

Exotic decays and collider signatures of pseudo-Nambu-Goldstone bosons in the $SU(5)/SO(5)$ composite Higgs model

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(Received 27 June 2025; accepted 23 December 2025; published 20 January 2026)

The nature of the Higgs boson, whether it is elementary or composite, will be investigated through precision measurements at the collider experiments. In composite Higgs scenarios, the Higgs may manifest as a pseudo-Nambu-Goldstone boson (pNGB) arising from a strongly interacting sector. The $SU(5)/SO(5)$ composite Higgs model features a rich scalar sector, with the decay patterns of the scalars being heavily influenced by the way fermions are embedded in various representations of $SU(5)$. We discuss how masses of the pNGB scalars and their couplings depend functionally on the compositeness scale and the parameters of the strong sector. Unique decay modes of the scalars emerge from the model when the mixing among the various pNGB scalars is non-negligible. We present a comprehensive and thorough analysis of the fermiophilic and fermiophobic decay modes of the pNGB scalars. Significant differences are observed in the decay patterns of the two singly charged scalars. Further, the decay of one pNGB to another on shell pNGB when masses exceed about 1 TeV presents a rich phenomenology, leading to distinctive signatures at the colliders. In this context, the future muon collider offers a promising avenue for detecting pNGB scalars with masses larger than 1 TeV, especially in final states involving W/Z fatjets.

DOI: [10.1103/PhysRevD.113.015022](https://doi.org/10.1103/PhysRevD.113.015022)

I. INTRODUCTION

The last missing piece of the Standard Model (SM), the Higgs boson, was discovered by ATLAS [1] and CMS [2] at the Large Hadron Collider (LHC) in 2012 and, the properties of the SM have been observed with great precision at the LHC so far, with hints of disagreement in some measurements [3–5]. Nevertheless, the question of whether the Higgs is a fundamental particle or composite persists. Moreover, in the Standard Model, the Higgs mass is not protected by any symmetry, thus, it receives large corrections at higher scales. Also, the SM does not provide any dynamical explanation for the mechanism of electroweak symmetry breaking (EWSB). Composite Higgs models [6–9] offer an elegant solution to the hierarchy problem in Higgs physics by predicting a new strongly interacting sector around the TeV energy scale. In these models, the Higgs emerges as a pseudo-Nambu-Goldstone boson of the strong sector [6]. The global symmetry is

spontaneously broken, thereby, protecting the Higgs mass from corrections above the compositeness scale (f). The angle of vacuum misalignment (θ) is expressed as $\sin\theta = v/f$, where $v = 246$ GeV, is the electroweak vacuum expectation value (VEV).

The minimal case, which is known as the minimal composite Higgs model, based on $SO(5)/SO(4)$ [10–13] predicts only one pNGB scalar. Depending upon the global symmetry, the nonminimal scenarios predict new composite pNGB scalars. For example, in $SO(6)/SO(5)$ [14,15], two pNGB scalars are obtained. In other popular nonminimal composite Higgs models such as $SU(4)/Sp(4)$ [16], $SU(5)/SO(5)$ [7], $SU(4)^2/SU(4)$ [17] etc., the breaking is introduced by the dynamics of the strong sector fermions [18]. These models not only offer a very rich scalar spectrum, they also predict many observations such as flavor changing neutral currents [17], dark matter [19], baryon asymmetry in the universe [20], anomalous magnetic moment measurements [21], etc. Moreover, these scalars exhibit unique decay modes. Thus, the signatures of these pNGB scalars may or may not match with the standard searches of the exotic scalars performed by the LHC [22,23].

We choose to study the specific model $SU(5)/SO(5)$ [7,24,25] which exhibits a very rich scalar sector. Here, in addition to a scalar doublet, the field content also includes one gauge singlet pseudoscalar and three $SU(2)_L$ triplets.

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At low energy, this model looks similar to the Georgi-Machacek (GM) model [26,27] with an additional singlet [28].¹ The composite sector and the SM fermion sector are coupled via a linear mixing between the SM fermions and composite spin 1/2 operators under the partial compositeness framework [29], which requires the existence of heavy fermionic resonances [30]. Depending upon the embedding of the SM fermions in the representations of $SU(5)$, the scalars may or may not have couplings with the fermions. This has been extensively studied in Refs. [24,29]. For example, if the left-handed and right-handed fermions are in the adjoint representation of $SU(5)$, the coupling of the pNGBs to the SM fermions vanishes, except with the SM Higgs. It is shown in Ref. [24] that in other possible representations, the couplings of the pNGB scalars to the SM fermions can be generated. Hence, depending upon the embedding of the fermions, the decays of the pNGB scalars are very distinct; either *fermiophilic* or *fermiophobic*.

Recently, Refs. [23,31] have studied a subset of the fermiophilic and fermiophobic scenarios in detail and discussed possible collider signatures and bounds from the LHC. The discussion in these papers is based on a simplified scenario where the value of the compositeness scale (f) and the masses of the scalars (hence, the mass differences among the pNGB scalars) are fixed at certain values. However, the value of the compositeness scale is generally fixed by the allowed fine tuning (measured in terms of the parameter $\xi = v^2/f^2$) in the model, which depends heavily on the embedding of the fermions in different representations [32]. Although the value of f is constrained from the electroweak precision data, it is safe to take the range of the compositeness scale from 1 TeV to 5 TeV [24]. We explicitly calculate the masses of the pNGB scalars as functions of f and C_g , where C_g is the contribution of the gauge loops in the pNGB potential and it encodes the dynamics of the strong sector. The parameter C_g is also influenced by the selected representation in the model being analyzed. The effect of these two parameters appears in the branching ratios of the pNGB scalars that we study and some unique decay patterns of the scalars are identified. In the previous studies of the pNGB scalars, the effects of mixing between the gauge and mass eigenstates of the scalars have been neglected. We derive the mass eigenstates and the couplings of the pNGBs using the mixing angles in the charged and neutral sectors. These considerations lead to interesting decay modes of the pNGB scalars with masses of order TeV.

ATLAS and CMS measurements for the Higgs coupling constant put a limit on the coupling of the standard Higgs

boson with the SM fermions and gauge bosons. For example, Ref. [18] combined the Run 1 and the recent Run 2 LHC data and found that different choices for fermion embedding lead to a spread of limits but a lower bound on the scale f can be set to 600 GeV. This limit, however, is obtained by combining different composite Higgs models. There is no dedicated search for pNGB scalars in the $SU(5)/SO(5)$ composite Higgs model. Nevertheless, ATLAS and CMS have placed limits on the charged and neutral scalars in other BSM models. The direct search limit on the charged Higgs comes from LEP, which is around 80 GeV [33]. Limits exist on the production cross section of singly-charged scalars where they directly decay to fermions [17,34–39]. Amongst the searches for fermiophobically decaying charged scalars, the searches exist where the singly-charged scalars decay to WZ [40,41] and the doubly-charged scalars decay to WW [22,41–43]. These experimental limits can be applied to the pNGB scalars when they have similar decay patterns. In all these scenarios, the limit on the scalars does not exceed 1 TeV. The mass spectrum of the scalars that we study for specific collider signatures is above 1 TeV. Hence, these limits are not applicable.

For a large value of f , around 5 TeV, the pNGB scalars have masses of order TeV. Hence, observation of these scalars at the LHC, at current center-of-mass energies, would be difficult due to low pair-production cross section. Although a high-energy pp machine [44] may be a great option, the presence of abundant gauge bosons in the final state makes it challenging to observe multijet signals (with or without leptons) due to the large QCD background. Given these limitations, the muon collider [45,46] emerges as a promising alternative for detecting pNGB scalars with unique signatures. The muon collider offers two main advantages. Unlike other lepton colliders that use electron beams, energy loss from synchrotron radiation is minimized in muon collider due to muon's greater mass. This feature allows the collider to achieve high energy as well as high luminosity. Further, the muon collider also offers a clean environment for studying multijets. We discuss the pair production cross sections of pNGB scalars at 3 TeV and 6 TeV center-of-mass energies of the muon collider. The unique decay modes of the pNGBs offer signals rich with multiple W/Z fatjets.

II. COMPOSITE HIGGS MODEL $SU(5)/SO(5)$

The nonminimal Composite Higgs Models predict additional pNGB scalars in addition to the standard Higgs doublet. The composite Higgs scenario based on the $SU(5)/SO(5)$ coset [24] leads to 14 pNGBs which decompose under the custodial $SU(2)_L \times SU(2)_R$ as

$$14 \rightarrow (3, 3) + (2, 2) + (1, 1), \quad (1)$$

i.e., it includes a bitriple, the Higgs bidoublet (H) and a singlet (η). The bitriple, further, decomposes as

¹However, there is a crucial difference between the scalars of the GM model and the $SU(5)/SO(5)$ scenario. The neutral component of the custodial triplet is CP -even, while the others are CP -odd, forbidding the singlet VEV and thus preventing tree-level coupling of new scalars to W and Z bosons in CP -conserving cases.

$$(3, 3) \rightarrow 5 + 3 + 1, \quad (2)$$

i.e., it includes a quintuplet $\eta_5 \equiv (\eta_5^0, \eta_5^\pm, \eta_5^{\pm\pm})$, a triplet $\eta_3 \equiv (\eta_3^0, \eta_3^\pm)$, and a singlet η_1^0 of custodial $SU(2)_C$.

First, we consider a particular scenario of the $SU(5)/SO(5)$ composite Higgs model in which the left- and right-handed fermions are embedded in the adjoint

representation of $SU(5)$. In this scenario, the minimal potential is generated at leading order (LO) and the coupling of the pNGB scalars (other than the SM Higgs) to SM fermions is forbidden. This situation is often termed as a *fermiophobic* scenario. On the other hand, their coupling to SM gauge bosons results from their covariant derivative which can be expressed through the Lagrangian as

$$\begin{aligned} \mathcal{L}_{\Phi\Phi V} = & \frac{ie}{s_W} W^{-\mu} \sum_{i,j} [\kappa_W^{\phi_i^0 \phi_j^+} \phi_i^0 \overleftrightarrow{\partial}_\mu \phi_j^+ + \kappa_W^{\phi_i^- \phi_j^{++}} \phi_i^- \overleftrightarrow{\partial}_\mu \phi_j^{++}] + \text{H.c.} \\ & + \frac{ie}{s_W c_W} Z^\mu \left[\sum_{i < j} \kappa_Z^{\phi_i^0 \phi_j^0} \phi_i^0 \overleftrightarrow{\partial}_\mu \phi_j^0 + \sum_{i,j} (\kappa_Z^{\phi_i^+ \phi_j^-} \phi_i^+ \overleftrightarrow{\partial}_\mu \phi_j^- + \kappa_Z^{\phi_i^{++} \phi_j^{--}} \phi_i^{++} \overleftrightarrow{\partial}_\mu \phi_j^{--}) \right] \\ & - ie A^\mu \sum_i [\phi_i^+ \overleftrightarrow{\partial}_\mu \phi_i^- + 2\phi_i^{++} \overleftrightarrow{\partial}_\mu \phi_i^{--}]. \end{aligned} \quad (3)$$

In the above equation, $\phi^0 = \{\eta, \eta_1^0, \eta_3^0, \eta_5^0\}$, $\phi^\pm = \{\eta_3^\pm, \eta_5^\pm\}$, and $\phi^{\pm\pm} = \eta_5^{\pm\pm}$. In these models, we also encounter additional gauge bosons. After integrating them out, we can derive couplings of the pNGB scalars to the Standard Model gauge bosons through a dimension-5 term such as

$$\begin{aligned} \mathcal{L}_{\Phi V \tilde{V}} = & \frac{e^2}{16\pi^2 v} \left[\phi^0 \left(\kappa_{\gamma\gamma}^{\phi^0} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{2}{s_W c_W} \kappa_{\gamma Z}^{\phi^0} F_{\mu\nu} \tilde{Z}^{\mu\nu} + \frac{1}{s_W^2 c_W^2} \kappa_{ZZ}^{\phi^0} Z_{\mu\nu} \tilde{Z}^{\mu\nu} + \frac{2}{s_W^2} \kappa_{WW}^{\phi^0} W_{\mu\nu}^+ \tilde{W}^{-\mu\nu} \right) \right. \\ & + \sum_i \phi^+ \left(\frac{2}{s_W} \kappa_{\gamma W}^{\phi^+} F_{\mu\nu} \tilde{W}^{-\mu\nu} + \frac{2}{s_W^2 c_W} \kappa_{ZW}^{\phi^+} Z_{\mu\nu} \tilde{W}^{-\mu\nu} \right) + \text{H.c.} \\ & \left. + \frac{1}{s_W^2} \sum_i \phi^{++} \kappa_{W^- W^-}^{\phi^{++}} W_{\mu\nu}^- \tilde{W}^{-\mu\nu} + \text{H.c.} \right]. \end{aligned} \quad (4)$$

The Lagrangian involving coupling between the SM Higgs boson and EW gauge bosons can be expressed as

$$\mathcal{L}_{hVV} = 2M_W^2 \frac{h}{v} \left[\kappa_{WW}^h W_\mu^+ W^{-\mu} + \kappa_{ZZ}^h \frac{1}{2c_W^2} Z_\mu Z^\mu \right]. \quad (5)$$

We refer to the complete *fermiophobic* Lagrangian as

$$\mathcal{L}_{F-\text{phobic}} = \mathcal{L}_{\Phi\Phi V} + \mathcal{L}_{\Phi V \tilde{V}}. \quad (6)$$

It is extensively discussed in Ref. [24] that if the fermions are embedded in other representations [such as Adjoint(L)+Symmetric (R)] of $SU(5)$, the scalar potential receives contributions from both the fermion and gauge loops. Hence, interaction Lagrangian in the *fermiophilic* scenario generates couplings between the pNGBs and the SM fermions which can be expressed as

$$\mathcal{L}_{F-\text{philic}} = \mathcal{L}_{\Phi\Phi V} + \mathcal{L}_{\Phi V \tilde{V}} + \mathcal{L}_{\Phi ff}, \quad (7)$$

where

$$\begin{aligned} \mathcal{L}_{\Phi ff} = & \phi^0 [\bar{t}(\kappa_t^{\Phi^0} + i\kappa_t^{\Phi^0} \gamma_5)t + \bar{b}(\kappa_b^{\Phi^0} + i\kappa_b^{\Phi^0} \gamma_5)b] \\ & + \Phi^+ [\kappa_{ib,L}^{\Phi^+} \bar{t} P_L b + (L \leftrightarrow R)] + \text{H.c.} \end{aligned} \quad (8)$$

Here, couplings (denoted by κ) to only the third generation of the SM fermions are considered. Note that we assume there is no Majorana-type coupling to the leptons. Hence, pNGB scalar decays into leptons are forbidden. In the next section, we discuss the gauge and fermionic couplings of the pNGB scalars in detail.

III. MASS SPECTRUM AND COUPLINGS OF THE PNGB SCALARS

In this section, we discuss the masses and mixing of the pNGBs in the scenario where both the left- and right-handed fermions are embedded in the adjoint representation of $SU(5)$. The mass matrices for the pNGBs are shown in Ref. [24] and the masses are computed under the assumption that the mixing between the pNGBs in neutral and charged sectors is negligible. Here, we do not neglect the mixing and we derive the masses of the pNGB in physical basis and their couplings as a function of the mixing angles.

A. Masses of the scalars

The doubly-charged scalar remains unmixed and has mass [24],

$$m_{\eta_5^{\pm\pm}}^2 = \frac{2}{3} m_h^2 \cot^2 2\theta + 4 \frac{C_g v^2}{s_\theta^2} (3g^2 + g'^2) + \frac{4}{3} \frac{C_g v^2}{s_\theta^2} (3 - c_{2\theta}) g'^2. \quad (9)$$

The mixing between the two singly charged custodial eigenstates, $\eta_3^\pm$ and $\eta_5^\pm$, can be parametrized by the following mass matrix [24]:

$$\mathcal{M}_\pm^2 = \begin{pmatrix} \frac{2}{3} m_h^2 \cot^2 2\theta + 4C_g \left(3g^2 + g'^2 - \frac{g'^2}{3} c_{2\theta}\right) \frac{v^2}{s_\theta^2} & 4C_g g'^2 \frac{v^2}{s_\theta^2} \\ 4C_g g'^2 \frac{v^2}{s_\theta^2} & \frac{2}{3} m_h^2 \cot^2 2\theta + 4C_g \left(3g^2 + \frac{g'^2}{2} + \frac{1}{6} g'^2 c_{2\theta}\right) \frac{v^2}{s_\theta^2} \end{pmatrix} \quad (10)$$

On diagonalization, we obtain the mass eigenstates, which we denote as, $\eta_1^\pm$ and $\eta_2^\pm$. The rotation between mass basis and the gauge basis are defined by angle k_+ , given by

$$\tan 2k_+ = \frac{2\mathcal{M}_{\pm 12}}{\mathcal{M}_{\pm 11} - \mathcal{M}_{\pm 22}}, \quad (11)$$

such that

$$\eta_1^\pm = \cos k_+ \eta_3^\pm - \sin k_+ \eta_5^\pm \quad \eta_2^\pm = \sin k_+ \eta_3^\pm + \cos k_+ \eta_5^\pm. \quad (12)$$

The masses, $m_{\eta_1^\pm}^2$ and $m_{\eta_2^\pm}^2$ are obtained, straightforwardly, from the mass matrix $\mathcal{M}_\pm^2$.

The neutral scalar, η_3^0 , being CP even, does not mix with other scalars. In addition, the other singlet scalar, η , mixes with η_1^0 and η_5^0 but the mixing is negligible. Hence, their masses are [24]

$$\begin{aligned} m_{\eta_3^0}^2 &\approx \frac{2}{3} m_h^2 \cot^2 2\theta + 4 \frac{C_g v^2}{\sin^2 \theta} \left((3g^2 + g'^2) + \frac{2}{3} g'^2 c_{2\theta} \right), \\ m_\eta^2 &\approx \frac{5}{6} \frac{m_h^2}{s_\theta^2} - \frac{20}{3} C_g (3g^2 + g'^2) \frac{v^2}{s_\theta^2}. \end{aligned} \quad (13)$$

On the other hand, there exists a significant mixing between η_1^0 and η_5^0 . The 2×2 mass matrix is [24],

$$\mathcal{M}^2 = \begin{pmatrix} \frac{1}{6} \frac{m_h^2}{s_\theta^2} + 4C_g (3g^2 + g'^2) \frac{v^2}{s_\theta^2} & \frac{8}{3} \sqrt{2} C_g g'^2 \frac{v^2}{s_\theta^2} \\ \frac{8}{3} \sqrt{2} C_g g'^2 \frac{v^2}{s_\theta^2} & \frac{1}{6} \frac{m_h^2}{s_\theta^2} + 4C_g (3g^2 + g'^2) \frac{v^2}{s_\theta^2} - \frac{8}{3} C_g g'^2 \frac{v^2}{s_\theta^2} \end{pmatrix}. \quad (14)$$

The mass eigenstates (η_1, η_2) are related to the gauge eigenstates (η_1^0, η_5^0) via a rotation by an angle, k_0 , given by

$$\tan 2k_0 = \frac{2\mathcal{M}_{12}}{\mathcal{M}_{11} - \mathcal{M}_{22}}, \quad (15)$$

and the mass eigenstates are

$$\eta_1 = \cos k_0 \eta_1^0 - \sin k_0 \eta_5^0, \quad \eta_2 = \sin k_0 \eta_1^0 + \cos k_0 \eta_5^0. \quad (16)$$

It is straightforward to obtain the masses, $m_{\eta_1}^2$ and $m_{\eta_2}^2$, from the mass matrix, $\mathcal{M}^2$.

B. Couplings of the scalars

Having obtained the mass spectrum, we derive the couplings of the pNGB scalars in terms of the model parameters and express them in the mass basis. The fermiophobic couplings of the pNGB scalars are listed in Tables I and II. Note that the mixing angles in the charged and the neutral sector depend on the parameters C_g and f , as $\sin \theta \sim v/f$. The couplings of the scalars are also function of the mixing angle k_0 and k_+ in the mass basis. In the fermiophilic scenario, the effective potential receives contributions from heavy fermions running in the loop. Consequently, the scalar masses and couplings acquire additional corrections. Nevertheless, because of the large

TABLE I. $\Phi\Phi V$ couplings in the mass basis of the pNGB scalars.

	$\phi_i^0 \phi_j^+$ κ_W		$\phi_i^- \phi_j^{++}$ κ_W	
	η_1^+	η_2^+	η_5^{++}	
η_3^0	$-\frac{i}{2} c_{k_+} - \frac{c_\theta}{2} s_{k_+}$	$-\frac{i}{2} s_{k_+} + \frac{c_\theta}{2} c_{k_+}$		
η_1	$\frac{c_\theta}{2\sqrt{3}} s_{k_0} c_{k_+} + \frac{i\sqrt{3}}{2} s_{k_0} s_{k_+} + \sqrt{\frac{2}{3}} c_\theta c_{k_0} c_{k_+}$	$\frac{c_\theta}{2\sqrt{3}} s_{k_0} s_{k_+} - \frac{i\sqrt{3}}{2} s_{k_0} c_{k_+} + \sqrt{\frac{2}{3}} c_\theta c_{k_0} s_{k_+}$		
η_2	$-\frac{c_\theta}{2\sqrt{3}} c_{k_0} c_{k_+} - \frac{i\sqrt{3}}{2} c_{k_0} s_{k_+} + \sqrt{\frac{2}{3}} c_\theta s_{k_0} c_{k_+}$	$\frac{c_\theta}{2\sqrt{3}} c_{k_0} s_{k_+} + \frac{i\sqrt{3}}{2} c_{k_0} c_{k_+} + \sqrt{\frac{2}{3}} c_\theta s_{k_0} s_{k_+}$		
η				
η_1^+				$\frac{c_\theta}{\sqrt{2}} c_{k_+} + \frac{i}{\sqrt{2}} s_{k_+}$
η_2^+				$\frac{c_\theta}{\sqrt{2}} s_{k_+} - \frac{i}{\sqrt{2}} c_{k_+}$
	$\phi_i^0 \phi_j^+$ κ_Z		$\phi_i^+ \phi_j^-$ κ_Z	
	η_1	η_2	η_1^-	η_2^-
η_3^0	$-\frac{ic_\theta}{\sqrt{3}} s_{k_0} + i\sqrt{\frac{2}{3}} c_\theta c_{k_0}$	$\frac{ic_\theta}{\sqrt{3}} c_{k_0} + i\sqrt{\frac{2}{3}} c_\theta s_{k_0}$	$-\frac{c_{2W}}{2}$	$-\frac{ic_\theta}{2}$
η_1^+			$\frac{ic_\theta}{2}$	$\frac{c_{2W}}{2} (c_{k_+}^2 - s_{k_+}^2)$
η_2^+				
η_5^{++}				$-c_{2W}$

masses of these fermions, their impact on the pNGB scalar masses remains negligible. However, in the limit of small mixing angles, that is small k_+ and k_0 , we get back the expressions where the mass eigenstates are assumed to be the same as the gauge eigenstates.

In the fermiophilic scenario, the effective potential receives contributions from heavy fermions running in the loop. Consequently, the scalar mass spectrum changes. However, Ref. [24] demonstrates that for specific values of parameters in the LO potential, the mass spectrum remains

identical to that in the fermiophobic case, where left-handed and right-handed fermions are embedded in the adjoint representation of $SU(5)$. The couplings of the pNGB scalars to the third generation SM quarks are parametrized as

$$k_t^{\phi^0} = c_t \frac{m_t}{f}, \quad k_b^{\phi^0} = c_b \frac{m_b}{f}, \quad k_{tb}^{\phi^+} = c_{tb} \frac{m_t}{f}, \quad (17)$$

at the leading order. The exact values of c_t , c_b , and c_{tb} are obtained after integrating out the heavy fermions in the loop

TABLE II. $\Phi V \tilde{V}$ couplings in the mass basis of the pNGB scalars.

	$\kappa_{\gamma\gamma}^{\phi^0}$	$\kappa_{\gamma Z}^{\phi^0}$	$\kappa_{ZZ}^{\phi^0}$	
η_1	$\frac{2s_\theta}{\sqrt{3}} s_{k_0} + \sqrt{\frac{2}{3}} s_\theta c_{k_0}$	$\frac{c_{2W}s_\theta}{\sqrt{3}} s_{k_0} + \frac{c_{2W}s_\theta}{\sqrt{6}} c_{k_0}$	$-\frac{(1-3c_{4W}+2c_{2\theta})s_\theta}{12\sqrt{3}} s_{k_0} + \frac{(1+6c_{4W}-7c_{2\theta})s_\theta}{24\sqrt{6}} c_{k_0}$	
η_2	$-\frac{2s_\theta}{\sqrt{3}} c_{k_0} + \sqrt{\frac{2}{3}} s_\theta s_{k_0}$	$-\frac{c_{2W}s_\theta}{\sqrt{3}} c_{k_0} + \frac{c_{2W}s_\theta}{\sqrt{6}} s_{k_0}$	$\frac{(1-3c_{4W}+2c_{2\theta})s_\theta}{12\sqrt{3}} c_{k_0} + \frac{(1+6c_{4W}-7c_{2\theta})s_\theta}{24\sqrt{6}} s_{k_0}$	
η	$\sqrt{\frac{2}{3}} s_\theta$	$-\frac{c_{2W}s_\theta}{\sqrt{10}}$	$\frac{(c_{4W}+3c_\theta^2)s_\theta}{4\sqrt{10}}$	
	$\kappa_{W^+W^-}^{\phi^0}$	$\kappa_{\gamma W^-}^{\phi^+}$	$\kappa_{ZW^-}^{\phi^+}$	$\kappa_{W^-W^-}^{\phi^{++}}$
η_1	$\frac{s_\theta^3}{6\sqrt{3}} s_{k_0} - \frac{7s_\theta^3}{12\sqrt{6}} c_{k_0}$			
η_2	$-\frac{s_\theta^3}{6\sqrt{3}} c_{k_0} - \frac{7s_\theta^3}{12\sqrt{6}} s_{k_0}$			
η	$-\frac{(3c_{2\theta}+5)s_\theta^2}{8\sqrt{10}}$			
η_1^+		$-\frac{s_{2\theta}}{4} c_{k_+} - \frac{is_{2\theta}}{2} s_{k_+}$	$\frac{s_W^2 s_{2\theta}}{4} c_{k_+} - \frac{i(c_{2W}-c_{2\theta}-2)s_\theta}{12} s_{k_+}$	
η_2^+		$-\frac{s_{2\theta}}{4} s_{k_+} + \frac{is_{2\theta}}{2} c_{k_+}$	$\frac{s_W^2 s_{2\theta}}{4} s_{k_+} + \frac{i(c_{2W}-c_{2\theta}-2)s_\theta}{12} c_{k_+}$	
η_5^{++}				$-\frac{s_\theta^3}{3\sqrt{2}}$

TABLE III. Fermionic ($\Phi f \bar{f}$) couplings of the pNGB scalars expressed in the mass basis at two different values of f and for $c_t = c_b = c_{tb} = 1$. The couplings for the conjugate fields have the same values.

Coupling	Value at $f = 2.5$ TeV	Value at $f = 5$ TeV
$k_{tb}^{\eta_1^+} = c_{tb}(c_{k_+} + s_{k_+}) \frac{m_t}{f}$	0.098	0.049
$k_{tb}^{\eta_2^+} = c_{tb}(c_{k_+} - s_{k_+}) \frac{m_t}{f}$	0.00017	0.000083
$k_t^{\eta_2} = c_t(c_{k_0} + s_{k_0}) \frac{m_t}{f}$	0.096	0.048
$k_t^{\eta_1} = c_t(c_{k_0} - s_{k_0}) \frac{m_t}{f}$	0.016	0.008
$k_b^{\eta_2} = c_b(c_{k_0} + s_{k_0}) \frac{m_b}{f}$	0.002	0.001
$k_b^{\eta_1} = c_b(c_{k_0} - s_{k_0}) \frac{m_b}{f}$	0.0004	0.0002
$k_b^{\eta_3^0}, k_b^{\eta} = c_b \frac{m_b}{f}$	0.0019	0.001
$k_t^{\eta_3^0}, k_t^{\eta} = c_t \frac{m_t}{f}$	0.069	0.034

and that depends on the details of the embedding. We want to focus on the highest reachable branching ratio of the pNGBs to SM fermions, in the fermiophilic scenario and hence, we chose $c_i = 1$ for all i . We list the fermiophilic couplings in Table III and show their approximate values at $f = 2.5$ TeV and 5 TeV in the mass basis.

It is important to observe that the fermiophilic couplings diminish in strength when f is large, as evident from Table III. We also observe that as a result of the mixing, the coupling of η_1^+ to $\bar{t}b$ is exceptionally suppressed, in contrast to the coupling of η_2^+ to $\bar{t}b$. In the neutral sector, the couplings of η_1 and η_2 with the SM quarks differ by only one order of magnitude. Hence, in the fermiophilic scenario, the decay modes of $\eta_1^\pm$ and $\eta_2^\pm$ are expected to be different whereas the decay modes of η_1 and η_2 should not differ much. Note that, the total decay width of the scalars in the fermiophilic case will be function of both c_i ($i = t, b, tb$) and the parameter, C_g along with other model parameters, mixing angles k_+ and k_0 , and f . Whereas, in the fermiophobic scenario, decay width depends on the gauge contribution C_g only along with k_+ and k_0 , and f .

We discuss the decays of the pNGB in the fermiophilic and fermiophobic scenario in the next section.

IV. DECAYS OF THE PNGB SCALARS

In the previous section, we demonstrated how the masses, mixing and couplings of the pNGB scalars are a function of the model parameters, C_g and f , where f is present through $\sin\theta = v/f$. The value of these model parameters leads to specific patterns of mixing, parametrized by k_+ and k_0 , and the mass splittings among the pNGB scalars which crucially affects the decay modes of these scalars. Reference [24] presented a comprehensive analysis of the mass differences among these pNGB scalars and explored how these differences are influenced by the value of C_g across all the possible representations.

In Table IV, we show the mass spectrum at different benchmark values of the parameters, C_g and f . In Fig. 1, we plot the mass differences between the scalars as a function of C_g (top panel), and, as a function of f (bottom panel). It is clear from both, Table IV and Fig. 1, that the masses and the mass differences are highly sensitive to the parameters, C_g and f , particularly f . Thus, while analyzing the decay modes, fixing the masses at one certain value or assuming a fixed mass splitting, is not always correct. A more insightful approach for analyzing the decays of the scalars is to include the effect of various mixing and mass splittings, which we discuss in the next subsection.

Before proceeding further, we reflect upon a few nuances. The singlet scalar, η , receives a negative mass correction from the gauge sector [24]. Hence, as shown in Table IV, as the value of C_g increases, the mass of η decreases. At a particular value of C_g , the singlet becomes tachyonic and this places an upper limit on C_g viz., $C_g < 0.023$. Hence, while observing the decay modes of the pNGB scalars, we vary C_g between 0.001–0.023. We also observe from Table IV that the lowest mass scalar is either η or η_1 . We, now, describe the decay modes of the pNGB scalars, in detail.

TABLE IV. Mass of the pNGB scalars in GeV at different benchmark scenarios.

Mass (GeV)	$f = 1$ TeV		$f = 2.5$ TeV		$f = 5$ TeV	
	$C_g = 0.01$	$C_g = 0.02$	$C_g = 0.01$	$C_g = 0.02$	$C_g = 0.01$	$C_g = 0.02$
$\dots$						
$\eta_1^\pm$	286.3	358.7	743.0	918.4	1493.8	1843.1
$\eta_2^\pm$	310.6	397.1	801.7	1012.3	1610.6	2030.4
$\eta_5^{\pm\pm}$	307.4	392.1	793.5	999.3	1594.1	2004.1
η	349.5	171.8	873.7	429.5	1747.5	859.0
η_1	302.8	374.8	757.0	936.9	1514.1	1873.8
η_2	319.3	401.2	798.2	1003	1596.5	2005.9
η_3^0	306.4	390.5	793.1	998.7	1593.9	2003.8

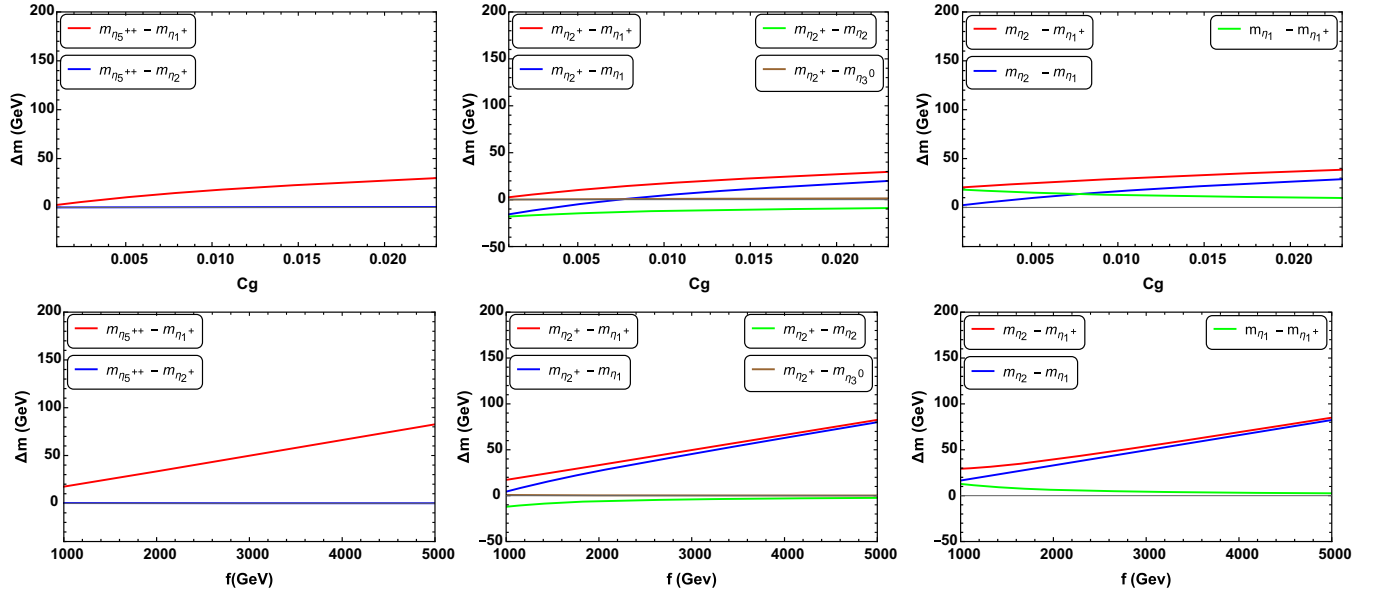


FIG. 1. Mass difference among the scalar pNGBs as a function of C_g with $f = 1$ TeV (top panel) and as a function of f with $C_g = 0.01$ (bottom panel).

A. Branching ratios of $\eta_5^{\pm\pm}$

The branching ratios of the doubly charged scalar $\eta_5^{\pm\pm}$ are plotted in Fig. 2, for the fermiophobic scenario (top panel) and the fermiophilic scenario (bottom panel). The branching ratios are varied as a function of the

pNGB mass and its various couplings, both affected by the parameters C_g and f . Even if f is fixed, the mass of the pNGB varies with C_g , as depicted by the auxiliary x -axes in these plots. We have fixed f at three different values; 1 TeV (left), 2.5 TeV (middle), and 5 TeV (right)

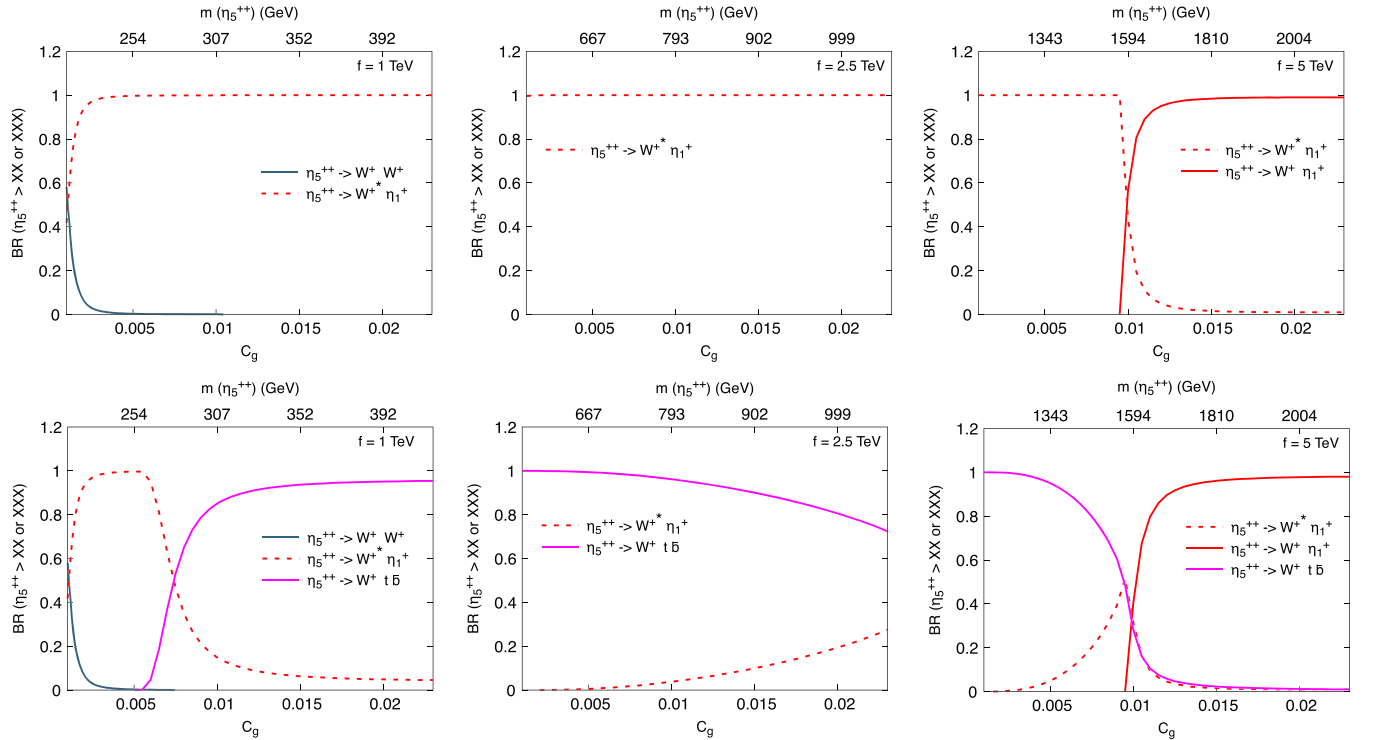


FIG. 2. Branching ratios of $\eta_5^{\pm\pm}$ for the *fermiophobic* scenario (top panel) and the *fermiophilic* scenario (bottom panel) at f ; 1 TeV (left), 2.5 TeV (middle), and 5 TeV (right).

which covers the range of pNGB mass $\sim 200\text{--}2000$ GeV. We discuss the fermiophobic scenario first (Fig. 2, top panel). In the absence of the fermiophilic couplings, $\eta_5^{\pm\pm}$ decays to two gauge bosons or a pNGB scalar and an off shell gauge boson, the latter dominating whenever sufficient phase space is available. From the mass differences depicted in Fig. 1, we see that the relation $m_{\eta_5^{\pm\pm}} > m_{\eta_1^\pm}$ holds true for any value of C_g and f . On the other hand, the mass splitting between $m_{\eta_5^{\pm\pm}}$ and $m_{\eta_2^\pm}$ is almost negligible. Hence, the doubly charged scalar preferentially decays to $\eta_1^\pm$ and an off shell W boson leading to the following three-body decays, at $f = 1$ TeV and 2.5 TeV:

$$\eta_5^{\pm\pm} \rightarrow W^{\pm*}\eta_1^\pm \rightarrow (q\bar{q})\eta_1^\pm, (l\nu)\eta_1^\pm. \quad (18)$$

At $f = 5$ TeV and $C_g \gtrsim 0.01$, the mass of $\eta_5^{\pm\pm}$ is $\gtrsim 1.6$ TeV and the mass difference between $\eta_5^{\pm\pm}$ and $\eta_1^\pm$ becomes sufficient to produce an on shell W boson,

$$\eta_5^{\pm\pm} \rightarrow W^\pm\eta_1^\pm, \quad (19)$$

and this decay mode dominates for $m_{\eta_5^{\pm\pm}} \gtrsim 1.6$ TeV. The couplings of the pNGB scalars to the gauge bosons are suppressed because these couplings are generated via five-dimensional operators. Hence, the branching ratio for

$$\eta_5^{\pm\pm} \rightarrow W^\pm W^\pm, \quad (20)$$

is suppressed. In principle, the decay via off shell pNGB i.e., $\eta_5^{\pm\pm} \rightarrow W^\pm\eta_i^{\pm*}$, $i = 1, 2$, and $\eta_5^{\pm\pm} \rightarrow Z\eta_5^{\pm\pm}$ are also possible, leading to three gauge bosons in the final states. However the decay width of these off shell scalars to the gauge bosons are suppressed.

The fermiophilic branching ratios for $\eta_5^{\pm\pm}$ are depicted in Fig. 2 (bottom panel). The decay of $\eta_5^{\pm\pm}$ to the gauge bosons are suppressed as before. At $f = 1$ TeV, for smaller masses, the three-body decay of the fermiophobic case [Eq. (18)] is dominant i.e.,

$$\eta_5^{\pm\pm} \rightarrow W^{\pm*}\eta_1^\pm \rightarrow (q\bar{q})\eta_1^\pm, (l\nu)\eta_1^\pm.$$

With increase in mass, however, the fermiophilic three-body decay to one gauge boson and two SM fermions via an off shell pNGB² i.e.,

$$\eta_5^{\pm\pm} \rightarrow W^\pm\eta_i^{\pm*} \rightarrow W^\pm tb, \quad (21)$$

²Here, in the writing, we have used the redefinition $t = t, \bar{t}$, $b = b, \bar{b}$, $q = q, \bar{q}$, and $l = l^+, l^-$, where particle and antiparticle are summed over.

where $i = 1, 2$, becomes feasible and is the dominating decay mode for most of the parameter space.³ However, with increasing f , the branching ratios of the pNGB scalars to the fermions get smaller since the fermiophilic coupling is proportional to m_t/f . Hence, at larger masses, when $f = 5$ TeV and $C_g \gtrsim 0.01$, the two-body on shell decay viz.,

$$\eta_5^{\pm\pm} \rightarrow W^\pm\eta_1^\pm, \quad (22)$$

comes to dominate.

The three-body decays where the off shell pNGB scalars decay to two gauge bosons are suppressed since coupling of the pNGB scalars with the SM gauge bosons occurs via dimension-5 terms. On the other hand, pNGB scalars coupling with the fermions occurs at tree level and hence, is not suppressed. The three-body decay to $W^\pm tb$ can also proceed via $W^\pm W^{\pm*}$ where the off shell gauge boson decays to two SM fermions. However, this contribution is negligible compared to the fermiophilic three-body decay via the off shell pNGB.

Therefore, in the decays of $\eta_5^{\pm\pm}$, the common decay products are the singly charged pNGBs. Next, we discuss the decays of these singly charged pNGBs.

B. Branching ratios of $\eta_2^\pm$

The branching ratios of the singly charged pNGB scalar $\eta_2^\pm$ are plotted in Fig. 3, for the fermiophobic scenario (top panel) and the fermiophilic scenario (bottom panel). The various parameters are the same as before. First, we discuss the *fermiophobic* case. The main decay channels of $\eta_2^\pm$ up to $f = 2.5$ TeV are

$$\begin{aligned} \eta_2^\pm &\rightarrow W^\pm Z, W^\pm \gamma, \\ \eta_2^\pm &\rightarrow Z^*\eta_1^\pm \rightarrow (l\bar{l})\eta_1^\pm, (q\bar{q})\eta_1^\pm, \\ \eta_2^\pm &\rightarrow W^{\pm*}\eta_1 \rightarrow (l\nu)\eta_1, (q\bar{q})\eta_1. \end{aligned} \quad (23)$$

At very small masses, in the absence of sufficient mass splitting with the pNGB scalars, $W^\pm \gamma$ emerges as the dominant decay mode. As the mass increases, decays to η_1 and $\eta_1^{\pm*}$ become possible accompanied by off shell gauge bosons and these decay modes start to dominate, which is shown in Fig. 3 (top left and middle). With increase in f , the coupling of $\eta_2^\pm$ with η_1 is enhanced compared to the coupling with $\eta_1^{\pm*}$. Effect of this in the branching ratio is depicted by the crossover of the red and green dashed curves in Fig. 3 (middle). With further increase in f , shown by the top-right plot in Fig. 3 ($f = 5$ TeV), the same decays

³The doubly charged pNGB does not have any direct coupling with the SM quarks and therefore, two-body decays of $\eta_5^{\pm\pm}$ to two SM quarks are absent.

⁴Considering the mass hierarchy, decay of $\eta_2^\pm$ to any other pNGB scalar is not allowed.

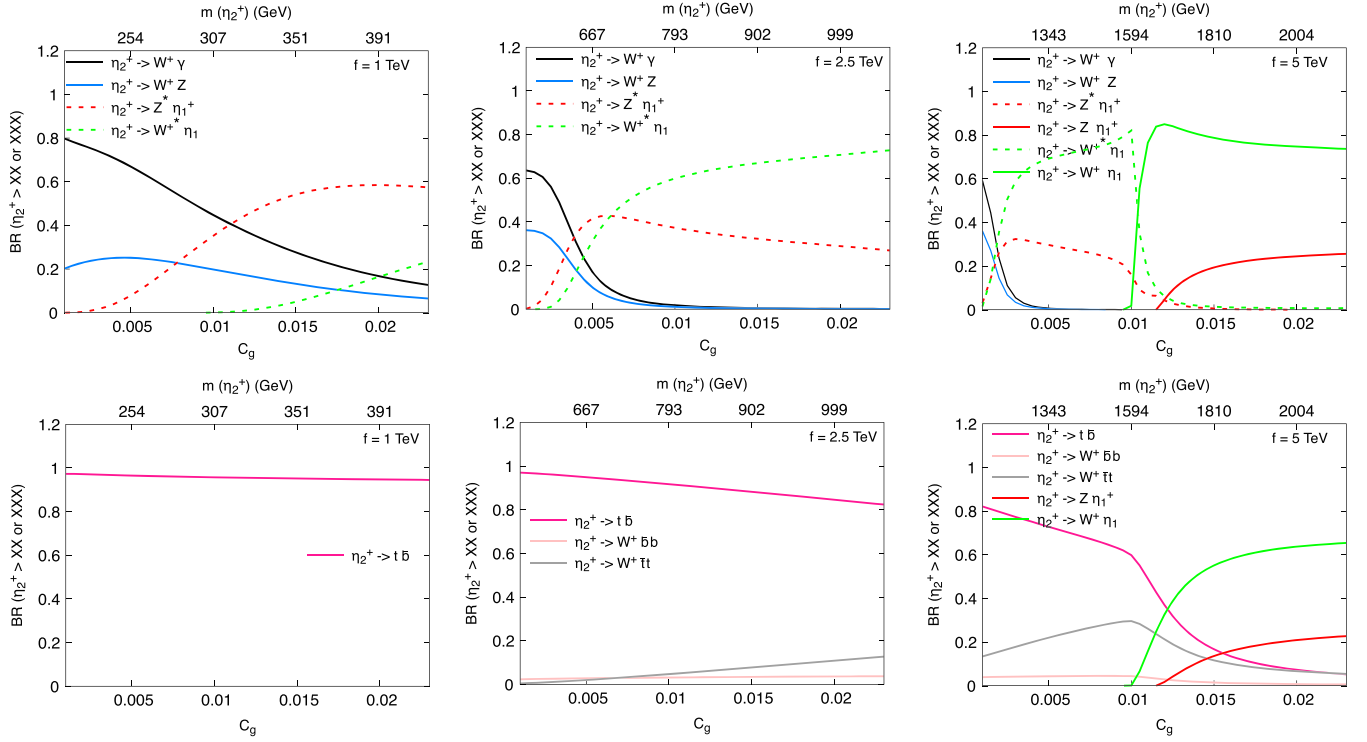


FIG. 3. Branching ratios of $\eta_2^\pm$ for the *fermiophobic* scenario (top panel) and the *fermiophilic* scenario (bottom panel) at f ; 1 TeV (left), 2.5 TeV (middle), and 5 TeV (right).

proceed with on shell gauge bosons as mass splittings among the scalars are increased i.e.,

$$\begin{aligned}\eta_2^\pm &\rightarrow Z\eta_1^\pm, \\ \eta_2^\pm &\rightarrow W^\pm\eta_1.\end{aligned}\quad (24)$$

Further, it is found that decays of $\eta_2^\pm$ to η_3^0 and $\eta_5^{\pm\pm}$, accompanied by a gauge boson, are suppressed throughout the parameter space.

If we allow for the fermiophilic couplings, decays of $\eta_2^\pm$ proceed as shown in Fig. 3 (bottom panel). For most of the parameter space, especially at $f = 1$ and 2.5 TeV, the two-body fermiophilic decay,

$$\eta_2^\pm \rightarrow tb, \quad (25)$$

emerges as the dominant decay mode, subjugating all the *fermiophobic* decay modes. Further, with increase in mass, the following three-body decays via off shell pNGB scalar or SM quark also arise,

$$\eta_2^\pm \rightarrow W^\pm\eta_i^* \rightarrow W^\pm tt, W^\pm bb, \quad (26)$$

where η_i^* represents all the neutral scalars viz., η_1 , η_2 , and η_3^0 . The branching ratio to $W^\pm tt$ increase as more phase space becomes available for them however branching ratio to $W^\pm bb$ is small if we consider only the off shell scalar

pNGB in the intermediate state.⁵ Hence, these branching ratios are much smaller compared to the two body final state tb . The branching ratios for $\eta_2^\pm \rightarrow W^\pm\eta_i$, $\eta_2^\pm \rightarrow W^\pm\eta_i^{\mp\mp}$, and $\eta_2^\pm \rightarrow Z\eta_1^{\pm*} \rightarrow Ztb$ are found to be very small. The branching ratio to the off shell $\eta_5^{\pm\pm}$ is also very small as $\eta_5^{\pm\pm}$ does not couple directly to the fermions.

The branching ratios for $\eta_2^\pm$ decays to the fermions fall with increase in f as the involved coupling is inversely proportional to f (see Fig. 3, bottom right). At $f = 5$ TeV and $C_g \gtrsim 0.01$, the dominating mode for $\eta_2^\pm$ decay is on shell gauge bosons and pNGB scalars i.e.,

$$\eta_2^\pm \rightarrow W^\pm\eta_1, \quad (27)$$

$$\eta_2^\pm \rightarrow Z\eta_1^\pm. \quad (28)$$

C. Branching ratios of $\eta_1^\pm$

The branching ratios of $\eta_1^\pm$ are very interesting as they differ from that of the other singly charged scalar, $\eta_2^\pm$. The graphs are plotted in Fig. 4, for the fermiophobic

⁵It is interesting to note that $W^\pm bb$ decay also occurs via off shell quark (t^*) and this contribution is larger than the contribution via off shell pNGB because $\eta_2^\pm$ coupling with quarks is enhanced due to mixing (see Table III). However we did not include the contribution from the off shell SM particles while calculating the contributions of the off shell scalars in the same final state. This applies to the rest of the studies as well.

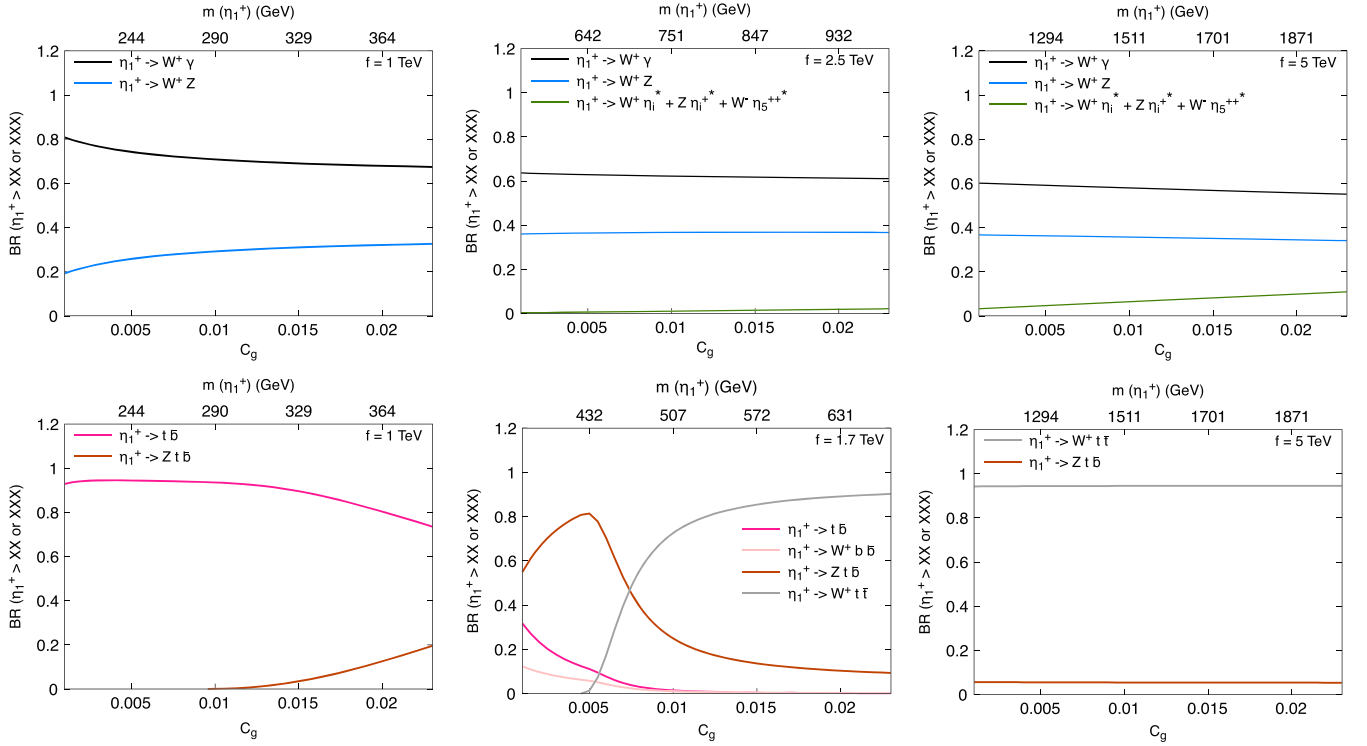


FIG. 4. Branching ratios of $\eta_1^\pm$ for the *fermiophobic* scenario (top panel) and the *fermiophilic* scenario (bottom panel) at f : 1 TeV (left), 2.5 TeV (middle), and 5 TeV (right).

scenario (top panel) and the fermiophilic scenario (bottom panel).

First, we discuss the fermiophobic scenario. From Table IV, we can observe that at lower values of C_g , $\eta_1^\pm$ has the lowest mass in the spectrum and therefore, any on shell decays involving other pNGB and SM gauge bosons are prohibited. At larger C_g , $\eta_1^\pm$ is the second lightest pNGB scalar, next to the lightest, η , but η does not couple to $\eta_1^\pm$ or any other pNGB scalars. As a result, $\eta_1^\pm$ cannot decay to an on shell pNGB scalar, at all, which is unlike the case of $\eta_2^\pm$. Instead, it prefers to decay to gauge bosons. The decay modes that are dominant at smaller masses ($f = 1$ and 2.5 TeV) are

$$\eta_1^\pm \rightarrow W^\pm Z, W^\pm \gamma, \quad (29)$$

as shown in Fig. 4 (top left and middle). At large masses [$f = 5$ TeV, Fig. 4 (top right)], three-body decays via off shell pNGB scalars become significant i.e.,

$$\eta_1^\pm \rightarrow W^\pm \eta_i^*, \quad (30)$$

$$\eta_1^\pm \rightarrow W^\mp \eta_5^{\pm\pm*}, \quad (31)$$

$$\eta_1^\pm \rightarrow Z \eta_2^{\pm*}, \quad (32)$$

where $\eta_i = \eta_1, \eta_2$, or η_3^0 . These off shell scalars finally decay to the gauge bosons leading to final states with three

gauge bosons. However branching ratios in these channels are suppressed, as shown in Fig. 4 (top right). Hence, we observe that in the fermiophobic case, the decay patterns of $\eta_1^\pm$ and $\eta_2^\pm$ are very different.

We discuss the fermiophilic decays of $\eta_1^\pm$, now. The dominating decay modes at $f = 1$ TeV is

$$\eta_1^\pm \rightarrow tb, \quad (33)$$

which is shown in Fig. 4 (bottom left). In Fig. 4 (bottom middle), we have plotted the branching ratios for $f = 1.7$ TeV. Unlike other pNGB scalar plots, we choose $f = 1.7$ TeV here for better depiction of the variation in branching ratios for intermediate masses. We find that in a very small region of mass range,

$$\eta_1^\pm \rightarrow Z \eta_2^{\pm*} \rightarrow Ztb, \quad (34)$$

dominates. This is because the $\eta_1^\pm tb$ coupling is suppressed compared to $\eta_2^\pm tb$ (see Table III). Hence, this is unlike the case of $\eta_2^\pm$ where due to its enhanced coupling with SM quarks, the decay to tb dominates the decay mode Ztb . The three-body branching ratio to the final state $W^\pm b\bar{b}$ is found to be very small throughout the parameter space. The three-body decays via off shell $\eta_5^{\pm\pm}$ are absent since $\eta_5^{\pm\pm}$ does not couple directly to the fermions.

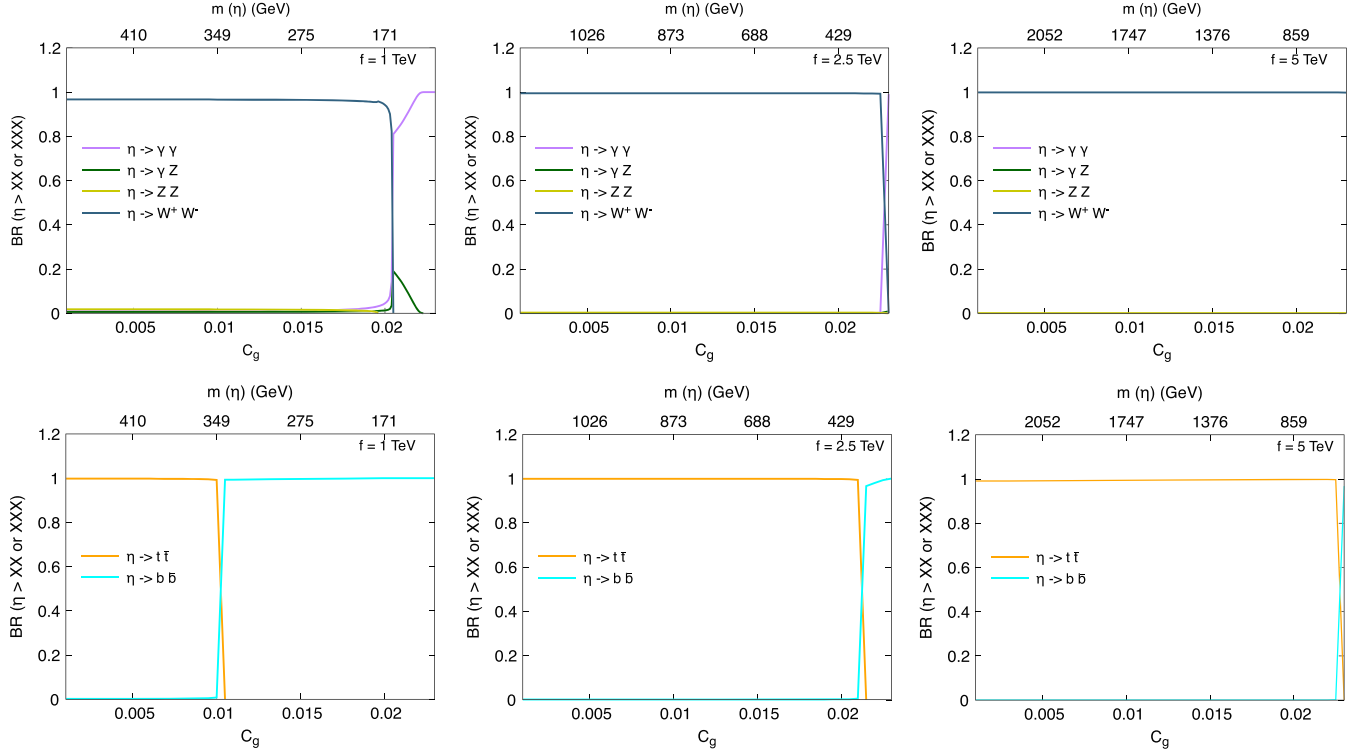


FIG. 5. Branching ratios of the singlet scalar η for the *fermiophobic* scenario (top panel) and the *fermiophilic* scenario (bottom panel) at f ; 1 TeV (left), 2.5 TeV (middle), and 5 TeV (right).

With further increase in the mass of $\eta_1^\pm$, the process

$$\eta_1^\pm \rightarrow W^\pm \eta_i^* \rightarrow W^\pm t\bar{t} \quad (35)$$

takes over and dominates its branching ratio, especially at $f = 5$ TeV (Fig. 4, bottom right). As discussed before, the mass of $\eta_1^\pm$ is always less than the mass of $\eta_2^\pm$ and η_i . Hence, we do not observe any on shell decay of $\eta_1^\pm$ to other pNGB scalar, especially at large values of f . The absence of these decays in $\eta_1^\pm$ can be useful to distinguish the signature of $\eta_1^\pm$ from $\eta_2^\pm$ and vice versa at the collider experiments.

D. Branching ratios of the singlet neutral scalar, η

The branching ratios of the neutral pNGB scalars are also very interesting which we discuss next. The branching ratios for the singlet scalar, η , are plotted in Fig. 5, for the *fermiophobic* scenario (top panel) and the *fermiophilic* scenario (bottom panel). Earlier, we discussed that the singlet scalar η couples to the gauge bosons but it does not have couplings to any other pNGB scalars. Hence, the decay modes of this neutral scalar, in the *fermiophobic* scenario, are

$$\eta \rightarrow \gamma Z, \gamma\gamma, ZZ, W^+W^-, \quad (36)$$

as shown in Fig. 5 (top panel). The dominating decay mode is W^+W^- , for most of the parameter space.

However, towards the upper limit of C_g i.e., when the mass of η is small, $\gamma\gamma$ becomes the dominating decay mode. In the *fermiophilic* scenario i.e., Fig. 5 (bottom panel), the major decay modes are

$$\eta \rightarrow bb, tt. \quad (37)$$

The preferential decay to tt at small values of C_g and large values of f may be easily understood from the dependence of η mass on these parameters, as depicted on the top axis, parallel x -axis of the plots. As the mass of η decreases with the increase of C_g , at smaller values of C_g , decay to tt dominates whereas, at larger values of C_g , decay to bb dominates when $f = 1$ TeV. However, as the compositeness scale f increases, the mass also increases for a fixed C_g . Hence, at large f , decay to tt starts to dominate again. As for the cases of other pNGB scalars, here as well, decays to the gauge bosons are suppressed in the *fermiophilic* scenario.

E. Branching ratios of η_2

The decays of the neutral scalar, η_2 , are shown in Fig. 6, for the *fermiophobic* scenario (top panel) and the *fermiophilic* scenario (bottom panel). We, first, discuss the *fermiophobic* scenario (Fig. 6, top panel). Considering the mass hierarchy among the pNGB scalars, the neutral scalar η_2 decays to lighter pNGB scalars. At small masses,

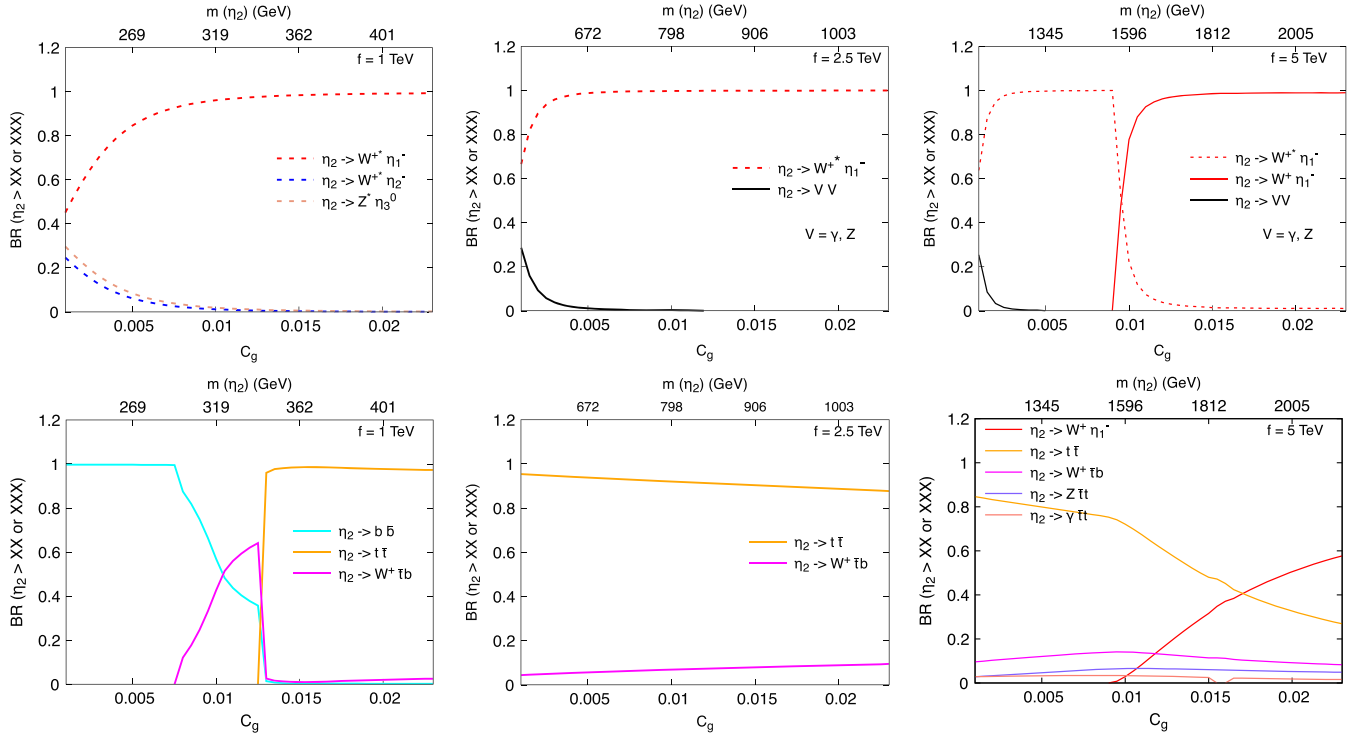


FIG. 6. Branching ratios of the singlet scalar η_2 for the *fermiophobic* scenario (top panel) and *fermiophilic* scenario (bottom panel) at f ; 1 TeV (left), 2.5 TeV (middle), and 5 TeV (right).

only three-body decays via off shell W/Z are possible i.e.,

$$\eta_2 \rightarrow W^{\mp*} \eta_i^{\pm} \rightarrow (l\nu) \eta_i^{\pm}, (qq) \eta_i^{\pm}, \quad (38)$$

$$\eta_2 \rightarrow Z^* \eta_3^0 \rightarrow (ll)(qq) \eta_3^0. \quad (39)$$

These are the dominating modes at $f = 1$ and 2.5 TeV, with the branching ratio for $\eta_2 \rightarrow W^{\pm*} \eta_1^{\pm}$ being larger due to large mass splitting between η_2 and $\eta_1^{\pm}$. The branching ratio to the gauge bosons viz.,

$$\eta_2 \rightarrow \gamma Z, \gamma\gamma, ZZ, W^{\pm} W^{\mp} \quad (40)$$

are suppressed. At $f = 5$ TeV, due to the increased phase space, the on shell decay $\eta_2 \rightarrow W^{\pm} \eta_1^{\mp}$ dominates when $C_g \gtrsim 0.01$.

In the fermiophilic scenario (Fig. 6, bottom panel), the following decays to SM fermions are dominant,

$$\eta_2 \rightarrow b\bar{b}, \quad (41)$$

$$\eta_2 \rightarrow t\bar{t}, \quad (42)$$

$$\eta_2 \rightarrow W^{\mp} \eta_i^{\pm*} \rightarrow W^{\mp} t\bar{b}, \quad (43)$$

where, $\eta_2 \rightarrow b\bar{b}$ is dominant at very small masses viz., for $f = 1$ TeV and $C_g \lesssim 0.01$. With increase in mass ($f = 1$ and 2.5 TeV), the branching ratio for $\eta_2 \rightarrow t\bar{t}$ become significant

(see Fig. 6, bottom left and middle). Branching ratio of $\eta_2 \rightarrow W^{\pm} t\bar{b}$ is significant in a small mass window at $f = 1$ TeV but is suppressed as soon as decay to $t\bar{t}$ becomes feasible. The branching ratio of $\eta_2 \rightarrow Z t\bar{t}$ is negligible.

At large f (5 TeV), however, the coupling of pNGB scalars with the SM fermions decreases. Hence, in the region, $f = 5$ TeV and $C_g \gtrsim 0.01$, the on shell decay mode $\eta_2 \rightarrow W^{\pm} \eta_1^{\mp}$ becomes the dominant decay channel (see Fig. 6, bottom right).

F. Branching ratios of η_1

Analogous to the case of charged scalars, the neutral scalar, η_1 , exhibits quite a different behavior than η_2 . We depict the branching ratios for the fermiophobic scenario in Fig. 7 (top panel). Considering the mass hierarchy and mass splittings among the pNGBs, the only decay of η_1 to another pNGB scalar is to $\eta_1^{\pm}$, accompanied by the off shell W boson i.e.,

$$\eta_1 \rightarrow W^{\pm*} \eta_1^{\pm} \rightarrow (l\nu) \eta_1^{\pm}, (qq) \eta_1^{\pm}. \quad (44)$$

This decay mode dominates at very small masses only because the mass splitting between η_1 and $\eta_1^{\pm}$ further diminishes with increase in f . As the mass increases, the dominant contribution comes from the decays to the gauge bosons, mainly from $\gamma\gamma$ (Fig. 7, top left and middle). In addition to the two-body decays, three-body decays via off shell singly charged scalars are also possible viz.,

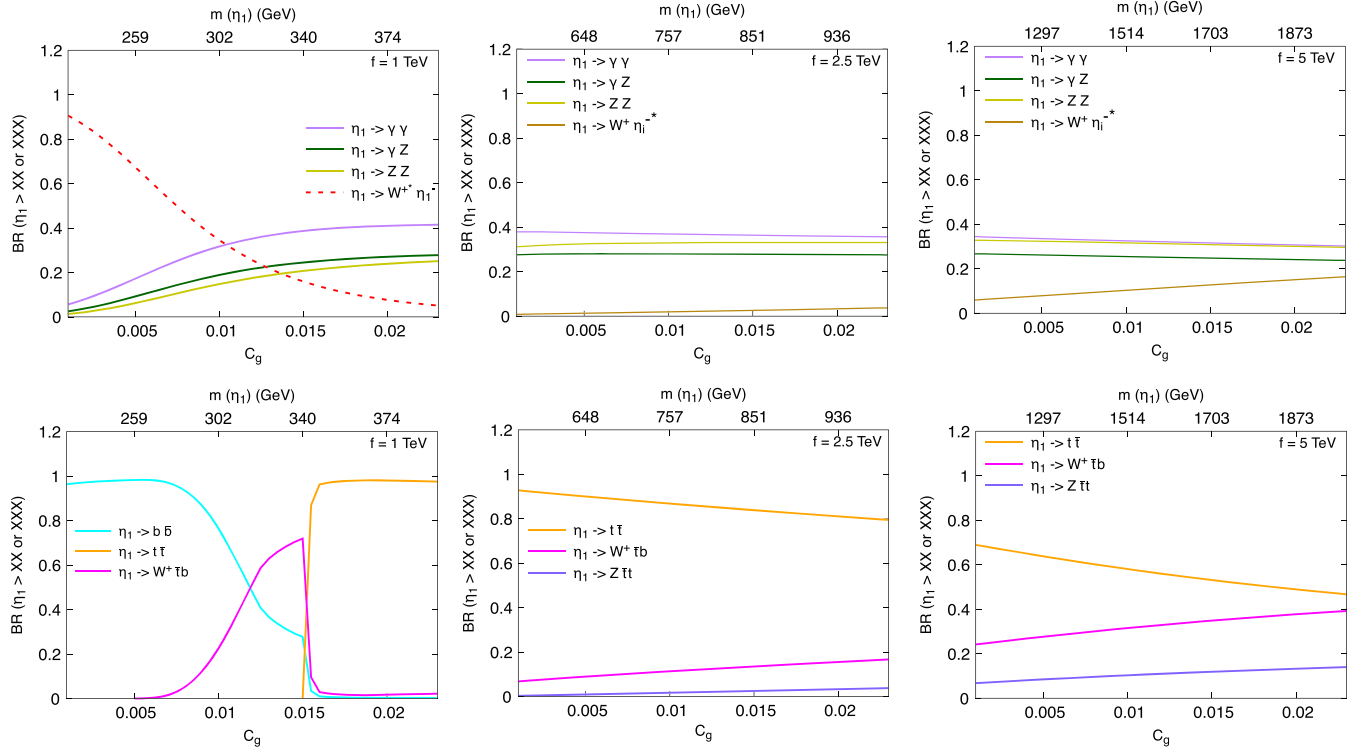


FIG. 7. Branching ratios of the singlet scalar η_1 for the *fermiophobic* scenario (top panel) and *fermiophilic* scenario (bottom panel) at f ; 1 TeV (left), 2.5 TeV (middle), and 5 TeV (right).

$$\eta_1 \rightarrow W^\pm \eta_i^{\mp*}, \quad (45)$$

where $i = 1, 2$, as shown in Fig. 7 (top middle and right).

In the fermiophilic scenario, the branching ratios of η_1 are almost similar to η_2 with the exception that at large f , the on shell decay to $\eta_1^\pm$ is not possible due to very small mass difference among the pNGB scalars.

G. Branching ratios of η_3^0

We, now, discuss the branching ratios of the CP -even scalar, η_3^0 . We start with the fermiophobic scenario (Fig. 8, top panel). The neutral pNGB, η_3^0 , does not decay to two gauge bosons. However, it couples to other pNGBs and one gauge boson. The mass of η_3^0 is close to that of $\eta_2, \eta_2^\pm$, and $\eta_5^{\pm\pm}$ and therefore, it can only decay to η_1 and $\eta_1^\pm$. At small masses ($f = 1$ and 2.5 TeV), however, the dominating decays are via off shell gauge bosons resulting into the three-body decays,

$$\eta_3^0 \rightarrow W^{\pm*} \eta_1^\mp \rightarrow (l\nu) \eta_1^\mp, (qq) \eta_1^\mp, \quad (46)$$

$$\eta_3^0 \rightarrow Z^* \eta_1 \rightarrow (ll) \eta_1, (qq) \eta_1. \quad (47)$$

For most of the parameter space, the decay mode $\eta_3^0 \rightarrow W^{\pm*} \eta_1^\mp + \text{H.c.}$ dominates (Fig. 8, top left and middle). At large f ($f = 5$ TeV), however, phase space becomes available for the fully on shell decays,

$$\eta_3^0 \rightarrow W^\pm \eta_1^\mp, \quad (48)$$

$$\eta_3^0 \rightarrow Z \eta_1. \quad (49)$$

The decay $\eta_3^0 \rightarrow W^\pm \eta_1^\mp$ comes to dominate in the region $C_g \gtrsim 0.01$ (Fig. 8, top right). The branching ratios in the *fermiophilic* case are shown in Fig. 8 (bottom panel). The decay patterns are similar to that of η_2 where decays to SM fermions dominate at small f while at large f , due to reduced coupling to fermions, on shell decays to lighter pNGB and gauge boson dominate. For η_1 , decay to another on shell pNGB is not possible due to the mass hierarchy.

V. COLLIDER SIGNATURES OF THE EXOTIC SCALARS

The collider signatures of pNGB scalars depend crucially upon their decay modes. For both charged and neutral scalars, their decays vary markedly depending upon whether their couplings with fermions are present or absent *vis-à-vis* the fermiophilic and fermiophobic scenarios. Working in the mass basis, we derived the masses and mixing of the pNGBs, as a function of the model parameters, f and C_g and mixing angles, κ_+ and κ_0 . Consequently, the branching ratios of the scalars, which are set by the mass spectrum and mixing structure, and are shown in Fig. 2 to Fig. 8 as functions of f and C_g , necessarily inherit a dependence on the mixing angles κ_+

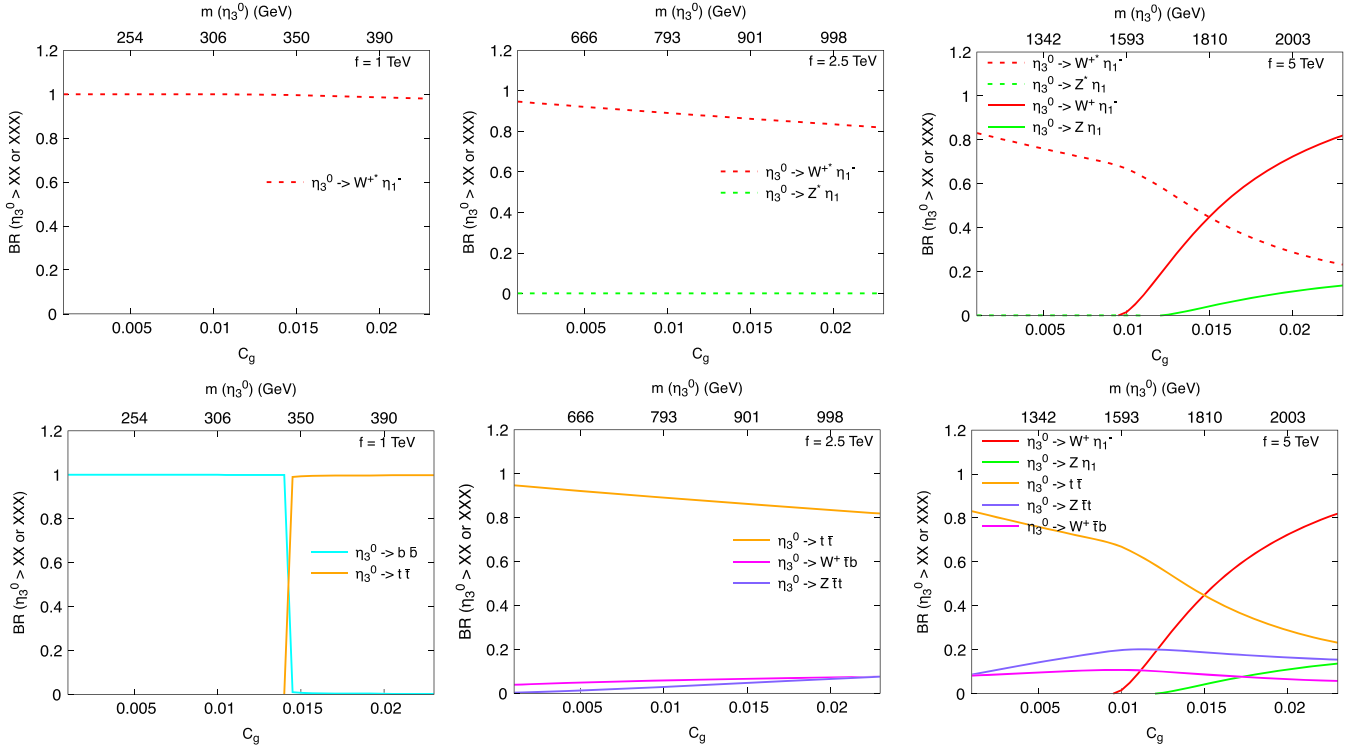


FIG. 8. Branching ratios of the singlet scalar η_3^0 for the *fermiophobic* scenario (top panel) and the *fermiophilic* scenario (bottom panel) at f ; 1 TeV (left), 2.5 TeV (middle), and 5 TeV (right).

and κ_0 . The decay modes of the pNGB scalars along with possible collider signatures were also discussed in [23], with the mass of the decaying scalar set to 600 GeV while other scalars were taken to be lighter or heavier. There, particular emphasis was placed on the three-body decay channels mediated by off shell gauge bosons, which occur in the regime where the scalar mass splittings lie below the W and Z boson masses. In this work, however, we explore a wider region of the (C_g, f) parameter space such that the masses extend up to ~ 2 TeV, with correspondingly larger mass splittings. As a result, we not only observe the above three-body decays but also two-body decays where one of the decay products is a lighter on shell pNGB scalar. For instance, in Fig. 6 (top panel), at $f = 1$ and 2.5 TeV, η_2 preferentially decays to $\eta_1^- W^{++}$, eventually leading to a three-body decay. At $f = 5$ TeV, however, there is sufficient mass splitting with η_1^- that the two-body decay, $\eta_2 \rightarrow \eta_1^- W^+$, can occur and eventually dominate.

We focus on this unexplored parameter space where the pNGB scalars decay to an on shell gauge boson and another pNGB scalar, in fermiophobic as well as fermiophilic scenarios. The daughter pNGBs continue to decay until the lightest pNGB finally decays into two gauge bosons, resulting in a cascade decay. In Tables V and VI, we express the branching ratios for a few of these decays, for the fermiophobic and fermiophilic cases, respectively. These decay modes lead to final states rich in gauge bosons, with large branching ratios. Hence, depending

on the subsequent decays of the W/Z bosons, multilepton and/or multijet final states can be observed at the collider experiments.

In Fig. 9, we plot the production cross section of the charged pNGB scalars at the 14 TeV LHC. The scalar masses are represented as a function of C_g while the value of f is fixed at 1 TeV. At this value of f , the masses of the scalars are below 500 GeV. The decays that we are concerned about occur mostly at $f = 5$ TeV and $C_g \gtrsim 0.01$ i.e., at pNGB scalar masses above 1 TeV. For charged scalars in the TeV range, the cross section at the LHC is estimated to be

TABLE V. Branching ratios for cascade decays of the charged pNGBs, when the parent pNGB scalar ($\eta_5^{\pm\pm}, \eta_2^{\pm}$) decays to an on shell gauge boson and another on shell pNGB scalar, for the fermiophobic case. We have fixed $f = 5$ TeV and all scalars have masses in the range ~ 1.5 –2 TeV. Note that, being the lightest scalar, $\eta_1^{\pm}$ has 100% branching ratio to gauge bosons, occurring via off shell pNGBs.

Cascade decay	Branching ratio
$\eta_5^{\pm\pm} \rightarrow W^{\pm} \eta_1^{\pm} \rightarrow W^{\pm} W^{\pm} \gamma$	0.33 – 0.55
$\eta_5^{\pm\pm} \rightarrow W^{\pm} \eta_1^{\pm} \rightarrow W^{\pm} W^{\pm} Z$	0.20 – 0.34
$\eta_2^{\pm} \rightarrow W^{\pm} \eta_1 \rightarrow W^{\pm} \gamma \gamma$	0 – 0.22
$\eta_2^{\pm} \rightarrow W^{\pm} \eta_1 \rightarrow W^{\pm} \gamma Z$	0 – 0.22
$\eta_2^{\pm} \rightarrow W^{\pm} \eta_1 \rightarrow W^{\pm} ZZ$	0 – 0.18
$\eta_1^{\pm} \rightarrow W^{\pm} V (V = \gamma, Z)$	~ 1

TABLE VI. Branching ratios for cascade decays of the charged pNGBs, when the parent pNGB scalar ($\eta_5^{\pm\pm}, \eta_2^{\pm}$) decays to an on shell gauge boson and another on shell pNGB scalar, for the fermiophilic case. We have fixed $f = 5$ TeV and all scalars have masses in the range ~ 1.5 – 2 TeV. Note that, being the lightest scalar, $\eta_1^{\pm}$ has 100% branching ratio to $W^{\pm}tt$, occurring via off shell pNGBs.

Cascade decay	Branching ratio
$\eta_5^{\pm\pm} \rightarrow W^{\pm}\eta_1^{\pm} \rightarrow W^{\pm}W^{\pm}tt$	0.39–0.94
$\eta_2^{\pm} \rightarrow W^{\pm}\eta_1 \rightarrow W^{\pm}tt$	0–0.3
$\eta_1^{\pm} \rightarrow W^{\pm}tt$	~ 1

$\sim \mathcal{O}(1)$ fb or even smaller. Further, the signatures of interest contain multiple W/Z bosons which lead to multilepton and/or multijet final states. When the W/Z bosons decay leptonically, the resulting signals are suppressed by the small

leptonic branching ratios. On the other hand, the hadronic decays avoid this suppression but suffer from substantial QCD backgrounds at the LHC.

For a cleaner environment, lepton colliders promise to be more effective than the pp collider. However, the future lepton colliders such as ILC and CLIC [47–49] will have their achievable center-of-mass energies limited to the sub-TeV regime. Since we are interested in the W/Z rich signatures of pNGB scalars with masses exceeding the TeV scale, the muon collider [45,46,50,51] will be a great alternative. Unlike the lepton colliders with electron beams, the energy loss due to the synchrotron radiation is suppressed in muon colliders because of the large mass of muon. Thus, achieving both high energy as well as high luminosity is possible. We plot the production cross section of the pNGB scalars at the 3 TeV and 6 TeV muon collider in Fig. 10. The pair production cross section of the two

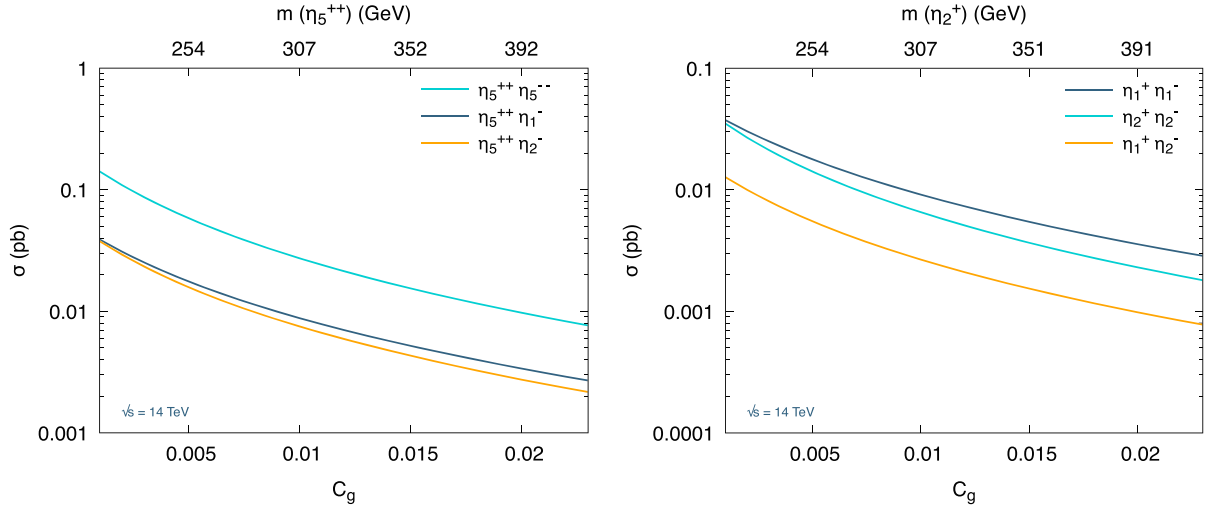


FIG. 9. Production cross sections of the charged scalars at the LHC 14 TeV with $f = 1$ TeV.

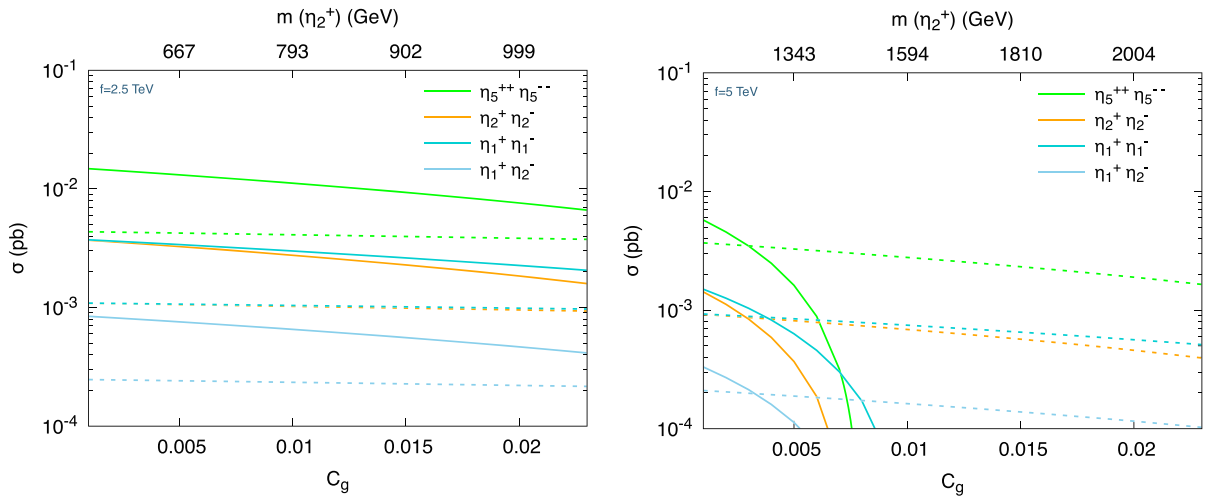


FIG. 10. Production cross sections of the pNGB scalars at $\mu^+\mu^-$ collider, at $f = 2.5$ TeV (left) and $f = 5$ TeV (right), for $\sqrt{s} = 3$ TeV (solid lines), and $\sqrt{s} = 6$ TeV (dashed lines).

singly charged scalars, $\eta_2^+\eta_2^-$ and $\eta_1^+\eta_1^-$, are different due to difference in their gauge couplings. The associated pair production cross section, $\eta_1^+\eta_2^-$, is comparatively smaller than the pair production cross sections.

The pair production of charged scalars, followed by cascade decays, gives rise to a variety of collider signatures such as monolepton, dilepton, and trilepton signals along with multiple jets. As discussed earlier, we are interested in the decays arising in the regime $f = 5$ TeV with scalar masses above 1.5 TeV. Hence, we adopt this benchmark setup for the signal analysis and choose $C_g = 0.015$ and $f = 5$ TeV. In the fermiophobic scenario, the two heavier charged pNGB scalars undergo two-body on shell decays viz., $\eta_2^\pm \rightarrow W^\pm \eta_1$ and $\eta_5^{\pm\pm} \rightarrow W^\pm \eta_\pm$. Considering further the decays of η_1 and $\eta_1^\pm$ via their dominating modes, the possible signatures are

- (1) $\mu^+\mu^- \rightarrow \eta_2^+\eta_2^- \rightarrow (W^+\eta_1)(W^-\eta_1) \rightarrow (W^+ZZ) \times (W^+ZZ) \rightarrow 4f_j + (\ell^+\ell^-) + MET$,
- (2) $\mu^+\mu^- \rightarrow \eta_5^{++}\eta_5^{--} \rightarrow (W^+\eta_1^+)(W^-\eta_1^-) \rightarrow (W^+W^+Z) \times (W^-W^-Z) \rightarrow 2f_j + 2(\ell^+\ell^-) + MET$.

These signals are rich in highly energetic W and Z bosons which can be identified as *fatjets* (f_j). In both cases, we consider the possibility when W decays to the leptons and Z decays to jets. Hence, Z will be identified as a fatjet. Up to the intermediate state consisting of all gauge bosons, the cross section $\times BR$ in these channels are ~ 0.02 fb and 0.2 fb, respectively, at the 6 TeV muon collider. If we consider the pair production of $\eta_1^+\eta_1^-$, we obtain the final state, W^+ZW^-Z that leads to dilepton $+2/3f_j$ signals. The dominant backgrounds for these channels are nV + jets, where n is the number of W/Z bosons. The leptons originate from the decay of the W/Z bosons and the highly energetic QCD jets are identified as fatjets. However, W/Z bosons in SM background processes, have lesser chance of being identified as fatjets as they have less energy. By imposing cuts on the fatjet mass and other jet-substructure variables, these backgrounds can be reduced. Events involving nV + jets + γ also contributes as a major background; however, the cross section is found to be typically an order of magnitude smaller than nV + jets.

In addition, when we consider the alternate decay modes of η_1 and $\eta_1^\pm$, the following signals arise:

- (3) $\mu^+\mu^- \rightarrow \eta_2^+\eta_2^- \rightarrow (W^+\eta_1)(W^-\eta_1) \rightarrow (W^+W^+W^-\gamma) \times (W^+W^+W^-\gamma) \rightarrow 4f_j + (\ell^+\ell^-) + (\gamma\gamma) + MET$,
- (4) $\mu^+\mu^- \rightarrow \eta_5^{++}\eta_5^{--} \rightarrow (W^+\eta_1^+)(W^-\eta_1^-) \rightarrow (W^+VVV) \times (W^-VVV) \rightarrow nf_j + m(\ell^+\ell^-) + MET$,

where $V = W^\pm, \gamma, Z$. These signals are also rich in fatjets, arising from the decays of W and/or Z . The resulting fatjet and lepton multiplicities in signal 4 depend upon the number of W and Z bosons present in the decay chain and their subsequent decays, and are hence, denoted by m and n , respectively. Up to the intermediate state consisting of all gauge bosons, the cross section $\times BR$ at the 6 TeV

muon collider are 0.002 fb and 0.014 fb, for channels 3 and 4, respectively. These are comparably smaller than for channels 1 and 2.

To analyze these fatjet-rich signals, we generate the signal samples for a muon collider operating at c.m. energy, $\sqrt{s} = 6$ TeV using MadGraph5_aMC@NLO [52], importing the corresponding UFO model file. Pythia 8.3 [53] is used for the hadronization of quarks and gluons, while Delphes [54], with the muon collider detector card, is used to simulate particle interactions in the detector. As the signals are rich in fatjets, the signal sensitivity will strongly depend on the fatjet reconstruction efficiency of the detector.

For studying the kinematical variables and their event distributions reliably, we apply initial cuts on the kinematical variables. We select leptons with $p_T(e, \mu) > 10$ GeV, $|\eta|(e, \mu) < 2.4$, $p_T(\tau) > 20$ GeV and $|\eta|(\tau) < 2.3$. The jets and the fatjets are reconstructed using the VLC (Valencia) jet algorithm [55]. Jets are selected with jet radius of 0.5, $p_T(j) > 30$ GeV, $|\eta|(j) < 2.4$ and $\Delta R(\ell j) > 0.4$ and fatjets are selected with jet radius $R = 1.0$, $p_T(f_j) > 200$ GeV and $|\eta(f_j)| < 2.4$ and $\Delta R(\ell f_j) > 0.4$. In Fig. 11, we plot the distributions of fatjet multiplicity (left), transverse momentum (p_T) (center) and invariant mass (right), for the signal 1 and 2. From Fig. 11 (top and bottom center), we can observe that the fatjet p_T is very high and the mass of the fatjets are also correctly reconstructed.

We find that the correct number of fatjets are identified in almost 70% of the events. However, we have found that the number of events with fatjet strongly depends on two factors. (i) Fatjet radius (R); if we choose different values of fatjet radius, say $R = 0.8$ or 1.2 , the fatjet identification efficiency changes, and the number of events with fatjets are less or more; (ii) Jet reconstruction algorithm; If we use other algorithms in Delphes such as anti- k_t [56] or N-subjettiness algorithm [57], the fatjet identification efficiency changes. Thus the signal efficiency is affected as the number of fatjets are very high in these signals. The fatjet masses are reconstructed successfully up to the third fatjet (f_2) for signal 1 (top right). Selection of jet pruning algorithm plays a major role in the fatjet mass reconstruction. Also, beam-induced backgrounds (BIB) are anticipated to be significant at a muon collider. A full treatment of BIB requires a dedicated simulation framework, optimized timing cuts, and detector-specific mitigation strategies. In addition, the cascade decays give rise to final states with a large number of particles, in contrast to the final states targeted in conventional searches of the scalars. Given the small signal as well as background cross sections in these channels, machine-learning techniques are expected to play a crucial role in efficient discrimination of signal from the background. Overall, the signal efficiency of the fatjet rich signals are found out to be promising but it depends on several parameters such as jet algorithm, jet radius, jet pruning algorithm and other jet substructure variables, thereby requiring a dedicated study.

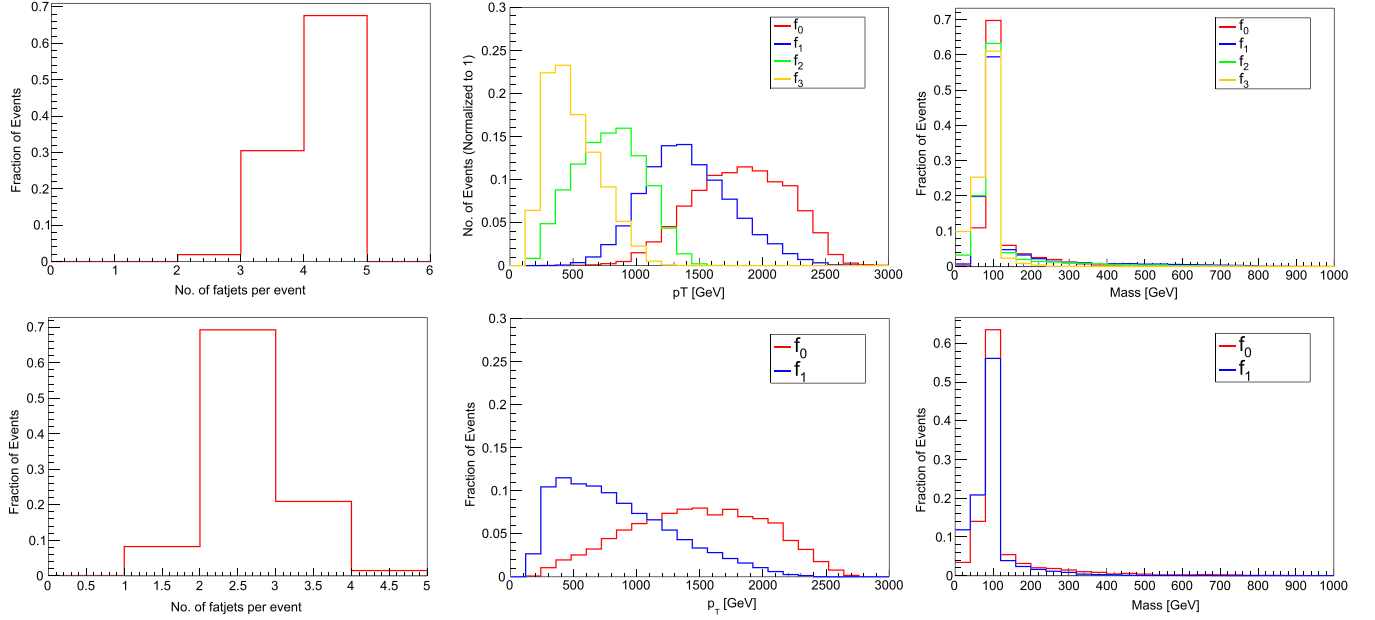


FIG. 11. Distribution of kinematic variables for Signal 1 (top) and Signal 2 (bottom), at the 6 TeV muon collider. In all plots, total number of events is normalized to 1.

VI. CONCLUSION

Key unanswered questions that remain after the discovery of Higgs are about the fundamental versus composite nature of Higgs, its unprotected mass from high-scale quantum corrections, and the absence of a dynamical EWSB mechanism in the SM. Composite Higgs models address these issues by introducing a TeV-scale strongly interacting sector where the Higgs emerges as a pNGB from spontaneous symmetry breaking. In the nonminimal composite model based on $SU(5)/SO(5)$ framework, resembling the Georgi-Machacek model at low energies, features a rich scalar sector. The fermiophilic and fermiophobic decay channels for pNGBs are influenced by fermion embedding choices in $SU(5)/SO(5)$ representations.

Recent studies have examined the fermiophilic and fermiophobic scenarios, analyzing collider signatures and the LHC constraints under simplified assumptions with fixed compositeness scale f and scalar masses. However, the compositeness scale f (varied from 1–5 TeV) is intrinsically linked to fine-tuning, which depends on the choice of representation for the fermions. In this work, we have shown that the masses and mixing of pNGB scalars, as functions of f and gauge-loop parameter C_g , have a key influence on the branching ratios of the pNGB scalars. Unlike the prior works, this study incorporates mixing between gauge and mass eigenstates, deriving couplings in terms of the mixing angles, which influences the scalar phenomenology.

In this paper, we have studied the fermiophilic and fermiophobic scenarios for the full scalar spectrum, where, the scalar masses fall in the range $\sim(200\text{--}2000)$ GeV. We find that the decay patterns of the scalars differ not only across the two scenarios but also within each scenario, as a

function of the mass without. The on shell and off shell decay modes are highly dependent on the mass differences among the pNGB scalars, which is calculated explicitly in terms of the model parameters f , C_g , and mixing angles. Further, we show that the branching ratios of the two singly charged scalars, $\eta_1^\pm$ and $\eta_2^\pm$ are very different. In the fermiophobic scenario, at small masses, both the scalars decay to the gauge bosons but at large masses, their decay patterns change. The heavier $\eta_2^\pm$ decays to another on shell pNGB scalar, accompanied by an off shell or on shell gauge boson. On the other hand, the scalar, $\eta_1^\pm$, due to its small mass splitting with the other pNGBs, cannot decay to an on shell pNGB scalar. Instead, it prefers decaying to two gauge bosons. In the fermiophilic scenario, $\eta_2^\pm$ decays dominantly to a top and a bottom quark, at small and intermediate masses and to an on shell pNGB and a W boson, at large masses. The scalar, $\eta_1^\pm$, instead, prefers decaying to a top and a bottom quark at small masses, to $Z\bar{t}b(Z\bar{b}t)$ at intermediate masses and $W^\pm\bar{t}t$ at large masses.

For the collider study, we focus on the decay channels where the parent pNGB scalar decays to a lighter pNGB scalar and an on shell gauge boson. The daughter pNGB scalar decays further to two gauge bosons leading to a cascade decay. These processes occur in the high-mass regime, with final states comprising multiple leptons and highly boosted jets. However if these jets are coming from the gauge bosons, then there is a greater probability of identifying W/Z bosons as fatjet. A muon collider is therefore, particularly well-suited for probing such signatures at large energy. We estimate the cross section $\times$ BR in the multilepton + fatjet channels when the charged pNGB scalars are pair produced at 6 TeV muon collider. We discuss

the backgrounds for these channels and the kinematical distributions which are crucial for the identification of the fatjets. Given that the signal involves multiple fatjets, it is crucial to implement a complete treatment using several fatjet reconstruction methods. A detailed examination of the beam-induced background is also necessary. Moreover, the kinematics feature cascade decays involving two distinct BSM particles, necessitating sophisticated approaches like machine learning techniques. Therefore, a thorough analysis is beyond the capacity of this study and will be pursued in subsequent work.

ACKNOWLEDGMENTS

N. K. expresses gratitude to A. M. Anirudhan for his contributions during the initial stages of this project, where

he served as the project associate (PA-1) under SRG/2022/000363. N. K. would like to thank Department of Science and Technology and Anusandhan National Research Foundation Government of India for the support from the Startup Research Grant, Grant Agreement No. SRG/2022/000363 and Core Research Grant with Grant Agreement No. CRG/2022/004120.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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