

Testing the type-II seesaw mechanism with gravitational waves

Yonghua Wang^{*} and Wei Chao[†]

Key Laboratory of Multi-scale Spin Physics, Ministry of Education, Beijing Normal University, Beijing 100875, China and Center of Advanced Quantum Studies, School of Physics and Astronomy, Beijing Normal University, Beijing, 100875, China



(Received 14 November 2025; accepted 23 March 2026; published 20 April 2026)

Traditional seesaw mechanisms provide an elegant theoretical framework in explaining the tiny yet nonzero masses of neutrinos. Nevertheless, they face significant challenges in experimental searches, primarily because the energy scale associated with the seesaw mechanism is too high to be directly reached by terrestrial experiments. In this paper, we explore the gravitational waves (GWs) generated via graviton bremsstrahlung during the decay of seesaw particles in the early Universe. Specifically, we compute the GW spectrum resulting from the decay of the Higgs triplet within the type-II seesaw model. Our results demonstrate that the resulting GW spectrum depends sensitively on the mass of the Higgs triplet and its couplings to the Standard Model Higgs doublet and the left-handed lepton doublets. The detection of such a high-frequency GW background could offer a unique experimental window into the seesaw mechanism and provide indirect evidence for its validity.

DOI: [10.1103/2q2-7fjv](https://doi.org/10.1103/2q2-7fjv)

I. INTRODUCTION

The origin of the tiny but nonzero neutrino masses is a longstanding puzzle in particle physics. The seesaw mechanism [1–5], which extends the standard model (SM) with seesaw particles, such as right-handed neutrino singlets or scalar triplet with $Y = 1$, has been so far the most elegant one to accommodate the active neutrino masses. Notably, the seesaw mechanism also provides explanation to the matter-antimatter asymmetry of the Universe via the leptogenesis mechanism [6]. All those observations make the testing of the seesaw mechanism a topic deserving meticulous investigation.

However, it is difficult to test the seesaw mechanism using the terrestrial experiments considering that the seesaw scale is approaching to the scale of the grand unification theory (GUT). Although many TeV scale seesaw mechanisms [7–11] have been proposed, they loss simplicity and naturalness in explaining the active neutrino masses. Given that a direct test is not possible, several indirect detection methods have been proposed, which includes examining the lepton-flavor-violating process in precision measurements [12], lepton number violation in

neutrino-less double beta decay [13], or testing leptogenesis with cosmological collider [14].

Since their discovery by ground-based interferometers [15], gravitational waves (GWs) have become a new probe of the early Universe physics, as they allow us to bypass the cosmic fog that obscures light and offer a direct window into epochs and physical processes—such as inflation [16–20, 20–30], GUT [31–33], dark matter [34–37], and cosmic phase transitions [34, 38–41]—that would otherwise be inaccessible. There are proposals for detecting GW spectrum in various frequency bands. Especially, efforts are underway in designing new facilities for testing high frequency GW, which mainly arise from the physics at high energy scale. It has been shown [42–46] that the thermal GW background, an analog of the cosmic microwave background, reflects the energy distribution of the SM plasma form which they are produced by microscopic collisions. Mapping these observations into neutrino physics, one can conclude that GW can be taken as an indirect probe of the seesaw mechanism. Actually, people have attempted to observing leptogenesis with GWs spectrum from graviton bremsstrahlung in decays of right-handed neutrinos during leptogenesis [47–50].

In this paper, we study the GW spectrum induced from the type-II seesaw mechanism, which extends the SM with a $Y = 1$ scalar triplet Δ that couples to left-handed lepton doublets ℓ_L as well as SM Higgs doublet H . Our results differ from the study of GWs induced by the type-I seesaw model in the following ways: (1) there are two decay channels in the type-II seesaw model, namely $\Delta \rightarrow HH$

^{*}Contact author: yonghuawang@mail.bnu.edu.cn

[†]Contact author: chaowei@bnu.edu.cn

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

and $\Delta \rightarrow \ell_L \ell_L$, the interplay of which may explicitly modify the decoupling temperature of the Δ in the early Universe; (2) the GW spectrum from the type-II seesaw mechanism is relevant to the match between these two-type interactions. Our illustrative numerical results show that the GW spectrum induced by the bremsstrahlung in the decay of the Higgs triplet is accessible by the proposed cavity facilities [51,52] which are designed for the direct detection of high frequency GWs.

The remaining the paper is organized as follows: In Secs. II and III, we give a brief overview on the type-II seesaw mechanism and the decay of the Higgs triplet as well as its evolution in the early Universe. Section IV is devoted to the calculation of the GW spectrum. The last part is conclusion remarks. The detail of the calculation, including the Feynman rules and the squared amplitudes are given in the Appendixes A and B.

II. THE TYPE-II SEESAW MODEL

The type-II seesaw mechanism extends the SM with a $SU(2)_L$ triplet scalar field Δ with hypercharge $Y = 1$. The matrix representation of Δ reads as [53]

$$\Delta = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix} \quad (1)$$

where Δ^{++} , Δ^+ , and Δ^0 are the doubly charged, singly charged and neutral components, whose charge conjugate states are denoted as Δ^{--} , Δ^- , and $\bar{\Delta}^0$, respectively. The relevant Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \text{Tr}[(D_\mu \Delta)^\dagger (D^\mu \Delta)] - V(H, \Delta) - \frac{1}{2} Y_\Delta^{ij} \bar{\ell}_{Li}^C i\sigma_2 \Delta \ell_{Lj} + \text{H.c.} \quad (2)$$

where D_μ is the covariant derivative, ℓ_L is the left-handed lepton doublet with Y_Δ the 3×3 symmetric Yukawa coupling between the Higgs triplet and lepton doublets, $C = i\gamma^2 \gamma^0$ being the charge conjugation operator. $V(H, \Delta)$ is the potential taking the following form

$$V(H, \Delta) = -\mu_H^2 H^\dagger H + \lambda (H^\dagger H)^4 + M_\Delta^2 \text{Tr}[\Delta^\dagger \Delta] + \lambda_s \text{Tr}[\Delta^\dagger \Delta]^2 + \lambda_{hs} \text{Tr}[\Delta^\dagger \Delta] (H^\dagger H) - \frac{1}{2} \mu H^T i\sigma^2 \Delta^\dagger H + \text{H.c.}, \quad (3)$$

where M_Δ is the mass of the Higgs triplet, λ , λ_s and λ_{hs} are quartic couplings, μ is a dimension one parameter describing triple coupling between the Higgs doublet and the triplet. After the electroweak symmetry breaking, the SM Higgs doublet acquires a vacuum expectation value (VEV) $v_0 = 246$ GeV [54]. The last term in the Lagrangian (3) induces a VEV for the neutral component of the Higgs triplet, v_Δ . In the limit of $M_\Delta \gg v_0$, the VEV of the Higgs triplet can be approximated as

$$v_\Delta \simeq \frac{\mu v_0^2}{4M_\Delta^2}. \quad (4)$$

The neutrino mass matrix is then generated as

$$M_\nu^{ij} = Y_\Delta^{ij} v_\Delta = Y_\nu^{ij} \frac{\mu v_0^2}{4M_\Delta^2}, \quad (5)$$

which can be diagonalized by the 3×3 Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix $\mathcal{U}$ [55–57],

$$\mathcal{U}^\dagger M_\nu \mathcal{U}^* = \text{diag}\{m_1, m_2, m_3\} \quad (6)$$

where m_1 , m_2 , and m_3 are the mass eigenvalues of active neutrinos, and $\mathcal{U}$ can be parametrized as

$$\mathcal{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix} \text{diag}(e^{i\Phi_1/2}, 1, e^{i\Phi_2/2}) \quad (7)$$

where $s_{ij} = \sin \theta_{ij}$, $c_{ij} = \cos \theta_{ij}$, δ_{CP} is the Dirac CP -violating phase, and Φ_1 and Φ_2 are the Majorana phases. Using data from various neutrino oscillation experiments, the global fit results for neutrino masses and mixing parameters are presented in Table I [58–60] for both normal hierarchy (NH) and inverted hierarchy (IH).

The recent DESI data [61,62] prefers the normal hierarchy of neutrino masses [63] and the lightest neutrino mass can be taken as massless, so the neutrino masses may be given as

$$0 \sim m_1 \ll m_2 \approx \sqrt{\Delta m_{21}^2} \approx 8.6 \times 10^{-3} \text{ eV} < m_3 \approx \sqrt{\Delta m_{32}^2 + \Delta m_{21}^2} \approx 5.0 \times 10^{-2} \text{ eV}. \quad (8)$$

Using the normal hierarchy data listed in Table I and setting the Majorana phases to $\Phi_1 = \Phi_2 = 0$ for simplicity, the Yukawa matrix Y_Δ is given by

TABLE I. Neutrino oscillation parameters with their best-fit values for both normal hierarchy (NH) and inverted hierarchy (IH) [59,60]. $\Delta m_{3\ell} = \Delta m_{31}$ for NH and $\Delta m_{3\ell} = \Delta m_{32}$ for IH.

Parameter	NH	IH
$\sin^2 \theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.304^{+0.013}_{-0.012}$
$\theta_{12}/^\circ$	$33.45^{+0.77}_{-0.75}$	$33.45^{+0.78}_{-0.75}$
$\sin^2 \theta_{23}$	$0.450^{+0.019}_{-0.016}$	$0.570^{+0.016}_{-0.022}$
$\theta_{23}/^\circ$	$42.1^{+1.1}_{-0.9}$	$49.0^{+0.9}_{-1.3}$
$\sin^2 \theta_{13}$	$0.02246^{+0.00062}_{-0.00062}$	$0.02241^{+0.00074}_{-0.00062}$
$\theta_{13}/^\circ$	$8.62^{+0.12}_{-0.12}$	$8.61^{+0.14}_{-0.12}$
$\delta_{CP}/^\circ$	230^{+36}_{-25}	278^{+22}_{-30}
$\Delta m_{21}^2/10^{-5} \text{ eV}^2$	$7.42^{+0.21}_{-0.20}$	$7.42^{+0.21}_{-0.20}$
$\Delta m_{3\ell}^2/10^{-3} \text{ eV}^2$	$+2.510^{+0.027}_{-0.027}$	$-2.490^{+0.026}_{-0.028}$

$$Y_\Delta \approx \frac{10^{-3} \text{ eV}}{v_\Delta} \begin{pmatrix} 2.4 - 1.1i & -0.13 + 4.0i & -6.0 + 4.4i \\ -0.13 + 4.0i & 26 + 0.48i & 21 + 0.074i \\ -6.0 + 4.4i & 21 + 0.074i & 29 - 0.42i \end{pmatrix}. \quad (9)$$

There are constraints on the v_Δ from the precision measurements of the ρ -parameter [64,65] and ensuring that the observed neutrino masses are generated while maintaining the perturbative Yukawa couplings [66], which give the upper bound and lower bound. One has

$$2.56 \text{ GeV} \gtrsim v_\Delta \gtrsim 0.05 \text{ eV}. \quad (10)$$

These bounds can be translated into constraints on the parameter μ as

$$10 \text{ GeV} \frac{M_\Delta^2}{v_\Delta^2} \gtrsim \mu \gtrsim 0.2 \text{ eV} \frac{M_\Delta^2}{v_0^2}. \quad (11)$$

Given these constraints one can analyze the decay of the Higgs triplet, which will be done in the next section.

III. HIGGS TRIPLET EVOLUTION IN THE EARLY UNIVERSE

Given Lagrangian (2), we can directly extract the Feynman rules for various vertices, which are presented in the Appendix A, and use them to calculate the decay widths of the Higgs-triplet components. Decay widths for the dilepton channel are given by [53,58,67]

$$\begin{aligned} \Gamma_{\Delta^{++} \rightarrow e_i^+ e_j^+} &= \frac{|Y_\Delta^{ij}|^2}{\delta_{ij} + 1} \frac{M_\Delta}{16\pi}, & \Gamma_{\Delta^+ \rightarrow e_i^+ \nu_j} &= |Y_\Delta^{ij}|^2 \frac{M_\Delta}{32\pi}, \\ \Gamma_{\Delta^0 \rightarrow \bar{\nu}_i \nu_j} &= \frac{|Y_\Delta^{ij}|^2}{\delta_{ij} + 1} \frac{M_\Delta}{16\pi} \end{aligned} \quad (12)$$

and decay widths for the di-Higgs channel are given by

$$\begin{aligned} \Gamma_{\Delta^{++} \rightarrow H^+ H^+} &= \frac{\mu^2}{32\pi M_\Delta}, & \Gamma_{\Delta^+ \rightarrow H^+ H^0} &= \frac{\mu^2}{32\pi M_\Delta}, \\ \Gamma_{\Delta^0 \rightarrow H^0 H^0} &= \frac{\mu^2}{32\pi M_\Delta} \end{aligned} \quad (13)$$

where δ_{ij} is the Kronecker delta function. The charge conjugate partner of the Higgs triplet components shares the same decay width. The total decay width, summed over rates for all decay channels, is given by

$$\begin{aligned} \Gamma_\Delta &= \sum_{i \geq j} \Gamma_{\Delta \rightarrow \ell_{L_i} \ell_{L_j}} + \Gamma_{\Delta \rightarrow HH} = \sum_{i,j} |Y_\Delta^{ij}|^2 \frac{M_\Delta}{32\pi} + \frac{\mu^2}{32\pi M_\Delta} \\ &= \frac{M_\Delta}{32\pi} \left(\frac{2.57 \times 10^{-3} \text{ eV}^2}{v_\Delta^2} + \frac{16M_\Delta^2 v_\Delta^2}{v_0^4} \right), \end{aligned} \quad (14)$$

where i and j run from 1 to 3. In the first row, the summation over i and j is restricted to $i \geq j$ to avoid double counting for symmetric final states under the exchange of i and j . In the second row, we have used numerical results given in the Eq. (9). Branching ratios are defined as

$$\text{BRL} = \frac{\sum_{i \geq j} \Gamma_{\Delta \rightarrow \ell_{L_i} \ell_{L_j}}}{\Gamma_\Delta}, \quad \text{BRH} = \frac{\Gamma_{\Delta \rightarrow HH}}{\Gamma_\Delta}. \quad (15)$$

We plot the total decay width and the branching ratios as functions of v_Δ for fixed value of M_Δ in the left-panel of the Fig. 1. It shows that the branching ratios for the dilepton and di-Higgs channels become equal and the total decay width is minimized when

$$v_\Delta = \sqrt{\frac{0.01 \text{ eV}}{M_\Delta}} v_0. \quad (16)$$

We show in the right-panel of the Fig. 1 two branching ratios as the function of v_Δ by setting $M_\Delta = 10^{14} \text{ GeV}$, which shows that two branching ratios equal with each other at $v_\Delta \sim 7.8 \times 10^{-2} \text{ eV}$.

Now we can evaluate the evolution of the Higgs triplets in the early Universe. Assuming that the mass of the Higgs triplet is smaller than the reheating temperature, $M_\Delta < T_{rh}$, the Higgs triplet is initially relativistic and remain in thermal equilibrium with the SM particles through gauge interactions [49]. Specifically, the Higgs triplet reaches thermal equilibrium with the SM plasma if its interaction rate Γ_g exceeds the Hubble expansion rate H_{rh} at the time of reheating, i.e., $\Gamma_g(T_{rh}) > H_{rh}$. Given that the Higgs triplet carries both isospin and hypercharge, it can interact with Standard Model particles through gauge interactions. The thermally averaged interaction rate at the reheating temperature is estimated as $\Gamma_g(T_{rh}) \sim g^2 T_{rh}$ [68] where g

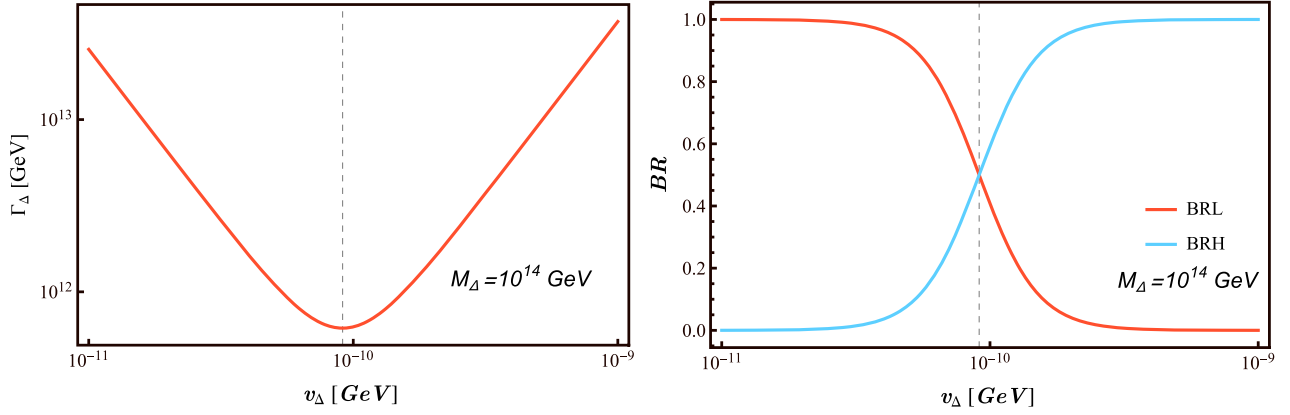


FIG. 1. Left: decay widths of the Higgs triplet as functions of v_Δ for $M_\Delta = 10^{14}$ GeV. Right: branching ratios for Higgs-triplet decays into leptons (BRL) and into Higgs bosons (BRH) as functions of v_Δ for the same mass. The vertical dashed line in the left panel marks the value of v_Δ at which the total decay width is minimized; this value coincides with the intersection of BRL and BRH in the right panel.

stands for relevant coupling, and $H_{rh} = \sqrt{\frac{g_*^2 \pi^2}{90} \frac{T_{rh}^2}{M_P}}$. Consequently, the Higgs triplet achieves thermal equilibrium provided that $g > \sqrt{T_{rh}/M_P}$. Considering that $g_Y \sim \mathcal{O}(0.1)$, the thermalization condition is easily satisfied. After reheating, the number density of the Higgs triplet is given by [68,69]

$$n_\Delta(T) = \frac{g_*^\Delta \zeta(3)}{\pi^2} T^3, \quad (17)$$

where $\zeta(3) \approx 1.202$ is the Riemann zeta function, and $g_*^\Delta = 6$ being the relativistic effective degrees of freedom of the Higgs triplet. Introducing comoving variable $Y_\Delta \equiv n_\Delta a^3$ where a is the scale factor of the Universe, the Boltzmann equation for the Higgs triplet can be written as

$$\frac{dY_\Delta}{dt} = -\langle \Gamma_\Delta \rangle (Y_\Delta - Y_\Delta^{\text{eq}}). \quad (18)$$

The thermally averaged decay width in this expression is given by

$$\langle \Gamma_\Delta \rangle = \Gamma_\Delta \frac{K_1(M_\Delta/T)}{K_2(M_\Delta/T)}, \quad (19)$$

where K_1 and K_2 are the modified Bessel functions of the second kind. The equilibrium number density of the Higgs triplet is denoted by n_Δ^{eq} . The Hubble parameter is defined as

$$H(T) = \sqrt{\frac{\rho_\Delta + \rho_{\text{SM}}}{3M_P^2}}, \quad (20)$$

where $M_P = 2.4 \times 10^{18}$ GeV being the reduced Planck mass. In the regime $T < M_\Delta$, the equilibrium term and

the Bessel functions can be safely neglected, and the Boltzmann equation can be solved analytically, yielding

$$\rho_\Delta(t) = \rho_\Delta(t_M) \left(\frac{a(t_M)}{a(t)} \right)^3 e^{-\Gamma_\Delta(t-t_M)}. \quad (21)$$

The time at which Higgs triplets decayed away can be estimated as

$$t_D = \Gamma_\Delta^{-1}, \quad (22)$$

which relates to the decay temperature T_D through the Hubble parameter as $H(T_D) \simeq 1/2t_D$ in the standard radiation-dominated Universe, and $H(T_D) \simeq 2/3t_D$ in an early matter-dominated era.

If the neutrino masses are solely generated by the type-II seesaw mechanism, the total decay width is minimized when the branching ratios for the dileptonic and di-Higgs channels equal with each other, resulting in the lowest decouple temperature. In this case, the parameters v_Δ and M_Δ are related with each other by Eq. (16), leaving only a single free parameter, which can be chosen as M_Δ . Consequently, the Boltzmann equation in Eq. (18) becomes independent of the model parameters. In this case, the decay temperature T_D is always larger than the mass of the Higgs triplet for all values of μ and M_Δ . Alternatively, if the neutrino masses are not solely generated by the type-II seesaw mechanism, the Yukawa couplings Y_Δ cannot be solely determined by the Eq. (9). In this case, we can redefine the total decay rate as

$$\Gamma_\Delta^{\text{tot}} = \frac{\zeta^2}{32\pi} M_\Delta. \quad (23)$$

where ζ is the function of the Yukawa coupling and the triple Higgs coupling constant. In this case, the Higgs triplets can decay after they become nonrelativistic as long

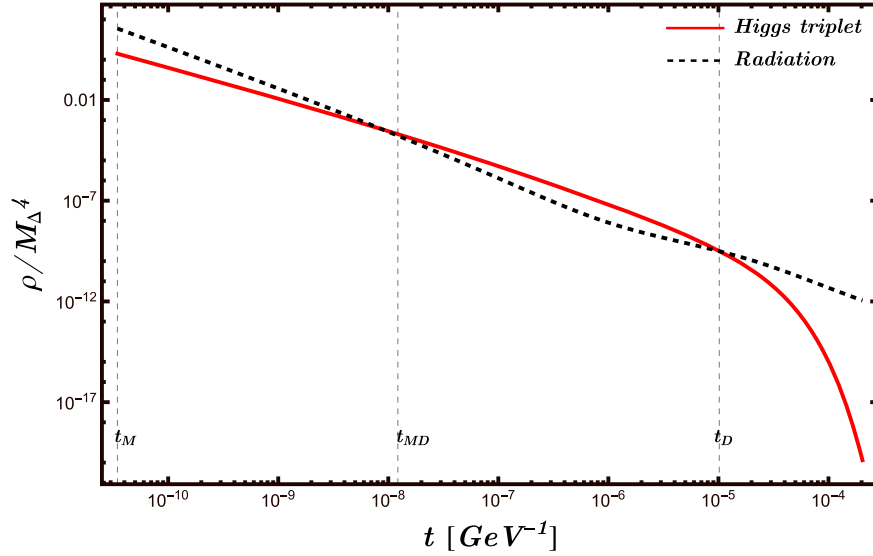


FIG. 2. Evolution of the energy densities for the Higgs triplet and radiation, with $M_\Delta = 10^{14}$ GeV and $\zeta = 10^{-4}$. The red solid line represents the energy density of the Higgs triplets, while the black dashed line represents that of radiation.

as $\zeta < 18\sqrt{M_\Delta/M_P}$. Furthermore, the Universe may experience an early matter-dominated era due to the existence of nonrelativistic Higgs triplets, if the energy density of the Higgs triplets ρ_Δ exceeds that of radiation ρ_R before they decay. The condition for this situation to occur is

$$\zeta < 0.38\sqrt{\frac{M_\Delta}{M_P}}. \quad (24)$$

Figure 2 shows the evolution of the energy density of the Higgs triplet and radiation for $M_\Delta = 10^{14}$ GeV and $\zeta = 10^{-4}$. Here, t_M marks the time when Higgs triplets become nonrelativistic, t_{MD} denotes the onset of the early matter-dominated era, and t_D is the decay time defined by Eq. (22), which also corresponds to the end of the early matter-dominated era. As can be seen from the figure, the Universe may undergo an early matter-dominated phase driven by the nonrelativistic Higgs triplet before it decays away. The presence of an early matter-dominated era implies that GW signals originating from matter decay can be distinguished from those produced by the SM thermal plasma [49].

IV. GRAVITATIONAL WAVE SPECTRUM

There are many sources for stochastic GWs in the early Universe, such as first order phase transitions [70–73], oscillation of cosmic strings [74], or thermal fluctuations of the plasma, etc. In the early Universe, gravitons can also be generated from graviton bremsstrahlung process [75,76] in the decays of the Higgs triplet. The Feynman diagrams that contribute to nonzero amplitudes of the graviton

bremsstrahlung [16] are shown in Fig. 3. The detailed Feynman rules are presented in the Appendix A.

The evolution of the energy density of GWs is governed by the following Boltzmann equation

$$\frac{d\rho_{\text{GW}}}{dt} + 4H\rho_{\text{GW}} = C_{\Delta \rightarrow LLg} + C_{\Delta \rightarrow HHg} \quad (25)$$

where $C_{\Delta \rightarrow LLg}$ and $C_{\Delta \rightarrow HHg}$ are the collision terms for generating graviton during the decay of the Higgs triplet into dilepton and di-Higgs, given by

$$C_{\Delta \rightarrow LLg} = \frac{\kappa^2 M_\Delta \zeta^2}{1024\pi^3} \text{BRL} \int d\omega n_\Delta \frac{K_1(M_\Delta/T)}{K_2(M_\Delta/T)} \left(1 - 2\frac{\omega}{M_\Delta}\right) \times \left(2 - 2\frac{M_\Delta}{\omega} + \frac{M_\Delta^2}{\omega^2}\right) \omega^2, \quad (26)$$

and

$$C_{\Delta \rightarrow HHg} = \frac{\kappa^2 M_\Delta \zeta^2}{1024\pi^3} \text{BRH} \int d\omega n_\Delta \frac{K_1(M_\Delta/T)}{K_2(M_\Delta/T)} \times \left(2 - \frac{M_\Delta}{\omega}\right)^2 \omega^2, \quad (27)$$

where n_Δ is the number density of the Higgs triplet, and $\kappa = \sqrt{2}/M_P$ being the coupling between graviton and SM particles. The detailed derivation of the above two equations can be found in the Appendix B.

Introducing the comoving variable $Y_{\text{GW}} = \rho_{\text{GW}} a^4$, the Boltzmann equation for the energy density of GWs can be written as

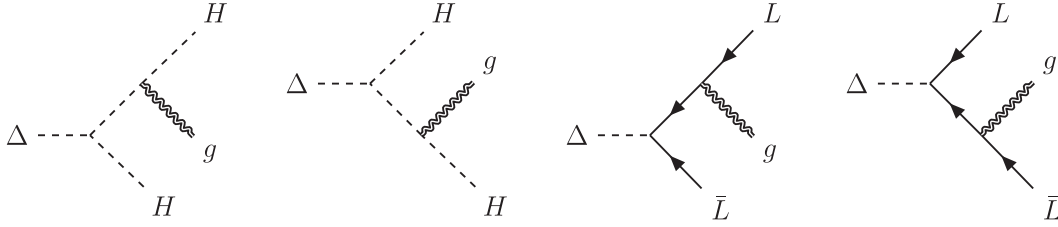


FIG. 3. Feynman diagrams for graviton bremsstrahlung in the decay of Higgs triplets. The left two diagrams are for the process $\Delta \rightarrow HHg$, and the right two diagrams are for the process $\Delta \rightarrow LLg$.

$$\frac{dY_{\text{GW}}}{dt} = a^4(\mathcal{C}_{\Delta \rightarrow LLg} + \mathcal{C}_{\Delta \rightarrow HHg}). \quad (28)$$

The GWs spectrum at present time is given by

$$\Omega_{\text{GW}} h^2(f) = \frac{1}{\rho_{cr,0}} \frac{d\rho_{\text{GW},0}}{d \ln f} h^2 = \frac{1}{\rho_{cr,0}} \frac{d(Y_{\text{GW}}(t_0) a^{-4}(t_0))}{d \ln f} h^2, \quad (29)$$

where $\rho_{cr,0} = 3H_0^2 M_P^2$ is the present-day critical energy density of the universe, with $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$ being the Hubble parameter today. $\rho_{\text{GW},0}$ denotes the present energy density of GWs, and f is the present-day frequency of GWs, which is related to the graviton energy by $f = \omega_0/2\pi$, where ω_0 is the graviton energy at present.

By solving the Boltzmann equation (28), we obtain the present-day GWs spectrum as

$$\begin{aligned} \Omega_{\text{GW}} h^2 &= \frac{h^2}{\rho_{cr,0}} \frac{d\rho_{\text{GW}}}{d \ln \omega}(t_0) = \frac{h^2}{\rho_{cr,0}} a^{-4}(t_0) \int_{t_M}^{t_D} dt \frac{d}{d \ln \omega} Y_{\text{GW}} \\ &= \frac{M_\Delta^2 \zeta^2}{512\pi^3 M_P^2 \rho_{cr,0}} \int_{t_M}^{t_D} dt \frac{a^4(t)}{a^4(t_0)} \rho_\Delta(t) \omega(t) \\ &\quad \times \left[\text{BRL} \left(1 - 2 \frac{\omega(t)}{M_\Delta} \right) \left(1 - 2 \frac{\omega(t)}{M_\Delta} + 2 \frac{\omega(t)^2}{M_\Delta^2} \right) \right. \\ &\quad \left. + \text{BRH} \left(1 - 2 \frac{\omega(t)}{M_\Delta} \right)^2 \right], \end{aligned} \quad (30)$$

where $\rho_\Delta \equiv n_\Delta M_\Delta$ is the energy density of the Higgs triplet. We integrate the collision term from time t_M , when the Higgs triplets start to decay, to t_D when they are mostly decayed out. Subsequently, the energy of the graviton is redshifted to present time t_0 . The frequency of the GWs at the present is given by

$$f = \frac{\omega(t_0)}{2\pi} = \frac{1}{2\pi} \left(\frac{g_*^S(T_0)}{g_*^S(T_D)} \right)^{1/3} \frac{T_0}{T_D} \omega(t_D). \quad (31)$$

During the decay of the Higgs triplets, if an early matter-dominated era does not occur, the scale factor evolves as $a(t) \propto t^{1/2}$. Conversely, if such an era arises, the evolution proceeds in two stages: $a(t) \propto t^{1/2}$ for $t_M < t < t_{MD}$, where t_{MD} marks the onset of matter domination, followed

by $a(t) \propto t^{2/3}$ for $t_{MD} < t < t_D$. After the Higgs triplets have effectively decayed ($t > t_D$), the universe returns to a radiation-dominated era.

We show the GW spectrum arising from the graviton bremsstrahlung in the decay of Higgs triplet in the Fig. 4. The plot in the left panel shows the GW spectra when the Higgs triplets trigger an early matter-dominated (EMD) era, while the plot in the right panel shows the GW spectra in a standard radiation-dominated scenario (without an EMD era), both in the case of equal branching ratios ($\text{BRH} = \text{BRL} = 1/2$). We show results for two mass scales: $M_\Delta = 10^{14} \text{ GeV}$ (red lines) and $M_\Delta = 10^{15} \text{ GeV}$ (blue lines). Solid and dashed lines distinguish between different values of ζ . In the EMD case (left), we set $\zeta = 10^{-3}$ and 10^{-4} , whereas in the radiation-dominated case (right), we choose $\zeta = 10^{-1}$ and $10^{-1.5}$.

To analyze how the GW spectrum scales with the coupling ζ and the Higgs triplet mass M_Δ , we assume that the decay occurs instantaneously at t_D . Under this assumption, the GW spectrum can be expressed as

$$\begin{aligned} \Omega_{\text{GW}} h^2 &= \frac{h^2}{\rho_{cr,0}} \frac{\zeta^2}{512\pi^3} \frac{M_\Delta^2}{M_P^2} \frac{1}{\Gamma_\Delta^{\text{tot}}} \rho_\Delta(t_D) \frac{a^4(t_D)}{a^4(t_0)} \omega(t_D) \\ &\quad \times \left[\text{BRL} \left(1 - 2 \frac{\omega(t_D)}{M_\Delta} \right) \left(1 - 2 \frac{\omega(t_D)}{M_\Delta} + 2 \frac{\omega(t_D)^2}{M_\Delta^2} \right) \right. \\ &\quad \left. + \text{BRH} \left(1 - 2 \frac{\omega(t_D)}{M_\Delta} \right)^2 \right], \end{aligned} \quad (32)$$

where all time dependent quantities in the integrand are evaluated at t_D . If the Higgs triplet does not trigger an early matter-dominated era, we have $\rho_\Delta(t_D) a^3(t_D) = e^{-1} \rho_\Delta(t_M) a^3(t_M) \propto M_\Delta$, where we have assumed $t_D \gg t_M$ in the first equality. In addition, since $a(t_D)/a(t_0) \propto T_D^{-1} \propto (\zeta \sqrt{M_\Delta})^{-1}$, the GW spectrum scales with M_Δ and ζ as $\Omega_{\text{GW}} h^2 \propto M_\Delta^{5/2} \zeta^{-1} \mathcal{F}(x_D)$, where $\mathcal{F}(x_D) \equiv x_D [\text{BRL}(1 - x_D)(1 - x_D + x_D^2/2) + \text{BRH}(1 - x_D)^2]$ with $x_D \equiv 2\omega(t_D)/M_\Delta$, which is maximized at $x_D = 0.36$ in the case of $\text{BRL} = \text{BRH} = 1/2$. Consequently, the peak frequency of the GW spectrum can be evaluated as

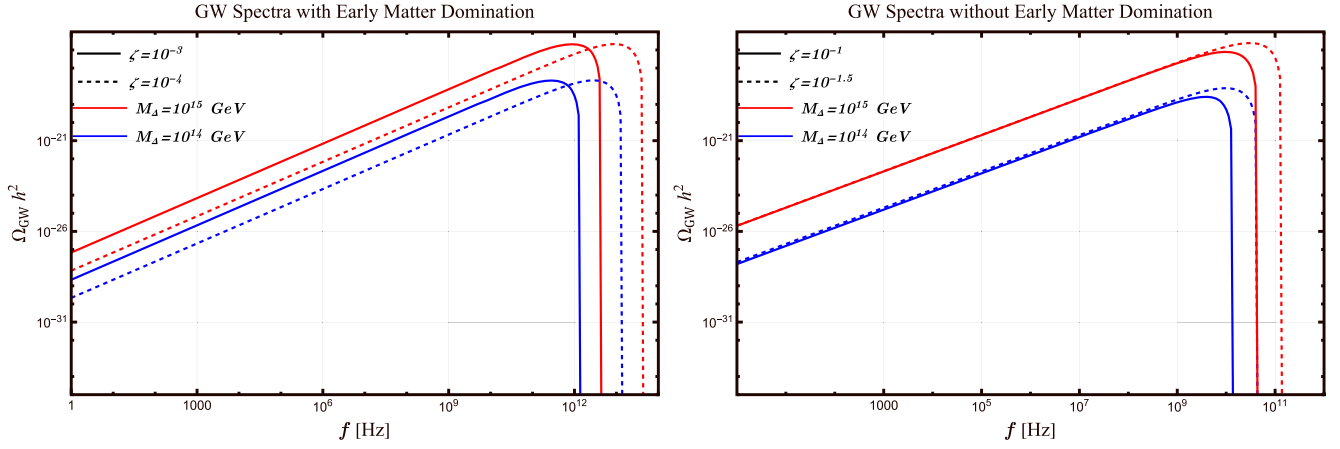


FIG. 4. GW spectra produced by graviton bremsstrahlung from Higgs triplet decays, assuming $\text{BRH} = \text{BRL} = 1/2$. The different colors correspond to various values of M_Δ , while dashed and solid lines represent different values of ζ . The plot in the left-panel displays the spectra in a scenario where the Higgs triplets trigger an early matter-dominated (EMD) era. Conversely, the one in the right-panel illustrates the spectra in a standard radiation-dominated scenario.

$$f_{\text{peak}} = 0.36 \times \frac{1}{2\pi} \left(\frac{g_*^S(T_0)}{g_*^S(T_D)} \right)^{1/3} \frac{T_0}{T_D} \frac{M_\Delta}{2}$$

$$= 1.3 \times 10^{10} \text{ Hz} \left(\frac{M_\Delta}{10^{15} \text{ GeV}} \right)^{1/2} \left(\frac{\zeta}{10^{-1}} \right)^{-1}. \quad (33)$$

The scaling laws for the GW spectrum and the peak frequency are consistent with the numerical results shown in the right panel of Fig. 4. On the other hand, if the Higgs triplet triggers an early matter-dominated era, this era ends at time t_D with $\rho_\Delta(t_D) = \rho_{\text{SM}}(t_D)$. Consequently, the GW

spectrum scales as $\Omega_{\text{GW}} h^2 \propto M_\Delta^2 \mathcal{F}(x_D)$. The scaling law for the peak frequency is the same as in the previous case, but the overall amplitude of the spectrum is independent of ζ and scales as M_Δ^2 . These scaling behaviors align with the numerical results presented in the left panel of Fig. 4.

We show the effects of different branching ratios on the GW spectrum in Fig. 5. The limiting cases of pure di-Higgs ($\text{BRH} = 1, \text{BRL} = 0$) and pure dilepton ($\text{BRH} = 0, \text{BRL} = 1$) decay channels are depicted by the red and blue lines, while the dashed line represents the case of equal

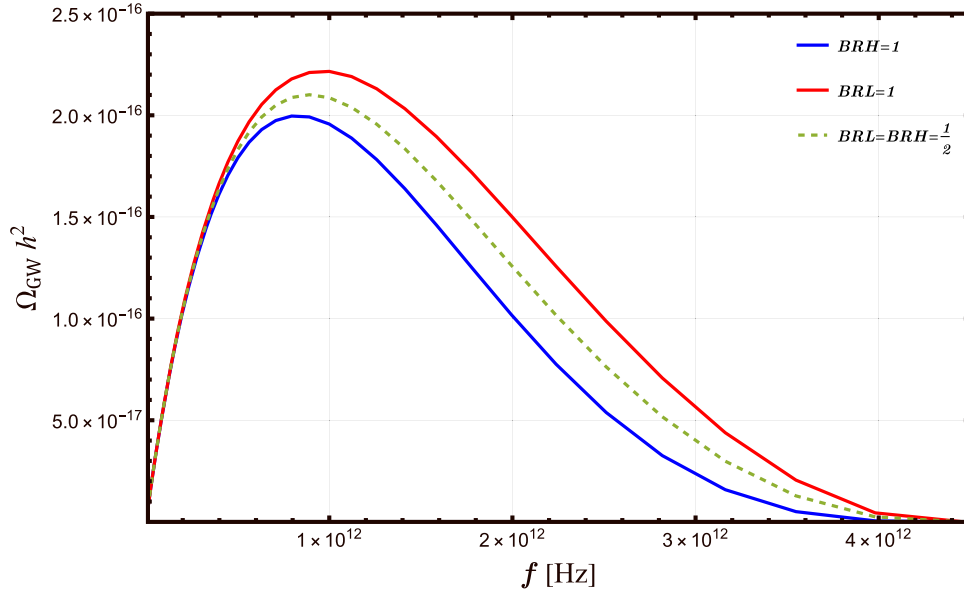


FIG. 5. GW spectra sourced by graviton bremsstrahlung in the Higgs triplet decay for $M_\Delta = 10^{14} \text{ GeV}$, $\zeta = 10^{-3}$ and various branching ratios. The solid red and blue lines represent the pure Higgs and leptonic decay channels, respectively. These two cases nearly overlap below the peak frequency but exhibit small distinction in the high-frequency regime. The dashed line corresponds to the case of equal branching ratios ($\text{BRH} = \text{BRL} = 1/2$). By varying the branching ratios, the total GW spectrum transitions between these two limiting cases.

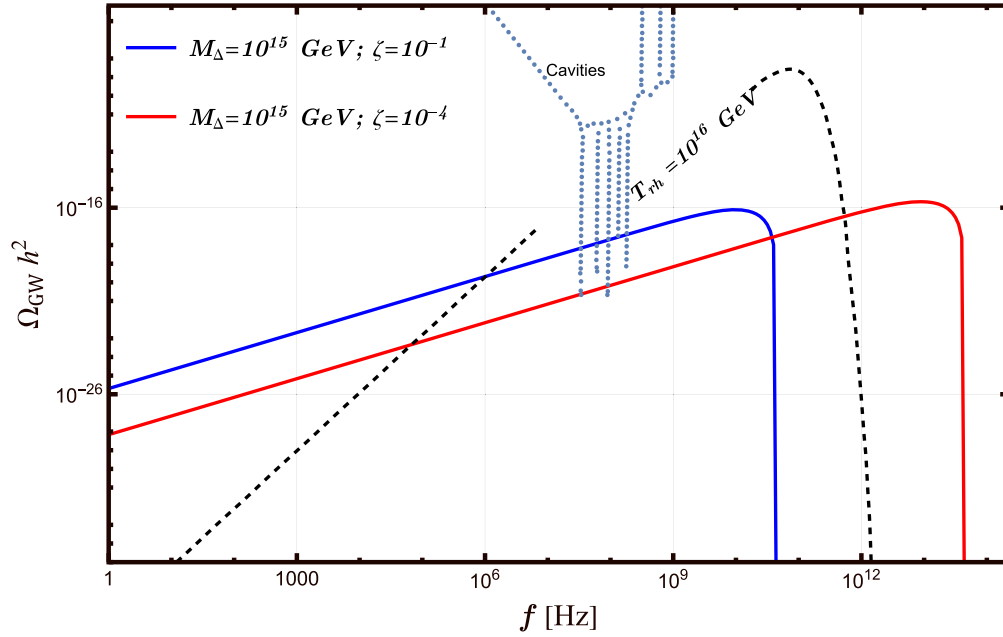


FIG. 6. GW spectra produced by graviton bremsstrahlung in the decays of Higgs triplets by assuming equal branching ratios for the leptonic and Higgs channels, i.e., $\text{BRL} = \text{BRH} = 1/2$. We fix the Higgs triplet mass $M_\Delta = 10^{15}$ GeV but vary the coupling: $\zeta = 10^{-1}$ (blue line) corresponds to the scenario without an early matter-dominated era, while $\zeta = 10^{-4}$ (red line) corresponds to the scenario with an early matter-dominated era. The sensitivity curves for the resonant-cavity experiments are depicted with dotted lines, and the GW spectrum from the SM thermal plasma for a reheating temperature $T_{rh} = 10^{16}$ GeV is plotted by black dashed line.

branching ratios ($\text{BRH} = \text{BRL} = 1/2$). We can see that varying the branching ratios does not affect the spectrum in the low frequency regime, but it changes the spectrum slightly in the vicinity of the peak frequency. By adjusting the branching ratios, the spectral curves vary smoothly between these two limiting cases.

In the Fig. 6, we show the total GW spectrum by setting branching ratios of the di-Higgs and dilepton processes equal with each other. As shown, the GW spectrum could be detected by the resonant cavities experiments [51,52,77–79]. Besides, these GW signals are clearly distinguishable from the GW background sourced by the SM thermal plasma in the high frequency regime provided that the Higgs triplets trigger an early matter dominated era, otherwise the GW signals would be overwhelmed by the thermal GW background. Additionally, the low frequency tail of the spectrum remains identifiable against the thermal GW background. Furthermore, future facilities for detecting higher frequency graviton may help to identify the signal of the Higgs triplet.

V. SUMMARY AND REMARKS

Seesaw are elegant mechanisms in addressing the tiny but nonzero active neutrino masses. Testing these mechanisms are longstanding challenges of the high energy physics. People have proposed a new strategy to test the physics of the very Universe via high frequency GWs. In this paper, we have calculated the GW spectrum arising

from the graviton bremsstrahlung process during the out of equilibrium decay of the Higgs triplet, which is the typical particle in the type-II seesaw model. We have studied the thermal history of the Higgs triplet, which may causes a prematter-dominated epoch in the early Universe for typical couplings between the Higgs triplet and the SM plasma. The GW spectrum is also derived, whose shape depends on the inputs of Y_Δ and μ .

We note that the early Universe hosts a plethora of potential sources for high-frequency GWs beyond the type-II seesaw model, including inflationary reheating, phase transitions associated with grand unified theories or extended gauge symmetries, and the decay of heavy scalars in the Peccei-Quinn model. A systematic derivation of the GW spectra for these diverse scenarios is a crucial direction for future research. By identifying unique spectral features—such as specific slopes or peak structures—we can aim to distinguish these sources from one another, thereby using high-frequency GW data to constrain the underlying new physics.

Although, currently there is no technology to observe GWs beyond kHz frequency, the GW spectrum emitted from the decay of the Higgs triplet can be reached by proposed cavity experiments designed for detecting high-frequency GWs. Understanding the origin of high-frequency GWs is a new pathway to explore the high energy scale physics. Developing new capability of measuring high frequency GWs may help us open the door of the

neutrino physics in the ultraviolet sector as well as the new physics beyond the SM. Future precision surveys of CMB and its spectral distortion missions, such as polarized radiation interferometer for spectral distortions and inflation exploration [80], the primordial inflation explorer (PIXIE) [81,82] and its next-generation concept Super-PIXIE, as well as ESA's Voyage 2050 program [83], hold the potential to probe the stochastic GW background as high as 10^{12} Hz. In parallel, we highlight substantial advances in direct- detection techniques employing high-Q resonant cavities, quantum sensors, and cryogenic technologies, which represent promising avenues for exploring the physics at the high-frequency GW frontier.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (NSFC) (Grants No. 12447105, No. 11775025 and No. 12175027), and the Fundamental Research Funds for the Central Universities.

DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: FEYNMAN RULES

In this appendix, we present the Feynman rules for the relevant vertices used in our calculations.

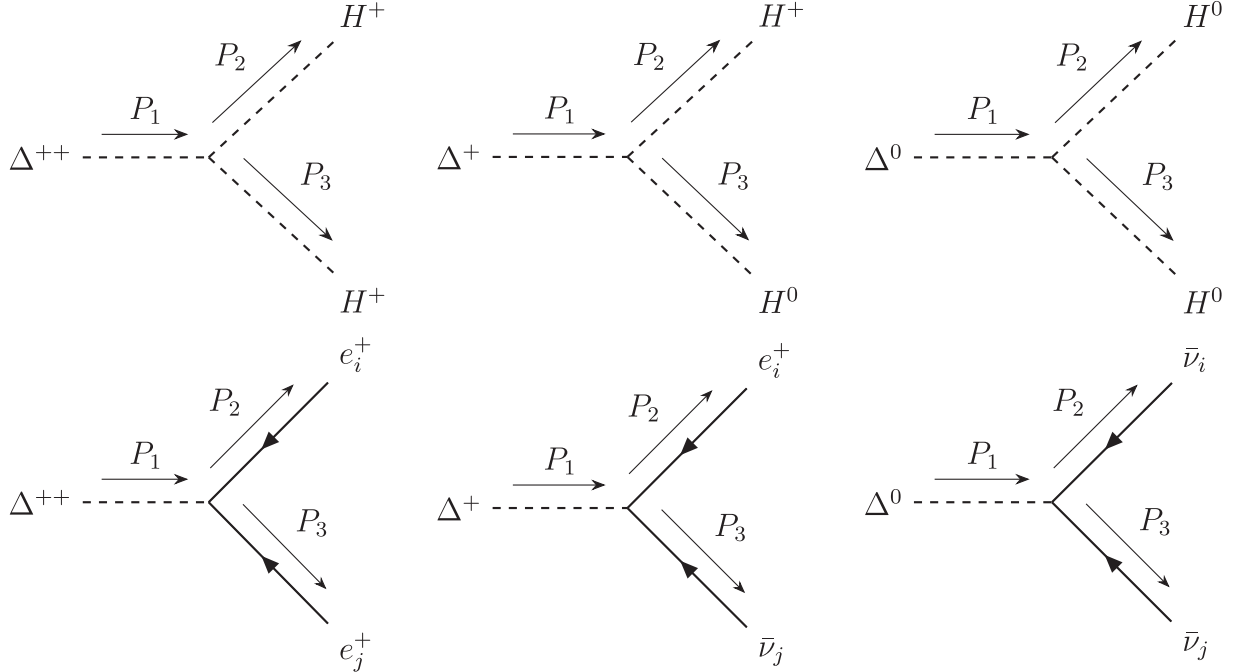


FIG. 7. Feynman diagrams for the interaction vertices for the Higgs triplet and leptons/Higgs bosons.

The interaction Lagrangian for the triplet and leptons/Higgs is given by Eq. (2), expanding it in terms of the component fields, we have [84]

$$\begin{aligned} \mathcal{L} \supset & -\frac{1}{2}\mu(\Delta^{--}H^+H^+ - \sqrt{2}\Delta^-H^0H^+ - \overline{\Delta^0}H^0H^0) + \text{H.c.} \\ & -\frac{1}{2}Y_\nu^{ij}(\Delta^0\nu_{iL}^TC\nu_{jL} - \sqrt{2}\Delta^+e_{iL}^TC\nu_{jL} - \Delta^{++}e_{iL}^TCe_{jL}) \\ & + \text{H.c.} \end{aligned} \quad (\text{A1})$$

where we have used the property that Y_ν^{ij} is symmetric. The Feynman diagrams for the interaction vertices are shown in Fig. 7, the corresponding vertex factors are

$$\begin{aligned} \Delta^{++} - H^+ - H^+ \text{ vertex: } & -i\mu \\ \Delta^+ - H^0 - H^+ \text{ vertex: } & i\frac{\sqrt{2}}{2}\mu \\ \Delta^0 - H^0 - H^0 \text{ vertex: } & i\mu \\ \Delta^{++} - e_i^+ - e_j^+ \text{ vertex: } & -iY_\nu^{ij}P_L \\ \Delta^+ - e_i^+ - \bar{\nu}_j \text{ vertex: } & i\frac{\sqrt{2}}{2}Y_\nu^{ij}P_L \\ \Delta^0 - \bar{\nu}_i - \bar{\nu}_j \text{ vertex: } & -iY_\nu^{ij}P_L \end{aligned} \quad (\text{A2})$$

where $P_L = (1 - \gamma^5)/2$ is the left-handed projection operator.

The graviton interacts with other particles through the following effective interaction Lagrangian

$$\mathcal{L}_{\text{int}} = -\frac{\kappa}{2}h_{\mu\nu}T^{\mu\nu} \quad (\text{A3})$$

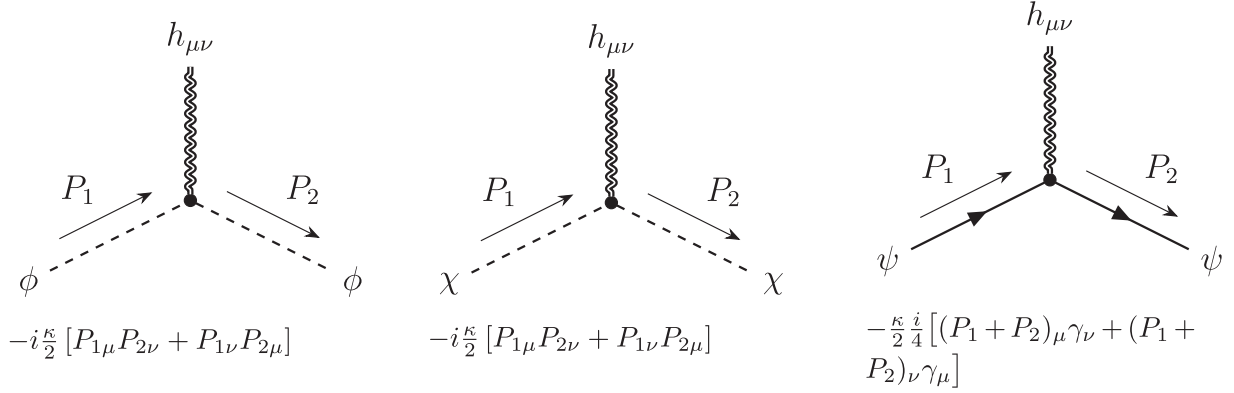


FIG. 8. Feynman rules for the graviton interaction with real scalar, complex scalar, and fermion fields.

where $h_{\mu\nu}$ is the graviton field, $\kappa = \sqrt{2}/M_P$ [85], and $T^{\mu\nu}$ is the energy-momentum tensor of the matter fields. For the scalar field ϕ , complex scalar field χ , and fermion field ψ , their energy-momentum tensors are given by [86]

$$\begin{aligned}
 T_{\mu\nu}^\phi &= \partial_\mu \phi \partial_\nu \phi - \eta_{\mu\nu} \mathcal{L}_\phi, \\
 T_{\mu\nu}^\chi &= \partial_\mu \chi^\dagger \partial_\nu \chi + \partial_\mu \chi \partial_\nu \chi^\dagger - \eta_{\mu\nu} \mathcal{L}_\chi, \\
 T_{\mu\nu}^\psi &= \frac{i}{4} \bar{\psi} (\gamma_\mu \overleftrightarrow{\partial}_\nu + \gamma_\nu \overleftrightarrow{\partial}_\mu) \psi - \eta_{\mu\nu} \mathcal{L}_\psi
 \end{aligned} \quad (\text{A4})$$

where $\mathcal{L}$ are the Lagrangians for the corresponding fields, and $\bar{\psi} \overleftrightarrow{\partial}_\mu \psi \equiv \bar{\psi} \partial_\mu \psi - (\partial_\mu \bar{\psi}) \psi$. The polarization tensor of the graviton $\varepsilon^{\mu\nu}$ satisfies the following conditions [87,88]:

$$\begin{aligned}
 \varepsilon^{i\mu\nu} &= \varepsilon^{i\nu\mu} \text{ symmetric} \\
 \omega_\mu \varepsilon^{i\mu\nu} &= 0 \text{ transverse} \\
 \eta_{\mu\nu} \varepsilon^{i\mu\nu} &= 0 \text{ traceless} \\
 \varepsilon^{i\mu\nu} \varepsilon_{\mu\nu}^{j*} &= \delta^{ij} \text{ orthonormal}
 \end{aligned} \quad (\text{A5})$$

where ω_μ is the four-momentum of the graviton. Due to the traceless condition, the nonzero interaction vertices are shown in Fig. 8.

The polarization sum of the graviton is given by

$$\sum_i \varepsilon_i^{\alpha\beta} \varepsilon_i^{*\mu\nu} = \frac{1}{2} (\hat{\eta}^{\alpha\mu} \hat{\eta}^{\beta\nu} + \hat{\eta}^{\alpha\nu} \hat{\eta}^{\beta\mu} - \hat{\eta}^{\mu\nu} \hat{\eta}^{\alpha\beta}) \quad (\text{A6})$$

with

$$\hat{\eta}^{\mu\nu} \equiv \eta^{\mu\nu} - \frac{\omega^\mu \bar{\omega}^\nu + \omega^\nu \bar{\omega}^\mu}{\omega \cdot \bar{\omega}} \quad (\text{A7})$$

where $\bar{\omega}^\mu \equiv (\omega, -\vec{\omega})$. We use ω to denote the energy of the graviton, $\vec{\omega}$ to denote its spatial momentum, ω^μ to denote its four-momentum and $\omega \cdot \bar{\omega}$ is the scalar product of two four-vectors.

APPENDIX B: COLLISION TERMS

In this section, we present the detailed derivation of the collision terms for generating GWs from the decay of Higgs triplets into a pair of leptons and a pair of Higgs bosons.

1. Collision term for $\Delta \rightarrow LLg$

The Higgs triplet has three components: a doubly charged scalar Δ^{++} , a singly charged scalar Δ^+ , and a neutral scalar Δ^0 . Each component can decay into a pair of leptons of any generation, accompanied by the emission of a graviton. The Hermitian conjugate of the Higgs triplet undergoes similar graviton-emission processes involving antiparticles, which give the same contribution as their particle counterparts. The collision term for generating GWs from the leptonic decay channels of the Higgs triplet involves all these processes, and is given by the sum of the collision terms for each individual process

$$\begin{aligned}
 \mathcal{C}_{\Delta \rightarrow LLg} &= 2 \left(\sum_{i \geq j} \mathcal{C}_{\Delta^{++} \rightarrow e_i^+ e_j^+ g} + \sum_{ij} \mathcal{C}_{\Delta^+ \rightarrow e_i^+ \bar{\nu}_j g} \right. \\
 &\quad \left. + \sum_{i \geq j} \mathcal{C}_{\Delta^0 \rightarrow \bar{\nu}_i \bar{\nu}_j g} \right) \quad (\text{B1})
 \end{aligned}$$

where the factor of 2 comes from the contributions of the antiparticles. The summations over i and j run over the three lepton generations. For processes symmetric under $i \leftrightarrow j$, we sum only over $i \geq j$ to avoid double counting.

We present a detailed calculation of the collision term for $\Delta^{++} \rightarrow e_i^+ e_j^+ g$, and the collision terms for the other two processes can be obtained analogously. The collision term for $\Delta^{++} \rightarrow e_i^+ e_j^+ g$ is given by

$$\begin{aligned}
 \mathcal{C}_{\Delta^{++} \rightarrow e_i^+ e_j^+ g} &= \int d\Pi_{\Delta^{++}} f_{\Delta^{++}} \frac{d\Pi_{e_i^+} d\Pi_{e_j^+}}{1 + \delta_{ij}} d\Pi_g (2\pi)^4 \delta^4 \\
 &\quad \times \left(\sum p_i - \sum p_f \right) |\mathcal{M}_{\Delta^{++} \rightarrow e_i^+ e_j^+ g}|^2 \omega. \quad (\text{B2})
 \end{aligned}$$

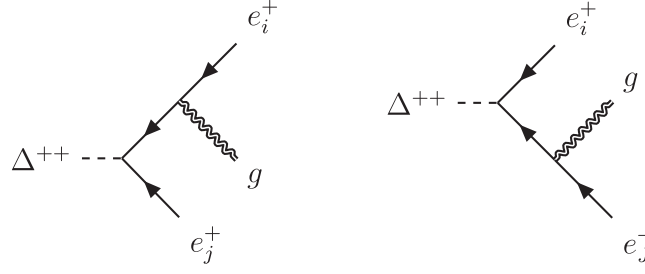


FIG. 9. Feynman diagrams for graviton bremsstrahlung in the decay of double charged Higgs triplets.

Here, $d\Pi = \frac{1}{(2\pi)^3 2E} d^3p$ is the Lorentz-invariant phase space volume element. The amplitude is the sum of the contributions from the two Feynman diagrams shown in Fig. 9, and is given by

$$i\mathcal{M}_{\Delta^{++} \rightarrow e_i^+ e_j^+ g} = \bar{v}(p_{e_i^+})(-iY_{\Delta}^{ij}P_L) \frac{i(p_{e_j^+} + \omega) \cdot \gamma}{(p_{e_j^+} + \omega)^2} \left(-\frac{\kappa}{2} i p_{e_j^+}^{\mu} \gamma^{\nu} \right) v(p_{e_j^+}) \epsilon_{\mu\nu}^* \\ + \bar{v}(p_{e_i^+}) \left(-\frac{\kappa}{2} i p_{e_i^+}^{\mu} \gamma^{\nu} \right) \frac{i(p_{e_j^+} + \omega) \cdot \gamma}{(p_{e_j^+} + \omega)^2} (-iY_{\Delta}^{ij}P_L) v(p_{e_j^+}) \epsilon_{\mu\nu}^*. \quad (\text{B3})$$

Here, the indices i and j are not summed. The Feynman diagram for emitting a graviton from the Δ^{++} line gives a vanishing contribution for the amplitude, because only moving particles can emit gravitons, and we choose the rest frame of Δ^{++} to calculate the Lorentz-invariant amplitude.

Squaring the amplitude and summing over the polarizations of the graviton and spins of the leptons, we have

$$|\mathcal{M}_{\Delta^{++} \rightarrow e_i^+ e_j^+ g}|^2 = \frac{\kappa^2}{8} |Y_{\Delta}^{ij}|^2 M_{\Delta}^2 \left(1 - 2 \frac{\omega}{M_{\Delta}} \right) \left(2 - 2 \frac{M_{\Delta}}{\omega} + \frac{M_{\Delta}^2}{\omega^2} \right). \quad (\text{B4})$$

The calculation of the amplitude was performed with the help of the package FeynCalc [89–92].

The integration over the phase space of final states gives

$$\int d\Pi_{e_i^+} d\Pi_{e_j^+} d\Pi_g (2\pi)^4 \delta^4(p_{\Delta^{++}} - p_{e_i^+} - p_{e_j^+} - p_g) = \int d\omega dE_{e_i^+} \frac{1}{32\pi^3} \quad (\text{B5})$$

The distribution function of the Higgs triplet in thermal equilibrium is approximated by the Maxwell-Boltzmann distribution, i.e., $f_{\Delta^{++}} = e^{-E_{\Delta^{++}}/T}$. The phase space integration over Δ^{++} can be performed as

$$\int d\Pi_{\Delta^{++}} f_{\Delta^{++}} = \frac{1}{2M_{\Delta}} \frac{n_{\Delta}}{g_{\Delta}} \frac{K_1(M_{\Delta}/T)}{K_2(M_{\Delta}/T)}, \quad (\text{B6})$$

where n_{Δ} is the number density of the Higgs triplet, $g_{\Delta} = 6$ is the internal degrees of freedom of the Higgs triplet, K_1 and K_2 are the modified Bessel functions of the second kind, and we have assumed that the Higgs triplet remains in kinetic equilibrium while it decays. With these ingredients, the collision term for $\Delta^{++} \rightarrow e_i^+ e_j^+ g$ can be written as

$$\mathcal{C}_{\Delta^{++} \rightarrow e_i^+ e_j^+ g} = \frac{\kappa^2 M_{\Delta}}{512\pi^3} \frac{|Y_{\Delta}^{ij}|^2}{1 + \delta_{ij}} \int d\omega dE_{e_i^+} \frac{n_{\Delta}}{g_{\Delta}} \frac{K_1(M_{\Delta}/T)}{K_2(M_{\Delta}/T)} \left(1 - 2 \frac{\omega}{M_{\Delta}} \right) \left(2 - 2 \frac{M_{\Delta}}{\omega} + \frac{M_{\Delta}^2}{\omega^2} \right) \omega \\ = \frac{\kappa^2 M_{\Delta}}{512\pi^3} \frac{|Y_{\Delta}^{ij}|^2}{1 + \delta_{ij}} \int d\omega \frac{n_{\Delta}}{g_{\Delta}} \frac{K_1(M_{\Delta}/T)}{K_2(M_{\Delta}/T)} \left(1 - 2 \frac{\omega}{M_{\Delta}} \right) \left(2 - 2 \frac{M_{\Delta}}{\omega} + \frac{M_{\Delta}^2}{\omega^2} \right) \omega^2. \quad (\text{B7})$$

In the second step, we integrated over $E_{e_i^+}$ from $M_{\Delta}/2 - \omega$ to $M_{\Delta}/2$.

Similarly, we can obtain the collision terms for the other two processes

$$\mathcal{C}_{\Delta^+ \rightarrow e^+ \bar{\nu}_j g} = \frac{\kappa^2 M_\Delta}{1024 \pi^3} |Y_\Delta^{ij}|^2 \int d\omega \frac{n_\Delta}{g_\Delta} \frac{K_1(M_\Delta/T)}{K_2(M_\Delta/T)} \frac{\kappa^2}{16} \times \left(1 - 2 \frac{\omega}{M_\Delta}\right) \left(2 - 2 \frac{M_\Delta}{\omega} + \frac{M_\Delta^2}{\omega^2}\right) \omega^2, \quad (\text{B8})$$

and

$$\mathcal{C}_{\Delta^0 \rightarrow \bar{\nu}_i \nu_j g} = \frac{\kappa^2 M_\Delta}{512 \pi^3} \frac{|Y_\Delta^{ij}|^2}{1 + \delta_{ij}} \int d\omega \frac{n_\Delta}{g_\Delta} \frac{K_1(M_\Delta/T)}{K_2(M_\Delta/T)} \times \left(1 - 2 \frac{\omega}{M_\Delta}\right) \left(2 - 2 \frac{M_\Delta}{\omega} + \frac{M_\Delta^2}{\omega^2}\right) \omega^2. \quad (\text{B9})$$

Summing over all the generations of leptons, we obtain the total collision term for $\Delta \rightarrow LLg$

$$\mathcal{C}_{\Delta \rightarrow LLg} = \frac{M_\Delta \kappa^2}{1024 \pi^3} \sum_{i,j} |Y_\Delta^{ij}|^2 n_\Delta \frac{K_1(M_\Delta/T)}{K_2(M_\Delta/T)} \times \int d\omega \left(1 - 2 \frac{\omega}{M_\Delta}\right) \left(2 - 2 \frac{M_\Delta}{\omega} + \frac{M_\Delta^2}{\omega^2}\right) \omega^2. \quad (\text{B10})$$

2. Collision term for $\Delta \rightarrow HHg$

Similarly, the collision term for generating GWs from the Higgs decay channels is given by

$$\mathcal{C}_{\Delta \rightarrow HHg} = 2(\mathcal{C}_{\Delta^{++} \rightarrow H^+ H^+ g} + \mathcal{C}_{\Delta^+ \rightarrow H^0 H^+ g} + \mathcal{C}_{\Delta^0 \rightarrow H^0 H^0 g}) \quad (\text{B11})$$

where the factor of 2 comes from the contributions of the antiparticles. In fact, the collision terms for these three processes are the same, we present a detailed calculation of the collision term for $\Delta^{++} \rightarrow H^+ H^+ g$, which is given by

$$\mathcal{C}_{\Delta^{++} \rightarrow H^+ H^+ g} = \frac{1}{2} \int d\Pi_{\Delta^{++}} f_{\Delta^{++}} d\Pi_{H_1^+} d\Pi_{H_2^+} d\Pi_g (2\pi)^4 \delta^4 \times \left(\sum p_i - \sum p_f\right) |\mathcal{M}_{\Delta^{++} \rightarrow H^+ H^+ g}|^2 \omega \quad (\text{B12})$$

Here, the subscripts 1 and 2 are used to distinguish the phase space variables of the two identical Higgs bosons in the final state, and the factor 1/2 arises from the identical particles in the final states. The amplitude is the sum of the contributions from the two Feynman diagrams shown in

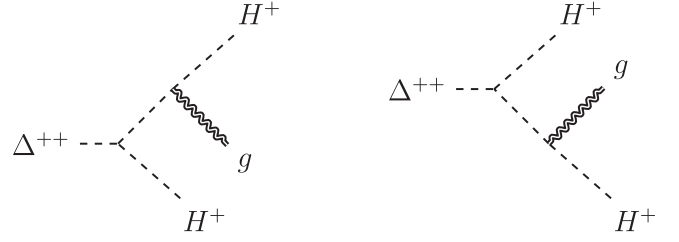


FIG. 10. Feynman diagrams for graviton bremsstrahlung in the decay of doubly charged Higgs triplets into a pair of singly charged Higgs bosons.

Fig. 10, and is given by

$$i\mathcal{M}_{\Delta^{++} \rightarrow H^+ H^+ g} = (-i\mu) \frac{i}{(p_{H_2^+} + \omega)^2} \left(-\frac{\kappa}{2} i 2 p_{H_2^+}^\mu p_{H_2^+}^\nu\right) \epsilon_{\mu\nu}^* + (-i\mu) \frac{i}{(p_{H_1^+} + \omega)^2} \left(-\frac{\kappa}{2} i 2 p_{H_1^+}^\mu p_{H_1^+}^\nu\right) \epsilon_{\mu\nu}^* \quad (\text{B13})$$

Squaring the amplitude and summing over the polarizations of the graviton, we have

$$|\mathcal{M}_{\Delta^{++} \rightarrow H^+ H^+ g}|^2 = \frac{\kappa^2}{8} \mu^2 \left(2 - \frac{M_\Delta}{\omega}\right)^2 \quad (\text{B14})$$

Therefore, the collision term for $\Delta^{++} \rightarrow H^+ H^+ g$ can be written as

$$\mathcal{C}_{\Delta^{++} \rightarrow H^+ H^+ g} = \frac{1}{2} \frac{\kappa^2}{8} \mu^2 \frac{1}{32 \pi^3} \frac{n_\Delta}{2 M_\Delta} \frac{K_1(M_\Delta/T)}{K_2(M_\Delta/T)} \times \int d\omega \left(2 - \frac{M_\Delta}{\omega}\right)^2 \omega^2 \quad (\text{B15})$$

There is no a identical particles factor 1/2 in the collision terms for $\Delta^+ \rightarrow H^0 H^+ g$, but its squared amplitude is half of that for $\Delta^{++} \rightarrow H^+ H^+ g$, so the collision term for $\Delta^+ \rightarrow H^0 H^+ g$ is the same as that for $\Delta^{++} \rightarrow H^+ H^+ g$, and same as the collision term for $\Delta^0 \rightarrow H^0 H^0 g$. Thus, the total collision term for $\Delta \rightarrow HHg$ is given by

$$\mathcal{C}_{\Delta \rightarrow HHg} = 6\mathcal{C}_{\Delta^{++} \rightarrow H^+ H^+ g} = \frac{1}{1024 \pi^3} \kappa^2 \mu^2 \frac{n_\Delta}{M_\Delta} \frac{K_1(M_\Delta/T)}{K_2(M_\Delta/T)} \times \int d\omega \left(2 - \frac{M_\Delta}{\omega}\right)^2 \omega^2 \quad (\text{B16})$$

- [1] P. Minkowski, $\mu \rightarrow e\gamma$ at a rate of one out of 10^9 muon decays?, *Phys. Lett.* **67B**, 421 (1977).
- [2] T. Yanagida, Horizontal gauge symmetry and masses of neutrinos, *Conf. Proc. C* **7902131**, 95 (1979).
- [3] M. Gell-Mann, P. Ramond, and R. Slansky, Complex spinors and unified theories, *Conf. Proc. C* **790927**, 315 (1979).
- [4] S. L. Glashow, The future of elementary particle physics, *NATO Sci. Ser. B* **61**, 687 (1980).
- [5] R. N. Mohapatra and G. Senjanovic, Neutrino mass and spontaneous parity nonconservation, *Phys. Rev. Lett.* **44**, 912 (1980).
- [6] M. Fukugita and T. Yanagida, Baryogenesis without grand unification, *Phys. Lett. B* **174**, 45 (1986).
- [7] Y. Cai, J. Herrero-García, M. A. Schmidt, A. Vicente, and R. R. Volkas, From the trees to the forest: A review of radiative neutrino mass models, *Front. Phys.* **5**, 63 (2017).
- [8] A. Zee, A theory of lepton number violation, neutrino Majorana mass, and oscillation, *Phys. Lett.* **93B**, 389 (1980); **95B**, 461(E) (1980).
- [9] K. S. Babu, Model of “calculable” Majorana neutrino masses, *Phys. Lett. B* **203**, 132 (1988).
- [10] E. Ma, Verifiable radiative seesaw mechanism of neutrino mass and dark matter, *Phys. Rev. D* **73**, 077301 (2006).
- [11] W. Chao, Neutrino masses and a TeV scale seesaw mechanism, *Phys. Rev. D* **82**, 016008 (2010).
- [12] M. Lindner, M. Platscher, and F. S. Queiroz, A call for new physics: The muon anomalous magnetic moment and lepton flavor violation, *Phys. Rep.* **731**, 1 (2018).
- [13] W. Rodejohann, Neutrino-less double beta decay and particle physics, *Int. J. Mod. Phys. E* **20**, 1833 (2011).
- [14] Y. Cui and Z.-Z. Xianyu, Probing leptogenesis with the cosmological collider, *Phys. Rev. Lett.* **129**, 111301 (2022).
- [15] B. P. Abbott *et al.*, Observation of gravitational waves from a binary black hole merger, *Phys. Rev. Lett.* **116**, 061102 (2016).
- [16] B. Barman, N. Bernal, Y. Xu, and Ó. Zapata, Gravitational wave from graviton bremsstrahlung during reheating, *J. Cosmol. Astropart. Phys.* **05** (2023) 019.
- [17] X.-J. Xu, Y. Xu, Q. Yin, and J. Zhu, Full-spectrum analysis of gravitational wave production from inflation to reheating, *J. High Energy Phys.* **10** (2025) 141.
- [18] D. Huang and L. Yin, Stochastic gravitational waves from inflaton decays, *Phys. Rev. D* **100**, 043538 (2019).
- [19] B. Barman, N. Bernal, Y. Xu, and Ó. Zapata, Bremsstrahlung-induced gravitational waves in monomial potentials during reheating, *Phys. Rev. D* **108**, 083524 (2023).
- [20] N. Bernal, S. Cléry, Y. Mambrini, and Y. Xu, Probing reheating with graviton bremsstrahlung, *J. Cosmol. Astropart. Phys.* **01** (2024) 065.
- [21] Y. Xu, Ultra-high frequency gravitational waves from scattering, Bremsstrahlung and decay during reheating, *J. High Energy Phys.* **10** (2024) 174.
- [22] Y. Xu, Gravitational wave from graviton Bremsstrahlung during reheating, *Proc. Sci. CORFU2023* (2024) 047.
- [23] N. Bernal and Y. Xu, Thermal gravitational waves during reheating, *J. High Energy Phys.* **01** (2025) 137.
- [24] N. Bernal, Q.-f. Wu, X.-J. Xu, and Y. Xu, Pre-thermalized gravitational waves, *J. High Energy Phys.* **08** (2025) 125.
- [25] A. Tokareva, Gravitational waves from inflaton decay and bremsstrahlung, *Phys. Lett. B* **853**, 138695 (2024).
- [26] S. Kanemura and K. Kaneta, Gravitational waves from particle decays during reheating, *Phys. Lett. B* **855**, 138807 (2024).
- [27] G. Montefalcone, B. Shams Es Haghi, T. Xu, and K. Freese, Thermal gravitons from warm inflation, *Phys. Rev. D* **112**, 063556 (2025).
- [28] Y. Ema, R. Jinno, and K. Nakayama, High-frequency graviton from inflaton oscillation, *J. Cosmol. Astropart. Phys.* **09** (2020) 015.
- [29] P. Klose, M. Laine, and S. Procacci, Gravitational wave background from non-Abelian reheating after axion-like inflation, *J. Cosmol. Astropart. Phys.* **05** (2022) 021.
- [30] P. Klose, M. Laine, and S. Procacci, Gravitational wave background from vacuum and thermal fluctuations during axion-like inflation, *J. Cosmol. Astropart. Phys.* **12** (2022) 020.
- [31] H. An, K.-F. Lyu, L.-T. Wang, and S. Zhou, Gravitational waves from an inflation triggered first-order phase transition, *J. High Energy Phys.* **06** (2022) 050.
- [32] X.-H. Hu and Y.-L. Zhou, Gravitational waves of GUT phase transition during inflation, *Phys. Rev. D* **111**, 115003 (2025).
- [33] W. Chao, W.-F. Cui, H.-K. Guo, and J. Shu, Gravitational wave imprint of new symmetry breaking, *Chin. Phys. C* **44**, 123102 (2020).
- [34] W. Chao, H.-K. Guo, and J. Shu, Gravitational wave signals of electroweak phase transition triggered by dark matter, *J. Cosmol. Astropart. Phys.* **09** (2017) 009.
- [35] W. Chao, J.-j. Feng, H.-k. Guo, and T. Li, Oscillations of ultralight dark photon into gravitational waves, *Nucl. Phys. B* **1009**, 116740 (2024).
- [36] Y. Wang and W. Chao, Gravitational wave spectrum from the production of dark matter via the freeze-in mechanism, *arXiv:2508.10665*.
- [37] P. Konar and S. Show, Unraveling freeze-in dark matter through the echoes of gravitational waves, *arXiv:2506.08106*.
- [38] C. Caprini, R. Durrer, and G. Servant, The stochastic gravitational wave background from turbulence and magnetic fields generated by a first-order phase transition, *J. Cosmol. Astropart. Phys.* **12** (2009) 024.
- [39] M. Hindmarsh, S. J. Huber, K. Rummukainen, and D. J. Weir, Shape of the acoustic gravitational wave power spectrum from a first order phase transition, *Phys. Rev. D* **96**, 103520 (2017); **101**, 089902(E) (2020).
- [40] P. Athron, C. Balázs, A. Fowlie, L. Morris, and L. Wu, Cosmological phase transitions: From perturbative particle physics to gravitational waves, *Prog. Part. Nucl. Phys.* **135**, 104094 (2024).
- [41] W. Chao, H.-K. Guo, and X.-F. Li, First order color symmetry breaking and restoration triggered by electroweak symmetry non-restoration, *Phys. Lett. B* **849**, 138430 (2024).
- [42] J. Ghiglieri and M. Laine, Gravitational wave background from standard model physics: Qualitative features, *J. Cosmol. Astropart. Phys.* **07** (2015) 022.
- [43] A. Ringwald and C. Tamarit, Revealing the cosmic history with gravitational waves, *Phys. Rev. D* **106**, 063027 (2022).

- [44] J. Ghiglieri, J. Schütte-Engel, and E. Speranza, Freezing-in gravitational waves, *Phys. Rev. D* **109**, 023538 (2024).
- [45] J. Ghiglieri, M. Laine, J. Schütte-Engel, and E. Speranza, Double-graviton production from standard model plasma, *J. Cosmol. Astropart. Phys.* **04** (2024) 062.
- [46] A. Ringwald, J. Schütte-Engel, and C. Tamarit, Gravitational waves as a big bang thermometer, *J. Cosmol. Astropart. Phys.* **03** (2021) 054.
- [47] A. Datta and A. Sil, Probing leptogenesis through gravitational waves, [arXiv:2410.01900](#).
- [48] K.-Y. Choi, E. Lkhagvadorj, and S. Mahapatra, Cosmological origin of the KM3-230213A event and associated gravitational waves, *J. Cosmol. Astropart. Phys.* **10** (2025) 079.
- [49] H. Murayama, B. Noether, and J. Schütte-Engel, Observing leptogenesis in action with gravitational waves, *J. Cosmol. Astropart. Phys.* **12** (2025) 027.
- [50] S. Kanemura, K. Kaneta, and D. Nanda, Gravitational waves from supermassive right-handed neutrinos produced at preheating, *Phys. Rev. D* **113**, 055046 (2026).
- [51] N. Herman, A. Füzfa, L. Lehoucq, and S. Clesse, Detecting planetary-mass primordial black holes with resonant electromagnetic gravitational-wave detectors, *Phys. Rev. D* **104**, 023524 (2021).
- [52] N. Herman, L. Lehoucq, and A. Füzfa, Electromagnetic antennas for the resonant detection of the stochastic gravitational wave background, *Phys. Rev. D* **108**, 124009 (2023).
- [53] P. Fileviez Perez, T. Han, G.-y. Huang, T. Li, and K. Wang, Neutrino masses and the CERN LHC: Testing type II seesaw, *Phys. Rev. D* **78**, 015018 (2008).
- [54] S. Navas *et al.*, Review of particle physics, *Phys. Rev. D* **110**, 030001 (2024).
- [55] B. Pontecorvo, Mesonium and antimesonium, *Sov. Phys. JETP* **6**, 429 (1958).
- [56] Z. Maki, M. Nakagawa, and S. Sakata, Remarks on the unified model of elementary particles, *Prog. Theor. Phys.* **28**, 870 (1962).
- [57] B. Pontecorvo, Neutrino experiments and the problem of conservation of leptonic charge, *Zh. Eksp. Teor. Fiz.* **53**, 1717 (1967).
- [58] T. Li, C.-Y. Yao, and M. Yuan, Revealing the origin of neutrino masses through the type II seesaw mechanism at high-energy muon colliders, *J. High Energy Phys.* **03** (2023) 137.
- [59] S. Ansarifard and Y. Farzan, Revisiting pseudo-Dirac neutrino scenario after recent solar neutrino data, *Phys. Rev. D* **107**, 075029 (2023).
- [60] I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, T. Schwetz, and A. Zhou, The fate of hints: Updated global analysis of three-flavor neutrino oscillations, *J. High Energy Phys.* **09** (2020) 178.
- [61] M. M. Ivanov, J. M. Sullivan, S.-F. Chen, A. Chudaykin, M. Maus, and O. H. E. Philcox, Reanalyzing DESI DR1: 4. Percent-level cosmological constraints from combined probes and robust evidence for the normal neutrino mass hierarchy, [arXiv:2601.16165](#).
- [62] A. G. Adame *et al.*, DESI 2024 VI: Cosmological constraints from the measurements of baryon acoustic oscillations, *J. Cosmol. Astropart. Phys.* **02** (2025) 021.
- [63] J.-Q. Jiang, W. Giarè, S. Gariazzo, M. G. Dainotti, E. Di Valentino, O. Mena, D. Pedrotti, S. S. da Costa, and S. Vagnozzi, Neutrino cosmology after DESI: Tightest mass upper limits, preference for the normal ordering, and tension with terrestrial observations, *J. Cosmol. Astropart. Phys.* **01** (2025) 153.
- [64] P. A. Zyla *et al.*, Review of particle physics, *Prog. Theor. Exp. Phys.* **2020**, 083C01 (2020).
- [65] R. Zhou, L. Bian, and Y. Du, Electroweak phase transition and gravitational waves in the type-II seesaw model, *J. High Energy Phys.* **08** (2022) 205.
- [66] N. D. Barrie, C. Han, and H. Murayama, Type II seesaw leptogenesis, *J. High Energy Phys.* **05** (2022) 160.
- [67] E. J. Chun, K. Y. Lee, and S. C. Park, Testing Higgs triplet model and neutrino mass patterns, *Phys. Lett. B* **566**, 142 (2003).
- [68] D. Hooper, *Particle Cosmology and Astrophysics* (Princeton University Press, Princeton, NJ, 2024).
- [69] E. W. Kolb and M. S. Turner, *The Early Universe* (Taylor and Francis, London, 2019), Vol. 69, [10.1201/9780429492860](#).
- [70] C. Caprini *et al.*, Science with the space-based interferometer eLISA. II: Gravitational waves from cosmological phase transitions, *J. Cosmol. Astropart. Phys.* **04** (2016) 001.
- [71] C. Caprini *et al.*, Detecting gravitational waves from cosmological phase transitions with LISA: An update, *J. Cosmol. Astropart. Phys.* **03** (2020) 024.
- [72] M. B. Hindmarsh, M. Lüben, J. Lumma, and M. Pauly, Phase transitions in the early universe, *SciPost Phys. Lect. Notes* **24**, 1 (2021).
- [73] Y. Gouttenoire, *Beyond the Standard Model Cocktail*, Springer Theses (Springer, Cham, 2022), [10.1007/978-3-031-11862-3](#).
- [74] G. Servant and P. Simakachorn, Ultrahigh frequency primordial gravitational waves beyond the kHz: The case of cosmic strings, *Phys. Rev. D* **109**, 103538 (2024).
- [75] S. Weinberg, Infrared photons and gravitons, *Phys. Rev.* **140**, B516 (1965).
- [76] B. R. Holstein, Graviton physics, *Am. J. Phys.* **74**, 1002 (2006).
- [77] A. Berlin, D. Blas, R. T. D'Agnolo, S. A. Ellis, R. Harnik, Y. Kahn, and J. Schütte-Engel, Detecting high-frequency gravitational waves with microwave cavities, *Phys. Rev. D* **105**, 116011 (2022).
- [78] A. Berlin, D. Blas, R. T. d'Agnolo, S. A. Ellis, R. Harnik, Y. Kahn, J. Schütte-Engel, and M. Wentzel, Electromagnetic cavities as mechanical bars for gravitational waves, *Phys. Rev. D* **108**, 084058 (2023).
- [79] R. Capdevilla, G. B. Gelmini, J. Hyman, A. J. Millar, and E. Vitagliano, Gravitational wave detection with plasma haloscopes, *Phys. Rev. D* **112**, 055011 (2025).
- [80] J. Chluba *et al.*, New horizons in cosmology with spectral distortions of the cosmic microwave background, *Exp. Astron.* **51**, 1515 (2021).
- [81] A. Kogut, D. Fixsen, D. Chuss, J. Dotson, E. Dwek, M. Halpern, G. Hinshaw, S. Meyer, S. Moseley, M. Seiffert *et al.*, The primordial inflation explorer (pixie): A nulling polarimeter for cosmic microwave background observations, *J. Cosmol. Astropart. Phys.* **07** (2011) 025.

- [82] A. Kogut, M. H. Abitbol, J. Chluba, J. Delabrouille, D. Fixsen, J. C. Hill, S. P. Patil, and A. Rotti, CMB spectral distortions: Status and prospects, *Bull. Am. Astron. Soc.* **51**, 113 (2019).
- [83] K. Basu *et al.*, A space mission to map the entire observable universe using the CMB as a backlight: Voyage 2050 science white paper, *Exp. Astron.* **51**, 1555 (2021).
- [84] E. Ma and U. Sarkar, Neutrino masses and leptogenesis with heavy Higgs triplets, *Phys. Rev. Lett.* **80**, 5716 (1998).
- [85] K. Nakayama and Y. Tang, Stochastic gravitational waves from particle origin, *Phys. Lett. B* **788**, 341 (2019); **839**, 137787(E) (2023).
- [86] P. de Aquino, K. Hagiwara, Q. Li, and F. Maltoni, Simulating graviton production at hadron colliders, *J. High Energy Phys.* **06** (2011) 132.
- [87] D. J. Gross and R. Jackiw, Low-energy theorem for graviton scattering, *Phys. Rev.* **166**, 1287 (1968).
- [88] T. Gleisberg, F. Krauss, K. T. Matchev, A. Schlicke, S. Schumann, and G. Soff, Helicity formalism for spin-2 particles, *J. High Energy Phys.* **09** (2003) 001.
- [89] R. Mertig, M. Böhm, and A. Denner, Feyn calc-computer-algebraic calculation of feynman amplitudes, *Comput. Phys. Commun.* **64**, 345 (1991).
- [90] V. Shtabovenko, R. Mertig, and F. Orellana, New developments in FeynCalc 9.0, *Comput. Phys. Commun.* **207**, 432 (2016).
- [91] V. Shtabovenko, R. Mertig, and F. Orellana, FeynCalc 9.3: New features and improvements, *Comput. Phys. Commun.* **256**, 107478 (2020).
- [92] V. Shtabovenko, R. Mertig, and F. Orellana, FeynCalc 10: Do multiloop integrals dream of computer codes?, *Comput. Phys. Commun.* **306**, 109357 (2025).