

Exploring the spectroscopic features of double-strangeness tetraquark states

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Since the discovery of the T_{cc} double-charm tetraquark by LHCb collaboration, the field of theoretical research on heavy quarks has advanced rapidly, with increasing interest in exploring the light quark sector. In this work, the quark delocalization color screening model, together with the resonating group method, is employed to systematically investigate the double-strangeness tetraquark system. Both meson-meson and diquark-antidiquark configurations are considered. The interactions between hadron pairs with different quantum numbers are studied, and possible bound-state and resonancelike solutions are examined. In the single-channel estimations, two weakly bound solutions, $\bar{K}^* \bar{K}$ and $\bar{K}^* \bar{K}^*$, are obtained in the $I(J^P) = 0(1^+)$ sector. After channel coupling between the two configurations is included, a deeply bound coupled-channel solution with the same quantum numbers and a mass of approximately 1310 MeV is found. In addition, the real-scaling stabilization analysis indicates a resonancelike structure in the $I(J^P) = 0(1^+) ss\bar{n}\bar{n}$ system, with an estimated mass of around 1783 MeV and a width of approximately 5.6 MeV.

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I. INTRODUCTION

Quantum chromodynamics (QCD) is the foundational theory describing the strong interactions between quarks and gluons. In the traditional quark model, hadrons consist of mesons, which are composed of a quark-antiquark pair, and baryons, which are composed of three quarks [1–3]. However, according to QCD theory, there exist hadronic states that go beyond the traditional quark model, such as multiquark states, hadronic molecules, glueballs, and hybrid states [4–11].

Since 2003, the experimental observation of several exotic hadronic states that lie beyond the traditional quark model has significantly revitalized interest in the field of exotic hadron research. These states not only expand the diversity of the hadron spectrum but also provide a valuable opportunity to probe the nonperturbative dynamics of the

strong interaction through detailed analyses of their properties and structures. To date, the majority of observed exotic states exhibit heavy flavor quantum numbers. For instance, many charmoniumlike states, referred to as XYZ states, possess hidden charm components [9]. Specific examples, such as $X(3872)$ [12–18] and $Z_c(3900)$ [19,20], are situated near two-hadron thresholds, yet their precise internal structures remain under debate [10,21–27]. A significant recent finding is the identification by LHCb collaboration of the double-charm $T_{cc}^+(3875)$ state in the $D^0 D^0 \pi^+$ invariant mass spectrum [28]. Further analysis of $T_{cc}^+(3875)$ state reveals its quark content of $cc\bar{u}\bar{d}$, identifying it as a doubly charmed tetraquark state. Unlike hidden-charm tetraquark states [such as $X(3872)$], $T_{cc}^+(3875)$ state does not contain a $c\bar{c}$ pair, and thus quark-antiquark annihilation effects are absent. This observation provides strong evidence supporting the existence of double-charm tetraquark states. Theoretically, there are multiple interpretations of $T_{cc}^+(3875)$ state, including it being the compact tetraquark [29–38], the hadronic molecule [39–45], the virtual state [46–48], or the Efimov state [49], etc.

From the experimental observations, most exotic states contain heavy flavor quarks, while research on the light flavor quark system is comparatively sparse in both experimental and theoretical aspects. Given this, if quark mass differences under an appropriate approximation are neglected, the discovery of $T_{cc}^+(3875)$ state raises the question of whether similar double-strange tetraquark states

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could also exist. This intriguing possibility deserves further exploration. These double-strange tetraquark states are of comparable importance to the double-charm tetraquark $T_{cc}^+(3875)$, as they offer essential insights into the formation mechanisms of tetraquark states in the light quark sector.

Although theoretical studies on double-strange tetraquark states are more restricted at present, there have been some preliminary advances. For example, in Ref. [50], the spectral properties of the $\bar{K}^*K^*$ system have been investigated by means of the one boson-exchange model, and it is suggested that $\bar{K}\bar{K}^*$ and $\bar{K}^*\bar{K}^*$ with $I(J^P) = 0(1^+)$ may be candidates of the double-strange tetraquark state. Similar double-strange tetraquark states have also been explored in Ref. [51]. Additionally, Refs. [52,53] utilized lattice QCD to calculate the scattering phase shifts and effective potential for the $I = 1$ system, finding the effective interaction to be repulsive. The mutual exclusion of $\bar{K}\bar{K}$ had also been verified in the investigation of systems $\bar{K}\bar{K}N$ [54,55] and $\bar{K}\bar{K}NN$ [56,57].

This paper aims to systematically examine the spectral properties of double-strange tetraquark systems. Utilizing the quark delocalization color screening model (QDCSM), we perform dynamical calculations to evaluate the effective interactions within these systems and solve the Schrödinger equation to identify prospective double-strange tetraquark states. Moreover, we apply the real-scaling method to predict resonance states within these systems. It is expected that the results of this study will provide valuable guidance for experimental efforts directed toward the discovery of double-strange tetraquark states, thereby advancing the broader investigation of tetraquark states in the light quark sector.

The paper is arranged as follows. In the next section, the theoretical framework of the QDCSM and the resonating group method are briefly introduced. The calculated results are listed in Sec. III, where some discussions are provided as well. Finally, the summary is given in Sec. IV.

II. QUARK DELOCALIZATION COLOR SCREENING MODEL AND THE RESONATING GROUP METHOD

A. Quark delocalization color screening model

The QDCSM is an extension of the naive quark cluster model [58–61] and was developed to address multi-quark systems. For the tetraquark system, the Hamiltonian reads

$$H = \sum_{i=1}^4 \left(m_i + \frac{\mathbf{p}_i^2}{2m_i} \right) - T_{\text{CM}} + \sum_{j>i=1}^4 V(r_{ij}), \quad (1)$$

where T_{CM} is the center-of-mass kinetic energy, which is usually subtracted without loss of generality because we focus on the internal relative motions of the multi-quark system. The interaction term is the sum of two-body

potentials, including the color-confining term V_{CON} , the one gluon-exchange potential V_{OGE} , and the Goldstone boson-exchange potential V_χ , i.e.,

$$V(r_{ij}) = V_{\text{CON}}(r_{ij}) + V_{\text{OGE}}(r_{ij}) + V_\chi(r_{ij}). \quad (2)$$

In the present work, we focus on the S -wave low-lying positive $ss\bar{q}\bar{q}$ tetraquark system with positive parity where q represents u or d . In this case, the spin-orbit and tensor interactions vanish and the potential $V_{\text{OGE}}(r_{ij})$ becomes

$$V_{\text{OGE}}(r_{ij}) = \frac{1}{4} \alpha_s^{q_i q_j} \lambda_i^c \cdot \lambda_j^c \times \left[\frac{1}{r_{ij}} - \frac{\pi}{2} \delta(r_{ij}) \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4\sigma_i \cdot \sigma_j}{3m_i m_j} \right) \right], \quad (3)$$

where m_i is the quark mass, while σ_i and λ_i^c are the Pauli matrices and SU(3) color matrices, respectively. The $\alpha_s^{q_i q_j}$ is the quark-gluon coupling constant, which offers a consistent description of mesons from the light to heavy quark sector. The values of α_{ij} are associated with the quark flavors and in the present work they are fixed by reproducing the mass difference of the low-lying mesons with $S = 0$ and $S = 1$.

The confining potential $V_{\text{CON}}(r_{ij})$ is

$$V_{\text{CON}}(r_{ij}) = -a_c \lambda_i^c \cdot \lambda_j^c \left[f(r_{ij}) + V_{0_{q_i q_j}} \right], \quad (4)$$

where the $V_{0_{q_i q_j}}$ is determined by the mass shift of the theoretical estimations and experimental measurement of each kind of meson, which is also a quark flavor-related parameter. In the QDCSM, the function $f(r_{ij})$ is

$$f(r_{ij}) = \begin{cases} r_{ij}^2 & \text{if } i, j \text{ occur in the same cluster,} \\ \frac{1 - e^{-\mu_{ij} r_{ij}^2}}{\mu_{ij}} & \text{if } i, j \text{ occur in different cluster,} \end{cases} \quad (5)$$

where the color screening parameter μ_{ij} relevant to the light quarks can be determined by fitting the deuteron properties, NN and NY scattering phase shifts [62–64].

The Goldstone boson-exchange interactions between light quarks appear because of the dynamical breaking of chiral symmetry. For the $ss\bar{n}\bar{n}$ system, the concrete form of the Goldstone boson-exchange potential becomes

$$V_{ij}^\chi = V_\pi(\mathbf{r}_{ij}) \sum_{a=1}^3 \lambda_i^a \cdot \lambda_j^a + V_K(\mathbf{r}_{ij}) \sum_{a=4}^7 \lambda_i^a \cdot \lambda_j^a + V_\eta(\mathbf{r}_{ij}) \left[(\lambda_i^8 \cdot \lambda_j^8) \cos \theta_P - (\lambda_i^0 \cdot \lambda_j^0) \sin \theta_P \right], \quad (6)$$

with

$$V_\chi(\mathbf{r}_{ij}) = \frac{g_{\text{ch}}^2}{4\pi} \frac{m_\chi^2}{12m_i m_j} \frac{\Lambda_\chi^2}{\Lambda_\chi^2 - m_\chi^2} m_\chi \times \left\{ (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \left[Y(m_\chi r_{ij}) - \frac{\Lambda_\chi^3}{m_\chi^3} Y(\Lambda_\chi r_{ij}) \right] \right\},$$

$$\chi = \{\pi, K, \eta\}, \quad (7)$$

where $Y(x) = e^{-x}/x$ is the standard Yukawa function. The λ^a is the SU(3) flavor Gell-Mann matrix. The masses of the π , K , and η mesons are taken from the experimental value [3], respectively. The chiral coupling constant, g_{ch} , is determined from the πNN coupling constant through

$$\frac{g_{\text{ch}}^2}{4\pi} = \left(\frac{3}{5}\right)^2 \frac{g_{\pi NN}^2}{4\pi} \frac{m_{u,d}^2}{m_N^2}, \quad (8)$$

where the SU(3) flavor symmetry is only broken by the different masses of the light quarks. All the other model parameters are the same as the ones in Ref. [65], which were determined by reproducing the mass spectrum of the ground mesons. The authors of Ref. [65] used the same model to investigate the tetraquarks with open charm flavor and found that the $X_0(2900)$ could be interpreted as the molecular state $\bar{D}K^*$ with $I(J^P) = 0(0^+)$.

In the QDCSM, the single-particle orbital wave functions in the ordinary quark cluster model are the left- and right-centered single Gaussian functions, which are

$$\phi_\alpha(\mathbf{S}_i) = \left(\frac{1}{\pi b^2}\right)^{\frac{3}{4}} e^{-\frac{(r_\alpha - \frac{1}{2}\mathbf{S}_i)^2}{2b^2}},$$

$$\phi_\beta(-\mathbf{S}_i) = \left(\frac{1}{\pi b^2}\right)^{\frac{3}{4}} e^{-\frac{(r_\beta + \frac{1}{2}\mathbf{S}_i)^2}{2b^2}}. \quad (9)$$

The quark delocalization is realized by writing the single-particle orbital wave function as a linear combination of the left and right Gaussians, which are

$$\psi_\alpha(\mathbf{S}_i, \epsilon) = (\phi_\alpha(\mathbf{S}_i) + \epsilon \phi_\alpha(-\mathbf{S}_i))/N(\epsilon),$$

$$\psi_\beta(-\mathbf{S}_i, \epsilon) = (\phi_\beta(-\mathbf{S}_i) + \epsilon \phi_\beta(\mathbf{S}_i))/N(\epsilon),$$

$$N(\epsilon) = \sqrt{1 + \epsilon^2 + 2\epsilon e^{-S_i^2/4b^2}}, \quad (10)$$

where $\epsilon(\mathbf{S}_i)$ is the delocalization parameter determined by the dynamics of the quark system rather than free parameters. In this way, the system can choose its most favorable configuration through its dynamics in a larger Hilbert space.

B. The resonating group method

In the present work, the resonating group method (RGM) is employed to carry out the dynamical calculation. The core of this approach is how to deal with the two-body problem [66,67]. When dealing with the two-cluster system in this method, one can only consider the relative motion

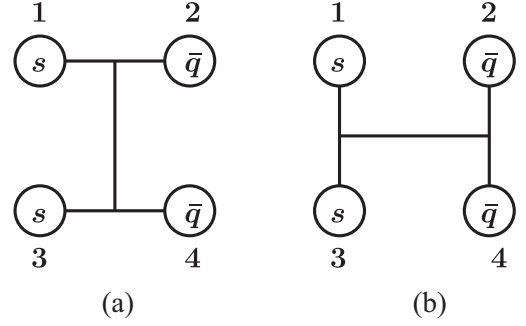


FIG. 1. The meson-meson configuration [diagram (a)] and diquark-antidiquark configuration [diagram (b)] in the $ss\bar{n}\bar{n}$ tetraquark system. “ $\bar{n}$ ” represents $\bar{u}$ and $\bar{d}$.

between the clusters, while the two clusters are frozen inside. So the wave function of the $ss\bar{n}\bar{n}$ system can be constructed as

$$\psi_{4q} = \mathcal{A} \left[[\psi_A(\boldsymbol{\rho}_A) \psi_B(\boldsymbol{\rho}_B)]^{[\sigma]IS} \otimes \chi_L(\mathbf{R}) \right]^J, \quad (11)$$

where the symbol $\mathcal{A}$ is the antisymmetry operator, which is defined as

$$\mathcal{A} = 1 - P_{13} - P_{24} + P_{13}P_{24}, \quad (12)$$

where the P_{ij} indicates the exchange of the particle positions with numbers i and j from Fig. 1, and $[\sigma] = [222]$ gives the total color symmetry. The symbols I , S , L , and J represent flavor, spin, orbit angular momentum, and total angular momentum of the $ss\bar{n}\bar{n}$ system, respectively. ψ_A and ψ_B are the wave functions of the two-quark cluster, which are

$$\psi_A = \left(\frac{1}{2\pi b^2}\right)^{3/4} e^{-\boldsymbol{\rho}_A^2/(4b^2)} \eta_{I_A} S_A \chi_A^c,$$

$$\psi_B = \left(\frac{1}{2\pi b^2}\right)^{3/4} e^{-\boldsymbol{\rho}_B^2/(4b^2)} \eta_{I_B} S_B \chi_B^c, \quad (13)$$

where η_I , S , and χ represent the flavor, spin, and internal color terms of the cluster wave functions, respectively. According to Fig. 1, we adopt different Jacobi coordinates for different diagrams. As for the meson-meson configuration in Fig. 1(a), the Jacobi coordinates are defined as

$$\boldsymbol{\rho}_A = \mathbf{r}_{q_1} - \mathbf{r}_{\bar{q}_2}, \quad \boldsymbol{\rho}_B = \mathbf{r}_{q_3} - \mathbf{r}_{\bar{q}_4},$$

$$\mathbf{R}_A = \frac{m_1 \mathbf{r}_{q_1} + m_2 \mathbf{r}_{\bar{q}_2}}{m_1 + m_2},$$

$$\mathbf{R}_B = \frac{m_3 \mathbf{r}_{q_3} + m_4 \mathbf{r}_{\bar{q}_4}}{m_3 + m_4},$$

$$\mathbf{R} = \mathbf{R}_A - \mathbf{R}_B,$$

$$\mathbf{R}_c = \frac{m_1 \mathbf{r}_{q_1} + m_2 \mathbf{r}_{\bar{q}_2} + m_3 \mathbf{r}_{q_3} + m_4 \mathbf{r}_{\bar{q}_4}}{m_1 + m_2 + m_3 + m_4}, \quad (14)$$

where the subscript $q/\bar{q}$ indicates the quark or antiquark particle, while the number indicates the quark position in Fig. 1(a). As for the diquark-antidiquark configuration, as shown in Fig. 1(b), the relevant Jacobi coordinates can be obtained by interchanging $\mathbf{r}_{q_3}$ with $\mathbf{r}_{\bar{q}_2}$ in Eq. (14).

Form the variational principle, after variation with respect to the relative motion wave function $\chi(R) = \sum_L \chi_L(R)$, one obtains the RGM equation, which is

$$\int H(\mathbf{R}, \mathbf{R}') \chi(\mathbf{R}') d(\mathbf{R}') = E \int N(\mathbf{R}, \mathbf{R}') \chi(\mathbf{R}') d(\mathbf{R}'), \quad (15)$$

where $H(\mathbf{R}, \mathbf{R}')$ and $N(\mathbf{R}, \mathbf{R}')$ are Hamiltonian and norm kernels, respectively. The eigenenergy E and the wave functions can be obtained by solving the RGM equation. In the present estimation, the function $\chi(\mathbf{R})$ can be expanded by Gaussian bases, which is

$$\chi(\mathbf{R}) = \frac{1}{\sqrt{4\pi}} \sum_L \left(\frac{1}{\pi b^2} \right)^{3/4} \sum_i^n C_{i,L} \times \int e^{-\frac{1}{2}(R-S_i)^2/b^2} Y^L(\hat{S}_i) d\hat{S}_i, \quad (16)$$

where $C_{i,L}$ is the expansion coefficient, and n is the number of Gaussian bases, which is determined by the stability of the results. S_i is the separation of two reference centers. $\mathbf{R}$ is the dynamic coordinate defined in Eq. (14). After including the motion of the center of mass, i.e.,

$$\phi_C(\mathbf{R}_c) = \left(\frac{4}{\pi b^2} \right)^{3/4} e^{-\frac{2R_c^2}{b^2}}, \quad (17)$$

one can rewrite Eq. (12) as

$$\psi_{4q} = \mathcal{A} \sum_{i,L} C_{i,L} \int \frac{d\hat{S}_i}{\sqrt{4\pi}} \prod_{\alpha=1}^2 \phi_\alpha(S_i) \prod_{\alpha=3}^4 \phi_\alpha(-S_i) \times \left[[\eta_{I_A S_A} \eta_{I_B S_B}]^{IS} Y^L(\hat{S}_i) \right]^J [\chi_{A S_A}^c \chi_{B S_B}^c]^{[\sigma]}, \quad (18)$$

where $\phi_\alpha(S_i)$ and $\phi_\beta(-S_i)$ are the single-particle orbital wave functions with different reference centers, whose specific expressions have been presented in Eq. (9).

With the reformulated ansatz, as shown in Eq. (18), the RGM equation becomes an algebraic eigenvalue equation, which is

$$\sum_{j,L} C_{j,L} H_{i,j}^{L,L'} = E \sum_j C_{j,L'} N_{i,j}^{L'}, \quad (19)$$

where $N_{i,j}^{L'}$ and $H_{i,j}^{L,L'}$ are the overlap of the wave functions and the matrix elements of the Hamiltonian, respectively. By solving the generalized eigenvalue problem, we can

obtain the energies of the tetraquark systems E and the corresponding expansion coefficients $C_{j,L}$. Finally, the relative motion wave function between two clusters can be obtained by substituting the $C_{j,L}$ into Eq. (16). As for the flavor, spin, and color wave functions of the tetraquark system, they are constructed in a two-step way. One can first construct the wave functions for the two clusters, and then couple the wave functions of two clusters to form the wave function for the tetraquark system. The details of the flavor, spin, and color wave functions for the tetraquark system are collected in the Appendix.

In addition, it is noteworthy that the quark composition of the $ss\bar{n}\bar{n}$ system exhibits uniqueness, as its structure includes two identical quarks and two identical antiquarks. Therefore, the wave function of the $ss\bar{n}\bar{n}$ tetraquark states, constructed from 4 degrees of freedom (orbital, color, spin, and flavor), should satisfy antisymmetry requirements. Since the present study is restricted to the S -wave case, the Pauli principle restricts the $ss\bar{n}\bar{n}$ system to the allowed channels with quantum numbers $I(J^P) = 0(1^+)$, $1(0^+)$, $1(1^+)$, and $1(2^+)$. The detailed derivation of these Pauli-allowed channels, discussed separately for the meson-meson and diquark-antidiquark configurations from the symmetry properties of the spatial, color, spin, and flavor wave functions, is presented in the Appendix. Table I lists the resulting color, spin, and flavor bases used in the present calculation.

III. NUMERICAL RESULTS AND DISCUSSIONS

In the present calculation, we work within the low-energy S -wave approximation. For each allowed quantum number sector, the RGM is used to search for bound-state solutions, while the real-scaling stabilization method is applied to identify possible resonancelike structures. As shown in Fig. 1, the two structural possibilities considered are meson-meson and diquark-antidiquark configurations. The allowed quantum numbers are $I(J^P) = 0(1^+)$, $1(0^+)$, $1(1^+)$, and $1(2^+)$ under the constraints of the Pauli principle for the $ss\bar{n}\bar{n}$ tetraquark states. In the present estimations, to assess whether bound states or resonance states exist in the doubly strange tetraquark system, the resonance group method is utilized to evaluate the existence of bound states for different quantum numbers, with the real-scaling stabilization method applied to search for possible resonancelike structures.

A. The bound states

For the tetraquark system containing $ss\bar{n}\bar{n}$, a systematic theoretical estimation of bound states is performed. The estimation results for the $ss\bar{n}\bar{n}$ tetraquark states under different quantum numbers are summarized in Table II. As shown in the table, the results include not only single-channel estimates for each configuration but also channel coupling estimates for the two configurations. The first

TABLE I. The constructed relevant channels for all possible quantum numbers. Square brackets [i,j,k] represent the flavor, spin, and color wave functions corresponding to the relevant channels, where “i,” “j,” and “k,” respectively, denote the selected flavor, spin, and color wave functions.

$I(J^P)$	Index	$F^i; S_s^j; \chi_k^c$ [i,j,k]	Channels
$0(1^+)$	1	[1,3,1]	$\bar{K}^0 K^{*-}$
	2	[1,4,1]	$\bar{K}^{*0} K^-$
	3	[1,5,1]	$\bar{K}^{*0} K^{*-}$
	4	[3,3,3]	$(ss)(\bar{n}\bar{n})$
	5	[4,4,3]	$(ss)(\bar{n}\bar{n})$
$1(0^+)$	1	[1,1,2]	$\bar{K}^0 \bar{K}^0$
	2	[1,2,2]	$\bar{K}^{*0} \bar{K}^{*0}$
	3	[3,1,4]	$(ss)(\bar{n}\bar{n})$
	4	[4,2,4]	$(ss)(\bar{n}\bar{n})$
$1(1^+)$	1	[1,3,2]	$\bar{K}^0 \bar{K}^{*0}$
	2	[1,4,2]	$\bar{K}^{*0} \bar{K}^0$
	3	[4,5,4]	$(ss)(\bar{n}\bar{n})$
$1(2^+)$	1	[1,6,2]	$\bar{K}^{*0} \bar{K}^{*0}$
	2	[4,6,4]	$(ss)(\bar{n}\bar{n})$

column, labeled “Index,” represents the symbols for each channel, while the second and third columns list all constructed relevant channels and their theoretical thresholds, respectively. Additionally, the fourth column, denoted as E_{sc} , represents the lowest eigenenergy obtained in the single-channel estimation, whereas E_{cc} and E_{mix} are the lowest estimated eigenenergies obtained by considering the coupled-channel effects of each kind of configuration and both configurations, respectively.

Additionally, it is worth noting that the binding energy of the $ss\bar{n}\bar{n}$ tetraquark state, E_b , is defined as $E_b = E_i - E_4(\infty)$, to evaluate the stability of the $ss\bar{n}\bar{n}$ tetraquark state under strong interactions. Here, $E_4(\infty)$ represents the lowest threshold of the two-meson configuration estimated within the QDCSM, and i denotes different channel coupling scenarios. With this definition, the eigenenergies and thresholds are treated consistently within one framework. A state with $E_b \geq 0$ is unstable against strong decay into two mesons, whereas a state with $E_b \leq 0$ is kinematically forbidden to decay through strong interactions and can only decay electromagnetically or weakly. For phenomenological comparison, the corresponding experimental thresholds are also considered. In the $I(J^P) = 0(1^+)$ sector, the difference between the QDCSM threshold and the corresponding Particle Data Group (PDG) value for the lowest $\bar{K}K^*$ channel is only a few MeV, so the qualitative conclusion is not affected.

For the system with $I(J^P) = 0(1^+)$, the estimated eigenenergies of five channels, including three channels in meson-meson configurations and two channels in diquark-antidiquark configurations, are presented in Table II. For the

TABLE II. The lowest-lying eigenenergies (in unit of MeV) of the $ss\bar{n}\bar{n}$ tetraquark states with all possible quantum numbers.

$I(J^P)$	Index	Channels	E_{th}	E_{sc}	E_{cc}	E_{mix}
$0(1^+)$	1	$\bar{K}^0 K^{*-}$	1386.2	1384.7	1324.8	1309.7
	2	$\bar{K}^{*0} K^-$	1386.2	1384.7		
	3	$\bar{K}^{*0} K^{*-}$	1784.2	1779.8		
	4	$(ss)(\bar{n}\bar{n})$		1818.0	1415.8	
	5	$(ss)(\bar{n}\bar{n})$		1513.6		
$1(0^+)$	1	$\bar{K}^0 \bar{K}^0$	988.2	992.4	992.4	991.7
	2	$\bar{K}^{*0} \bar{K}^{*0}$	1784.2	1787.3		
	3	$(ss)(\bar{n}\bar{n})$		1982.1	1391.3	
	4	$(ss)(\bar{n}\bar{n})$		1711.9		
$1(1^+)$	1	$\bar{K}^0 \bar{K}^{*0}$	1386.2	1390.5	1390.5	1390.2
	2	$\bar{K}^{*0} \bar{K}^0$	1386.2	1390.5		
	3	$(ss)(\bar{n}\bar{n})$		1812.1		
$1(2^+)$	1	$\bar{K}^{*0} \bar{K}^{*0}$	1784.2	1786.9	1786.9	1786.4
	2	$(ss)(\bar{n}\bar{n})$		1964.2		

meson-meson configuration, the single-channel estimations show that the lowest eigenenergies for the $\bar{K}^0 K^{*-}$, $\bar{K}^{*0} K^-$, and $\bar{K}^{*0} K^{*-}$ channels are all below their respective theoretical thresholds, suggesting weakly bound solutions. Their binding energies are approximately -1.5 MeV, -1.5 MeV, and -4.4 MeV, respectively. Therefore, the isolated meson-meson channels are only weakly bound in the present estimations. When considering the channel coupling effects in the meson-meson configurations, the bound state with binding energy of approximately -61.4 MeV can be obtained, which indicates that the coupled-channel effect in the meson-meson configuration is rather strong. For the diquark-antidiquark configuration, both the single-channel and coupled-channel results remain above the lowest $\bar{K}K^*$ threshold, however, the channel coupling estimation lowers the energy by about 100 MeV relative to the lowest single-channel diquark-antidiquark result. This indicates that channel coupling within the diquark-antidiquark sector is also non-negligible. Moreover, when the meson-meson and diquark-antidiquark configurations are further coupled, a bound-state solution with a binding energy of about -76.5 MeV is obtained in the $ss\bar{n}\bar{n}$ tetraquark system. Since the difference between the model threshold and the corresponding PDG threshold is only at the level of a few MeV, replacing the model threshold by the experimental one changes the numerical binding energy only slightly and does not alter the qualitative conclusion that this state is deeply bound. This estimated result is also qualitatively similar to the channel coupling results reported in Refs. [50,51], which also support the appearance of a bound-state solution in the $\bar{K}K^*$ channel.

Furthermore, the above estimations show that most of the deep attraction already emerges from the meson-meson coupled-channel dynamics, whereas the diquark-antidiquark sector provides an additional but smaller contribution

TABLE III. Threshold-subtracted Hamiltonian contributions (in MeV) for the $I(J^P) = 0(1^+)$ state in the single-channel, meson-meson coupled-channel, and full coupled-channel calculations. Here, “SC” denotes the single-channel $\bar{K}K^*$ result, “CC(MM)” denotes the meson-meson coupled-channel result, and “CC(MM + DA)” denotes the full coupled-channel result including both meson-meson and diquark-antidiquark channels. All quantities are given relative to the $\bar{K}K^*$ threshold.

Scenario	ΔM_{mass}	ΔT_m	ΔV_{CON}	ΔV_{OGE}	ΔV_π	ΔV_K	ΔV_η	ΔE
SC: $\bar{K}K^*$	0	43.8	-22.3	-22.5	-1.8	0	1.4	-1.5
CC(MM)	0	214.8	-184.6	-61.6	-36.5	0	6.5	-61.4
CC(MM + DA)	0	322.0	-181.4	-148.2	-78.0	0	9.1	-76.5

of about 15 MeV. To make the origin of this deep binding more transparent, the contributions of each interaction are further given in Table III. The single-channel $\bar{K}^0 K^{*-}$ result is only weakly bound, while the meson-meson coupled-channel calculation makes the confinement, one gluon-exchange, and pion-exchange terms substantially more attractive. After the diquark-antidiquark channels are included, the energy is lowered further mainly through the additional enhancement of the one gluon-exchange and pion-exchange contributions, whereas the confinement term changes only moderately. Therefore, the 1309.7 MeV state should be understood primarily as a coupled-channel effect generated by the combined action of confinement, one-gluon exchange, and pion exchange in the present QDCSM framework, rather than as a simple single-channel molecular binding. At the same time, since the precise depth of the binding may still depend quantitatively on the short-range dynamics of the model, this result should be regarded conservatively as a model-dependent prediction within the present framework.

For the system with $I(J^P) = 1(0^+)$, there are four channels, which are $\bar{K}^0 \bar{K}^0$, $\bar{K}^{*0} \bar{K}^{*0}$, and two color structures with $\mathbf{6}_c \otimes \mathbf{6}_c$ and $\mathbf{\bar{3}}_c \otimes \mathbf{3}_c$ in the diquark-antidiquark configuration. From Table II, it can be seen that the single-channel estimated eigenenergies are all above the corresponding threshold in the meson-meson configuration, indicating that the single-channel estimations do not support the existence of the bound states. Additionally, the coupling of channels $\bar{K}^0 \bar{K}^0$ and $\bar{K}^{*0} \bar{K}^{*0}$ in the meson-meson configuration resulted in a minimum eigenenergy estimate of 992.4 MeV, which is still above the theoretical threshold for channel $\bar{K}^0 \bar{K}^0$. It implies that the significant mass difference between the two channels leads to diminished coupling effects. A similar phenomenon occurs in the diquark-antidiquark configuration, as both single-channel and channel coupling estimations yield the lowest eigenenergies, surpassing the theoretical threshold of $\bar{K}^0 \bar{K}^0$. Comparing the channel coupling estimates of the meson-meson and diquark-antidiquark configurations reveals that the coupling effects of the color structure are more pronounced than that of the meson-meson configuration. Furthermore, the coupling estimates of both configurations, as illustrated in Table II, provide the lowest eigenenergy of

991.7 MeV, still exceeding the lowest theoretical threshold of 988.2 MeV, which suggests no bound state exists in the $ss\bar{n}\bar{n}$ tetraquark system with $I(J^P) = 1(0^+)$.

For the system with $I(J^P) = 1(1^+)$, the lowest eigenenergies derived from single-channel estimations within both the meson-meson and diquark-antidiquark configurations exceed the $\bar{K}^0 \bar{K}^{*0}$ threshold. Additionally, when considering the full channel coupling between the meson-meson and diquark-antidiquark configurations, the lowest resulting eigenenergy is approximately 1390.2 MeV, which is significantly higher than the $\bar{K}^0 \bar{K}^{*0}$ channel threshold of 1386.2 MeV. These findings suggest that a bound state does not exist in the $ss\bar{n}\bar{n}$ tetraquark system with $I(J^P) = 1(1^+)$. Analogous conclusions can be drawn for the $ss\bar{n}\bar{n}$ tetraquark system with $I(J^P) = 1(2^+)$. In this case, all estimations, including channel coupling within meson-meson configurations, within diquark-antidiquark configurations, and the full coupling between both types, yield the lowest eigenenergies that consistently exceed the minimum threshold of $\bar{K}^{*0} \bar{K}^{*0}$. A comparison with estimation results from the one-boson-exchange (OBE) model for systems with quantum number $I = 1$ reveals nearly identical conclusions. Both this study and the OBE model find no bound states in these channels [50].

The conclusions drawn from the bound-state estimation prompt a more rigorous investigation into the interactions between the two clusters, with the aim of determining whether a mechanism for bound-state formation exists. For the $ss\bar{n}\bar{n}$ tetraquark system, Figs. 2 and 3 present the effective potentials of the meson-meson and diquark-antidiquark configurations under various quantum numbers. In the meson-meson configuration, when the quantum number is $0(1^+)$, as shown in Fig. 2(a), the effective potentials for channels $\bar{K}^0 K^{*-}$, $\bar{K}^{*0} K^-$, and $\bar{K}^{*0} K^{*-}$ exhibit attractive interactions. This clearly explains why these channels can form bound states in the single-channel estimation. Notably, bound states form even though the attractive interactions for $\bar{K}^0 K^{*-}$ and $\bar{K}^{*0} K^-$ are weaker than those for $\bar{K}^{*0} K^{*-}$. From Figs. 2(b) and (c), it can be observed that the interactions for the $\bar{K}^0 \bar{K}^0$ with $1(0^+)$, and $\bar{K}^0 \bar{K}^{*0}$ and $\bar{K}^{*0} \bar{K}^0$ with $1(1^+)$, are repulsive. Furthermore, the repulsive interaction between $\bar{K}^0 \bar{K}^0$ with $1(0^+)$ has been consistently verified in lattice QCD calculations and

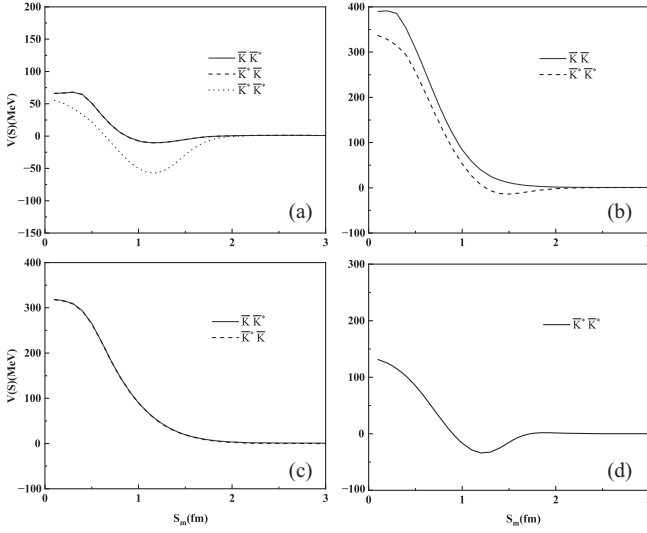


FIG. 2. Effective potentials for the $ss\bar{n}\bar{n}$ system in the meson-meson configuration. Panels (a), (b), (c), and (d) correspond to $I(J^P) = 0(1^+)$, $1(0^+)$, $1(1^+)$, and $1(2^+)$, respectively. The channel labels shown in each panel denote the corresponding meson-meson channels listed in Table I.

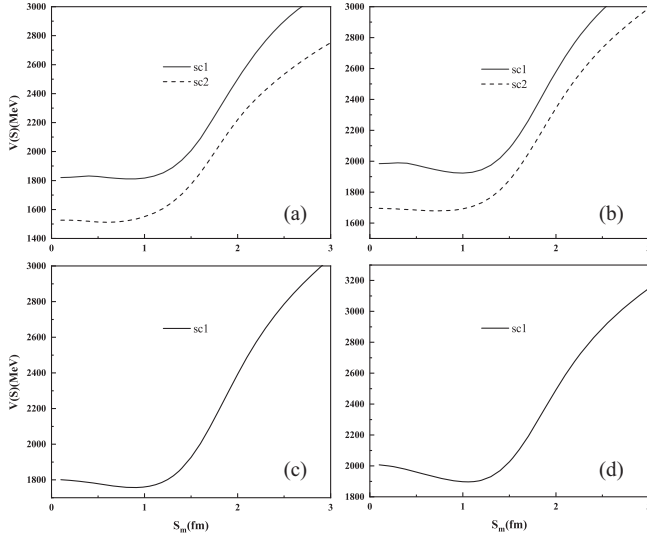


FIG. 3. Effective potentials for the $ss\bar{n}\bar{n}$ system in the diquark-antidiquark configuration. Panels (a), (b), (c), and (d) correspond to $I(J^P) = 0(1^+)$, $1(0^+)$, $1(1^+)$, and $1(2^+)$, respectively. The label sci denotes the i th diquark-antidiquark basis channel in the corresponding quantum number sector, following the channel definitions in Table I.

the OBE model [50,52,53]. Therefore, bound states cannot form in these channels within single-channel estimations. In Fig. 2(b) and Fig. 2(d), although the interactions of the remaining channels $\bar{K}^{*0}\bar{K}^{*0}$ with $1(0^+)$ and $1(2^+)$ display attraction, the bound-state analysis reveals that the attraction in these channels is too weak to support bound states in

single-channel calculations. Consequently, these channels transition into scattering states.

In the context of the diquark-antidiquark configuration, subclusters, each associated with specific color structures, cannot separate directly due to the constraints of the confinement potentials. This implies that channels in this configuration are likely to form resonance states. Figure 3 presents the effective potential behaviors for various quantum numbers in this configuration. As shown in the figure, the effective potential for each single channel exhibits a characteristic behavior: the energy obtained is higher when the two subclusters are either very close or very far apart. Specifically, for systems with quantum numbers $0(1^+)$, the minimum potential energy occurs at distances of approximately 0.6–0.8 fm, while for $1(0^+)$, it occurs at 0.7–1 fm. For systems with $1(1^+)$ and $1(2^+)$, the minimum energy is generally found at a separation of around 1 fm. The presence of these minima at intermediate distances suggests the possible formation of resonance states, which are likely to be compact tetraquark states. This provides additional insight into the interaction mechanisms within the diquark-antidiquark configuration and their potential physical significance.

B. Resonance states

From the aforementioned analysis, it can be inferred that resonance states might exist in the $ss\bar{n}\bar{n}$ tetraquark system. To further explore such possible structures, the real-scaling method is applied within a multichannel framework that incorporates both meson-meson and diquark-antidiquark configurations [66,67]. This method has been successfully employed in electron-atom, electron-molecule, and atom-antiatom systems and is extended to explore genuine resonance states in different systems. A key aspect of this method is to monitor the variation of estimated energy as the distance S_m between two clusters, which serves as a scaling parameter for the basis functions used to represent the system, increases incrementally. With the increased distance between the two clusters, the continuous states, bound states, and resonance states display unique behaviors: continuous states will fall off toward their respective thresholds, while the energies of bound and resonance states will remain stable. Regarding the resonance states, if the scattering state energy is far away from one of the resonance states, the weak coupling will ensure the stability of the resonance state's energy. Conversely, as S_m increases, the energy of a scattering state may approach the resonance energy. In this region, the coupling effect between the resonance and scattering state becomes stronger. If S_m increases further, the energy gap between these states widens again, and the coupling weakens. This interaction leads to the characteristic “avoided-crossing” behavior, illustrated in Fig. 4, where the energy levels appear to repel each other instead of crossing. Such avoided-crossing phenomenon recurs with the increasing

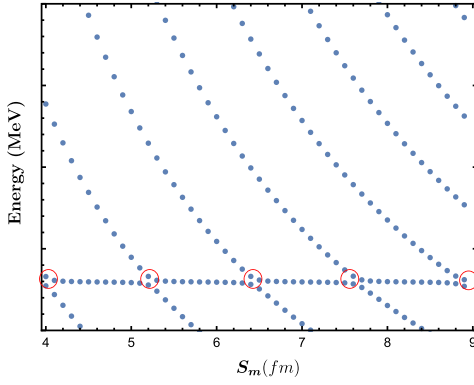


FIG. 4. A sketch diagram of the resonance shape in the real-scaling method.

of S_m due to the continuum nature of the scattering state and the stable energy plateau exhibited by the resonance state before and after the avoided crossing corresponds to the resonance energy. Additionally, the decay width of the resonance state can also be derived from

$$\Gamma = 4|V_{\min}(S)| \frac{\sqrt{|k_r||k_c|}}{|k_r - k_c|}, \quad (20)$$

where k_r and k_c are the slopes of the resonance and scattering states, respectively. Here, $V_{\min}(S)$ represents the minimal energy difference between the resonance and the scattering state at the avoided-crossing point. This method has been successfully applied to investigate the pentaquark [68,69], the dibaryon [70], and the tetraquark systems [71–73]. These research findings indirectly validate the rationality and reliability of this method, offering a practical theoretical framework for exploring possible exotic state properties.

In the present work, the spatial wave function of the $ss\bar{n}\bar{n}$ tetraquark system with different quantum numbers is expanded using basis functions scaled by the distance S_m ($m = 1, 2, 3, \dots, n$) between the two constituent clusters. The estimated eigenenergies of the $ss\bar{n}\bar{n}$ tetraquark system with different quantum numbers can be acquired by varying the distance parameter S_m from 4 fm to 10 fm, allowing for the identification of possible resonance states through their characteristic behavior as a function of S_m . The relevant results with different quantum numbers in the $ss\bar{n}\bar{n}$ tetraquark system are presented in Figs. 5, 8, 9, and 10, respectively. In these figures, black horizontal lines represent the relevant physical thresholds of the involved meson-meson configurations. Red horizontal lines indicate identified bound states, while blue horizontal lines indicate identified genuine resonances. The corresponding estimated masses for these bound and resonance states are marked on the plots.

For the $ss\bar{n}\bar{n}$ tetraquark system with $I(J^P) = 0(1^+)$, the channel coupling estimation, including meson-meson and

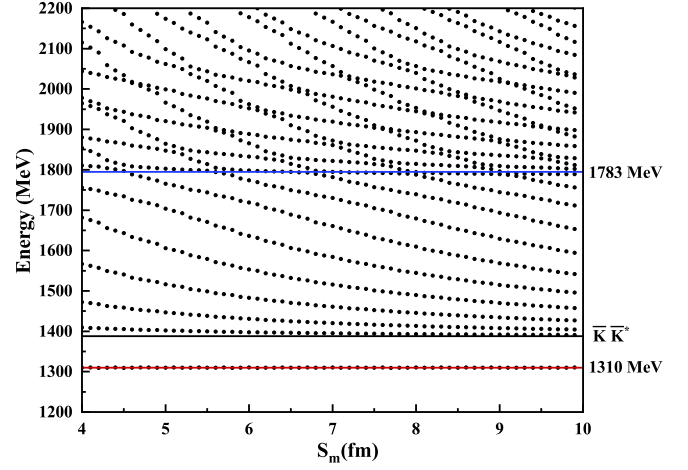


FIG. 5. Stabilization plot of the energies of the $ss\bar{n}\bar{n}$ tetraquark system with $I(J^P) = 0(1^+)$ obtained using the real-scaling method. The black horizontal lines indicate the relevant meson-meson thresholds, the red horizontal line denotes the bound-state solution, and the blue horizontal line denotes the resonancelike structure. The channel contents follow the channel indices listed in Table I.

diquark-antidiquark configurations, is performed by gradually increasing the distance parameter S_m . As illustrated in Fig. 5, with the increasing distance between the two clusters, a bound state and a resonance state progressively emerge. For the identified bound state attributed to the channel coupling, Fig. 5 indicates that the energy of the bound state remains stable at approximately 1310 MeV as the spatial range is enlarged. To further characterize this state, we perform detailed calculations of the component proportions for each involved channel and the root-mean-square (RMS) radius of this state. The results show that the wave function is dominated by meson-meson basis components, with about 59% from the $\bar{K}^0 K^{*-}$ and $\bar{K}^{*0} K^-$ channels, about 38% from the $\bar{K}^{*0} K^{*-}$ channel, and about 3% from the diquark-antidiquark channels. Its RMS radius is approximately 0.81 fm. However, we note that within the QDCSM the meson-meson and diquark-antidiquark channels should be understood primarily as coupling bases rather than as physical cluster assignments at short distances. When the clusters overlap, the antisymmetrization operator and quark delocalization mechanism allow quark exchange and short-range multi-quark mixing between the two clusters [70]. In particular, previous QDCSM studies have emphasized that, once intercluster quark exchange and delocalization become important, the channel labels should be interpreted as coupling sequences rather than literal two-hadron assignments. In the same framework, the color screening phenomenology has long been interpreted as an effective description of hidden color-related channel coupling effects and short-range multi-quark dynamics [74,75]. Therefore, the dominance of meson-meson basis components does not necessarily imply a loosely bound

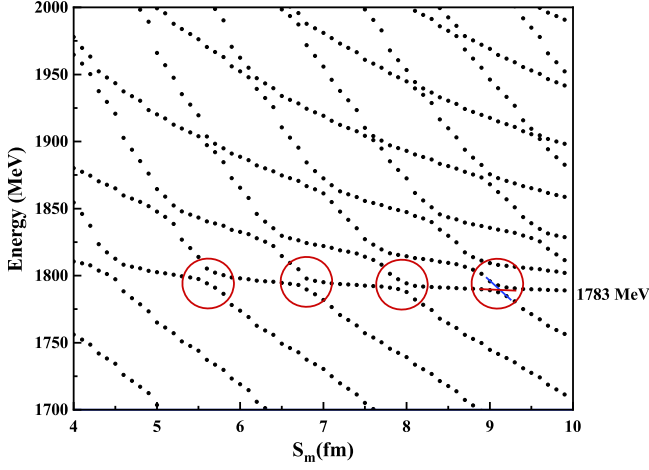


FIG. 6. Magnified stabilization plot for the $I(J^P) = 0(1^+)$ $ss\bar{n}\bar{n}$ system in the energy range from 1700 MeV to 2000 MeV. The black horizontal line marks the relevant meson-meson threshold, while the avoided-crossing behavior around 1783 MeV indicates the resonancelike structure. The blue and red guide lines denote the slopes of the scattering and resonancelike levels used in the width extraction, respectively.

molecular geometry in the present framework. Instead, the relatively small root-mean-square radius indicates substantial short-range overlap and nontrivial multi-quark dynamics. At the same time, since we have not separately quantified an independent hidden-color fraction inside the meson-meson basis, we do not take the small radius alone as sufficient evidence to identify the state unambiguously as a compact tetraquark. For this reason, the 1310 MeV state is interpreted here more conservatively as a deeply bound coupled-channel solution with dominant meson-meson basis components and significant short-range multi-quark dynamics, rather than being identified unambiguously as a compact tetraquark state.

Additionally, as illustrated in Fig. 5, it is evident that the avoided-crossing resonance behavior repeatedly emerges around an energy of approximately 1783 MeV as the finite volume increases, indicating the possible existence of a resonance state in the $ss\bar{n}\bar{n}$ tetraquark system. To provide a more intuitive visualization of the avoided-crossing behavior associated with the resonance state, a magnified view of the energy range from 1700 MeV to 2000 MeV in Fig. 6 is presented. As depicted in Fig. 6, the avoided-crossing behavior consistently appears near 1783 MeV. Subsequently, Eq. (20) is applied to estimate the decay width of the resonance state. Notably, the estimated width stabilizes with increasing spatial range, indicating that larger spatial ranges produce more reliable results. The estimated decay width is approximately 5.6 MeV.

To examine the dependence of the extracted resonance parameters on the generator-coordinate range, we further enlarge the spatial basis space by varying the maximum intercluster distance $S_{\max}$. The results are summarized

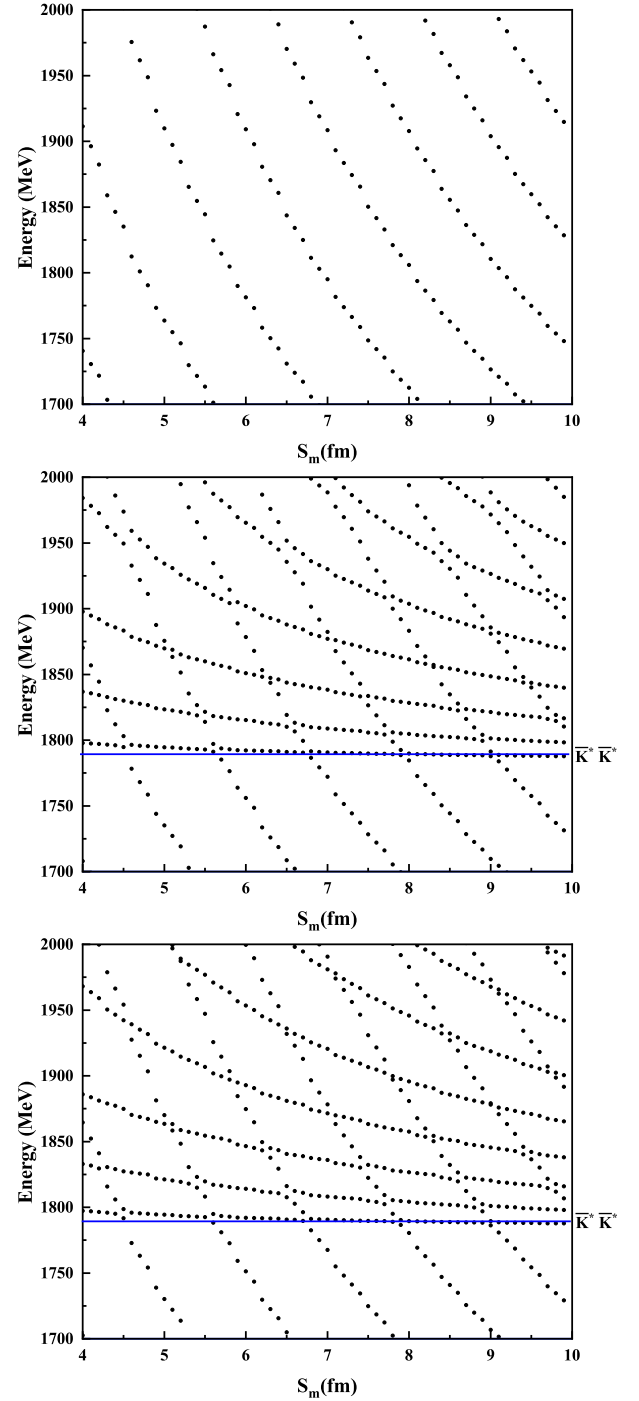


FIG. 7. Channel-space dependence of the real-scaling analysis for the $I(J^P) = 0(1^+)$ system. The top, middle, and bottom panels correspond to the coupling of channels 1 and 2, the coupling of channels 1, 2, and 3, and the coupling of channels 1, 2, 4, and 5, respectively. The horizontal blue line in the middle and bottom panels denotes the $\bar{K}^* \bar{K}^*$ threshold. No stable avoided-crossing structure is found near 1783 MeV in any of these reduced spaces. By contrast, the resonancelike signal appears only in the full coupled-channel space with all five channels included (see Fig. 6), indicating that the 1783 MeV structure is generated by the collective interplay of the full coupled-channel dynamics in the present QDCSM calculation.

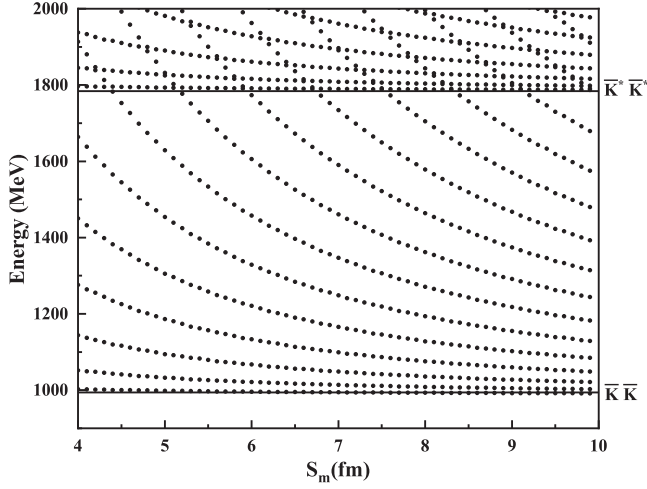


FIG. 8. Stabilization plot of the energies of the $ss\bar{n}\bar{n}$ tetraquark system with $I(J^P) = 1(0^+)$. The black horizontal lines indicate the relevant meson-meson thresholds. No stable bound-state or resonancelike signal is found in this channel.

in Table V and illustrated in Fig. 7. As S_{max} increases from 8 fm to 12 fm, the extracted resonance mass changes only from 1782.4 MeV to 1782.8 MeV, while the width changes from 6.1 MeV to 5.6 MeV. Therefore, both the resonance mass and the decay width remain stable within the present numerical precision, indicating that the 1783 MeV resonancelike structure is not an artifact of a particular spatial basis range. We also examine the dependence of the resonance signal on the selected channel space. To this end, the real-scaling analysis is performed in three reduced coupled-channel spaces, namely, the coupling of channels 1 and 2, the coupling of channels 1, 2, and 3, and the coupling of channels 1, 2, 4, and 5. The corresponding results are shown in Fig. 7. No stable avoided-crossing structure is found near 1783 MeV in any of these reduced spaces. By contrast, the repeated avoided-crossing behavior near 1783 MeV appears only in the full coupled-channel calculation with channels 1, 2, 3, 4, and 5 included, as shown in Fig. 6. This indicates that the 1783 MeV resonancelike structure is not generated by a smaller truncated channel space, but is a collective coupled-channel effect that emerges only when all five channels are included. Moreover, the internal structure of the resonance state is also analyzed. The results indicate that the resonance state with a mass of roughly 1783 MeV is composed of approximately 10% $\bar{K}^0 K^{*-}$, 10% $\bar{K}^{*0} K^-$, 50% $\bar{K}^{*0} K^{*-}$, and 30% diquark-antidiquark configuration. The RMS radius is approximately 0.58 fm. Such a relatively small radius suggests substantial short-range overlap and non-trivial multi-quark dynamics in the present QDCSM framework. We, therefore, describe this state more conservatively as a resonancelike coupled-channel solution with significant short-range multi-quark dynamics, rather than identifying it straightforwardly as a compact tetraquark state.

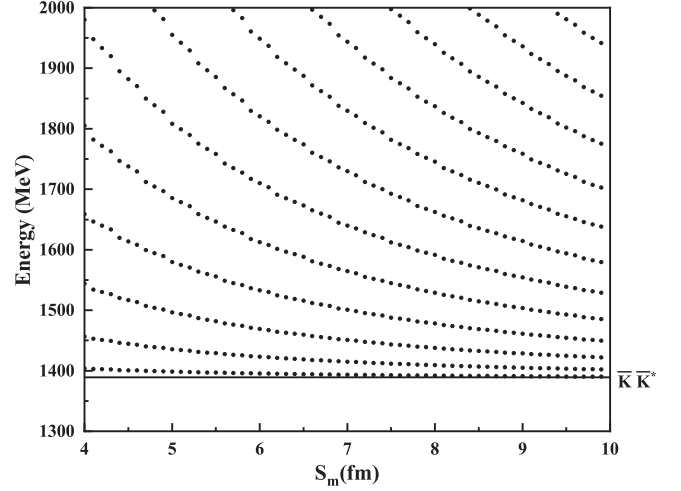


FIG. 9. Stabilization plot of the energies of the $ss\bar{n}\bar{n}$ tetraquark system with $I(J^P) = 1(1^+)$. The black horizontal lines indicate the relevant meson-meson thresholds. No stable bound-state or resonancelike signal is found in this channel.

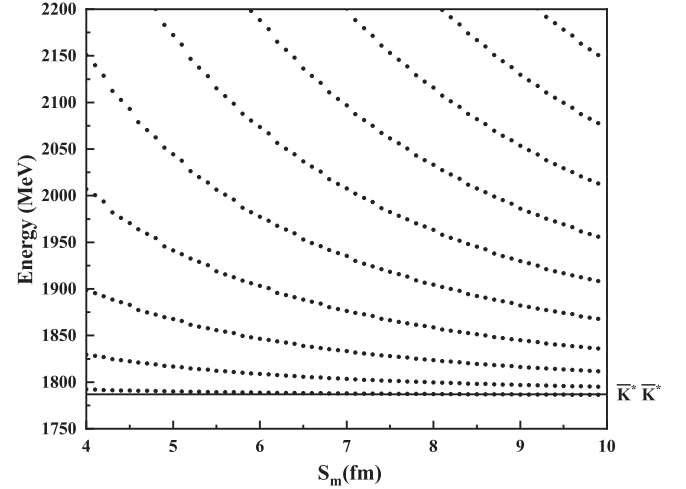


FIG. 10. Stabilization plot of the energies of the $ss\bar{n}\bar{n}$ tetraquark system with $I(J^P) = 1(2^+)$. The black horizontal lines indicate the relevant meson-meson thresholds. No stable bound-state or resonancelike signal is found in this channel.

For the $ss\bar{n}\bar{n}$ tetraquark system with $I(J^P) = 1(0^+)$, shown in Fig. 8, one can find that only two black horizontal lines representing the approximate thresholds for $\bar{K}^0 \bar{K}^*$ and $\bar{K}^{*0} \bar{K}^{*0}$ are visible, and the channel coupling estimations yield no bound states or resonance states. Similar situations are observed for the $ss\bar{n}\bar{n}$ tetraquark system with $I(J^P) = 1(1^+)$ and $I(J^P) = 1(2^+)$. From Figs. 9 and 10, a notable feature is that the energy of the continuum state gradually converges toward the respective $\bar{K}^0 \bar{K}^{*0}$ and $\bar{K}^{*0} \bar{K}^{*0}$ thresholds as the distance parameter S_m increases. Again, no bound states or resonance states are found in these channels.

TABLE IV. Comparison of representative theoretical predictions for the double-strangeness tetraquark candidates in the $I(J^P) = 0(1^+)$ sector.

Approach	Binding energy (MeV)	Bound-state mass (MeV)
Present work (QDCSM)	~ -76.5	~ 1309.7
OBE model [50]	$\bar{K}\bar{K}^*: \sim -(3.9 - 30.5); \bar{K}^*\bar{K}^*: \sim -(4.0 - 32.6)$	$\bar{K}\bar{K}^*: \sim 1359 - 1385; \bar{K}^*\bar{K}^*: \sim 1751 - 1779$
Chiral quark model [51]	~ -64	~ 1328

TABLE V. Stability of the extracted resonance parameters for the $I(J^P) = 0(1^+)$ resonancelike structure under enlargement of the generator-coordinate space. Here, $S_{\max}$ denotes the maximum intercluster distance used in the real-scaling calculation.

$S_{\max}$ (fm)	M_R (MeV)	Γ (MeV)
8	1782.4	6.1
9	1783.0	5.7
10	1782.8	5.6
12	1782.8	5.6

To place the present results in a broader theoretical context, a quantitative comparison with other representative studies of the double-strangeness tetraquark system is given in Table IV. Since existing single-boson exchange and chiral quark models provide direct predictions for the $I(J^P) = 0(1^+)$ bound-state region, they are included here in the comparison with the current QDCSM result. As shown in Table IV, different model calculations consistently identify the region $I(J^P) = 0(1^+)$ as the most favorable region, although the predicted binding depth still exhibits a significant model dependence. In particular, the OBE model tends to favor loosely bound near-threshold molecular states, whereas both the present QDCSM calculation and the recent chiral quark model study support a more deeply bound $0(1^+)$ state. It should be noted that the currently available lattice QCD result is not included in Table IV, because it does not yet provide a direct spectrum for the $ss\bar{q}\bar{q}$ tetraquark candidates [52,53]. Instead, lattice QCD has so far mainly constrained the relevant kaonic subsystem: the negative (K^+K^+) scattering length indicates that the corresponding ($I=1$)($\bar{K}\bar{K}$) interaction is repulsive, which is consistent with the absence of bound states in the $I=1$ sector in the present calculation.

IV. SUMMARY

The properties of the $ss\bar{n}\bar{n}$ tetraquark system with various quantum numbers are systematically investigated using the QDCSM, with emphasis on effective hadron interactions and the presence of bound or resonance states. Two configurations, meson-meson and diquark-antidiquark, are primarily considered in current studies of the $ss\bar{n}\bar{n}$ tetraquark system. For these configurations, we perform single-channel estimations, channel coupling

within each configuration, and channel coupling between the two configurations. Bound-state calculations are conducted using the RGM. Additionally, the stabilization calculation is used to identify potential resonance states in the $ss\bar{n}\bar{n}$ tetraquark system with various quantum numbers.

In the $I(J^P) = 0(1^+)$ $ss\bar{n}\bar{n}$ tetraquark system, bound states $\bar{K}^0K^{*-}$, $\bar{K}^{*0}K^-$, and $\bar{K}^{*0}K^{*-}$ with shallow binding energies of approximately -1.5 MeV, -1.5 MeV, and -4.4 MeV, respectively, are revealed by single-channel estimations. These bound states, specifically $\bar{K}^0K^{*-}$ and $\bar{K}^{*0}K^-$, have been identified in the one boson-exchange model [50] and the chiral quark model [51], while the $\bar{K}^{*0}K^{*-}$ state is corroborated by the one boson-exchange model. After channel coupling is included, a deeply bound state with a mass of about 1310 MeV is obtained in the present QDCSM calculation. Our analysis shows that most of the deep attraction already arises from the meson-meson coupled-channel dynamics, while the diquark-antidiquark sector provides an additional but smaller contribution. Accordingly, this 1310 MeV state is interpreted more conservatively as a model-dependent coupled-channel bound-state solution with dominant meson-meson basis components and significant short-range multiquark dynamics, rather than being identified unambiguously as a compact tetraquark state. In addition, the real-scaling stabilization analysis indicates a resonancelike structure with a mass of approximately 1783 MeV and a width of about 5.6 MeV. For the $I=1$ sectors, no bound or resonancelike states are found in the present calculation. The present results may serve as a useful theoretical reference for future studies of double-strangeness tetraquark candidates.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX: THE WAVE FUNCTION OF THE DOUBLE-STRANGENESS TETRAQUARK

1. The color wave function

Plenty of color structures in multiquark systems will be available with respect to those of conventional hadrons such as $q\bar{q}$ mesons and qqq baryons. In this appendix, the goal is to construct the colorless wave function of a 4-quark system.

For the meson-meson configurations, the color wave functions of a $q\bar{q}$ cluster are listed as

$$\begin{aligned} C_{[111]}^1 &= \sqrt{\frac{1}{3}}(r\bar{r} + g\bar{g} + b\bar{b}), \\ C_{[21]}^2 &= r\bar{b}, \quad C_{[21]}^3 = -r\bar{g}, \\ C_{[21]}^4 &= g\bar{b}, \quad C_{[21]}^5 = -b\bar{g}, \\ C_{[21]}^6 &= g\bar{r}, \quad C_{[21]}^7 = b\bar{r}, \\ C_{[21]}^8 &= \sqrt{\frac{1}{2}}(r\bar{r} - g\bar{g}), \\ C_{[21]}^9 &= \sqrt{\frac{1}{6}}(-r\bar{r} - g\bar{g} + 2b\bar{b}), \end{aligned} \quad (\text{A1})$$

where the subscripts [111] and [21] stand for color-singlet ($\mathbf{1}_c$) and color-octet ($\mathbf{8}_c$), respectively. So, the $SU(3)_{\text{color}}$ wave functions of color-singlet (two color-singlet clusters, $\mathbf{1}_c \otimes \mathbf{1}_c$) and hidden-color (two color-octet clusters, $\mathbf{8}_c \otimes \mathbf{8}_c$) channels are given, respectively, by

$$\begin{aligned} \chi_1^c &= C_{[111]}^1 C_{[111]}^1, \\ \chi_2^c &= \sqrt{\frac{1}{8}} \left(C_{[21]}^2 C_{[21]}^7 - C_{[21]}^4 C_{[21]}^5 - C_{[21]}^3 C_{[21]}^6 \right. \\ &\quad + C_{[21]}^8 C_{[21]}^8 - C_{[21]}^6 C_{[21]}^3 + C_{[21]}^9 C_{[21]}^9 \\ &\quad \left. - C_{[21]}^5 C_{[21]}^4 + C_{[21]}^7 C_{[21]}^2 \right). \end{aligned} \quad (\text{A2})$$

For the diquark-antidiquark structure, the color wave functions of the diquark clusters are given as

$$\begin{aligned} C_{[2]}^1 &= rr, \quad C_{[2]}^2 = \sqrt{\frac{1}{2}}(rg + gr), \\ C_{[2]}^3 &= gg, \quad C_{[2]}^4 = \sqrt{\frac{1}{2}}(rb + br), \\ C_{[2]}^5 &= \sqrt{\frac{1}{2}}(gb + bg), \quad C_{[2]}^6 = bb, \\ C_{[11]}^7 &= \sqrt{\frac{1}{2}}(rg - gr), \quad C_{[11]}^8 = \sqrt{\frac{1}{2}}(rb - br), \\ C_{[11]}^9 &= \sqrt{\frac{1}{2}}(gb - bg), \end{aligned} \quad (\text{A3})$$

while the color wave functions of the antidiquark clusters can be written as

$$\begin{aligned} C_{[22]}^1 &= \bar{r}\bar{r}, \quad C_{[22]}^2 = -\sqrt{\frac{1}{2}}(\bar{r}\bar{g} + \bar{g}\bar{r}), \\ C_{[22]}^3 &= \bar{g}\bar{g}, \quad C_{[22]}^4 = \sqrt{\frac{1}{2}}(\bar{r}\bar{b} + \bar{b}\bar{r}), \\ C_{[22]}^5 &= -\sqrt{\frac{1}{2}}(\bar{g}\bar{b} + \bar{b}\bar{g}), \quad C_{[22]}^6 = \bar{b}\bar{b}, \\ C_{[211]}^7 &= \sqrt{\frac{1}{2}}(\bar{r}\bar{g} - \bar{g}\bar{r}), \quad C_{[211]}^8 = -\sqrt{\frac{1}{2}}(\bar{r}\bar{b} - \bar{b}\bar{r}), \\ C_{[211]}^9 &= \sqrt{\frac{1}{2}}(\bar{g}\bar{b} - \bar{b}\bar{g}). \end{aligned} \quad (\text{A4})$$

The color-singlet wave functions of the diquark-antidiquark configuration can be the product of color-sextet and antisextet clusters ($\mathbf{6}_c \otimes \bar{\mathbf{6}}_c$) or the product of color-triplet and antitriplet clusters ($\mathbf{3}_c \otimes \bar{\mathbf{3}}_c$), which read

$$\begin{aligned} \chi_3^c &= \sqrt{\frac{1}{6}} \left(C_{[2]}^1 C_{[22]}^1 - C_{[2]}^2 C_{[22]}^2 + C_{[2]}^3 C_{[22]}^3 \right. \\ &\quad \left. + C_{[2]}^4 C_{[22]}^4 - C_{[2]}^5 C_{[22]}^5 + C_{[2]}^6 C_{[22]}^6 \right), \\ \chi_4^c &= \sqrt{\frac{1}{3}} \left(C_{[11]}^7 C_{[211]}^7 - C_{[11]}^8 C_{[211]}^8 + C_{[11]}^9 C_{[211]}^9 \right). \end{aligned} \quad (\text{A5})$$

2. The flavor wave function

For the flavor degree of freedom, the different coupling methods generate different flavor wave functions. From Table 1, the $ss\bar{n}\bar{n}$ tetraquark flavor wave function can be categorized as F_i^j , where the subscript “T” means the total isospin of $ss\bar{n}\bar{n}$ tetraquark states. Distinctive structures make the quark coupling arrange.

For the $ss\bar{n}\bar{n}$ system, according to Fig. 1, the isospins of the $ss\bar{n}\bar{n}$ system are 0 and 1, respectively. For the meson-meson structure, the coupling orders can be accessed, which are

$$F_0^1 = -\sqrt{\frac{1}{2}}(s\bar{d}s\bar{u} - s\bar{u}s\bar{d}),$$

$$F_1^2 = (s\bar{d}) - (s\bar{d}).$$

For the diquark-antidiquark structure, the flavor wave function should be written as

$$F_0^3 = -\sqrt{\frac{1}{2}}(ss\bar{d}\bar{u} - ss\bar{u}\bar{d}),$$

$$F_1^4 = (ss) - (\bar{d}\bar{d}).$$

3. The spin wave function

For the spin, the total spin S of tetraquark states ranges from 0 to 2. All of them are considered. The wave functions of two-body clusters are

$$\begin{aligned}\chi_{11} &= \alpha\alpha, \\ \chi_{10} &= \sqrt{\frac{1}{2}}(\alpha\beta + \beta\alpha), \\ \chi_{1-1} &= \beta\beta, \\ \chi_{00} &= \sqrt{\frac{1}{2}}(\alpha\beta - \beta\alpha).\end{aligned}\quad (\text{A6})$$

Then, the total spin wave functions S_s^i are obtained by considering the coupling of two subcluster spin wave functions with SU(2) algebra, and the total spin wave functions of four-quark states can be read as

$$\begin{aligned}S_0^1 &= \chi_{00}\chi_{00}, \\ S_0^2 &= \sqrt{\frac{1}{3}}(\chi_{11}\chi_{1-1} - \chi_{10}\chi_{10} + \chi_{1-1}\chi_{11}), \\ S_1^3 &= \chi_{00}\chi_{11}, \\ S_1^4 &= \chi_{11}\chi_{00}, \\ S_1^5 &= \sqrt{\frac{1}{2}}(\chi_{11}\chi_{10} - \chi_{10}\chi_{11}), \\ S_2^6 &= \chi_{11}\chi_{11}.\end{aligned}\quad (\text{A7})$$

4. Pauli constraints on the allowed channels

For the $ss\bar{n}\bar{n}$ tetraquark system, particles 1 and 3 are two identical s quarks, while particles 2 and 4 are two identical light antiquarks. Therefore, the total wave function must satisfy

$$P_{13}\Psi = -\Psi, \quad P_{24}\Psi = -\Psi.$$

Since the present work is restricted to the S -wave case, the spatial wave function is symmetric under both exchanges. Accordingly, the required antisymmetry must be provided by the combined color-spin-flavor part.

For the meson-meson configuration, each meson cluster is a color singlet, so the color part is symmetric under the exchange of the two mesons. Since the spatial part is also symmetric in the S -wave case, the combined flavor-spin part must be symmetric for channels composed of two identical mesons. This immediately explains the excluded channels. In the $I = 0$ sector, the flavor wave function is antisymmetric, whereas the spin wave functions of the $\bar{K}\bar{K}$ basis and of the $\bar{K}^*\bar{K}^*$ bases with total spin $S = 0$ or $S = 2$ are symmetric. Therefore, the $I(J^P) = 0(0^+)$ and $0(2^+)$ meson-meson channels are not allowed. In the $I = 1$ sector, the flavor wave function is symmetric, but for two-vector mesons the spin wave function with $S_1 \otimes S_2 = 1 \otimes 1 \rightarrow 1$ is antisymmetric. Hence, no $\bar{K}^*\bar{K}^*$ meson-meson basis is retained in the $I = 1(1^+)$ sector after antisymmetrization.

For the diquark-antidiquark configuration, the Pauli constraints can be analyzed explicitly at the pair level. For the ss pair, the flavor and spatial wave functions are both symmetric, so the color-spin part must be antisymmetric. This allows only

$$(\bar{3}_c, S_{ss} = 1), \quad (6_c, S_{ss} = 0).$$

For the $\bar{n}\bar{n}$ pair, the allowed combinations depend on the isospin. When $I = 0$, the flavor wave function is antisymmetric, so the color-spin part must be symmetric, leading to

$$(3_c, S_{\bar{n}\bar{n}} = 0), \quad (\bar{6}_c, S_{\bar{n}\bar{n}} = 1).$$

When $I = 1$, the flavor wave function is symmetric, so the color-spin part must be antisymmetric, leading to

$$(3_c, S_{\bar{n}\bar{n}} = 1), \quad (\bar{6}_c, S_{\bar{n}\bar{n}} = 0).$$

These symmetry constraints explain the allowed channels in Table I. In the $I = 0$ sector, only the combinations

$$\begin{aligned}(\bar{3}_c, S_{ss} = 1) \otimes (3_c, S_{\bar{n}\bar{n}} = 0), \\ (6_c, S_{ss} = 0) \otimes (\bar{6}_c, S_{\bar{n}\bar{n}} = 1)\end{aligned}$$

can form a color-singlet tetraquark state, and both of them give only total spin $S = 1$. Therefore, only $I(J^P) = 0(1^+)$ is Pauli-allowed, while $0(0^+)$ and $0(2^+)$ are excluded. In the $I = 1$ sector, the basis

$$(ss)_{\bar{3}_c}^{S=1}(\bar{n}\bar{n})_{3_c}^{S=1}$$

with $S_1 \otimes S_2 = 1 \otimes 1 \rightarrow 1$ is Pauli-allowed and can form a color-singlet tetraquark state. By contrast, the basis

$$(ss)_{\bar{3}_c}^{S=1}(\bar{n}\bar{n})_{6_c}^{S=0}$$

with $S_1 \otimes S_2 = 1 \otimes 0 \rightarrow 1$ is excluded because it cannot couple to a color-singlet tetraquark state. Therefore, the channels retained in Table I are precisely those satisfying the Pauli principle in the present S -wave calculation.

- [1] M. Gell-Mann, *Phys. Lett.* **8**, 214 (1964).
- [2] *Developments in the Quark Theory of Hadrons. Vol. 1. 1964–1978*, edited by D. B. Lichtenberg and S. P. Rosen (Hadronic Press, Nonantum, 1980).
- [3] R. L. Workman *et al.* (Particle Data Group), *Prog. Theor. Exp. Phys.* **2022**, 083C01 (2022).
- [4] C. A. Meyer and E. S. Swanson, *Prog. Part. Nucl. Phys.* **82**, 21 (2015).
- [5] H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, *Phys. Rep.* **639**, 1 (2016).
- [6] H. Clement, *Prog. Part. Nucl. Phys.* **93**, 195 (2017).
- [7] F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, and B.-S. Zou, *Rev. Mod. Phys.* **90**, 015004 (2018); **94**, 029901(E) (2022).
- [8] Y.-R. Liu, H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, *Prog. Part. Nucl. Phys.* **107**, 237 (2019).
- [9] N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C.-P. Shen, C. E. Thomas, A. Vairo, and C.-Z. Yuan, *Phys. Rep.* **873**, 1 (2020).
- [10] H.-X. Chen, W. Chen, X. Liu, Y.-R. Liu, and S.-L. Zhu, *Rep. Prog. Phys.* **86**, 026201 (2023).
- [11] M.-Z. Liu, Y.-W. Pan, Z.-W. Liu, T.-W. Wu, J.-X. Lu, and L.-S. Geng, *Phys. Rep.* **1108**, 1 (2025).
- [12] S. K. Choi *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **91**, 262001 (2003).
- [13] B. Aubert *et al.* (BABAR Collaboration), *Phys. Rev. D* **71**, 071103 (2005).
- [14] D. Acosta *et al.* (CDF Collaboration), *Phys. Rev. Lett.* **93**, 072001 (2004).
- [15] T. Aaltonen *et al.* (CDF Collaboration), *Phys. Rev. Lett.* **103**, 152001 (2009).
- [16] V. M. Abazov *et al.* (D0 Collaboration), *Phys. Rev. Lett.* **93**, 162002 (2004).
- [17] R. Aaij *et al.* (LHCb Collaboration), *Eur. Phys. J. C* **72**, 1972 (2012).
- [18] S. Chatrchyan *et al.* (CMS Collaboration), *J. High Energy Phys.* **04** (2013) 154.
- [19] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. Lett.* **111**, 242001 (2013).
- [20] Z. Q. Liu *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **110**, 252002 (2013); **111**, 019901(E) (2013).
- [21] F.-L. Wang, X.-D. Yang, R. Chen, and X. Liu, *Phys. Rev. D* **104**, 094010 (2021).
- [22] X. Liu, *Chin. Sci. Bull.* **59**, 3815 (2014).
- [23] A. Hosaka, T. Iijima, K. Miyabayashi, Y. Sakai, and S. Yasui, *Prog. Theor. Exp. Phys.* **2016**, 062C01 (2016).
- [24] J.-M. Richard, *Few Body Syst.* **57**, 1185 (2016).
- [25] R. F. Lebed, R. E. Mitchell, and E. S. Swanson, *Prog. Part. Nucl. Phys.* **93**, 143 (2017).
- [26] S. L. Olsen, T. Skwarnicki, and D. Zieminska, *Rev. Mod. Phys.* **90**, 015003 (2018).
- [27] L. Meng, B. Wang, G.-J. Wang, and S.-L. Zhu, *Phys. Rep.* **1019**, 1 (2023).
- [28] R. Aaij *et al.* (LHCb Collaboration), *Nat. Phys.* **18**, 751 (2022).
- [29] W.-X. Zhang, H. Xu, and D. Jia, *Phys. Rev. D* **104**, 114011 (2021).
- [30] X.-Z. Weng, W.-Z. Deng, and S.-L. Zhu, *Chin. Phys. C* **46**, 013102 (2022).
- [31] T. Guo, J. Li, J. Zhao, and L. He, *Phys. Rev. D* **105**, 014021 (2022).
- [32] X.-Y. Liu, W.-X. Zhang, and D. Jia, *Phys. Rev. D* **108**, 054019 (2023).
- [33] Q. Meng, E. Hiyama, M. Oka, A. Hosaka, and C. Xu, *Phys. Lett. B* **846**, 138221 (2023).
- [34] S. Noh and W. Park, *Phys. Rev. D* **108**, 014004 (2023).
- [35] H. Mutuk, *Eur. Phys. J. C* **84**, 395 (2024).
- [36] D. Wang, K.-R. Song, W.-L. Wang, and F. Huang, *Phys. Rev. D* **109**, 074026 (2024).
- [37] D. Park and S. H. Lee, [arXiv:2406.00756](https://arxiv.org/abs/2406.00756).
- [38] S.-Y. Li, Y.-R. Liu, Z.-L. Man, Z.-G. Si, and J. Wu, *Phys. Rev. D* **110**, 094044 (2024).
- [39] L. Meng, Y.-K. Chen, Y. Ma, and S.-L. Zhu, *Phys. Rev. D* **108**, 114016 (2023).
- [40] Y. Ma, L. Meng, Y.-K. Chen, and S.-L. Zhu, *Phys. Rev. D* **109**, 074001 (2024).
- [41] T. Asanuma, Y. Yamaguchi, and M. Harada, *Phys. Rev. D* **110**, 074030 (2024).
- [42] M. Sakai and Y. Yamaguchi, *Phys. Rev. D* **109**, 054016 (2024).
- [43] L. Qiu, C. Gong, and Q. Zhao, *Phys. Rev. D* **109**, 076016 (2024).
- [44] M.-J. Zhao, Z.-Y. Wang, C. Wang, and X.-H. Guo, *Phys. Rev. D* **105**, 096016 (2022).
- [45] H.-W. Ke, X.-H. Liu, and X.-Q. Li, *Eur. Phys. J. C* **82**, 144 (2022).
- [46] M. Padmanath and S. Prelovsek, *Phys. Rev. Lett.* **129**, 032002 (2022).
- [47] Y. Lyu, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, and J. Meng, *Phys. Rev. Lett.* **131**, 161901 (2023).
- [48] L.-Y. Dai, X. Sun, X.-W. Kang, A. P. Szczepaniak, and J.-S. Yu, *Phys. Rev. D* **105**, L051507 (2022).
- [49] Y. Liu, M. A. Nowak, and I. Zahed, *Phys. Rev. D* **105**, 054021 (2022).
- [50] F.-L. Wang, S.-Q. Luo, R.-Q. Qian, and X. Liu, *Phys. Rev. D* **110**, 114041 (2024).
- [51] J. Ji, Y. Xing, X. Wu, N. Xu, and Y. Tan, *Chin. Phys. C* **49**, 013101 (2025).
- [52] S. R. Beane, T. C. Luu, K. Orginos, A. Parreno, M. J. Savage, A. Torok, and A. Walker-Loud (NPLQCD Collaboration), *Phys. Rev. D* **77**, 094507 (2008).
- [53] Y. Kanada-En'yo and D. Jido, *Phys. Rev. C* **78**, 025212 (2008).
- [54] N. V. Shevchenko and J. Haidenbauer, *Phys. Rev. C* **92**, 044001 (2015).
- [55] R. Y. Kezerashvili, S. M. Tsiklauri, I. N. Filikhin, V. M. Suslov, and B. Vlahovic, *EPJ Web Conf.* **113**, 07005 (2016).
- [56] S. Marri, S. Z. Kalantari, and J. Esmaili, *Eur. Phys. J. A* **52**, 361 (2016).
- [57] N. Barnea, A. Gal, and E. Z. Liverts, *Phys. Lett. B* **712**, 132 (2012).
- [58] A. De Rujula, H. Georgi, and S. L. Glashow, *Phys. Rev. D* **12**, 147 (1975).
- [59] N. Isgur and G. Karl, *Phys. Rev. D* **18**, 4187 (1978).
- [60] N. Isgur and G. Karl, *Phys. Rev. D* **19**, 2653 (1979); **23**, 817 (E) (1981).
- [61] N. Isgur and G. Karl, *Phys. Rev. D* **20**, 1191 (1979).
- [62] M. Chen, H. Huang, J. Ping, and F. Wang, *Phys. Rev. C* **83**, 015202 (2011).

- [63] J. L. Ping, F. Wang, G. H. Wu, L. J. Teng, and J. T. Goldman, in *13th International Conference on Particles and Nuclei* (World Scientific, Singapore, 1993), pp. 526–528.
- [64] F. Wang, D. Qing, P. Xu, and J.-L. Ping, *Nucl. Phys.* **A631**, 462C (1998).
- [65] Y. Xue, X. Jin, H. Huang, and J. Ping, *Phys. Rev. D* **103**, 054010 (2021).
- [66] M. Kamimura, *Nucl. Phys.* **A351**, 456 (1981).
- [67] M. Kamimura, *Prog. Theor. Phys. Suppl.* **62**, 236 (1977).
- [68] E. Hiyama, M. Kamimura, A. Hosaka, H. Toki, and M. Yahiro, *Phys. Lett. B* **633**, 237 (2006).
- [69] E. Hiyama, A. Hosaka, M. Oka, and J.-M. Richard, *Phys. Rev. C* **98**, 045208 (2018).
- [70] Z. Xia, S. Fan, X. Zhu, H. Huang, and J. Ping, *Phys. Rev. C* **105**, 025201 (2022).
- [71] X. Jin, Y. Xue, H. Huang, and J. Ping, *Eur. Phys. J. C* **80**, 1083 (2020).
- [72] X. Liu, H. Huang, J. Ping, D. Chen, and X. Zhu, *Eur. Phys. J. C* **81**, 950 (2021).
- [73] X. Liu, D. Chen, H. Huang, J. Ping, X. Chen, and Y. Yang, *Sci. China Phys. Mech. Astron.* **66**, 221012 (2023).
- [74] H. Huang, P. Xu, J. Ping, and F. Wang, *Phys. Rev. C* **84**, 064001 (2011).
- [75] H. Huang, J. Ping, and F. Wang, *Phys. Rev. C* **89**, 034001 (2014).