

Perturbative approach to the infrared gluon propagator in the maximal Abelian gauge

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The inclusion of a masslike term for the gluon in Yang-Mills theories quantized in the Landau gauge has proven to be an effective way of reproducing lattice results for gauge-fixed correlation functions within perturbative computations. Since those quantities are gauge dependent, it is natural to question how general this prescription is for describing the infrared behavior of gluon and Faddeev-Popov ghost propagators in different gauges. In this work, we provide a systematic investigation of this issue in the maximal Abelian gauge, which cannot be deformed into the Landau gauge and has been investigated in gauge-fixed lattice simulations. We compute the one-loop non-Abelian and diagonal gluon propagators and perform fits to lattice data in the case of $SU(2)$. Our results show that the transverse component of the non-Abelian gluon propagator as well as the diagonal gluon propagator, are in good agreement with lattice data in the infrared.

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I. INTRODUCTION

Accessing the infrared (IR) regime of Yang-Mills (YM) theories in the continuum remains a challenging task. Since these theories are strongly coupled at low energies, non-perturbative methods are required. Concrete calculations in the continuum require fixing the gauge which is usually done through the Faddeev-Popov (FP) procedure [1]. For this reason, many explicit computations are focused on gauge-dependent correlators that in turn build up the gauge-invariant correlation functions associated with observables. In recent years, important progress has been made in understanding gauge-fixed correlators through numerical simulations [2–12] and (semi)analytical approaches [13–19] with the Landau gauge being employed in most studies. Gauge-fixed lattice

simulations offer, among other things, an independent benchmark for the computations performed in the continuum, and such a synergetic approach has led to considerable progress in the comprehension of the infrared behavior of correlation functions in Yang-Mills theories over the past decades.

It is well known that gauge fixing in non-Abelian gauge theories suffers from a subtle issue in the nonperturbative regime due to the presence of Gribov copies [20,21]; see, e.g., [22–24] for reviews on the Gribov problem. The FP procedure is not able to fix completely the gauge redundancy, and spurious configurations, the Gribov copies, remain present in the path integral. It is still unknown how to completely eliminate those configurations, and only a partial resolution of the issue is understood for some particular gauge choices. In fact, most of the works in this direction are restricted to the Landau gauge [17,20,25] where a systematic elimination of infinitesimal Gribov copies is achieved. Another gauge in which the Gribov problem has been partially dealt with is the so-called maximal Abelian gauge (MAG); see, e.g., [26,27]. It treats non-Abelian (off diagonal) and Abelian (diagonal) components of the gauge field differently. This feature makes the MAG well suited to study color confinement via the dual superconductivity mechanism [28,29], where low-energy Yang-Mills theories reduce to an effective Abelian theory with monopoles [30–38].

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For all gauges in which a concrete (partial) elimination of Gribov copies was implemented, the analysis was restricted to infinitesimal copies. Those are configurations that are related by an infinitesimal gauge transformation and satisfy the same gauge condition. The action that implements the elimination of the infinitesimal copies is the so-called Gribov-Zwanziger (GZ) action; see [25]. Originally, it was constructed in the Landau gauge leading to a local and renormalizable framework which effectively takes into account the elimination of infinitesimal Gribov copies. It was found that the GZ action features IR instabilities that are stabilized by the inclusion of condensates. The resulting theory is the so-called Refined Gribov-Zwanziger (RGZ) action; see [17]. Later, the construction of the analog of the GZ action in the MAG was achieved in [26]. The dynamical formation of condensates, whose inclusion led to the so-called RGZ action was worked out in [39,40]. Remarkably, the RGZ action (both in the Landau gauge and in the MAG) is perturbatively renormalizable and local. More recently, the construction of the RGZ action in linear covariant and Curci-Ferrari gauges was achieved; see [41–46]. It is worth mentioning that different strategies to deal with Gribov copies were employed in [47–50] where instead of eliminating the spurious configurations, an averaging procedure is used. Recently, attempts to establish a connection between both approaches were carried out in [51,52]. The RGZ action can be viewed as an improved version of the gauge-fixed FP action since it partially deals with the presence of Gribov copies. Nevertheless, the removal of infinitesimal Gribov copies leads to an effective modification of the tree-level gluon propagator as well as new interactions with auxiliary localizing fields. As a result, already at tree-level the gluon propagator carries nonperturbative information and its form is such that it fits very well with the lattice data in the infrared. This is well explored in the Landau gauge [10–12,16] and reasonably understood in the MAG [39].

Recent lattice simulations have revealed a massivelike behavior for the gluon propagator in the deep IR [2–6]. This has also been seen in continuum approaches as in the RGZ, and in functional methods (see, e.g., [14,15,53–55]) and in the context of center vortices [56]. From the RGZ perspective, for instance, the tree-level gluon propagator can be written as a Yukawa-like propagator with a momentum-dependent mass. This feature led [57,58] to the proposal of an effective massive model for the gluon, the Curci-Ferrari (CF) model, to describe the IR behavior of the gluon propagator obtained in lattice simulations in the Landau gauge. First, by including one-loop corrections, they were able to fit the gluon and ghost propagators in the IR regime. Later, these results were improved with the inclusion of two-loop corrections [59]. Alongside this model, a renormalization scheme was proposed, the infrared-safe scheme, in which the effective coupling does not develop an IR Landau pole. We refer to [18] for a review and to [60] for a pedagogical introduction to the CF model. More developments, including dynamical quarks [61], such as the computation of the (un)quenched

ghost-gluon and three-gluon vertices [62,63], quark-gluon vertex [64,65], and the four-point gluon vertex [66] were carried out within the CF model. The inclusion of quarks yields more realistic investigations, which can be related to hadronic phenomenology; see [67,68]. At finite temperature, the model can be employed to probe the breaking of center symmetry which is associated with the confinement/deconfinement transition of quarks and gluons; see [69–71]. At this stage, a comment is in order: The CF model consists in introducing a masslike term on top of the gauge-fixing term (originally, in the Landau gauge). It should not be thought of as a massive Proca-like theory. Effectively, the introduction of such a masslike term can be thought as the most economical model to deal with the existence of (infinitesimal) Gribov copies.

After gauge-fixing through the FP procedure, the gauge(-parameter) independence of physical quantities is controlled by the Becchi-Rouet-Stora-Tyutin (BRST) symmetry [72,73]. It allows for a systematic definition a physical subspace through its associated charge. However, beyond the perturbative regime this construction can be challenged due to the existence of Gribov copies.¹ The CF model contains the main elements of the BRST quantization, namely, the gauge-fixing term with the corresponding FP ghost fields. In fact, the masslike term for the gluons is added together with the gauge-fixed YM action in the Landau gauge. However, the masslike term explicitly breaks BRST symmetry. In this model there is a symmetry which resembles the BRST, but it is not nilpotent. This later property is crucial to guarantee that ghost states with a negative norm decouple [74]. This poses a limitation to the CF model. For instance, if one adds a simple masslike term for the gluons in the gauge-fixed YM action in linear covariant gauges, the lack of BRST invariance will translate into gauge-parameter dependence of gauge-invariant correlator quantities. Progress in building a CF-like model in linear covariant gauges can be found in [75]. The construction of a CF-like model in the Curci-Ferrari-Delbourgo-Jarvis gauge has also been worked out in [49,76] with quarks.

In light of these considerations, we believe it interesting to investigate whether CF-like models in other gauges that are not simply related to the Landau gauge by a suitable choice of gauge parameter are able to reproduce lattice data as effectively as in the Landau gauge. The main reason behind this is simplicity, i.e., the possibility of having an effective model that accesses the IR dynamics of YM theories through explicit perturbative computations. Therefore, we propose an analogous CF-like model in the MAG with gauge group $SU(2)$. This gauge brings new challenges with respect to the Landau gauge due to its nonlinearity. In particular, this engenders the

¹In fact, as originally formulated, the (R)GZ action breaks BRST symmetry explicitly but in a soft manner. More recently, a BRST-invariant formulation of the RGZ action in linear covariant gauges was proposed (see, e.g., [41,45]) thanks to the use of dressed variables.

introduction of ghost quartic interactions and a fiducial parameter α to be renormalizable. Only after explicit computations are performed one takes the limit $\alpha \rightarrow 0$ and the MAG is recovered. Moreover, it breaks the global color symmetry explicitly. Interestingly, however, this gauge can be implemented in the lattice, as was done in the four-dimensional case for $SU(2)$ in [77–79] and has also been explored by functional methods; see [80]. More recent simulations for $SU(3)$ were also reported in [81]. Unfortunately, moving from $SU(2)$ to $SU(3)$ in the case of the MAG is not so simple due to the existence of extra vertices, and we focus on the $SU(2)$ case in this work for simplicity.

The effective model that we propose for the $SU(2)$ MAG contains the same elements of the CF model. We keep the gauge-fixing term of the renormalizable MAG, i.e., we include the quartic ghost terms with the gauge parameter α , and add masslike terms for the gauge fields. These masses should play the crucial role of capturing the behavior of the gluon propagators for $p \rightarrow 0$ seen on the lattice. Since the global color symmetry is explicitly broken in this gauge, we consider different masses for the components, namely, M and m for the off diagonal and diagonal components, respectively. In fact, to properly describe the Abelian dominance [79] observed in the IR region, we need $M^2 > m^2$. We also add a mass term for the off diagonal ghosts, which is necessary to have a renormalizable model as will be clear later. Since on the lattice the original MAG ($\alpha = 0$) can be simulated, in the end of any calculation with this model the limit $\alpha \rightarrow 0$ should be taken for meaningful comparisons.

This article is organized as follows. In Sec. II, we present and provide the ground for the IR-effective model for the $SU(2)$ MAG. In Sec. III, we present the one-loop results for the off diagonal gluon propagator and the diagonal gluon propagator, alongside a comparison with the lattice results. Finally, in Sec. IV, we collect our conclusions and indicate some directions for future research.

II. THE MODEL

A. Yang-Mills theories in the MAG

The model we consider in this work corresponds to Euclidean four-dimensional Yang-Mills theories in the MAG with $SU(2)$ gauge group equipped with masslike terms for the Abelian and non-Abelian components of the gauge field. Before discussing the introduction of the mass terms, we start with a short overview of the quantization of YM theories in the MAG. The starting point is the YM action given by²

$$\begin{aligned} S_{\text{YM}} &= \frac{1}{4} \int_x F_{\mu\nu}^A F_{\mu\nu}^A \\ &= \frac{1}{4} \int_x (F_{\mu\nu}^a F_{\mu\nu}^a + F_{\mu\nu} F_{\mu\nu}), \end{aligned} \quad (1)$$

where $F_{\mu\nu}^a = \mathcal{D}_{\mu}^{ab} A_{\nu}^b - \mathcal{D}_{\nu}^{ab} A_{\mu}^b$ and $F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} + g\epsilon^{ab} A_{\mu}^a A_{\nu}^b$. The generators of the gauge group are $\{T^A\}$ with $A = 1, 2, 3$, the Cartan subgroup is generated by T^3 , and therefore we employ small latin indices $a, b, \dots = 1, 2$ to represent the non-Abelian components of the algebra. Since we just have one Abelian component, we omit its associated index, i.e., $A_{\mu}^3 \equiv A_{\mu}$. The covariant derivative $\mathcal{D}_{\mu}^{ab}$ is defined only with respect to the Abelian component, i.e., $\mathcal{D}_{\mu}^{ab} \equiv \delta^{ab} \partial_{\mu} - g\epsilon^{ab} A_{\mu}$. Finally, the antisymmetric symbol ϵ^{ab} is defined by $[T^A, T^B] = i\epsilon^{ABC} T^C$ where ϵ^{ABC} is the Levi-Civita symbol and $\epsilon^{ab3} \equiv \epsilon^{ab}$.

The BRST quantization of YM theories in the MAG requires the introduction of the following BRST-exact term,

$$\begin{aligned} S_{\text{FP}} &= \mathbf{s} \int_x \bar{c}^a \left(\mathcal{D}_{\mu}^{ab} A_{\mu}^b - \frac{\alpha}{2} i b^a + g \frac{\alpha}{2} \epsilon^{ab} \bar{c}^b c \right) \\ &\quad + \mathbf{s} \int_x \bar{c} \partial_{\mu} A_{\mu}. \end{aligned} \quad (2)$$

The nilpotent BRST operator $\mathbf{s}$ acts on the elementary fields as

$$\begin{aligned} \mathbf{s} A_{\mu}^a &= -\mathcal{D}_{\mu}^{ab} c^b - g\epsilon^{ab} A_{\mu}^b c, & \mathbf{s} A_{\mu} &= -\partial_{\mu} c - g\epsilon^{ab} A_{\mu}^a c^b, \\ \mathbf{s} c^a &= g\epsilon^{ab} c^b c, & \mathbf{s} c &= \frac{g}{2} \epsilon^{ab} c^a c^b, \\ \mathbf{s} \bar{c}^a &= i b^a, & \mathbf{s} \bar{c} &= i b, \\ \mathbf{s} b^a &= 0, & \mathbf{s} b &= 0, \end{aligned} \quad (3)$$

where $(\bar{c}^a, c^a)$ and $(\bar{c}, c)$ are the non-Abelian and Abelian FP ghosts. The fields b^a and b are the non-Abelian and Abelian Nakanishi-Lautrup fields, respectively. The resulting FP action is

$$\begin{aligned} S_{\text{FP}} &= \int_x \left[i b^a \mathcal{D}_{\mu}^{ab} A_{\mu}^b + \frac{\alpha}{2} b^a b^a + i b \partial_{\mu} A_{\mu} - \bar{c}^a \mathcal{M}^{ab} c^b \right. \\ &\quad \left. + g\epsilon^{ab} \bar{c}^a (\mathcal{D}_{\mu}^{bc} A_{\mu}^c) c + \bar{c} \partial_{\mu} (\partial_{\mu} c + g\epsilon^{ab} A_{\mu}^a c^b) \right. \\ &\quad \left. + \alpha g \epsilon^{ab} i b^a \bar{c}^b c - \frac{\alpha}{2} g^2 (\bar{c}^a c^a)^2 \right], \end{aligned} \quad (4)$$

with $\mathcal{M}^{ab} \equiv -\mathcal{D}_{\mu}^{ac} \mathcal{D}_{\mu}^{cb} - g^2 \epsilon^{ac} \epsilon^{bd} A_{\mu}^c A_{\mu}^d$ being the Faddeev-Popov operator in the MAG, and α is a gauge parameter. Some comments are in order. The “true” MAG is attained at vanishing value for the gauge parameter, i.e., $\alpha = 0$. This means, that the MAG selects gauge field configurations which satisfy

$$\mathcal{D}_{\mu}^{ab} A_{\mu}^b = 0, \quad (5)$$

and the residual $U(1)$ symmetry is fixed by a Landau-type condition, i.e., $\partial_{\mu} A_{\mu} = 0$. The condition (5) is achieved by demanding the minimization of the functional

²We employ the short-hand notation $\int d^4x \equiv \int_x$.

$$\mathcal{R}[A] = \int_x A_\mu^a A_\mu^a, \quad (6)$$

along each gauge orbit, which makes the search for gauge fields satisfying (5) algorithmic for gauge-fixed lattice simulations. Yet condition (5) is nonlinear and requires the introduction of quartic interactions in the FP ghosts, and thereby we define the FP action containing α -dependent terms that are necessary for renormalizability [82–84]. The nonlinear structure of the MAG also leaves imprints in the formation of condensates. In particular, the off diagonal components of ghosts and gluons acquire a dynamical mass; see [85–87]. This leads to a hint toward the Abelian dominance phenomenon which nicely fits the picture of non-Abelian degrees of freedom decoupling below the dynamical-mass scale and rendering an effective Abelian theory in the deep infrared; see, e.g., [88–91]. The analytic treatment of the formation of condensates was achieved by the introduction of a BRST-on-shell invariant dimension two ghost-gluon operator [85,87], i.e.,

$$\mathcal{O}(\alpha) = \frac{1}{2} A_\mu^a A_\mu^a - \alpha \bar{c}^a c^a. \quad (7)$$

This operator can be included in a BRST-invariant manner by means of the so-called local composite operator (LCO) technique as follows: two external sources λ and J are introduced as BRST doublets, i.e.,

$$s\lambda = J, \quad \text{and} \quad sJ = 0. \quad (8)$$

Hence, the Euclidean Yang-Mills action quantized in the MAG and in the presence of the operator (7) has the following action:

$$S_{\text{MAG}}^\alpha = S_{\text{YM}} + S_{\text{FP}} + s \int_x \lambda \left(\mathcal{O}(\alpha) + \frac{\xi}{2} J \right). \quad (9)$$

The parameter ξ is the so-called LCO parameter, and it is necessary to absorb divergences arising from correlation functions of $\mathcal{O}(\alpha)$. Upon the action of the BRST operator on the last term in Eq. (9) one obtains,

$$S_{\text{MAG}}^\alpha = S_{\text{YM}} + S_{\text{FP}} + \int_x \left[J \mathcal{O}(\alpha) + \frac{\xi}{2} J^2 + \lambda (A_\mu^a \mathcal{D}_\mu^{ab} c^b + \alpha i b^a c^a - \alpha g \epsilon^{ab} \bar{c}^a c^b c) \right]. \quad (10)$$

The addition of the sources (λ, J) has the purpose of introducing the ghost-gluon operator in a BRST-invariant fashion into the initial action. Yet the sources should attain their physical values in concrete computations, i.e., $\lambda = 0$ and $J = M^2$ with M^2 being a mass-squared parameter. Thus, in the physical limit, the operator $\mathcal{O}(\alpha)$ provides an effective mass term for the off diagonal gluons and ghosts.

Remarkably, the term

$$S_{M^2} = M^2 \int_x \left(\frac{1}{2} A_\mu^a A_\mu^a - \alpha \bar{c}^a c^a \right) \quad (11)$$

is BRST-invariant on shell.

The condensation of the operator (7) engenders a dynamical mass generation for the off diagonal gluons and ghosts. Such a feature is very welcome for the Abelian dominance paradigm since such a mass will decouple the off diagonal degrees of freedom for sufficiently low momentum scales and lead to an effective Abelian theory in the deep infrared. Yet the MAG is not a complete gauge-fixing due to the Gribov problem [20,21]. Even after the complete implementation of the MAG, gauge field configurations which satisfy the MAG condition and are connected by gauge transformations exist. The Gribov problem in the MAG was investigated in, e.g., [26,27,39,40,92,93]. In the next subsection, we provide a quick overview of such an issue and explain how the removal of Gribov copies affects the dynamics of the Abelian components of the gauge field in the infrared.

B. Infinitesimal Gribov copies in the MAG

Gribov copies are defined by normalizable field configurations that satisfy the gauge condition and are connected by a gauge transformation. In the MAG this would correspond to the following statement: Given the field configuration (A_μ^a, A_μ) that satisfies

$$\mathcal{D}_\mu^{ab}(A) A_\mu^b = 0, \quad \text{and} \quad \partial_\mu A_\mu = 0, \quad (12)$$

a configuration $(\tilde{A}_\mu^a, \tilde{A}_\mu)$ is a Gribov copy associated with (A_μ^a, A_μ) if

$$\mathcal{D}_\mu^{ab}(\tilde{A}) \tilde{A}_\mu^b = 0, \quad \text{and} \quad \partial_\mu \tilde{A}_\mu = 0, \quad (13)$$

and $(\tilde{A}_\mu^a, \tilde{A}_\mu)$ is related to (A_μ^a, A_μ) by a gauge transformation. In particular, for infinitesimal gauge transformations generated by (ξ^a, ξ) ,

$$\begin{aligned} \tilde{A}_\mu^a &= A_\mu^a - \mathcal{D}_\mu^{ab}(A) \xi^b - g \epsilon^{ab} A_\mu^b \xi, \\ \tilde{A}_\mu &= A_\mu - \partial_\mu \xi - g \epsilon^{ab} A_\mu^a \xi^b. \end{aligned} \quad (14)$$

Plugging Eqs. (14) into (13) leads to

$$\begin{aligned} -\mathcal{D}_\mu^{ac} \mathcal{D}_\mu^{cb} \xi^b - g^2 \epsilon^{ac} \epsilon^{bd} A_\mu^c A_\mu^d \xi^b &= 0, \\ -\partial^2 \xi - g \epsilon^{ab} \partial_\mu (A_\mu^a \xi^b) &= 0. \end{aligned} \quad (15)$$

Therefore if Eq. (15) have nontrivial solutions, then there exist infinitesimal Gribov copies $(\tilde{A}_\mu^a, \tilde{A}_\mu)$ generated by (ξ^a, ξ) . In practice, the second equation in (15) expresses the Abelian component ξ as a function of the non-Abelian

components of the gauge field and the gauge parameter of the infinitesimal transformation. Hence, if the first equation is solved and since it does not involve ξ , the second one automatically constrains the form of ξ . As such, for the analysis of the existence of infinitesimal Gribov copies in the MAG, investigating the existence of normalizable solutions to the first equation in (15) is what effectively matters. Moreover, the operator that acts on ξ^b in the first equation in (15) is precisely the FP operator in the MAG. This operator is Hermitian and hence has real spectrum. Thus, very much like in the Landau gauge, one can propose a region which is free of infinitesimal Gribov copies, the so-called Gribov region, in which the FP operator is positive and thereby restrict the path integral to this region. Such a restriction can be achieved by two different methods: the implementation of the so-called no-pole condition, similarly to the original work [20] in the Landau gauge, or the horizon condition, similarly to [25] in the Landau gauge. This was explored in [26,27]. Effectively, the Gribov region in the MAG satisfies important properties such as the following: it is bounded by the so-called Gribov horizon in every direction in the off diagonal components of the gauge field; it is unbounded in the diagonal direction; and every configuration inside the Gribov region and sufficiently close to the Gribov horizon has a copy that lies on the other side of the horizon and also close to it (see [40]).

The restriction of the path integral to the Gribov region in the MAG was worked out in [26] at leading order by means of the no-pole condition. Its all-order generalization was proposed by demanding consistency with localizability and renormalizability in [92]. Its all-order construction based on the horizon condition was worked out in [27]. As far as we are concerned, a proof that the no-pole condition at all orders is equivalent to the horizon condition in the MAG was not provided up to now. This is in contrast to the situation in the Landau gauge; see, e.g., [94].

The restriction of the gauge-fixed YM action in the MAG to the Gribov region is effectively achieved by the introduction of the following nonlocal term:

$$S_h = \gamma^4 H(A) \equiv g^2 \gamma^4 \int_x \epsilon^{ab} A_\mu [\mathcal{M}^{-1}]^{ac} \epsilon^{cb} A_\mu, \quad (16)$$

with $\mathcal{M}^{-1}$ denoting the inverse FP operator and $H(A)$ known as the horizon function. The mass-squared parameter γ^2 is known as the Gribov parameter, and it is not free but fixed by a gap equation. For further details, we refer the reader to [26]. Albeit nonlocal, the horizon function can be localized by the introduction of a suitable set of auxiliary fields. The resulting local action is renormalizable at all orders in perturbation theory and implements the restriction of the path integral to the Gribov region in the MAG thereby removing infinitesimal Gribov copies.

From Eq. (16), it is clear that the horizon function introduces a masslike term for the Abelian component of

the gauge field. This is a nonperturbative effect arising from the removal of Gribov copies. Yet this is not a standard mass term since it is also momentum dependent due to the nonlocal structure of the inverse FP operator. Moreover, the introduction of the localizing auxiliary fields is such that their own dynamics renders the formation of condensates. This is due to dynamical infrared instabilities of the model leading to the RGZ action. If we call the mass-squared parameter associated to this condensate μ^2 , then the tree-level Abelian gluon propagator in the RGZ theory is written as [39]

$$\langle A_\mu(p) A_\nu(-p) \rangle = \frac{p^2 + \mu^2}{p^2(p^2 + \mu^2) + 4g^2\gamma^4} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right). \quad (17)$$

The tree-level non-Abelian gluon propagator is not affected by the restriction of the path integral to the Gribov region. Hence, if one includes the operator (7), its propagator reads as

$$\langle A_\mu^a(p) A_\nu^b(-p) \rangle = \frac{\delta^{ab}}{p^2 + M^2} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right). \quad (18)$$

Some comments are in order. (i) At vanishing momentum, the tree-level Abelian propagator attains a finite value hinting to a massive behavior. (ii) The mass parameters that provide such a behavior for the Abelian gauge field propagator arise from the restriction of the path integral to the Gribov region. Thus the removal of infinitesimal Gribov copies and the dynamics of the localizing fields were essential to display such a nontrivial behavior of the propagator of the Abelian components of the gauge field. (iii) The diagonal gauge fields have an exactly transverse propagator while the nondiagonal components have a transverse propagator at tree level, but such a property is lost at higher orders. (iv) Although the Abelian propagator reaches a finite value at vanishing momentum, its momentum dependence is very different from a Yukawa-like propagator. In particular, it can be expressed as a sum of Yukawa propagators as follows:

$$\langle A_\mu(p) A_\nu(-p) \rangle = \left(\frac{R_+}{p^2 + m_+^2} + \frac{R_-}{p^2 + m_-^2} \right) \mathcal{P}_{\mu\nu}, \quad (19)$$

with

$$\mathcal{P}_{\mu\nu}(p) = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \quad (20)$$

standing for the transverse projector. The poles $m_\pm^2$ and the residues $R_\pm$ are given by

$$m_\pm^2 = \frac{1}{2}(\mu^2 \pm \Omega^2), \quad (21)$$

and

$$R_{\pm} = \frac{1}{2} \left(1 \mp \frac{\mu^2}{\Omega^2} \right), \quad (22)$$

with $\Omega^2 = \sqrt{\mu^4 - 16g^2\gamma^4}$.

With such an understanding of the effects of the removal of infinitesimal Gribov copies in the MAG, it is clear that the infrared dynamics of the Abelian sector is affected by mass parameters that have a geometrical nature due to the restriction of the path integral to the Gribov region. Since the RGZ action is not particularly simple due to the large set of fields introduced to localize the Horizon function (16) and the complicated set of Feynman rules which involve mixings and the proliferation of new diagrams with respect to standard perturbative YM theories, one might raise a legitimate question of practical importance: Can the effects produced by the removal of infinitesimal Gribov copies be mimicked by a simple mass term for the Abelian sector? Such a strategy was pursued in the Landau gauge (see, e.g., [57–59]) leading to the so-called CF model. It provides a highly effective framework that captures the behavior of correlation functions in YM theories quantized in the Landau gauge without the introduction of extra fields, propagators, and vertices. In particular, the low-momentum behavior of the underlying correlation functions in the Landau gauge is well described within perturbation theory. It is clear that this model, i.e., the gauge-fixed YM action in the Landau gauge supplemented by a masslike term for the gluon, is an effective description. One possible first-principle explanation for the introduction of masslike terms comes precisely from the elimination of Gribov copies. One might wonder if this is just a peculiarity of the Landau gauge or if in other gauges, the introduction of a masslike term effectively describes the restriction of the path integral to the Gribov region. In this work we provide the first evidence that such extension of gauge-fixed YM theories constitutes a good candidate to describe the infrared behavior of correlation functions in the MAG.

C. Curci-Ferrari-like extension in the MAG

We define the CF-like model in the MAG by the following action:

$$S = S_{\text{YM}} + S_{\text{FP}} + S_{M^2} + S_{m^2}, \quad (23)$$

with

$$S_{m^2} = \frac{m^2}{2} \int_x A_{\mu} A_{\mu}, \quad (24)$$

and S_{M^2} is given by Eq. (11). As previously described, the α -dependent terms are introduced for renormalization purposes, but one should take $\alpha \rightarrow 0$ after the computations of the desired quantity in order to recover the MAG. For a

systematic analysis of the renormalization properties of (23), see [95]. The tree-level non-Abelian gauge field propagator, in the presence of the parameter α is given by

$$\langle A_{\mu}^a(p) A_{\nu}^b(-p) \rangle = \frac{\delta^{ab}}{p^2 + M^2} \mathcal{P}_{\mu\nu}(p) + \frac{\alpha \delta^{ab}}{p^2 + \alpha M^2} \mathcal{L}_{\mu\nu}(p), \quad (25)$$

where

$$\mathcal{L}_{\mu\nu}(p) = \frac{p_{\mu} p_{\nu}}{p^2}. \quad (26)$$

For nonvanishing α , the tree-level off diagonal gluon propagator acquires a longitudinal component as shown in Eq. (25). However, it should be stressed that, although the tree-level longitudinal component drops for $\alpha = 0$, it is not expected that such a propagator remains transverse upon inclusion of quantum corrections. This is due to the non-linear nature of the MAG. As for the Abelian gauge field propagator, it reads as

$$\langle A_{\mu}(p) A_{\nu}(-p) \rangle = \frac{1}{p^2 + m^2} \mathcal{P}_{\mu\nu}(p). \quad (27)$$

This is rather different from the expression (19). Thus, as in the Landau gauge, it is expected that one needs to compute loop corrections to such a tree-level propagator in order to have a chance to describe the lattice data in the infrared. We will work out the one-loop correction to the gauge-field two-point functions in the next sections.

The masslike term (24) is not invariant under BRST transformations (3). In fact,

$$\text{ss} S_{m^2} = m^2 \int_x (c \partial_{\mu} A_{\mu} + g \epsilon^{ab} c^a A_{\mu}^b A_{\mu}), \quad (28)$$

and thus it is not invariant even on shell. As such, the mass parameter m^2 is not akin to a gauge parameter and can enter physical correlation functions. Consequently, the action defining the model is not invariant under BRST transformations, i.e., $\text{ss} S \neq 0$ (even on shell). One important remark on this matter is that the RGZ action in the MAG [26] also displays a BRST breaking due to the restriction of the path integral to the Gribov region. Such a breaking as in the present case is explicit but soft. In fact, a proposal for the reformulation of the RGZ action in the MAG that is BRST invariant was put forward in [96].

As discussed in Sec. II A, it is possible to introduce the masslike term for the Abelian components of the gauge field in a BRST-invariant fashion upon inclusion of external sources that form a BRST doublet. In the end of concrete computations, the sources attain their physical values recovering the initial model (23). Given the sources τ and ρ such that

$$\mathbf{s}\tau = \rho \quad \text{and} \quad \mathbf{s}\rho = 0, \quad (29)$$

one can write

$$S_{\tau\rho} = \mathbf{s} \int_x \frac{\tau}{2} (A_\mu A_\mu + \zeta\rho) = \int_x \frac{\rho}{2} (A_\mu A_\mu + \zeta\rho) + \int_x \tau (A_\mu \partial_\mu c + g\epsilon^{ab} A_\mu^a A_\mu^b c^b), \quad (30)$$

where the parameter ζ is the LCO parameter and plays the analog role of the parameter ξ in Eq. (9). In the physical limit, the sources attain the corresponding values: $\tau = 0$ and $\rho = m^2$. This is useful, for instance, to prove the renormalizability of the model.

III. GLUON PROPAGATORS AT ONE-LOOP ORDER

In this section, we discuss the essential elements for the explicit evaluation of the non-Abelian and diagonal gluon propagators at one-loop order. We integrate out the Lautrup-Nakanishi fields (b^a, b) and take the physical limits of the corresponding sources (λ, J, τ, ρ) , i.e., $\lambda = \tau = 0$, $J = M^2$ and $\rho = m^2$. For the integration of the Abelian field b we introduce a quadratic term on b together with a gauge parameter β that must be set to zero in order to recover the condition $\partial_\mu A_\mu = 0$. The resulting action is written as

$$S = \frac{1}{4} \int_x (F_{\mu\nu}^a F_{\mu\nu}^a + F_{\mu\nu} F_{\mu\nu}) + \frac{1}{2\alpha} \int_x (\mathcal{D}_\mu^{ab} A_\mu^b)^2 + \frac{1}{2\beta} \int_x (\partial_\mu A_\mu)^2 + \int_x \bar{c}^a (\mathcal{D}_\mu^{ac} \mathcal{D}_\mu^{cb} + g^2 \epsilon^{ac} \epsilon^{bd} A_\mu^c A_\mu^d) c^d + \int_x \bar{c} \partial_\mu (\partial_\mu c + g\epsilon^{ab} A_\mu^a c^b) - \frac{\alpha}{2} \int_x g^2 (\bar{c}^a c^a)^2 + M^2 \int_x \left(\frac{1}{2} A_\mu^a A_\mu^a - \alpha \bar{c}^a c^a \right) + \frac{m^2}{2} \int_x A_\mu A_\mu. \quad (31)$$

In the next subsections, we shall present the one-loop corrections to the off diagonal and diagonal gluon propagators. Our conventions and Feynman rules are collected in Appendixes A and B.

A. Renormalization conditions

We calculate the diagonal gauge-field propagator

$$G_{\mu\nu}(p) = \langle A_\mu(p) A_\nu(-p) \rangle \quad (32)$$

and the off diagonal propagator

$$\mathbb{G}_{\mu\nu}^{ab}(p) = \langle A_\mu^a(p) A_\nu^b(-p) \rangle \quad (33)$$

up to one-loop order. This requires the calculation of the Feynman diagrams collected in Figs. 1 and 2.

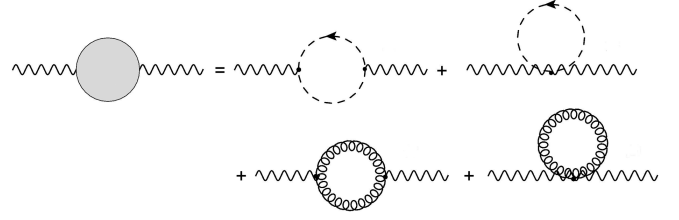


FIG. 1. One-loop diagrams that contribute to $\langle A_\mu(p) A_\nu(-p) \rangle$.

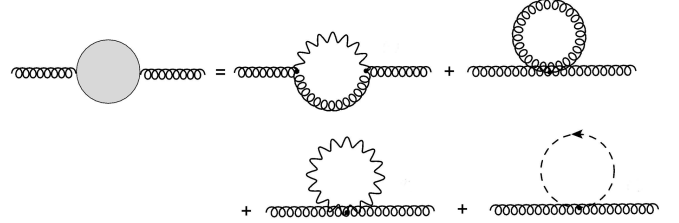


FIG. 2. One-loop diagrams that contribute to $\langle A_\mu^a(p) A_\nu^b(-p) \rangle$.

The tree-level propagators and vertex functions necessary for this calculation are set out in Appendix B. The most general structure of both diagonal and off diagonal propagators is, schematically,

$$\mathcal{G}_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \mathcal{G}_T(p) + \frac{p_\mu p_\nu}{p^2} \mathcal{G}_L(p), \quad (34)$$

with $\mathcal{G}_{T,L}$ being scalar functions representing the transverse and longitudinal parts of the propagators, respectively. Calligraphic letters are employed to represent that (34) is valid both for the non-Abelian and diagonal propagators with the necessary adjustments. The tree-level expressions for $\mathbb{G}_{\mu\nu}^{ab}$ and $G_{\mu\nu}$ are given in Eq. (25) and (27), respectively. It follows from the Landau-type condition for the diagonal component of the gauge field that the longitudinal part of the diagonal gluon propagator exactly vanishes, that is,

$$G_L(p) = 0. \quad (35)$$

Thus we have three nontrivial form factors to be computed: G_T and $\mathbb{G}_{T,L}$. Notice that the tree-level contribution of $\mathbb{G}_L$ is zero for $\alpha = 0$; see Eq. (25). For renormalization purposes, we keep α and just take the limit $\alpha \rightarrow 0$ at the very end of the computation. After resummation, the form factors look like

$$G_T(p) = \frac{1}{p^2 + m^2 - \Pi_T^d(p^2)}, \quad (36)$$

$$\mathbb{G}_T(p) = \frac{1}{p^2 + M^2 - \Pi_T^{\text{off}}(p^2)}, \quad (37)$$

$$\mathbb{G}_L(p) = \frac{\alpha}{p^2 + \alpha M^2 - \Pi_L^{\text{off}}(p^2)}, \quad (38)$$

with $\Pi(p^2)$ representing the corresponding self-energies. Employing dimensional regularization with $d = 4 - \epsilon$, we get the following divergent parts of the one-loop corrections:

$$\begin{aligned}\Pi_T^d(p^2) \Big|_{\text{div}}^{1\text{-loop}} &= \frac{g^2}{(4\pi)^2\epsilon} \frac{44p^2}{3} \\ \Pi_T^{\text{off}}(p^2) \Big|_{\text{div}}^{1\text{-loop}} &= \frac{g^2}{(4\pi)^2\epsilon} \left[\left(\frac{17}{3} - \alpha \right) p^2 - \frac{3}{2} m^2 (3 + \alpha) \right. \\ &\quad \left. - 3M^2 (1 + \alpha) \right] \\ \Pi_l^{\text{off}}(p^2) \Big|_{\text{div}}^{1\text{-loop}} &= \frac{g^2}{(4\pi)^2\epsilon} \left[\left(\alpha + 3 + \frac{6}{\alpha} \right) p^2 - \frac{3}{2} m^2 (3\alpha + \alpha^2) \right. \\ &\quad \left. - 3M^2 (\alpha + \alpha^2) \right].\end{aligned}\quad (39)$$

Notice that all divergences are local in the external momentum, p , and in the mass parameters, m^2 and M^2 .

All of these divergences can be absorbed by Z factors relating the bare fields and parameters to their renormalized counterparts. Here, we implement the same momentum subtraction (MOM)-like renormalization conditions as those used in [57,58] for the one-loop calculation in the Landau gauge-fixed CF model for better comparison with lattice data.³ They are

$$\Pi(p^2 = \mu^2) = 0 \quad \text{and} \quad \Pi(p^2 = 0) = 0, \quad (40)$$

for all $\Pi(p^2)$ given in Eqs. (36)–(38). The renormalization factors are defined by

$$\begin{aligned}A_\mu &= Z_A^{1/2} A_{R\mu}, & A_\mu^a &= Z_A^{1/2} A_{R\mu}^a, & \alpha &= Z_\alpha \alpha_R \\ m^2 &= Z_{m^2} m_R^2, & M^2 &= Z_{M^2} M_R^2,\end{aligned}\quad (41)$$

and from Eq. (39) we can read off their divergent part,

$$\begin{aligned}Z_A &= 1 + \frac{g^2}{(4\pi)^2\epsilon} \frac{44}{3}, \\ Z_{A^a} &= 1 + \frac{g^2}{(4\pi)^2\epsilon} \left(\frac{17}{3} - \alpha \right), \\ Z_\alpha &= 1 + \frac{g^2}{(4\pi)^2\epsilon} \left(\frac{8}{3} - 2\alpha - \frac{6}{\alpha} \right), \\ Z_{M^2} &= 1 - \frac{g^2}{(4\pi)^2\epsilon} \left(\frac{3}{2} (\alpha + 3) \frac{m^2}{M^2} + \frac{26}{3} + 2\alpha \right), \\ Z_{m^2} &= Z_A^{-1}.\end{aligned}\quad (42)$$

Notice that the parameter m^2 does not get renormalized up to first order in g^2 . However, it does appear in the renormalization of M^2 . The renormalization factors Z_A , Z_{A^a} , and Z_α coincide with those of [97], where the authors considered the MAG without any masses, i.e. $m = M = 0$. This means that the addition of a mass term does not change the renormalization of the original fields and parameters, something that was also found in the Landau gauge [57,58].

B. Results

After renormalizing the fields and parameters, we can take $\alpha \rightarrow 0$ which corresponds to the “true” MAG. The renormalized functions $G_T(p^2)$ and $\mathbb{G}_T(p^2)$ then read as

$$\begin{aligned}\frac{G_T^{-1}(p)}{m^2} &= t + 1 - \frac{g^2 t}{576\pi^2} \left\{ -288s^{-1} + 3(s^2 - 4s - 4) \ln(s) - 6 \left(s^2 - 9s + \frac{1}{s} - 9 \right) \ln(1 + s) \right. \\ &\quad \left. - \frac{3\sqrt{s(s+4)}(s^3 - 16s^2 - 68s + 48)}{s^2} \ln \left(\frac{s - \sqrt{s(s+4)} + 2}{2} \right) - (p^2 \rightarrow \mu^2) \right\} \\ \frac{(\mathbb{G}_T)^{-1}(p)}{M^2} &= s + 1 - \frac{g^2 s}{192\pi^2} \left\{ \frac{s^2(-327t - 36) - 6st(47t - 19) - 48t^2}{6s^2 t^2} - \frac{((s+1)^3(s^2 - 10s + 1)t)}{s^4} \ln(s+1) \right. \\ &\quad + \left(\frac{((s-10)s+1)(s+1)^3 t}{2s^4} + \frac{7}{2s^3} + \frac{13s^2 - 25s + 13}{st^2} + \frac{25}{s^2} - \frac{13}{s^2 t} - \frac{s}{2t^4} + \frac{7(s-1)}{2t^3} + \frac{36}{t-s} + \frac{7s}{2} \right) \ln \frac{s}{t} \\ &\quad - \frac{((t^3 - 9t^2 - 9t + 1)(st + s + t)^2)}{st^4} \ln(t+1) + \frac{((s+1)^2 t)}{s} \ln(t) \\ &\quad + \frac{((s^2(t+1)^2 + 2s(t-1)t + t^2)^{3/2}(s^2((t-10)t+1) - 10s(t-1)t + t^2))}{2s^4 t^4} \\ &\quad \left. \times \ln \left(\frac{\sqrt{s^2(t+1)^2 + 2s(t-1)t + t^2} + st + s + t}{-\sqrt{s^2(t+1)^2 + 2s(t-1)t + t^2} + st + s + t} \right) - (p^2 \rightarrow \mu^2) \right\},\end{aligned}$$

³Although we employ dimensional regularization, the use of a MOM-like renormalization scheme (instead of minimal subtraction) allows for renormalization of correlation functions at specified momenta, which is what is operationally done in lattice calculations. Since lattice regularization does not access the usual poles found in dimensional regularization, minimal subtraction is not suitable for a direct comparison between analytic continuum computations and lattice data.

with $s = p^2/M^2$ and $t = p^2/m^2$. We checked that for $m = M = 0$ the results, before implementing our renormalization conditions, coincide with the results of [97]. As it was reported in [97], in the massless case the longitudinal correction Π_L^{off} blows up, while the tree-level propagator vanishes. We found that in the case of finite m, M , this effect remains, making an analysis up to first loop order of the longitudinal propagator impossible in the $\alpha \rightarrow 0$ limit. In contrast, lattice simulations of [79] show a finite longitudinal off diagonal propagator, saturating in the deep infrared; see Fig. 5.

C. Comparison with the lattice results

We compare our results with those obtained by lattice simulation in [79]. In Figs. 3 and 4, we fit the dressing functions $p^2 G_T(p)$ and $p^2 \mathbb{G}_T(p)$ with the lattice data from the simulations reported in [79]. The fits are obtained with the constraint $M > m$. We find that the best parameter fits are $Z_{\text{diag}} = 13.4$, $Z_{\text{off}} = 3.9$, $g = 8.1$, $M = 1.7$ GeV, $\mu = 1.4$ GeV, and $m = 0.4$ GeV. The parameters Z_{diag} and Z_{off} are global multiplicative parameters necessary to adjust to the normalization of lattice data. Notice that these values are within the same order as those obtained in the Landau gauge [57,58]. We find a satisfactory fit in the IR regime, as can be seen from Figs. 3 and 4. We also compared our fitting with the lattice data on the transverse components of the propagators, which was given in [79] for larger values of p ; see Fig. 5. It is clear that for larger values of p , the fitting does not work that well. In this case, we certainly need a renormalization group improvement before drawing a conclusion.

From these results, it is clear that neither propagator vanishes or diverges in the deep IR, i.e., $p \rightarrow 0$. However, at low momenta the diagonal transverse propagator is much larger than the off diagonal propagator. This can be interpreted as a hint toward Abelian dominance. In Fig. 6,

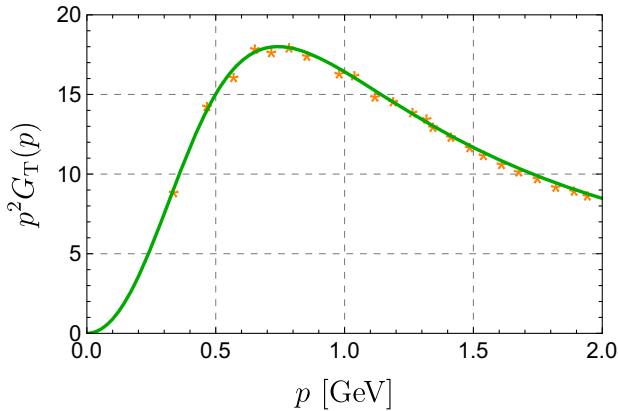


FIG. 3. Gluon dressing of the diagonal transverse propagator in the MAG for the $SU(2)$ gauge group. The results of the present work (green line) are compared with lattice data of [79] (orange stars).

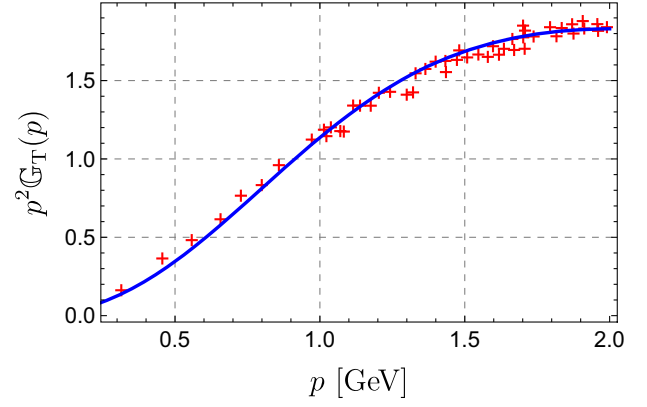


FIG. 4. Gluon dressing of the off diagonal transverse propagator in the MAG for the $SU(2)$ gauge group. The results of the present work (blue line) are compared with lattice data of [79] (red crosses).

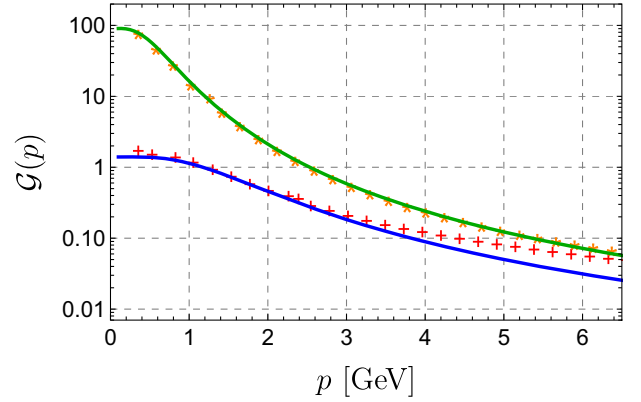


FIG. 5. Transverse gluon propagator in the MAG for the $SU(2)$ gauge group. The results of the present work are compared with lattice data of [79]. For the diagonal transverse propagator, green line (present work) and orange stars (lattice data). For the off diagonal transverse propagator, blue line (present work) and red crosses (lattice data).

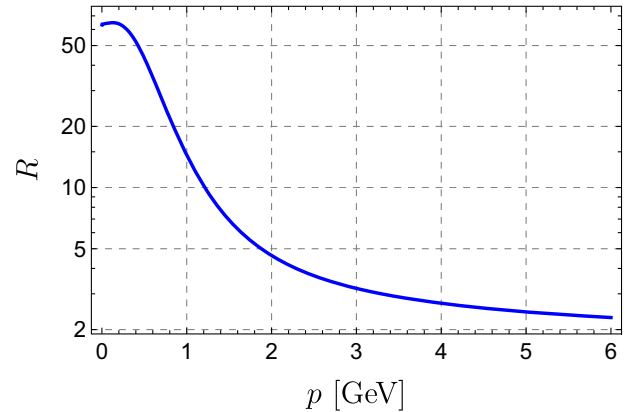


FIG. 6. Ratio of the transverse diagonal and off diagonal propagators.

we compare the ratio of G_T and $\mathbb{G}_T$. As can be seen, the suppression of the off diagonal propagator with respect to the diagonal propagator increases as the momentum decreases. Such an observation can be taken as an indication that, in the IR, the diagonal gluons are the dominant degrees of freedom. Notice that since for our fitting parameters $M > m$, this effect is already present at tree level.

At this stage, a comment is in order: The comparison of the present analytical one-loop results with lattice data relies on the fitting of six parameters. Thus, it is logically conceivable that the good agreement is just a consequence of the substantial freedom one has in choosing those parameters. Ultimately, a definite answer regarding the predictive power of this model requires the computation of other correlation functions and the use of the fitted parameters to “predict” their behavior. This is particularly challenging in the MAG since lattice data for correlation function is scarce. Yet, in the present work, we have fitted the Abelian and non-Abelian gauge-field propagators simultaneously, a fact that makes such good agreement less coincidental. We hope that more lattice results for correlation functions in the MAG become available in the near future so that one can establish whether this effective model is indeed predictive. Alternatively, functional methods such as the functional renormalization group or Dyson-Schwinger equations could be used as computational frameworks to evaluate higher-order correlation functions and provide a template for comparison.

IV. CONCLUSIONS

In this work, we have shown that the CF-like model quantized in the MAG reproduces the transverse components of the gluonic two-point correlation functions in $SU(2)$ YM theories for small momenta. We calculated up to first-loop order the off diagonal gluon propagator and the diagonal gluon propagator, $\langle A_\mu^a(p) A_\nu^b(-p) \rangle$ and $\langle A_\mu^3(p) A_\nu^3(-p) \rangle$, respectively. We employed dimensional regularization and MOM-type renormalization conditions for the three nonzero form factors. By fitting the available free parameters, we are able to obtain a rather satisfactory description of the transverse components of the propagators obtained in lattice simulations in the IR regime. This seems promising, since there is room for many improvements as the inclusion of the runnings of the coupling and anomalous dimensions. Unfortunately, the longitudinal part of the off diagonal component is inaccessible in our approach, simply because in the limit $\alpha \rightarrow 0$ the tree-level contribution vanishes and, at the limit $p \rightarrow 0$, the one-loop correction blows up.

The results obtained in this work are comparable to those in the CF model in the Landau gauge, which has been highly successful in reproducing a range of nonperturbative features of pure Yang-Mills theories and QCD. Of course, many improvements were achieved in the Landau gauge and such an effective model has shown to be rather robust. It is unknown if the same will hold in the MAG, and further

investigations are necessary. The importance of this result is twofold. First, it reinforces the status of the CF model as a relevant phenomenological framework for low-energy QCD, demonstrating that its effectiveness is potentially not limited to the Landau gauge. Second, although the MAG is undoubtedly more involved than the Landau gauge from a computational standpoint, it brings to light certain features of Yang-Mills theories, e.g., Abelian dominance, that remain obscure in the Landau gauge.

These results are encouraging for future investigations of other nonperturbative features of gauge-fixed correlation functions in the MAG. For example, one can consider a renormalization group (RG) improvement following the IR-safe RG scheme proposed in [57,58]. We expect such an improvement leads to better results for larger values of p , which, for our simple approach, are not very satisfying. It remains to be investigated how to consistently account for the longitudinal sector of the off diagonal gauge-field propagator which seems difficult to be meaningfully accessed at one-loop order in the IR. This also relates to the issue that, as in the CF model in the Landau gauge, we have to explicitly verify that a consistent perturbative expansion parameter is available in the present case. In addition, lattice data are available for $SU(3)$ YM theories; see [98]. Studying the gluon propagator up to first loop order in the $SU(3)$ CF-like model with MAG is a nontrivial extension of the $SU(2)$ calculations presented here, due to the fact that there are now two Abelian components to the gauge field, A_μ^3 and A_μ^8 , which means that the residual symmetry is an $U(1) \times U(1)$ symmetry and more vertices are involved. One topic that is particularly scarce in the MAG literature is the inclusion of matter, and, in the present framework, we can investigate the coupling of scalars and fermions which are of great phenomenological importance.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: CONVENTIONS

In this paper we work on four-dimensional flat Euclidean space and adopt the following convention for Fourier transform for any field $\Phi(x)$:

$$\Phi(x) = \int \frac{d^4 p}{(2\pi)^4} e^{-ix \cdot p} \tilde{\Phi}(p). \quad (\text{A1})$$

For convenience, we shall employ the short-hand notation $\int \frac{d^4 p}{(2\pi)^4} = \int_p$ or $\int \frac{d^d p}{(2\pi)^d} = \int_{p^d}$ when an analytic extension of the space dimensionality is employed.

APPENDIX B: FEYNMAN RULES

1. Propagators

The propagators of the theory are determined by the quadratic part of S , namely,

$$S_{\text{quad}} = \int_x \left[\frac{1}{4} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)^2 + \frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \frac{1}{2\alpha} (\partial_\mu A_\mu^a)^2 + \frac{1}{2\beta} (\partial_\mu A_\mu)^2 + \frac{M^2}{2} A_\mu^a A_\mu^a + \frac{m^2}{2} A_\mu A_\mu \right] + \int_x (\bar{c}^a \partial^2 c^a - \alpha M^2 \bar{c}^a c^a + \bar{c} \partial^2 c). \quad (\text{B1})$$

Notice that the bosonic sector can be written as $\int_{x,y} \frac{1}{2} \Phi^T(x) \Delta(x-y) \Phi(y)$, where

$$\Phi = \begin{pmatrix} A_\mu^a \\ A_\mu \end{pmatrix} \quad (\text{B2})$$

and

$$\Delta(x-y) = \begin{pmatrix} \delta^{ab} \left[-\delta_{\mu\nu} (\partial^2 - M^2) + \left(1 - \frac{1}{\alpha} \right) \partial_\mu \partial_\nu \right] & 0 \\ 0 & \left[-\delta_{\mu\nu} (\partial^2 - m^2) + \left(1 - \frac{1}{\beta} \right) \partial_\mu \partial_\nu \right] \end{pmatrix}. \quad (\text{B3})$$

We obtain the propagators by inverting $\Delta(x-y)$, which is not a difficult task, as there are no mixing terms. By doing that and in the end taking the limits $\alpha \rightarrow 0$, $\beta \rightarrow 0$, we get

$$\langle A_\mu^a(p) A_\nu^b(-p) \rangle = \frac{\delta^{ab}}{p^2 + M^2} \mathcal{P}_{\mu\nu}(p), \quad \langle A_\mu(p) A_\nu(-p) \rangle = \frac{1}{p^2 + m^2} \mathcal{P}_{\mu\nu}(p). \quad (\text{B4})$$

Ghost propagators are obtained in a similar way, but taking into account the Grassmannian nature of the fields. It follows that

$$\begin{aligned} \langle A_\mu^a(p) A_\nu^b(-p) \rangle &= \text{diagram with curly line, indices } a\mu \text{ and } b\nu \\ \langle A_\mu(p) A_\nu(-p) \rangle &= \text{diagram with wavy line, indices } \mu \text{ and } \nu \\ \langle c^a(p) \bar{c}^b(-p) \rangle &= \text{diagram with dashed line and arrow, indices } a \text{ and } b \\ \langle c(p) \bar{c}(-p) \rangle &= \text{diagram with dotted line and arrow} \end{aligned}$$

FIG. 7. Diagrammatic representation for the propagators.

$$\langle \bar{c}^a(p) c^b(-p) \rangle = \frac{\delta^{ab}}{p^2}, \quad \langle \bar{c}(p) c(-p) \rangle = \frac{1}{p^2}. \quad (\text{B5})$$

The diagrammatic notation for the propagators is shown in Fig. 7.

2. Vertices

The vertices of the theory are determined by the interacting action, which is

$$\begin{aligned} S_{\text{int}} &= S - S_{\text{quad}} \\ &= \int_x \left[g\epsilon^{ab}(\partial_\mu A_\nu^a) A_\mu^b A_\nu - g\epsilon^{ab}(\partial_\mu A_\nu^a) A_\mu A_\nu^b + g\epsilon^{ab}(\partial_\mu A_\nu) A_\mu^b A_\nu^c \right. \\ &\quad + \frac{g^2}{4} (A_\mu^a A_\mu^a A_\nu^b A_\nu^b + 2A_\mu^a A_\mu^a A_\nu A_\nu - A_\mu^a A_\nu^a A_\mu^b A_\nu^b - 2A_\mu^a A_\nu^a A_\mu A_\nu) - g\epsilon^{ab} \bar{c}^a \partial_\mu (A_\mu c^b) - g\epsilon^{ab} \bar{c}^a A_\mu \partial_\mu c^b \\ &\quad \left. - g^2 \bar{c}^a c^b A_\mu A_\mu + g^2 \epsilon^{ac} \epsilon^{bd} \bar{c}^a c^b A_\mu^c A_\mu^d + g\epsilon^{ab} \bar{c} \partial_\mu (A_\mu^c c^b) - \frac{\alpha g^2}{2} \bar{c}^a c^a \bar{c}^b c^b \right]. \end{aligned} \quad (\text{B6})$$

A vertex factor, $V_{\phi_{i_1} \dots \phi_{i_n}}(p_1, \dots, p_n)$, corresponding to an interaction between the fields $\phi_{i_1}, \dots, \phi_{i_n}$, is defined in momentum space by

$$(2\pi)^d \delta^d(p_1 + \dots + p_n) V_{\phi_{i_1} \dots \phi_{i_n}}(p_1, \dots, p_n) = - \int_{x_1, \dots, x_n} e^{i(x_1 \cdot p_1 + \dots + x_n \cdot p_n)} \frac{\delta^n S_{\text{int}}}{\delta \phi_{i_1}(x_1) \dots \delta \phi_{i_n}(x_n)} \Big|_{\phi=0}. \quad (\text{B7})$$

All vertex factors present in this theory are shown in Fig. 8.

Figure 8 displays eight Feynman diagrams representing different vertex factors, each with its corresponding mathematical expression. The diagrams are arranged in two columns. The left column shows vertices with one wavy line and three straight lines, and vertices with two wavy lines and two straight lines. The right column shows vertices with two wavy lines and two straight lines, and vertices with two wavy lines and two straight lines. The expressions involve various tensors and delta functions.

Left Column Vertices:

- Top: $-ig\epsilon^{ab} [\delta_{\mu\nu} (p_\rho - q_\rho) + \delta_{\rho\nu} (q_\mu - r_\mu) + \delta_{\rho\mu} (r_\nu - p_\nu) - \frac{1}{\alpha} (q_\nu \delta_{\mu\rho} - p_\mu \delta_{\nu\rho})]$
- Middle: $-ig\epsilon^{ab} (r_\mu - q_\mu)$
- Bottom: $ig\epsilon^{ab} q_\mu$

Right Column Vertices:

- Top: $g^2 [\epsilon^{ab} \epsilon^{cd} (\delta_{\mu\rho} \delta_{\nu\sigma} - \delta_{\mu\sigma} \delta_{\nu\rho}) + \epsilon^{ad} \epsilon^{cb} (\delta_{\mu\rho} \delta_{\nu\sigma} - \delta_{\rho\sigma} \delta_{\mu\nu}) + \epsilon^{ac} \epsilon^{db} (\delta_{\mu\sigma} \delta_{\nu\rho} - \delta_{\rho\sigma} \delta_{\mu\nu})]$
- Middle: $g^2 \epsilon^{ab} \left(\delta_{\rho\sigma} \delta_{\mu\nu} - \delta_{\mu\sigma} \delta_{\nu\rho} + \frac{1}{\alpha} \delta_{\nu\sigma} \delta_{\mu\rho} + \delta_{\rho\sigma} \delta_{\mu\nu} - \delta_{\nu\sigma} \delta_{\mu\rho} + \frac{1}{\alpha} \delta_{\mu\sigma} \delta_{\nu\rho} \right)$
- Bottom: $-2g^2 \delta^{ab} \delta_{\mu\nu}$

Bottom Row Vertices:

- Left: $g^2 (\epsilon^{da} \epsilon^{cb} + \epsilon^{db} \epsilon^{ca}) \delta_{\mu\nu}$
- Right: $-\alpha g^2 (\epsilon^{cd} \epsilon^{ba} - \epsilon^{ca} \epsilon^{bd})$

FIG. 8. Vertex factors.

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