

Gauge-independent gravitational waves fromogenesis in a $B-L$ conserving universe

Wan-Zhe Feng^{1,*}, Jinzheng Li^{2,†}, Pran Nath^{2,‡} and Zong-Huan Ye^{1,§}

¹Center for Joint Quantum Studies and Department of Physics, School of Science, Tianjin University, Tianjin 300350, People's Republic of China

²Department of Physics, Northeastern University, Boston, Massachusetts 02115-5000, USA



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An analysis of baryogenesis and stochastic gravitational wave production is presented for an extension of the Standard Model where the dark sector consists of dark matter particles charged under a $U(1)_x$ gauge symmetry, while a subset of dark fields also carry lepton number but no $U(1)_x$ charge. We demonstrate that, with CP violation induced by Yukawa couplings, equal and opposite lepton asymmetries are generated in the visible and hidden sectors. Subsequent evolution preserves the lepton number separately in each sector, and sphaleron interactions partially convert the lepton asymmetry into baryon asymmetry near the temperature of the first-order phase transition. Furthermore, we discuss stochastic gravitational wave background production for the first-order phase transition using a gauge-independent bubble nucleation dynamics which yields spectra also valid in the supercooled low-temperature regime with $T_p/m_{A_x} \ll 1$, where T_p is the percolation temperature and m_{A_x} is the dark photon mass. A parameter-space scan identifies regions that simultaneously account for cogenesis of baryon asymmetry and dark matter and predict stochastic gravitational wave signals within reach of current (NANOGrav, EPTA, and PPTA) and future detectors at higher frequencies, providing a unified framework for cogenesis and associated gravitational wave production.

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I. INTRODUCTION

After the discovery of gravitational waves by the LIGO Collaboration in 2016 [1], the study of gravitational waves has established itself as a new tool for the exploration of fundamental physics. Thus, while the observation by LIGO was confined to the observation of gravitational waves arising from black hole mergers, the broader study now is related to stochastic backgrounds of gravitational waves. Recently, several pulsar timing array (PTA) collaborations, including NANOGrav [2], EPTA [3], and PPTA [4], have reported strong evidence for a stochastic gravitational wave background at nanohertz frequencies. These observations have sparked significant interest in cosmological interpretations involving new physics beyond the Standard Model [5,6]. Thus, such backgrounds can arise from a variety of

sources such as from primordial black holes and neutron stars, the end of inflation decay of the inflaton [7–9], and cosmic phase transitions in the early Universe [10–14]. An interesting possibility that arises in this case is that such cosmic phase transitions near the electroweak scale [15–59] may lead to the production of gravitational waves. Another important phenomena at this scale is related to the possibility for baryogenesis with sufficient strength consistent with observation [60–65]. The Standard Model alone fails to provide adequate solutions to either phenomena. Thus, the electroweak baryogenesis within the Standard Model cannot generate a sufficient CP violation to explain the observed baryon asymmetry in the Universe, and the first-order phase transitions in the Standard Model do not produce gravitational waves at detectable levels [66–68]. These inadequacies motivate extensions of physics beyond the Standard Model involving hidden sectors, which appear naturally in supergravity, string theory, D-brane constructions, and quiver theories [69]. In this case, the analysis of first-order phase transitions at the electroweak scale is significantly modified and a possibility exists for achieving both observable gravitational waves as well as the desired matter-antimatter asymmetry in the Universe in such a setup [70]. Furthermore, it is entirely possible that the hidden sector and the visible sector have interactions beyond the gravitational interactions. However, the success

*Contact author: vicf@tju.edu.cn

†Contact author: li.jinzh@northeastern.edu

‡Contact author: p.nath@northeastern.edu

§Contact author: y2953083702@tju.edu.cn

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of the Standard Model in explaining a large amount of electroweak data indicates that such interactions must remain small. Such small interactions between the hidden sector and the visible sector can come about via a variety of portals: the Higgs portal [71], kinetic mixing [72], Stueckelberg mass mixing [73], and mixed kinetic and Stueckelberg mechanisms [74].¹

In this work, we present an analysis ofogenesis, i.e., the simultaneous generation of baryon asymmetry and dark matter, within a $U(1)_x$ extension of the Standard Model where $B - L$ symmetry is conserved [80]. In this framework, both the observed matter-antimatter asymmetry and the dark matter abundance arise from out-of-equilibrium decays of heavy Majorana fermions that generate equal and opposite $B - L$ asymmetries in the visible and hidden sectors. Cogenesis mechanisms naturally yield asymmetric dark matter with a mass of the order of the proton mass, thereby explaining the observed similarity between the cosmic abundances of baryons and dark matter, $\Omega_{\text{DM}}/\Omega_B \simeq 5.5$ [81–87]. In the above framework, we further investigate production of gravitational waves from first-order phase transitions, which applies broadly to dark matter models featuring asymmetric dark matter with masses around $\mathcal{O}(\text{GeV})$. The analysis utilizes gauge-invariant computation of nucleation and gravitational wave production [88–93], particularly in the low-temperature regime ($T/m \lesssim 1$). In the analyses, we use the Metaxas-Weinberg approach and obtain robust predictions for bubble nucleation dynamics and the resulting gravitational wave power spectrum. Our parameter-space analysis indicates regions where the model can account for both the observed matter-antimatter asymmetry and the gravitational wave signals that may be within reach of the current and future experiments. We observe that parameter choices of particular phenomenological interest often involve supercooled or near-supercooled phase transitions with $T_p/m_{A_x} \ll 1$, where T_p is the percolation temperature.

The paper is organized as follows: Section II presents the $U(1)_x$ extended model and demonstrates the cogenesis mechanism, including asymmetry generation, neutrino mass generation, and dark matter dynamics. Section III develops the gauge-invariant analysis of first-order phase transitions and presents parameter-space scans showing regions of observability. Section IV concludes with a summary of our findings and their implications for future gravitational wave observations. In Appendix A, we derive the full Boltzmann equations for the decays of heavy Majorana fermions, including washout effects. In Appendix B, we present a benchmark model demonstrating

that an extended neutrino sector can naturally account for the observed neutrino masses and mixings. Appendix C discusses the kinetic mixing between the $U(1)_x$ dark photon and the Standard Model hypercharge gauge boson, including experimental constraints on the kinetic mixing parameter. Appendix D provides additional discussion of the gauge-invariant effective action for first-order phase transitions. Appendix E presents further details on phase transition physics, hydrodynamics, and gravitational wave production.

II. EXTENSION OF THE STANDARD MODEL TO INCLUDE A HIDDEN SECTOR

A. The model with a $B - L$ conserving hidden sector

The observed matter-antimatter asymmetry, along with the fact that dark matter and baryonic matter have abundances of the same order of magnitude $\Omega_{\text{DM}}h^2/\Omega_Bh^2 \approx 5.5$ [94], suggests a potential common origin in the early Universe. As proposed in 2014 [80], both the baryon asymmetry and dark matter relic abundance can arise in a framework where $B - L$ symmetry is conserved. In this paper, we extend this line of investigation by exploring the associated gravitational wave signatures and other phenomenological implications of such a scenario. We consider a $U(1)_x$ gauge extension of the Standard Model (SM), which consists of at least two Majorana fermions N_i , a massive Dirac field ψ , a complex scalar dark matter ϕ , a dark $U(1)_x$ Higgs field Φ_x , the $U(1)_x$ dark photon A_x , and two dark matter candidates X and X' which are Dirac fermions charged under $U(1)_x$ and also carry nonvanishing lepton number. The quantum numbers of these fields are summarized in the following table:

	L	H	N	ψ	ϕ	X	X'	Φ_x
L No.	+1	0	0	+1	-1	$+\frac{1}{2}$	$+\frac{1}{2}$	0
$B - L$	-1	0	0	-1	+1	$-\frac{1}{2}$	$-\frac{1}{2}$	0
$U(1)_x$	0	0	0	0	0	+1	-1	+1

The complete Lagrangian of the model is given by

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{\text{SM}} + \lambda_i \bar{N}_i \psi \phi + y_i^{\mu} \bar{\psi} L_i H + y_x \phi \bar{X}^c X' + \text{H.c.} \\ & + \frac{1}{2} \bar{N}_i (i \not{\partial} - M_i) N_i + \bar{\psi} i \not{\partial} \psi - m_{\psi} \bar{\psi} \psi + |\partial_{\mu} \phi|^2 \\ & - m_{\phi}^2 \phi^* \phi + \bar{X} (i \not{\partial} - m_{\text{DM}}) X + \bar{X}' (i \not{\partial} - m_{\text{DM}}) X' \\ & - \frac{1}{4} F_{x\mu\nu} F_x^{\mu\nu} - \frac{\epsilon}{2} F_{x\mu\nu} F^{\mu\nu} + |D_{\mu} \Phi_x|^2 - V_{\text{eff}}(\Phi_x), \quad (2.1) \end{aligned}$$

where $F_x^{\mu\nu}$ is the $U(1)_x$ field strength for the field A_x^{μ} and the corresponding covariant derivative of $U(1)_x$ is given by $D_{\mu} = \partial_{\mu} - i g_x A_{x\mu}$. Both X and X' are dark matter candidates which have the same mass m_{DM} , have similar lepton number and $B - L$ number, but are oppositely charged under $U(1)_x$. Their masses will be determined in Sec. II D.

¹The Stueckelberg mass mixing mechanism produces milli-charged dark matter [73–76] in the hidden sector, and a small amount ($\sim 0.3\%$) might explain the experiment to detect the global epoch of reionization signature (EDGES) anomaly [77], Stueckelberg-Higgs mixing [78], extra-weak interactions [79], or higher-dimensional operators.

In a universe where $B - L$ symmetry (not necessarily a gauge symmetry) is exactly conserved, theogenesis proceeds as follows: At high energies, heavy Majorana fermions N_i decay out of equilibrium, generating an asymmetry between the fields ψ and ϕ and their anti-particles. Because of the chargeless nature of N_i , and the fact that both ψ and ϕ carry nonzero $B - L$ charges, these decays produce equal and opposite $B - L$ asymmetries. The asymmetry is subsequently transferred to the visible sector via the decay $\psi \rightarrow L_i H$ and to the dark sector via $\phi \rightarrow XX'$, resulting in equal and opposite $B - L$ asymmetries stored in each sector. In the visible sector, the lepton asymmetry is partially converted into a baryon asymmetry through sphaleron processes. The similarity in the cosmic abundances of dark matter and baryons is naturally explained if the dark matter mass is of the same order as that of the proton. The symmetric components of $\mathcal{O}(\text{GeV})$ dark matter primarily annihilate into two $U(1)_x$ dark photons A_x , leaving the relic density dominated by the asymmetric component.

B. Baryogenesis via cogenesis

The key step for the cogenesis mechanism is relying on the heavy Majorana right-handed neutrinos decay into ψ and ϕ at high energies. The Yukawa coupling constant λ_i being complex ensures that the CP -violating decay of such a process produces the same amount of asymmetry in the ψ and in ϕ . The asymmetry of ψ then transforms to the Standard Model via the interaction term $\bar{\psi} L_i H$ and eventually disappears, and there would be no vestige left of them in the current Universe. In the dark sector, all ϕ particles eventually decay into dark matter. At later stages of the Universe, the symmetric component of dark matter X, X' efficiently annihilates, leaving the residual asymmetric component as the dominant constituent of dark matter. In Eq. (2.1), the couplings λ_i are assumed to be complex and the couplings y_i^ψ and y_x are assumed to be real. Thus, the only couplings that enter in generating the asymmetry are λ_i . The decay of chargeless Majorana fermion N_i will generate an equal and opposite net $B - L$ asymmetry in the field ψ and ϕ . The asymmetry generated via the decaying processes $N_i \rightarrow \psi + \phi$ is through the interference of the tree amplitude and loop diagrams and is given by [86,95]

$$\epsilon_L = \frac{\Gamma(N_1 \rightarrow \psi\phi) - \Gamma(N_1 \rightarrow \bar{\psi}\phi^*)}{\Gamma(N_1 \rightarrow \psi\phi) + \Gamma(N_1 \rightarrow \bar{\psi}\phi^*)} \simeq -\frac{1}{8\pi} \frac{\text{Im}(\lambda_1^2 \lambda_2^{*2})}{|\lambda_1|^2} \frac{M_1}{M_2}, \quad (2.2)$$

where we assume there are two Majorana fields N_1 and N_2 with N_2 mass M_2 being much larger than the N_1 mass M_1 , i.e., $M_2 \gg M_1$. When the temperature drops down below the mass of N_i , the out-of-equilibrium decay of N_i will occur, which will generate the asymmetry in ψ and

ϕ . The evolution of N_1 and the created net lepton number L , which is equal to the net Ψ number generated ($Y_L = Y_\Psi$), are governed by the following set of Boltzmann equations:

$$\begin{aligned} \frac{dY_{N_1}}{dT} = & \frac{1}{T\bar{H}} \left[\langle \Gamma_{N_1} \rangle (Y_{N_1} - Y_{N_1}^{\text{EQ}}) - \epsilon_L \langle \Gamma_{N_1} \rangle Y_{N_1}^{\text{EQ}} \times \frac{4\pi^2 g_{*S} Y_L}{15} \right] \\ & + \frac{2s}{T\bar{H}} (\langle \sigma v \rangle_{N_1 N_1 \rightarrow \phi\bar{\phi}} \\ & + \langle \sigma v \rangle_{N_1 N_1 \rightarrow \psi\bar{\psi}}) (Y_{N_1} Y_{N_1} - Y_{N_1}^{\text{EQ}} Y_{N_1}^{\text{EQ}}), \end{aligned} \quad (2.3)$$

$$\begin{aligned} \frac{dY_L}{dT} = & -\frac{1}{T\bar{H}} \left(\epsilon_L \langle \Gamma_{N_1} \rangle (Y_{N_1} - Y_{N_1}^{\text{EQ}}) - \frac{4\pi^2 g_{*S} Y_L}{15} \langle \Gamma_{N_1} \rangle Y_{N_1}^{\text{EQ}} \right) \\ & + \frac{s}{T\bar{H}} \frac{4\pi^2 g_{*S} Y_L}{15} (8 \langle \sigma v \rangle_{\psi\psi \rightarrow \bar{\phi}\bar{\phi}}^{\text{RIS}} n_\psi^{\text{EQ}} n_\psi^{\text{EQ}} \\ & + 4 \langle \sigma v \rangle_{\psi\phi \rightarrow \bar{\psi}\bar{\phi}}^{\text{RIS}} n_\psi^{\text{EQ}} n_\phi^{\text{EQ}}), \end{aligned} \quad (2.4)$$

where $s = 2\pi^2 h_{\text{eff}} T^3 / 45$ and

$$\bar{H} = \frac{H}{1 + \frac{1}{3} \frac{T}{h_{\text{eff}}} \frac{dh_{\text{eff}}}{dT}} = \sqrt{\frac{\pi^2 g_{\text{eff}}}{90}} \frac{T^2 / M_{\text{Pl}}}{1 + \frac{1}{3} \frac{T}{h_{\text{eff}}} \frac{dh_{\text{eff}}}{dT}}. \quad (2.5)$$

The detailed derivations of Eqs. (2.3) and (2.4), including all washout effects, are provided in Appendix A. The cross sections of processes $\psi\psi \rightarrow \bar{\phi}\bar{\phi}$ and $\psi\phi \rightarrow \bar{\psi}\bar{\phi}$ indicated by the superscript “RIS” have been RIS-subtracted to eliminate the double counting associated with the on-shell contribution of the intermediate N_1 propagator. After the sphaleron processes go out of equilibrium, at a temperature ~ 100 GeV by which the top quark has already decoupled from the thermal bath, the baryon number and lepton number become separately conserved, corresponding to the values observed in the present Universe. By analyzing the chemical potentials of the relevant particle species [85,96], one can determine that the current value of B_f in term of $(B - L)_v$ is given by $B_f / (B - L)_v = 30/97$, where B_f denotes the final (the currently observed) value of the baryon number density.² Hence, the converted baryon asymmetry is given by

$$Y_B = \frac{30}{97} Y_{B-L} = \frac{30}{97} Y_L, \quad (2.6)$$

²The commonly used conversion factor $\frac{28}{79}$ is derived under the assumption that all Standard Model fermions, including the top quark, remain in thermal equilibrium while sphaleron processes are active. However, it is known that the top quark decouples prior to the electroweak crossover, whereas sphaleron processes typically remain efficient down to temperatures around 100–130 GeV. Consequently, the top quark should be excluded from the chemical potential relations used to derive the baryon-to-lepton conversion factor, leading to a revised coefficient of $\frac{30}{97}$ [85,96].

which should match the experimental value $B/n_\gamma \sim 6 \times 10^{-10}$ [97]. The current entropy $s_0 = 7.04n_\gamma^0$ leads to $(B/s)_{\text{ex}} \simeq 8.6 \times 10^{-11}$, and, thus, $Y_L \simeq 3 \times 10^{-10}$. For $m_\psi, m_\phi \sim \mathcal{O}(\text{TeV})$, if $M_i \gg 10^6 \text{ GeV}$ and $|\lambda_i| \sim 10^{-5}$, the washout effect is less than 1% and is, thus, negligible. From above, we estimate the size of $\epsilon_L \sim 10^{-8}$, which can be easily manufactured using Eq. (2.2).

C. The active neutrino masses

The observed tiny neutrino mass is explained by introducing three right-handed Dirac neutrinos with the Yukawa interactions

$$\mathcal{L}_\nu = -y_i^\nu \bar{\nu}_{iR} L_i H + \text{H.c.} \quad (2.7)$$

We notice that the Yukawa coupling $\bar{\psi} L_i H$ also contributes to the neutrino mass term, and the right-handed neutrino can also mix with the left-handed component of ψ . Hence, the full mass terms in the neutrino sector can be written as

$$\mathcal{L}_\nu^{\text{mass}} = -y_{ij}^\nu \bar{\nu}_{jR} L_i H - y_i^\psi \bar{\psi}_R L_i H - \mu_i \bar{\nu}_{iR} \psi_L + \text{H.c.} - m \bar{\psi} \psi, \quad (2.8)$$

which generate the neutrino mass terms

$$\begin{aligned} \mathcal{L}_\nu^{\text{mass}} = & -m_{ij}^\nu (\bar{\nu}_{iL} \nu_{jR} + \bar{\nu}_{jR} \nu_{iL}) - m_i^\psi (\bar{\psi}_R \nu_{iL} + \bar{\nu}_{iL} \psi_R) \\ & - \mu_i (\bar{\psi}_L \nu_{iR} + \bar{\nu}_{iR} \psi_L) - m_\psi (\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R), \end{aligned} \quad (2.9)$$

where $m_{ij}^\nu = \frac{1}{\sqrt{2}} y_{ij}^\nu v_{\text{SM}}$ and $m_i^\psi = \frac{1}{\sqrt{2}} y_i^\psi v_{\text{SM}}$, and can be rewritten in the matrix form

$$\begin{aligned} \mathcal{L}_\nu^{\text{mass}} = & - \begin{pmatrix} \bar{\nu}_{eL} & \bar{\nu}_{\mu L} & \bar{\nu}_{\tau L} & \bar{\psi}_L \end{pmatrix} \\ & \times \begin{pmatrix} m_{ee}^\nu & m_{e\mu}^\nu & m_{e\tau}^\nu & \mu_e \\ m_{\mu e}^\nu & m_{\mu\mu}^\nu & m_{\mu\tau}^\nu & \mu_\mu \\ m_{\tau e}^\nu & m_{\tau\mu}^\nu & m_{\tau\tau}^\nu & \mu_\tau \\ m_e^\psi & m_\mu^\psi & m_\tau^\psi & m_\psi \end{pmatrix} \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \\ \psi_R \end{pmatrix} + \text{H.c.} \end{aligned} \quad (2.10)$$

A 4×4 unitary matrix U transforms the neutrino flavor eigenbasis to mass eigenbasis, and the top-left 3×3 block of the rotation matrix U should match the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix determined from neutrino oscillation data:

$$|U_{\text{PMNS}}^{\text{ex}}| = \begin{pmatrix} 0.803\text{--}0.845 & 0.514\text{--}0.578 & 0.142\text{--}0.155 \\ 0.233\text{--}0.505 & 0.460\text{--}0.693 & 0.630\text{--}0.799 \\ 0.262\text{--}0.525 & 0.473\text{--}0.702 & 0.610\text{--}0.762 \end{pmatrix}. \quad (2.11)$$

The masses of the three light neutrinos should also satisfy (i) the upper bound of neutrino mass sum, i.e.,

$$\sum_i m_i^\nu \lesssim 0.12 \text{ eV}, \quad (2.12)$$

and (ii) the experimental fitting values of the mass-square differences

$$\Delta m_{12}^2 = m_{\nu 2}^2 - m_{\nu 1}^2 = 7.54 \times 10^{-5} \text{ eV}^2, \quad (2.13)$$

$$\Delta m_{13}^2 = m_{\nu 3}^2 - m_{\nu 1}^2 = 2.47 \times 10^{-3} \text{ eV}^2. \quad (2.14)$$

We apply the particle swarm optimization algorithm to find the values of all the input parameters. The optimization is done by requiring the calculated values from seesaw mixing matrix matching all above-mentioned experiment values. To this end, we define six fitting functions, the first four of which capture the deviation of the calculated $|U_{\text{PMNS}}|$, neutrino mass sum, two mass-square differences to the respective experimental values. The fifth fitting function ensures that the nonunitary constraint is satisfied, which sets limits on the size of the mixing between the Standard Model neutrinos and the massive fermion ψ . Such a requirement is needed for considerable large mixings between the Standard Model neutrinos and extra massive fermions. We further define a sixth fitting function implementing the measured Dirac phase, resulting in complex-valued couplings y_i^ψ .

A set of parameters that reproduce all experimentally measured neutrino data is provided in Appendix B. In particular, we require the 4×4 neutrino mass matrix given in Eq. (2.10) to take the following form:

$$\begin{pmatrix} m_{ee}^\nu & 0 & 0 & \mu_e \\ 0 & m_{\mu\mu}^\nu & 0 & \mu_\mu \\ 0 & 0 & m_{\tau\tau}^\nu & \mu_\tau \\ m_e^\psi & m_\mu^\psi & m_\tau^\psi & m_\psi \end{pmatrix}, \quad (2.15)$$

which is remarkable in that the three Standard Model neutrino flavors initially exhibit no mixing in the upper-left 3×3 block of the mass matrix, and all observed mixing arises solely from the presence of the new particle ψ . Although the couplings y_i^ψ are complex, they do not generate additional asymmetry in the Standard Model leptonic sector via the decay $\psi \rightarrow LH$, due to the extremely strong washout effects arising from the mass of ψ that lies not far from the electroweak scale.

D. Dark matter budget

In the model, there are two dark matter candidates X and X' , which are produced equally from the decay of the heavy ϕ particle. The asymmetric component of X and X' constitutes the dominant contribution to the total dark

TABLE I. Benchmark models illustrating annihilation of the symmetric X component into pairs of sub-GeV dark photons. We list $\Omega_{\text{sym}} h^2$ and $\Omega_{\text{sym}'} h^2$, the relic abundances of the X symmetric component computed with and without the dark matter asymmetry. We consider two representative values for the kinetic mixing parameter, $\epsilon = 10^{-4}$ and $\epsilon = 10^{-9}$, which are shown as black lines within the allowed regions in Fig. 11. The kinetic mixing leads to a dark photon lifetime much shorter than 1 sec, so the dark photon decays before big bang nucleosynthesis (BBN).

Model	v_x [GeV]	g_x	m_{A_x}	ϵ	τ_{A_x} [sec]	$\Omega_{\text{sym}'} h^2$	$\Omega_{\text{sym}} h^2$
A	1	0.2	0.2	10^{-4}	7×10^{-16}	8.3×10^{-7}	$\ll 10^{-10}$
B	1	0.1	0.1	10^{-4}	2×10^{-15}	8.5×10^{-7}	$\ll 10^{-10}$
C	1	0.2	0.2	10^{-9}	7×10^{-6}	8.6×10^{-7}	$\ll 10^{-10}$
D	1	0.1	0.1	10^{-9}	2×10^{-5}	8.9×10^{-7}	$\ll 10^{-10}$

matter relic abundance, $\Omega_X h^2 = \Omega_{X'} h^2 \approx 0.06$. Using $\Omega_{\text{DM}} h^2 / \Omega_B h^2 \approx 5.5$ and Eq. (2.6), one finds $m_{\text{DM}} = 5.5 \times (30/97) \times (1/2) \approx 0.85$ GeV, where the $1/2$ factor arises because both X and X' carry a $B-L$ number $-1/2$. The symmetric component of X and X' can be efficiently reduced from the annihilation processes

$$\bar{X}X \rightarrow A_x A_x, \quad \bar{X}'X' \rightarrow A_x A_x, \quad (2.16)$$

into sub-GeV $U(1)_x$ dark photons A_x . Such a dark photon will decay into Standard Model fermion and antifermion pairs ($\bar{f}f$) because of the kinetic mixing between the Standard Model, detailed in Appendix C. The Boltzmann equations that govern the evolution of the

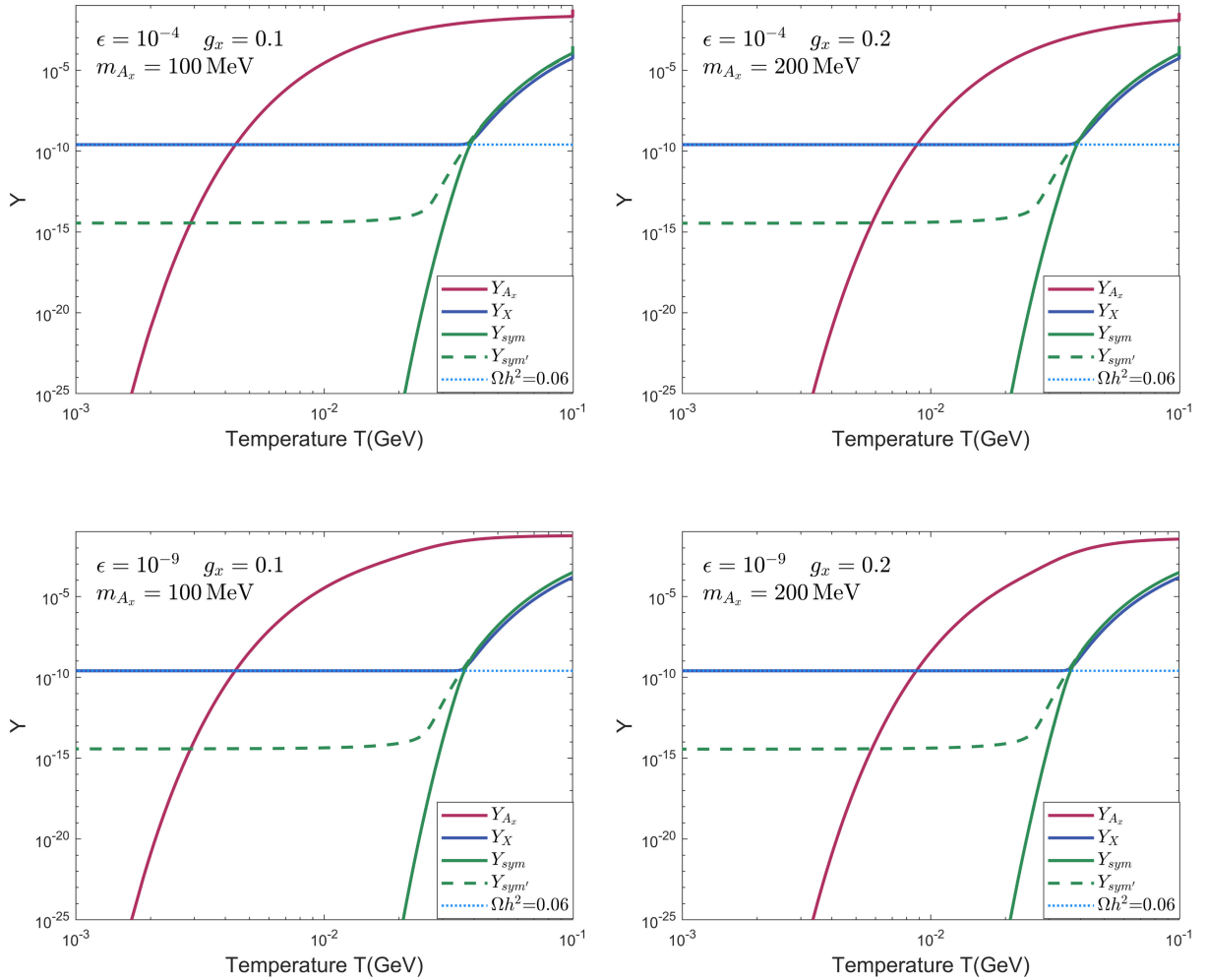


FIG. 1. Thermal evolution of Y_{A_x} , Y_X , Y_{sym} , and $Y_{\text{sym}'}$ for four benchmark models labeled by ϵ , g_x , and m_{A_x} as given in each of the four panels. The plots show the comoving number densities of asymmetric dark matter X , its symmetric component ($Y_{\text{sym}} = 2Y_{\bar{X}}$), and the dark photon A_x as functions of the universe temperature. $Y_{\text{sym}'}$ denotes the symmetric abundance of the dark matter in the absence of dark matter asymmetry, i.e., when $Y_X - Y_{\bar{X}} = 0$. The final abundance of Y_X corresponds to a relic density $\Omega_X h^2 = 0.06$, as shown by the light-blue dotted line, while X' contributes the other half. In the presence of dark matter asymmetry, the symmetric component is more efficiently annihilated through the process $\bar{X}X \rightarrow A_x A_x$. The $U(1)_x$ dark photon A_x which mixes with the hypercharge gauge field via a kinetic term with kinetic parameter ϵ will eventually decay into Standard Model fermion pairs before BBN.

dark matter particle X are given by (X' constitutes the other half of the dark matter relic density, and the computation of X' evolution is similar)

$$\frac{dY_X}{dT} = \frac{s}{T\bar{H}} (Y_X Y_{\bar{X}} \langle \sigma v \rangle_{\bar{X}X \rightarrow A_x A_x} - Y_{A_x}^2 \langle \sigma v \rangle_{A_x A_x \rightarrow \bar{X}X}), \quad (2.17)$$

$$\frac{dY_{\bar{X}}}{dT} = \frac{s}{T\bar{H}} (Y_X Y_{\bar{X}} \langle \sigma v \rangle_{\bar{X}X \rightarrow A_x A_x} - Y_{A_x}^2 \langle \sigma v \rangle_{A_x A_x \rightarrow \bar{X}X}), \quad (2.18)$$

$$\begin{aligned} \frac{dY_{A_x}}{dT} = \frac{s}{T\bar{H}} & \left(2Y_{A_x}^2 \langle \sigma v \rangle_{A_x A_x \rightarrow \bar{X}X} - 2Y_X Y_{\bar{X}} \langle \sigma v \rangle_{\bar{X}X \rightarrow A_x A_x} \right. \\ & \left. + \frac{1}{s} \langle \Gamma \rangle_{A_x \rightarrow \bar{f}f} + Y_f Y_{\bar{f}} \langle \sigma v \rangle_{\bar{f}f \rightarrow A_x} \right), \end{aligned} \quad (2.19)$$

where $Y_X - Y_{\bar{X}} = Y_L \simeq 3 \times 10^{-10}$, determined by the cogenesis and computed in Eq. (2.4).

We present the dark matter evolution for the four representative benchmarks in Table I, and the evolution of X and $\bar{X}$ and the dark photon A_x for the four models are shown in Fig. 1. The symmetric component of dark matter can be efficiently annihilated through the secluded process $\bar{X}X \rightarrow A_x A_x$, leaving the asymmetric component as the dominant contributor to the total dark matter relic density. Compared to the scenario without a dark matter asymmetry ($Y_X - Y_{\bar{X}} = 0$), the symmetric component is more efficiently depleted in the presence of an asymmetry.

Furthermore, we note that kinetic mixing between the hidden sector and the visible sector is tightly constrained for dark photon masses below 20 MeV [98,99]. In the analysis here, we compute the minimal $U(1)_x$ gauge coupling that yields a sufficient annihilation of the symmetric dark matter component, so that the constraint $\Omega_{\text{sym}} h^2 \lesssim 1\% \times 0.12$, for dark photon masses in the range 20 MeV–1 GeV is satisfied. The result of our analysis is

shown in Fig. 2, where the light-green regions indicate the allowed values of g_x for the case when the kinetic mixing parameter $\epsilon = 10^{-4}$, a value consistent with experimental bounds for dark photon masses in the range 20 MeV–1 GeV.

III. GAUGE-INVARIANT ANALYSIS OF FIRST-ORDER PHASE TRANSITION FROM A $U(1)_x$ GAUGE-INVARIANT HIDDEN SECTOR

While the model of Eq. (2.1) provides a possible cogenesis mechanism that explains the matter-antimatter asymmetry, a first-order phase transition could also occur, potentially generating observable stochastic gravitational waves. Since gravitational wave production is gauge dependent, we adopt the approach developed by Metaxas and Weinberg [88] to obtain a gauge-independent effective bounce action, as detailed in Appendix D. Using this approach, we compute the leading and the subleading actions, which are both gauge independent. Thus, the leading terms in the total action S_{eff} are given by

$$S_{\text{eff}} = S_0 + S_1, \quad (3.1)$$

$$S_0 = \beta \int d^3x \left[\frac{1}{2} (\partial_\mu \phi_b)^2 + V_{g_x^A}(\phi_b) \right], \quad (3.2)$$

$$S_1 = \beta \int d^3x \left[\frac{1}{2} Z_{g_x^2}(\phi_b, \xi) (\partial_\mu \phi_b)^2 + V_{g_x^S}(\phi_b, \xi) \right], \quad (3.3)$$

where the definitions of the quantities that enter in Eqs. (3.2) and (3.3) are given in Appendix D. Here, S_0 is gauge invariant by construction, and S_1 is gauge invariant due to Nielsen identity. Details are given in Appendix D.

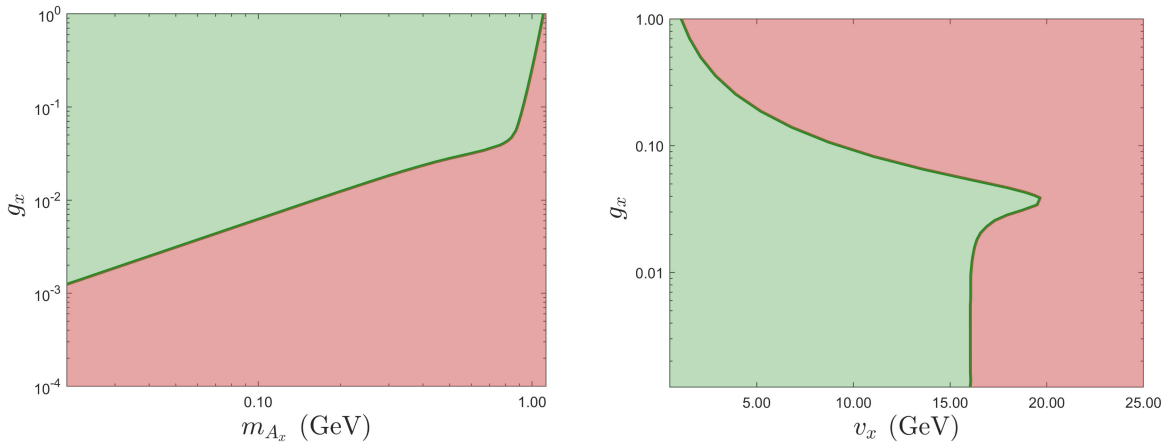


FIG. 2. Left panel: g_x versus the dark photon mass m_{A_x} . Right panel: g_x versus the vacuum expectation value of the $U(1)_x$ Higgs field, v_x . The shaded light-green regions in both panels indicate the parameter space of g_x that yields sufficient secluded annihilation of the symmetric dark matter component, for dark photon masses in the range 20 MeV–1 GeV. The kinetic mixing between $U(1)_x$ and the hypercharge gauge field is fixed at $\epsilon = 10^{-4}$, consistent with current experimental constraints.

TABLE II. Benchmark models for test of gauge invariance action up to one-loop order. Here, g_x is the $U(1)_x$ gauge coupling, λ_x is the Higgs self-coupling, and v_x is the vacuum expectation value, which are free parameters. T_p is the percolation temperature defined by Eq. (E2). T_p/m_{A_x} gives the temperature-mass ratio. We note that the model labeled * originates from a supercooled phase transition which remains valid after all the relevant constraints are satisfied.

Model	g_x	λ_x	v_x [GeV]	T_p [GeV]	T_p/m_{A_x}
*	0.58	2.21×10^{-3}	0.239	0.01	0.074
a	0.691	4.6×10^{-3}	1	0.093	0.13
b	0.400	1×10^{-3}	1	0.128	0.32
c	0.145	1×10^{-4}	1	0.135	0.93

For comparative analysis, we calculate the gauge-dependent effective bounce action employing the original effective potential given in Eq. (D39), with $Z(\phi) = 1$ and utilizing the tree-level mass for m_G , such that the gauge-dependent effective bounce action $S_{\text{eff}}^{\text{GD}}$ is defined so that

$$S_{\text{eff}}^{\text{GD}} = \beta \int dx^3 \left[\frac{1}{2} (\partial_\mu \phi)^2 + V_{\text{eff}}(\phi, \xi) \right]. \quad (3.4)$$

A standard gauge choice is the Landau gauge ($\xi = 0$), which is adopted in the majority of studies in the literature.

A. Numerical results using gauge-invariant effective bounce action

To implement this methodology, it is not necessary to solve for the complete profile of $\phi_b(x)$ with the renormalization factor $Z(\phi)$, as only the bounce solution for the leading-order term is required, where the leading order of $Z(\phi)$ is unity. Therefore, only a minor modification to the action S_1 calculation is needed. We, thus, use the CosmoTransition package with appropriate modifications for this analysis. Once we determined this gauge-invariant effective action, we follow the pipeline given in our previous paper [100] to finish the rest of the analysis on gravitational wave prediction. A brief summary is given in

Appendix E. One of the major differences here is that, in this paper, we have the hidden sector in thermalization with the visible sector during the phase transition, i.e., $T_h = T$.

We first examine the validity of this method under low-temperature conditions, i.e., $T/m_{A_x} < 1$. We evaluate the associated uncertainties of this approach across different temperature regimes, particularly for cases where $T/m_{A_x} \ll 1$ and $T/m_{A_x} \sim 1$. In Table II, we provide three benchmark models with different temperature to mass ratios. Also, we provide a benchmark model (*) which is an example of supercooled phase transition that stayed valid after all kind of constraints.

Using the parameter values specified in Table II, we present the computational results for $S_{\text{eff}}^{\text{GD}}$ with Landau gauge ($\xi = 0$), S_0 , and $S_0 + S_1$ in Fig. 3. The temperature is set to the percolation temperature T_p , which represents a critical parameter in gravitational wave analysis since it determines the onset of efficient bubble nucleation and sets the scale for the phase transition dynamics. Our calculations show that the sum $S_0 + S_1$ remains approximately constant across different values of the parameter ξ for all three examined cases, with a maximum deviation of only about 5%. This stability provides clear evidence for the successful derivation of a gauge-invariant effective action across all investigated temperature conditions where $T/m_{A_x} < 1$. By contrast, S_{eff} exhibits a pronounced and systematic dependence on ξ , with values that can vary by nearly an order of magnitude as ξ increases from 0 to 10. This spread illustrates the potential pitfalls of relying on gauge-dependent quantities, since even modest changes in the gauge parameter can lead to large shifts in the predicted nucleation dynamics and, by extension, in the resulting gravitational wave signals.

B. Gravitational wave prediction sensitivity on gauge dependence and gauge-invariant result

We perform further analysis of bubble nucleation and gravitational wave production using benchmark models in Table II, with the gauge-fixing parameter ξ chosen from the range $(-1, 1)$. The results are shown in Fig. 4. Our analysis demonstrates that variations in ξ across this parameter

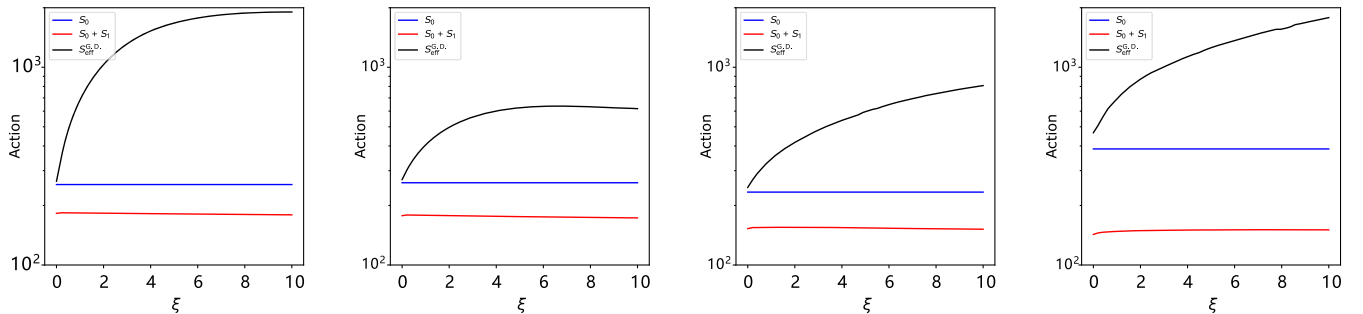


FIG. 3. A display of effective bounce actions $S_{\text{eff}}^{\text{GD}}$ defined in Eq. (3.4) in Landau gauge ($\xi = 0$), along with S_0 and $S_0 + S_1$ with different choices of ξ . From left to right, the models are *, a, b, and c, where the models are defined in Table II.

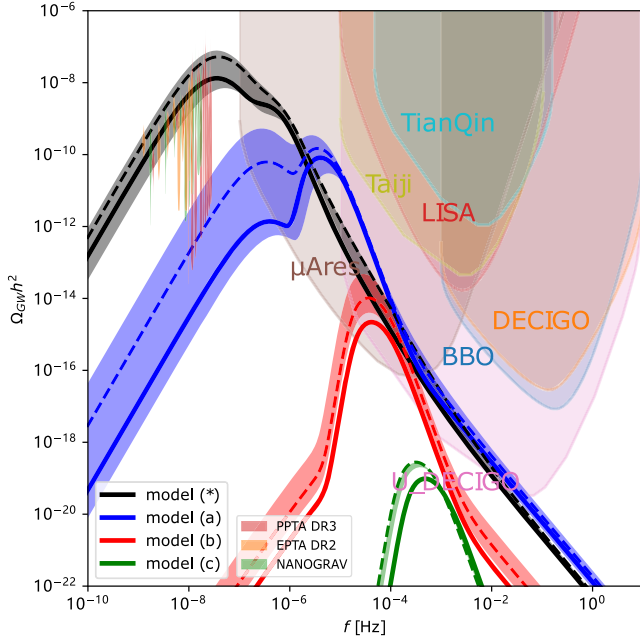


FIG. 4. Analysis of gravitational wave power spectra for models *, a, b, and c defined in Table II, which shows variations in the gravitational wave power spectrum $\Omega_{\text{GW}} h^2$ on the gauge parameter ξ across the range $(-1, 1)$ exhibited by the color bands for each model. The solid line displays the gauge-invariant solution. The dashed line displays the Landau gauge solution. Shaded regions represent pulsar timing array signals from NANOGrav [2], PPTA [4], and EPTA [3], while detection regions for planned space-based gravitational wave detectors are shown, including LISA [101–103], BBO [104], Decigo [105], Taiji [106], TianQin [107], and μ Ares [108].

range produce power spectrum curves that differ by several orders of magnitude. The Landau gauge selection typically employed in most of studies shows substantial differences from our gauge-invariant findings. These results emphasize the critical importance of implementing gauge-invariant approaches in such computations. From Fig. 4, we see that the spread in predictions due to gauge choice is not uniform across the spectrum. The gauge dependence is particularly large in regions where the gravitational wave power is the highest, i.e., where signals would be the most promising for detection by pulsar timing arrays or future space-based interferometers. This implies that the very signals most relevant for near-future experiments are also the most vulnerable to spurious gauge effects. Without gauge-invariant treatment, the predicted amplitude and peak frequency in these regions could be significantly misestimated, leading to false expectations about their observability. By contrast, the solid line in Fig. 4 represents the gauge-invariant result, which eliminates the ξ -dependent effects and provides a robust prediction.

The impact of gauge dependence on phase transition dynamics becomes particularly significant in the case of strongly supercooled transitions. At leading order, the

partial cancellation between ghosts and Goldstone boson contributions leads to only modest gauge dependence at next-to-leading order. However, when the potential barrier is extremely small, as in supercooled cases, even small gauge-induced variations can dramatically affect the nucleation rate and the resulting gravitational wave signal. This behavior can be clearly seen in Fig. 4: For the non-supercooled benchmark models (b and c), the spread due to gauge choice is minimal, whereas in the supercooled benchmarks (models * and a), gauge dependence becomes large. Previous studies such as Ref. [109] have analyzed theoretical uncertainties including gauge dependence mainly in the context of general nonsupercooled transitions where the effect is less pronounced. Our research highlights that, in the supercooled regime, gauge dependence can lead to significant error. This underscores the critical necessity of adopting gauge-invariant methods to obtain reliable predictions, especially for supercooled phase transitions.

C. Parameter scan for the $U(1)_x$ extended model

We examine the cosmological phase transition in a minimal $U(1)_x$ extension of the Standard Model. The one-loop finite-temperature effective potential depends on three parameters:

$$g_x, \quad \lambda_x, \quad v_x = \frac{\mu_x}{\sqrt{\lambda_x}}, \quad (3.5)$$

We conduct a parameter-space scan over the following ranges: $g_x \in (0.06, 0.8)$, $\lambda_x \in (10^{-5}, 10^{-2})$, and $v_x = 1 \text{ GeV}$. For comparison purposes, we apply the scan using the gauge-dependent effective action and select the Landau gauge ($\xi = 0$), which is the gauge choice employed in most other similar studies. Then, we apply the scan with the gauge-invariant analysis. The result is shown in Fig. 5. The scans illustrate how gauge treatment reshapes both the allowed region in the (g_x, λ_x) plane and the distribution of corresponding gravitational wave spectra. In the top panels, colored points indicate parameters for which a first-order transition nucleates and percolates; notably, the envelope and internal texture of the viable band change when moving from Landau gauge ($\xi = 0$) to the gauge-invariant analysis. In particular, part of the parameter space that appears allowed in the gauge-dependent case is removed once the gauge-invariant construction is imposed. The bottom panels convert these identical parameter values into gravitational wave power spectra. A notable observation is that the color gradient additionally demonstrates that numerous phenomenologically significant points concentrate near $T_p/m_{A_x} \leq 1$, confirming that these transitions take place at comparatively low temperatures. This indicates that the frequently employed high-temperature expansion ($T/m \gg 1$) may not be applicable in this context, and a low-temperature, gauge-invariant approach should be implemented instead.

The four panels in Fig. 6 show gauge-invariant gravitational wave spectra obtained with the same

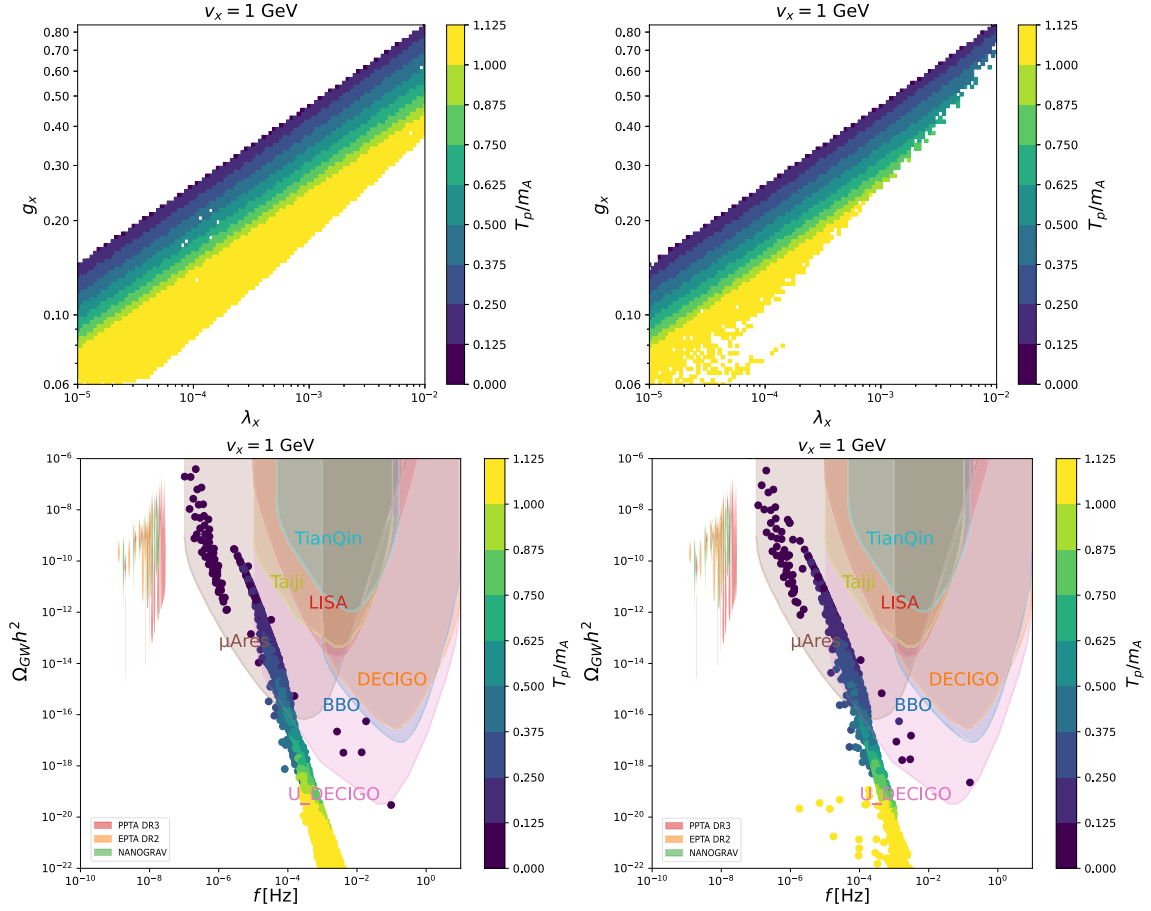


FIG. 5. Upper left panel: parameter scan within the $g_x - \lambda_x$ parameter plane, with v_x fixed at 1 GeV. It presents results obtained using gauge-dependent effective action with Landau gauge selection ($\xi = 0$). Upper right panel: parameter scan within the $g_x - \lambda_x$ parameter plane, with v_x fixed at 1 GeV. It presents results obtained using gauge-invariant analysis. The color coding along the left vertical axis indicates the ratio T_p/m_{A_x} for both left and right panels. Lower panels: theoretical predictions for gravitational wave power spectra $\Omega_{\text{GW}} h^2$ as a function of the frequency corresponding to the parameter configurations displayed in the upper panels. Each point marks the peak of the power spectrum.

scan ranges in (g_x, λ_x) but different symmetry-breaking scales $v_x = \{0.1, 1, 10, 100\}$ GeV. Across all choices of v_x , the colored points concentrate near $T_p/m_{A_x} \lesssim 1$, indicating that the relevant transitions occur in a low-

temperature regime. Varying v_x primarily reorganizes the distribution of peak frequencies and amplitudes across PTA-to-millihertz bands, which falls at different detection regions.

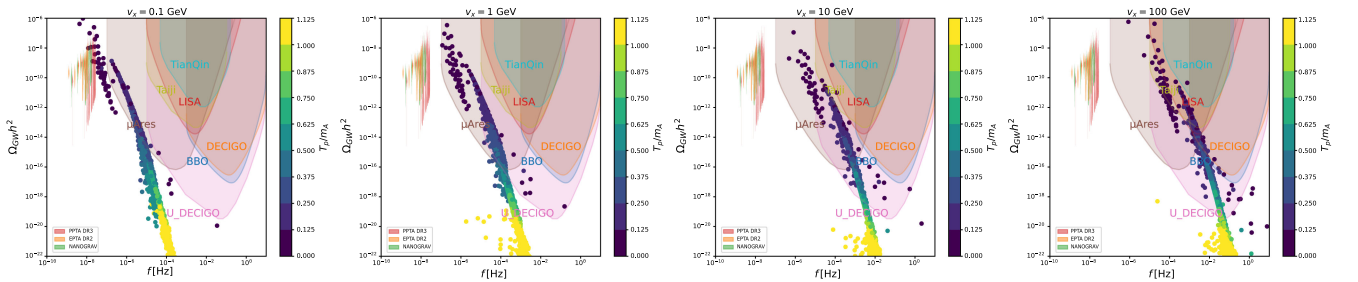


FIG. 6. Exhibition of the dependence of the gravitational wave power spectrum for four choices for the hidden sector vacuum expectation value v_x , where $v_x = (0.1, 1, 10, 100)$ GeV. Aside from using different values of v_x in each for the four cases, the same parameter scan range is chosen using gauge-invariant analysis. Each point marks the peak of the power spectrum. The color coding on the right vertical axis indicates the ratio T_p/m_{A_x} .

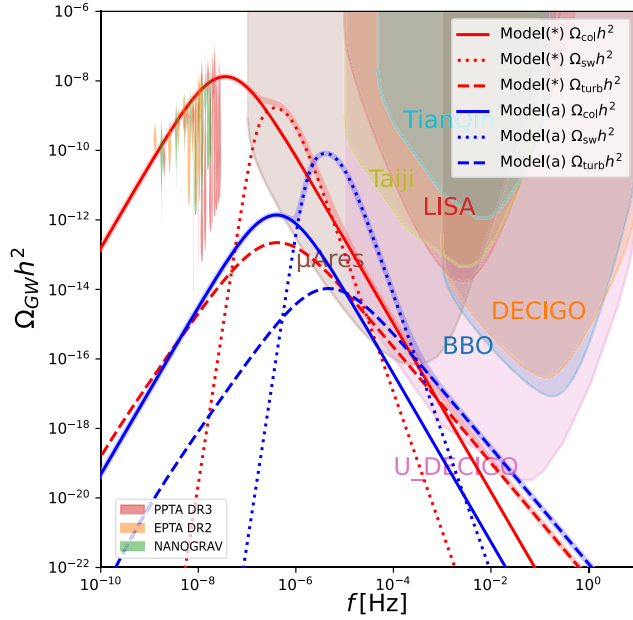


FIG. 7. Individual gravitational wave source contributions for two benchmark models (*) and (a) defined in Table II. For these model points, the plots show the relative contribution of bubble collisions (“col”), sound waves (“sw”), and turbulence (“turb”). Model * exemplifies a supercooled phase transition dominated by bubble collisions, while model a illustrates a case where sound waves dominate.

D. Contributions to the gravitational wave power spectrum

From Figs. 5 and 6 in the previous section, one may notice that the peak points of the gravitational wave power spectra tend to separate into two distinct patterns: one group characterized by lower frequencies but higher amplitudes and another with higher frequencies but lower amplitudes. This difference in patterns arises because the

dominant gravitational wave sources differ between these two cases. In the first case, the signal is dominated by direct bubble collisions, while in the second case the sound wave contribution plays the leading role.

Figure 7 illustrates this behavior explicitly by showing the relative contributions of different gravitational wave sources for benchmark model * and model a in Table II. For model *, the spectrum is largely shaped by the bubble collision component, while the sound wave and turbulence contributions remain subdominant. In contrast, for model a, the sound wave component provides the dominant contribution to the gravitational wave spectrum, yielding a higher-frequency peak but with reduced overall amplitude compared to model *. This finding is consistent with our earlier work [100], where it was shown that strongly supercooled phase transitions capable of generating signals within the sensitivity range of pulsar timing arrays are generally dominated by bubble-collision-induced gravitational waves. Meanwhile, scenarios with less supercooling and weaker latent heat tend to fall into the sound-wave-dominated regime, producing higher-frequency but comparatively weaker signals.

E. Validation range of the perturbative gauge-invariant method

As detailed in Appendix D, the gauge-invariant framework adopted in this work is constructed as a perturbative expansion, whose accuracy is expected to improve in the weak-coupling limit. In particular, the expansion is formally justified when the gauge coupling g_x is small, ensuring that higher-order corrections to the effective bounce action remain under control. However, parts of the parameter space relevant to our study extend into the regime of comparatively large couplings, $g_x \sim \mathcal{O}(1)$. This raises the question of whether the perturbative construction continues to provide a quantitatively reliable description in

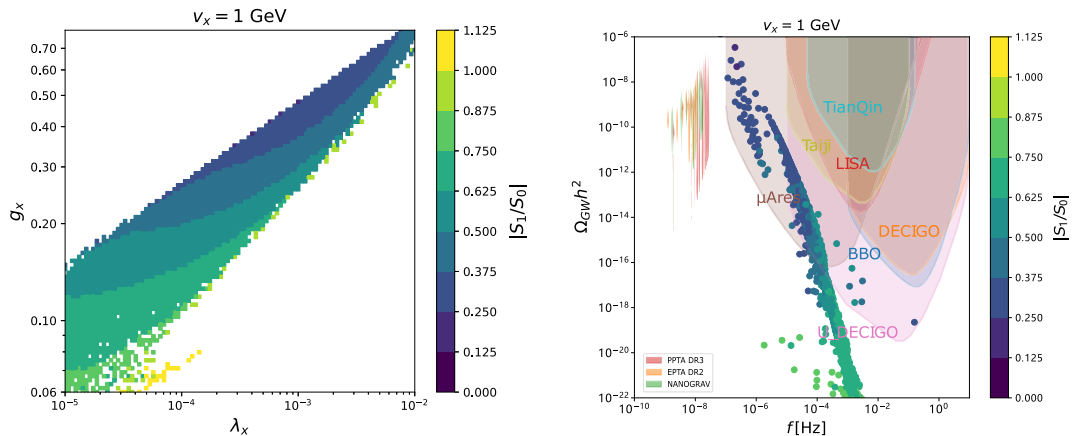


FIG. 8. Left panel: ratio $|S_1/S_0|$ at T_p across (g_x, λ_x) with $v_x = 1$ GeV. The phase-transition-completed region satisfies $|S_1/S_0| < 1$, indicating that next-to-leading corrections remain subdominant to the leading bounce action. Right panel: correlation between gravitational wave spectra and $|S_1/S_0|$ at $v_x = 1$ GeV. Models yielding the largest gravitational wave peak amplitudes preferentially exhibit smaller $|S_1/S_0|$, showing that perturbative control is strongest in the phenomenologically most relevant regions.

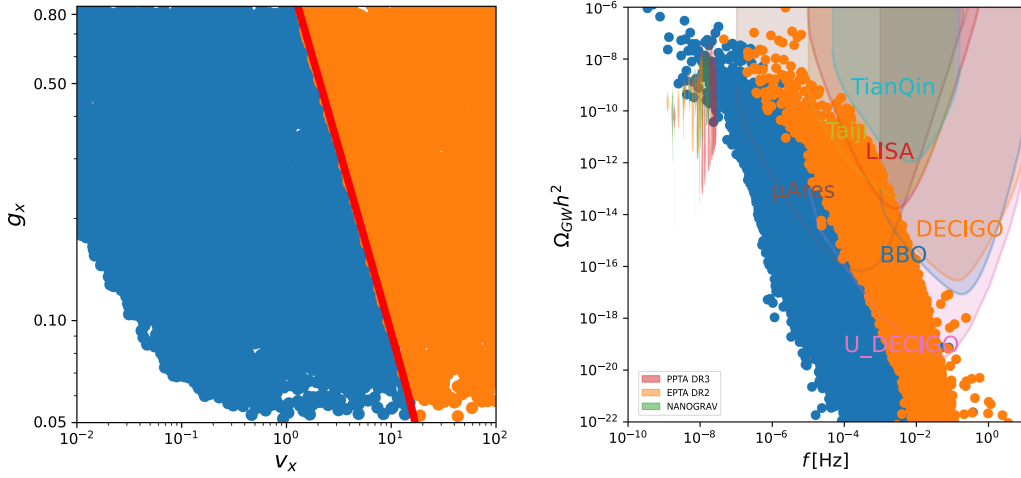


FIG. 9. Left panel: exhibition of the parameter space in the $(g_x - v_x)$ plane) allowed by the cogenesis constraint, ensuring adequate annihilation of the symmetric dark matter component. Blue points represent allowed models, while orange points are excluded. Right panel: gravitational wave power spectrum $\Omega_{\text{GW}} h^2$ corresponding to the left panel where the power spectrum is plotted vs the frequency f (Hz) and where the experimental reach of several experiments are also shown. It is seen that the blue region has intercept with the NANOGrav, EPTA, and PPTA data as well as can probe future data in the higher-frequency region.

such regions. To address this, a pragmatic consistency check is to verify that the next-to-leading correction S_1 to the bounce action remains subdominant relative to the leading-order term S_0 at the percolation temperature.

As shown in Fig. 8, the perturbative expansion is well controlled across the region in which the phase transition completes, with $|S_1/S_0| < 1$ broadly satisfied. Across the scanned range of g_x , we do not observe a monotonic decrease toward smaller g_x ; rather, bands of reduced $|S_1/S_0|$ appear at multiple g_x values (depending on λ_x), indicating that perturbative control is not determined by g_x alone. Even toward moderately strong coupling, $g_x \sim \mathcal{O}(1)$, the ratio remains below unity in the phenomenologically relevant swath of parameter space, supporting the quantitative reliability of the gauge-invariant perturbative framework. Importantly, the right panel shows that the subset of models producing the largest gravitational wave power systematically resides in the lower $|S_1/S_0|$ region. In other words, the points of greatest phenomenological interest are also those for which the perturbative expansion is most reliable. While the condition $|S_1/S_0| < 1$ is not by itself a strict proof of convergence, it provides a practical and widely adopted check that higher-order terms remain subdominant. The combined evidence in Fig. 8, together with the stability of observables across the explored parameter space, indicates that the perturbative gauge-invariant framework remains under good control, particularly in the regions of greatest phenomenological relevance.

F. Monte Carlo analysis and cogenesis constraint

In the end, we run a Monte Carlo analysis with the same scan range of g_x and λ_x as in Sec. III C, but with v_x extended to the range (0.01,100) GeV. The resulting

parameter points are then subjected to the cogenesis constraint shown in Fig. 2, which delineates the region of parameter space consistent with efficient secluded annihilation of the symmetric dark matter component. As illustrated in Fig. 9, the cogenesis requirement significantly narrows the viable parameter space. In particular, models that would otherwise reach the detection range of space-based interferometers such as LISA are excluded once the annihilation condition is imposed. However, a subset of models in the lower-frequency, higher-power region precisely those overlapping the PTA sensitivity bands remain viable.

TABLE III. Benchmark models that accommodate pulsar timing array signals while satisfying other experimental and cosmological constraints. The relic densities of the symmetric components of X and X' are computed to be $\Omega_{\text{sym}}^X h^2$ and $\Omega_{\text{sym}}^{X'} h^2 \ll 10^{-10}$. The dark matter abundance is, thus, overwhelmingly set by the asymmetric components. The BBN constraints are satisfied as all hidden sector species freeze out before $T = 0.1$ GeV.

Model	g_x	λ_x	v_x [GeV]	m_{A_x} [GeV]	T_p [GeV]	T_p/m_{A_x}
*	0.58	2.21×10^{-3}	0.239	0.14	0.01	0.074
1	0.629	3.07×10^{-3}	0.186	0.12	0.011	0.094
2	0.684	4.24×10^{-3}	0.183	0.13	0.013	0.11
3	0.752	6.01×10^{-3}	0.996	0.75	0.064	0.086
4	0.529	1.54×10^{-3}	0.565	0.3	0.023	0.076
5	0.597	2.50×10^{-3}	0.255	0.15	0.014	0.09
6	0.638	3.24×10^{-3}	0.454	0.29	0.027	0.092
7	0.575	2.13×10^{-3}	0.643	0.37	0.026	0.071
8	0.541	1.70×10^{-3}	0.653	0.35	0.03	0.085
9	0.781	6.91×10^{-3}	0.107	0.084	0.01	0.12

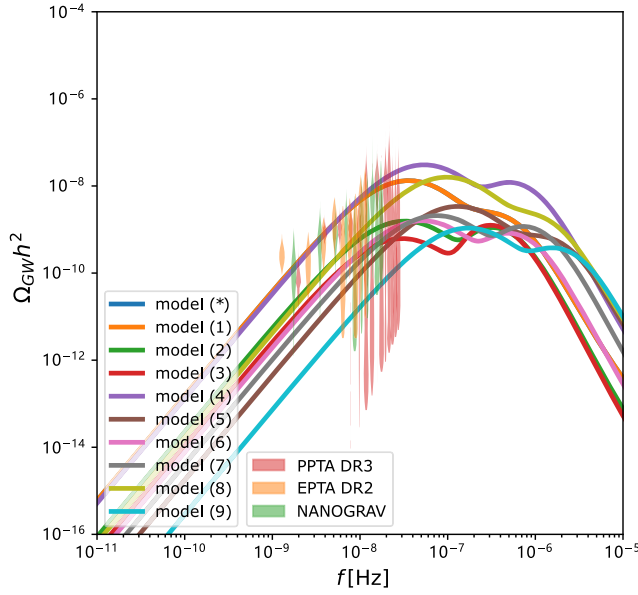


FIG. 10. Plots of the gravitational wave power spectrum $\Omega_{\text{GW}} h^2$ as a function of the frequency f (Hz) for the ten benchmark points in Table III. All the benchmark curves intercept the observed PTA signals.

In Table III, we list a series of benchmark models that explain the observed PTA signals while all relevant experimental and cosmological constraints are simultaneously satisfied, including the relic density requirement, cogenesis, bounds from BBN, and collider searches. The gravitational wave power spectrum curves of these models are also shown in Fig. 10. Thus, the analysis demonstrates that, while the LISA-accessible region is ruled out under the cogenesis condition, parameter points capable of producing PTA-scale signals survive. These surviving scenarios constitute a phenomenologically consistent and observationally motivated region of parameter space, offering a potential explanation for the stochastic gravitational wave background hinted at in recent PTA observations.

IV. CONCLUSION

In this work, we have presented an extension of the Standard Model to include a dark sector with a $U(1)_x$ gauge symmetry which includes matter which is $B - L$ conserving, which can account of several phenomena outside the realm of a possible Standard Model explanation. These include the possibility of explaining baryon asymmetry and observable gravitational wave power spectrum via a first-order phase transition in the dark sector. The baryon asymmetry is generated via cogenesis which also leads to a dark matter relic density consistent with current data. Thus, in the $U(1)_x$ extension, out-of-equilibrium decays of heavy Majorana fermions generate equal and opposite $B - L$ asymmetries in the visible and in the hidden sectors. Sphaleron interactions partially convert the visible sector lepton asymmetry

into a baryon asymmetry, while the dark sector asymmetry is deposited in two Dirac fermions, X and X' , yielding an asymmetric dark matter population. This setup naturally accounts for the observed similarity between the cosmic abundances of baryons and dark matter, implying a characteristic dark matter mass $m_{\text{DM}} \sim \mathcal{O}(\text{GeV})$. The symmetric component efficiently annihilates through secluded channels $\bar{X}X \rightarrow A_x A_x$ and $\bar{X}'X' \rightarrow A_x A_x$, with the dark photon subsequently decaying into Standard Model states, consistent with existing kinetic mixing constraints. We further use this framework for analysis of first-order phase transitions and production of their stochastic gravitational wave backgrounds. Building on the Metaxas-Weinberg approach, we constructed a gauge-invariant effective action that controls bubble nucleation in the low-temperature regime relevant for observable signals. We demonstrate that gauge-invariant analysis is relevant for reliable predictions for the gravitational wave backgrounds, since, in general, the gravitational wave power spectrum is sensitively dependent of the gauge choice. Our parameter scans reveal broad regions in which successful cogenesis coincides with supercooled or near-supercooled transitions ($T_p/m_{A_x} \lesssim 1$). In these regions, the gauge-invariant prediction yields robust gravitational wave signals, with amplitudes and peak frequencies that can fall within the reach of pulsar timing arrays experiments and future space-based interferometers. Taken together, these results provide correlated targets for multimessenger searches: gravitational wave observations of first-order phase transitions, laboratory probes of sub-GeV dark photons with small kinetic mixing, direct detection of $\mathcal{O}(\text{GeV})$ asymmetric dark matter, and cosmological tests of neutrino properties and nonunitarity associated with the extended fermion sector. The framework, thus, links otherwise disparate phenomena and their observability.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: THE GENERATION OF BARYON ASYMMETRY INCLUDING WASHOUT

In the early Universe, the heavy N_i decay generates an asymmetry in ψ and ϕ , which is subsequently transferred to both the visible and dark sectors. The corresponding Lagrangian is given by

$$\mathcal{L}_{\text{asy}} = \lambda_i \bar{N}_i \psi \phi + \lambda_i^* \bar{\psi} N_i \phi^*, \quad (\text{A1}) \quad \text{and, thus,}$$

which gives rise to the following decays:

$$N \rightarrow \psi + \phi, \quad N \rightarrow \bar{\psi} + \bar{\phi}, \quad (\text{A2})$$

where we use ψ and ϕ to denote particles and $\bar{\psi}$ and $\bar{\phi}$ antiparticles. We further define

$$|\mathcal{M}_N|^2 \equiv |\mathcal{M}_{N \rightarrow \psi \phi}|^2 + |\mathcal{M}_{N \rightarrow \bar{\psi} \bar{\phi}}|^2 \quad (\text{A3})$$

The Boltzmann equation governing the evolution of N reads

$$\begin{aligned} n_N + 3Hn_N = & - \int d\Pi_p d\Pi_{k_1} d\Pi_{k_2} (2\pi)^4 |\mathcal{M}_{N \rightarrow \psi \phi}|^2 \delta^4(p - k_1 - k_2) [f_N(p) - f_\psi(k_1) f_\phi(k_2)] \\ & - \int d\Pi_p d\Pi_{k_1} d\Pi_{k_2} (2\pi)^4 |\mathcal{M}_{N \rightarrow \bar{\psi} \bar{\phi}}|^2 \delta^4(p - k_1 - k_2) [f_N(p) - f_{\bar{\psi}}(k_1) f_{\bar{\phi}}(k_2)], \end{aligned} \quad (\text{A6})$$

where the distribution functions are given by

$$f_\psi = e^{-(E_\psi - \mu_\psi)/T}, \quad f_{\bar{\psi}} = e^{-(E_{\bar{\psi}} + \mu_\psi)/T}, \quad (\text{A7})$$

$$f_\phi = e^{-(E_\phi - \mu_\phi)/T}, \quad f_{\bar{\phi}} = e^{-(E_{\bar{\phi}} + \mu_\phi)/T}. \quad (\text{A8})$$

At high temperatures, ψ denotes the fermionic particle carrying two degrees of freedom, while ϕ denotes the complex scalar particle carrying one degree of freedom. Their corresponding antiparticles $\bar{\psi}$ and $\bar{\phi}$ also carry two and one degree of freedom, respectively. Given that the decay of N will generate an equal amount of asymmetry in both ψ and ϕ ,

$$n_\psi - n_{\bar{\psi}} = n_\phi - n_{\bar{\phi}}, \quad (\text{A9})$$

one thus derives

$$\mu_\psi = \mu_\phi. \quad (\text{A10})$$

The net ψ and ϕ comoving number densities are expressed by, respectively,³

$$Y_\Psi \equiv \frac{n_\psi - n_{\bar{\psi}}}{s} = Y_\psi - Y_{\bar{\psi}} = \frac{15g_\psi}{4\pi^2 g_{*S}} \frac{\mu_\psi}{T} = \frac{15}{2\pi^2 g_{*S}} \frac{\mu_\psi}{T}, \quad (\text{A13})$$

$$Y_\Phi \equiv \frac{n_\phi - n_{\bar{\phi}}}{s} = Y_\phi - Y_{\bar{\phi}} = \frac{15g_\phi}{2\pi^2 g_{*S}} \frac{\mu_\psi}{T} = \frac{15}{2\pi^2 g_{*S}} \frac{\mu_\phi}{T}. \quad (\text{A14})$$

Thus, the product $f_\psi f_\phi$ can be further expressed by

$$f_\psi f_\phi = e^{-(E_\psi + E_\phi)/T} e^{(\mu_\psi + \mu_\phi)/T} = f_N^{\text{EQ}} e^{2\mu_\psi/T} \approx f_N^{\text{EQ}} \left[1 + \frac{4\pi^2 g_{*S}}{15} Y_\Psi \right], \quad (\text{A15})$$

$$f_{\bar{\psi}} f_{\bar{\phi}} = e^{-(E_{\bar{\psi}} + E_{\bar{\phi}})/T} e^{-(\mu_\psi + \mu_\phi)/T} = f_N^{\text{EQ}} e^{-2\mu_\psi/T} \approx f_N^{\text{EQ}} \left[1 - \frac{4\pi^2 g_{*S}}{15} Y_\Psi \right]. \quad (\text{A16})$$

³We adopt the standard notation for the number density, defined as (+ for fermion and − for bosons)

$$n_i^{\text{EQ}} = \frac{g_i}{2\pi^2} \int_m^\infty dE \frac{E \sqrt{E^2 - m^2}}{e^{(E - \mu_i)/T} \pm 1} = \frac{\zeta(3)}{\pi^2} g_i T^3 \text{ (bosons)}, \quad \frac{3}{4} \frac{\zeta(3)}{\pi^2} g_i T^3 \text{ (fermions)}, \quad (\text{A11})$$

where g_i is the degrees of freedom of only the particle i but not the antiparticle of i . For high temperatures ($T \gg m$), the net particle number density is given by

$$n_i - n_{\bar{i}} = \frac{g_i}{2\pi^2} \int_0^\infty dE E \sqrt{E^2 - m_i^2} \{ [e^{(E - \mu_i)/T} \pm 1]^{-1} - [e^{(E + \mu_i)/T} \pm 1]^{-1} \} \approx \frac{g_i T^3 c \mu_i}{6 T}, \quad (\text{A12})$$

where the bar denotes the antiparticle, $c = 2$ for bosons, and $c = 1$ for fermions.

The Boltzmann equation of N then becomes

$$\begin{aligned} n_N + 3Hn_N &= - \int d\Pi_p d\Pi_{k_1} d\Pi_{k_2} (2\pi)^4 |\mathcal{M}_N|^2 \delta^4(p - k_1 - k_2) \left\{ [f_N(p) - f_N^{\text{EQ}}(p)] - \epsilon f_N^{\text{EQ}}(p) \times \frac{4\pi^2 g_{*S}}{15} Y_\Psi \right\} \\ &= -\langle \Gamma_N \rangle (n_N - n_N^{\text{EQ}}) + \epsilon \langle \Gamma_N \rangle n_N^{\text{EQ}} \times \frac{4\pi^2 g_{*S}}{15} Y_\Psi. \end{aligned} \quad (\text{A17})$$

We now rewrite the above Boltzmann equation in terms of the comoving number density Y_N as

$$\frac{dY_N}{dT} = \frac{1}{T\bar{H}} \left\{ \langle \Gamma_N \rangle (Y_N - Y_N^{\text{EQ}}) - \epsilon \langle \Gamma_N \rangle Y_N^{\text{EQ}} \times \frac{4\pi^2 g_{*S}}{15} Y_\Psi \right\}. \quad (\text{A18})$$

The $2 \rightarrow 2$ scattering processes contributing to the change of N number density are $NN \leftrightarrow \bar{\psi}\psi$ and $NN \leftrightarrow \bar{\phi}\phi$, and their corresponding collision terms are derived as follows:

$$\begin{aligned} C_{NN \leftrightarrow \bar{\psi}\psi} &= -2 \int d\Pi_{p_1} d\Pi_{p_2} d\Pi_{k_1} d\Pi_{k_2} (2\pi)^4 \frac{1}{2} |\mathcal{M}_{NN \rightarrow \bar{\psi}\psi}|^2 \delta^4(k_1 + k_2 - p_1 - p_2) [f_N(p_1) f_N(p_2) - f_\psi(k_1) f_{\bar{\psi}}(k_2)] \\ &= -2 \int d\Pi_{p_1} d\Pi_{p_2} d\Pi_{k_1} d\Pi_{k_2} (2\pi)^4 \frac{1}{2} |\mathcal{M}_{NN \rightarrow \bar{\psi}\psi}|^2 \delta^4(k_1 + k_2 - p_1 - p_2) [f_N(p_1) f_N(p_2) - f_N^{\text{EQ}}(p_1) f_N^{\text{EQ}}(p_2)] \\ &= -2 \langle \sigma v \rangle_{NN \rightarrow \bar{\psi}\psi} (n_N n_N - n_N^{\text{EQ}} n_N^{\text{EQ}}), \end{aligned} \quad (\text{A19})$$

$$\begin{aligned} C_{NN \leftrightarrow \bar{\phi}\phi} &= -2 \int d\Pi_{p_1} d\Pi_{p_2} d\Pi_{k_1} d\Pi_{k_2} (2\pi)^4 \frac{1}{2} |\mathcal{M}_{NN \rightarrow \bar{\phi}\phi}|^2 \delta^4(k_1 + k_2 - p_1 - p_2) [f_N(p_1) f_N(p_2) - f_\phi(k_1) f_{\bar{\phi}}(k_2)] \\ &= -2 \int d\Pi_{p_1} d\Pi_{p_2} d\Pi_{k_1} d\Pi_{k_2} (2\pi)^4 \frac{1}{2} |\mathcal{M}_{NN \rightarrow \bar{\phi}\phi}|^2 \delta^4(k_1 + k_2 - p_1 - p_2) [f_N(p_1) f_N(p_2) - f_N^{\text{EQ}}(p_1) f_N^{\text{EQ}}(p_2)] \\ &= -2 \langle \sigma v \rangle_{NN \rightarrow \bar{\phi}\phi} (n_N n_N - n_N^{\text{EQ}} n_N^{\text{EQ}}). \end{aligned} \quad (\text{A20})$$

In the above equations, the coefficient -2 in the front indicates that two units of N are annihilated. The factor of $1/2$ in front of the squared amplitude accounts for the avoidance of double counting when integrating over the phase space of the two identical N particles, which is, however, absorbed into the definition of the cross section σ .

The evolution of the generated net ψ number can be obtained by subtracting the two Boltzmann equations $n_\psi + 3Hn_\psi$ and $\dot{n}_{\bar{\psi}} + 3Hn_{\bar{\psi}}$, and we obtain

$$\begin{aligned} \dot{n}_\Psi + 3Hn_\Psi &= \dot{n}_\psi + 3Hn_\psi - (\dot{n}_{\bar{\psi}} + 3Hn_{\bar{\psi}}) \\ &= C_{\psi\phi \leftrightarrow N} + C_{\bar{\psi}\bar{\phi} \leftrightarrow N} + C_{\psi\psi \leftrightarrow \bar{\phi}\bar{\phi}} + C_{\bar{\psi}\bar{\psi} \leftrightarrow \phi\phi} + C_{\psi\phi \leftrightarrow \bar{\psi}\bar{\phi}}, \end{aligned} \quad (\text{A21})$$

where the collision terms are

$$\begin{aligned} C_{\psi\phi \leftrightarrow N} + C_{\bar{\psi}\bar{\phi} \leftrightarrow N} &= - \int d\Pi_p d\Pi_{k_1} d\Pi_{k_2} (2\pi)^4 |\mathcal{M}_{\psi\phi \leftrightarrow N}|^2 \delta^4(p - k_1 - k_2) [f_\psi(k_1) f_\phi(k_2) - f_N(p)] \\ &\quad + \int d\Pi_p d\Pi_{k_1} d\Pi_{k_2} (2\pi)^4 |\mathcal{M}_{\bar{\psi}\bar{\phi} \leftrightarrow N}|^2 \delta^4(p - k_1 - k_2) [f_{\bar{\psi}}(k_1) f_{\bar{\phi}}(k_2) - f_N(p)] \\ &= -\epsilon \langle \Gamma_N \rangle (n_N^{\text{EQ}} - n_N) - \langle \Gamma_N \rangle n_N^{\text{EQ}} \times \frac{4\pi^2 g_{*S}}{15} Y_\Psi \end{aligned} \quad (\text{A22})$$

and

$$\begin{aligned} C_{\psi\psi \leftrightarrow \bar{\phi}\bar{\phi}} + C_{\bar{\psi}\bar{\psi} \leftrightarrow \phi\phi} &= -2 \int d\Pi_{p_1} d\Pi_{p_2} d\Pi_{k_1} d\Pi_{k_2} (2\pi)^4 \frac{1}{4} |\mathcal{M}_{\psi\psi \rightarrow \bar{\phi}\bar{\phi}}|^2 \delta^4(k_1 + k_2 - p_1 - p_2) [f_\psi(p_1) f_\psi(p_2) - f_{\bar{\phi}}(k_1) f_{\bar{\phi}}(k_2)] \\ &\quad + 2 \int d\Pi_{p_1} d\Pi_{p_2} d\Pi_{k_1} d\Pi_{k_2} (2\pi)^4 \frac{1}{4} |\mathcal{M}_{\bar{\psi}\bar{\psi} \rightarrow \phi\phi}|^2 \delta^4(k_1 + k_2 - p_1 - p_2) [f_{\bar{\psi}}(p_1) f_{\bar{\psi}}(p_2) - f_\phi(k_1) f_\phi(k_2)] \\ &= -2 \langle \sigma v \rangle_{\psi\psi \rightarrow \bar{\phi}\bar{\phi}} n_\psi^{\text{EQ}} n_{\bar{\psi}}^{\text{EQ}} \times \frac{8\pi^2 g_{*S}}{15} + 2 \langle \sigma v \rangle_{\bar{\psi}\bar{\psi} \rightarrow \phi\phi} n_{\bar{\psi}}^{\text{EQ}} n_\psi^{\text{EQ}} \times \left(-\frac{8\pi^2 g_{*S}}{15} \right) \\ &= -2 \langle \sigma v \rangle_{\psi\psi \rightarrow \bar{\phi}\bar{\phi}} n_\psi^{\text{EQ}} n_{\bar{\psi}}^{\text{EQ}} \times \frac{16\pi^2 g_{*S}}{15}, \end{aligned} \quad (\text{A23})$$

$$\begin{aligned}
C_{\psi\phi\leftrightarrow\bar{\psi}\bar{\phi}} &= - \int d\Pi_{p_1} d\Pi_{p_2} d\Pi_{k_1} d\Pi_{k_2} (2\pi)^4 |\mathcal{M}_{\psi\phi\rightarrow\bar{\psi}\bar{\phi}}|^2 \delta^4(k_1 + k_2 - p_1 - p_2) [f_\psi(p_1) f_\phi(p_2) - f_{\bar{\psi}}(k_1) f_{\bar{\phi}}(k_2)] \\
&\quad + \int d\Pi_{p_1} d\Pi_{p_2} d\Pi_{k_1} d\Pi_{k_2} (2\pi)^4 |\mathcal{M}_{\bar{\psi}\bar{\phi}\rightarrow\psi\phi}|^2 \delta^4(k_1 + k_2 - p_1 - p_2) [f_{\bar{\psi}}(p_1) f_{\bar{\phi}}(p_2) - f_\psi(k_1) f_\phi(k_2)] \\
&= -\langle\sigma v\rangle_{\psi\phi\rightarrow\bar{\psi}\bar{\phi}} n_\psi^{\text{EQ}} n_\phi^{\text{EQ}} \times \frac{8\pi^2 g_{*S}}{15} + \langle\sigma v\rangle_{\bar{\psi}\bar{\phi}\rightarrow\psi\phi} n_\psi^{\text{EQ}} n_\phi^{\text{EQ}} \times \left(-\frac{8\pi^2 g_{*S}}{15}\right) \\
&= -\langle\sigma v\rangle_{\psi\phi\rightarrow\bar{\psi}\bar{\phi}} n_\psi^{\text{EQ}} n_\phi^{\text{EQ}} \times \frac{16\pi^2 g_{*S}}{15}. \tag{A24}
\end{aligned}$$

Combining all the above collision terms, the Boltzmann equation tracking the generation of the asymmetries are written as

$$\frac{dY_N}{dT} = \frac{1}{T\bar{H}} \left[\langle\Gamma_N\rangle (Y_N - Y_N^{\text{EQ}}) - \epsilon \langle\Gamma_N\rangle Y_N^{\text{EQ}} \times \frac{4\pi^2 g_{*S}}{15} Y_\Psi \right] + \frac{2s}{T\bar{H}} (\langle\sigma v\rangle_{NN\rightarrow\phi\bar{\phi}} + \langle\sigma v\rangle_{NN\rightarrow\psi\bar{\psi}}) (Y_N Y_N - Y_N^{\text{EQ}} Y_N^{\text{EQ}}), \tag{A25}$$

$$\frac{dY_\Psi}{dT} = \frac{d(\frac{n_\Psi - n_{\bar{\Psi}}}{s})}{dT} = -\frac{1}{T\bar{H}} \epsilon \langle\Gamma_N\rangle (Y_N - Y_N^{\text{EQ}}) + \frac{s}{T\bar{H}} \frac{4\pi^2 g_{*S}}{15} Y_\Psi \left(\frac{\langle\Gamma_N\rangle}{s} Y_N^{\text{EQ}} + 8\langle\sigma v\rangle_{\psi\psi\rightarrow\bar{\phi}\bar{\phi}} n_\psi^{\text{EQ}} n_\psi^{\text{EQ}} + 4\langle\sigma v\rangle_{\psi\phi\rightarrow\bar{\psi}\bar{\phi}} n_\psi^{\text{EQ}} n_\phi^{\text{EQ}} \right). \tag{A26}$$

Since N is much heavier than the combined masses of ψ and ϕ , the thermally averaged cross section $\langle\sigma v\rangle_{\psi\psi\rightarrow\bar{\phi}\bar{\phi}}$ and $\langle\sigma v\rangle_{\psi\phi\rightarrow\bar{\psi}\bar{\phi}}$ should be replaced by the real intermediate state (RIS)-subtracted cross sections, denoted with the superscript “RIS,” to avoid the double counting the contribution from the on-shell N propagator, as was shown in Eq. (2.4).

The Boltzmann equation governing the net ϕ number density Y_ϕ takes the same form as that for Y_ψ . As a result,

the decay of N generates equal amounts of asymmetry in both ψ and ϕ , including all washout processes.

APPENDIX B: A SET OF PARAMETERS GENERATING THE NEUTRINO MIXING

We provide a representative set of parameters that yields three light neutrinos in agreement with current experimental observations (including the observed Dirac phase), where the neutrino mass matrix given by Eq. (2.15) reads

$$\begin{pmatrix} 2.056 \times 10^{-11} & 0 & 0 & 3.198 \times 10^{-5} \\ 0 & 1.361 \times 10^{-11} & 0 & 3.805 \times 10^{-4} \\ 0 & 0 & 2.560 \times 10^{-11} & 3.314 \times 10^{-4} \\ (-9.681 - 2.343i) \times 10^{-5} & (-1.084 + 3.700i) \times 10^{-4} & (-3.168 + 13.23i) \times 10^{-5} & 4.708 \times 10^3 \end{pmatrix}, \tag{B1}$$

with all elements expressed in units of GeV.

APPENDIX C: $U(1)_x$ -HYPERCHARGE KINETIC MIXING

An additional $U(1)_x$ gauge symmetry can mix with hypercharge via gauge kinetic terms, induced by, e.g., loop effects of heavy bifundamental fermions charged under both $U(1)_x$ and $U(1)_Y$ [72]. After integrating out the heavy states, in low energies one has

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4} F_{Y\mu\nu} F_Y^{\mu\nu} - \frac{1}{4} F_{x\mu\nu} F_x^{\mu\nu} - \frac{\epsilon}{2} F_{x\mu\nu} F_Y^{\mu\nu}, \tag{C1}$$

where ϵ is the kinetic mixing parameter. The kinetic mixing, thus, provides a portal that connects the $U(1)_x$ dark sector to the Standard Model. After simultaneously diagonalizing the kinetic and mass mixing matrices, in the mass eigenbasis the dark photon couples to the Standard Model fermions.

The neutral current couplings of A_x and Z to the SM fermions f_i and to the dark matter fields X and X' can be written as [74]

$$\begin{aligned}
-\mathcal{L}_{A_x, Z} &= \frac{1}{2} \bar{f}_i \gamma_\mu [(v'_i - a'_i \gamma^5) f_i A_x^\mu + (v_i - a_i \gamma^5) f_i Z^\mu] \\
&\quad + g_x (R_{11} A_x^\mu + R_{13} Z^\mu) (\bar{X} \gamma_\mu X - \bar{X}' \gamma_\mu X'), \tag{C2}
\end{aligned}$$

where

$$v_i = (g_2 R_{33} - g_Y R_{23}) T_i^3 + 2g_Y R_{23} Q_i, \quad (C3)$$

$$a_i = (g_2 R_{33} - g_Y R_{23}) T_i^3, \quad (C4)$$

$$v'_i = (g_2 R_{31} - g_Y R_{21}) T_i^3 + 2g_Y R_{21} Q_i, \quad (C5)$$

$$a'_i = (g_2 R_{31} - g_Y R_{21}) T_i^3, \quad (C6)$$

g_Y , g_2 , and g_x are the hypercharge and weak and dark gauge couplings, respectively, and the rotation matrix is given by

$$R = \begin{pmatrix} c_\delta \cos \psi & 0 & c_\delta \sin \psi \\ -s_\delta \cos \psi + \sin \psi \sin \theta_W & \cos \theta_W & -s_\delta \sin \psi - \cos \psi \sin \theta_W \\ -\sin \psi \cos \theta_W & \sin \theta_W & \cos \psi \cos \theta_W \end{pmatrix}. \quad (C7)$$

After mixing, the photon couplings to Standard Model fermions remain unchanged, and the photon does not couple to the dark matter fields.

In the literature, constraints on the kinetic mixing parameter ϵ is obtained from a simplified model that the dark photon mixed directly with the photon field. The experimental constraints on the $U(1)_x$ -hypercharge kinetic mixing is obtained by the relation [76,110]

$$\epsilon = \frac{\sqrt{g_2^2 + g_Y^2}}{g_2} \epsilon \quad (C8)$$

and are shown in Fig. 11. In this paper, we focus on $\mathcal{O}(1)$ GeV dark matter annihilating into dark photons with masses in the sub-GeV to $\mathcal{O}(1)$ GeV range. Accordingly, we consider two representative values for the kinetic mixing parameter, $\epsilon = 10^{-4}$ and $\epsilon = 10^{-9}$, which are shown as black lines within the allowed regions in Fig. 11.

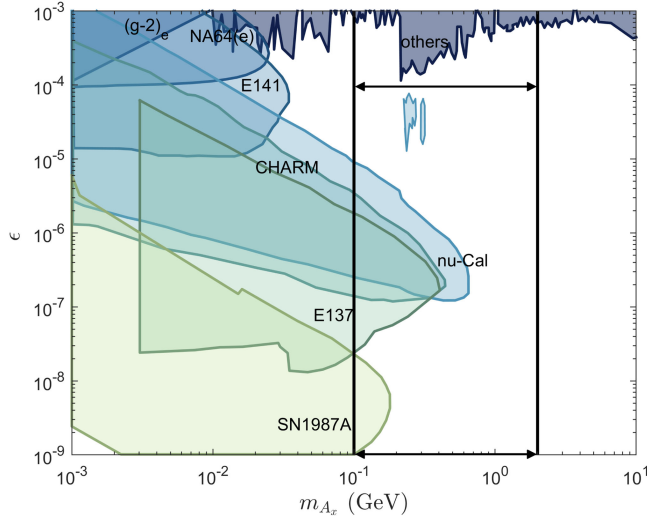


FIG. 11. Experimental constraints for the kinetic mixing parameter ϵ given in Eqs. (C1) and (C8), for the mass of the dark $U(1)_x$ gauge boson A_x ranging from 1 MeV to 10 GeV. For all benchmark points, the kinetic mixing parameters $\epsilon = 10^{-4}$ and $\epsilon = 10^{-9}$, shown as black lines, are consistent with current experimental constraints. Constraints on the kinetic mixing parameter ϵ are shown in the shaded regions, including bounds from SN1987A [111], E137 [112–114], CHARM [115], ν -Cal [116,117], E141 [118], $(g-2)_e$ [119], NA64(e) [120], and others (A1 [121], BABAR [122], NA48/2 [123], KLOE [124–127], LHCb [128], and CMS [129]).

APPENDIX D: GAUGE-INVARIANT EFFECTIVE ACTION FOR FIRST-ORDER PHASE TRANSITION

1. Generating functional and effective potential or action

We start with the generating functional and connected Green's functions with J to be the source, ϕ to be the scale field, and $\mathcal{L}$ to be Eq. (2.1), and then

$$Z[J] = \int \mathcal{D}\phi \exp \left[i \int d^4x (\mathcal{L} + J\phi) \right], \quad (D1)$$

$$W[J] = -i \ln Z[J].$$

The effective action or potential is obtained through the Legendre transform of $W[J]$ that

$$\Gamma[\phi_c] = W[J] - \int d^4x J(x) \phi_c(x), \quad \phi_c(x) = \frac{\partial W[J]}{\partial J}. \quad (D2)$$

Write it as the expansion in powers of a scalar field:

$$S_{\text{eff}}(\phi_c) = \Gamma(\phi_c) = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^4x_1 \cdots d^4x_n \Gamma^n(x_1, \dots, x_n) \times \phi_c(x_1) \cdots \phi_c(x_n), \quad (D3)$$

where Γ^n are n -point 1PI Green's functions. Following [130,131], we take Fourier transformation on the classic background field ϕ_c and Green's function

$$\phi_c(x) = \int \frac{d^4 p}{(2\pi)^4} \bar{\phi}(p) e^{ipx}, \quad \Gamma^n(x_i) = \int \prod_{i=1}^n \frac{d^4 p_i}{(2\pi)^4} e^{ip_i x_i} (2\pi)^4 \delta^4\left(\sum_i p_i\right) \Gamma^n(p_i). \quad (D4)$$

Then, our effective action will be

$$\begin{aligned} S_{\text{eff}}(\bar{\phi}) &= \sum_{n=1}^{\infty} \frac{1}{n!} \int \prod_{i=1}^n \frac{d^4 p_i}{(2\pi)^4} \bar{\phi}(-p_i) (2\pi)^4 \delta^4\left(\sum_i p_i\right) \Gamma^n(p_i) \\ &= \sum_{n=1}^{\infty} \frac{1}{n!} \int \prod_{i=1}^n \frac{d^4 p_i}{(2\pi)^4} \bar{\phi}(-p_i)|_{p_i=0} (2\pi)^4 \delta^4(0) \Gamma^n(p_i)|_{p_i=0} + \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \frac{\partial \Gamma^2(p^2)}{\partial p^2} \Big|_{p^2=0} p^2 \bar{\phi}(-p) \bar{\phi}(p) + \dots, \end{aligned} \quad (D5)$$

where the first term represents the zero-order derivative expansion and the second term represents the leading term of the first-order expansion. Let us define the quantity

$$Z = -\frac{\partial \Gamma^2(p^2)}{\partial p^2} \Big|_{p^2=0}. \quad (D6)$$

Upon applying the Fourier transformation again, we obtain

$$\begin{aligned} S_{\text{eff}}(\phi_c) &= \int d^4 x \sum_{n=0}^{\infty} \frac{(2\pi)^4}{n!} \phi_c^n(x) \Gamma^n(p)|_{p=0} + \frac{Z(\phi_c)}{2} \int d^4 x \partial_\mu \phi_c \partial^\mu \phi_c \\ &= \int d^4 x \left[V_{\text{eff}}(\phi_c) + \frac{Z(\phi_c)}{2} \partial_\mu \phi_c \partial^\mu \phi_c \right]. \end{aligned} \quad (D7)$$

2. Gauge-invariant effective action using the derivative expansion

Here, we follow the method by Metaxas and Weinberg [88,90] to get a gauge-independent effective bounce action. According to Nielsen identity, which is directly from Becchi-Rouet-Stora-Tyupin invariance of effective action, we have

$$\xi \frac{\partial S_{\text{eff}}}{\partial \xi} = - \int d^4 x C(x) \frac{\delta S_{\text{eff}}}{\delta \phi(x)}, \quad (D8)$$

$$C(x) = -\frac{ie}{2} \int d^4 y \langle \bar{c}(x) G(x) c(y) [\partial_\mu A^\mu(y) - g_x \xi \bar{\phi} G(y)] \rangle. \quad (D9)$$

Performing gradient expansion on S_{eff} at finite temperature, $C(x)$ and $\delta S_{\text{eff}}/\delta \phi$, we get

$$S_{\text{eff}} = \beta \int d^3 x \left[V_{\text{eff}}(\phi) + \frac{1}{2} Z(\phi) (\partial_\mu \phi)^2 + \dots \right], \quad (D10)$$

$$C(x) = C_0(\phi) + D(\phi) (\partial_\mu \phi)^2 + \dots, \quad (D11)$$

$$\frac{\delta S_{\text{eff}}}{\delta \phi} = \frac{\partial V_{\text{eff}}}{\partial \phi} + \frac{1}{2} \frac{\partial Z}{\partial \phi} (\partial_\mu \phi)^2 - \partial_\mu [Z(\phi) \partial_\mu \phi] + \dots, \quad (D12)$$

where $\beta = 1/T$. By expanding $V_{\text{eff}}(\phi)$ and $Z(\phi)$ in power series of coupling constant g_x ,

$$V_{\text{eff}} = V_{g_x^4} + V_{g_x^6} + \dots, \quad (D13)$$

$$Z = 1 + Z_{g_x^2} + \dots. \quad (D14)$$

Nielsen identity for effective action Eq. (D8) turns into the following form for leading and subleading contributions:

$$\xi \frac{\partial V_{g_x^4}}{\partial \xi} = 0, \quad (D15)$$

$$\xi \frac{\partial V_{g_x^6}}{\partial \xi} = -C_{g_x^2} \frac{\partial V_{g_x^4}}{\partial \phi}, \quad (D16)$$

$$\xi \frac{\partial Z_{g_x^2}}{\partial \xi} = -2 \frac{\partial C_{g_x^2}}{\partial \phi}. \quad (D17)$$

Meanwhile, we have the leading- and subleading-order terms of effective action to be

$$S_{\text{eff}} = S_0 + S_1, \quad (D18)$$

$$S_0 = \beta \int d^3 x \left[\frac{1}{2} (\partial_\mu \phi_b)^2 + V_{g_x^4}(\phi_b) \right], \quad (D19)$$

$$S_1 = \beta \int d^3x \left[\frac{1}{2} Z_{g_x^2}(\phi_b, \xi) (\partial_\mu \phi_b)^2 + V_{g_x^2}(\phi_b, \xi) \right], \quad (D20)$$

where $\phi_b(x)$ is a bounce solution for the leading approximation to the effective action given by

$$\square \phi_b = \frac{\partial V_{g_x^2}}{\partial \phi}. \quad (D21)$$

By Eq. (D15), we directly have S_0 to be gauge independent. For S_1 , we have

$$\begin{aligned} \xi \frac{\partial S_1}{\partial \xi} &= \xi \frac{\partial}{\partial \xi} \beta \int d^3x \left[\frac{1}{2} Z_{g_x^2}(\phi_b, \xi) (\partial_\mu \phi_b)^2 + V_{g_x^2}(\phi_b, \xi) \right] \\ &= \beta \int d^3x \left[C_{g_x^2} \frac{\partial V_{g_x^2}}{\partial \phi} + \frac{\partial C_{g_x^2}}{\partial \phi} (\partial_\mu \phi_b)^2 \right] \\ &= \beta \int d^3x \left[C_{g_x^2} \frac{\partial V_{g_x^2}}{\partial \phi} + (\partial_\mu C_{g_x^2}) (\partial_\mu \phi_b) \right] \\ &= \beta \int d^3x C_{g_x^2} \left[\frac{\partial V_{g_x^2}}{\partial \phi} - \square \phi_b \right] \\ &= 0. \end{aligned} \quad (D22)$$

Thus, S_1 is also gauge independent.

3. Gauge-dependent effective potential

Before discussion of how to generate a gauge-invariant effective action, we introduce the gauge-dependent effective potential which provides the starting point for further analysis. We begin by exhibiting the gauge dependence of the effective potential in R_ξ gauge fixing [132]. According to Eq. (2.1), the part involved in gravitational wave generation is

$$\Delta \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\delta}{2} F_{\mu\nu} B^{\mu\nu} - |(\partial_\mu - i g_x A_\mu) \Phi|^2 - V_0^x(\Phi), \quad (D23)$$

where

$$V_0^x = -\mu_x^2 \Phi \Phi^* + \lambda_x (\Phi^* \Phi)^2, \quad \Phi = \frac{1}{\sqrt{2}} (\phi_c + \phi + iG). \quad (D24)$$

By expanding the covariant derivative term, we have that

$$\begin{aligned} |(\partial_\mu - i g_x A_\mu) \Phi|^2 &= \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} (\partial_\mu G)^2 + \frac{1}{2} g_x^2 \phi_c^2 A_\mu A^\mu \\ &\quad + g_x \phi_c A^\mu \partial_\mu G + \mathcal{O}(\text{cubic}). \end{aligned} \quad (D25)$$

The last term $g_x \phi_c A^\mu \partial_\mu G$ gives the mixing. In order to cancel this mixing and fix the gauge, we introduce R_ξ gauge that

$$\mathcal{L}_{\text{gf}} = -\frac{1}{2\xi} F^2, \quad (D26)$$

where

$$F = \partial_\mu A^\mu - \xi g_A \phi_c G. \quad (D27)$$

Expanding it gives us

$$\begin{aligned} \mathcal{L}_{\text{gf}} &= -\frac{1}{2\xi} (\partial_\mu A^\mu - \xi g_x v G)^2 \\ &= -\frac{1}{2\xi} (\partial_\mu A^\mu)^2 + g_x \phi_c G \partial_\mu A^\mu - \frac{1}{2} \xi g_x^2 \phi_c^2 G^2. \end{aligned} \quad (D28)$$

Here, $g_x \phi_c G \partial_\mu A^\mu$ cancel out $g_x \phi_c A^\mu \partial_\mu G$ in Eq. (D25) with integral by parts. Beside this, we also have ghost Lagrangian

$$\mathcal{L}_{\text{gh}} = \bar{c}(-\partial^2 - \xi g_x^2 v^2) c + g f^{abc} (\partial^\mu c^{\dagger a}) c^b A_\mu^c + \dots \quad (D29)$$

Adding all Lagrangians together, we have the expanded form to be

$$\begin{aligned} \Delta \mathcal{L} + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}} &= \frac{1}{2} A_\mu [(\partial^2 g^{\mu\nu} - (1 - 1/\xi) \partial^\mu \partial^\nu) + m_{A_x}^2 g^{\mu\nu}] A_\nu \\ &\quad + \frac{1}{2} \phi(-\partial^2 - m_\phi^2) \phi + \frac{1}{2} G(-\partial^2 - m_G^2 - \xi m_{A_x}^2) G \\ &\quad + \bar{c}(-\partial^2 - \xi m_{A_x}^2) c + \dots, \end{aligned} \quad (D30)$$

where we have field-dependent masses to be

$$m_{A_x}^2 = g_x^2 \phi_c^2, \quad m_\phi^2 = -\mu_x^2 + 3\lambda_x \phi_c^2, \quad m_G^2 = -\mu_x^2 + \lambda_x \phi_c^2. \quad (D31)$$

At one loop, we have the Coleman-Weinberg correction to be

$$\begin{aligned} V_{\text{CW}}(\phi_c, \xi) &= \frac{1}{4(4\pi)^2} \left\{ (m_\phi^2)^2 \left[\ln \left(\frac{m_\phi^2}{\Lambda^2} \right) - \frac{3}{2} \right] \right. \\ &\quad + 3(m_{A_x}^2)^2 \left[\ln \left(\frac{m_{A_x}^2}{\Lambda^2} \right) - \frac{5}{6} \right] \\ &\quad + \frac{1}{4(4\pi)^2} (m_G^2 + \xi m_{A_x}^2)^2 \left[\ln \left(\frac{m_G^2 + \xi m_{A_x}^2}{\Lambda^2} \right) - \frac{3}{2} \right] \\ &\quad \left. - \frac{1}{4(4\pi)^2} (\xi m_{A_x}^2)^2 \left[\ln \left(\frac{\xi m_{A_x}^2}{\Lambda^2} \right) - \frac{3}{2} \right] \right\}. \end{aligned} \quad (D32)$$

In this paper, we take on-shell-like schemes, which means renormalization conditions are defined such that the single and double derivatives of the one-loop effective potential vanish, i.e.,

$$\left. \frac{\partial}{\partial \phi} (V_{\text{CW}} + V_{\text{CT}}) \right|_{\phi=v_0} = 0, \quad (\text{D33})$$

$$\left. \frac{\partial^2}{\partial \phi^2} (V_{\text{CW}} + V_{\text{CT}}) \right|_{\phi=v_0} = 0. \quad (\text{D34})$$

By resolving these two equations, we can then determine the counterterms $\delta\mu_x^2$ and $\delta\lambda_x$ so that

$$V_{\text{CT}}(\phi_c, \xi) = \frac{\delta\mu_x^2}{2} \phi_c^2 + \frac{\delta\lambda_h}{4} \phi_c^4. \quad (\text{D35})$$

Also, we have the finite-temperature correction to be

$$V_T(\phi_c, T, \xi) = \frac{T^4}{2\pi^2} \left[J_B \left(\frac{m_\phi^2}{T^2} \right) + 3J_B \left(\frac{m_{A_x}^2}{T^2} \right) + J_B \left(\frac{m_G^2 + \xi m_{A_x}^2}{T^2} \right) - J_B \left(\frac{\xi m_{A_x}^2}{T^2} \right) \right]. \quad (\text{D36})$$

The one-loop corrections given above are unambiguous and well established. However, for V_{daisy} corrections, a further detailed discussion is required. Infrared divergences appear at high temperatures and break down finite-temperature perturbation theory. To resolve this problem, daisy resummation is applied. Using the Arnold-Espinosa approach, which employs a high-temperature approximation, we have

$$V_{\text{daisy}}(\phi, T) = -\frac{T}{12} \sum_B [(m_B^2 + \Pi_B)^{3/2} - (m_B^2)^{3/2}], \quad (\text{D37})$$

where for the $U(1)_x$ model we have

$$\begin{aligned} \Pi_A(T) &= \frac{2}{3} g_x^2 T^2, & \Pi_\phi(T) &= \frac{1}{4} g_x^2 T^2 + \frac{1}{3} \lambda_x T^2, \\ \Pi_G(T) &= \frac{1}{4} \lambda_x T^2. \end{aligned} \quad (\text{D38})$$

The significance of daisy resummation emerges in the high-temperature regime where $T \gg m$, as it captures essential infrared effects within the finite-temperature field theory framework. In scenarios characterized by supercooled or near-supercooled conditions, where the temperature satisfies $T \leq m$, the contribution of daisy resummation to the effective potential becomes negligible. Consequently, we adopt the approximation of neglecting this term throughout the present analysis for computational simplicity [130,133]. For a more rigorous treatment of finite-temperature mass resummation in the low-temperature regime, we refer the reader to the comprehensive methodologies of “full dressing” and “partial dressing” as detailed in Refs. [20,134]. All together, we have

$$V_{\text{eff}}(\phi_c, T, \xi) = V_0(\phi_c) + V_{\text{CW}}(\phi_c, \xi) + V_{\text{CT}}(\phi_c, \xi) + V_T(\phi_c, T, \xi). \quad (\text{D39})$$

4. Gauge-invariant effective action for the $U(1)_x$ sector at low temperature

We proceed to implement the gauge invariance analysis within the framework of our $U(1)$ extension model. This approach follows methodologies analogous to those employed in previous investigations of gauge-invariant formulations for $U(1)$ extension models, wherein the effective potential incorporates contributions from both tree-level diagrams and Coleman-Weinberg radiative corrections [90]. To extend this theoretical framework to the present investigation, we must incorporate temperature-dependent corrections into our analysis. Previous investigations have established the requisite formalism for the high-temperature approximation regime, characterized by the condition $m/T \ll 1$ [91,92]. However, the supercooled and near-supercooled phase transitions that may account for gravitational wave signals detectable by next-generation instruments operate in a regime where the high-temperature approximation fails (see Sec. III C). Specifically, these phenomena require consideration of the low-temperature regime where $m/T > 1$. As demonstrated in our preceding analysis, daisy summation corrections become negligible in the low-temperature limit and may, therefore, be omitted from the gauge-invariant treatment. This represents a fundamental departure from the methodologies employed in Refs. [91,92], wherein such corrections constitute the dominant contribution to the effective potential. Consequently, a novel analytical approach tailored to the low-temperature regime is required.

To ensure the validity of our gauge invariance analysis, we first clarify the definitions of the ξ -independent term $V_{g_x}^\xi$ and the ξ -dependent term $V_{g_x}^\phi$. We impose the conditions that $\lambda_x + \delta\lambda_x \sim g_x^4$ and $\mu_x^2 + \delta\mu_x^2 \sim g_x^4 \langle \phi_c \rangle^2$, where $\langle \phi_c \rangle$ is the vacuum expectation value. Since we are working in the on-shell scheme, we have $\langle \phi_c \rangle \sim v_0$. It is important to note that, for certain choices of g_x and λ_x , the relation $\lambda_x \sim g_x^4$ may not strictly hold; however, the inclusion of counterterms ensures that the power counting remains consistent. Under these conditions, the tree-level potential with counterterms maintains order g_x^4 and exhibits ξ independence. From Eq. (D32), we observe that the correction attributable to the dark photon scales as $m_{A_x}^4 \sim g_x^4 \langle \phi_c \rangle^4$, while the correction arising from the dark Higgs scales as $m_\phi^4 \sim \lambda_x^2 \sim g_x^8 \langle \phi_c \rangle^4$, which is, thus, negligible. Regarding the remaining two ξ -dependent correction terms, we note that

$$\begin{aligned} V_{\text{cw}}^\xi &= X(m_G^2 + \xi m_{A_x}^2) - X(\xi m_{A_x}^2), \\ X(m^2) &= \frac{1}{4(4\pi)^2} [m^2]^2 \left[\ln \left(\frac{m^2}{\Lambda^2} \right) - \frac{3}{2} \right]. \end{aligned} \quad (\text{D40})$$

In the regime where $\xi m_{A_x}^2 \ll m_G^2$, we obtain $V_{\text{cw}}^\xi \sim X(m_G^2) \sim m_G^2 \sim g_x^8$, where $m_G^2 \sim g_x^4 \phi^2$. This contribution is, therefore, negligible within our leading- and subleading-order analysis. Conversely, in the alternative regime, we have

$$V_{\text{cw}}^\xi = X(m_G^2 + \xi m_{A_x}^2) - X(\xi m_{A_x}^2) \simeq m_G^2 X'(\xi m_{A_x}^2) \propto m_G^2 m_{A_x}^2 \sim \mathcal{O}(g_x^6). \quad (\text{D41})$$

Thus, this ξ -dependent term exhibits the anticipated scaling of order g_x^6 . In this study, we examine the low-temperature regime characterized by $m/T > 1$. We begin by analyzing two distinct cases based on the scaling of temperature relative to the scalar field expectation value ϕ and the gauge coupling g_x .

a. Case 1: $T \sim g_x \phi$

This corresponds to the limiting condition $m/T \sim 1$. Under this condition, the ξ -independent thermal correction

term, which includes contributions from both the dark Higgs ϕ and dark photon A , scales as

$$V_T \sim T^4 \sim g_x^4 \phi^4, \quad (\text{D42})$$

which is of the order of $\mathcal{O}(g_x^4)$ and contributes to the leading-order potential. The gauge-dependent component can be written as

$$V_T^\xi = \frac{T^4}{2\pi^2} \left[J_B \left(\frac{m_G^2 + \xi m_{A_x}^2}{T^2} \right) - J_B \left(\frac{\xi m_{A_x}^2}{T^2} \right) \right] \simeq \frac{T^4}{2\pi^2} \frac{m_G^2}{T^2} \left[J'_B \left(\frac{\xi m_{A_x}^2}{T^2} \right) \right] \sim T^2 m_G^2 \sim (g_x \phi)^2 (g_x^4 \phi^2) \sim \mathcal{O}(g_x^6), \quad (\text{D43})$$

contributing to the subleading potential as expected. Combining these results, we define our leading and subleading potentials as

$$V_{g_x^4}(\phi, T) = -\frac{\mu_x^2 + \delta\mu_x^2}{2} \phi^2 + \frac{\lambda + \delta\lambda}{4} \phi^4 + \frac{3m_{A_x}(\phi)^4}{64\pi^2} \left[\ln \left(\frac{m_{A_x}^2}{\Lambda^2} \right) - \frac{5}{6} \right] + \frac{T^4}{2\pi^2} \left[J_B \left(\frac{m_\phi^2}{T^2} \right) + 3J_B \left(\frac{m_{A_x}^2}{T^2} \right) \right], \quad (\text{D44})$$

$$V_{g_x^6}(\phi, \xi, T) = \frac{(m_G^2 + \xi m_{A_x}^2)^2}{64\pi^2} \left[\ln \left(\frac{m_G^2 + \xi m_{A_x}^2}{\Lambda^2} \right) - \frac{3}{2} \right] - \frac{\xi^2 m_{A_x}^4}{64\pi^2} \left[\ln \left(\frac{\xi m_{A_x}^2}{\Lambda^2} \right) - \frac{3}{2} \right] + \frac{T^4}{2\pi^2} \left[J_B \left(\frac{m_G^2 + \xi m_{A_x}^2}{T^2} \right) - J_B \left(\frac{\xi m_{A_x}^2}{T^2} \right) \right]. \quad (\text{D45})$$

b. Case 2: $T \ll g_x \phi$

This corresponds to the very low-temperature or strongly supercooled regime, where $m/T \gg 1$. In this limit, all temperature-dependent corrections, including the thermal potential and daisy resummation, are exponentially suppressed by Boltzmann factors of the form $e^{-m/T}$. These contributions are, therefore, negligible to all relevant orders of perturbation theory. The effective potential reduces to the zero-temperature Coleman-Weinberg potential, and our analysis simplifies to the treatment presented in [90].

As in [90], three key modifications are required to ensure the gauge invariance of the effective action. In the low-temperature regime, these modifications are slightly different, and we will now discuss each of them in detail.

- (i) First, we use the bounce solution derived from the leading-order potential for the calculation of both S_0 and S_1 . In the supercooled regime (case 2), the bounce solution is highly sensitive to thermal corrections, even though they are suppressed and do not contribute to the leading-order potential ($V_{g_x^4}$). Therefore, we always include the leading temperature correction term when solving for the

bounce solution with Eq. (D21); i.e., Eq. (D44) will be used for both cases. The leading temperature correction is ξ independent and, thus, we make sure Eq. (D15) is satisfied.

- (ii) Second, we use the dressed Goldstone boson mass, which is defined as

$$m_G^2 = \frac{1}{\phi} \frac{\partial V_{g_x^4}}{\partial \phi}, \quad (\text{D46})$$

instead of the expression in Eq. (D31). This ensures that the Nielsen identity [Eq. (D16)] is satisfied, preserving gauge invariance. This was verified at zero temperature in [90]. The generalization to finite temperature is straightforward, requiring the replacement of the zero-temperature four-momentum integral with the Matsubara sum integral:

$$\int \frac{d^4 p}{(2\pi)^4} \rightarrow T \sum_{n \in \mathbb{Z}} \int \frac{d^3 p}{(2\pi)^3}, \quad \omega_n \equiv 2\pi n T. \quad (\text{D47})$$

Define $E_{G'} = \sqrt{p^2 + m_{G'}^2}$ and $E_{\text{FP}} = \sqrt{p^2 + m_{\text{FP}}^2}$ with the finite-temperature ghost and dressed Goldstone masses

$$m_{\text{FP}}^2 = 2\xi g_x^2 \phi^2, \quad m_{G'}^2 = \frac{1}{\phi} \frac{\partial V_{g_x^4}(\phi, T)}{\partial \phi} + 2\xi g_x^2 \phi^2. \quad (\text{D48})$$

The one-loop ghost-Goldstone sector then gives

$$C_{g_x}^T(\phi, T) = -g_x^2 \xi \phi^2 T \sum_{n \in \mathbb{Z}} \int \frac{d^3 p}{(2\pi)^3} \times \frac{1}{(\omega_n^2 + E_{G'}^2)(\omega_n^2 + E_{\text{FP}}^2)}. \quad (\text{D49})$$

The ξ -dependent part of the finite-temperature correction in $V_{g_x^6}$ is the difference between the Goldstone and ghost determinants:

$$V_{g_x^6}^{\xi, T}(\phi, T) = \frac{T}{2} \sum_{n \in \mathbb{Z}} \int \frac{d^3 p}{(2\pi)^3} \times [\ln(\omega_n^2 + E_{G'}^2) - \ln(\omega_n^2 + E_{\text{FP}}^2)]. \quad (\text{D50})$$

Differentiating Eq. (D50) with respect to ξ and using $\partial_\xi m_{G'}^2 = \partial_\xi m_{\text{FP}}^2 = 2g_x^2 \phi^2$ gives

$$\xi \frac{\partial V_{g_x^6}^{\xi, T}(\phi, T)}{\partial \xi} = -\xi g_x^2 \phi^2 T \sum_n \int \frac{d^3 p}{(2\pi)^3} \times \left(\frac{1}{\omega_n^2 + E_{\text{FP}}^2} - \frac{1}{\omega_n^2 + E_{G'}^2} \right). \quad (\text{D51})$$

Using the relation $\frac{1}{A} - \frac{1}{B} = \frac{B-A}{AB}$ with

$$B - A = m_{G'}^2 - m_{\text{FP}}^2 = \frac{1}{\phi} \frac{\partial V_{g_x^4}(\phi, T)}{\partial \phi} \quad (\text{D52})$$

and identifying the sum integral defining $C_{g_x}^T(\phi, T)$ leads to

$$\xi \frac{\partial V_{g_x^6}^{\xi, T}(\phi, T)}{\partial \xi} = -C_{g_x}^T(\phi, T) \frac{\partial V_{g_x^4}(\phi, T)}{\partial \phi}, \quad (\text{D53})$$

which is the thermal analog of the zero-temperature identity.

- (iii) Third, we calculate the normalization factor $Z(\phi, \xi)$ and its perturbative expansion. This quantity is determined through the relation

$$Z = \left. -\frac{\partial \Gamma^2(p^2)}{\partial p^2} \right|_{p^2=0}, \quad (\text{D54})$$

where Γ denotes the one-particle-irreducible (1PI) Green's function, which is evaluated from the sum of one-particle-irreducible diagrams. We should expect that

$$Z(\phi) = 1 + Z_{g_x^2}(\phi, T=0) + \delta Z_{g_x^2}(\phi, T) + \mathcal{O}(g_x^4), \quad (\text{D55})$$

where the zero-temperature factor $Z_{g_x^2}(\phi, T=0)$ has been provided in [90]:

$$Z_{g_x^2}(\phi, T=0) = \frac{g_x^2}{16\pi^2} \left[\xi \ln \left(\frac{g_x^2 \phi^2 \xi}{\Lambda^2} \right) + 3 \ln \left(\frac{g_x^2 \phi^2}{\Lambda^2} \right) + \xi \right]. \quad (\text{D56})$$

For the leading thermal correction $\delta Z_{g_x^2}(\phi, T)$, one must again reevaluate these loop diagrams using the Matsubara formalism following [89]. In the low-temperature regime considered here, these corrections take the general form of a sum over the particle species, with each term proportional to a Boltzmann suppression factor $e^{-m/T}$ and, therefore, do not contribute to the subleading-order term $Z_{g_x^2}$.

APPENDIX E: FIRST-ORDER PHASE TRANSITIONS, HYDRODYNAMICS, AND GRAVITATIONAL WAVE DETAILS

1. Percolation temperature

As established in the literature [135–138], the percolation temperature T_p serves as the characteristic temperature scale relevant for gravitational wave generation during phase transitions. The quantity T_p represents the hidden sector temperature at which 71% of the Universe volume remains in the false vacuum state and is determined by the condition

$$P_f(T_p) = 0.71, \quad (\text{E1})$$

$$P_f(T) = \exp \left[-\frac{4\pi}{3} v_w^3 \int_T^{T_c} dT' \frac{\Gamma(T')}{T'^4 H(T')} \times \left(\int_T^{T'} dT'' \frac{1}{H(T'')} \right)^3 \right], \quad (\text{E2})$$

$$\Gamma(T) \simeq T^4 \left[\frac{S_{\text{eff}}(T)}{2\pi T} \right]^{3/2} e^{-S_{\text{eff}}(T)/T}, \quad (\text{E3})$$

where $S_{\text{eff}}(T)$ denotes the three-dimensional gauge-invariant effective action calculated in the previous section, T_c is the critical temperature, v_w represents the bubble wall velocity, and H is the Hubble parameter.

2. Transition strength

Two primary types of transition strength parameters are defined, as described in [139,140]:

$$\alpha_{\text{tot}} = \frac{\Delta\bar{\theta}(T_{h,p})}{\rho_{\text{rad}}^v(T_p) + \rho_{\text{rad}}^h(T_p)}, \quad (\text{E4})$$

$$\alpha_h = \frac{\Delta\bar{\theta}(T_p)}{\rho_{\text{rad}}^h(T_p)}. \quad (\text{E5})$$

The quantity $\Delta\bar{\theta}$ represents the vacuum energy released during the phase transition [141,142] and is expressed as

$$\Delta\bar{\theta}(T_h) = \bar{\theta}_f(T_h) - \bar{\theta}_t(T_h), \quad (\text{E6})$$

$$\bar{\theta}_i(T_h) = \frac{1}{4} \left[\rho_i^h - \frac{p_i^h(T_h)}{c_{s,i}^2(T_h)} \right], \quad (\text{E7})$$

where

$$p_i^h(\phi_i, T_h) = \frac{\pi^2}{90} g_{\text{eff}}^h(T_h) T_h^4 - V_{\text{eff}}^h(\phi_i, T_h), \quad (\text{E8})$$

$$\rho_i^h(\phi_i, T_h) = T_h \frac{\partial p_i^h}{\partial T_h} - p_i^h, \quad (\text{E9})$$

$$c_{s,i}^2(T_h) = \frac{\partial p_i^h}{\partial \rho_i^h} = \frac{\partial p_i^h / \partial T_h}{\partial \rho_i^h / \partial T_h}, \quad (\text{E10})$$

where g_{eff}^h is the effective degrees of freedom of the hidden sector. The subscripts “f” and “t” denote the false vacuum state and the true vacuum state, respectively. The parameter α_{tot} is utilized in gravitational wave power spectrum calculations, while α_h is employed in fluid dynamics calculations for the hidden sector to determine the efficiency parameter κ and the bubble wall velocity v_w .

3. Transition rate

The mean bubble separation distance R_* is used to characterize the transition rate instead of action derivative $dS_{\text{eff}}/d \log T|_{T_p}$, which does not provide a good approximation for supercooled phase transitions [100]. The value of R_* is calculated from the bubble number density according to

$$R_*(T_h) = [n_B(T_h)]^{-\frac{1}{3}} = \left[T_h^3 \int_{T_h}^{T_{h,c}} dT'_h \frac{\Gamma(T'_h) P_f(T'_h)}{T_h'^4 H(T'_h)} \right]^{-\frac{1}{3}}. \quad (\text{E11})$$

The transition rate parameter is then defined in relation to this quantity as

$$\frac{\beta}{H_*} = \frac{(8\pi)^{1/3} v_w}{R_* H_*}. \quad (\text{E12})$$

4. Reheating temperature

After bubble percolation, the vacuum energy released during the phase transition is converted into radiation, reheating the plasma. The reheating occurs primarily in the hidden sector where the phase transition takes place. Following [100], assuming reheating completes instantaneously around the time of percolation, energy conservation in the hidden sector gives

$$\rho_{\text{rad}}^h(T_{h,\text{reh}}) \simeq \rho_{\text{rad}}^h(T_{h,p}) + \rho_{\text{vac}}(T_{h,p}), \quad (\text{E13})$$

where $\rho_{\text{rad}}^h = (\pi^2/30) g_{\text{eff}}^h T_h^4$ is the hidden sector radiation energy density and ρ_{vac} is the vacuum energy density released during the transition. Using the definition of the hidden sector transition strength α_h [Eq. (E5)], the hidden sector reheating temperature is

$$T_{h,\text{reh}} = (1 + \alpha_h)^{1/4} \left(\frac{g_{\text{eff}}^h(T_{h,p})}{g_{\text{eff}}^h(T_{h,\text{reh}})} \right)^{1/4} T_{h,p}. \quad (\text{E14})$$

For strong phase transitions with $\alpha_h \gg 1$, reheating significantly increases the hidden sector temperature: $T_{h,\text{reh}} \gg T_{h,p}$.

Although the hidden sector is in thermal equilibrium with the visible sector before the phase transition ($T_h = T_v$), the phase transition completes on a timescale much shorter than the heat transfer timescale between sectors. Consequently, the vacuum energy is initially deposited entirely in the hidden sector, and the visible sector temperature remains essentially unchanged during reheating: $T_{v,\text{reh}} \simeq T_{v,p}$.

5. Redshift factors for gravitational wave spectrum

The gravitational wave spectrum produced at the phase transition must be redshifted to the present day. Following [100], the frequency and amplitude redshift factors are defined as

$$\mathcal{R}_f \equiv \frac{a_*}{a_0} = \left(\frac{h_{\text{eff}}^0}{h_{\text{eff}}(T_{h,\text{reh}})} \right)^{1/3} \frac{T^0}{T_{v,\text{reh}}}, \quad (\text{E15})$$

$$\begin{aligned} \mathcal{R}_\Omega &\equiv \left(\frac{a_*}{a_0} \right)^4 \left(\frac{H_*}{H_0} \right)^2 \\ &\simeq 2.473 \times 10^{-5} h^{-2} \left(\frac{h_{\text{eff}}^0}{h_{\text{eff}}(T_{h,\text{reh}})} \right)^{4/3} \frac{g_{\text{eff}}(T_{h,\text{reh}})}{2}, \end{aligned} \quad (\text{E16})$$

where $T^0 = 2.35 \times 10^{-13}$ GeV is the present cosmic microwave background temperature and $h_{\text{eff}}^0 \simeq 3.91$ is the current effective entropy degrees of freedom. The effective degrees of freedom account for contributions from both sectors, weighted by the temperature ratio $\xi_{\text{reh}} \equiv T_{h,\text{reh}}/T_{v,\text{reh}}$:

$$g_{\text{eff}}(T_{h,\text{reh}}) = g_{\text{eff}}^h(T_{h,\text{reh}}) + \xi_{\text{reh}}^{-4} g_{\text{eff}}^v(T_{v,\text{reh}}), \quad (\text{E17})$$

$$h_{\text{eff}}(T_{h,\text{reh}}) = h_{\text{eff}}^h(T_{h,\text{reh}}) + \xi_{\text{reh}}^{-3} h_{\text{eff}}^v(T_{v,\text{reh}}), \quad (\text{E18})$$

where the superscripts v and h denote the visible and hidden sector contributions, respectively.

6. Hydrodynamics: Efficiency factor and bubble wall velocity

The efficiency factor κ quantifies the fraction of vacuum energy converted into bulk kinetic energy of the plasma [141–147]. In this paper, we follow the calculation in [141,142] to get $\kappa(c_{s,f}^2, c_{s,t}^2, \alpha, v_w)$. Following [100], we use the hidden sector transition strength α_h [Eq. (E5)] to determine the hydrodynamic properties. The bubble expansion proceeds in two regimes distinguished by $\alpha_{h,\infty}$, the critical parameter characterizing maximum plasma friction on the bubble wall.

a. Nonrunaway regime ($\alpha_h < \alpha_{h,\infty}$)

The bubble wall reaches a terminal velocity $v_w < 1$. The efficiency factors are

$$\kappa_{\text{col}} \simeq 0, \quad \kappa_f \simeq \kappa(c_{s,f}^2, c_{s,t}^2, \alpha_h, v_J), \quad (\text{E19})$$

where the Chapman-Jouguet velocity is [14,144]

$$v_J = \frac{c_{s,f} \left(1 + \sqrt{3\alpha_h(1 + c_{s,f}^2\alpha_h)} \right)}{1 + 3c_{s,f}^2\alpha_h}, \quad (\text{E20})$$

with $c_{s,f}/c_{s,t}$ the speed of sound in the false or true vacuum.

b. Runaway regime ($\alpha_h > \alpha_{h,\infty}$)

The bubble wall accelerates toward $v_w \rightarrow 1$, with most vacuum energy going into the wall. The efficiency factors are [143,147]

$$\kappa_{\text{col}} \simeq 1 - \frac{\alpha_{h,\infty}}{\alpha_h}, \quad \kappa_f \simeq \frac{\alpha_{h,\infty}}{\alpha_h} \kappa(c_{s,f}^2, c_{s,t}^2, \alpha_{h,\text{inf}}, 1), \quad (\text{E21})$$

for bubble collisions and plasma motion, respectively.

The bubble collision contribution becomes the dominant source of gravitational waves when $\alpha_h \gg \alpha_{h,\infty}$, which leads to $\kappa_{\text{col}} \gg \kappa_f$. The critical parameter $\alpha_{h,\infty}$ characterizing maximum plasma friction is determined by the mass spectrum of particles coupling to the bubble wall [143]:

$$\alpha_{h,\infty} = \frac{T_p^2}{\rho_{\text{rad}}(T_p)} \left(\sum_{i=\text{boson}} n_i \frac{\Delta m_i^2}{24} + \sum_{i=\text{fermion}} n_i \frac{\Delta m_i^2}{48} \right), \quad (\text{E22})$$

where n_i counts the degrees of freedom and Δm_i^2 is the change in the squared mass across the phase transition.

In our hidden sector model, only two bosons participate in the phase transition: the dark photon and the dark scalar. Since both have small, sub-GeV masses, the value of $\alpha_{h,\infty}$ remains relatively small. Consequently, the collision-dominated condition $\alpha_h \gg \alpha_{h,\infty}$ is readily satisfied with moderate values of $\alpha_h \sim 10^1\text{--}10^3$, without requiring extreme supercooling. This contrasts with the classically conformal $U(1)_{B-L}$ models studied in Refs. [147,148], where the phase transition involves Standard Model particles and additional TeV-scale bosons.

7. Details of the gravitational wave power spectrum

The total gravitational wave power spectrum produced by a first-order phase transition receives contributions from three distinct sources—bubble wall collisions, sound waves in the plasma, and turbulence [136,137,149–153]:

$$\Omega_{\text{GW}} h^2 = \Omega_{\text{col}} h^2 + \Omega_{\text{sw}} h^2 + \Omega_{\text{turb}} h^2. \quad (\text{E23})$$

Following [136], the redshifted spectra are expressed in terms of the mean bubble separation R_* and the kinetic energy fraction $K = \kappa_f \alpha_{\text{tot}} / (1 + \alpha_{\text{tot}})$.

a. Bubble collision contribution

For runaway bubble walls, the contribution from bubble collisions is [136,154]

$$\Omega_{\text{col}}(f) = \mathcal{R}_{\Omega} A \left(\frac{H_* R_*}{(8\pi)^{1/3} v_w} \right)^2 \kappa_{\text{col}}^2 \left(\frac{\alpha_{\text{tot}}}{1 + \alpha_{\text{tot}}} \right)^2 S_{\text{col}}(f), \quad (\text{E24})$$

where $A = 0.049$ and the spectral shape is a broken power law:

$$S_{\text{col}}(f) = \frac{(a+b)^c}{[b(f/f_{\text{col}})^{-a/c} + a(f/f_{\text{col}})^{b/c}]^c}, \quad (\text{E25})$$

with spectral indices $(a, b, c) = (2.4, 2.3, 2.0)$ and peak frequency $f_{\text{col}} = \mathcal{R}_f \cdot 0.77(8\pi)^{1/3} v_w / (2\pi R_*)$.

b. Sound wave contribution

The sound shell model gives [136,152]

$$h^2 \Omega_{\text{sw}}(f) = 3 \mathcal{R}_{\Omega} (H_* R_*) K^2 \frac{M(s, r_b, b)}{\mu_f(r_b)} \Upsilon(\tau_{\text{sw}}) \tilde{\Omega}_{\text{gw}}, \quad (\text{E26})$$

where $\tilde{\Omega}_{\text{gw}} \approx 0.01$ and $M(s, r_b, b)$ is the spectral shape function with $s = 2\pi f R_* / \mathcal{R}_f$ being the dimensionless frequency. The suppression factor accounting for finite sound wave lifetime is [153]

$$\Upsilon(\tau_{\text{sw}}) = 1 - \frac{1}{\sqrt{1 + 2H_*\tau_{\text{sw}}}}, \quad (\text{E27})$$

where $\tau_{\text{sw}} \sim R_*/\bar{U}_f$ is the sound wave lifetime with $\bar{U}_f$ being the root-mean-square fluid velocity.

c. Turbulence contribution

The magnetohydrodynamics turbulence contribution is [136,155]

$$\Omega_{\text{turb}}(f) = 9.0\mathcal{R}_\Omega(H_*R_*)(\kappa_{\text{turb}}K)^{3/2}S_{\text{turb}}(f), \quad (\text{E28})$$

where $\kappa_{\text{turb}} \simeq 0.05$ is the turbulence efficiency factor and the spectral shape is

$$S_{\text{turb}}(f) = \frac{(f/f_{\text{turb}})^3}{[1 + (f/f_{\text{turb}})]^{11/3}(1 + 8\pi f/(\mathcal{R}_f H_*))}, \quad (\text{E29})$$

with peak frequency $f_{\text{turb}} = 3.5\mathcal{R}_f/R_*$.

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- [1] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), *Phys. Rev. Lett.* **116**, 061102 (2016).
 - [2] G. Agazie *et al.* (NANOGrav Collaboration), *Astrophys. J. Lett.* **951**, L8 (2023).
 - [3] J. Antoniadis *et al.* (EPTA and InPTA Collaborations), *Astron. Astrophys.* **678**, A50 (2023).
 - [4] D. J. Reardon, A. Zic, R. M. Shannon, G. B. Hobbs, M. Bailes, V. Di Marco, A. Kapur, A. F. Rogers, E. Thrane, J. Askew *et al.*, *Astrophys. J. Lett.* **951**, L6 (2023).
 - [5] N. Agarwal, G. Agazie, A. Anumalapudi, A. M. Archibald, Z. Arzoumanian, J. G. Baier, P. T. Baker, B. Bessy, L. Blecha, A. Brazier *et al.*, *Astrophys. J. Lett.* **998**, L11 (2026).
 - [6] Q. Zheng, C. M. F. Mingarelli, W. DeRocco, J. Nay, K. K. Boddy, and J. A. Dror, *Phys. Rev. D* **113**, 023037 (2026).
 - [7] S. Y. Khlebnikov and I. I. Tkachev, *Phys. Rev. D* **56**, 653 (1997).
 - [8] R. Easther, J. T. Giblin, Jr., and E. A. Lim, *Phys. Rev. Lett.* **99**, 221301 (2007).
 - [9] J. Garcia-Bellido, D. G. Figueroa, and A. Sastre, *Phys. Rev. D* **77**, 043517 (2008).
 - [10] D. A. Kirzhnits, *JETP Lett.* **15**, 529 (1972).
 - [11] D. A. Kirzhnits and A. D. Linde, *Phys. Lett.* **42B**, 471 (1972).
 - [12] E. Witten, *Nucl. Phys.* **B177**, 477 (1981).
 - [13] A. H. Guth and E. J. Weinberg, *Phys. Rev. D* **23**, 876 (1981).
 - [14] P. J. Steinhardt, *Phys. Rev. D* **25**, 2074 (1982).
 - [15] F. C. Adams, *Phys. Rev. D* **48**, 2800 (1993).
 - [16] R. R. Parwani, *Phys. Rev. D* **45**, 4695 (1992); **48**, 5965(E) (1993).
 - [17] P. B. Arnold and O. Espinosa, *Phys. Rev. D* **47**, 3546 (1993); **50**, 6662(E) (1994).
 - [18] J. R. Espinosa, M. Quiros, and F. Zwirner, *Phys. Lett. B* **314**, 206 (1993).
 - [19] M. Quiros, [arXiv:hep-ph/9304284](https://arxiv.org/abs/hep-ph/9304284).
 - [20] D. Curtin, P. Meade, and H. Ramani, *Eur. Phys. J. C* **78**, 787 (2018).
 - [21] J. R. Espinosa and M. Quiros, *Phys. Rev. D* **76**, 076004 (2007).
 - [22] J. R. Espinosa, T. Konstandin, J. M. No, and M. Quiros, *Phys. Rev. D* **78**, 123528 (2008).
 - [23] D. Azevedo, P. M. Ferreira, M. M. Muhlleitner, S. Patel, R. Santos, and J. Wittbrodt, *J. High Energy Phys.* **11** (2018) 091.
 - [24] A. Mohamadnejad, *J. High Energy Phys.* **03** (2022) 188.
 - [25] L. Biermann, M. Muhlleitner, and J. Müller, *Eur. Phys. J. C* **83**, 439 (2023).
 - [26] W. Wang, W. L. Xu, and J. M. Yang, *Eur. Phys. J. Plus* **138**, 781 (2023).
 - [27] A. Addazi, *Mod. Phys. Lett. A* **32**, 1750049 (2017).
 - [28] Y. Zhu, J. Liu, R. Qin, and L. Bian, *Phys. Rev. D* **112**, 015018 (2025).
 - [29] M. Aoki, H. Goto, and J. Kubo, *Phys. Rev. D* **96**, 075045 (2017).
 - [30] R. Pasechnik, M. Reichert, F. Sannino, and Z. W. Wang, *J. High Energy Phys.* **02** (2024) 159.
 - [31] P. Schwaller, *Phys. Rev. Lett.* **115**, 181101 (2015).
 - [32] A. Addazi and A. Marciano, *Chin. Phys. C* **42**, 023107 (2018).
 - [33] M. Fairbairn, E. Hardy, and A. Wickens, *J. High Energy Phys.* **07** (2019) 044.
 - [34] R. T. Co, K. Harigaya, and A. Pierce, *J. High Energy Phys.* **12** (2021) 099.
 - [35] D. Borah, A. Dasgupta, and S. K. Kang, *Phys. Rev. D* **104**, 063501 (2021).
 - [36] T. Abe and K. Hashino, *Prog. Theor. Exp. Phys.* **2024**, 063B02 (2024).
 - [37] B. Imtiaz, Y. F. Cai, and Y. Wan, *Eur. Phys. J. C* **79**, 25 (2019).
 - [38] A. Paul, U. Mukhopadhyay, and D. Majumdar, *J. High Energy Phys.* **05** (2021) 223.
 - [39] M. Breitbach, J. Kopp, E. Madge, T. Opferkuch, and P. Schwaller, *J. Cosmol. Astropart. Phys.* **07** (2019) 007.
 - [40] F. Ertas, F. Kahlhoefer, and C. Tasillo, *J. Cosmol. Astropart. Phys.* **02** (2022) 014.
 - [41] K. Freese and M. W. Winkler, *Phys. Rev. D* **106**, 103523 (2022).
 - [42] T. Bringmann, T. E. Gonzalo, F. Kahlhoefer, J. Matuszak, and C. Tasillo, *J. Cosmol. Astropart. Phys.* **05** (2024) 065.
 - [43] A. Banik, Y. Cui, Y. D. Tsai, and Y. Tsai, [arXiv:2412.16282](https://arxiv.org/abs/2412.16282).
 - [44] J. Jaeckel, V. V. Khoze, and M. Spannowsky, *Phys. Rev. D* **94**, 103519 (2016).
 - [45] S. P. Li and K. P. Xie, *Phys. Rev. D* **108**, 055018 (2023).

- [46] T. Ghosh, A. Ghoshal, H. K. Guo, F. Hajkarim, S. F. King, K. Sinha, X. Wang, and G. White, *J. Cosmol. Astropart. Phys.* **05** (2024) 100.
- [47] P. Di Bari, D. Marfatia, and Y. L. Zhou, *J. High Energy Phys.* **10** (2021) 193.
- [48] Z. Chen, K. Ye, and M. Zhang, *Phys. Rev. D* **107**, 095027 (2023).
- [49] Y. Gouttenoire and T. Volansky, *Phys. Rev. D* **110**, 043514 (2024).
- [50] S. Kanemura and S. P. Li, *J. Cosmol. Astropart. Phys.* **03** (2024) 005.
- [51] M. Kierkla, N. Ramberg, P. Schicho, and D. Schmitt, *arXiv:2506.15496*.
- [52] P. Ghosh, T. Ghosh, and S. Roy, *J. High Energy Phys.* **10** (2023) 057.
- [53] S. Roy, *Phys. Rev. D* **111**, 015037 (2025).
- [54] S. Balan, T. Bringmann, F. Kahlhoefer, J. Matuszak, and C. Tasillo, *J. Cosmol. Astropart. Phys.* **08** (2025) 062.
- [55] A. Gonstal, M. Lewicki, and B. Swiezewska, *J. High Energy Phys.* **08** (2025) 039.
- [56] M. Carena, A. Ireland, T. Ou, and I. R. Wang, *J. High Energy Phys.* **09** (2025) 175.
- [57] A. Dasgupta, M. Knauss, and M. Sher, *Phys. Rev. D* **112**, 055034 (2025).
- [58] T. Biekötter and M. O. Olea-Romacho, *J. High Energy Phys.* **12** (2025) 040.
- [59] M. Chala, L. Gil, and Z. Ren, *Chin. Phys.* **49**, 123105 (2025).
- [60] S. Roy, *arXiv:2509.19982*.
- [61] A. G. Cohen, D. B. Kaplan, and A. E. Nelson, *Phys. Lett. B* **245**, 561 (1990).
- [62] M. Carena, J. M. Moreno, M. Quiros, M. Seco, and C. E. M. Wagner, *Nucl. Phys.* **B599**, 158 (2001).
- [63] J. M. Cline, *arXiv:hep-ph/0609145*.
- [64] G. A. White, *A Pedagogical Introduction to Electroweak Baryogenesis* (Morgan & Claypool, 2016), ISBN 978-1-68174-456-8, 978-1-68174-457-5, [10.1088/978-1-6817-4457-5](https://doi.org/10.1088/978-1-6817-4457-5).
- [65] J. M. Cline, *Proc. Sci., TASI2018* (2019) 001 [*arXiv:1807.08749*].
- [66] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, *Phys. Rev. Lett.* **77**, 2887 (1996).
- [67] M. Gurtler, E. M. Ilgenfritz, and A. Schiller, *Phys. Rev. D* **56**, 3888 (1997).
- [68] F. Csikor, Z. Fodor, and J. Heitger, *Phys. Rev. Lett.* **82**, 21 (1999).
- [69] C. T. Hill, S. Pokorski, and J. Wang, *Phys. Rev. D* **64**, 105005 (2001).
- [70] S. Girmohanta, Y. Nakai, and Z. Zhang, *Phys. Rev. D* **112**, 075028 (2025).
- [71] B. Patt and F. Wilczek, *arXiv:hep-ph/0605188*.
- [72] B. Holdom, *Phys. Lett.* **166B**, 196 (1986).
- [73] B. Kors and P. Nath, *Phys. Lett. B* **586**, 366 (2004).
- [74] D. Feldman, Z. Liu, and P. Nath, *Phys. Rev. D* **75**, 115001 (2007).
- [75] K. Cheung and T. C. Yuan, *J. High Energy Phys.* **03** (2007) 120.
- [76] W. Z. Feng, Z. H. Zhang, and K. Y. Zhang, *J. Cosmol. Astropart. Phys.* **05** (2024) 112.
- [77] A. Aboubrahim, P. Nath, and Z. Y. Wang, *J. High Energy Phys.* **12** (2021) 148.
- [78] M. Du, Z. Liu, and P. Nath, *Phys. Lett. B* **834**, 137454 (2022).
- [79] D. Feldman, B. Kors, and P. Nath, *Phys. Rev. D* **75**, 023503 (2007).
- [80] W. Z. Feng and P. Nath, *Phys. Lett. B* **731**, 43 (2014).
- [81] D. E. Kaplan, M. A. Luty, and K. M. Zurek, *Phys. Rev. D* **79**, 115016 (2009).
- [82] T. Cohen, D. J. Phalen, A. Pierce, and K. M. Zurek, *Phys. Rev. D* **82**, 056001 (2010).
- [83] M. L. Graesser, I. M. Shoemaker, and L. Vecchi, *J. High Energy Phys.* **10** (2011) 110.
- [84] M. Ibe, S. Matsumoto, and T. T. Yanagida, *Phys. Lett. B* **708**, 112 (2012).
- [85] W. Z. Feng, P. Nath, and G. Peim, *Phys. Rev. D* **85**, 115016 (2012).
- [86] W. Z. Feng, A. Mazumdar, and P. Nath, *Phys. Rev. D* **88**, 036014 (2013).
- [87] W. Z. Feng and P. Nath, *Mod. Phys. Lett. A* **32**, 1740005 (2017).
- [88] D. Metaxas and E. J. Weinberg, *Phys. Rev. D* **53**, 836 (1996).
- [89] M. Garny and T. Konstandin, *J. High Energy Phys.* **07** (2012) 189.
- [90] S. Arunasalam and M. J. Ramsey-Musolf, *J. High Energy Phys.* **08** (2022) 138.
- [91] J. Löfgren, M. J. Ramsey-Musolf, P. Schicho, and T. V. I. Tenkanen, *Phys. Rev. Lett.* **130**, 251801 (2023).
- [92] J. Hirvonen, J. Löfgren, M. J. Ramsey-Musolf, P. Schicho, and T. V. I. Tenkanen, *J. High Energy Phys.* **07** (2022) 135.
- [93] M. Kierkla, B. Swiezewska, T. V. I. Tenkanen, and J. van de Vis, *J. High Energy Phys.* **02** (2024) 234.
- [94] P. A. R. Ade *et al.* (Planck Collaboration), *Astron. Astrophys.* **571**, A1 (2014).
- [95] L. Covi, E. Roulet, and F. Vissani, *Phys. Lett. B* **384**, 169 (1996).
- [96] J. A. Harvey and M. S. Turner, *Phys. Rev. D* **42**, 3344 (1990).
- [97] E. Komatsu *et al.* (WMAP Collaboration), *Astrophys. J. Suppl. Ser.* **192**, 18 (2011).
- [98] M. Fabbrihesi, E. Gabrielli, and G. Lanfranchi, *arXiv:2005.01515*.
- [99] A. Aboubrahim, M. M. Altakach, M. Klasen, P. Nath, and Z. Y. Wang, *J. High Energy Phys.* **03** (2023) 182.
- [100] J. Li and P. Nath, *Phys. Rev. D* **111**, 123007 (2025).
- [101] P. Amaro-Seoane *et al.* (LISA Collaboration), *arXiv:1702.00786*.
- [102] J. Baker, J. Bellovary, P. L. Bender, E. Berti, R. Caldwell, J. Camp, J. W. Conklin, N. Cornish, C. Cutler, R. DeRosa *et al.*, *arXiv:1907.06482*.
- [103] P. Amaro-Seoane, S. Aoudia, S. Babak, P. Binetruy, E. Berti, A. Bohe, C. Caprini, M. Colpi, N. J. Cornish, K. Danzmann *et al.*, *GW Notes* **6**, 4 (2013).
- [104] C. Grojean and G. Servant, *Phys. Rev. D* **75**, 043507 (2007).
- [105] S. Kawamura, T. Nakamura, M. Ando, N. Seto, K. Tsubono, K. Numata, R. Takahashi, S. Nagano, T. Ishikawa, M. Musha *et al.*, *Classical Quantum Gravity* **23**, S125 (2006).
- [106] W. H. Ruan, Z. K. Guo, R. G. Cai, and Y. Z. Zhang, *Int. J. Mod. Phys. A* **35**, 2050075 (2020).

- [107] J. Luo *et al.* (TianQin Collaboration), *Classical Quantum Gravity* **33**, 035010 (2016).
- [108] A. Sesana, N. Korsakova, M. A. Sedda, V. Baibhav, E. Barausse, S. Barke, E. Berti, M. Bonetti, P. R. Capelo, C. Caprini *et al.*, *Exp. Astron.* **51**, 1333 (2021).
- [109] D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen, and G. White, *J. High Energy Phys.* **04** (2021) 055.
- [110] W. Z. Feng and Z. H. Zhang, *Phys. Rev. D* **112**, 035004 (2025).
- [111] J. H. Chang, R. Essig, and S. D. McDermott, *J. High Energy Phys.* **01** (2017) 107.
- [112] J. D. Bjorken, S. Ecklund, W. R. Nelson, A. Abashian, C. Church, B. Lu, L. W. Mo, T. A. Nunamaker, and P. Rassmann, *Phys. Rev. D* **38**, 3375 (1988).
- [113] B. Batell, R. Essig, and Z. Surujon, *Phys. Rev. Lett.* **113**, 171802 (2014).
- [114] L. Marsicano, M. Battaglieri, M. Bondi', C. D. R. Carvajal, A. Celentano, M. De Napoli, R. De Vita, E. Nardi, M. Raggi, and P. Valente, *Phys. Rev. D* **98**, 015031 (2018).
- [115] S. N. Gninenko, *Phys. Lett. B* **713**, 244 (2012).
- [116] J. Blümlein and J. Brunner, *Phys. Lett. B* **731**, 320 (2014).
- [117] J. Blümlein and J. Brunner, *Phys. Lett. B* **701**, 155 (2011).
- [118] E. M. Riordan, M. W. Krasny, K. Lang, P. De Barbaro, A. Bodek, S. Dasu, N. Varelas, X. Wang, R. G. Arnold, D. Benton *et al.*, *Phys. Rev. Lett.* **59**, 755 (1987).
- [119] M. Pospelov, *Phys. Rev. D* **80**, 095002 (2009).
- [120] NA64 Collaboration, CERN Report No. CERN-SPSC-2018-024.
- [121] H. Merkel, P. Achenbach, C. Ayerbe Gayoso, T. Beranek, J. Bericic, J. C. Bernauer, R. Böhm, D. Bosnar, L. Correa, L. Debenjak *et al.*, *Phys. Rev. Lett.* **112**, 221802 (2014).
- [122] J. P. Lees *et al.* (BABAR Collaboration), *Phys. Rev. Lett.* **113**, 201801 (2014).
- [123] R. Piandani, *EPJ Web Conf.* **126**, 04035 (2016).
- [124] F. Archilli *et al.* (KLOE-2 Collaboration), *Phys. Lett. B* **706**, 251 (2012).
- [125] D. Babusci *et al.* (KLOE-2 Collaboration), *Phys. Lett. B* **720**, 111 (2013).
- [126] D. Babusci *et al.* (KLOE-2 Collaboration), *Phys. Lett. B* **736**, 459 (2014).
- [127] A. Anastasi *et al.* (KLOE-2 Collaboration), *Phys. Lett. B* **757**, 356 (2016).
- [128] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **124**, 041801 (2020).
- [129] CMS Collaboration, CERN Report No. CMS-PAS-EXO-19-018.
- [130] B. Hua and J. Zhu, *Comput. Phys. Commun.* **316**, 109764 (2025).
- [131] M. Sher, *Phys. Rep.* **179**, 273 (1989).
- [132] H. H. Patel and M. J. Ramsey-Musolf, *J. High Energy Phys.* **07** (2011) 029.
- [133] H. Bahl, M. Carena, A. Ireland, and C. E. M. Wagner, *J. High Energy Phys.* **09** (2024) 153.
- [134] P. Bittar, S. Roy, and C. E. M. Wagner, *J. High Energy Phys.* **12** (2025) 021.
- [135] P. Athron, C. Balázs, and L. Morris, *J. Cosmol. Astropart. Phys.* **03** (2023) 006.
- [136] P. Athron, A. Fowlie, C. T. Lu, L. Morris, L. Wu, Y. Wu, and Z. Xu, *Phys. Rev. Lett.* **132**, 221001 (2024).
- [137] C. Caprini, M. Chala, G. C. Dorsch, M. Hindmarsh, S. J. Huber, T. Konstandin, J. Kozaczuk, G. Nardini, J. M. No, K. Rummukainen *et al.*, *J. Cosmol. Astropart. Phys.* **03** (2020) 024.
- [138] X. Wang, F. P. Huang, and X. Zhang, *J. Cosmol. Astropart. Phys.* **05** (2020) 045.
- [139] T. Bringmann, P. F. Depta, T. Konstandin, K. Schmidt-Hoberg, and C. Tasillo, *J. Cosmol. Astropart. Phys.* **11** (2023) 053.
- [140] Y. Bai and M. Korwar, *Phys. Rev. D* **105**, 095015 (2022).
- [141] F. Giese, T. Konstandin, K. Schmitz, and J. van de Vis, *J. Cosmol. Astropart. Phys.* **01** (2021) 072.
- [142] F. Giese, T. Konstandin, and J. van de Vis, *J. Cosmol. Astropart. Phys.* **07** (2020) 057.
- [143] D. Bodeker and G. D. Moore, *J. Cosmol. Astropart. Phys.* **05** (2017) 025.
- [144] J. R. Espinosa, T. Konstandin, J. M. No, and G. Servant, *J. Cosmol. Astropart. Phys.* **06** (2010) 028.
- [145] W. Y. Ai, B. Garbrecht, and C. Tamarit, *J. Cosmol. Astropart. Phys.* **03** (2022) 015.
- [146] W. Y. Ai, B. Laurent, and J. van de Vis, *J. High Energy Phys.* **02** (2025) 119.
- [147] J. Ellis, M. Lewicki, J. M. No, and V. Vaskonen, *J. Cosmol. Astropart. Phys.* **06** (2019) 024.
- [148] J. Ellis, M. Lewicki, and V. Vaskonen, *J. Cosmol. Astropart. Phys.* **11** (2020) 020.
- [149] C. Caprini, M. Hindmarsh, S. Huber, T. Konstandin, J. Kozaczuk, G. Nardini, J. M. No, A. Petiteau, P. Schwaller, G. Servant *et al.*, *J. Cosmol. Astropart. Phys.* **04** (2016) 001.
- [150] M. Hindmarsh, *Phys. Rev. Lett.* **120**, 071301 (2018).
- [151] M. Hindmarsh and M. Hijazi, *J. Cosmol. Astropart. Phys.* **12** (2019) 062.
- [152] M. Hindmarsh, S. J. Huber, K. Rummukainen, and D. J. Weir, *Phys. Rev. D* **96**, 103520 (2017); **101**, 089902(E) (2020).
- [153] H. K. Guo, K. Sinha, D. Vagie, and G. White, *J. Cosmol. Astropart. Phys.* **01** (2021) 001.
- [154] S. J. Huber and T. Konstandin, *J. Cosmol. Astropart. Phys.* **09** (2008) 022.
- [155] C. Caprini, R. Durrer, and G. Servant, *J. Cosmol. Astropart. Phys.* **12** (2009) 024.