination of the baryon asymmetry within the LTE approximation.

The outcome of the computation of the baryon asymmetry of the Universe (BAU) in the SSM with $\lambda_s = 1$, modified with the dimension five operator in Eq. (62) with $|c_5|/\Lambda = 1 \text{ TeV}^{-1}$, is shown in Fig. 21. The results are normalized in terms of the observed baryon asymmetry, $\eta_{\text{obs}} \sim 8.7 \times 10^{-11}$, so that successful baryogenesis requires

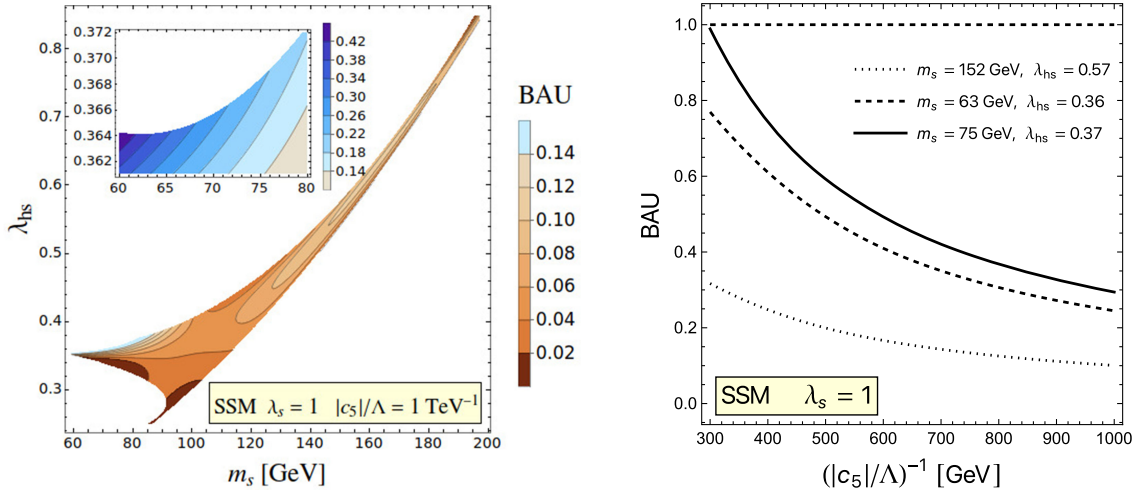


FIG. 21. *Left panel.* Contour plot of the BAU in the (m_s, λ_{hs}) plane for the SSM with $\lambda_s = 1$ augmented with the dimension-5 operator in Eq. (62). The scale Λ is set to $\Lambda = 1 \text{ TeV}$, and the Wilson coefficient is fixed to $c_5 = \pm 1$, with the sign chosen in order to realize a positive BAU. The results are normalized in terms of the observed baryon asymmetry. The inside figure is an enlargement of the largest one in a specific corner. *Right panel.* Plot of the BAU versus the $|c_5|/\Lambda$. The dotted, dashed and thick lines refer to the three benchmark points indicated in the figure, while the horizontal dashed line is for successful baryogenesis.

BAU = 1. In most of the parameter space, however, we find that the BAU is $\mathcal{O}(10^{-1}) - \mathcal{O}(10^{-2})$. The largest values are found in the leftmost upper corner of the parameter space, where the BAU grows up to ~ 0.4 . Differently from what one might naively expect, this is the region where the largest values for v_w are found. The reason for these results is that the BAU not only depends on the wall velocity, but more generally on the static (nucleation temperature T_n and VEVs h_n, s_n) and dynamical (v_w, δ_s, L_h, L_s) parameters of the transition, which vary nontrivially across the parameter space.

By consistently extracting their values for each point of the parameter space for which the EWPhT is strongly first order, it appears that, for the “augmented” SSM with $\lambda_s = 1$ and $|c_5|/\Lambda = 1 \text{ TeV}^{-1}$, a complete baryogenesis hardly can be obtained.

The right panel of Fig. 21 shows the dependence of the baryon asymmetry, within the LTE approximation, on the scale Λ (with $|c_5| = 1$). Since the BAU decreases with the latter, successful baryogenesis could be obtained for sufficiently small values of Λ but only in a corner of the parameter space.

This result highlights the importance of including out-of-equilibrium effects, which generally lead to a reduction in wall velocity across the entire parameter space, thereby facilitating the achievement of the BAU. This will be explored in a forthcoming work [98].

VII. CONCLUSIONS

In this paper, we performed a detailed and systematic investigation of the bubble wall dynamics during a first-order EWPhT within three representative extensions of the Higgs sector of the SM: the real singlet scalar model, the real triplet model, and the inert doublet model. Our study was performed under the assumption of local thermal equilibrium, using an optimized numerical algorithm, originally introduced in previous works [48,56,64] and further refined here, to extract the key dynamical properties of the bubble wall: the velocity v_w , the Higgs and scalar wall thicknesses (L_h and L_s), and the scalar field displacement δ_s . The full determination of the field and fluid profiles were crucial in the calculation of cosmological observables related to the transition, such as the EW baryogenesis.

Our approach relied on the solution of the full set of coupled scalar equations of motion along with the hydrodynamic conservation laws. Our results confirm that, within the LTE framework, steady-state solutions are of deflagration type. Detonations could be found when out-of-equilibrium effects are included.

An outcome of our study is the emergence of an approximate model-independence in the correlation between the bubble wall velocity and global phase transition parameters, such as the critical and nucleation temperatures. This behavior points to possible universal features in the hydrodynamic response of the plasma during EWPhTs, and may

simplify efforts to estimate bubble dynamics across a wide class of models without requiring a full microscopic analysis.

Our comparative study highlights several consistent trends across models. In particular, we observe a clear correlation between the wall velocity and the parameters of the phase transition, namely the critical temperature T_c and the amount of supercooling T_n/T_c . We showed that this also leads to an upper bound on T_n/T_c . An inverse relation between wall thicknesses and the strength of the phase transition is also observed, with thinner walls associated with stronger transitions. This trend is consistent with theoretical expectations.

Looking forward, the results presented here provide a step toward a fully out-of-equilibrium analysis, which we will pursue in a future work [98] by applying the complete numerical solution of the Boltzmann equation developed in [48,56,64]. Given that out-of-equilibrium effects introduce additional frictional forces, our LTE-based wall velocities should be interpreted as conservative upper bounds. We also expect corrections to the other wall properties including δ_s and $L_{h,s}$.

Capturing these effects will be essential for accurately assessing the viability of EW baryogenesis and the gravitational wave signatures of these transitions that we have preliminarily studied here within the LTE approximation. Overall, this work offers a robust framework for studying bubble dynamics in first-order phase transitions and sets the stage for more comprehensive kinetic treatments that are crucial for connecting theoretical models to cosmological observables.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX: FIELD-DEPENDENT THERMAL MASSES

In the following we give the expressions of the field-dependent masses and thermal masses used in the calculation of the one-loop finite temperature effective potential for the three models studied in this work.

1. SSM

The field-dependent masses $m_i^2(h, s)$ for the W and Z bosons, the top, and the Goldstones are given by

$$m_W^2 = \frac{g^2}{4} h^2, \quad m_Z^2 = \frac{g^2 + g'^2}{4} h^2, \quad m_t^2 = \frac{1}{2} y_t^2 h^2, \\ m_\chi^2 = \mu_h^2 + \lambda_h h^2 + \lambda_{hs} s^2. \quad (\text{A1})$$

As for the h and s fields, the mass matrix reads

$$\mathcal{M}_{hs}^2 = \begin{pmatrix} \mu_h^2 + 3\lambda_h h^2 + \lambda_{hs} s^2 & 2\lambda_{hs} hs \\ 2\lambda_{hs} hs & \mu_s^2 + 3\lambda_s s^2 + \lambda_{hs} h^2 \end{pmatrix}. \quad (\text{A2})$$

The structure of the perturbative expansion at finite temperature requires resummation of the leading self-energy daisy diagrams. In the Parwani scheme adopted in the paper, this is achieved by shifting the tree-level masses $m_i^2 \rightarrow m_i^2 + \Pi_i(T)$, where $\Pi_i(T)$ denotes the thermal contribution to the zero external-momentum self-energy that gives the thermal mass. In particular, comparing with the notation used in (60), we have,

$$\Pi_W(T) = \frac{11}{6} g^2 T^2, \quad (\text{A3})$$

$$\Pi_B(T) = \frac{11}{6} (g^2 + g'^2) T^2, \quad (\text{A4})$$

$$\Pi_h(T) = \left(\frac{3g^2 + g'^2}{16} + \frac{y_t^2}{4} + \frac{\lambda_h}{2} + \frac{\lambda_{hs}}{12} \right) T^2, \quad (\text{A5})$$

$$\Pi_s(T) = \left(\frac{\lambda_{hs}}{3} + \frac{\lambda_s}{4} \right) T^2, \quad (\text{A6})$$

with $\Pi_h = c_h T^2$ and $\Pi_s = c_s T^2$.

2. RTSM

We report here the analogous expressions for the RTSM. The field-dependent masses $m_i^2(h, \sigma_3)$ are obtained as

$$m_W^2 = \frac{g^2}{4} h^2 + g^2 \sigma_3^2, \quad m_Z^2 = \frac{g^2 + g'^2}{4} h^2, \quad m_t^2 = \frac{1}{2} y_t^2 h^2, \\ m_\chi^2 = \mu_h^2 + \lambda_h h^2 + \lambda_{h\sigma} \sigma_3^2. \quad (\text{A7})$$

$$m_\chi^2 = \mu_h^2 + \lambda_h h^2 + \lambda_{h\sigma} \sigma_3^2, \\ m_{\sigma_{1,2}}^2 = \mu_\sigma^2 + \lambda_\sigma \sigma_3^2 + \lambda_{h\sigma} h^2. \quad (\text{A8})$$

Concerning the fields h and σ_3 , the mass matrix reads

$$\mathcal{M}_{h\sigma_3}^2 = \begin{pmatrix} \mu_h^2 + 3\lambda_h h^2 + \lambda_{h\sigma} \sigma_3^2 & 2\lambda_{h\sigma} h \sigma_3 \\ 2\lambda_{h\sigma} h \sigma_3 & \mu_\sigma^2 + 3\lambda_\sigma \sigma_3^2 + \lambda_{h\sigma} h^2 \end{pmatrix}. \quad (\text{A9})$$

Finally, the thermal masses are obtained from

$$\Pi_W(T) = \frac{13}{6} g^2 T^2, \quad (\text{A10})$$

$$\Pi_B(T) = \frac{11}{6} (g^2 + g'^2) T^2, \quad (\text{A11})$$

$$\Pi_h(T) = \left(\frac{3g^2 + g'^2}{16} + \frac{y_t^2}{4} + \frac{\lambda_h}{2} + \frac{\lambda_{h\sigma}}{4} \right) T^2, \quad (\text{A12})$$

$$\Pi_\sigma(T) = \left(\frac{5}{12} \lambda_\sigma + \frac{g^2}{2} + \frac{\lambda_{h\sigma}}{3} \right) T^2. \quad (\text{A13})$$

3. IDM

We give here the expression of the field-dependent and thermal masses for the IDM. Again, field-dependent masses are obtained as

$$m_W^2 = \frac{g^2}{4} (h^2 + h'^2), \quad m_Z^2 = \frac{g^2 + g'^2}{4} (h^2 + h'^2), \\ m_t^2 = \frac{1}{2} y_t^2 h^2, \quad (\text{A14})$$

for the W , Z and the top, and from the following mass matrices for the scalar fields,

$$\mathcal{M}_{hh'}^2 = \begin{pmatrix} \mu_1^2 + 3\lambda_1 h^2 + \frac{\lambda_{345}}{2} h'^2 & \lambda_{345} h h' \\ \lambda_{345} h h' & \mu_2^2 + 3\lambda_2 h'^2 + \frac{\lambda_{345}}{2} h^2 \end{pmatrix}, \quad (\text{A15})$$

$$\mathcal{M}_{\chi^1 \chi^3 \chi^6}^2 = \begin{pmatrix} \mu_1^2 + \lambda_1 h^2 + \frac{\tilde{\lambda}_{345}}{2} h'^2 & \lambda_5 h h' \\ \lambda_5 h h' & \mu_2^2 + \lambda_2 h'^2 + \frac{\tilde{\lambda}_{345}}{2} h^2 \end{pmatrix}, \quad (\text{A16})$$

$$\mathcal{M}_{\chi^1 \chi^4, \chi^2 \chi^5}^2 = \begin{pmatrix} \mu_1^2 + \lambda_1 h^2 + \frac{\lambda_3}{2} h'^2 & \frac{\lambda_4 + \lambda_5}{2} h h' \\ \frac{\lambda_4 + \lambda_5}{2} h h' & \mu_2^2 + \lambda_2 h'^2 + \frac{\lambda_3}{2} h^2 \end{pmatrix}. \quad (\text{A17})$$

The thermal contributions to the zero external momentum self-energy are

$$\Pi_W(T) = 2g^2 T^2, \quad (\text{A18})$$

$$\Pi_B(T) = 2g^2 T^2, \quad (\text{A19})$$

$$\Pi_h(T) = \left(\frac{3g^2 + g'^2}{16} + \frac{y_t^2}{4} + \frac{6\lambda_1 + 2\lambda_3 + \lambda_4}{12} \right) T^2, \quad (\text{A20})$$

$$\Pi_{h'}(T) = \left(\frac{3g^2 + g'^2}{16} + \frac{6\lambda_2 + 2\lambda_3 + \lambda_4}{12} \right) T^2. \quad (\text{A21})$$

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