

$D^{(*)}\bar{B}^{(*)}$ dynamics in chiral effective field theoryZhe Liu^{1,2,5,*}, Hao Xu^{4,†} and Xiang Liu^{1,2,3,5,‡}¹*School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China*²*Lanzhou Center for Theoretical Physics, Key Laboratory of Theoretical Physics of Gansu Province, Key Laboratory of Quantum Theory and Applications of MoE, Gansu Provincial Research Center for Basic Disciplines of Quantum Physics, Lanzhou University, Lanzhou 730000, China*³*MoE Frontiers Science Center for Rare Isotopes, Lanzhou University, Lanzhou 730000, China*⁴*Institute of Theoretical Physics, College of Physics and Electronic Engineering, Northwest Normal University, Lanzhou 730070, China*⁵*Research Center for Hadron and CSR Physics, Lanzhou University and Institute of Modern Physics of CAS, Lanzhou 730000, China*

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In this work, we systematically study the interactions of the S -wave $D^{(*)}\bar{B}^{(*)}$ systems within the framework of chiral effective field theory in heavy hadron formalism. We calculate the $D^{(*)}\bar{B}^{(*)}$ effective potentials up to next-to-leading order, explore the bound state formations, and investigate the $D^{(*)}\bar{B}^{(*)}$ scattering properties, such as scattering rate, scattering length, and effective range. Our results show that all $I = 1$ $D^{(*)}\bar{B}^{(*)}$ potentials are repulsive, preventing the formation of bound states, while the $I = 0$ potentials are generally attractive. Specifically, we get two important observations: first, the shallow bound state is more likely to exist in the $D\bar{B}[I(J^P) = 0(0^+)]$ system than in the $D\bar{B}^*[I(J^P) = 0(1^+)]$ system; second, $D^*\bar{B}^*[I(J^P) = 0(0^+)]$ and $D^*\bar{B}^*[I(J^P) = 0(1^+)]$ systems possess relatively large binding energies and positive scattering lengths, which suggests strong bound state formations in these channels. So, the attractions in the $D^*\bar{B}^*[I = 0]$ systems are deeper than those in the $D\bar{B}^{(*)}[I = 0]$ systems, thus we strongly recommend the future experiment to search for the $D^*\bar{B}^*[I = 0]$ tetraquark systems. We also investigate the coupled-channel effects on the $J = 0, 1$ systems and conclude that the inclusion of the coupled channels introduces small but visible influences. In addition, we also investigate the dependencies of the $D\bar{B}^{(*)}$ binding energies on the contact low-energy coupling constants.

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I. INTRODUCTION

The study of exotic hadrons has become a vibrant frontier in particle physics, offering profound insights into the nonperturbative regime of strong interactions. In recent years, LHCb collaboration has made groundbreaking discoveries that have significantly advanced our understanding of multiquark states. In 2020, LHCb reported the observation of two resonances, $X_0(2900)$ and $X_1(2900)$, in the D^-K^+ invariant mass spectrum of the $B^+ \rightarrow D^+D^-K^+$ decay process [1,2]. These states, with quantum numbers $J^P = 0^+$ and $J^P = 1^+$, respectively, were

identified as the first tetraquark candidates with the exotic quark content $ud\bar{s}\bar{c}$. This discovery was followed in 2021 by the observation of the double-charm tetraquark state T_{cc}^+ in the $D^0D^0\pi^+$ invariant mass spectrum [3,4]. Shortly thereafter, LHCb reported two additional open heavy-flavor tetraquark candidates, $T_{c\bar{s}0}^a(2900)^{++}$ and $T_{c\bar{s}0}^a(2900)^0$, in the $D_s^+\pi^\pm$ final states of the $B^+ \rightarrow D^-D_s^+\pi^+$ and $B^0 \rightarrow \bar{D}^0D_s^+\pi^+$ decays [5,6]. These states are believed to have the quark content $c\bar{s}u\bar{d}$.

These experimental discoveries have spurred extensive theoretical investigations into the nature of open heavy-flavor multiquark states. Various approaches, including QCD sum rules, lattice QCD, and effective field theories, have been employed to explore their properties and structures (see Refs. [7–34] for comprehensive reviews). These studies have deepened our understanding of the exotic hadron spectrum and highlighted the importance of investigating the interactions of meson systems, particularly those involving doubly heavy quarks.

Doubly heavy meson systems have attracted significant theoretical interest due to their potential to form stable

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or quasistable molecular states. For instance, Li *et al.* explored the possibility of deuteronlike molecular states in $D^{(*)}D^{(*)}$, $\bar{B}^{(*)}\bar{B}^{(*)}$, and $D^{(*)}\bar{B}^{(*)}$ systems, identifying several promising candidates [35]. QCD sum rules have been applied to study exotic open-flavor tetraquark states, such as $bc\bar{q}\bar{q}$, $bc\bar{s}\bar{s}$, $qc\bar{q}\bar{b}$, and $sc\bar{s}\bar{b}$, revealing that some of these states lie below the $D^{(*)}B^{(*)}$ and $D_s^{(*)}B_s^{(*)}$ thresholds, making them susceptible to decay via fall-apart mechanisms [36]. The color-magnetic interaction model has also been used to calculate mass splittings for $qq\bar{Q}\bar{Q}$ tetraquark states [37], while the molecular picture has been employed to investigate the binding energies of T_{cc}^+ and its bottom and strange partners [38]. Additionally, the complex scaling method has been utilized to study doubly heavy tetraquark bound and resonant states [39].

The stability of doubly heavy tetraquarks, particularly those involving bottom and charm quarks, remains a topic of active debate. While some studies suggest that states such as $bb\bar{u}\bar{d}$, $bb\bar{u}\bar{s}$, and $bb\bar{d}\bar{s}$ are stable against strong decay [40], others argue that bottom-charm tetraquarks may not be stable [41–47]. The production mechanisms and detection prospects for these states at colliders have also been explored, with estimates suggesting significant production cross sections for double-bottom and mixed bottom-charm tetraquarks at the LHC [48].

Recent lattice QCD studies have provided further insights into the properties of doubly heavy tetraquarks. Meinel *et al.* found no evidence for QCD-stable $\bar{b}\bar{c}ud$ tetraquark states [49], while Alexandrou *et al.* and Padmanath *et al.* reported shallow bound states for isoscalar $\bar{b}\bar{c}ud$ systems [50,51]. These findings, along with the identification of subthreshold poles in the $D\bar{B}$ scattering amplitude [52], significantly advanced our understanding of the weak decay behaviors of $D\bar{B}$ molecules [53].

In this work, we investigate the interactions of S -wave $D\bar{B}$, $D\bar{B}^*$, and $D^*\bar{B}^*$ systems using chiral effective field theory (ChEFT) in the heavy hadron formalism. ChEFT, a low-energy effective theory of QCD, has proven to be a powerful tool for studying hadronic interactions, particularly in the context of nucleon-nucleon systems [54,55]. By incorporating heavy-quark and chiral symmetries, ChEFT provides a robust framework for analyzing the interactions of heavy mesons. Previous studies have applied ChEFT to systems such as $D^{(*)}D^{(*)}$ [56,57], $\bar{B}^{(*)}\bar{B}^{(*)}$ [58,60], and hidden-charm systems [59], as well as to the interactions of heavy pentaquark molecular states [61–63]. These investigations have demonstrated the effectiveness of ChEFT in describing the low-energy dynamics of heavy hadrons.

This paper is organized as follows: In Sec. II, we introduce the chiral effective Lagrangians with $SU(2)$ flavor symmetry used in our calculations. In Sec. III, we present the scattering amplitudes and derive the effective meson-meson potentials. In Sec. IV, we analyze the numerical results, examining the behaviors of the effective

potentials and searching for possible bound states by solving the Schrödinger equation. We also examine the coupled-channel effects. We discuss the scattering T -matrix and extract physical quantities, such as the scattering rate, scattering length, and effective range. Finally, in Sec. V, we summarize our findings and discuss their implications for future studies of doubly heavy tetraquark systems.

II. EFFECTIVE LAGRANGIANS

Within the framework of ChEFT in the heavy hadron formalism, the low-energy $D^{(*)} - \bar{B}^{(*)}$ scattering amplitudes are expanded order by order in terms of a small parameter $\epsilon = q/\Lambda_\chi$, where q represents the momentum of a Goldstone boson, the mass difference between D^* and D mesons (or $\bar{B}^*$ and $\bar{B}$ mesons), or the residual momentum of a heavy meson. The parameter Λ_χ denotes the chiral symmetry broken scale or the mass of the heavy mesons.

A. Lagrangians at the leading order

At leading order (LO), the Lagrangian describing the LO contact $D^{(*)}\bar{B}^{(*)}$ interaction is

$$\begin{aligned}\mathcal{L}_{4H}^{(0)} = & D_a \text{Tr}[H\gamma_\mu \bar{H}] \text{Tr}[H\gamma^\mu \bar{H}] \\ & + D_b \text{Tr}[H\gamma_\mu \gamma_5 \bar{H}] \text{Tr}[H\gamma^\mu \gamma_5 \bar{H}] \\ & + E_a \text{Tr}[H\gamma_\mu \tau^a \bar{H}] \text{Tr}[H\gamma^\mu \tau_a \bar{H}] \\ & + E_b \text{Tr}[H\gamma_\mu \gamma_5 \tau^a \bar{H}] \text{Tr}[H\gamma^\mu \gamma_5 \tau_a \bar{H}],\end{aligned}\quad (1)$$

where D_a , D_b , E_a , and E_b are independent low-energy coupling constants (LECs), and τ^a represents the Pauli matrix in isospin space. The heavy meson doublet (D, D^*) or $(\bar{B}, \bar{B}^*)$ is described by the H field in the heavy hadron formalism [56,58,59],

$$\begin{aligned}H &= \frac{1 + \not{v}}{2} (P_\mu^* \gamma^\mu + i P \gamma_5), \\ \bar{H} &= \gamma^0 H^\dagger \gamma^0 = (P_\mu^{\dagger*} \gamma^\mu + i P^\dagger \gamma_5) \frac{1 + \not{v}}{2}, \\ P &= (D^0, D^+) \quad \text{or} \quad (B^-, \bar{B}^0), \\ P_\mu^* &= (D^{*0}, D^{*+})_\mu \quad \text{or} \quad (B^{*-}, \bar{B}^{*0})_\mu,\end{aligned}\quad (2)$$

where $v = (1, 0, 0, 0)$ is the four-velocity of the heavy mesons. For the $D^*\bar{B}^*$ system, the scattering amplitudes receive contributions from the one-pion-exchange interactions at LO. The Lagrangian for the LO $D^*D^*\pi$ vertex or $\bar{B}^*\bar{B}^*\pi$ vertex is given by [56,58,59]

$$\begin{aligned}\mathcal{L}_{H\phi}^{(1)} = & -\langle (iv \cdot \partial H) \bar{H} \rangle + \langle H v \cdot \Gamma \bar{H} \rangle + g \langle H \not{\epsilon} \gamma_5 \bar{H} \rangle \\ & - \frac{1}{8} \Delta \langle H \sigma^{\mu\nu} \bar{H} \sigma_{\mu\nu} \rangle,\end{aligned}\quad (3)$$

$$\Gamma_\mu = \frac{i}{2} [\xi^\dagger, \partial_\mu \xi], \quad u_\mu = \frac{i}{2} \{\xi^\dagger, \partial_\mu \xi\}, \quad (4)$$

where Δ in the last term represents the mass splitting between D and D^* (or $\bar{B}$ and $\bar{B}^*$), Γ_μ is the chiral connection, u_μ is the axial vector current, and $\xi = \exp(i\phi/2f)$. Here, f is the bare pion decay constant, and the pion field ϕ is

$$\phi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}. \quad (5)$$

B. Lagrangians at next-to-leading order

At next-to-leading order (NLO), the scattering amplitudes receive contributions from one-loop corrections to the LO contact interaction, one-loop corrections to LO one-pion exchange (OPE), and the two-pion-exchange (TPE) terms. To renormalize these loop contributions, we employ the contact Lagrangians at NLO as follows:

$$\begin{aligned} \mathcal{L}_{AH}^{(2,h)} = & D_a^h \text{Tr}[H\gamma_\mu \bar{H}] \text{Tr}[H\gamma^\mu \bar{H}] \text{Tr}(\chi_+) \\ & + D_b^h \text{Tr}[H\gamma_\mu \gamma_5 \bar{H}] \text{Tr}[H\gamma^\mu \gamma_5 \bar{H}] \text{Tr}(\chi_+) \\ & + E_a^h \text{Tr}[H\gamma_\mu \tau^a \bar{H}] \text{Tr}[H\gamma^\mu \tau_a \bar{H}] \text{Tr}(\chi_+) \\ & + E_b^h \text{Tr}[H\gamma_\mu \gamma_5 \tau^a \bar{H}] \text{Tr}[H\gamma^\mu \gamma_5 \tau_a \bar{H}] \text{Tr}(\chi_+), \end{aligned} \quad (6)$$

$$\begin{aligned} \mathcal{L}_{4H}^{(2,v)} = & \{ D_{a1}^v \text{Tr}[(v \cdot DH)\gamma_\mu (v \cdot D\bar{H})] \text{Tr}[H\gamma^\mu \bar{H}] \\ & + D_{a2}^v \text{Tr}[(v \cdot DH)\gamma_\mu \bar{H}] \text{Tr}[(v \cdot DH)\gamma^\mu \bar{H}] \\ & + D_{a3}^v \text{Tr}[(v \cdot DH)\gamma_\mu \bar{H}] \text{Tr}[H\gamma^\mu (v \cdot D\bar{H})] \\ & + D_{a4}^v \text{Tr}[(v \cdot D)^2 H]\gamma_\mu \bar{H}] \text{Tr}[H\gamma^\mu \bar{H}] \\ & + D_{b1}^v \text{Tr}[(v \cdot DH)\gamma_\mu \gamma_5 (v \cdot D\bar{H})] \text{Tr}[H\gamma^\mu \gamma_5 \bar{H}] + \dots \\ & + E_{a1}^v \text{Tr}[(v \cdot DH)\gamma_\mu \tau^a (v \cdot D\bar{H})] \text{Tr}[H\gamma^\mu \tau_a \bar{H}] + \dots \\ & + E_{b1}^v \text{Tr}[(v \cdot DH)\gamma_\mu \gamma_5 \tau^a (v \cdot D\bar{H})] \text{Tr}[H\gamma^\mu \gamma_5 \tau_a \bar{H}] \\ & + \dots \} + \text{H.c.}, \end{aligned} \quad (7)$$

$$\begin{aligned} \mathcal{L}_{4H}^{(2,q)} = & \{ D_1^q \text{Tr}[(D^\mu H)\gamma_\mu \gamma_5 (D^\nu \bar{H})] \text{Tr}[H\gamma_\nu \gamma_5 \bar{H}] \\ & + D_2^q \text{Tr}[(D^\mu H)\gamma_\mu \gamma_5 \bar{H}] \text{Tr}[(D^\nu H)\gamma_\nu \gamma_5 \bar{H}] \\ & + D_3^q \text{Tr}[(D^\mu H)\gamma_\mu \gamma_5 \bar{H}] \text{Tr}[H\gamma_\nu \gamma_5 (D^\nu \bar{H})] \\ & + D_4^q \text{Tr}[(D^\mu D^\nu H)\gamma_\mu \gamma_5 \bar{H}] \text{Tr}[H\gamma_\nu \gamma_5 \bar{H}] \\ & + E_1^q \text{Tr}[(D^\mu H)\gamma_\mu \gamma_5 \tau^a (D^\nu \bar{H})] \text{Tr}[H\gamma_\nu \gamma_5 \tau_a \bar{H}] \\ & + \dots \} + \text{H.c.}, \end{aligned} \quad (8)$$

where

$$\tilde{\chi}_\pm = \chi_\pm - \frac{1}{2} \text{Tr}[\chi_\pm], \quad \chi_\pm = \xi^\dagger \chi \xi^\dagger \pm \xi \chi \xi, \quad \chi = m_\pi^2. \quad (9)$$

The low-energy constants appearing in the above equations are split into finite and infinite parts. The infinite parts

are used to cancel the divergences of the loop contributions at NLO. The finite parts also contribute but are not determined here due to the lack of data input. Therefore, we just discard them.

After the scattering amplitudes for the $D^{(*)}\bar{B}^{(*)}$ systems are calculated using the above Lagrangians, the effective potentials in momentum space can be obtained via the relation

$$\mathcal{V}(q) = -\frac{\mathcal{M}(q)}{4}, \quad (10)$$

where the factor $-1/4$ comes from the Breit approximation. To understand the interactions and further investigate whether the $D^*\bar{B}^{(*)}$ systems can form stable molecular states, we need the potentials in coordinate space. Fourier transformations of the momentum-space potentials allow for their derivation,

$$\mathcal{V}(r) = \int \frac{d^3\mathbf{q}}{(2\pi)^3} \mathcal{V}(\mathbf{q}) e^{-i\mathbf{q}\cdot\mathbf{r}} \mathcal{F}(\mathbf{q}), \quad (11)$$

where $\mathcal{F}(\mathbf{q})$ is a regulator function. To avoid ultraviolet (UV) divergence in the integral, we use a Gaussian regulator function $\mathcal{F}(\mathbf{q}) = \exp(-\mathbf{q}^2/\Lambda^{2n})$, and an adequately large n is adopted so that the powers generated by the regulator are beyond the order at which our calculation is performed so that it does not affect the accuracy at the given order [56,58,59,64,65]. Therefore, we chose $2n > \epsilon$ for the contact terms at order ϵ . Here, we take $n = 2$. Based on the potentials in coordinate space, we can solve the Schrödinger equations to search for the possible bound states.

Simultaneously, according to the effective potentials in momentum space, we can also solve the Lippmann-Schwinger equations to calculate the partial-wave two-body scattering amplitudes T ,

$$\begin{aligned} T_l(k, k') = & V_l(k, k') \\ & + \int \frac{q^2 dq}{(2\pi)^3} \mathcal{V}_l(k, q) \mathcal{G}(E, q) T_l(q, k'), \end{aligned} \quad (12)$$

where $E = p^2/(2\mu)$ is the energy and μ is the reduced mass. The Green's function $\mathcal{G}(E, q)$ is given by

$$\mathcal{G}(E, q) = \frac{1}{E - (q^2/2\mu) + i\epsilon}. \quad (13)$$

The relation between the T matrix and the phase shift δ is

$$T_l = e^{i\delta} \sin \delta_l. \quad (14)$$

Considering the S -wave, in the low-energy limit the $k \cot \delta_0(k)$ can be expanded in powers of k^2 ,

$$k \cot \delta_0(k) = -\frac{1}{a} + \frac{1}{2} r_0 k^2 + \dots, \quad (15)$$

where a is the scattering length and r_0 is the effective range. The ellipsis represents higher-order terms in k . The S -wave scattering cross section is proportional to the phase shift,

$$\sigma_0(k) = \frac{4\pi}{k^2} \sin^2 \delta_0(k). \quad (16)$$

To facilitate a comparison with lattice QCD results [50], we will also compute the scattering rate $k\sigma(k)$.

III. EFFECTIVE POTENTIALS OF THE $D^{(*)}\bar{B}^{(*)}$ SYSTEMS

A. $D\bar{B}$ systems

As shown in Fig. 1, at LO, there is only a tree-level contact diagram for the scattering process $D(p_1)\bar{B}(p_2) \rightarrow D(p_3)\bar{B}(p_4)$. Using the Lagrangians in Eq. (1), the amplitudes can be written as follows:

$$\mathcal{M}_{a1}^{(0)} = 4(D_a + E_a) \quad (17)$$

for isospin $I = 1$, and for isospin $I = 0$,

$$\mathcal{M}_{a1}^{(0)} = 4(D_a - 3E_a). \quad (18)$$

Next, we consider NLO contributions illustrated in Fig. 1. There are two types of NLO diagrams. The diagrams in Fig. 1 represent one-loop corrections to the LO contact interaction, and the corresponding amplitudes are given by

$$\mathcal{M}_{a1.1}^{(2)} = -4A(d-1)\frac{g_2^2}{f^2}J_{22}^g, \quad (19)$$

$$\mathcal{M}_{a1.2}^{(2)} = -4A(d-1)\frac{g_1^2}{f^2}J_{22}^g, \quad (20)$$

$$\mathcal{M}_{a1.3}^{(2)} = 4A(d-1)\frac{g_1g_2}{f^2}J_{22}^h, \quad (21)$$

$$\mathcal{M}_{a1.4}^{(2)} = 4A(d-1)\frac{g_1g_2}{f^2}J_{22}^h, \quad (22)$$

$$\mathcal{M}_{a1.5}^{(2)} = -\frac{3}{2}A(d-1)\frac{g_1^2}{f^2}\partial_\omega J_{22}^b, \quad (23)$$

$$\mathcal{M}_{a1.6}^{(2)} = -\frac{3}{2}A(d-1)\frac{g_1^2}{f^2}\partial_\omega J_{22}^b. \quad (24)$$

Here, d represents the space-time dimension, and the coefficient A depends on each diagram and isospin I , which is listed in Table I. The coupling constants g_1 and g_2 refer to the bare coupling constants for the $DD^*\pi$ and $\bar{B}\bar{B}^*\pi$ vertices, respectively.

Using the Lagrangians in Eq. (3), the amplitudes for the TPE diagrams in Fig. 1 can be written as

$$\begin{aligned} \mathcal{M}_{c1.1}^{(2)} = & -4\frac{1}{f^4}[A_1(q_0^2J_0^F + J_{22}^F) \\ & - A_{15}(q_0^2J_{11}^F + q_0^2J_{21}^F + J_{22}^F) \\ & - A_{51}(q_0^2J_{11}^F + q_0^2J_{21}^F + J_{22}^F) \\ & + A_5(q_0^2J_0^F + 2q_0^2J_{11}^F + q_0^2J_{21}^F + J_{22}^F)], \end{aligned} \quad (25)$$

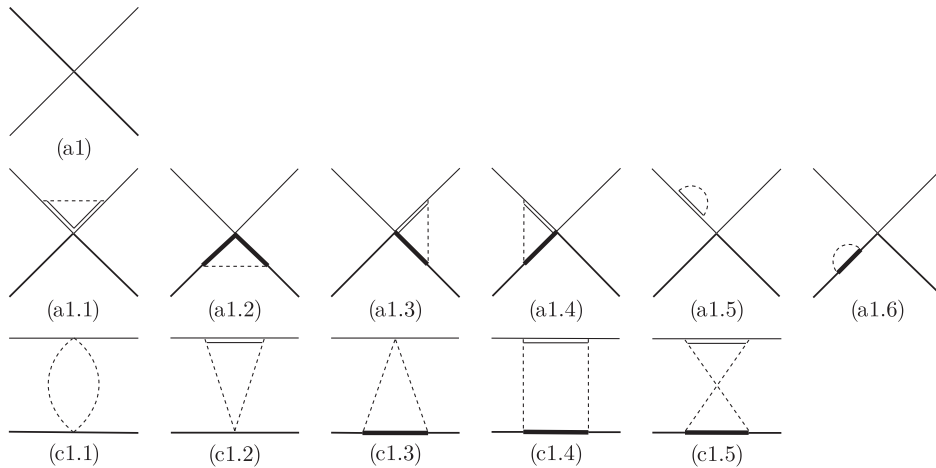


FIG. 1. LO contact (a1), NLO contact (a1.1–a1.6) and NLO TPE (c1.1–c1.5) diagrams of the process $D\bar{B} \rightarrow D\bar{B}$. The thin solid, double-thin solid, solid, thick solid, and dashed lines stand for the $\bar{B}$, $\bar{B}^*$, D , D^* , and a pion, respectively.

TABLE I. The coefficients appearing in the contact amplitudes [Eqs. (19)–(24)].

	$I = 1$	$I = 0$	ω_1	ω_2
	A	A		
$A_{a1.1}$	0	0	$-\delta_2$	$-\delta_2$
$A_{a1.2}$	0	0	$-\delta_1$	$-\delta_1$
$A_{a1.3}$	$\frac{1}{4}(D_b + E_b)$	$\frac{1}{4}(-3D_b + 9E_b)$	$-\delta_1$	$-\delta_2$
$A_{a1.4}$	$\frac{1}{4}(D_b + E_b)$	$\frac{1}{4}(-3D_b + 9E_b)$	$-\delta_1$	$-\delta_2$
$A_{a1.5}$	$D_a + E_a$	$D_a - 3E_a$	$-\delta_1$	0
$A_{a1.6}$	$D_a + E_a$	$D_a - 3E_a$	$-\delta_2$	0

$$\begin{aligned} \mathcal{M}_{c1.2}^{(2)} = & -4i \frac{g_2^2}{f^4} \{ A_1 [q_0 \vec{q}^2 J_{22}^T + \vec{q}^2 J_{24}^T - (d-1)q_0 J_{31}^T \\ & + q_0 \vec{q}^2 J_{32}^T + \vec{q}^2 J_{33}^T - (d-1)J_{34}^T] \\ & - A_5 [q_0 \vec{q}^2 J_{11}^T - (d-1)q_0 J_{21}^T \\ & + 2q_0 \vec{q}^2 J_{22}^T + \vec{q}^2 J_{24}^T - (d-1)q_0 J_{31}^T \\ & + q_0 \vec{q}^2 J_{32}^T + \vec{q}^2 J_{33}^T - (d-1)J_{34}^T] \}, \end{aligned} \quad (26)$$

$$\begin{aligned} \mathcal{M}_{c1.3}^{(2)} = & -4i \frac{g_2^2}{f^4} \{ A_1 [q_0 \vec{q}^2 J_{22}^T + \vec{q}^2 J_{24}^T - (d-1)q_0 J_{31}^T \\ & + q_0 \vec{q}^2 J_{32}^T + \vec{q}^2 J_{33}^T - (d-1)J_{34}^T] \\ & - A_5 [q_0 \vec{q}^2 J_{11}^T - (d-1)q_0 J_{21}^T \\ & + 2q_0 \vec{q}^2 J_{22}^T + \vec{q}^2 J_{24}^T - (d-1)q_0 J_{31}^T \\ & + q_0 \vec{q}^2 J_{32}^T + \vec{q}^2 J_{33}^T - (d-1)J_{34}^T] \}, \end{aligned} \quad (27)$$

$$\begin{aligned} \mathcal{M}_{c1.4}^{(2)} = & -4 \frac{g_1^2 g_2^2}{f^4} A [-\vec{q}^2 J_{21}^B + \vec{q}^4 J_{22}^B - 2(d+1)\vec{q}^2 J_{31}^B \\ & + 2\vec{q}^4 J_{32}^B + (d^2 - 1)J_{41}^B \\ & - 2(d+1)\vec{q}^2 J_{42}^B + \vec{q}^4 J_{43}^B], \end{aligned} \quad (28)$$

$$\begin{aligned} \mathcal{M}_{c1.5}^{(2)} = & -4 \frac{g_1^2 g_2^2}{f^4} A [-\vec{q}^2 J_{21}^R + \vec{q}^4 J_{22}^R - 2(d+1)\vec{q}^2 J_{31}^R \\ & + 2\vec{q}^4 J_{32}^R + (d^2 - 1)J_{41}^R \\ & - 2(d+1)\vec{q}^2 J_{42}^R + \vec{q}^4 J_{43}^R], \end{aligned} \quad (29)$$

where coefficients A_1 , A_{15} , A_{51} , and A_5 for each amplitude can be found in Table II.

In the above amplitudes (19)–(29), the loop functions $J_{ij}^{a/b}(m, \omega)$, $J_{ij}^{g/h}(m, \omega_1, \omega_2)$, $J_{ij}^F(m_1, m_2, q)$, $J_{ij}^{T/S}(m_1, m_2, \omega, q)$, and $J_{ij}^{B/R}(m_1, m_2, \omega_1, \omega_2, q)$ are abbreviated as $J_{ij}^{a/b}$, $J_{ij}^{g/h}$, J_{ij}^F , $J_{ij}^{T/S}$, and $J_{ij}^{B/R}$, respectively. The variables m , m_1 , and m_2 denote the pion mass, while the mass-splitting-dependent variables ω_1 and ω_2 for every amplitude

TABLE II. The coefficients appearing in the TPE amplitudes [Eqs. (25)–(29)]. Note that we have $A_{51} = A_{15}$.

	$I = 1$			$I = 0$			ω_1	ω_2
	A_1	A_5	A_{15}	A_1	A_5	A_{15}		
$A_{c1.1}$	$\frac{1}{16}$	$\frac{1}{16}$	$-\frac{1}{16}$	$-\frac{3}{16}$	$-\frac{3}{16}$	$\frac{3}{16}$	0	0
$A_{c1.2}$	$\frac{i}{8}$	$-\frac{i}{8}$	0	$-\frac{3i}{8}$	$\frac{3i}{8}$	0	$-\delta_2$	0
$A_{c1.3}$	$\frac{i}{8}$	$-\frac{i}{8}$	0	$-\frac{3i}{8}$	$\frac{3i}{8}$	0	$-\delta_1$	0
$A_{c1.4}$	$\frac{1}{16}$	0	0	$\frac{9}{16}$	0	0	$-\delta_1$	$-\delta_2$
$A_{c1.5}$	$\frac{5}{16}$	0	0	$-\frac{3}{16}$	0	0	$-\delta_1$	$-\delta_2$

are collected in Tables I and II. Here, we define $\delta_1 = M_{D^*} - M_D$ and $\delta_2 = M_{\bar{B}^*} - M_{\bar{B}}$ as the $D^* - D$ and $\bar{B}^* - \bar{B}$ mass differences, respectively. We list the definitions of the loop functions in Appendix B, and the calculation procedures of these loop functions follow the Refs. [56,59].

B. $D\bar{B}^*$ systems

We now focus on the $D\bar{B}^*$ system. As in the $D\bar{B}$ system, a contact term contributes to the $D\bar{B}^*$ scattering amplitudes at LO, as shown in Fig. 2. For the process $D(p_1)\bar{B}^*(p_2) \rightarrow D(p_3)\bar{B}^*(p_4)$, the scattering amplitude is given by

$$\mathcal{M}_{a2}^{(0)} = -4(D_a + E_a)(\epsilon_2 \cdot \epsilon_4^*), \quad (30)$$

with isospin $I = 1$, and

$$\mathcal{M}_{a2}^{(0)} = -4(D_a - 3E_a)(\epsilon_2 \cdot \epsilon_4^*), \quad (31)$$

with isospin $I = 0$, where ϵ_2 and ϵ_4^* are the polarization vectors of the initial $\bar{B}^*$ and final $\bar{B}^*$, respectively.

Next, in Fig. 2, we present the one-loop corrections to the contact diagram and the TPE diagrams of the $D\bar{B}^*$ system at NLO. We can calculate the amplitudes for these diagrams using the Feynman rules derived from the Lagrangian (3). The one-loop correction terms are

$$\mathcal{M}_{a2.1}^{(2)} = -4 \frac{g_2^2}{f^2} A J_{22}^g(\epsilon_2 \cdot \epsilon_4^*), \quad (32)$$

$$\mathcal{M}_{a2.2}^{(2)} = 4 \frac{g_2^2}{f^2} (d-3)(d-2) A J_{22}^g(\epsilon_2 \cdot \epsilon_4^*), \quad (33)$$

$$\mathcal{M}_{a2.3}^{(2)} = 4 \frac{g_1^2}{f^2} (d-1) A J_{22}^g(\epsilon_2 \cdot \epsilon_4^*), \quad (34)$$

$$\mathcal{M}_{a2.4}^{(2)} = -4 \frac{g_1 g_2}{f^2} A J_{22}^h(\epsilon_2 \cdot \epsilon_4^*), \quad (35)$$

$$\mathcal{M}_{a2.5}^{(2)} = -4 \frac{g_1 g_2}{f^2} A J_{22}^h(\epsilon_2 \cdot \epsilon_4^*), \quad (36)$$

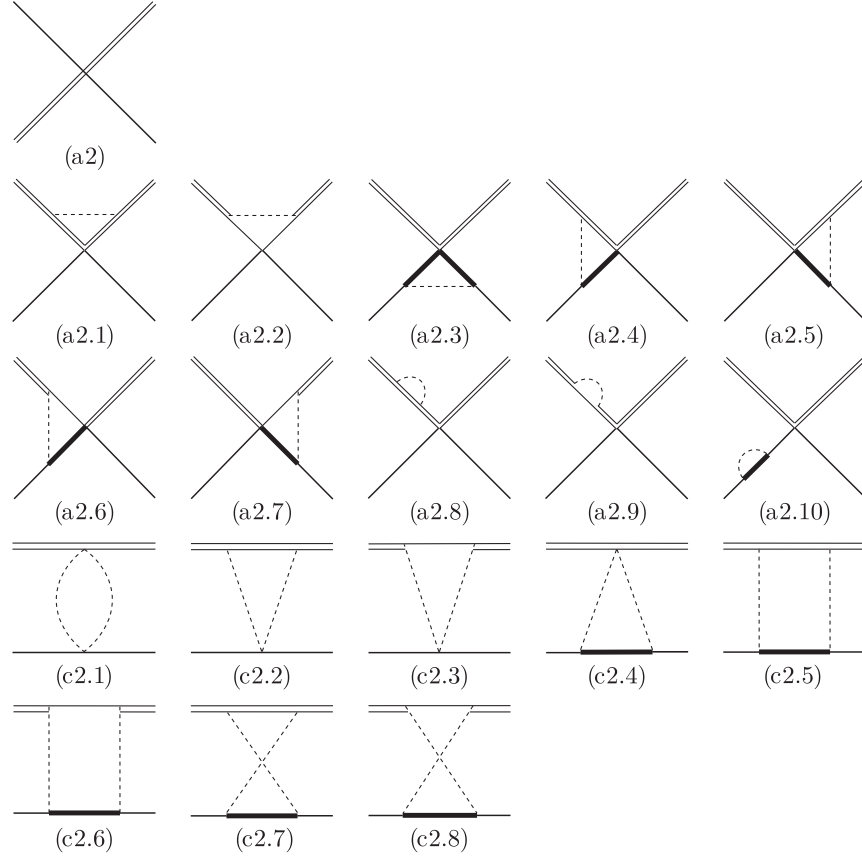


FIG. 2. LO contact (a2), NLO contact (a2.1–a2.10) and NLO TPE (c2.1–c2.8) diagrams of the process $D\bar{B}^* \rightarrow D\bar{B}^*$. The thin solid, double-thin solid, solid, thick solid, and dashed lines stand for the $\bar{B}$, $\bar{B}^*$, D , D^* , and a pion, respectively.

$$\mathcal{M}_{a2.6}^{(2)} = -4 \frac{g_1 g_2}{f^2} (d-3)(d-2) A J_{22}^h (\epsilon_2 \cdot \epsilon_4^*), \quad (37)$$

$$\mathcal{M}_{a2.7}^{(2)} = -4 \frac{g_1 g_2}{f^2} (d-3)(d-2) A J_{22}^h (\epsilon_2 \cdot \epsilon_4^*), \quad (38)$$

$$\begin{aligned} \mathcal{M}_{a2.(8+9)}^{(2)} &= \frac{3g_2^2}{2f^2} A [(d-2) \partial_\omega J_{22}^b(\omega_1) + \partial_\omega J_{22}^b(\omega_2)] \\ &\times (\epsilon_2 \cdot \epsilon_4^*), \end{aligned} \quad (39)$$

$$\mathcal{M}_{a2.10}^{(2)} = \frac{3g_1^2}{2f^2} (d-2) A \partial_\omega J_{22}^b (\epsilon_2 \cdot \epsilon_4^*). \quad (40)$$

The TPE diagrams consist of football, triangle, planar, and crossed box diagrams. The corresponding amplitudes are given by

$$\begin{aligned} \mathcal{M}_{c2.1}^{(2)} &= 4 \frac{1}{f^4} [A_1 (q_0^2 J_0^F + J_{22}^F) \\ &+ A_{15} (q_0^2 J_{11}^F + q_0^2 J_{21}^F + J_{22}^F) \\ &- A_{51} (q_0^2 J_{11}^F + q_0^2 J_{21}^F + J_{22}^F) \\ &+ A_5 (q_0^2 J_0^F + 2q_0^2 J_{11}^F \\ &+ q_0^2 J_{21}^F + J_{22}^F)] \epsilon_2 \cdot \epsilon_4^*, \end{aligned} \quad (41)$$

$$\begin{aligned} \mathcal{M}_{c2.2}^{(2)} &= i4 \frac{g_2^2}{f^4} \{ [A_1 (q_0 J_{22}^S + J_{24}^S + q_0 J_{32}^S + J_{33}^S) \\ &- A_5 (q_0 J_{11}^S + 2q_0 J_{22}^S + J_{24}^S \\ &+ q_0 J_{32}^S + J_{33}^S)] (q \cdot \epsilon_2) (q \cdot \epsilon_4^*) \\ &+ [A_1 (q_0 J_{31}^S + J_{34}^S) \\ &- A_5 (q_0 J_{21}^S + q_0 J_{31}^S + J_{34}^S)] (\epsilon_2 \cdot \epsilon_4^*) \}, \end{aligned} \quad (42)$$

$$\begin{aligned} \mathcal{M}_{c2.3}^{(2)} &= -i4 \frac{g_2^2}{f^4} (d-3) [A_1 (q_0 J_{22}^S + J_{24}^S + q_0 J_{32}^S + J_{33}^S) \\ &- A_5 (q_0 J_{11}^S + 2q_0 J_{22}^S + J_{24}^S \\ &+ q_0 J_{32}^S + J_{33}^S)] (q \cdot \epsilon_2) (q \cdot \epsilon_4^*) \\ &- i4 \frac{g_2^2}{f^4} (d-3) \{ A_1 [q_0 \vec{q}^2 J_{22}^S + \vec{q}^2 J_{24}^S \\ &+ (2-d) q_0 J_{31}^S + q_0 \vec{q}^2 J_{32}^S + \vec{q}^2 J_{33}^S \\ &+ (2-d) J_{34}^S] - A_5 [q_0 \vec{q}^2 J_{11}^S \\ &+ (2-d) q_0 (J_{21}^S + J_{31}^S) + 2q_0 \vec{q}^2 J_{22}^S + \vec{q}^2 J_{24}^S \\ &+ q_0 \vec{q}^2 J_{32}^S + \vec{q}^2 J_{33}^S + (2-d) J_{34}^S] \} (\epsilon_2 \cdot \epsilon_4^*), \end{aligned} \quad (43)$$

$$\begin{aligned} \mathcal{M}_{c2.4}^{(2)} = & -i4\frac{g_1^2}{f^4}\{A_1[q_0\vec{q}^2J_{22}^T + \vec{q}^2J_{24}^T + (1-d)q_0J_{31}^T \\ & + q_0\vec{q}^2J_{32}^T + \vec{q}^2J_{33}^T + (1-d)J_{34}^T] - A_5[q_0\vec{q}^2J_{11}^T \\ & + (1-d)q_0(J_{21}^T + J_{31}^T) + 2q_0\vec{q}^2J_{22}^T + \vec{q}^2J_{24}^T \\ & + q_0\vec{q}^2J_{32}^T + \vec{q}^2J_{33}^T + (1-d)J_{34}^T]\}(\epsilon_2 \cdot \epsilon_4^*), \quad (44) \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{c2.5}^{(2)} = & 4\frac{g_1^2g_2^2}{f^4}A_1\{[J_{21}^B - \vec{q}^2J_{22}^B + (d+3)(J_{31}^B + J_{42}^B) \\ & - 2\vec{q}^2J_{32}^B - \vec{q}^2J_{43}^B](q \cdot \epsilon_2)(q \cdot \epsilon_4^*) \\ & - [\vec{q}^2(J_{31}^B + J_{42}^B) - (1+d)J_{41}^B](\epsilon_2 \cdot \epsilon_4^*)\}, \quad (45) \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{c2.6}^{(2)} = & 4\frac{g_1^2g_2^2}{f^4}(d-3)A_1\{[J_{21}^B - \vec{q}^2J_{22}^B + (d+3)J_{31}^B \\ & - 2\vec{q}^2J_{32}^B + (d+3)J_{42}^B - \vec{q}^2J_{43}^B](q \cdot \epsilon_2)(q \cdot \epsilon_4^*) \\ & + [-\vec{q}^2J_{21}^B + \vec{q}^4J_{22}^B - (2d+1)\vec{q}^2(J_{31}^B + J_{42}^B) \\ & + 2\vec{q}^4J_{32}^B + (d+1)(d-2)J_{41}^B + \vec{q}^4J_{43}^B](\epsilon_2 \cdot \epsilon_4^*)\}, \quad (46) \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{c2.7}^{(2)} = & 4\frac{g_1^2g_2^2}{f^4}A_1\{[J_{21}^R - \vec{q}^2J_{22}^R + (d+3)(J_{31}^R + J_{42}^R) \\ & - 2\vec{q}^2J_{32}^R - \vec{q}^2J_{43}^R](q \cdot \epsilon_2)(q \cdot \epsilon_4^*) \\ & - [\vec{q}^2J_{31}^R - (1+d)J_{41}^R + \vec{q}^2J_{42}^R](\epsilon_2 \cdot \epsilon_4^*)\}, \quad (47) \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{c2.8}^{(2)} = & 4\frac{g_1^2g_2^2}{f^4}(d-3)A_1\{[J_{21}^R - \vec{q}^2J_{22}^R + (d+3)J_{31}^R \\ & - 2\vec{q}^2J_{32}^R + 3J_{42}^R - \vec{q}^2J_{43}^R](q \cdot \epsilon_2)(q \cdot \epsilon_4^*) \\ & + [-\vec{q}^2J_{21}^R + \vec{q}^4J_{22}^R - (2d+1)\vec{q}^2(J_{31}^R + J_{42}^R) \\ & + 2\vec{q}^4J_{32}^R + (d+1)(d-2)J_{41}^R + \vec{q}^4J_{43}^R](\epsilon_2 \cdot \epsilon_4^*)\}. \quad (48) \end{aligned}$$

TABLE III. The coefficients in the NLO contact amplitudes in Eqs. (32)–(40).

	$I = 1$	$I = 0$	ω_1	ω_2
	A	A		
$A_{a2.1}$	$\frac{1}{4}(3D_a - E_a)$	$\frac{3}{4}(D_a + E_a)$	δ_2	δ_2
$A_{a2.2}$	$\frac{1}{4}(3D_a - E_a)$	$\frac{3}{4}(D_a + E_a)$	0	0
$A_{a2.3}$	0	0	$-\delta_1$	$-\delta_1$
$A_{a2.4}$	$\frac{1}{4}(D_b + E_b)$	$\frac{-3}{4}(D_b - 3E_b)$	0	$-\delta_1$
$A_{a2.5}$	$\frac{1}{4}(D_b + E_b)$	$\frac{-3}{4}(D_b - 3E_b)$	$-\delta_1$	0
$A_{a2.6}$	$\frac{1}{4}(D_b + E_b)$	$\frac{-3}{4}(D_b - 3E_b)$	δ_2	$-\delta_1$
$A_{a2.7}$	$\frac{1}{4}(D_b + E_b)$	$\frac{-3}{4}(D_b - 3E_b)$	$-\delta_1$	δ_2
$A_{a2.(8+9)}$	$D_a + E_a$	$D_a - 3E_a$	0	δ_2
$A_{a2.10}$	$D_a + E_a$	$D_a - 3E_a$	$-\delta_1$	0

TABLE IV. The coefficients in the TPE amplitudes [Eqs. (41)–(48)]. Note that we have $A_{51} = A_{15}$.

	$I = 1$			$I = 0$			ω_1	ω_2
	A_1	A_5	A_{15}	A_1	A_5	A_{15}		
$A_{c2.1}$	$\frac{1}{16}$	$\frac{1}{16}$	$-\frac{1}{16}$	$-\frac{3}{16}$	$-\frac{3}{16}$	$\frac{3}{16}$	0	0
$A_{c2.2}$	$\frac{i}{8}$	$\frac{-i}{8}$	0	$-\frac{3i}{8}$	$\frac{3i}{8}$	0	0	0
$A_{c2.3}$	$\frac{i}{8}$	$-\frac{i}{8}$	0	$-\frac{3i}{8}$	$\frac{3i}{8}$	0	δ_2	0
$A_{c2.4}$	$\frac{i}{8}$	$-\frac{i}{8}$	0	$-\frac{3i}{8}$	$\frac{3i}{8}$	0	$-\delta_1$	0
$A_{c2.5}$	$\frac{1}{16}$	0	0	$\frac{9}{16}$	0	0	$-\delta_1$	0
$A_{c2.6}$	$\frac{1}{16}$	0	0	$\frac{9}{16}$	0	0	$-\delta_1$	δ_2
$A_{c2.7}$	$\frac{5}{16}$	0	0	$-\frac{3}{16}$	0	0	$-\delta_1$	0
$A_{c2.8}$	$\frac{5}{16}$	0	0	$\frac{-3}{16}$	0	0	$-\delta_1$	δ_2

The coefficients in the above expressions are listed in Tables III and IV. In this work, we focus on the S -wave interactions, so we replace the terms $\epsilon_2 \cdot \epsilon_4^*$ and $(q \cdot \epsilon_2)(q \cdot \epsilon_4^*)$ in the equations with [56,58,59]

$$(\epsilon_2 \cdot \epsilon_4^*) \mapsto -1, \quad (q \cdot \epsilon_2)(q \cdot \epsilon_4^*) \mapsto \frac{1}{d-1}\vec{q}^2. \quad (49)$$

C. $D^*\bar{B}^*$ systems

Next, we consider the $D^*\bar{B}^*$ systems. At LO, both the contact and OPE diagrams contribute to the scattering amplitudes, as shown in Fig. 3. At NLO, there are one-loop corrections to the LO diagrams and newly appeared TPE diagrams shown in Fig. 3. Compared to the $D\bar{B}$ and $D\bar{B}^*$ systems, the $D^*\bar{B}^*$ systems include significantly more diagrams, resulting in more complex interactions. The corresponding scattering amplitudes for these diagrams are listed in Eqs. (50)–(53) and (57)–(98).

At LO, utilizing the Lagrangian presented in Eqs. (1) and (3), the contact and OPE contributions of the scattering process $D^*(p_1)\bar{B}^*(p_2) \rightarrow D^*(p_3)\bar{B}^*(p_4)$ depicted in Fig. 3 read

$$\begin{aligned} \mathcal{M}_{a3}^{(0)} = & 4[(D_a + E_a)\mathcal{O}_1 - (D_b + E_b)\mathcal{O}_2 \\ & + (D_b + E_b)\mathcal{O}_3], \quad (50) \end{aligned}$$

$$\mathcal{M}_{b3}^{(0)} = -\frac{g_1g_2}{f^2}\frac{\mathcal{G}(q, \epsilon_1, \epsilon_2, \epsilon_3^*, \epsilon_4^*)}{q^2 - m_\pi^2} \quad (51)$$

for isospin $I = 1$, and we have

$$\begin{aligned} \mathcal{M}_{(a3)}^{(0)} = & 4[(D_a + E_a)\mathcal{O}_1 - (D_b + E_b)\mathcal{O}_2 \\ & + (D_b + E_b)\mathcal{O}_3], \quad (52) \end{aligned}$$

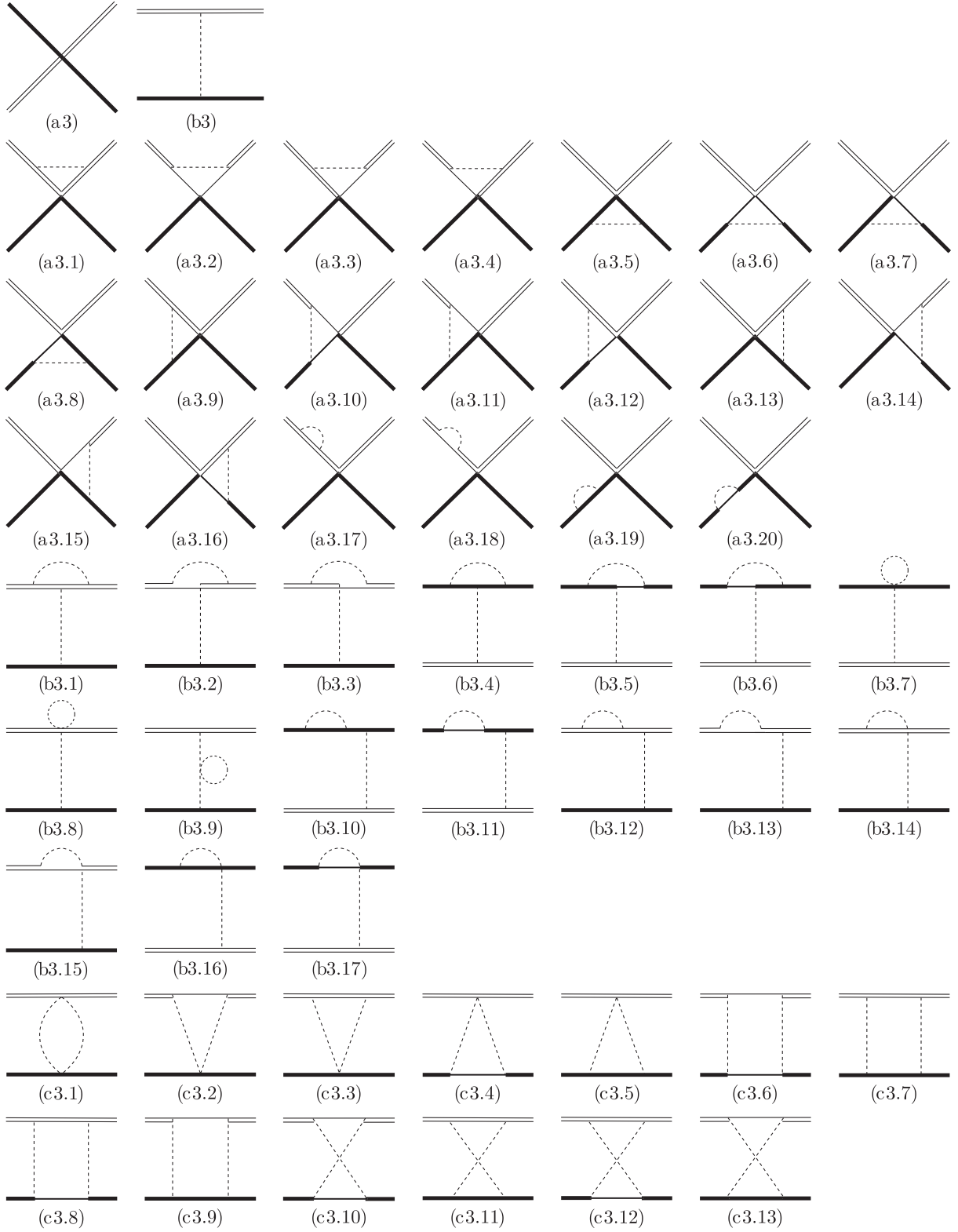


FIG. 3. LO contact (a3), LO OPE (b3), NLO contact (a3.1–a3.20), NLO OPE (b3.1–b3.17), and NLO TPE (c3.1–c3.13) diagrams of the process $D^* \bar{B}^* \rightarrow D^* \bar{B}^*$. The thin solid, double-thin solid, solid, thick solid, and dashed lines stand for the $\bar{B}$, $\bar{B}^*$, D , D^* , and a pion, respectively.

$$\mathcal{M}_{(b3)}^{(0)} = -\frac{g_1 g_2}{f^2} \frac{\mathcal{G}(q, \epsilon_1, \epsilon_2, \epsilon_3^*, \epsilon_4^*)}{q^2 - m_\pi^2} \quad (53)$$

for isospin $I = 0$.

Here, $\epsilon_1(\epsilon_2)$ and $\epsilon_3^*(\epsilon_4^*)$ stand for the polarization vectors of the initial $D^*(\bar{B}^*)$ and final $D^*(\bar{B}^*)$ mesons, respectively. For convenience, we define

$$\begin{aligned} \mathcal{O}_1 &= (\epsilon_1 \cdot \epsilon_3^*)(\epsilon_2 \cdot \epsilon_4^*), & \mathcal{O}_2 &= (\epsilon_1 \cdot \epsilon_4^*)(\epsilon_2 \cdot \epsilon_3^*), \\ \mathcal{O}_3 &= (\epsilon_1 \cdot \epsilon_2)(\epsilon_3^* \cdot \epsilon_4^*), & \mathcal{O}_2^a &= (q \cdot \epsilon_3^*)(q \cdot \epsilon_2)(\epsilon_1 \cdot \epsilon_4^*), \\ \mathcal{O}_2^b &= (q \cdot \epsilon_1)(q \cdot \epsilon_4^*)(\epsilon_2 \cdot \epsilon_3^*), \\ \mathcal{O}_3^a &= (q \cdot \epsilon_3^*)(q \cdot \epsilon_4^*)(\epsilon_1 \cdot \epsilon_2), \\ \mathcal{O}_3^b &= (q \cdot \epsilon_1)(q \cdot \epsilon_2)(\epsilon_3^* \cdot \epsilon_4^*), \end{aligned} \quad (54)$$

and

$$\begin{aligned} \mathcal{G}(q, \epsilon_1, \epsilon_2, \epsilon_3^*, \epsilon_4^*) &= \vec{q}^2(\mathcal{O}_2 - \mathcal{O}_3) + (\mathcal{O}_2^a - \mathcal{O}_3^a) \\ &\quad + (\mathcal{O}_2^b - \mathcal{O}_3^b). \end{aligned} \quad (55)$$

Similar to the definitions in (54), we also define

$$\begin{aligned} \mathcal{O}_1^a &= (q \cdot \epsilon_1)(q \cdot \epsilon_3^*)(\epsilon_2 \cdot \epsilon_4^*), \\ \mathcal{O}_1^b &= (q \cdot \epsilon_2)(q \cdot \epsilon_4^*)(\epsilon_1 \cdot \epsilon_3^*), \\ \mathcal{O}_1^c &= (q \cdot \epsilon_1)(q \cdot \epsilon_3^*)(q \cdot \epsilon_2)(q \cdot \epsilon_4^*), \\ \mathcal{O}_3^c &= (q \cdot \epsilon_1)(q \cdot \epsilon_2)(q \cdot \epsilon_3^*)(q \cdot \epsilon_4^*). \end{aligned} \quad (56)$$

The one-loop corrections to the contact amplitudes are listed as follows:

$$\begin{aligned} \mathcal{M}_{a_{3,1}}^{(2)} &= -4 \frac{g_2^2}{f^2} [(d-3)(d-2)A_1\mathcal{O}_1 - (d-3)A_2\mathcal{O}_2 \\ &\quad + (d-3)A_3\mathcal{O}_3]J_{22}^g, \end{aligned} \quad (57)$$

$$\mathcal{M}_{a_{3,2}}^{(2)} = -4 \frac{g_2^2}{f^2} A_1\mathcal{O}_1 J_{22}^g, \quad (58)$$

$$\mathcal{M}_{a_{3,3}}^{(2)} = -4 \frac{g_2^2}{f^2} (d-3)[A_1\mathcal{O}_2 - A_2\mathcal{O}_3]J_{22}^g, \quad (59)$$

$$\mathcal{M}_{a_{3,4}}^{(2)} = -4 \frac{g_2^2}{f^2} (d-3)[A_1\mathcal{O}_2 - A_2\mathcal{O}_3]J_{22}^g, \quad (60)$$

$$\begin{aligned} \mathcal{M}_{a_{3,5}}^{(2)} &= -4 \frac{g_1^2}{f^2} [(d-3)(d-2)A_1\mathcal{O}_1 - (d-3)A_2\mathcal{O}_2 \\ &\quad + (d-3)A_3\mathcal{O}_3]J_{22}^g, \end{aligned} \quad (61)$$

$$\mathcal{M}_{a_{3,6}}^{(2)} = -4 \frac{g_1^2}{f^2} A_1\mathcal{O}_1 J_{22}^g, \quad (62)$$

$$\mathcal{M}_{a_{3,7}}^{(2)} = -4 \frac{g_1^2}{f^2} (d-3)[A_1\mathcal{O}_2 - A_2\mathcal{O}_3]J_{22}^g, \quad (63)$$

$$\mathcal{M}_{a_{3,8}}^{(2)} = -4 \frac{g_1^2}{f^2} (d-3)[A_1\mathcal{O}_2 - A_2\mathcal{O}_3]J_{22}^g, \quad (64)$$

$$\mathcal{M}_{a_{3,9}}^{(2)} = 4 \frac{g_1 g_2}{f^2} (d-3)[A_1\mathcal{O}_1 - A_2\mathcal{O}_2 + A_3\mathcal{O}_3]J_{22}^h, \quad (65)$$

$$\mathcal{M}_{a_{3,10}}^{(2)} = 4 \frac{g_1^2}{f^2} A_1\mathcal{O}_3 J_{22}^h, \quad (66)$$

$$\mathcal{M}_{a_{3,11}}^{(2)} = 4 \frac{g_1^2}{f^2} (d-3)[A_1\mathcal{O}_1 - A_2\mathcal{O}_2]J_{22}^h, \quad (67)$$

$$\mathcal{M}_{a_{3,12}}^{(2)} = 4 \frac{g_1^2}{f^2} (d-3)[A_1\mathcal{O}_1 - A_2\mathcal{O}_2]J_{22}^h, \quad (68)$$

$$\mathcal{M}_{a_{3,13}}^{(2)} = 4 \frac{g_1 g_2}{f^2} (d-3)[A_1\mathcal{O}_1 - A_2\mathcal{O}_2 + A_3\mathcal{O}_3]J_{22}^h, \quad (69)$$

$$\mathcal{M}_{a_{3,14}}^{(2)} = 4 \frac{g_1 g_2}{f^2} A_1\mathcal{O}_3 J_{22}^h, \quad (70)$$

$$\mathcal{M}_{a_{3,15}}^{(2)} = 4 \frac{g_1 g_2}{f^2} (d-3)[A_1\mathcal{O}_1 - A_2\mathcal{O}_2]J_{22}^h, \quad (71)$$

$$\mathcal{M}_{a_{3,16}}^{(2)} = 4 \frac{g_1 g_2}{f^2} (d-3)[A_1\mathcal{O}_1 - A_2\mathcal{O}_2]J_{22}^h, \quad (72)$$

$$\mathcal{M}_{a_{3,(17+18)}}^{(2)} = -\frac{3g_2^2}{2f^2} A_1[(d-2)\partial_\omega J_{22}^b(\omega_1) + \partial_\omega J_{22}^b(\omega_2)], \quad (73)$$

$$\mathcal{M}_{a_{3,(19+20)}}^{(2)} = -\frac{3g_1^2}{2f^2} A_1[(d-2)\partial_\omega J_{22}^b(\omega_1) + \partial_\omega J_{22}^b(\omega_2)], \quad (74)$$

where coefficients appearing in the above contact amplitudes are shown in Table VI.

Next, we consider the NLO one-pion-exchange interactions in the $D^*\bar{B}^*$ systems. The corresponding diagrams are illustrated in Fig. 3, and the amplitudes

$$\mathcal{M}_{b_{3,1}}^{(2)} = 4 \frac{g_1^3 g_2}{f^2} A_1 J_{22}^g \frac{\mathcal{G}(q, \epsilon_1, \epsilon_2, \epsilon_3^*, \epsilon_4^*)}{q^2 - m_\pi^2}, \quad (75)$$

$$\mathcal{M}_{b_{3,2}}^{(2)} = -4 \frac{g_1^3 g_2}{f^2} A_1 J_{22}^g \frac{\mathcal{G}(q, \epsilon_1, \epsilon_2, \epsilon_3^*, \epsilon_4^*)}{q^2 - m_\pi^2}, \quad (76)$$

$$\mathcal{M}_{b_{3,3}}^{(2)} = -4 \frac{g_1^3 g_2}{f^2} A_1 J_{22}^g \frac{\mathcal{G}(q, \epsilon_1, \epsilon_2, \epsilon_3^*, \epsilon_4^*)}{q^2 - m_\pi^2}, \quad (77)$$

$$\mathcal{M}_{b3.4}^{(2)} = 4 \frac{g_1 g_2^3}{f^2} A_1 J_{22}^g \frac{\mathcal{G}(q, \epsilon_1, \epsilon_2, \epsilon_3^*, \epsilon_4^*)}{q^2 - m_\pi^2}, \quad (78)$$

$$\mathcal{M}_{b3.5}^{(2)} = 4 \frac{g_1 g_2^3}{f^2} A_1 J_{22}^g \frac{\mathcal{G}(q, \epsilon_1, \epsilon_2, \epsilon_3^*, \epsilon_4^*)}{q^2 - m_\pi^2}, \quad (79)$$

$$\mathcal{M}_{b3.6}^{(2)} = 4 \frac{g_1 g_2^3}{f^2} A_1 J_{22}^g \frac{\mathcal{G}(q, \epsilon_1, \epsilon_2, \epsilon_3^*, \epsilon_4^*)}{q^2 - m_\pi^2}, \quad (80)$$

$$\mathcal{M}_{b3.7}^{(2)} = -\frac{8g_1 g_2}{3f^2} A_1 J_0^g \frac{\mathcal{G}(q, \epsilon_1, \epsilon_2, \epsilon_3^*, \epsilon_4^*)}{q^2 - m_\pi^2}, \quad (81)$$

$$\mathcal{M}_{b3.8}^{(2)} = -\frac{8g_1 g_2}{3f^2} A_1 J_0^c \frac{\mathcal{G}(q, \epsilon_1, \epsilon_2, \epsilon_3^*, \epsilon_4^*)}{q^2 - m_\pi^2}, \quad (82)$$

$$\begin{aligned} \mathcal{M}_{b3.9}^{(2)} = & -\frac{8g_1 g_2}{3f^2} A_1 \left[2m^2 L + \frac{m^2}{16\pi^2} \log\left(\frac{m^2}{\mu^2}\right) \right] \\ & \times \frac{\mathcal{G}(q, \epsilon_1, \epsilon_2, \epsilon_3^*, \epsilon_4^*)}{q^2 - m_\pi^2}, \end{aligned} \quad (83)$$

$$\begin{aligned} \mathcal{M}_{b3.(10+11)}^{(2)} = & \frac{3g_1 g_2^3}{2f^2} A_1 [(d-2)\partial_\omega J_{22}^b(\omega_1) + \partial_\omega J_{22}^b(\omega_2)] \\ & \times \frac{\mathcal{G}(q, \epsilon_1, \epsilon_2, \epsilon_3^*, \epsilon_4^*)}{q^2 - m_\pi^2}, \end{aligned} \quad (84)$$

$$\begin{aligned} \mathcal{M}_{b3.(12+13)}^{(2)} = & \frac{3g_1 g_2^3}{2f^2} A_1 [(d-2)\partial_\omega J_{22}^b(\omega_1) + \partial_\omega J_{22}^b(\omega_2)] \\ & \times \frac{\mathcal{G}(q, \epsilon_1, \epsilon_2, \epsilon_3^*, \epsilon_4^*)}{q^2 - m_\pi^2}, \end{aligned} \quad (85)$$

$$\mathcal{M}_{b3.14}^{(2)} = \mathcal{M}_{b3.15}^{(2)} = \mathcal{M}_{b3.16}^{(2)} = \mathcal{M}_{b3.17}^{(2)} = 0. \quad (86)$$

The amplitudes for the TPE diagrams shown in Fig. 3 are calculated to be

$$\begin{aligned} \mathcal{M}_{c3.1}^{(2)} = & -4 \frac{1}{f^4} [A_1(q_0^2 J_{21}^F + J_{22}^F) \\ & - (A_{15} + A_{51} + A_5)(q_0^2 J_{11}^F + q_0^2 J_{21}^F + J_{22}^F) \\ & + A_5(q_0^2 J_0^F + q_0^2 J_{11}^F)] \mathcal{O}_1, \end{aligned} \quad (87)$$

$$\begin{aligned} \mathcal{M}_{c3.2}^{(2)} = & -i4 \frac{g_2^2}{f^4} \{ [A_1(q_0 J_{22}^S + J_{24}^S + q_0 J_{32}^S + J_{33}^S) \\ & - A_5(q_0 J_{11}^S + 2q_0 J_{22}^S + J_{24}^S + q_0 J_{32}^S + J_{33}^S)] \mathcal{O}_1^a \\ & + [A_1(q_0 J_{31}^S + J_{34}^S) \\ & - A_5(q_0 J_{21}^S + q_0 J_{31}^S + J_{34}^S)] \mathcal{O}_1 \}, \end{aligned} \quad (88)$$

$$\begin{aligned} \mathcal{M}_{c3.3}^{(2)} = & i4 \frac{g_2^2}{f^4} (d-3) [A_1(q_0 J_{22}^S + J_{24}^S + q_0 J_{32}^S + J_{33}^S) \\ & - A_5(q_0 J_{11}^S + 2q_0 J_{22}^S + J_{24}^S + q_0 J_{32}^S + J_{33}^S)] \mathcal{O}_1^a \\ & + i4 \frac{g_2^2}{f^4} (d-3) \{ A_1 [(q_0 \vec{q}^2 J_{22}^S + \vec{q}^2 J_{24}^S + (2-d)q_0 J_{31}^S \\ & + q_0 \vec{q}^2 J_{32}^S + \vec{q}^2 J_{33}^S + (2-d)q_0 J_{34}^S)] \\ & - A_5 [q_0 \vec{q}^2 J_{11}^S + (2-d)q_0 J_{21}^S + \vec{q}^2 J_{24}^S + (2-d)q_0 J_{31}^S \\ & + q_0 \vec{q}^2 J_{32}^S + \vec{q}^2 J_{33}^S + (2-d)q_0 J_{34}^S] \} \mathcal{O}_1, \end{aligned} \quad (89)$$

$$\begin{aligned} \mathcal{M}_{c3.4}^{(2)} = & i4 \frac{g_1^2}{f^4} \{ [A_1(q_0 J_{22}^T + J_{24}^T + q_0 J_{32}^T + J_{33}^T) \\ & - A_5(q_0 J_{11}^T + 2q_0 J_{22}^T + J_{24}^T + q_0 J_{32}^T + J_{33}^T)] \mathcal{O}_1^b \\ & + [A_1(q_0 J_{31}^T + J_{34}^T) \\ & - A_5(q_0 J_{21}^T + q_0 J_{31}^T + J_{34}^T)] \mathcal{O}_1 \}, \end{aligned} \quad (90)$$

$$\begin{aligned} \mathcal{M}_{c3.5}^{(2)} = & -i4 \frac{g_1^2}{f^4} (d-3) [A_1(q_0 J_{22}^T + J_{24}^T + q_0 J_{32}^T + J_{33}^T) \\ & - A_5(q_0 J_{11}^T + 2q_0 J_{22}^T + J_{24}^T + q_0 J_{32}^T + J_{33}^T)] \mathcal{O}_1^a \\ & - i4 \frac{g_1^2}{f^4} (d-3) \{ A_1 [(q_0 \vec{q}^2 J_{22}^T + \vec{q}^2 J_{24}^T \\ & + (2-d)q_0 J_{31}^T + q_0 \vec{q}^2 J_{32}^T + \vec{q}^2 J_{33}^T + (2-d)q_0 J_{34}^T)] \\ & - A_5 [q_0 \vec{q}^2 J_{11}^T + (2-d)q_0 J_{21}^T + \vec{q}^2 J_{24}^T \\ & + (2-d)q_0 J_{31}^T + q_0 \vec{q}^2 J_{32}^T + \vec{q}^2 J_{33}^T \\ & + (2-d)q_0 J_{34}^T] \} \mathcal{O}_1, \end{aligned} \quad (91)$$

$$\begin{aligned} \mathcal{M}_{c3.6}^{(2)} = & -4 \frac{g_1^2 g_2^2}{f^4} A_1 [(\mathcal{O}_1 + \mathcal{O}_2 + \mathcal{O}_3) J_{41}^B \\ & + \mathcal{O}_3^b (J_{21}^B + 2J_{31}^B + J_{42}^B) \\ & + (\mathcal{O}_1^a + \mathcal{O}_1^b + \mathcal{O}_2^a + \mathcal{O}_2^b) (J_{31}^B + J_{42}^B) + \mathcal{O}_3^a J_{42}^B \\ & + \mathcal{O}_3^c (J_{22}^B + 2J_{32}^B + J_{43}^B)], \end{aligned} \quad (92)$$

$$\begin{aligned} \mathcal{M}_{c3.7}^{(2)} = & -4 \frac{g_1^2 g_2^2}{f^4} (d-3)^2 A_1 \{ [(d-2)(d-1) \mathcal{O}_1 \\ & + \mathcal{O}_2 + \mathcal{O}_3] J_{41}^B + (\vec{q}^4 \mathcal{O}_1 + \vec{q}^2 \mathcal{O}_1^a \\ & + \vec{q}^2 \mathcal{O}_1^b + \mathcal{O}_1^c) (J_{22}^B + 2J_{32}^B + J_{43}^B) \\ & - (\vec{q}^2 \mathcal{O}_1 + \mathcal{O}_1^a + \mathcal{O}_1^b - \mathcal{O}_3^a) J_{21}^B \\ & - [2d\vec{q}^2 \mathcal{O}_1 + (d+2)\mathcal{O}_1^a + (d+2)\mathcal{O}_1^b - \mathcal{O}_2^a \\ & - \mathcal{O}_2^b - \mathcal{O}_3^b] (J_{31}^B + J_{42}^B) + \mathcal{O}_3^b J_{31}^B + \mathcal{O}_3^c J_{42}^B \}, \end{aligned} \quad (93)$$

$$\begin{aligned}\mathcal{M}_{c3.8}^{(2)} = & 4 \frac{g_1^2 g_2^2}{f^4} (d-3) A_1 \{ (-d\mathcal{O}_1 + \mathcal{O}_2 + \mathcal{O}_3) J_{41}^B \\ & + (\vec{q}^2 \mathcal{O}_1^b + \mathcal{O}_3^c) (J_{22}^B + 2J_{32}^B + J_{43}^B) \\ & + (\mathcal{O}_2^b - \mathcal{O}_1^b) J_{21}^B + [\vec{q}^2 \mathcal{O}_1 + \mathcal{O}_1^a - (d+2)\mathcal{O}_1^b \\ & + \mathcal{O}_2^b + \mathcal{O}_3^a + \mathcal{O}_3^b] (J_{31}^B + J_{42}^B) + \mathcal{O}_2^b J_{31}^B + \mathcal{O}_2^a J_{42}^B \},\end{aligned}\quad (94)$$

$$\begin{aligned}\mathcal{M}_{c3.9}^{(2)} = & 4 \frac{g_1^2 g_2^2}{f^4} (d-3) A_1 \{ (-d\mathcal{O}_1 + \mathcal{O}_2 + \mathcal{O}_3) J_{41}^B \\ & + (\vec{q}^2 \mathcal{O}_1^a + \mathcal{O}_3^c) (J_{22}^B + 2J_{32}^B + J_{43}^B) \\ & + (\mathcal{O}_2^a - \mathcal{O}_1^a) J_{21}^B + [\vec{q}^2 \mathcal{O}_1 - (d+2)\mathcal{O}_1^a \\ & + \mathcal{O}_1^b + \mathcal{O}_2^a + \mathcal{O}_3^a + \mathcal{O}_3^b] (J_{31}^B + J_{42}^B) \\ & + \mathcal{O}_2^a J_{31}^B + \mathcal{O}_2^b J_{42}^B \},\end{aligned}\quad (95)$$

$$\begin{aligned}\mathcal{M}_{c3.10}^{(2)} &= 5\mathcal{M}_{c3.6}^{(2)} / \cdot \{J^B \rightarrow J^R\}, \\ \mathcal{M}_{c3.11}^{(2)} &= 5\mathcal{M}_{c3.7}^{(2)} / \cdot \{J^B \rightarrow J^R\},\end{aligned}\quad (96)$$

$$\begin{aligned}\mathcal{M}_{c2.12}^{(2)} = & 4 \frac{g_1^2 g_2^2}{f^4} (d-3) A_1 \{ (-d\mathcal{O}_1 + \mathcal{O}_2 + \mathcal{O}_3) J_{41}^R \\ & + (\vec{q}^2 \mathcal{O}_1^b + \mathcal{O}_3^c) (J_{22}^R + 2J_{32}^R + J_{43}^R) \\ & + (\mathcal{O}_2^b - \mathcal{O}_1^b) J_{21}^R + [\vec{q}^2 \mathcal{O}_1 + \mathcal{O}_1^a - (d+2)\mathcal{O}_1^b \\ & + \mathcal{O}_2^a + \mathcal{O}_2^b + \mathcal{O}_3^b] (J_{31}^R + J_{42}^R) \\ & + \mathcal{O}_3^b J_{31}^R + \mathcal{O}_3^a J_{42}^R \},\end{aligned}\quad (97)$$

TABLE V. The values of the products of polarization vectors in the S -wave effective potentials with total angular momentum $J = 0, 1$, and 2 , respectively [58].

Terms	$J = 0$	$J = 1$	$J = 2$
$\mathcal{O}_1$	1	1	1
$\mathcal{O}_2$	1	-1	1
$\mathcal{O}_3$	3	0	0

$$\begin{aligned}\mathcal{M}_{c3.13}^{(2)} = & 4 \frac{g_1^2 g_2^2}{f^4} (d-3) A_1 \{ (-d\mathcal{O}_1 + \mathcal{O}_2 + \mathcal{O}_3) J_{41}^R \\ & + (\vec{q}^2 \mathcal{O}_1^b + \mathcal{O}_3^c) (J_{22}^R + 2J_{32}^R + J_{43}^R) \\ & + (\mathcal{O}_2^b - \mathcal{O}_1^b) J_{21}^R + [\vec{q}^2 \mathcal{O}_1 + \mathcal{O}_1^a - (d+2)\mathcal{O}_1^b \\ & + \mathcal{O}_2^a + \mathcal{O}_2^b + \mathcal{O}_3^a] (J_{31}^R + J_{42}^R) \\ & + \mathcal{O}_3^a J_{31}^R + \mathcal{O}_3^b J_{42}^R \}.\end{aligned}\quad (98)$$

The coefficients in the above NLO OPE and TPE amplitudes are listed in Tables VII and VIII, respectively. To further evaluate the calculated amplitudes, we should deal with the terms in (54) according to the partial wave we considered. In the S -wave, for the terms like $(q \cdot \epsilon_i)(q \cdot \epsilon_j^*)(\epsilon_k \cdot \epsilon_l^*)$, we can make the following substitutions:

$$\begin{aligned}(\epsilon_i \cdot \epsilon_j)(q \cdot \epsilon_k)(q \cdot \epsilon_l) &\mapsto -\frac{1}{d-1} \vec{q}^2 (\epsilon_i \cdot \epsilon_j)(\epsilon_k \cdot \epsilon_l), \\ (q \cdot \epsilon_i)(q \cdot \epsilon_j)(q \cdot \epsilon_k)(q \cdot \epsilon_l) &\mapsto \frac{1}{(d-1)^2} \vec{q}^4 (\epsilon_i \cdot \epsilon_j)(\epsilon_k \cdot \epsilon_l),\end{aligned}$$

together with Table V.

TABLE VI. The coefficients appearing in the contact term amplitudes at NLO [Eqs. (57)–(74)].

	$I = 1$			$I = 0$			ω_1 ω_2	
	A_1	A_2	A_3	A_1	A_2	A_3		
$A_{a3.1}$	$\frac{1}{4}(3D_a - E_a)$	$\frac{1}{4}(3D_b - E_b)$	$\frac{1}{4}(3D_b - E_b)$	$\frac{3}{4}(D_a + E_a)$	$\frac{3}{4}(D_b + E_b)$	$\frac{3}{4}(D_b + E_b)$	0	0
$A_{a3.2}$	$\frac{1}{4}(3D_a - E_a)$	0	0	$\frac{3}{4}(D_a + E_a)$	0	0	δ_2	δ_2
$A_{a3.3(4)}$	$\frac{1}{4}(3D_b - E_b)$	$\frac{1}{4}(3D_b - E_b)$	0	$\frac{3}{4}(D_b + E_b)$	$\frac{3}{4}(D_b + E_b)$	0	$0(\delta_2)$	$\delta_2(0)$
$A_{a3.5}$	0	0	0	0	0	0	0	0
$A_{a3.6}$	0	0	0	0	0	0	δ_1	δ_1
$A_{a3.7(8)}$	0	0	0	0	0	0	$0(\delta_1)$	$\delta_1(0)$
$A_{a3.9}$	$\frac{1}{4}(D_b + E_b)$	$\frac{1}{4}(D_a + E_a)$	$\frac{1}{4}[(d-3)(D_b + E_b) + (D_a + E_a)]$	$\frac{-3}{4}(D_b - 3E_b)$	$\frac{-3}{4}(D_a - 3E_a)$	$\frac{-3}{4}[(d-3)(D_b - 3E_b) + (D_a - 3E_a)]$	0	0
$A_{a3.10}$	$\frac{1}{4}(D_b + E_b)$	0	0	$\frac{-3}{4}(D_b - 3E_b)$	0	0	δ_1	δ_2
$A_{a3.11(12)}$	$\frac{1}{4}(D_b + E_b)$	$\frac{1}{4}(D_b + E_b)$	0	$\frac{-3}{4}(D_b - 3E_b)$	$\frac{-3}{4}(D_b - 3E_b)$	0	$0(\delta_1)$	$\delta_2(0)$
$A_{a3.13}$	$\frac{1}{4}(D_b + E_b)$	$\frac{1}{4}(D_a + E_a)$	$\frac{1}{4}[(d-3)(D_b + E_b) + (D_a + E_a)]$	$\frac{-3}{4}(D_b - 3E_b)$	$\frac{-3}{4}(D_a - 3E_a)$	$\frac{-3}{4}[(d-3)(D_b - 3E_b) + (D_a - 3E_a)]$	0	0
$A_{a3.14}$	$\frac{1}{4}(D_b + E_b)$	0	0	$\frac{-3}{4}(D_b - 3E_b)$	0	0	δ_1	δ_2
$A_{a3.15(16)}$	$\frac{1}{4}(D_b + E_b)$	$\frac{1}{4}(D_b + E_b)$	0	$\frac{-3}{4}(D_b - 3E_b)$	$\frac{-3}{4}(D_b - 3E_b)$	0	$0(\delta_1)$	$\delta_2(0)$
$A_{a3.(17+18)}$	$D_a + E_a$	$D_b + E_b$	$D_b + E_b$	$D_a - 3E_a$	$D_b - 3E_b$	$D_b - 3E_b$	0	δ_2
$A_{a3.(19+20)}$	$D_a + E_a$	$D_b + E_b$	$D_b + E_b$	$D_a - 3E_a$	$D_b - 3E_b$	$D_b - 3E_b$	0	δ_1

TABLE VII. The constants A (as well as $\omega_{1,2}$) appearing in the OPE amplitudes [Eqs. (75)–(86)].

	$A_{b3.1}$	$A_{b3.2(3)}$	$A_{b3.4}$	$A_{b3.5(6)}$	$A_{b3.7}$	$A_{b3.8}$	$A_{b3.9}$	$A_{b3.(10+11)}$	$A_{b3.(12+13)}$	$A_{b3.14(15)}$	$A_{b3.16(17)}$
$I = 1$	$-\frac{1}{16}$	$-\frac{1}{16}$	$-\frac{1}{16}$	$-\frac{1}{16}$	$-\frac{1}{12}$	$-\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0
$I = 0$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{3}{4}$	$-\frac{3}{4}$	$-\frac{3}{4}$	0	0
ω_1	0	$\delta_2(0)$	0	$0(\delta_1)$	0	0	0	0	0	0	0
ω_2	0	$0(\delta_2)$	0	$\delta_1(0)$	0	0	0	δ_1	δ_2	0	0

TABLE VIII. The constants appearing in the TPE amplitudes [Eqs. (87)–(98)]. Note that we have $A_{51} = A_{15}$.

	$I = 1$			$I = 0$			ω_1	ω_2
	A_1	A_5	A_{15}	A_1	A_5	A_{15}		
$A_{c3.1}$	$\frac{1}{16}$	$\frac{1}{16}$	$-\frac{1}{16}$	$-\frac{3}{16}$	$-\frac{3}{16}$	$\frac{3}{16}$	0	0
$A_{c3.2}$	$\frac{i}{8}$	$-\frac{i}{8}$	0	$-\frac{3i}{8}$	$\frac{3i}{8}$	0	δ_2	0
$A_{c3.3}$	$\frac{i}{8}$	$-\frac{i}{8}$	0	$-\frac{3i}{8}$	$\frac{3i}{8}$	0	0	0
$A_{c3.4}$	$\frac{i}{8}$	$-\frac{i}{8}$	0	$-\frac{3i}{8}$	$\frac{3i}{8}$	0	δ_1	0
$A_{c3.5}$	$\frac{i}{8}$	$-\frac{i}{8}$	0	$-\frac{3i}{8}$	$\frac{3i}{8}$	0	0	0
$A_{c3.6}$	$\frac{1}{16}$	0	0	$\frac{9}{16}$	0	0	δ_1	δ_2
$A_{c3.7}$	$\frac{1}{16}$	0	0	$\frac{9}{16}$	0	0	0	0
$A_{c3.8}$	$\frac{1}{16}$	0	0	$\frac{9}{16}$	0	0	δ_1	0
$A_{c3.9}$	$\frac{1}{16}$	0	0	$\frac{9}{16}$	0	0	0	δ_2
$A_{c3.10}$	$\frac{5}{16}$	0	0	$-\frac{3}{16}$	0	0	δ_1	δ_2
$A_{c3.11}$	$\frac{5}{16}$	0	0	$-\frac{3}{16}$	0	0	0	0
$A_{c3.12}$	$\frac{5}{16}$	0	0	$-\frac{3}{16}$	0	0	δ_1	0
$A_{c3.13}$	$\frac{5}{16}$	0	0	$-\frac{3}{16}$	0	0	0	δ_2

IV. NUMERICAL RESULTS AND DISCUSSIONS

A. Potentials in coordinate space and possible bound states

After obtaining the scattering amplitudes, we now evaluate the effective potentials and analyze their behavior in coordinate space. To obtain the numerical results, we use the following LECs, determined by the resonance

saturation model [56,58,59,66–69]:

$$D_a = -13.23 \text{ GeV}^{-2}, \quad E_a = -11.49 \text{ GeV}^{-2}. \quad (99)$$

Other parameters include $m_\pi = 0.139 \text{ GeV}$, the pion decay constant $f_\pi = 0.086 \text{ GeV}$, the renormalization scale $\mu = 4\pi f_\pi$, $\delta_1 = 0.142 \text{ GeV}$, $\delta_2 = 0.045 \text{ GeV}$, and the coupling constants $g_1 = 0.65$ and $g_2 = 0.52$ [56,58].

Then, we will substitute the potentials into the Schrödinger equation and search for the bound states. In this work, we regularize the effective potentials using the Gauss regulator $\mathcal{F}(\mathbf{q}) = \exp(-\mathbf{q}^2/\Lambda^2)$ to prevent the divergence at high-momentum transfer. Usually, the value of the cutoff parameter Λ in chiral effective field theory is around 0.5 GeV [54,61,70].

1. $D\bar{B}$ systems

Firstly, we analyze the behaviors of the effective potentials of the $D\bar{B}$ systems, which receive contributions from the contact and the TPE terms. There are two channels: $I(J^P) = 1(0^+)$ and $I(J^P) = 0(0^+)$, and their effective potentials in coordinate space are shown in Figs. 4(a) and 4(b), respectively. Here, we adopt the cutoff parameter Λ with 0.5 GeV .

For the $I(J^P) = 1(0^+)$ state in Fig. 4(a), we observe that although attractive, the TPE interaction is much weaker than the repulsive contact interaction, resulting in an overall repulsive potential. Accordingly, no bound states are found in this system.

For the $I(J^P) = 0(0^+)$ system, the effective potentials are shown in Fig. 4(b). Compared to the $I(J^P) = 1(0^+)$

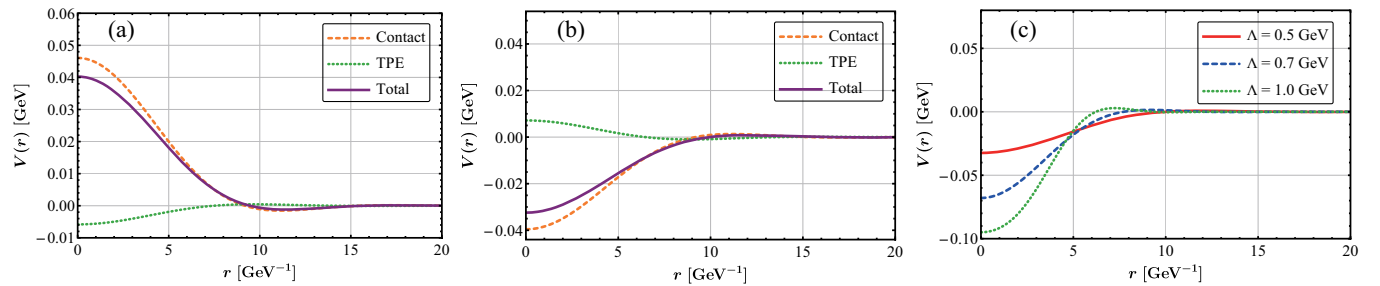


FIG. 4. S -wave potential of the $D\bar{B}$ systems and the dependence of the total potential on the cutoff parameter, namely, (a) the potential of the $I(J^P) = 1(0^+)$ $D\bar{B}$ system at $\Lambda = 0.5 \text{ GeV}$, (b) the potential of the $I(J^P) = 0(0^+)$ $D\bar{B}$ system at $\Lambda = 0.5 \text{ GeV}$, and (c) the total potential of the $D\bar{B}[1(0^+)]$ system with different cutoffs. The green dashed, red dashed, and purple solid lines represent the contact, TPE, and full potential, respectively.

system, we can find that the TPE potential is weakly repulsive, while the contact potential is attractive. However, the summed attraction is too weak, so no bound state is found in this system.

It is worth noting that the effective potentials of the $I(J^P) = 1(0^+)$ $D\bar{B}$ system have similar behaviors with the potentials of the $I(J^P) = 1(0^+)$ $\bar{B}\bar{B}$ system in Ref. [58], which is a consequence of heavy flavor symmetry between the charm and bottom quarks. As shown in Fig. 4(c), the total potentials become more and more attractive as the cutoff parameter Λ increases. The bound states emerge at $\Lambda = 0.7$ GeV and $\Lambda = 1.0$ GeV, lying 0.8 MeV and 3.6 MeV below the $D\bar{B}$ mass threshold, respectively. However, if we change the cutoff to 0.54 GeV, the very shallow bound state can emerge in the $D\bar{B}[0(0^+)]$ channel.

2. $D\bar{B}^*$ systems

In this section, we investigate the interactions of the $D\bar{B}^*$ systems with $I(J^P) = 1(1^+)$ and $I(J^P) = 0(1^+)$. As shown in Fig. 2 before, only the contact diagram contributes to the amplitudes at LO. The one-loop corrections to the contact diagrams and TPE diagrams appear at NLO. Their results are shown in Figs. 5(a) and 5(b), where we adopt the cutoff parameter Λ with 0.5 GeV.

Comparing Fig. 5(a) with Fig. 4(a), we observe that the behavior of the $D\bar{B}^*[1(1^+)]$ effective potential is similar to the $D\bar{B}[1(0^+)]$ effective potential. For the $D\bar{B}^*[1(1^+)]$ system, the repulsive contact interaction is much stronger than the attractive TPE interaction, which leads to a net repulsive potential. As a result, no bound state exists in this system.

In the case of the $D\bar{B}^*[0(1^+)]$ system, the effective potentials are shown in Fig. 5(b). Compared to the $1(1^+)$ system, the behaviors of the $0(1^+)$ contact and TPE potential are both reversed, which results in an overall attractive interaction. However, this potential is weaker even than the one in the previous $D\bar{B}[0(0^+)]$ system. As shown in Fig. 5(c), the total potentials at $\Lambda = 0.7$ GeV and at $\Lambda = 0.5$ GeV are of similar strength. However, when

$\Lambda = 1.0$ GeV, the effective potential is repulsive near the origin but becomes attractive at intermediate distances. This behavior appears because the TPE contribution at large momentum is more sensitive to cutoff Λ variations, which leads to a repulsive potential at short distances. In our calculation, no bound state is found in the $D\bar{B}^*[0(1^+)]$ system.

Therefore, we conclude that the interaction in the $D\bar{B}^*[0(1^+)]$ system is weaker than that in the $D\bar{B}[0(0^+)]$ system. This is consistent with the predictions of the one-boson-exchange potential model [35], but contrasts with the conclusions drawn from lattice QCD calculations [50].

3. $D^*\bar{B}^*$ systems

In this section, we explore the possibility of bound states in the $D^*\bar{B}^*$ systems, which includes six different isospin-spin states: $1(0^+)$, $0(0^+)$, $1(1^+)$, $0(1^+)$, $1(2^+)$, and $0(2^+)$. At LO, both contact and OPE contribute to the effective potential. At NLO, the one-loop corrections to the contact and OPE diagrams shown in Fig. 3 as well as TPE diagrams all contribute.

First, we focus on the $J^P = 0^+$ $D^*\bar{B}^*$ state. The effective potentials are shown in Figs. 6(a) and 6(b), where we adopt the cutoff parameter Λ with 0.5 GeV. In Fig. 6(a), we observe that both the contact and OPE interactions are repulsive, while the TPE interaction is attractive. However, the total potential is repulsive, meaning no bound state can exist in the $1(0^+)$ $D^*\bar{B}^*$ system. In contrast, for the $0(0^+)$ system in Fig. 6(b), the contact and OPE interactions provide an attractive force, while the TPE interaction is repulsive. The repulsive TPE contribution partially counteracts the attractive contact interaction, but the OPE interaction remains strong enough to form a bound state. The resulting binding energy is $\Delta E = 6.0$ MeV. As shown in Fig. 6, the OPE terms can contribute to the total interaction of $D^*\bar{B}^*$ systems, which is a departure from the $D\bar{B}^{(*)}$ systems.

The Λ dependence of the $I(J^P) = 0(0^+)$ total potential is shown in Fig. 6(c). We find that the total interaction is sensitive to the cutoff, particularly near the origin, which

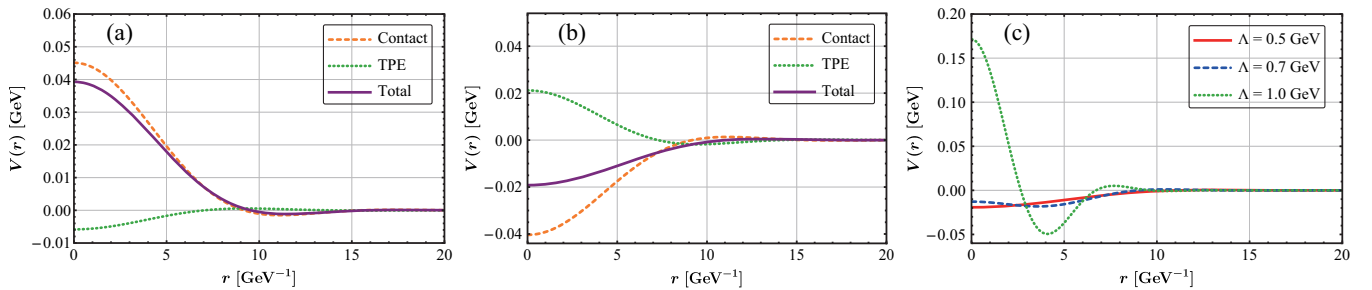


FIG. 5. S-wave potential of the $D\bar{B}^*$ systems and the dependence of the total potential on the cutoff parameter, namely, (a) the potential of the $I(J^P) = 1(1^+)$ $D\bar{B}^*$ system at $\Lambda = 0.5$ GeV, (b) the potential of the $I(J^P) = 0(1^+)$ $D\bar{B}^*$ system at $\Lambda = 0.5$ GeV, and (c) the total potential of the $D\bar{B}^*[0(1^+)]$ system with different cutoffs. The green dashed and red dashed lines represent the contact and TPE contributions, respectively. The purple solid lines stand for the full potential.

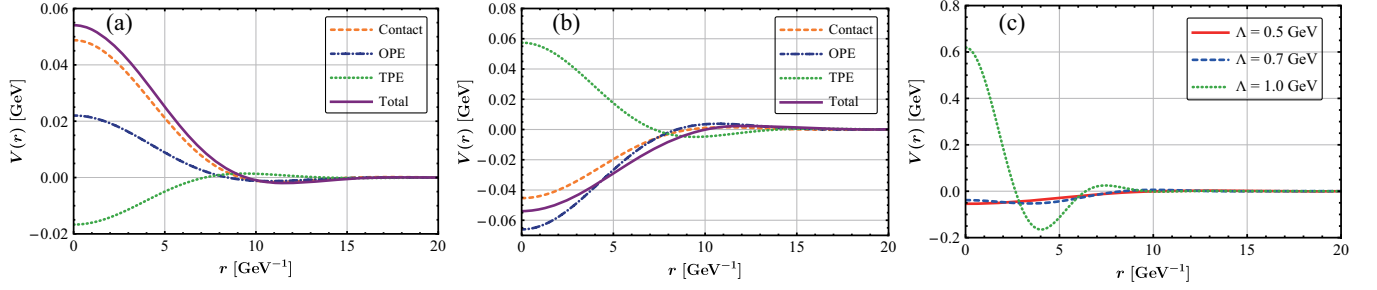


FIG. 6. S -wave potential of $J = 0$ $D^*\bar{B}^*$ systems and the dependence of the total potential on the cutoff parameter, namely, (a) the potential of the $I(J^P) = 1(0^+)D^*\bar{B}^*$ system at $\Lambda = 0.5$ GeV, (b) the potential of the $I(J^P) = 0(0^+)D^*\bar{B}^*$ system at $\Lambda = 0.5$ GeV, and (c) the total potential of the $D^*\bar{B}^*[0(0^+)]$ system with different cutoffs. The green dashed, blue dashed, and red dashed lines represent the contact, OPE, and TPE contributions, respectively. The purple solid lines stand for the full potential.

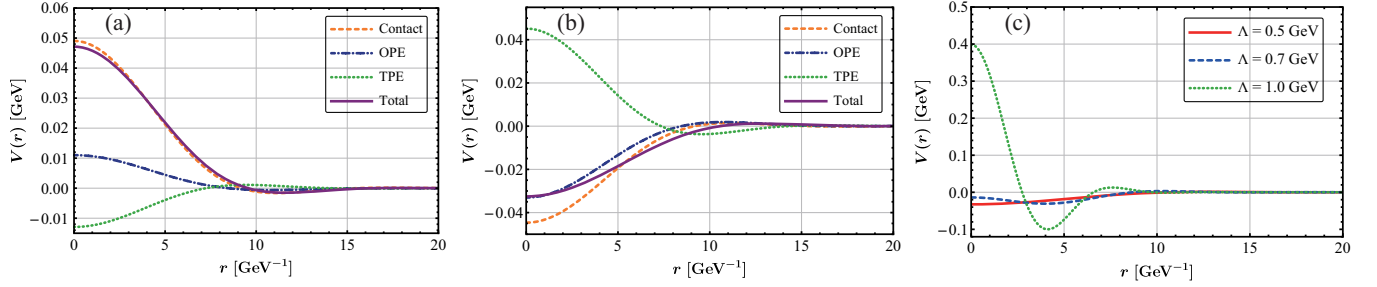


FIG. 7. S -wave potential of $J = 1$ $D^*\bar{B}^*$ systems and the dependence of the total potential on the cutoff parameter, namely, (a) the potential of the $I(J^P) = 1(1^+)D^*\bar{B}^*$ system at $\Lambda = 0.5$ GeV, (b) the potential of the $I(J^P) = 0(1^+)D^*\bar{B}^*$ system at $\Lambda = 0.5$ GeV, and (c) the total potential of the $D^*\bar{B}^*[0(1^+)]$ system with different cutoffs. The green dashed, blue dashed, and red dashed lines represent the contact, OPE, and TPE contributions, respectively. The purple solid lines stand for the full potential.

stems from the TPE contribution. As the cutoff increases to 1.0 GeV, the binding energy becomes 35.9 MeV.

Next, we analyze the interactions in the $D^*\bar{B}^*[1^+]$ system. For the $1(1^+)$ state, as seen in Fig. 7(a), the OPE contribution is largely offset by the TPE contribution, with the repulsive contact potential remaining. It results in a fully repulsive total potential, meaning no bound state exists in the $D^*\bar{B}^*[1(1^+)]$ system. For the $D^*\bar{B}^*[0(1^+)]$ system, the effective potentials, shown in Fig. 7(b), exhibit similar behaviors to those of the $D^*\bar{B}^*[0(0^+)]$ system. Although the attraction is not as strong as in the $D^*\bar{B}^*[0(0^+)]$ system,

it is still sufficient to form a shallow bound state. By solving the Schrödinger equation, we obtain the $D^*\bar{B}^*[0(1^+)]$ binding energy $\Delta E = 0.6$ MeV at $\Lambda = 0.5$ GeV.

Finally, we examine the possibility of bound state formations in the $J^P = 2^+ D^*\bar{B}^*$ system. In Fig. 8(a), for the $1(2^+)D^*\bar{B}^*$ system, both the OPE and TPE potentials are mildly attractive, while the dominant contact potential remains repulsive, resulting in a repulsive total potential.

We also calculate the effective potentials for the $D^*\bar{B}^*[0(2^+)]$ system, with the potential profiles shown in

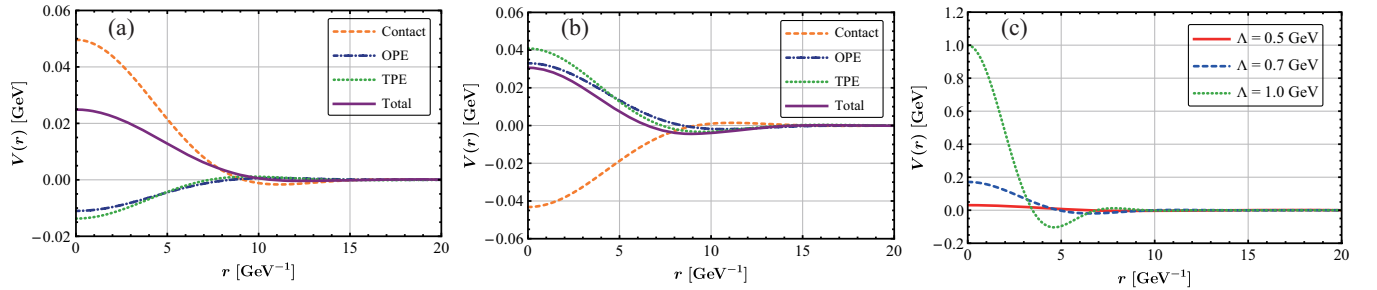


FIG. 8. S -wave potential of $J = 2$ $D^*\bar{B}^*$ systems and the dependence of the total potential on the cutoff parameter, namely, (a) the potential of the $I(J^P) = 1(2^+)D^*\bar{B}^*$ system at $\Lambda = 0.5$ GeV, (b) the potential of the $I(J^P) = 0(2^+)D^*\bar{B}^*$ system at $\Lambda = 0.5$ GeV, and (c) the total potential of the $D^*\bar{B}^*[0(2^+)]$ system with the different cutoffs. The green dashed, blue dashed, and red dashed lines represent the contact, OPE, and TPE contributions, respectively. The purple solid lines stand for the full potential.

TABLE IX. The bound states in the five $D^{(*)}\bar{B}^{(*)}$ systems with $I = 0$. The cutoff parameter Λ , state mass M , and binding energy E are in units of GeV, MeV, and MeV respectively. “...” means that there is no bound state.

Λ	$D\bar{B}$		$D\bar{B}^*$		$D^*\bar{B}^*(J=0)$		$D^*\bar{B}^*(J=1)$		$D^*\bar{B}^*(J=2)$	
	M	E	M	E	M	E	M	E	M	E
0.5	...	...	...	...	7235.7	6.0	7241.1	0.6	...	...
0.7	7143.6	0.8	...	...	7230.6	11.1	7240.0	1.7	...	...
1.0	7140.8	3.6	...	...	7205.8	35.9	7232.3	9.4	7237.4	4.3

Fig. 8(b). In this situation, both the OPE and TPE act as repulsive forces that counter the attractive force from the contact term, leading to an overall repulsive potential. This behavior contrasts with the attractive effective potential of the other $D^{(*)}\bar{B}^{(*)}$ systems with $I = 0$. Consequently, there is no evident bound state in the $D^*\bar{B}^*[0(2^+)]$ system.

Similarly, the effective potentials of $D^*\bar{B}^*[0(2^+)]$ at different cutoff parameters are shown in Fig. 8(c). Although the total interaction is repulsive at small distances, as seen in other channels, the potential becomes attractive in the intermediate range at $\Lambda = 1.0$ GeV. This attraction allows the $D^* - \bar{B}^*$ mesons to form a bound state, which lies 4.3 MeV below the threshold.

Based on the above calculations and analyses, we have obtained the coordinate potentials and determined the possible bound states by solving the Schrödinger equation. The total interactions are repulsive in the $I = 1$ channels and attractive in the $I = 0$ channels. Then, we focus on the $I = 0$ $D^{(*)}\bar{B}^{(*)}$ systems to search for bound states. For the $D\bar{B}[0(0^+)]$, $D\bar{B}^*[0(1^+)]$, and $D^*\bar{B}^*[0(2^+)]$ systems, the attractive potentials are too weak to bind the mesons, and no definite bound states are found in these channels at a cutoff of $\Lambda = 0.5$ GeV. However, if we change the cutoff to 0.54 GeV, the very shallow bound state can emerge in the $D\bar{B}[0(0^+)]$ channel. As for the $D\bar{B}^*[0(1^+)]$, the bound state appears at $\Lambda = 1.02$ GeV, and the binding energy will increase as the Λ gets larger.

Besides, in the $D^*\bar{B}^*[0(0^+)]$ and $D^*\bar{B}^*[0(1^+)]$ channels, two shallow bound states are found with binding energies of 6.0 MeV and 0.6 MeV, respectively. Therefore, we suggest that future theoretical and experimental studies should focus on these two systems to find new heavy tetraquark states. Compared to the total potentials of the five $I = 0$ channels, we can also conclude that the interaction in the $D^*\bar{B}^*[0(0^+)]$ channel is more attractive than the other four channels. The mass and the binding energy of each $D^{(*)}\bar{B}^{(*)}[I = 0]$ system under different cutoffs are listed in Table IX.

B. Coupled-channel effects

We have considered interactions of the $D^{(*)}\bar{B}^{(*)}$ systems in the single-channel case. In this section, we will investigate how much their coupled-channel effects affect the above results. We categorize the $I = 0$ coupled-channel

$D^{(*)}\bar{B}^{(*)}$ systems into two groups according to their $0(J^P)$ quantum numbers: $\{D\bar{B}(^1S_0), D^*\bar{B}^*(^1S_0)\}[0(0^+)]$ and $\{D\bar{B}^*(^3S_1), D^*\bar{B}^*(^3S_1), D^*\bar{B}^*(^3S_1)\}[0(1^+)]$.

We first consider the $D^{(*)}\bar{B}^{(*)}[0(0^+)]$ system composed of $\{D\bar{B}(^1S_1), D^*\bar{B}^*(^1S_1)\}$. In this case, the effective potentials of the inelastic scattering process $D\bar{B} \rightarrow D^*\bar{B}^*$ and $D^*\bar{B}^* \rightarrow D\bar{B}$ are needed. We depict the $D\bar{B} \rightarrow D^*\bar{B}^*$ diagrams in Fig. 12. At LO, the contact and OPE diagrams contribute to the scattering amplitudes. At NLO, there are one-loop corrections to these LO diagrams and TPE diagrams. The calculated scattering amplitudes for these diagrams are listed in Appendix A 1. The $D^{(*)}\bar{B}^{(*)}[0(0^+)]$ coupled-channel potential in the coordinate space is

$$\begin{pmatrix} V_{D\bar{B}-D\bar{B}} & V_{D\bar{B}-D^*\bar{B}^*} \\ V_{D^*\bar{B}^*-D\bar{B}} & V_{D^*\bar{B}^*-D^*\bar{B}^*} \end{pmatrix},$$

where the inelastic scattering $D\bar{B} \rightarrow D^*\bar{B}^*$ and $D^*\bar{B}^* \rightarrow D\bar{B}$ appear as nondiagonal elements in the 2×2 potential matrix. Then, we solve the coupled-channel Schrödinger equation and obtain the numerical results, including the binding energy E and the probability of the individual channel P_i , which are shown in Table X.

Here, we consider the variation on the cutoff Λ just as in the single-channel case. Compared with the single-channel $D\bar{B}[0(0^+)]$ results in Table IX, we find coupled channels somewhat strengthen the binding energy: from 0.8 GeV in the single channel to 1.1 GeV in the coupled channels at $\Lambda = 0.7$ GeV, for example. Also, because of the large mass gap (about 190 MeV) between the $D\bar{B}$ and $D^*\bar{B}^*$ thresholds, the contribution of $D^*\bar{B}^*(^1S_0)$ is 0.1% at $\Lambda = 0.5$ GeV. If we raise the cutoff to 0.7 GeV and

TABLE X. The numerical results for $D^{(*)}\bar{B}^{(*)}[0(0^+)]$ systems in the S -wave. The cutoff parameter Λ and binding energy E relative to the threshold of $D\bar{B}$ are in units of GeV and MeV, respectively. “...” means that there is no bound solution.

Λ	0.5	0.7	1.0
E	...	1.1	7.3
P_1	99.9%	99.7%	99.1%
P_2	0.1%	0.3%	0.9%

TABLE XI. The numerical results for $D^{(*)}\bar{B}^{(*)}[0(1^+)]$ systems in the S -wave. The cutoff parameter Λ and binding energy E relative to the threshold of $D\bar{B}^*$ are in units of GeV and MeV, respectively. “...” means that there is no bound solution.

Λ	0.5	0.7	1.0
E	...	...	...
P_1	99.9%	99.7%	99.5%
P_2	0.1%	0.3%	0.5%
P_3	0.0%	0.0%	0.0%

1.0 GeV, the probability of $D^*\bar{B}^*(^1S_0)$ increases to 0.3% and 0.9%, respectively.

For the $D^{(*)}\bar{B}^{(*)}[0(1^+)]$ state, all the coupled-channel diagrams and corresponding scattering amplitudes are shown in Appendix A 1–A 5. Using these amplitudes, we can obtain a 3×3 coupled-channel potential matrix,

$$\begin{pmatrix} V_{D\bar{B}^*-D\bar{B}^*} & V_{D\bar{B}^*-D^*\bar{B}} & V_{D\bar{B}^*-D^*\bar{B}^*} \\ V_{D^*\bar{B}-D\bar{B}^*} & V_{D^*\bar{B}-D^*\bar{B}} & V_{D^*\bar{B}-D^*\bar{B}^*} \\ V_{D^*\bar{B}^*-D\bar{B}^*} & V_{D^*\bar{B}^*-D^*\bar{B}} & V_{D^*\bar{B}^*-D^*\bar{B}^*} \end{pmatrix}.$$

We still take the values of the cutoff parameter at 0.5 GeV, 0.7 GeV, and 1.0 GeV to evaluate binding energy and probability P_i , respectively. The numerical results are listed in Table XI. We find no bound state in the $D^{(*)}\bar{B}^{(*)}[0(1^+)]$ system, which is consistent with the single-channel $D\bar{B}^*[0(1^+)]$ calculation. The dominant channel is $D\bar{B}^*(^3S_1)$, with a probability of 99.9%–99.5%.

According to the above analyses, we can find that the coupled-channel effects do not change our main conclusions obtained in the single-channel case.

C. Two-body scattering of $D^{(*)}\bar{B}^{(*)}$ systems

In addition to the binding energies, we further derive information about $D^{(*)}\bar{B}^{(*)}$ elastic scattering by solving the Lippmann-Schwinger equation. In quantum theory, the scattering rate represents the probability per unit time for a scattering event, and it is proportional to the cross section, the number of targets, and the flux [71]. Using Eqs. (12)–(16), we now calculate the scattering rates between $D^{(*)}$ and $\bar{B}^{(*)}$. The variations of the scattering rates $k\sigma(k)$ with cutoffs are shown in Figs. 9 and 10, as functions of the center-of-mass energy.

The scattering length and effective range can be related to the interaction between particles through the scattering amplitude or phase shift [73]. Based on Eq. (15), we also evaluate the scattering lengths and effective ranges at different cutoff values Λ , with numerical results listed in Table XII.

Let us focus on the scattering rates $k\sigma(k)$ for the $D\bar{B}[0(0^+)]$ and $D\bar{B}^*[0(1^+)]$ systems in Figs. 9(a) and 9(b). The enhancement of $k\sigma(k)$ near the threshold is gradual, which is insufficient to confirm the presence of a bound or virtual state. Looking at the scattering lengths and effective ranges in Table XII, we find that the scattering lengths a for different cutoffs are negative, which indicates that the phase shifts satisfy $0 \leq \delta < \pi/2$ [72]. As a result, there are no evident bound states in the two systems, which

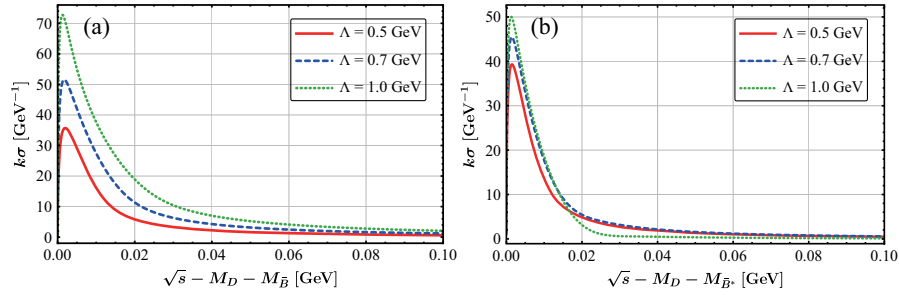


FIG. 9. The dependencies of the S -wave scattering rates for the $D\bar{B}[0(0^+)]$ system and $D\bar{B}^*[0(1^+)]$ system on the cutoff Λ .

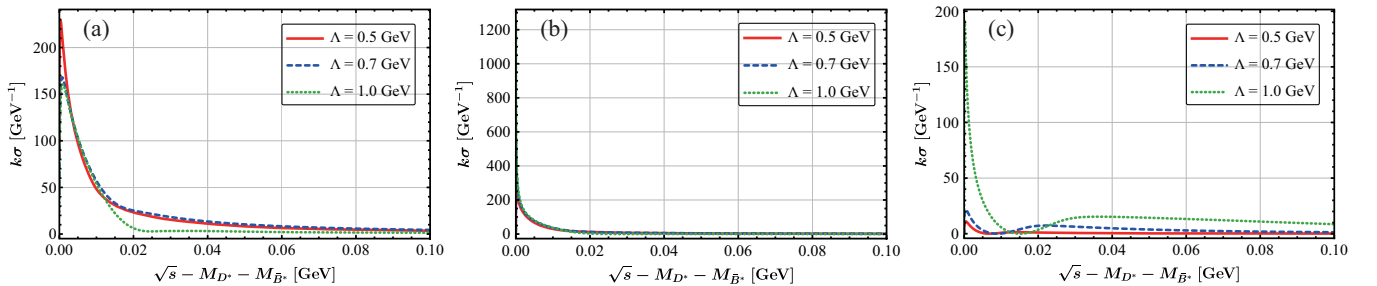


FIG. 10. The dependence of the S -wave scattering rate $k\sigma$ for the $D^*\bar{B}^*[0(0^+)]$, $D^*\bar{B}^*[0(1^+)]$, and $D^*\bar{B}^*[0(2^+)]$ systems on the cutoff Λ .

TABLE XII. The scattering length a and effective range r in the five $D^{(*)}\bar{B}^{(*)}[I = 0]$ scattering channels. The cutoff parameters Λ , a , and r are in units of GeV, fm, and fm, respectively.

Λ	$D\bar{B}$		$D\bar{B}^*$		$D^*\bar{B}^*(J = 0)$		$D^*\bar{B}^*(J = 1)$		$D^*\bar{B}^*(J = 2)$	
	a	r	a	r	a	r	a	r	a	r
0.5	-1.64	2.60	-1.86	2.05	6.37	1.91	-9.87	0.54	-1.25	11.05
0.7	-2.12	2.21	-2.03	1.97	4.48	1.97	-24.06	0.71	-1.76	7.97
1.0	-2.74	1.85	-2.22	2.08	4.38	0.92	140.94	1.19	-7.31	3.67

aligns with the outcomes found in the previous section by solving the Schrödinger equation.

We now move to the scattering of the two vector meson systems. The scattering rates for the $D^*\bar{B}^*[0(0^+)]$, $D^*\bar{B}^*[0(1^+)]$, and $D^*\bar{B}^*[0(2^+)]$ systems are shown in Fig. 10. Near the thresholds, there are significant enhancements, which may indicate the presence of possible bound or virtual states.

In Table XII, we find that, in the $D^*\bar{B}^*[0(0^+)]$ system, the scattering lengths are positive for different cutoff values, indicating the presence of unambiguous bound states, which supports the calculations in the above section.

For the $D^*\bar{B}^*[0(1^+)]$ system, the scattering length is negative at $\Lambda = 0.5$ GeV and $\Lambda = 0.7$ GeV, but it takes on a large positive value at $\Lambda = 1.0$ GeV, signaling the existence of a shallow bound state.

Additionally, we evaluate the corresponding phase shifts for these S -wave bound systems according to Eq. (14). We find that

$$\lim_{k \rightarrow 0} \delta_0 = \pi, \quad (100)$$

which is consistent with Levinson's theorem [73], describing the physical connection between phase shifts and the existence of bound states.

For the $D^*\bar{B}^*[0(2^+)]$ system at different cutoffs, the effective interaction is attractive at small momentum (large distance) but repulsive at large momentum (small distance). It initially causes a positive phase shift at low momentum and then a negative phase shift at high momentum, so that the scattering rate drops to zero, and then gradually increases against the center-of-momentum energy. The negative scattering length indicates that the attractive effect is not strong enough to bind D^* and $\bar{B}^*$ together in the $0(2^+)$ channel.

D. Estimation of the contact term LECs

For the $D\bar{B}[0(0^+)]$ and $D\bar{B}^*[0(1^+)]$ systems, the above calculations show that the total potentials are not strong enough to form bound states for the heavy mesons. However, this does not imply that no bound state can exist in these systems. Note that the contact terms are determined using the resonance saturation model, therefore, we now explore alternative methods to redetermine the LECs of the contact terms in this work.

Recently, a lattice QCD study conducted by Alexandrou *et al.* [50] found shallow bound states for both $J = 0$ and $J = 1$ $\bar{b}\bar{c}ud$ tetraquarks, with binding energies of $0.5_{-1.5}^{+0.4}$ MeV and $2.4_{-0.7}^{+2.0}$ MeV, respectively. Padmanath *et al.* reported a lattice QCD study indicating that an $I(J^P) = 0(1^+) \bar{b}\bar{c}ud$ bound state with a binding energy of $43_{-6}^{+7}({}_{-14}^{+24})$ MeV can exist below the $B^*\bar{D}$ threshold [51]. And, they presented another study searching for tetraquark candidates with exotic quark content $bc\bar{u}\bar{d}$ in the $I = 0$ and $J^P = 0^+$ channels, and found a subthreshold pole in the S -wave $D\bar{B}$ scattering amplitude, which corresponds to the binding energy of $39_{-6}^{+4}({}_{-18}^{+8})$ MeV below the $D\bar{B}$ threshold [52]. The above results provide valuable references for redetermining the contact terms' LECs.

To ascertain the possible parameter region which allows bound states to exist in the $D\bar{B}[0(0^+)]$ and $D\bar{B}^*[0(1^+)]$ systems at $\Lambda = 0.5$ GeV, we can vary the LECs D_a and E_a within the ranges $[-80, 10]$ GeV⁻² and $[-50, -10]$ GeV⁻², respectively. We find that when the LECs satisfy the relation

$$D_a - 3E_a = 22.2 \text{ GeV}^2, \quad (101)$$

a bound state forms with almost zero binding energy. When the LECs satisfy

$$D_a - 3E_a = 69.0 \text{ GeV}^2, \quad (102)$$

a bound state with a binding energy of 30 MeV below the $D\bar{B}$ threshold can be obtained. The green band in Fig. 11 represents the corresponding parameter region. Similarly, we can also determine the contact term LECs for the $D\bar{B}^*[0(1^+)]$ system. For binding energy of 0 MeV, we have

$$D_a - 3E_a = 27.3 \text{ GeV}^2, \quad (103)$$

and for a binding energy of 30 MeV,

$$D_a - 3E_a = 72.6 \text{ GeV}^2. \quad (104)$$

The corresponding parameter variations are shown as the purple band in Fig. 11. The arrow indicates the direction in

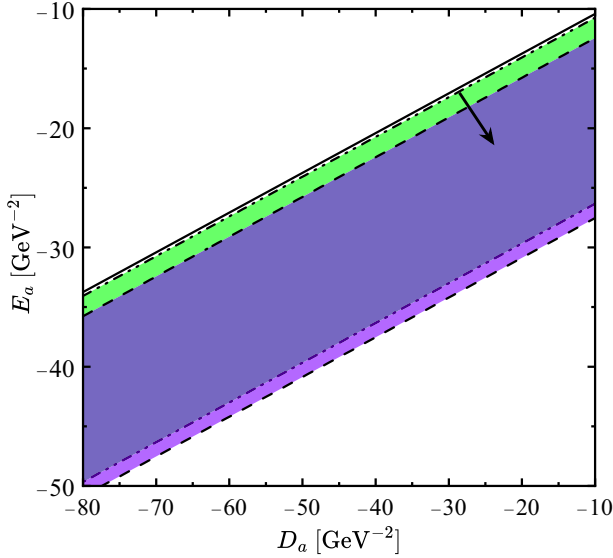


FIG. 11. The binding energies of the $D\bar{B}[0(0^+)]$ and $D^*\bar{B}^*[0(1^+)]$ states vary with the redetermined contact LECs D_a and E_a with cutoff $\Lambda = 0.5$ GeV. The purple and green bands correspond to the $D\bar{B}$ and $D^*\bar{B}^*$ systems, respectively. The black solid line represents the contact LECs determined by the resonance saturation model. The parallel dashed (dotted) lines at the boundaries of the purple (green) band stand for the regions of parameters within the binding energies 0 MeV and 30 MeV. The arrow stands for the direction that binding energy increases in for both $D\bar{B}[0(0^+)]$ and $D^*\bar{B}^*[0(1^+)]$ systems.

which the binding energy increases for both the $D\bar{B}[0(0^+)]$ and $D^*\bar{B}^*[0(1^+)]$ systems.

Notably, in this analysis, the attraction between D and $\bar{B}$ is stronger than that between D and $\bar{B}^*$. This contrasts the results reported in Ref. [50] but is consistent with the results obtained using the one-boson-exchange model [35].

V. SUMMARY

In this work, we systematically investigated the interactions of the S -wave $D^{(*)}\bar{B}^{(*)}$ systems within the framework of ChEFT using the heavy hadron formalism. We calculated the effective potentials, including contributions from the contact, OPE, and TPE terms, up to NLO at the one-loop level, adopting Weinberg's formalism. By performing a Fourier transformation, we analyzed the behaviors of the effective potentials in coordinate space in detail and employed the Gauss form factor to regularize the divergence in the integral at large momentum. We then inserted the coordinate space potentials into the Schrödinger equation to search for potential $D^{(*)}\bar{B}^{(*)}$ bound states.

The results showed that all total potentials in the $I = 1$ channels were repulsive, whereas those in the $I = 0$ channels were attractive. Further calculations examined the variation of interactions with the cutoff parameter Λ across five $I = 0$ $D^{(*)}\bar{B}^{(*)}$ channels. We found that

bound states are more likely to exist in the $D\bar{B}[0(0^+)]$, $D^*\bar{B}^*[0(0^+)]$, and $D^*\bar{B}^*[0(1^+)]$ channels. Notably, the TPE contributions were more sensitive to the cutoff in the $I = 0$ $D^*\bar{B}^*$ channels and played a dominant role in these channels, unlike the case in $D\bar{B}[0(0^+)]$. We list the mass and binding energies in Table IX.

We also discussed the S -wave coupled-channel effects of $D^{(*)}\bar{B}^{(*)}[0(0^+)]$ and $D^{(*)}\bar{B}^{(*)}[0(1^+)]$ systems. The numerical results listed in Table X indicated that, for the $D^{(*)}\bar{B}^{(*)}[0(0^+)]$ system, the $D\bar{B}[0(0^+)]$ channel is dominant, and the coupled channels can somewhat strengthen the binding of the single channel. For the $D^{(*)}\bar{B}^{(*)}[0(1^+)]$ system, $D^*\bar{B}^*[0(1^+)]$ is dominant, and we obtained no binding solution, which is consistent with the single-channel calculation. In a word, the inclusion of the coupled channels did not change our main conclusions given in the single-channel case.

By substituting the momentum space potentials into the Lippmann-Schwinger equation, we calculated the scattering T -matrices and the associated scattering phase shifts. To gain further insight into the S -wave $D^{(*)}\bar{B}^{(*)}$ interactions, we calculated the scattering rate, scattering length, and effective range, and provided their numerical results in Table XII based on the T -matrix and scattering phase. Accordingly, we studied the dependence of these physical quantities on the cutoff parameter Λ . In our investigation, the shallow bound state was more likely to exist in the $D\bar{B}[I(J^P) = 0(0^+)]$ system than in the $D^*\bar{B}^*[I(J^P) = 0(1^+)]$ system.

Based on the above calculations, we found that the interactions in the $D^*\bar{B}^*[I = 0]$ system were more attractive than those in the $D\bar{B}^{(*)}[I = 0]$ systems, and $D^*\bar{B}^*[I(J^P) = 0(0^+)]$ and $D^*\bar{B}^*[I(J^P) = 0(1^+)]$ systems possessed large binding energies and positive scattering lengths, which suggests strong bound state formations in these channels. Therefore, we strongly recommend the experiment to find the $D^*\bar{B}^*[I = 0]$ tetraquark systems.

Considering other theoretical and lattice QCD studies, we redetermined the contact term LECs and calculated the effective potentials for the $0(0^+)$ $D\bar{B}$ system and the $0(1^+)$ $D^*\bar{B}^*$ system. In Fig. 11, we present the dependencies of the binding energies of these two charm-bottom systems on the redetermined contact LECs D_a and E_a .

Building on our calculations and analysis, we can comprehensively compare previous theoretical and lattice QCD studies. Our results provide valuable insights to inform future experimental research.

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No. 12005168, the “111 Center” under Grant No. B20063, the Natural Science Foundation of Gansu Province (No. 22JR5RA389 and No. 22JR5RA171), the fundamental Research Funds for the Central Universities (Grant No. lzujbky-2023-stlt01), and the project for top-notch innovative talents of Gansu Province.

DATA AVAILABILITY

The data that support the findings of this article are openly available [74].

APPENDIX A: COUPLED-CHANNEL CALCULATIONS

In this appendix, we show the coupled-channel calculations of $D^{(*)}\bar{B}^{(*)}$ systems. The effective potentials derived from Lagrangians in Eqs. (1) and (6) for $D\bar{B} \rightarrow D^*\bar{B}^*$ and $D^{(*)}\bar{B}^{(*)} \rightarrow D^*\bar{B}^*$ scattering are presented in this appendix.

1. $D\bar{B} \rightarrow D^*\bar{B}^*$

For the $D(p_1)\bar{B}(p_2) \rightarrow D^*(p_3)\bar{B}^*(p_4)$ process, the corresponding diagrams are illustrated in Fig. 12. The scattering amplitudes of LO diagrams are given by

$$\mathcal{M}_{a4}^{(0)} = -4(D_a + E_a)(\epsilon_3^* \cdot \epsilon_4^*), \quad (\text{A1})$$

$$\mathcal{M}_{b4}^{(0)} = \frac{g_1 g_2}{f^2} \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_3^{*\mu}(p_3) \epsilon_4^{*\nu}(p_4), \quad (\text{A2})$$

with isospin $I = 1$, and

$$\mathcal{M}_{a4}^{(0)} = -4(D_a - 3E_a)(\epsilon_3^* \cdot \epsilon_4^*), \quad (\text{A3})$$

$$\mathcal{M}_{b4}^{(0)} = -3 \frac{g_1 g_1}{f^2} \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_3^{*\mu}(p_3) \epsilon_4^{*\nu}(p_4), \quad (\text{A4})$$

with isospin $I = 0$, where ϵ_3^* and ϵ_4^* are the polarization vectors of the final D^* and $\bar{B}^*$, respectively.

The one-loop corrections to the contact amplitudes are listed as

$$\mathcal{M}_{a4.1}^{(2)} = -4 \frac{g_2^2}{f^2} (d-3)(d-2) A J_{22}^g (\epsilon_3^* \cdot \epsilon_4^*), \quad (\text{A5})$$

$$\mathcal{M}_{a4.2}^{(2)} = 4 \frac{g_2^2}{f^2} A J_{22}^g (\epsilon_3^* \cdot \epsilon_4^*), \quad (\text{A6})$$

$$\mathcal{M}_{a4.3}^{(2)} = -4 \frac{g_1^2}{f^2} (d-3)(d-2) A J_{22}^g (\epsilon_3^* \cdot \epsilon_4^*), \quad (\text{A7})$$

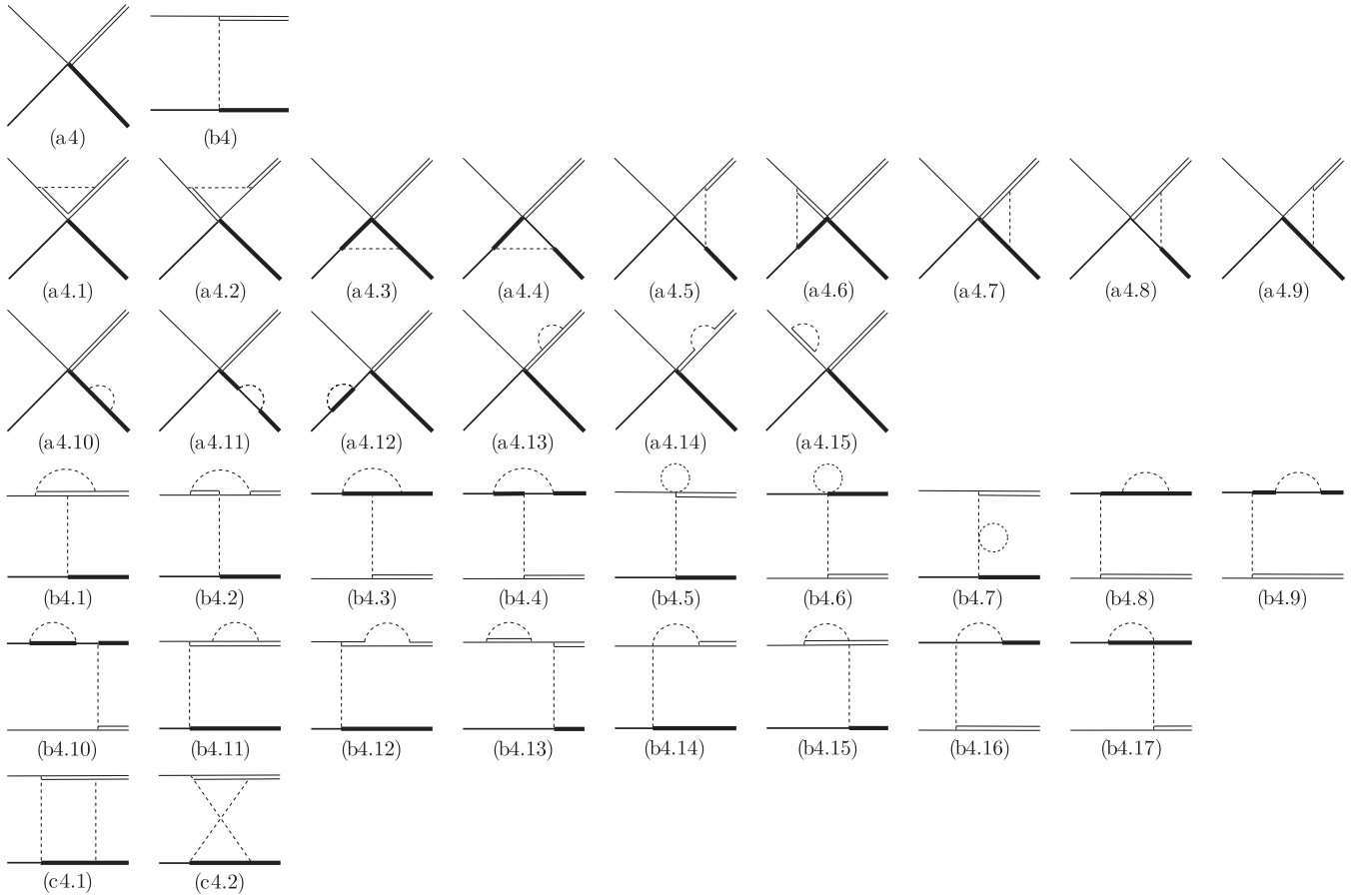


FIG. 12. LO contact (a4), LO OPE (b4), NLO contact (a4.1–a4.15), NLO OPE (b4.1–b4.17), and NLO TPE (c4.1–c4.2) diagrams of the process $D\bar{B} \rightarrow D^*\bar{B}^*$.

$$\mathcal{M}_{a4.4}^{(2)} = 4 \frac{g_1^2}{f^2} A J_{22}^g (\epsilon_3^* \cdot \epsilon_4^*), \quad (\text{A8})$$

$$\mathcal{M}_{a4.5}^{(2)} = -4 \frac{g_1 g_2}{f^2} A J_{22}^h (\epsilon_3^* \cdot \epsilon_4^*), \quad (\text{A9})$$

$$\mathcal{M}_{a4.6}^{(2)} = -4 \frac{g_1 g_2}{f^2} A J_{22}^h (\epsilon_3^* \cdot \epsilon_4^*), \quad (\text{A10})$$

$$\mathcal{M}_{a4.7}^{(2)} = -4 \frac{g_1 g_2}{f^2} (d-3)(d-2) A J_{22}^h (\epsilon_3^* \cdot \epsilon_4^*), \quad (\text{A11})$$

$$\mathcal{M}_{a4.8}^{(2)} = \mathcal{M}_{a4.9}^{(2)} = 0, \quad (\text{A12})$$

$$\begin{aligned} \mathcal{M}_{a4.(10+11)}^{(2)} &= \frac{3g_1^2}{2f^2} A [(d-2)\partial_\omega J_{22}^b(\omega_1) + \partial_\omega J_{22}^b(\omega_2)] \\ &\times (\epsilon_3^* \cdot \epsilon_4^*), \end{aligned} \quad (\text{A13})$$

$$\mathcal{M}_{a4.12}^{(2)} = \frac{3g_1^2}{2f^2} (d-1) A \partial_\omega J_{22}^b(\omega_1) (\epsilon_3^* \cdot \epsilon_4^*), \quad (\text{A14})$$

$$\begin{aligned} \mathcal{M}_{a4.(13+14)}^{(2)} &= \frac{3g_2^2}{2f^2} A [(d-2)\partial_\omega J_{22}^b(\omega_1) + \partial_\omega J_{22}^b(\omega_2)] \\ &\times (\epsilon_3^* \cdot \epsilon_4^*), \end{aligned} \quad (\text{A15})$$

$$\mathcal{M}_{a4.15}^{(2)} = \frac{3g_2^2}{2f^2} (d-1) A \partial_\omega J_{22}^b(\omega_1) (\epsilon_3^* \cdot \epsilon_4^*), \quad (\text{A16})$$

where coefficients appearing in the above contact amplitudes are shown in Table XIII. The NLO OPE diagrams are illustrated in Fig. 12, and the corresponding amplitudes

$$\mathcal{M}_{b4.1}^{(2)} = -4 \frac{g_1 g_2^3}{f^2} A J_{22}^g \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_3^{*\mu} \epsilon_4^{*\nu}, \quad (\text{A17})$$

$$\mathcal{M}_{b4.2}^{(2)} = 4 \frac{g_1 g_2^3}{f^2} (d-3)(d-2) A J_{22}^g \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_3^{*\mu} \epsilon_4^{*\nu}, \quad (\text{A18})$$

$$\mathcal{M}_{b4.3}^{(2)} = -4 \frac{g_1^3 g_2}{f^2} A J_{22}^g \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_3^{*\mu} \epsilon_4^{*\nu}, \quad (\text{A19})$$

$$\mathcal{M}_{b4.4}^{(2)} = 4 \frac{g_1^3 g_2}{f^2} (d-3)(d-2) A J_{22}^g \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_3^{*\mu} \epsilon_4^{*\nu}, \quad (\text{A20})$$

$$\mathcal{M}_{b4.5}^{(2)} = 4 \frac{g_1 g_2}{f^2} A J_0^c \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_3^{*\mu} \epsilon_4^{*\nu}, \quad (\text{A21})$$

$$\mathcal{M}_{b4.6}^{(2)} = 4 \frac{g_1 g_2}{f^2} A J_0^c \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_3^{*\mu} \epsilon_4^{*\nu}, \quad (\text{A22})$$

$$\begin{aligned} \mathcal{M}_{b4.7}^{(2)} &= \frac{8g_1 g_2}{3f^2} A \left[2m^2 L + \frac{m^2}{8\pi^2} \log\left(\frac{m}{\mu}\right) \right] \frac{q_\mu q_\nu}{q^2 - m^2} \\ &\times \epsilon_3^{*\mu} \epsilon_4^{*\nu}, \end{aligned} \quad (\text{A23})$$

TABLE XIII. The coefficients in the NLO contact amplitudes for the $D\bar{B} \rightarrow D^* \bar{B}^*$ process.

	$I = 1$		$I = 0$	
	A	A	ω_1	ω_2
$A_{a4.1(2)}$	0	0	$-\delta_2$	$0(\delta_2)$
$A_{a4.3(4)}$	0	0	$-\delta_1$	$0(\delta_1)$
$A_{a4.5}$	$\frac{1}{4}(D_b + E_b)$	$\frac{-3}{4}(D_b - 3E_b)$	δ_1	δ_2
$A_{a4.6}$	$\frac{1}{4}(D_a + E_a + 2D_b + 2E_b)$	$\frac{-3}{4}(D_a - 3E_a + 2D_b - 6E_b)$	$-\delta_1$	$-\delta_2$
$A_{a4.7}$	0	0	0	0
$A_{a4.(10+11)}$	$D_b + E_b$	$D_b - 3E_b$	0	δ_1
$A_{a4.12}$	$D_b + E_b$	$D_b - 3E_b$	$-\delta_1$	0
$A_{a4.(13+14)}$	$D_b + E_b$	$D_b - 3E_b$	δ_2	0
$A_{a4.15}$	$D_b + E_b$	$D_b - 3E_b$	0	$-\delta_2$

$$\begin{aligned} \mathcal{M}_{b4.(8+9)}^{(2)} &= -4 \frac{g_1^3 g_2}{f^2} A [(d-2)\partial_\omega J_{22}^b(\omega_1) + \partial_\omega J_{22}^b(\omega_2)] \\ &\times \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_3^{*\mu} \epsilon_4^{*\nu}, \end{aligned} \quad (\text{A24})$$

$$\begin{aligned} \mathcal{M}_{b4.10}^{(2)} &= -\frac{3g_1^3 g_2}{2f^2} (d-1) A \partial_\omega J_{22}^b(\omega_1) \frac{q_\mu q_\nu}{q^2 - m^2} \\ &\times \epsilon_3^{*\mu} \epsilon_4^{*\nu}, \end{aligned} \quad (\text{A25})$$

$$\begin{aligned} \mathcal{M}_{b4.(11+12)}^{(2)} &= -4 \frac{g_1 g_2^3}{f^2} A [(d-2)\partial_\omega J_{22}^b(\omega_1) + \partial_\omega J_{22}^b(\omega_2)] \\ &\times \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_3^{*\mu} \epsilon_4^{*\nu}, \end{aligned} \quad (\text{A26})$$

$$\begin{aligned} \mathcal{M}_{b4.13}^{(2)} &= -\frac{3g_1 g_2^3}{2f^2} (d-1) A \partial_\omega J_{22}^b(\omega_1) \frac{q_\mu q_\nu}{q^2 - m^2} \\ &\times \epsilon_3^{*\mu} \epsilon_4^{*\nu}, \end{aligned} \quad (\text{A27})$$

$$\begin{aligned} \mathcal{M}_{b4.14}^{(2)} &= \frac{8g_1 g_2}{f^2} A \left[2m^2 L + \frac{m^2}{8\pi^2} \log\left(\frac{m}{\mu}\right) \right] \frac{q_\mu q_\nu}{q^2 - m^2} \\ &\times \epsilon_3^{*\mu} \epsilon_4^{*\nu}, \end{aligned} \quad (\text{A28})$$

$$\begin{aligned} \mathcal{M}_{b4.15}^{(2)} &= \frac{8g_1 g_2}{f^2} A \left[2m^2 L + \frac{m^2}{8\pi^2} \log\left(\frac{m}{\mu}\right) \right] \frac{q_\mu q_\nu}{q^2 - m^2} \\ &\times \epsilon_3^{*\mu} \epsilon_4^{*\nu}, \end{aligned} \quad (\text{A29})$$

$$\begin{aligned} \mathcal{M}_{b4.16}^{(2)} &= \frac{8g_1 g_2}{f^2} A \left[2m^2 L + \frac{m^2}{8\pi^2} \log\left(\frac{m}{\mu}\right) \right] \frac{q_\mu q_\nu}{q^2 - m^2} \\ &\times \epsilon_3^{*\mu} \epsilon_4^{*\nu}, \end{aligned} \quad (\text{A30})$$

TABLE XIV. The coefficients in the NLO OPE amplitudes for the $D\bar{B} \rightarrow D^*\bar{B}^*$ process.

	$I = 1$	$I = 0$	ω_1	ω_2
	A	A		
$A_{b4,1(2)}$	$-\frac{1}{16}$	$\frac{3}{16}$	$-\delta_2$	$0(\delta_2)$
$A_{b4,3(4)}$	$-\frac{1}{16}$	$\frac{3}{16}$	$-\delta_1$	$0(\delta_1)$
$A_{b4,5(6)}$	$-\frac{1}{12}$	$\frac{1}{4}$	0	0
$A_{b4,7}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	0
$A_{b4,(8+9)}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	δ_1
$A_{b4,10}$	$\frac{1}{4}$	$-\frac{3}{4}$	$-\delta_1$	0
$A_{b4,(11+12)}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	δ_2
$A_{b4,13}$	$\frac{1}{4}$	$-\frac{3}{4}$	$-\delta_2$	0
$A_{b4,14(15)}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	0
$A_{b4,16(17)}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	0

$$\mathcal{M}_{b4,17}^{(2)} = \frac{8g_1g_2}{f^2}A \left[2m^2L + \frac{m^2}{8\pi^2} \log\left(\frac{m}{\mu}\right) \right] \frac{q_\mu q_\nu}{q^2 - m^2} \times \epsilon_3^{*\mu} \epsilon_4^{*\nu}. \quad (\text{A31})$$

The TPE amplitudes at NLO are given by

$$\mathcal{M}_{c4,1}^{(2)} = -4 \frac{g_1^2 g_2^2}{f^4} (d-3) A [(q \cdot \epsilon_3^*)(q \cdot \epsilon_4^*) + q^2 (\epsilon_3^* \cdot \epsilon_4^*)] J_{21}^B (-\delta_1, -\delta_2), \quad (\text{A32})$$

$$\mathcal{M}_{c4,2}^{(2)} = 4 \frac{g_1^2 g_2^2}{f^4} (d-3) A [(q \cdot \epsilon_3^*)(q \cdot \epsilon_4^*) + q^2 (\epsilon_3^* \cdot \epsilon_4^*)] J_{21}^R (-\delta_1, -\delta_2). \quad (\text{A33})$$

In Table XIV, we list the coefficients of the OPE amplitudes in the $D\bar{B} \rightarrow D^*\bar{B}^*$ process. For the TPE amplitudes, the coefficients are

$$A_{c4,1} = 1/16, \quad A_{c4,2} = 5/16, \quad (\text{A34})$$

with $I = 1$, and

$$A_{c4,1} = 9/16, \quad A_{c4,2} = -3/16, \quad (\text{A35})$$

with $I = 0$.

2. $D^*\bar{B} \rightarrow D^*\bar{B}$

For the $D^*(p_1)\bar{B}(p_2) \rightarrow D^*(p_3)\bar{B}(p_4)$ process, the scattering amplitude of the LO contact diagram is

$$\mathcal{M}_{a5}^{(0)} = -4(D_a + E_a)(\epsilon_1 \cdot \epsilon_3^*), \quad (\text{A36})$$

with isospin $I = 1$, and

$$\mathcal{M}_{a5}^{(0)} = -4(D_a - 3E_a)(\epsilon_1 \cdot \epsilon_3^*), \quad (\text{A37})$$

with isospin $I = 0$, where ϵ_1 and ϵ_4^* are the polarization vectors of the initial D and final $\bar{B}^*$, respectively. Next, we consider the NLO interactions in the $D^*(p_1)\bar{B}(p_2) \rightarrow D^*(p_3)\bar{B}(p_4)$ process. The corresponding diagrams are illustrated in Fig. 13. The one-loop corrections to the contact term are

$$\mathcal{M}_{a5,1}^{(2)} = 4 \frac{g_1^2}{f^2} (d-3)(d-2) A J_{22}^g (\epsilon_1 \cdot \epsilon_3^*), \quad (\text{A38})$$

$$\mathcal{M}_{a5,2}^{(2)} = -4 \frac{g_1^2}{f^2} A J_{22}^g (\epsilon_1 \cdot \epsilon_3^*), \quad (\text{A39})$$

$$\mathcal{M}_{a5,3}^{(2)} = 4 \frac{g_2^2}{f^2} (d-1) A J_{22}^g (\epsilon_1 \cdot \epsilon_3^*), \quad (\text{A40})$$

$$\mathcal{M}_{a5,4}^{(2)} = -4 \frac{g_1 g_2}{f^2} (d-3)(d-2) A J_{22}^h (\epsilon_1 \cdot \epsilon_3^*), \quad (\text{A41})$$

$$\mathcal{M}_{a5,5}^{(2)} = -4 \frac{g_1 g_2}{f^2} (d-3)(d-2) A J_{22}^h (\epsilon_1 \cdot \epsilon_3^*), \quad (\text{A42})$$

$$\mathcal{M}_{a5,6}^{(2)} = -4 \frac{g_1 g_2}{f^2} A J_{22}^h (\epsilon_1 \cdot \epsilon_3^*), \quad (\text{A43})$$

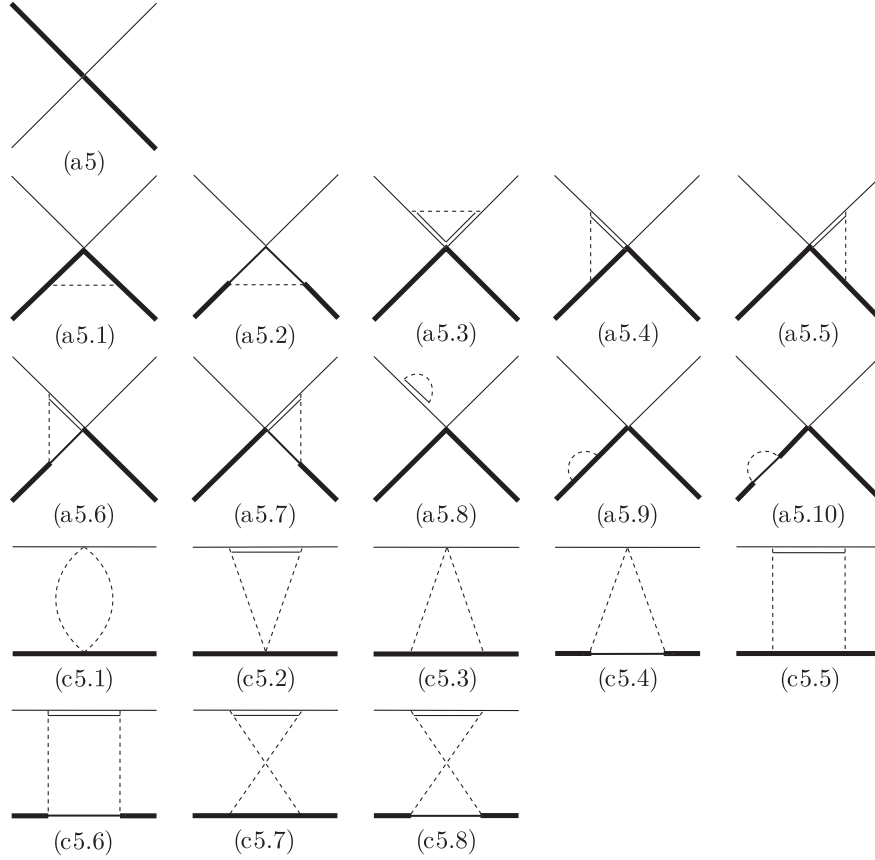
$$\mathcal{M}_{a5,7}^{(2)} = -4 \frac{g_1 g_2}{f^2} A J_{22}^h (\epsilon_1 \cdot \epsilon_3^*), \quad (\text{A44})$$

$$\mathcal{M}_{a5,8}^{(2)} = \frac{3g_2^2}{2f^2} (d-1) A \partial_\omega J_{22}^b (\epsilon_1 \cdot \epsilon_3^*), \quad (\text{A45})$$

$$\mathcal{M}_{a5,(9+10)}^{(2)} = \frac{3g_1^2}{2f^2} A [(d-2) \partial_\omega J_{22}^b (\omega_1) + \partial_\omega J_{22}^b (\omega_2)] \times (\epsilon_1 \cdot \epsilon_3^*), \quad (\text{A46})$$

where coefficients appearing in the above contact amplitudes are shown in Table XV. The TPE amplitudes at NLO are given by

$$\begin{aligned} \mathcal{M}_{c5,1}^{(2)} = & 4 \frac{1}{f^4} [A_1 (q_0^2 J_0^F + J_{22}^F) \\ & + A_{15} (q_0^2 J_{11}^F + q_0^2 J_{21}^F + J_{22}^F) \\ & - A_{51} (q_0^2 J_{11}^F + q_0^2 J_{21}^F + J_{22}^F) \\ & + A_5 (q_0^2 J_0^F + 2q_0^2 J_{11}^F + q_0^2 J_{21}^F + J_{22}^F)] \epsilon_1 \cdot \epsilon_3^*, \end{aligned} \quad (\text{A47})$$

FIG. 13. LO contact (a5), NLO contact (a5.1–a5.10) and NLO TPE (c5.1–c5.8) diagrams of the process $D^* \bar{B} \rightarrow D^* \bar{B}$.

$$\begin{aligned} \mathcal{M}_{c5.2}^{(2)} = & -i4 \frac{g_1^2}{f^4} \{ A_1 [q_0 \vec{q}^2 J_{22}^T + \vec{q}^2 J_{24}^T + (1-d) q_0 J_{31}^T \\ & + q_0 \vec{q}^2 J_{32}^T + \vec{q}^2 J_{33}^T + (1-d) J_{34}^T] \\ & - A_5 [q_0 \vec{q}^2 J_{11}^T + (1-d) q_0 (J_{21}^T + J_{31}^T) \\ & + 2q_0 \vec{q}^2 J_{22}^T + \vec{q}^2 J_{24}^T + q_0 \vec{q}^2 J_{32}^T + \vec{q}^2 J_{33}^T \\ & + (1-d) J_{34}^T] \} (\epsilon_1 \cdot \epsilon_3^*), \end{aligned} \quad (\text{A48})$$

$$\begin{aligned} \mathcal{M}_{c5.3}^{(2)} = & -i4 \frac{g_1^2}{f^4} (d-3) [A_1 (q_0 J_{22}^S + J_{24}^S + q_0 J_{32}^S + J_{33}^S) \\ & - A_5 (q_0 J_{11}^S + 2q_0 J_{22}^S + J_{24}^S \\ & + q_0 J_{32}^S + J_{33}^S)] (q \cdot \epsilon_1) (q \cdot \epsilon_3^*) \\ & - i4 \frac{g_1^2}{f^4} (d-3) \{ A_1 [q_0 \vec{q}^2 J_{22}^S + \vec{q}^2 J_{24}^S \\ & + (2-d) q_0 J_{31}^S + q_0 \vec{q}^2 J_{32}^S + \vec{q}^2 J_{33}^S + (2-d) J_{34}^S] \\ & - A_5 [q_0 \vec{q}^2 J_{11}^S + (2-d) q_0 (J_{21}^S + J_{31}^S) \\ & + 2q_0 \vec{q}^2 J_{22}^S + \vec{q}^2 J_{24}^S + q_0 \vec{q}^2 J_{32}^S + \vec{q}^2 J_{33}^S \\ & + (2-d) J_{34}^S] \} (\epsilon_1 \cdot \epsilon_3^*), \end{aligned} \quad (\text{A49})$$

$$\begin{aligned} \mathcal{M}_{c5.4}^{(2)} = & i4 \frac{g_1^2}{f^4} \{ [A_1 (q_0 J_{22}^S + J_{24}^S + q_0 J_{32}^S + J_{33}^S) \\ & - A_5 (q_0 J_{11}^S + 2q_0 J_{22}^S + J_{24}^S + q_0 J_{32}^S \\ & + J_{33}^S)] (q \cdot \epsilon_1) (q \cdot \epsilon_3^*) + [A_1 (q_0 J_{31}^S + J_{34}^S) \\ & - A_5 (q_0 J_{21}^S + q_0 J_{31}^S + J_{34}^S)] (\epsilon_1 \cdot \epsilon_3^*) \}, \end{aligned} \quad (\text{A50})$$

$$\begin{aligned} \mathcal{M}_{c5.5}^{(2)} = & 4 \frac{g_1^2 g_2^2}{f^4} (d-3) A_1 \{ [J_{21}^B - \vec{q}^2 J_{22}^B + (d+3) J_{31}^B \\ & - 2\vec{q}^2 J_{32}^B + (d+3) J_{42}^B - \vec{q}^2 J_{43}^B] (q \cdot \epsilon_1) (q \cdot \epsilon_3^*) \\ & + [-\vec{q}^2 J_{21}^B + \vec{q}^4 J_{22}^B - (2d+1) \vec{q}^2 (J_{31}^B + J_{42}^B) \\ & + 2\vec{q}^4 J_{32}^B + (d+1)(d-2) J_{41}^B + \vec{q}^4 J_{43}^B] (\epsilon_1 \cdot \epsilon_3^*) \}, \end{aligned} \quad (\text{A51})$$

$$\begin{aligned} \mathcal{M}_{c5.6}^{(2)} = & 4 \frac{g_1^2 g_2^2}{f^4} A_1 \{ [J_{21}^B - \vec{q}^2 J_{22}^B + (d+3) (J_{31}^B + J_{42}^B) \\ & - 2\vec{q}^2 J_{32}^B - \vec{q}^2 J_{43}^B] (q \cdot \epsilon_1) (q \cdot \epsilon_3^*) \\ & - [\vec{q}^2 (J_{31}^B + J_{42}^B) - (1+d) J_{41}^B] (\epsilon_1 \cdot \epsilon_3^*) \}, \end{aligned} \quad (\text{A52})$$

TABLE XV. The coefficients in the NLO contact amplitudes for the $D^*\bar{B} \rightarrow D^*\bar{B}$ process.

	$I = 1$		$I = 0$	
	A	A	ω_1	ω_2
$A_{a5.1}$	0	0	0	0
$A_{a5.2}$	0	0	δ_1	δ_1
$A_{a5.3}$	0	0	$-\delta_2$	$-\delta_2$
$A_{a5.4}$	$\frac{1}{4}(D_b + E_b)$	$\frac{-3}{4}(D_b - 3E_b)$	$-\delta_2$	0
$A_{a5.5}$	$\frac{1}{4}(D_b + E_b)$	$\frac{-3}{4}(D_b - 3E_b)$	0	$-\delta_2$
$A_{a5.6}$	$\frac{1}{4}(D_b + E_b)$	$\frac{-3}{4}(D_b - 3E_b)$	$-\delta_2$	δ_1
$A_{a5.7}$	$\frac{1}{4}(D_b + E_b)$	$\frac{-3}{4}(D_b - 3E_b)$	δ_1	$-\delta_2$
$A_{a5.8}$	$D_a + E_a$	$D_a - 3E_a$	0	$-\delta_2$
$A_{a5.(9+10)}$	$D_a + E_a$	$D_a - 3E_a$	δ_1	0

$$\begin{aligned} \mathcal{M}_{c5.7}^{(2)} = & 4 \frac{g_1^2 g_2^2}{f^4} (d-3) A_1 \{ [J_{21}^R - \vec{q}^2 J_{22}^R + (d+3) J_{31}^R \\ & - 2\vec{q}^2 J_{32}^B + 3J_{42}^B - \vec{q}^2 J_{43}^B] (q \cdot \epsilon_1) (q \cdot \epsilon_3^*) \\ & + [-\vec{q}^2 J_{21}^R + \vec{q}^4 J_{22}^R - (2d+1) \vec{q}^2 (J_{31}^R + J_{42}^R) \\ & + 2\vec{q}^4 J_{32}^R + (d+1)(d-2) J_{41}^R + \vec{q}^4 J_{43}^R] (\epsilon_1 \cdot \epsilon_3^*) \}, \end{aligned} \quad (\text{A53})$$

$$\begin{aligned} \mathcal{M}_{c5.8}^{(2)} = & 4 \frac{g_1^2 g_2^2}{f^4} A_1 \{ [J_{21}^R - \vec{q}^2 J_{22}^R + (d+3) (J_{31}^R + J_{42}^R) \\ & - 2\vec{q}^2 J_{32}^R - \vec{q}^2 J_{43}^R] (q \cdot \epsilon_1) (q \cdot \epsilon_3^*) \\ & - [\vec{q}^2 J_{31}^R - (1+d) J_{41}^R + \vec{q}^2 J_{42}^R] (\epsilon_1 \cdot \epsilon_3^*) \}, \end{aligned} \quad (\text{A54})$$

where coefficients appearing in the above contact amplitudes are shown in Table XVI.

TABLE XVI. The coefficients in the TPE amplitudes for the $D^*(p_1)\bar{B}(p_2) \rightarrow D^*(p_3)\bar{B}(p_4)$ process. Note that we have $A_{51} = A_{15}$.

	$I = 1$			$I = 0$			ω_1	ω_2
	A_1	A_5	A_{15}	A_1	A_5	A_{15}		
$A_{c5.1}$	$\frac{1}{16}$	$\frac{1}{16}$	$-\frac{1}{16}$	$-\frac{3}{16}$	$-\frac{3}{16}$	$\frac{3}{16}$	0	0
$A_{c5.2}$	$\frac{i}{8}$	$-\frac{i}{8}$	0	$-\frac{3i}{8}$	$\frac{3i}{8}$	0	$-\delta_2$	0
$A_{c5.3}$	$\frac{i}{8}$	$-\frac{i}{8}$	0	$-\frac{3i}{8}$	$\frac{3i}{8}$	0	0	0
$A_{c5.4}$	$\frac{i}{8}$	$-\frac{i}{8}$	0	$-\frac{3i}{8}$	$\frac{3i}{8}$	0	δ_1	0
$A_{c5.5}$	$\frac{1}{16}$	0	0	$\frac{9}{16}$	0	0	0	$-\delta_2$
$A_{c5.6}$	$\frac{1}{16}$	0	0	$\frac{9}{16}$	0	0	δ_1	$-\delta_2$
$A_{c5.7}$	$\frac{5}{16}$	0	0	$-\frac{3}{16}$	0	0	0	$-\delta_2$
$A_{c5.8}$	$\frac{5}{16}$	0	0	$-\frac{3}{16}$	0	0	δ_1	$-\delta_2$

3. $D\bar{B}^* \rightarrow D^*\bar{B}$

Now, we focus on the process $D(p_1)\bar{B}^*(p_2) \rightarrow D^*(p_3)\bar{B}(p_4)$, and the corresponding diagrams are illustrated in Fig. 14. The scattering amplitudes of LO diagrams are given by

$$\mathcal{M}_{a6}^{(0)} = 4(D_b + E_b)(\epsilon_2 \cdot \epsilon_3^*), \quad (\text{A55})$$

$$\mathcal{M}_{b6}^{(0)} = -\frac{g_1 g_2}{f^2} \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_2^\mu \epsilon_3^{*\nu}, \quad (\text{A56})$$

with isospin $I = 1$, and

$$\mathcal{M}_{a6}^{(0)} = 4(D_b - 3E_b)(\epsilon_2 \cdot \epsilon_3^*), \quad (\text{A57})$$

$$\mathcal{M}_{b6}^{(0)} = \frac{3g_1 g_2}{f^2} \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_2^\mu \epsilon_3^{*\nu}, \quad (\text{A58})$$

with isospin $I = 0$.

The one-loop corrections to the contact amplitudes are

$$\mathcal{M}_{a6.1}^{(2)} = 4 \frac{g_2^2}{f^2} (d-3)(d-2) A J_{22}^g (\epsilon_2 \cdot \epsilon_3^*), \quad (\text{A59})$$

$$\mathcal{M}_{a6.2}^{(2)} = -4 \frac{g_2^2}{f^2} A J_{22}^g (\epsilon_2 \cdot \epsilon_3^*), \quad (\text{A60})$$

$$\mathcal{M}_{a6.3}^{(2)} = 4 \frac{g_1^2}{f^2} (d-3)(d-2) A J_{22}^g (\epsilon_2 \cdot \epsilon_3^*), \quad (\text{A61})$$

$$\mathcal{M}_{a6.4}^{(2)} = -4 \frac{g_1^2}{f^2} A J_{22}^g (\epsilon_2 \cdot \epsilon_3^*), \quad (\text{A62})$$

$$\mathcal{M}_{a6.5}^{(2)} = 4 \frac{g_1 g_2}{f^2} (d-3)(d-2) A J_{22}^h (\epsilon_2 \cdot \epsilon_3^*), \quad (\text{A63})$$

$$\mathcal{M}_{a6.6}^{(2)} = 4 \frac{g_1 g_2}{f^2} A J_{22}^h (\epsilon_2 \cdot \epsilon_3^*), \quad (\text{A64})$$

$$\mathcal{M}_{a6.7}^{(2)} = 4 \frac{g_1 g_2}{f^2} (d-3)(d-2) A J_{22}^h (\epsilon_2 \cdot \epsilon_3^*), \quad (\text{A65})$$

$$\mathcal{M}_{a6.8}^{(2)} = 4 \frac{g_1 g_2}{f^2} A J_{22}^h (\epsilon_2 \cdot \epsilon_3^*), \quad (\text{A66})$$

$$\begin{aligned} \mathcal{M}_{a6.(9+10)}^{(2)} = & -\frac{3g_1^2}{2f^2} A [(d-2) \partial_\omega J_{22}^b(\omega_1) + \partial_\omega J_{22}^b(\omega_2)] \\ & \times (\epsilon_2 \cdot \epsilon_3^*), \end{aligned} \quad (\text{A67})$$

$$\mathcal{M}_{a6.11}^{(2)} = -\frac{3g_1^2}{2f^2} (d-1) A \partial_\omega J_{22}^b (\epsilon_2 \cdot \epsilon_3^*), \quad (\text{A68})$$

$$\begin{aligned} \mathcal{M}_{a6.(12+13)}^{(2)} = & -\frac{3g_2^2}{2f^2} A [(d-2) \partial_\omega J_{22}^b(\omega_1) + \partial_\omega J_{22}^b(\omega_2)] \\ & \times (\epsilon_2 \cdot \epsilon_3^*), \end{aligned} \quad (\text{A69})$$

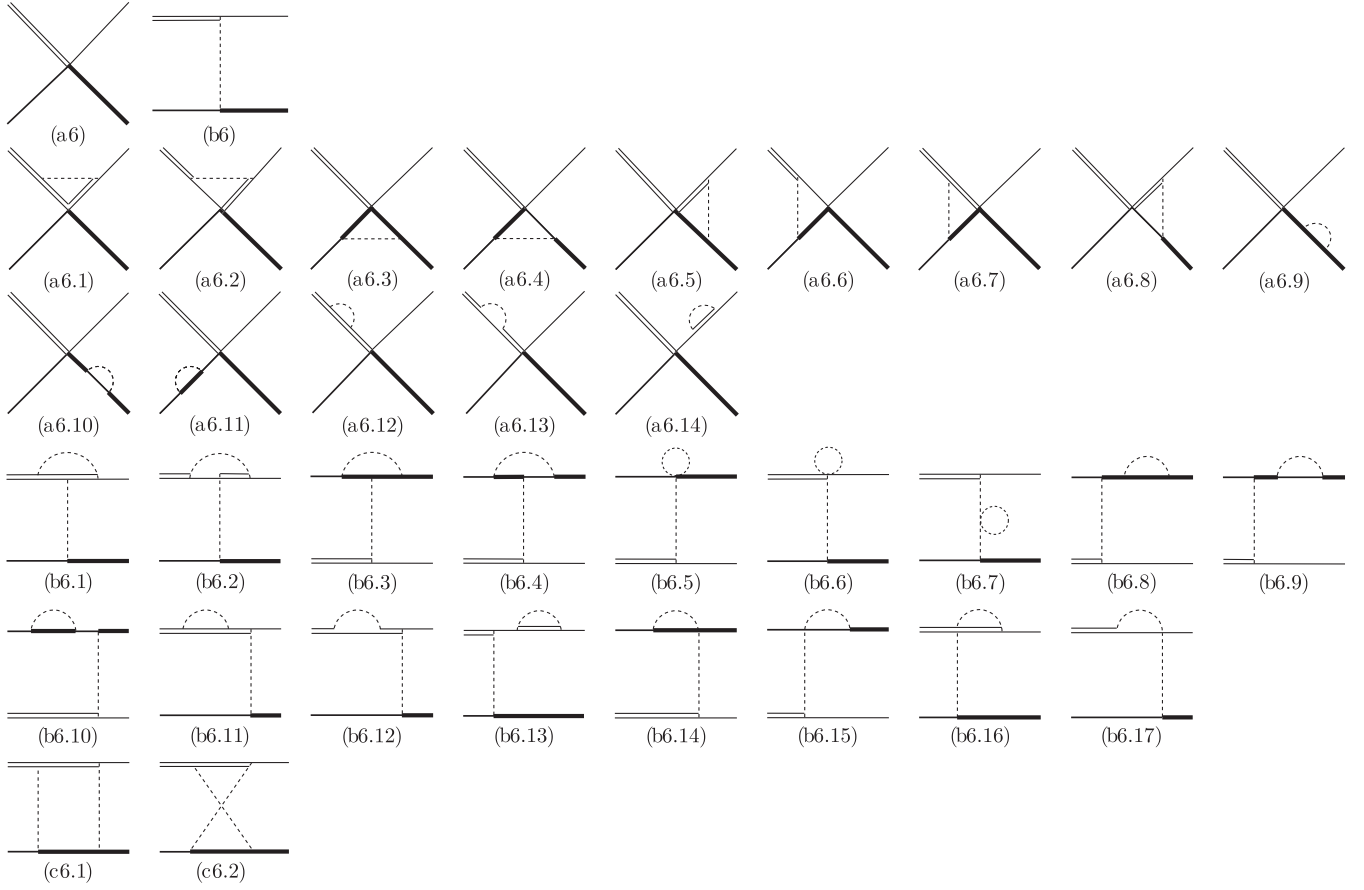


FIG. 14. LO contact (a6), LO OPE (b6), NLO contact (a6.1–a6.14), NLO OPE (b6.1–b6.17), and NLO TPE (c6.1–c6.2) diagrams of the process $D\bar{B}^* \rightarrow D^*\bar{B}$.

$$\mathcal{M}_{a6.14}^{(2)} = -\frac{3g_2^2}{2f^2} (d-1) A \partial_\omega J_{22}^b (\epsilon_2 \cdot \epsilon_3^*). \quad (\text{A70})$$

$$\mathcal{M}_{b6.7}^{(2)} = -\frac{8g_1g_2}{f^2} A \left[2m^2 L + \frac{m^2}{8\pi^2} \log\left(\frac{m}{\mu}\right) \right] \frac{q_\mu q_\nu}{q^2 - m^2} \times \epsilon_2^\mu \epsilon_3^{*\nu}, \quad (\text{A77})$$

From Fig. 14, we can obtain the scattering amplitudes of NLO OPE diagrams,

$$\mathcal{M}_{b6.1}^{(2)} = -4 \frac{g_1g_2^3}{f^2} (d-3)(d-2) A J_{22}^g \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_2^\mu \epsilon_3^{*\nu}, \quad (\text{A71})$$

$$\mathcal{M}_{b6.2}^{(2)} = 4 \frac{g_1g_2^3}{f^2} A J_{22}^g \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_2^\mu \epsilon_3^{*\nu}, \quad (\text{A72})$$

$$\mathcal{M}_{b6.3}^{(2)} = -4 \frac{g_1g_2^3}{f^2} (d-3)(d-2) A J_{22}^g \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_2^\mu \epsilon_3^{*\nu}, \quad (\text{A73})$$

$$\mathcal{M}_{b6.4}^{(2)} = 4 \frac{g_1g_2^3}{f^2} A J_{22}^g \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_2^\mu \epsilon_3^{*\nu}, \quad (\text{A74})$$

$$\mathcal{M}_{b6.5}^{(2)} = -4 \frac{g_1g_2}{f^2} A J_0^c \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_2^\mu \epsilon_3^{*\nu}, \quad (\text{A75})$$

$$\mathcal{M}_{b6.6}^{(2)} = -4 \frac{g_1g_2}{f^2} A J_0^c \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_2^\mu \epsilon_3^{*\nu}, \quad (\text{A76})$$

$$\mathcal{M}_{b6.(8+9)}^{(2)} = -\frac{3g_1g_2}{2f^2} A [(d-2) \partial_\omega J_{22}^b(\omega_1) + \partial_\omega J_{22}^b(\omega_2)] \times \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_2^\mu \epsilon_3^{*\nu}, \quad (\text{A78})$$

$$\mathcal{M}_{b6.10}^{(2)} = \frac{3g_1g_2}{2f^2} (d-1) A \partial_\omega J_{22}^b \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_2^\mu \epsilon_3^{*\nu}, \quad (\text{A79})$$

$$\mathcal{M}_{b6.(11+12)}^{(2)} = -\frac{3g_1g_2}{2f^2} A [(d-2) \partial_\omega J_{22}^b(\omega_1) + \partial_\omega J_{22}^b(\omega_2)] \times \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_2^\mu \epsilon_3^{*\nu}, \quad (\text{A80})$$

$$\mathcal{M}_{b6.13}^{(2)} = \frac{3g_1g_2}{2f^2} (d-1) A \partial_\omega J_{22}^b \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_2^\mu \epsilon_3^{*\nu}, \quad (\text{A81})$$

$$\mathcal{M}_{b6.14}^{(2)} = -\frac{8g_1g_2}{f^2}A \left[2m^2L + \frac{m^2}{8\pi^2} \log\left(\frac{m}{\mu}\right) \right] \frac{q_\mu q_\nu}{q^2 - m^2} \times \epsilon_2^\mu \epsilon_3^{*\nu}, \quad (\text{A82})$$

$$\mathcal{M}_{b6.15}^{(2)} = -\frac{8g_1g_2}{f^2}A \left[2m^2L + \frac{m^2}{8\pi^2} \log\left(\frac{m}{\mu}\right) \right] \frac{q_\mu q_\nu}{q^2 - m^2} \times \epsilon_2^\mu \epsilon_3^{*\nu}, \quad (\text{A83})$$

$$\mathcal{M}_{b6.16}^{(2)} = -\frac{8g_1g_2}{f^2}A \left[2m^2L + \frac{m^2}{8\pi^2} \log\left(\frac{m}{\mu}\right) \right] \frac{q_\mu q_\nu}{q^2 - m^2} \times \epsilon_2^\mu \epsilon_3^{*\nu}, \quad (\text{A84})$$

$$\mathcal{M}_{b6.17}^{(2)} = -\frac{8g_1g_2}{f^2}A \left[2m^2L + \frac{m^2}{8\pi^2} \log\left(\frac{m}{\mu}\right) \right] \frac{q_\mu q_\nu}{q^2 - m^2} \times \epsilon_2^\mu \epsilon_3^{*\nu}. \quad (\text{A85})$$

The TPE amplitudes are given by

$$\mathcal{M}_{c6.1}^{(2)} = -4 \frac{g_1^2 g_2^2}{f^4} (d-3) A [(q \cdot \epsilon_2)(q \cdot \epsilon_3^*) + q^2 (\epsilon_2 \cdot \epsilon_3^*)] J_{21}^B (-\delta_1, -\delta_2), \quad (\text{A86})$$

$$\mathcal{M}_{c6.2}^{(2)} = 4 \frac{g_1^2 g_2^2}{f^4} (d-3) A [(q \cdot \epsilon_2)(q \cdot \epsilon_3^*) + q^2 (\epsilon_2 \cdot \epsilon_3^*)] J_{21}^R (-\delta_1, -\delta_2). \quad (\text{A87})$$

In Tables XVII and XVIII, we list the coefficients of the contact amplitudes and OPE amplitudes at NLO, respectively. For the TPE terms, the coefficients are

$$A_{c6.1} = 1/16, \quad A_{c6.2} = 5/16, \quad (\text{A88})$$

with $I = 1$, and

TABLE XVII. The coefficients in the NLO contact amplitudes for the $D\bar{B}^* \rightarrow D^*\bar{B}$ process.

	$I = 1$	$I = 0$		
	A	A	ω_1	ω_2
$A_{a6.1(2)}$	0	0	$0(\delta_2)$	$-\delta_2$
$A_{a6.3(4)}$	0	0	$-\delta_1$	$0(\delta_1)$
$A_{a6.5}$	$\frac{1}{4}(D_b + E_b)$	$\frac{-3}{4}(D_b - 3E_b)$	0	$-\delta_2$
$A_{a6.6}$	$\frac{1}{4}(D_a + E_a)$	$\frac{-3}{4}(D_a - 3E_a)$	δ_2	$-\delta_1$
$A_{a6.7}$	$\frac{1}{4}(D_b + E_b)$	$\frac{-3}{4}(D_b - 3E_b)$	0	$-\delta_1$
$A_{a6.8}$	$\frac{1}{4}(D_a + E_a)$	$\frac{-3}{4}(D_a - 3E_a)$	δ_1	$-\delta_2$
$A_{a6.(9+10)}$	$D_b + E_b$	$D_b - 3E_b$	0	δ_1
$A_{a6.11}$	$D_b + E_b$	$D_b - 3E_b$	$-\delta_1$	0
$A_{a6.(12+13)}$	$D_b + E_b$	$D_b - 3E_b$	0	δ_2
$A_{a6.14}$	$D_b + E_b$	$D_b - 3E_b$	$-\delta_2$	0

TABLE XVIII. The coefficients in the NLO OPE amplitudes for the $D\bar{B}^* \rightarrow D^*\bar{B}$ process.

	$I = 1$	$I = 0$		
	A	A	ω_1	ω_2
$A_{b6.1(2)}$	$-\frac{1}{16}$	$\frac{3}{16}$	$0(\delta_2)$	$-\delta_2$
$A_{b6.3(4)}$	$-\frac{1}{16}$	$\frac{3}{16}$	$-\delta_1$	$0(\delta_1)$
$A_{b6.5(6)}$	$-\frac{1}{12}$	$\frac{1}{4}$	0	0
$A_{b6.7}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	0
$A_{b6.(8+9)}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	δ_1
$A_{b6.10}$	$\frac{1}{4}$	$-\frac{3}{4}$	$-\delta_1$	0
$A_{b6.(11+12)}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	δ_2
$A_{b6.13}$	$\frac{1}{4}$	$-\frac{3}{4}$	$-\delta_2$	0
$A_{b6.14(15)}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	0
$A_{b6.16(17)}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	0

$$A_{c6.1} = 9/16, \quad A_{c6.2} = -3/16, \quad (\text{A89})$$

with $I = 0$.

4. $D\bar{B}^* \rightarrow D^*\bar{B}^*$

For the process $D(p_1)\bar{B}^*(p_2) \rightarrow D^*(p_3)\bar{B}^*(p_4)$, the contact, OPE, and TPE diagrams are shown in Fig. 15. We can write the scattering amplitudes of LO diagrams as

$$\mathcal{M}_{a7}^{(0)} = 4(D_b + E_b)\epsilon_3^{*\mu}(\epsilon_2 \times \epsilon_4^*)_\mu, \quad (\text{A90})$$

$$\mathcal{M}_{b7}^{(0)} = \frac{g_1g_2}{f^2} \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_2^\mu (\epsilon_4^* \times \epsilon_3^*)^\nu, \quad (\text{A91})$$

with isospin $I = 1$, and

$$\mathcal{M}_{a7}^{(0)} = 4(D_b - 3E_b)\epsilon_3^{*\mu}(\epsilon_2 \times \epsilon_4^*)_\mu, \quad (\text{A92})$$

$$\mathcal{M}_{b7}^{(0)} = -\frac{3g_1g_2}{f^2} \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_2^\mu (\epsilon_4^* \times \epsilon_3^*)^\nu, \quad (\text{A93})$$

with isospin $I = 0$.

The one-loop corrections to the contact amplitudes are listed as

$$\mathcal{M}_{a7.1}^{(2)} = 4 \frac{g_2^2}{f^2} (d-3) A J_{22}^g \epsilon_3^{*\mu} (\epsilon_2 \times \epsilon_4^*)_\mu, \quad (\text{A94})$$

$$\mathcal{M}_{a7.2}^{(2)} = 0, \quad (\text{A95})$$

$$\mathcal{M}_{a7.3}^{(2)} = -4 \frac{g_2^2}{f^2} A J_{22}^g \epsilon_3^{*\mu} (\epsilon_2 \times \epsilon_4^*)_\mu, \quad (\text{A96})$$

$$\mathcal{M}_{a7.4}^{(2)} = -4 \frac{g_2^2}{f^2} A J_{22}^g \epsilon_3^{*\mu} (\epsilon_2 \times \epsilon_4^*)_\mu, \quad (\text{A97})$$

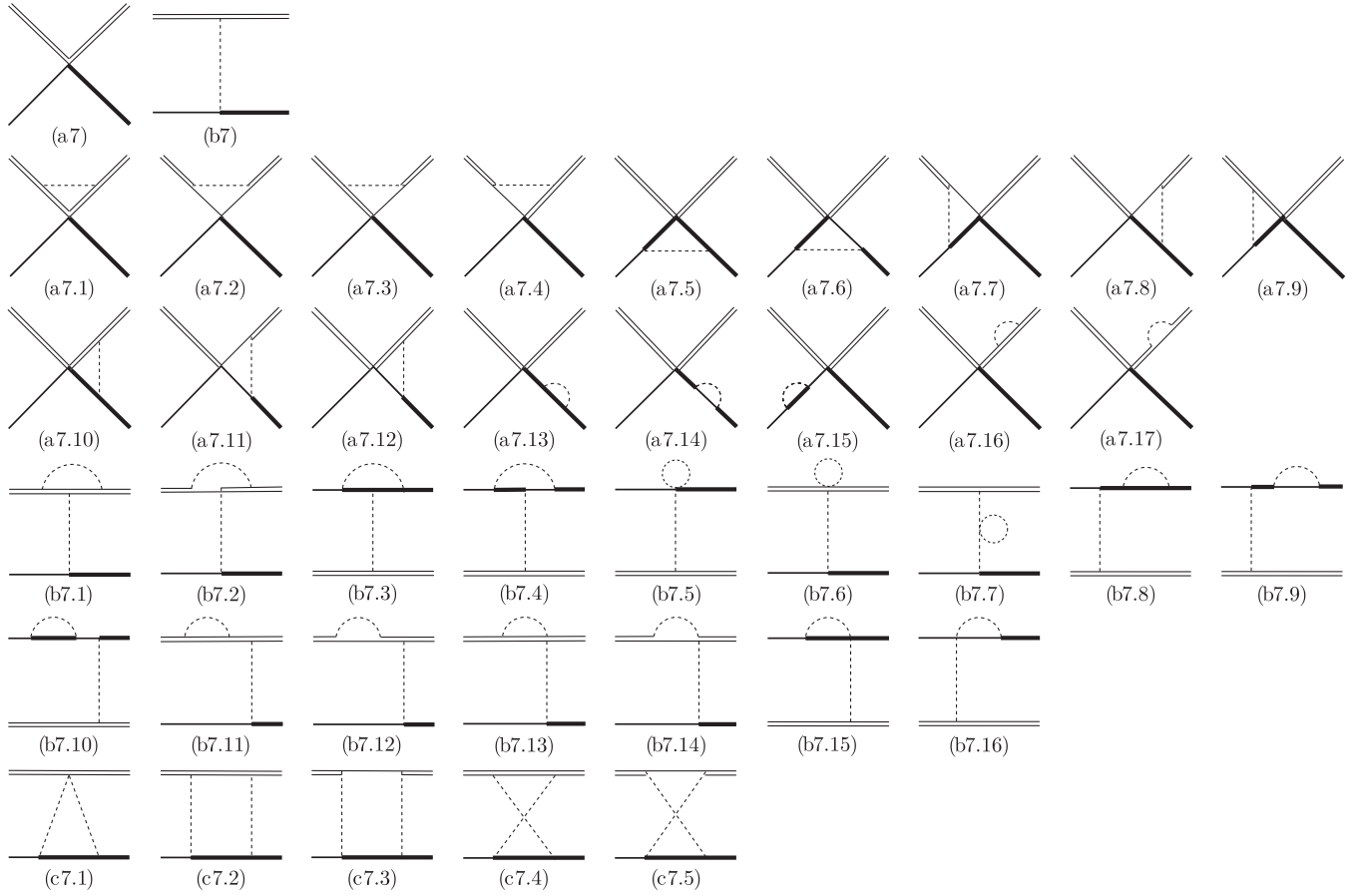


FIG. 15. LO contact (a7), LO OPE (b7), NLO contact (a7.1-a7.17), NLO OPE (b7.1-b7.16), and NLO TPE (c7.1-c7.5) diagrams of the process $D\bar{B}^* \rightarrow D^*\bar{B}^*$.

$$\mathcal{M}_{a7.5}^{(2)} = -8 \frac{g_1^2}{f^2} A J_{22}^g \epsilon_3^{*\mu} (\epsilon_2 \times \epsilon_4^*)_\mu, \quad (\text{A98})$$

$$\mathcal{M}_{a7.6}^{(2)} = 4 \frac{g_1^2}{f^2} A J_{22}^g \epsilon_3^{*\mu} (\epsilon_2 \times \epsilon_4^*)_\mu, \quad (\text{A99})$$

$$\mathcal{M}_{a7.7}^{(2)} = -4 \frac{g_1 g_2}{f^2} A J_{22}^h \epsilon_3^{*\mu} (\epsilon_2 \times \epsilon_4^*)_\mu, \quad (\text{A100})$$

$$\mathcal{M}_{a7.8}^{(2)} = -4 \frac{g_1 g_2}{f^2} A J_{22}^h \epsilon_3^{*\mu} (\epsilon_2 \times \epsilon_4^*)_\mu, \quad (\text{A101})$$

$$\mathcal{M}_{a7.9}^{(2)} = -4 \frac{g_1 g_2}{f^2} A J_{22}^h \epsilon_3^{*\mu} (\epsilon_2 \times \epsilon_4^*)_\mu, \quad (\text{A102})$$

$$\mathcal{M}_{a7.10}^{(2)} = -4 \frac{g_1 g_2}{f^2} (d-3) A J_{22}^h \epsilon_3^{*\mu} (\epsilon_2 \times \epsilon_4^*)_\mu, \quad (\text{A103})$$

$$\mathcal{M}_{a7.11}^{(2)} = 0, \quad (\text{A104})$$

$$\mathcal{M}_{a7.12}^{(2)} = -4 \frac{g_1 g_2}{f^2} A J_{22}^h \epsilon_3^{*\mu} (\epsilon_2 \times \epsilon_4^*)_\mu, \quad (\text{A105})$$

$$\begin{aligned} \mathcal{M}_{a7.(13+14)}^{(2)} &= \frac{3g_2^2}{2f^2} A [(d-2)\partial_\omega J_{22}^b(\omega_1) + \partial_\omega J_{22}^b(\omega_2)] \\ &\quad \times [\epsilon_3^{*\mu} (\epsilon_2 \times \epsilon_4^*)_\mu], \end{aligned} \quad (\text{A106})$$

$$\mathcal{M}_{a7.15}^{(2)} = \frac{3g_1^2}{2f^2} (d-1) A \partial_\omega J_{22}^b \epsilon_3^{*\mu} (\epsilon_2 \times \epsilon_4^*)_\mu, \quad (\text{A107})$$

$$\begin{aligned} \mathcal{M}_{a7.(16+17)}^{(2)} &= \frac{3g_2^2}{2f^2} A [(d-2)\partial_\omega J_{22}^b(\omega_1) + \partial_\omega J_{22}^b(\omega_2)] \\ &\quad \times [\epsilon_3^{*\mu} (\epsilon_2 \times \epsilon_4^*)_\mu]. \end{aligned} \quad (\text{A108})$$

For the one-loop corrections to the OPE amplitudes, we have

$$\mathcal{M}_{b7.1}^{(2)} = -4 \frac{g_1 g_2^3}{f^2} (d-3) A J_{22}^g \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_2^\mu (\epsilon_4^* \times \epsilon_3^*)^\nu, \quad (\text{A109})$$

$$\mathcal{M}_{b7.2}^{(2)} = 4 \frac{g_1 g_2^3}{f^2} A J_{22}^g \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_2^\mu (\epsilon_4^* \times \epsilon_3^*)^\nu, \quad (\text{A110})$$

$$\mathcal{M}_{b7.3}^{(2)} = -8 \frac{g_1^3 g_2}{f^2} A J_{22}^g \frac{1}{q^2 - m^2} [q^2 \epsilon_3^{*\mu} (\epsilon_2 \times \epsilon_4^*)_\mu + q_\mu q_\nu \epsilon_3^{*\mu} (\epsilon_2 \times \epsilon_4^*)^\nu - q_\mu q_\nu \epsilon_3^{*\mu} (\epsilon_4^* \times \epsilon_2)^\nu], \quad (\text{A111})$$

$$\mathcal{M}_{b7.4}^{(2)} = -4 \frac{g_1^3 g_2}{f^2} A J_{22}^g \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_2^\mu (\epsilon_4^* \times \epsilon_3^*)^\nu, \quad (\text{A112})$$

$$\mathcal{M}_{b7.5}^{(2)} = 4 \frac{g_1 g_2}{f^2} A J_0^c \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_2^\mu (\epsilon_4^* \times \epsilon_3^*)^\nu, \quad (\text{A113})$$

$$\mathcal{M}_{b7.6}^{(2)} = 4 \frac{g_1 g_2}{f^2} A J_0^c \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_2^\mu (\epsilon_4^* \times \epsilon_3^*)^\nu, \quad (\text{A114})$$

$$\mathcal{M}_{b7.7}^{(2)} = \frac{8g_1 g_2}{3f^2} A \left[2m^2 L + \frac{m^2}{8\pi^2} \log\left(\frac{m}{\mu}\right) \right] \frac{q_\mu q_\nu}{q^2 - m^2} \times [\epsilon_2^\mu (\epsilon_4^* \times \epsilon_3^*)^\nu], \quad (\text{A115})$$

$$\mathcal{M}_{b7.(8+9)}^{(2)} = -\frac{3g_1^3 g_2}{2f^2} A [(d-2) \partial_\omega J_{22}^b(\omega_1) + \partial_\omega J_{22}^b(\omega_2)] \times \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_2^\mu (\epsilon_4^* \times \epsilon_3^*)^\nu, \quad (\text{A116})$$

$$\mathcal{M}_{b7.10}^{(2)} = -\frac{3g_1^3 g_2}{2f^2} (d-1) A \partial_\omega J_{22}^b \frac{q_\mu q_\nu}{q^2 - m^2} \times [\epsilon_2^\mu (\epsilon_4^* \times \epsilon_3^*)^\nu], \quad (\text{A117})$$

$$\mathcal{M}_{b7.(11+12)}^{(2)} = -\frac{3g_1 g_2^3}{2f^2} A [(d-2) \partial_\omega J_{22}^b(\omega_1) + \partial_\omega J_{22}^b(\omega_2)] \times \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_2^\mu (\epsilon_4^* \times \epsilon_3^*)^\nu, \quad (\text{A118})$$

$$\mathcal{M}_{b7.13(14)}^{(2)} = \frac{8g_1 g_2}{f^2} A \left[2m^2 L + \frac{m^2}{8\pi^2} \log\left(\frac{m}{\mu}\right) \right] \frac{q_\mu q_\nu}{q^2 - m^2} \times [\epsilon_2^\mu (\epsilon_4^* \times \epsilon_3^*)^\nu], \quad (\text{A119})$$

$$\mathcal{M}_{b7.15(16)}^{(2)} = \frac{8g_1 g_2}{f^2} A \left[2m^2 L + \frac{m^2}{8\pi^2} \log\left(\frac{m}{\mu}\right) \right] \frac{q_\mu q_\nu}{q^2 - m^2} \times [\epsilon_2^\mu (\epsilon_4^* \times \epsilon_3^*)^\nu]. \quad (\text{A120})$$

In Tables XIX and XX, we list the coefficients of the above contact amplitudes at NLO and OPE amplitudes at NLO, respectively. The TPE amplitudes are given by

$$\mathcal{M}_{c7.1}^{(2)} = 0, \quad (\text{A121})$$

$$\mathcal{M}_{c7.2}^{(2)} = -4 \frac{g_1^2 g_2^2}{f^4} A \{ (J_{21}^B + J_{31}^B) q^2 \epsilon_3^{*\mu} (\epsilon_2 \times \epsilon_4^*)_\mu (J_{21}^B + J_{31}^B) q_\mu q_\nu \epsilon_2^\mu (\epsilon_4^* \times \epsilon_3^*)^\nu + [J_{21}^B + (d-2) J_{31}^B] \times q_\mu q_\nu \epsilon_3^{*\mu} (\epsilon_2 \times \epsilon_4^*)^\nu \}, \quad (\text{A122})$$

TABLE XIX. The coefficients in the NLO contact amplitudes for the $D\bar{B}^* \rightarrow D^* \bar{B}^*$ process.

	$I = 1$	$I = 0$	ω_1	ω_2
	A	A		
$A_{a7.1}$	0	0	0	0
$A_{a7.3(4)}$	0	0	$0(\delta_2)$	$\delta_2(0)$
$A_{a7.5(6)}$	0	0	$-\delta_1$	$0(\delta_1)$
$A_{a7.7}$	$\frac{1}{4}(D_a + E_a)$	$\frac{-3}{4}(D_a - 3E_a)$	δ_2	$-\delta_1$
$A_{a7.8}$	$\frac{1}{4}(D_b + E_b)$	$\frac{-3}{4}(D_b - 3E_b)$	0	δ_2
$A_{a7.9}$	$\frac{1}{4}(D_a + E_a + D_b + E_b)$	$\frac{-3}{4}(D_a - 3E_a + D_b - 3E_b)$	0	$-\delta_1$
$A_{a7.10}$	$\frac{1}{4}(D_b + E_b)$	$\frac{-3}{4}(D_b - 3E_b)$	0	0
$A_{a7.12}$	$\frac{1}{4}(D_a + E_a)$	$\frac{-3}{4}(D_a - 3E_a)$	0	δ_1
$A_{a7.(13+14)}$	$D_b + E_b$	$D_b - 3E_b$	0	δ_1
$A_{a7.15}$	$D_b + E_b$	$D_b - 3E_b$	$-\delta_1$	0
$A_{a7.(16+17)}$	$D_b + E_b$	$D_b - 3E_b$	0	δ_2

TABLE XX. The coefficients in the NLO OPE amplitudes for the $D\bar{B}^* \rightarrow D^* \bar{B}^*$ process.

	$I = 1$	$I = 0$	ω_1	ω_2
	A	A		
$A_{b7.1(2)}$	$-\frac{1}{16}$	$\frac{3}{16}$	$0(\delta_2)$	0
$A_{b7.3(4)}$	$-\frac{1}{16}$	$\frac{3}{16}$	$-\delta_1$	$0(\delta_1)$
$A_{b7.5(6)}$	$-\frac{1}{12}$	$\frac{1}{4}$	0	0
$A_{b7.7}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	0
$A_{b7.(8+9)}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	δ_1
$A_{b7.10}$	$\frac{1}{4}$	$-\frac{3}{4}$	$-\delta_1$	0
$A_{b7.(11+12)}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	δ_2
$A_{b7.13(14)}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	0
$A_{b7.15(16)}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	0

$$\mathcal{M}_{c7.3}^{(2)} = 4 \frac{g_1^2 g_2^2}{f^4} A [(J_{21}^R + J_{31}^R) q_\mu q_\nu \epsilon_3^{*\mu} (\epsilon_2 \times \epsilon_4^*)^\nu + J_{31}^R q_\mu q_\nu \epsilon_3^{*\mu} (\epsilon_4^* \times \epsilon_2)^\nu], \quad (\text{A123})$$

$$\mathcal{M}_{c7.4}^{(2)} = 4 \frac{g_1^2 g_2^2}{f^4} A \{ (J_{21}^R + J_{31}^R) q^2 \epsilon_3^{*\mu} (\epsilon_2 \times \epsilon_4^*)_\mu + (J_{21}^R + J_{31}^R) q_\mu q_\nu \epsilon_2^\mu (\epsilon_4^* \times \epsilon_3^*)^\nu - [J_{21}^R + (d-2) J_{31}^R] \times q_\mu q_\nu \epsilon_3^{*\mu} (\epsilon_4^* \times \epsilon_2)^\nu \}, \quad (\text{A124})$$

$$\mathcal{M}_{c7.5}^{(2)} = 4 \frac{g_1^2 g_2^2}{f^4} A [(J_{21}^R + J_{31}^R) q_\mu q_\nu \epsilon_3^{*\mu} (\epsilon_4^* \times \epsilon_2)^\nu + J_{31}^R q_\mu q_\nu \epsilon_3^{*\mu} (\epsilon_2 \times \epsilon_4^*)^\nu]. \quad (\text{A125})$$

TABLE XXI. The coefficients in the TPE amplitudes for the $D\bar{B}^* \rightarrow D^*\bar{B}^*$ process.

	$I = 1$	$I = 0$		
	A	A	ω_1	ω_2
$A_{c7.2}$	$\frac{1}{16}$	$\frac{5}{16}$	$-\delta_1$	0
$A_{c7.3}$	$\frac{1}{16}$	$\frac{5}{16}$	$-\delta_1$	δ_2
$A_{c7.4}$	$\frac{9}{16}$	$-\frac{3}{16}$	$-\delta_1$	0
$A_{c7.5}$	$\frac{9}{16}$	$-\frac{3}{16}$	$-\delta_1$	δ_2

The coefficients in the above TPE amplitudes are listed in Table XXI.

5. $D^*\bar{B} \rightarrow D^*\bar{B}^*$

For the process $D^*(p_1)\bar{B}(p_2) \rightarrow D^*(p_3)\bar{B}^*(p_4)$, the contact, OPE, and TPE diagrams are shown in Fig. 16. The scattering amplitudes of LO diagrams are given by

$$\mathcal{M}_{a8}^{(0)} = -4(D_b + E_b)\epsilon_1^\mu(\epsilon_3^* \times \epsilon_4^*)_\mu, \quad (\text{A126})$$

$$\mathcal{M}_{b8}^{(0)} = \frac{g_1 g_2}{f^2} \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_1^\mu(\epsilon_4^* \times \epsilon_3^*)^\nu, \quad (\text{A127})$$

with isospin $I = 1$, and

$$\mathcal{M}_{a8}^{(0)} = -4(D_b - 3E_b)\epsilon_1^\mu(\epsilon_3^* \times \epsilon_4^*)_\mu, \quad (\text{A128})$$

$$\mathcal{M}_{b8}^{(0)} = -\frac{3g_1 g_2}{f^2} \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_1^\mu(\epsilon_4^* \times \epsilon_3^*)^\nu, \quad (\text{A129})$$

with isospin $I = 0$.

The one-loop corrections to the contact amplitudes are listed as follows:

$$\mathcal{M}_{a8.1}^{(2)} = 4\frac{g_2^2}{f^2}(d-3)AJ_{22}^g\epsilon_1^\mu(\epsilon_3^* \times \epsilon_4^*)_\mu, \quad (\text{A130})$$

$$\mathcal{M}_{a8.2}^{(2)} = 0, \quad (\text{A131})$$

$$\mathcal{M}_{a8.3}^{(2)} = -4\frac{g_2^2}{f^2}AJ_{22}^g\epsilon_1^\mu(\epsilon_3^* \times \epsilon_4^*)_\mu, \quad (\text{A132})$$

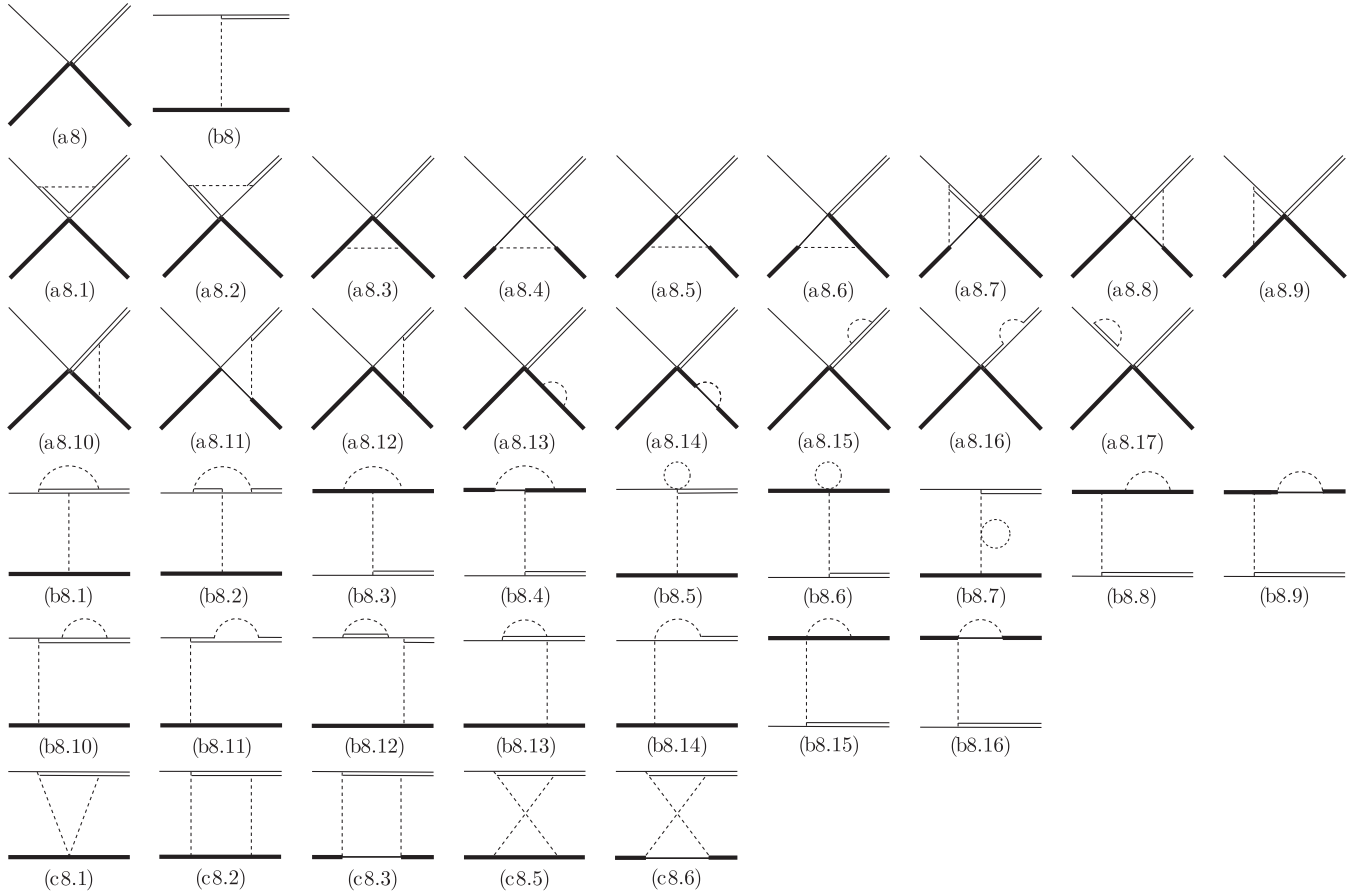


FIG. 16. LO contact (a8), LO OPE (b8), NLO contact (a8.1–a8.17), NLO OPE (b8.1–b8.16), and NLO TPE (c8.1–c8.5) diagrams of the process $D^*\bar{B} \rightarrow D^*\bar{B}^*$.

$$\mathcal{M}_{a8.4}^{(2)} = -4 \frac{g_2^2}{f^2} A J_{22}^g \epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)_\mu, \quad (\text{A133})$$

$$\mathcal{M}_{a8.5}^{(2)} = -8 \frac{g_1^2}{f^2} A J_{22}^g \epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)_\mu, \quad (\text{A134})$$

$$\mathcal{M}_{a8.6}^{(2)} = 4 \frac{g_1^2}{f^2} A J_{22}^g \epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)_\mu, \quad (\text{A135})$$

$$\mathcal{M}_{a8.7}^{(2)} = -4 \frac{g_1 g_2}{f^2} A J_{22}^h \epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)_\mu, \quad (\text{A136})$$

$$\mathcal{M}_{a8.8}^{(2)} = -4 \frac{g_1 g_2}{f^2} A J_{22}^h \epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)_\mu, \quad (\text{A137})$$

$$\mathcal{M}_{a8.9}^{(2)} = -4 \frac{g_1 g_2}{f^2} A J_{22}^h \epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)_\mu, \quad (\text{A138})$$

$$\mathcal{M}_{a8.10}^{(2)} = -4 \frac{g_1 g_2}{f^2} (d-3) A J_{22}^h \epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)_\mu, \quad (\text{A139})$$

$$\mathcal{M}_{a8.11}^{(2)} = 0, \quad (\text{A140})$$

$$\mathcal{M}_{a8.12}^{(2)} = -4 \frac{g_1 g_2}{f^2} A J_{22}^h \epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)_\mu, \quad (\text{A141})$$

$$\begin{aligned} \mathcal{M}_{a8.(13+14)}^{(2)} &= \frac{3g_2^2}{2f^2} A [(d-2)\partial_\omega J_{22}^b(\omega_1) + \partial_\omega J_{22}^b(\omega_2)] \\ &\times [\epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)_\mu], \end{aligned} \quad (\text{A142})$$

$$\mathcal{M}_{a8.15}^{(2)} = \frac{3g_1^2}{2f^2} (d-1) A \partial_\omega J_{22}^b \epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)_\mu, \quad (\text{A143})$$

$$\begin{aligned} \mathcal{M}_{a8.(16+17)}^{(2)} &= \frac{3g_2^2}{2f^2} A [(d-2)\partial_\omega J_{22}^b(\omega_1) + \partial_\omega J_{22}^b(\omega_2)] \\ &\times [\epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)_\mu]. \end{aligned} \quad (\text{A144})$$

The one-loop corrections to the OPE amplitudes are given by

$$\begin{aligned} \mathcal{M}_{b8.1}^{(2)} &= -8 \frac{g_1 g_2^3}{f^2} A J_{22}^g \frac{1}{q^2 - m^2} [q^2 \epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)_\mu \\ &- q_\mu q_\nu \epsilon_1^\mu (\epsilon_4^* \times \epsilon_3^*)^\nu + q_\mu q_\nu \epsilon_3^{*\mu} (\epsilon_4^* \times \epsilon_1)^\nu], \end{aligned} \quad (\text{A145})$$

$$\mathcal{M}_{b8.2}^{(2)} = -4 \frac{g_1 g_2^3}{f^2} A J_{22}^g \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)^\nu, \quad (\text{A146})$$

$$\mathcal{M}_{b8.3}^{(2)} = -4 \frac{g_1^3 g_2}{f^2} (d-3) A J_{22}^g \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)^\nu, \quad (\text{A147})$$

$$\mathcal{M}_{b8.4}^{(2)} = 4 \frac{g_1^3 g_2}{f^2} A J_{22}^g \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)^\nu, \quad (\text{A148})$$

$$\mathcal{M}_{b8.5}^{(2)} = 4 \frac{g_1 g_2}{f^2} A J_0^c \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)^\nu, \quad (\text{A149})$$

$$\mathcal{M}_{b8.6}^{(2)} = 4 \frac{g_1 g_2}{f^2} A J_0^c \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)^\nu, \quad (\text{A150})$$

$$\begin{aligned} \mathcal{M}_{b8.7}^{(2)} &= \frac{8g_1 g_2}{3f^2} A_1 \left[2m^2 L + \frac{m^2}{8\pi^2} \log\left(\frac{m}{\mu}\right) \right] \frac{q_\mu q_\nu}{q^2 - m^2} \\ &\times [\epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)^\nu], \end{aligned} \quad (\text{A151})$$

$$\begin{aligned} \mathcal{M}_{b8.(8+9)}^{(2)} &= -\frac{3g_1^3 g_2}{2f^2} A [(d-2)\partial_\omega J_{22}^b(\omega_1) + \partial_\omega J_{22}^b(\omega_2)] \\ &\times \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)^\nu, \end{aligned} \quad (\text{A152})$$

TABLE XXII. The coefficients in the NLO contact amplitudes for the process $D^* \bar{B} \rightarrow D^* \bar{B}^*$.

	$I = 1$	$I = 0$	ω_1	ω_2
	A	A		
$A_{a8.1}$	0	0	$-\delta_2$	0
$A_{a8.2(3)}$	0	0	$-\delta_2(0)$	$\delta_2(0)$
$A_{a8.5(6)}$	0	0	$0(\delta_1)$	δ_1
$A_{a8.7}$	$\frac{1}{4}(D_a + E_a)$	$\frac{-3}{4}(D_a - 3E_a)$	δ	$-\delta_2$
$A_{a8.8}$	$\frac{1}{4}(D_b + E_b)$	$\frac{-3}{4}(D_b - 3E_b)$	0	δ_1
$A_{a8.9}$	$\frac{1}{4}(D_a + E_a + D_b + E_b)$	$\frac{-3}{4}(D_a - 3E_a + D_b - 3E_b)$	0	$-\delta_2$
$A_{a8.10}$	$\frac{1}{4}(D_b + E_b)$	$\frac{-3}{4}(D_b - 3E_b)$	0	0
$A_{a8.12}$	$\frac{1}{4}(D_a + E_a)$	$\frac{-3}{4}(D_a - 3E_a)$	δ_2	0
$A_{a8.(13+14)}$	$D_b + E_b$	$D_b - 3E_b$	0	δ_1
$A_{a8.(15+16)}$	$D_b + E_b$	$D_b - 3E_b$	0	δ_2
$A_{a8.(16+17)}$	$D_b + E_b$	$D_b - 3E_b$	$-\delta_2$	0

TABLE XXIII. The coefficients in the NLO OPE amplitudes for the process $D^* \bar{B} \rightarrow D^* \bar{B}^*$.

	$I = 1$	$I = 0$	ω_1	ω_2
	A	A		
$A_{b8.1(2)}$	$-\frac{1}{16}$	$\frac{3}{16}$	$-\delta_2$	$0(\delta_2)$
$A_{b8.3(4)}$	$-\frac{1}{16}$	$\frac{3}{16}$	$0(\delta_1)$	0
$A_{b8.5(6)}$	$-\frac{1}{12}$	$\frac{1}{4}$	0	0
$A_{b8.7}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	0
$A_{b8.(8+9)}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	δ_1
$A_{b8.(10+11)}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	δ_2
$A_{b8.12}$	$\frac{1}{4}$	$-\frac{3}{4}$	$-\delta_2$	0
$A_{b8.13}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	0
$A_{b8.14}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	0
$A_{b8.15}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	0
$A_{b8.16}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	0

$$\mathcal{M}_{b8.(10+11)}^{(2)} = -\frac{3g_1g_2^3}{2f^2}A[(d-2)\partial_\omega J_{22}^b(\omega_1) + \partial_\omega J_{22}^b(\omega_2)] \times \frac{q_\mu q_\nu}{q^2 - m^2} \epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)^\nu, \quad (\text{A153})$$

$$\mathcal{M}_{b8.12}^{(2)} = -\frac{3g_1g_2^3}{2f^2}(d-1)A\partial_\omega J_{22}^b \frac{q_\mu q_\nu}{q^2 - m^2} \times [\epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)^\nu], \quad (\text{A154})$$

$$\mathcal{M}_{b8.13(14)}^{(2)} = \frac{8g_1g_2}{f^2}A\left[2m^2L + \frac{m^2}{8\pi^2}\log\left(\frac{m}{\mu}\right)\right] \frac{q_\mu q_\nu}{q^2 - m^2} \times [\epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)^\nu], \quad (\text{A155})$$

$$\mathcal{M}_{b8.15(16)}^{(2)} = \frac{8g_1g_2}{f^2}A\left[2m^2L + \frac{m^2}{8\pi^2}\log\left(\frac{m}{\mu}\right)\right] \frac{q_\mu q_\nu}{q^2 - m^2} \times [\epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)^\nu]. \quad (\text{A156})$$

The coefficients appearing in the NLO contact and OPE amplitudes are listed in Tables XXII and XXIII, respectively, and the TPE amplitudes

$$\mathcal{M}_{c8.1}^{(2)} = 0, \quad (\text{A157})$$

$$\mathcal{M}_{c8.2}^{(2)} = -4\frac{g_1^2g_2^2}{f^4}A\{(J_{21}^B + J_{31}^B)q^2\epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)_\mu + (J_{21}^B + J_{31}^B)q_\mu q_\nu \epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)^\nu + [J_{21}^B + (d-2)J_{31}^B]q_\mu q_\nu \epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)^\nu\}, \quad (\text{A158})$$

TABLE XXIV. The coefficients in the NLO TPE amplitudes for the process $D^*\bar{B} \rightarrow D^*\bar{B}^*$.

	$I = 1$	$I = 0$	ω_1	ω_2
	A	A		
$A_{c8.2}$	$\frac{1}{16}$	$\frac{5}{16}$	$-\delta_2$	0
$A_{c8.3}$	$\frac{1}{16}$	$\frac{5}{16}$	$-\delta_2$	δ_1
$A_{c8.4}$	$\frac{9}{16}$	$-\frac{3}{16}$	$-\delta_2$	0
$A_{c8.5}$	$\frac{9}{16}$	$-\frac{3}{16}$	$-\delta_2$	δ_1

$$\mathcal{M}_{c8.3}^{(2)} = -4\frac{g_1^2g_2^2}{f^4}A[(J_{21}^B + J_{31}^B)q_\mu q_\nu \epsilon_1^\mu (\epsilon_4^* \times \epsilon_3^*)^\nu + J_{31}^B q_\mu q_\nu \epsilon_3^{\mu*} (\epsilon_4^* \times \epsilon_1)^\nu], \quad (\text{A159})$$

$$\mathcal{M}_{c8.4}^{(2)} = 4\frac{g_1^2g_2^2}{f^4}A\{(J_{21}^R + J_{31}^R)q^2\epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)_\mu + (J_{21}^R + J_{31}^R)q_\mu q_\nu \epsilon_1^\mu (\epsilon_3^* \times \epsilon_4^*)^\nu + [J_{21}^R + (d-2)J_{31}^R]q_\mu q_\nu \epsilon_3^{\mu*} (\epsilon_4^* \times \epsilon_1)^\nu\}, \quad (\text{A160})$$

$$\mathcal{M}_{c8.5}^{(2)} = -4\frac{g_1^2g_2^2}{f^4}A[(J_{21}^R + J_{31}^R)q_\mu q_\nu \epsilon_3^{\mu*} (\epsilon_4^* \times \epsilon_1)^\nu + J_{31}^R q_\mu q_\nu \epsilon_1^\mu (\epsilon_4^* \times \epsilon_3^*)^\nu]. \quad (\text{A161})$$

The coefficients in the above TPE amplitudes are listed in Table XXIV.

APPENDIX B: DEFINITIONS OF SOME LOOP FUNCTIONS

The various loop functions used in amplitudes are defined as

$$i \int \frac{d^D l \mu^{4-D}}{(2\pi)^D} \frac{\{1, l^\alpha, l^\alpha l^\beta, l^\alpha l^\beta l^\gamma\}}{[(+/-)v \cdot l + \omega + i\epsilon](l^2 - m^2 + i\epsilon)} \equiv \{J_0^{a/b}, v^\alpha J_{11}^{a/b}, v^\alpha v^\beta J_{21}^{a/b} + g^{\alpha\beta} J_{22}^{a/b}, (g \vee v) J_{31}^{a/b} + v^\alpha v^\beta v^\gamma J_{32}^{a/b}\}(m, \omega), \quad (\text{B1})$$

$$i \int \frac{d^D l \mu^{4-D}}{(2\pi)^D} \frac{\{1, l^\alpha, l^\alpha l^\beta, l^\alpha l^\beta l^\gamma\}}{(v \cdot l + \omega_1 + i\epsilon)[(+/-)v \cdot l + \omega_2 + i\epsilon](l^2 - m^2 + i\epsilon)} \equiv \{J_0^{g/h}, v^\alpha J_{11}^{g/h}, v^\alpha v^\beta J_{21}^{g/h} + g^{\alpha\beta} J_{22}^{g/h}, (g \vee v) J_{31}^{g/h} + v^\alpha v^\beta v^\gamma J_{32}^{g/h}\}(m, \omega_1, \omega_2), \quad (\text{B2})$$

$$i \int \frac{d^D l \mu^{4-D}}{(2\pi)^D} \frac{\{1, l^\alpha, l^\alpha l^\beta, l^\alpha l^\beta l^\gamma\}}{(l^2 - m_1^2 + i\epsilon)[(q+l)^2 - m_2^2 + i\epsilon]} \equiv \{J_0^F, q^\alpha J_{11}^F, q^\alpha q^\beta J_{21}^F + g^{\alpha\beta} J_{22}^F, (g \vee q) J_{31}^F + q^\alpha q^\beta q^\gamma J_{32}^F\}(m_1, m_2, q), \quad (\text{B3})$$

$$\begin{aligned}
 & i \int \frac{d^D l \mu^{4-D}}{(2\pi)^D} \frac{\{1, l^\alpha, l^\alpha l^\beta, l^\alpha l^\beta l^\gamma, l^\alpha l^\beta l^\gamma l^\delta\}}{[(+/-)v \cdot l + \omega + i\epsilon](l^2 - m_1^2 + i\epsilon)[(q+l)^2 - m_2^2 + i\epsilon]} \\
 & \equiv \{J_0^{T/S}, q^\alpha J_{11}^{T/S} + v^\alpha J_{12}^{T/S}, g^{\alpha\beta} J_{21}^{T/S} + q^\alpha q^\beta J_{22}^{T/S} + v^\alpha v^\beta J_{23}^{T/S} + (q \vee v) J_{24}^{T/S}, (g \vee q) J_{31}^{T/S} + q^\alpha q^\beta q^\gamma J_{32}^{T/S} \\
 & + (q^2 \vee v) J_{33}^{T/S} + (g \vee v) J_{34}^{T/S} + (q \vee v^2) J_{35}^{T/S} + v^\alpha v^\beta v^\gamma J_{36}^{T/S}, (g \vee g) J_{41}^{T/S} + (g \vee q^2) J_{42}^{T/S} + q^\alpha q^\beta q^\gamma q^\delta J_{43}^{T/S} \\
 & + (g \vee v^2) J_{44}^{T/S} + v^\alpha v^\beta v^\gamma v^\delta J_{45}^{T/S} + (q^3 \vee v) J_{46}^{T/S} + (q^2 \vee v^2) J_{47}^{T/S} + (q \vee v^3) J_{48}^{T/S} \\
 & + (g \vee q \vee v) J_{49}^{T/S}\}(m_1, m_2, \omega, q), \tag{B4}
 \end{aligned}$$

$$\begin{aligned}
 & i \int \frac{d^D l \mu^{4-D}}{(2\pi)^D} \frac{\{1, l^\alpha, l^\alpha l^\beta, l^\alpha l^\beta l^\gamma, l^\alpha l^\beta l^\gamma l^\delta\}}{(v \cdot l + \omega_1 + i\epsilon)[(+/-)v \cdot l + \omega_2 + i\epsilon](l^2 - m_1^2 + i\epsilon)[(q+l)^2 - m_2^2 + i\epsilon]} \\
 & \equiv \{J_0^{R/B}, q^\alpha J_{11}^{R/B} + v^\alpha J_{12}^{R/B}, g^{\alpha\beta} J_{21}^{R/B} + q^\alpha q^\beta J_{22}^{R/B} + v^\alpha v^\beta J_{23}^{R/B} + (q \vee v) J_{24}^{R/B}, (g \vee q) J_{31}^{R/B} + q^\alpha q^\beta q^\gamma J_{32}^{R/B} \\
 & + (q^2 \vee v) J_{33}^{R/B} + (g \vee v) J_{34}^{R/B} + (q \vee v^2) J_{35}^{R/B} + v^\alpha v^\beta v^\gamma J_{36}^{R/B}, (g \vee g) J_{41}^{R/B} + (g \vee q^2) J_{42}^{R/B} + q^\alpha q^\beta q^\gamma q^\delta J_{43}^{R/B} \\
 & + (g \vee v^2) J_{44}^{R/B} + v^\alpha v^\beta v^\gamma v^\delta J_{45}^{R/B} + (q^3 \vee v) J_{46}^{R/B} + (q^2 \vee v^2) J_{47}^{R/B} + (q \vee v^3) J_{48}^{R/B} \\
 & + (g \vee q \vee v) J_{49}^{R/B}\}(m_1, m_2, \omega_1, \omega_2, q), \tag{B5}
 \end{aligned}$$

with

$$\begin{aligned}
 & q \vee v \equiv q^\alpha v^\beta + q^\beta v^\alpha, \quad g \vee q \equiv g^{\alpha\beta} q^\gamma + g^{\alpha\gamma} q^\beta + g^{\gamma\beta} q^\alpha, \quad g \vee v \equiv g^{\alpha\beta} v^\gamma + g^{\alpha\gamma} v^\beta + g^{\gamma\beta} v^\alpha, \\
 & q^2 \vee v \equiv q^\beta q^\gamma v^\alpha + q^\alpha q^\gamma v^\beta + q^\alpha q^\beta v^\gamma, \quad q \vee v^2 \equiv q^\gamma v^\alpha v^\beta + q^\beta v^\alpha v^\gamma + q^\alpha v^\beta v^\gamma, \\
 & g \vee g \equiv g^{\alpha\beta} g^{\gamma\delta} + g^{\alpha\delta} g^{\beta\gamma} + g^{\alpha\gamma} g^{\beta\delta}, \quad g \vee q^2 \equiv q^\alpha q^\beta g^{\gamma\delta} + q^\alpha q^\delta g^{\beta\gamma} + q^\alpha q^\gamma g^{\beta\delta} + q^\gamma q^\delta g^{\alpha\beta} + q^\beta q^\delta g^{\alpha\gamma} + q^\beta q^\gamma g^{\alpha\delta}, \\
 & g \vee v^2 \equiv v^\alpha v^\beta g^{\gamma\delta} + v^\alpha v^\delta g^{\beta\gamma} + v^\alpha v^\gamma g^{\beta\delta} + v^\gamma v^\delta g^{\alpha\beta} + v^\beta v^\delta g^{\alpha\gamma} + v^\beta v^\gamma g^{\alpha\delta}, \\
 & q^3 \vee v \equiv q^\beta q^\gamma q^\delta v^\alpha + q^\alpha q^\gamma q^\delta v^\beta + q^\alpha q^\beta q^\delta v^\gamma + q^\alpha q^\beta q^\gamma v^\delta, \quad q \vee v^3 \equiv q^\delta v^\alpha v^\beta v^\gamma + q^\gamma v^\alpha v^\beta v^\delta + q^\beta v^\alpha v^\gamma v^\delta + q^\alpha v^\beta v^\gamma v^\delta, \\
 & q^2 \vee v^2 \equiv q^\gamma q^\delta v^\alpha v^\beta + q^\beta q^\delta v^\alpha v^\gamma + q^\alpha q^\delta v^\beta v^\gamma + q^\beta q^\gamma v^\alpha v^\delta + q^\alpha q^\gamma v^\beta v^\delta + q^\alpha q^\beta v^\gamma v^\delta, \\
 & g \vee q \vee v \equiv q^\beta v^\alpha g^{\gamma\delta} + q^\alpha v^\beta g^{\gamma\delta} + q^\delta v^\alpha g^{\beta\gamma} + q^\gamma v^\alpha g^{\beta\delta} + q^\alpha v^\delta g^{\beta\gamma} + q^\alpha v^\gamma g^{\beta\delta} + q^\delta v^\gamma g^{\alpha\beta} + q^\delta v^\beta g^{\alpha\gamma} + q^\gamma v^\delta g^{\alpha\beta} \\
 & + q^\gamma v^\beta g^{\alpha\delta} + q^\beta v^\delta g^{\alpha\gamma} + q^\beta v^\gamma g^{\alpha\delta}. \tag{B6}
 \end{aligned}$$

J^b is related to J^a ,

$$\begin{aligned}
 J_0^b &= J_0^a, & J_{11}^b &= -J_{11}^a, & J_{21}^b &= J_{21}^a, \\
 J_{22}^b &= J_{22}^a, & J_{31}^b &= -J_{31}^a, & J_{32}^b &= -J_{32}^a. \tag{B7}
 \end{aligned}$$

J^g and J^h can be deduced to

$$J^g(\omega_1, \omega_2) = \frac{1}{\omega_2 - \omega_1} [J^a(\omega_1) - J^a(\omega_2)], \tag{B8}$$

$$J^h(\omega_1, \omega_2) = \frac{1}{\omega_2 + \omega_1} [J^a(\omega_1) + J^b(\omega_2)]. \tag{B9}$$

J^S is related to J^T ,

$$\begin{aligned}
 J_0^S(v \cdot q) &= J_0^T(-v \cdot q), & J_{11}^S(v \cdot q) &= J_{11}^T(-v \cdot q), \\
 J_{12}^S(v \cdot q) &= -J_{12}^T(-v \cdot q), & J_{21}^S &= J_{21}^T(-v \cdot q), \\
 J_{22}^S(v \cdot q) &= J_{22}^T(-v \cdot q), & J_{23}^S(v \cdot q) &= J_{23}^T(-v \cdot q), \\
 J_{24}^S(v \cdot q) &= -J_{24}^T(-v \cdot q), & J_{31}^S(v \cdot q) &= J_{31}^T(-v \cdot q), \\
 J_{32}^S(v \cdot q) &= J_{32}^T(-v \cdot q), & J_{33}^S(v \cdot q) &= -J_{33}^T(-v \cdot q), \\
 J_{34}^S(v \cdot q) &= -J_{34}^T(-v \cdot q), & J_{35}^S(v \cdot q) &= J_{35}^T(-v \cdot q), \\
 J_{36}^S(v \cdot q) &= -J_{36}^T(-v \cdot q), & J_{41}^S(v \cdot q) &= J_{41}^T(-v \cdot q), \\
 J_{42}^S(v \cdot q) &= J_{42}^T(-v \cdot q), & J_{43}^S(v \cdot q) &= J_{43}^T(-v \cdot q), \\
 J_{44}^S(v \cdot q) &= J_{44}^T(-v \cdot q), & J_{45}^S(v \cdot q) &= J_{45}^T(-v \cdot q), \\
 J_{46}^S(v \cdot q) &= -J_{46}^T(-v \cdot q), & J_{47}^S(v \cdot q) &= J_{47}^T(-v \cdot q), \\
 J_{48}^S(v \cdot q) &= -J_{48}^T(-v \cdot q), & J_{49}^S(v \cdot q) &= -J_{49}^T(-v \cdot q). \tag{B10}
 \end{aligned}$$

J^R and J^B can be deduced to

$$J^R(\omega_1, \omega_2) = \frac{1}{\omega_2 - \omega_1} [J^T(\omega_1) - J^T(\omega_2)], \quad (\text{B11})$$

$$J^B(\omega_1, \omega_2) = \frac{1}{\omega_2 + \omega_1} [J^T(\omega_1) + J^S(\omega_2)]. \quad (\text{B12})$$

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- [1] R. Aaij *et al.* (LHCb Collaboration), A model-independent study of resonant structure in $B^+ \rightarrow D^+ D^- K^+$ decays, *Phys. Rev. Lett.* **125**, 242001 (2020).
- [2] R. Aaij *et al.* (LHCb Collaboration), Amplitude analysis of the $B^+ \rightarrow D^+ D^- K^+$ decay, *Phys. Rev. D* **102**, 112003 (2020).
- [3] R. Aaij *et al.* (LHCb Collaboration), Observation of an exotic narrow doubly charmed tetraquark, *Nat. Phys.* **18**, 751 (2022).
- [4] R. Aaij *et al.* (LHCb Collaboration), Study of the doubly charmed tetraquark T_{cc}^+ , *Nat. Commun.* **13**, 3351 (2022).
- [5] R. Aaij *et al.* (LHCb Collaboration), First observation of a doubly charged tetraquark and its neutral partner, *Phys. Rev. Lett.* **131**, 041902 (2023).
- [6] R. Aaij *et al.* (LHCb Collaboration), Amplitude analysis of $B^+ \rightarrow D^- D_s^+ \pi^+$ and $B^0 \rightarrow \bar{D}^0 D_s^+ \pi^+$ decays, *Phys. Rev. D* **108**, 012017 (2023).
- [7] Y. Cui, X. L. Chen, W. Z. Deng, and S. L. Zhu, The possible heavy tetraquarks $qq\bar{q}\bar{Q}$, $qq\bar{Q}\bar{Q}$ and $qQ\bar{Q}\bar{Q}$, *High Energy Phys. Nucl. Phys.* **31**, 7 (2007), <https://scispace.com/papers/the-possible-heavy-tetraquarks-qq-bar-q-bar-q-q-q-bar-q-bar-q-51dg6lskhm>.
- [8] W. Detmold, K. Orginos, and M. J. Savage, BB potentials in quenched lattice QCD, *Phys. Rev. D* **76**, 114503 (2007).
- [9] M. V. Carlucci, F. Giannuzzi, G. Nardulli, M. Pellicoro, and S. Stramaglia, AdS-QCD quark-antiquark potential, meson spectrum and tetraquarks, *Eur. Phys. J. C* **57**, 569 (2008).
- [10] F. S. Navarra, M. Nielsen, and S. H. Lee, QCD sum rules study of $QQ - \bar{u}\bar{d}$ mesons, *Phys. Lett. B* **649**, 166 (2007).
- [11] Y. Yang, C. Deng, J. Ping, and T. Goldman, S -wave $QQ\bar{q}\bar{q}$ state in the constituent quark model, *Phys. Rev. D* **80**, 114023 (2009).
- [12] T. F. Carames, A. Valcarce, and J. Vijande, Doubly charmed exotic mesons: A gift of nature?, *Phys. Lett. B* **699**, 291 (2011).
- [13] R. Molina, T. Branz, and E. Oset, A new interpretation for the $D_{s2}^*(2573)$ and the prediction of novel exotic charmed mesons, *Phys. Rev. D* **82**, 014010 (2010).
- [14] M. L. Du, W. Chen, X. L. Chen, and S. L. Zhu, Exotic $QQ\bar{q}\bar{q}$, $QQ\bar{q}\bar{s}$ and $QQ\bar{s}\bar{s}$ states, *Phys. Rev. D* **87**, 014003 (2013).
- [15] T. Hyodo, Y. R. Liu, M. Oka, K. Sudoh, and S. Yasui, Production of doubly charmed tetraquarks with exotic color configurations in electron-positron collisions, *Phys. Lett. B* **721**, 56 (2013).
- [16] S. Ohkoda, Y. Yamaguchi, S. Yasui, K. Sudoh, and A. Hosaka, Exotic mesons with double charm and bottom flavor, *Phys. Rev. D* **86**, 034019 (2012).
- [17] J. Vijande, A. Valcarce, and J. M. Richard, Adiabaticity and color mixing in tetraquark spectroscopy, *Phys. Rev. D* **87**, 034040 (2013).
- [18] Y. Ikeda, B. Charron, S. Aoki, T. Doi, T. Hatsuda, T. Inoue, N. Ishii, K. Murano, H. Nemura, and K. Sasaki, Charmed tetraquarks T_{cc} and T_{cs} from dynamical lattice QCD simulations, *Phys. Lett. B* **729**, 85 (2014).
- [19] P. Bicudo, K. Cichy, A. Peters, and M. Wagner, BB interactions with static bottom quarks from Lattice QCD, *Phys. Rev. D* **93**, 034501 (2016).
- [20] P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, and M. Wagner, Evidence for the existence of $ud\bar{b}\bar{b}$ and the non-existence of $ss\bar{b}\bar{b}$ and $cc\bar{b}\bar{b}$ tetraquarks from lattice QCD, *Phys. Rev. D* **92**, 014507 (2015).
- [21] A. Peters, P. Bicudo, K. Cichy, B. Wagenbach, and M. Wagner, Exploring possibly existing $qq\bar{b}\bar{b}$ tetraquark states with $qq = ud, ss, cc$, *Proc. Sci. LATTICE2015* (2016) 095, <https://pos.sissa.it/251/095/>.
- [22] A. Peters, P. Bicudo, L. Leskovec, S. Meinel, and M. Wagner, Lattice QCD study of heavy-heavy-light-light tetraquark candidates, *Proc. Sci. LATTICE2016* (2016) 104.
- [23] A. Francis, R. J. Hudspith, R. Lewis, and K. Maltman, Lattice prediction for deeply bound doubly heavy tetraquarks, *Phys. Rev. Lett.* **118**, 142001 (2017).
- [24] P. Bicudo, M. Cardoso, A. Peters, M. Pflaumer, and M. Wagner, $ud\bar{b}\bar{b}$ tetraquark resonances with lattice QCD potentials and the Born-Oppenheimer approximation, *Phys. Rev. D* **96**, 054510 (2017).
- [25] W. Chen, H. X. Chen, X. Liu, T. G. Steele, and S. L. Zhu, Open-flavor charm and bottom $s\bar{q}\bar{q}\bar{Q}$ and $qq\bar{q}\bar{Q}$ tetraquark states, *Phys. Rev. D* **95**, 114005 (2017).
- [26] K. Azizi and U. Özdem, The electromagnetic multipole moments of the charged open-flavor $Z_{\bar{c}q}$ states, *J. Phys. G* **45**, 055003 (2018).
- [27] P. Bicudo, M. Cardoso, A. Peters, M. Pflaumer, and M. Wagner, Doubly heavy tetraquark resonances in lattice QCD, *J. Phys. Conf. Ser.* **1137**, 012039 (2019).
- [28] Z. G. Wang, Analysis of the axialvector B_c -like tetraquark states with the QCD sum rules, *Europhys. Lett.* **128**, 11001 (2019).
- [29] Q. F. Lü, D. Y. Chen, and Y. B. Dong, Open charm and bottom tetraquarks in an extended relativized quark model, *Phys. Rev. D* **102**, 074021 (2020).
- [30] X. G. He, W. Wang, and R. Zhu, Open-charm tetraquark X_c and open-bottom tetraquark X_b , *Eur. Phys. J. C* **80**, 1026 (2020).

- [31] J. B. Cheng, S. Y. Li, Y. R. Liu, Y. N. Liu, Z. G. Si, and T. Yao, Spectrum and rearrangement decays of tetraquark states with four different flavors, *Phys. Rev. D* **101**, 114017 (2020).
- [32] R. M. Albuquerque, S. Narison, D. Rabetiarivony, and G. Randriamanatrika, $X_{0,1}(2900)$ and (D^-K^+) invariant mass from QCD Laplace sum rules at NLO, *Nucl. Phys. A* **1007**, 122113 (2021).
- [33] T. Guo, J. Li, J. Zhao, and L. He, Mass spectra and decays of open-heavy tetraquark states, *Phys. Rev. D* **105**, 054018 (2022).
- [34] H. X. Chen, W. Chen, X. Liu, Y. R. Liu, and S. L. Zhu, An updated review of the new hadron states, *Rep. Prog. Phys.* **86**, 026201 (2023).
- [35] N. Li, Z. F. Sun, X. Liu, and S. L. Zhu, Coupled-channel analysis of the possible $D^{(*)}D^{(*)}$, $\bar{B}^{(*)}\bar{B}^{(*)}$ and $D^{(*)}\bar{B}^{(*)}$ molecular states, *Phys. Rev. D* **88**, 114008 (2013).
- [36] W. Chen, T. G. Steele, and S. L. Zhu, Exotic open-flavor $bc\bar{q}\bar{q}$, $bc\bar{s}\bar{s}$ and $qc\bar{q}\bar{b}$, $sc\bar{s}\bar{b}$ tetraquark states, *Phys. Rev. D* **89**, 054037 (2014).
- [37] S. Q. Luo, K. Chen, X. Liu, Y. R. Liu, and S. L. Zhu, Exotic tetraquark states with the $qq\bar{Q}\bar{Q}$ configuration, *Eur. Phys. J. C* **77**, 709 (2017).
- [38] C. Deng and S. L. Zhu, T_{cc}^+ and its partners, *Phys. Rev. D* **105**, 054015 (2022).
- [39] W. L. Wu, Y. Ma, Y. K. Chen, L. Meng, and S. L. Zhu, Doubly heavy tetraquark bound and resonant states, *Phys. Rev. D* **110**, 094041 (2024).
- [40] E. J. Eichten and C. Quigg, Heavy-quark symmetry implies stable heavy tetraquark mesons $Q_i Q_j \bar{q}_k \bar{q}_l$, *Phys. Rev. Lett.* **119**, 202002 (2017).
- [41] B. Silvestre-Brac and C. Semay, Systematics of $L = 0$ $q^2\bar{q}^2$ systems, *Z. Phys. C* **57**, 273 (1993).
- [42] D. Ebert, R. N. Faustov, V. O. Galkin, and W. Lucha, Masses of tetraquarks with two heavy quarks in the relativistic quark model, *Phys. Rev. D* **76**, 114015 (2007).
- [43] W. Park, S. Noh, and S. H. Lee, Masses of the doubly heavy tetraquarks in a constituent quark model, *Nucl. Phys. A* **983**, 1 (2019).
- [44] Q. F. Lü, D. Y. Chen, and Y. B. Dong, Masses of doubly heavy tetraquarks $T_{QQ'}$ in a relativized quark model, *Phys. Rev. D* **102**, 034012 (2020).
- [45] E. Braaten, L. P. He, and A. Mohapatra, Masses of doubly heavy tetraquarks with error bars, *Phys. Rev. D* **103**, 016001 (2021).
- [46] Y. Kim, M. Oka, and K. Suzuki, Doubly heavy tetraquarks in a chiral-diquark picture, *Phys. Rev. D* **105**, 074021 (2022).
- [47] Y. Song and D. Jia, Mass spectra of doubly heavy tetraquarks in diquark-antidiquark picture, *Commun. Theor. Phys.* **75**, 055201 (2023).
- [48] A. Ali, Q. Qin, and W. Wang, Discovery potential of stable and near-threshold doubly heavy tetraquarks at the LHC, *Phys. Lett. B* **785**, 605 (2018).
- [49] S. Meinel, M. Pflaumer, and M. Wagner, Search for $\bar{b}\bar{b}us$ and $\bar{b}\bar{c}ud$ tetraquark bound states using lattice QCD, *Phys. Rev. D* **106**, 034507 (2022).
- [50] C. Alexandrou, J. Finkenrath, T. Leontiou, S. Meinel, M. Pflaumer, and M. Wagner, Shallow bound states and hints for broad resonances with quark content $\bar{b}\bar{c}ud$ in $B - \bar{D}$ and $B^* - \bar{D}$ scattering from lattice QCD, *Phys. Rev. Lett.* **132**, 151902 (2024).
- [51] M. Padmanath, A. Radhakrishnan, and N. Mathur, Bound isoscalar axial-vector $bc\bar{u}\bar{d}$ tetraquark T_{bc} from lattice QCD using two-meson and diquark-antidiquark variational basis, *Phys. Rev. Lett.* **132**, 20 (2024).
- [52] A. Radhakrishnan, M. Padmanath, and N. Mathur, Study of the isoscalar scalar $bc\bar{u}\bar{d}$ tetraquark T_{bc} with lattice QCD, *Phys. Rev. D* **110**, 034506 (2024).
- [53] M. Z. Liu and L. S. Geng, Investigations of the weak decays of $D\bar{B}$ molecules, *Phys. Rev. D* **110**, 053002 (2024).
- [54] R. Machleidt and D. R. Entem, Chiral effective field theory and nuclear forces, *Phys. Rep.* **503**, 1 (2011).
- [55] H. W. Hammer, S. König, and U. van Kolck, Nuclear effective field theory: Status and perspectives, *Rev. Mod. Phys.* **92**, 025004 (2020).
- [56] H. Xu, B. Wang, Z. W. Liu, and X. Liu, DD^* potentials in chiral perturbation theory and possible molecular states, *Phys. Rev. D* **99**, 014027 (2019); **104**, 119903(E) (2021).
- [57] B. Wang and L. Meng, Revisiting the DD^* chiral interactions with the local momentum-space regularization up to the third order and the nature of T_{cc}^+ , *Phys. Rev. D* **107**, 094002 (2023).
- [58] B. Wang, Z. W. Liu, and X. Liu, $\bar{B}^{(*)}\bar{B}^{(*)}$ interactions in chiral effective field theory, *Phys. Rev. D* **99**, 036007 (2019).
- [59] H. Xu, Study of the hidden charm $D\bar{D}^*$ interactions in chiral effective field theory, *Phys. Rev. D* **105**, 034013 (2022).
- [60] L. M. Abreu, A note on the possible bound $D^{(*)}D^{(*)}$, $\bar{B}^{(*)}\bar{B}^{(*)}$ and $D^{(*)}\bar{B}^{(*)}$ states, *Nucl. Phys. B* **985**, 115994 (2022).
- [61] L. Meng, B. Wang, G. J. Wang, and S. L. Zhu, The hidden charm pentaquark states and $\Sigma_c\bar{D}^{(*)}$ interaction in chiral perturbation theory, *Phys. Rev. D* **100**, 014031 (2019).
- [62] B. Wang, L. Meng, and S. L. Zhu, Hidden-charm and hidden-bottom molecular pentaquarks in chiral effective field theory, *J. High Energy Phys.* **11** (2019) 108.
- [63] B. Wang, L. Meng, and S. L. Zhu, Molecular tetraquarks and pentaquarks in chiral effective field theory, *Nucl. Part. Phys. Proc.* **324–329**, 45 (2023).
- [64] D. R. Entem and R. Machleidt, Accurate charge-dependent nucleon-nucleon potential at fourth order of chiral perturbation theory, *Phys. Rev. C* **68**, 041001 (2003).
- [65] D. R. Entem, R. Machleidt, and Y. Nosyk, High-quality two-nucleon potentials up to fifth order of the chiral expansion, *Phys. Rev. C* **96**, 024004 (2017).
- [66] M. L. Du, F. K. Guo, U. G. Meißner, and D. L. Yao, Aspects of the low-energy constants in the chiral Lagrangian for charmed mesons, *Phys. Rev. D* **94**, 094037 (2016).
- [67] G. Ecker, J. Gasser, A. Pich, and E. de Rafael, The role of resonances in chiral perturbation theory, *Nucl. Phys. B* **321**, 311 (1989).
- [68] G. Ecker, J. Gasser, H. Leutwyler, A. Pich, and E. de Rafael, Chiral Lagrangians for massive spin 1 fields, *Phys. Lett. B* **223**, 425 (1989).

- [69] J. F. Donoghue, C. Ramirez, and G. Valencia, The spectrum of QCD and chiral Lagrangians of the strong and weak interactions, *Phys. Rev. D* **39**, 1947 (1989).
- [70] E. Epelbaum, H. Krebs, and U. G. Meißner, Improved chiral nucleon-nucleon potential up to next-to-next-to-next-to-leading order, *Eur. Phys. J. A* **51**, 53 (2015).
- [71] M. Thomson, *Modern Particle Physics* (Cambridge University Press, Cambridge, England, 2013).
- [72] H. Zhai, *Ultracold Atomic Physics* (Cambridge University Press, Cambridge, England, 2021).
- [73] J. R. Taylor, *Scattering Theory: The Quantum Theory of Nonrelativistic Collisions* (John Wiley & Sons, Inc., New York, 1972).
- [74] Z. Liu, H. Xu, and X. Liu, S-wave potentials and scattering rates of $D^{(*)}\bar{B}^{(*)}$ systems in chiral effective field theory, Zenodo, [10.5281/zenodo.16628036](https://zenodo.org/record/16628036) (2025)