

Quantum rods and clock in a gravitational universe

Hao Geng 

*Gravity, Spacetime, and Particle Physics (GRASP) Initiative, Harvard University,
17 Oxford Street, Cambridge Massachusetts 02138, USA*



(Received 16 December 2024; accepted 29 May 2025; published 2 July 2025)

Local operators are the basic observables in quantum field theory which encode the physics observed by a local experimentalist. However, when gravity is dynamical, diffeomorphism symmetries are gauged which apparently obstruct a sensible definition of local operators, as different locations in spacetime are connected by these gauged symmetries. This consideration brings in the puzzle of reconciling our empirical world with quantum gravity. Intuitively, this puzzle can be avoided using relatively defined observables when there exists a natural reference system such as a distribution of galaxies in our Universe. Nevertheless, this intuition is classical as the rods and clock defined in this way may also have quantum fluctuations, so it is not *a priori* clear if it can be realized in the quantum regime. In this paper, we provide an affirmative answer to this question. Interestingly, we notice that the quantum fluctuations of the reference system are in fact essential for the realization of the above intuition in the quantum regime.

DOI: [10.1103/kpqb-jrc8](https://doi.org/10.1103/kpqb-jrc8)

I. INTRODUCTION

Locality is one of the basic principles in quantum field theory which reconciles special relativity and quantum mechanics and correctly describes all phenomena we observed so far. In quantum field theory the basic observables are local field operators $\hat{O}(x)$ which are defined at each point $x^\mu = (t, \vec{x})$ in the spacetime $\mathcal{M}$. These operators and the smearing of them in a compact spacetime region $\mathcal{U} \subset \mathcal{M}$ encode all the measurements an experimentalist could carry out at the locations and during the moments within $\mathcal{U}$. Nevertheless, when gravity is dynamical the diffeomorphism symmetries

$$x^\mu \rightarrow x'^\mu(x) \quad (1)$$

are gauged which apparently obstructs the local field operators as gauge invariant observables. More specifically, this obstruction can be seen as follows. Let us consider the infinitesimal version of the diffeomorphism transforms, which in fact generate all local diffeomorphism transforms:

$$x^\mu \rightarrow x^\mu + \kappa \epsilon^\mu(x), \quad (2)$$

where $\epsilon^\mu(x)$ is a local vector field and κ is a small parameter indicating the infinitesimal nature of this transform. A scalar field operator $\hat{O}(x)$ transforms under Eq. (2) as

$$\hat{O}(x) \rightarrow \hat{O}(x) + \kappa \epsilon^\mu(x) \partial_\mu \hat{O}(x) + \mathcal{O}(\kappa^2), \quad (3)$$

which therefore is not invariant as being a local operator its dependence on x^μ is necessarily nontrivial.

The above issue has its roots more or less back in the early days of Einstein's theory of general relativity where Einstein visualized the fact that matter curves spacetime by exploiting Mach's principle. Einstein basically realized that gravitational theory should be a relative theory as a distribution of matter curves spacetime and the curved spacetime tells the observer which frame is inertial, i.e., there is no absolute spacetime.¹ This classical consideration shades much light on its quantum counterpart discussed in the previous paragraph. In simple spacetimes with an asymptotic boundary, it has been shown that we can define quasilocal diffeomorphism invariant observables by dressing them to the asymptotic boundary using the gravitational Wilson lines [1–5]. These gravitational Wilson lines are specific line integrals of the graviton field $\hat{h}_{\mu\nu}(x)$ from the spacetime point x^μ to the asymptotic boundary, analogous to Wilson lines in ordinary gauge theories, which have only been constructed for highly symmetric spacetime backgrounds. However, such boundary-dressed operators are

¹In other words, the spacetime diffeomorphism symmetry has to be gauged.

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

only quasilocal as two spacelike separated operators dressed by crossing gravitational Wilson lines do not commute [1]. Furthermore, guided by the intuition that a nontrivial background matter distribution $\phi_0(x)$ can be identified as a natural reference frame, it was proposed [6,7] that a spacetime integral of local operators against a delta function in the form $\delta(\phi_0(x) - a)$, i.e., the delta function localizes the spacetime integral to the place x^μ where $\phi(x) = a$, should be diffeomorphism invariant. Nevertheless, as an integrated expression, it is not clear if such construction leads to truly localized operators in general backgrounds, as the delta function might have multiple spacelike separated supports. Therefore, a systematic understanding of whether sensible local operators can be constructed in a gravitational universe is still lacking.

Interesting recent progress in [8–10] realizes that in gravitational theories where the graviton is massive due to the spontaneous breaking of diffeomorphism symmetries, one can easily construct diffeomorphism invariant local operators. The construction is done in the fully covariant description and makes use of the Goldstone vector field $\hat{V}^\mu(x)$ associated with the spontaneous breaking of the diffeomorphism symmetry. The Goldstone vector field $\hat{V}^\mu$, under a proper normalization, transforms under Eq. (2) as

$$\hat{V}^\mu(x) \rightarrow \hat{V}^\mu(x) + \kappa \epsilon^\mu(x), \quad (4)$$

so one can visualize $\hat{V}^\mu(x)$ as a quantum field of clock and rods and use it to construct diffeomorphism invariant operators. As an explicit example, given a scalar operator $\hat{O}(x)$ we can construct a diffeomorphism invariant operator

$$\hat{O}^{\text{Phys}}(x) = \hat{O}(x - \hat{V}(x)). \quad (5)$$

Nonetheless, this construction is rather formal; for example, it was not so clear how the vector field $\hat{V}^\mu(x)$ emerges in a general scenario with spontaneously broken diffeomorphism symmetries.²

In this paper, we show that in the appearance of enough nontrivial matter distributions, which spontaneously break the diffeomorphism symmetry, one is able to construct such a vector field whose components are the quantum fluctuations of these matter distributions. We provide the construction to the leading order in κ , and we expect that our construction could be extended to all orders in κ . In the end, we clarify the nature of this vector field in different

scenarios and their physical implications. This vector field is ubiquitous in realistic cosmological scenarios such as strongly time dependent Friedman-Robertson-Walker (FRW) universes during the inflation.³

II. THE ADM FORMALISM AND DIFFEOMORPHISM CONSTRAINTS

The Arnowitt-Deser-Misner (ADM) formalism [11] in general relativity is the natural framework to study the question we discussed in the Introduction, as diffeomorphism invariance is formulated as a set of algebraic constraint equations in this formalism. Diffeomorphism invariant observables are solutions of these constraint equations. In this section, we set up the stage of the calculations in this paper with a lightening review of the ADM formalism.

The ADM formalism starts with the following decomposition of metric on a $(d + 1)$ -dimensional spacetime:

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \quad (6)$$

where N is called the *lapse function*, and the vector N^i is called the *shift vector*. The matter-coupled Einstein-Hilbert action can be written as

$$S = \frac{1}{16\pi G_N} \int dt d^d x N \sqrt{g} (R[g] - 2\Lambda + K_{ij}K^{ij} - K^2) + S_{\text{matter}} + S_{\text{bdy}}, \quad (7)$$

with Λ as the cosmological constant, $R[g]$ as the Ricci scalar of the metric g_{ij} , S_{matter} as the action of the minimally coupled matter field, and S_{bdy} denoting the boundary term of the action. In the above formula, K_{ij} is the extrinsic curvature of the constant- t slices, and it is given by

$$K_{ij} = \frac{1}{2N} (-\dot{g}_{ij} + D_j N_i + D_i N_j), \quad (8)$$

where D_i is the torsion-free and metric-compatible covariant derivative with respect to g_{ij} . From Eqs. (7) and (8), we can see that the following canonical momenta are zero:

$$\Pi = \frac{1}{\sqrt{g}} \frac{\delta S}{\delta \dot{N}} = 0, \quad \Pi_i = \frac{1}{\sqrt{g}} \frac{\delta S}{\delta \dot{N}^i} = 0, \quad (9)$$

which are the *primary constraints* [12]. Meanwhile, the canonical Hamiltonian of this system can be written as

$$H_{\text{tot}} = \int d^d x \sqrt{g} [N \mathcal{H} + N^i \mathcal{H}_i] + H_{\text{bdy}}, \quad (10)$$

²This is understood recently in [10] in the so-called *island model*, a setup in AdS/CFT correspondence that enables the calculation of the Page curve of black hole radiation, which reveals the remarkable fact that operators in island, the interior of the black hole, are in fact nonlocally dressed to the external bath, the early-time Hawking radiation. This articulated how the information in the island is encoded in the external bath.

³We thank an anonymous referee for pointing this point out.

where

$$\begin{aligned}\mathcal{H} &= 16\pi G_N \left(\Pi_{ij} \Pi^{ij} - \frac{1}{d-1} (\Pi_i^i)^2 \right) \\ &\quad - \frac{1}{16\pi G_N} (R[g] - 2\Lambda) + \mathcal{H}_{\text{matter}}, \\ \mathcal{H}_i &= -2g_{ij} D_k \Pi^{jk} + \mathcal{H}_{i,\text{matter}}.\end{aligned}\quad (11)$$

In Eq. (11), we denote the Hamiltonian density of the matter field as $\mathcal{H}_{\text{matter}}$, the momentum density of the matter fields as $\mathcal{H}_{i,\text{matter}}$, and the canonical momentum of g_{ij} as

$$\Pi^{ij} = \frac{1}{\sqrt{g}} \frac{\delta S}{\delta g_{ij}} = -\frac{1}{16\pi G_N} (K^{ij} - g^{ij} K). \quad (12)$$

The primary constraints in Eq. (9) have to be preserved under the time evolution generated by the Hamiltonian, Eq. (10), and this consideration generates the *secondary constraints*

$$\mathcal{H} = 0, \quad \mathcal{H}_i = 0. \quad (13)$$

After we promote $\mathcal{H}$ and $\mathcal{H}_i$ to operators $\hat{\mathcal{H}}$ and $\hat{\mathcal{H}}_i$, these secondary constraints constrain physical states and observables. Physical states have to be annihilated by $\hat{\mathcal{H}}$ and $\hat{\mathcal{H}}_i$ and physical, i.e., gauge invariant, observables have to commute with them. These constraints imposed on the states are also called the Wheeler-de Witt equations.

III. THE QUANTUM RODS AND CLOCK

We are interested in a universe with nontrivial matter distributions as this is the situation in our Universe, and we want to articulate how these matter distributions can be used as a reference system to define local operators. An ideal such reference system should have enough features that we can use it to define both time x^0 and locations x^i where $i = 1, \dots, d$. Let us model the matter distributions using $(d+1)$ free scalar fields $\Phi^\mu(x)$, and the matter distributions are background configurations $\phi_0^\mu(x^\mu)$, i.e., each configuration depends only on one spacetime coordinate. These background configurations and the corresponding backreacted metric $G_{\mu\nu}^{(0)}(x)^4$ specify the universe we are considering. Intuitively, we can think of these matter distributions $\phi_0^\mu(x^\mu)$ as the rods and clock in the universe with metric $G_{\mu\nu}^{(0)}$. Our final goal is to construct local operators in this universe which satisfy the diffeomorphism constraints.

In this section, we will study the quantum fluctuations of these rod and clock fields $\Phi^\mu(x)$ in the universe $\{G_{\mu\nu}^{(0)}(x), \phi_0^\mu(x^\mu)\}$. We can write

$$\Phi^\mu(x) = \phi_0^\mu(x^\mu) + \phi^\mu(x), \quad (14)$$

where the fields $\phi^\mu(x)$ denote the fluctuations. The full action of these fields in any spacetime is

$$S[\Phi] = -\frac{1}{2} \int d^{d+1}x \sqrt{-G} [G^{\alpha\beta} \partial_\alpha \Phi^\mu \partial_\beta \Phi^\mu + m_\mu^2 (\Phi^\mu)^2], \quad (15)$$

where m_μ is the mass of the field $\Phi^\mu(x)$, and the summation over μ is implicit. For the sake of convenience, we will ignore the indices μ of the field Φ^μ and restore them when appropriate from now on. Under the ADM decomposition, we can write the action, Eq. (15), as

$$\frac{1}{2} \int d^{d+1}x \sqrt{g} N \left[\frac{1}{N^2} (\dot{\Phi} - N^i \partial_i \Phi)^2 - g^{ij} \partial_i \Phi \partial_j \Phi - m^2 \Phi^2 \right], \quad (16)$$

from which we have the canonical momenta density

$$\pi_{\Phi^\mu} = \frac{1}{\sqrt{g}} \frac{\delta S}{\delta \dot{\Phi}^\mu(x)} = \frac{1}{N} (\dot{\Phi}^\mu - N^i \partial_i \Phi^\mu), \quad (17)$$

which obeys the equal-time commutation relations

$$[\hat{\pi}_{\Phi^\mu}(t, \vec{x}), \Phi^\nu(t, \vec{y})] = -i \frac{1}{\sqrt{g}} \delta^d(\vec{x} - \vec{y}) \delta_\mu^\nu. \quad (18)$$

The corresponding Hamiltonian is

$$\begin{aligned}H[\Phi] &= \int d^d x \sqrt{-G} [\pi(x) \dot{\Phi}(x) - \mathcal{L}], \\ &= \int d^d x \sqrt{g} \left[\frac{N}{2} (\pi^2 + g^{ij} \partial_i \Phi \partial_j \Phi + m^2 \Phi^2) + N^i \pi \partial_i \Phi \right],\end{aligned}\quad (19)$$

from which and Eq. (10) we have

$$\mathcal{H}[\Phi] = \frac{1}{2} (\pi^2 + g^{ij} \partial_i \Phi \partial_j \Phi + m^2 \Phi^2), \quad \mathcal{H}_i[\Phi] = \pi \partial_i \Phi. \quad (20)$$

Therefore, under the linearization of Eq. (14) and

$$G_{\mu\nu}(x) = G_{\mu\nu}^{(0)}(x) + \kappa h_{\mu\nu}(x), \quad (21)$$

we have

$$\begin{aligned}\pi_{\phi^\mu} &= \frac{1}{N} (\dot{\phi}_0^\mu - N^i \partial_i \phi_0^\mu + \dot{\phi}^\mu - N^i \partial_i \phi^\mu), \\ [\hat{\pi}_\phi(t, \vec{x}), \hat{\phi}^\nu(t, \vec{y})] &= -i \frac{1}{\sqrt{g_0}} \delta^d(\vec{x} - \vec{y}) \delta_\mu^\nu,\end{aligned}\quad (22)$$

⁴That is, $\{G_{\mu\nu}^{(0)}(x), \phi_0^\mu(x^\mu)\}$ exactly solves the matter sourced Einstein's equations, and so they satisfy the classical diffeomorphism constraints, Eq. (13).

which tells us that

$$\begin{aligned}
[\hat{\mathcal{H}}[\Phi](t, \vec{x}), \hat{\phi}^\mu(t, \vec{y})] &= -i \frac{1}{N\sqrt{g_0}} \delta^d(\vec{x} - \vec{y}) (\dot{\phi}_0^\mu - N^i \partial_i \phi_0^\mu), \\
&= -i \frac{1}{N\sqrt{g_0}} \delta^d(\vec{x} - \vec{y}) (\delta^{\mu 0} \phi_0^{0'} - N^i \phi_0^{i'} \delta_i^\mu), \\
[\hat{\mathcal{H}}_i[\Phi](t, \vec{x}), \hat{\phi}^\mu(t, \vec{y})] &= -i \frac{1}{N\sqrt{g_0}} \delta^d(\vec{x} - \vec{y}) \partial_i \phi^\mu, \\
&= -i \frac{1}{N\sqrt{g_0}} \delta^d(\vec{x} - \vec{y}) \delta_i^\mu \phi^{i'}, \quad (23)
\end{aligned}$$

where we are working at the leading order in ϕ^μ and $h_{\mu\nu}$. Last but not least, the parameter κ in Eq. (21) is chosen such that the graviton field $h_{\mu\nu}(x)$ is canonically normalized, i.e., in terms of Newton's constant $\kappa = \sqrt{32\pi G_N}$.

IV. DEFINING LOCAL OPERATORS USING THE QUANTUM RODS AND CLOCK

In the previous section, we introduced the quantum fluctuations $\hat{\phi}^\mu(x)$ of the rod and clock fields in the Universe $\{G_{\mu\nu}^{(0)}(x), \phi_0^\mu(x)\}$. Let us call these fluctuations the *quantum rod and clock fields*. In this section, we will show that one can use these quantum rod and clock fields to construct local operators in the Universe, $\{G_{\mu\nu}^{(0)}(x), \phi_0^\mu(x)\}$. For the sake of simplicity, we focus on scalar operators $\hat{O}(x)$.

Intuitively, under the diffeomorphism transform

$$x^\mu \rightarrow x^\mu + \kappa \epsilon^\mu, \quad (24)$$

we have

$$\hat{O}(x) \rightarrow \hat{O}(x) + \kappa \epsilon^\mu \partial_\mu \hat{O}(x), \quad (25)$$

and

$$\Phi^\mu(x) \rightarrow \Phi^\mu(x) + \kappa \epsilon^\nu \partial_\nu \Phi^\mu(x). \quad (26)$$

Therefore, using Eq. (14) we have that, to the leading order of the perturbation, for the quantum rod and clock fields

$$\hat{\phi}^\mu(x) \rightarrow \hat{\phi}^\mu(x) + \kappa \epsilon^\mu \phi_0^{\mu'}(x^\mu). \quad (27)$$

Let us define a more convenient quantum rod and clock field as

$$\delta\phi^\mu(x) \equiv \frac{\hat{\phi}^\mu(x)}{\phi_0^{\mu'}(x^\mu)}, \quad (28)$$

which under the diffeomorphism transforms as

$$\delta\phi^\mu(x) \rightarrow \delta\phi^\mu(x) + \kappa \epsilon^\mu(x). \quad (29)$$

Therefore, $\delta\phi^\mu(x)$ is really the Goldstone fields associated with the spontaneously broken diffeomorphism symmetry, and the combinations $x^\mu - \delta\phi^\mu$ are diffeomorphism invariant. Hence we expect the following operator to be diffeomorphism invariant:

$$\hat{O}^{\text{Phys}}(x) = \hat{O}(x - \delta\phi), \quad (30)$$

where $x - \delta\phi$ is a shorthand notation for $x^\mu - \delta\phi^\mu(x)$.

Let us denote the constraint operators associated with $\hat{O}(x)$ as $\hat{\mathcal{H}}[O]$ and $\hat{\mathcal{H}}_i[O]$ which gives⁵

$$\begin{aligned}
[\hat{\mathcal{H}}[O](t, \vec{x}), \hat{O}(t, \vec{y})] &= -i \frac{1}{N\sqrt{g_0}} \delta^d(\vec{x} - \vec{y}) (\dot{O} - N^i \partial_i O), \\
[\hat{\mathcal{H}}_i[O](t, \vec{x}), \hat{O}(t, \vec{y})] &= -i \frac{1}{N\sqrt{g_0}} \delta^d(\vec{x} - \vec{y}) \partial_i O. \quad (31)
\end{aligned}$$

Using Eq. (23), we can explicitly see that

$$\begin{aligned}
[\hat{\mathcal{H}}_{\text{matter}}, \hat{O}^{\text{Phys}}(x)] &= [\hat{\mathcal{H}}[O] + \hat{\mathcal{H}}[\phi], \hat{O}^{\text{Phys}}(x)] = 0, \\
[\hat{\mathcal{H}}_{i,\text{matter}}, \hat{O}^{\text{Phys}}(x)] &= [\hat{\mathcal{H}}_i[O] + \hat{\mathcal{H}}_i[\phi], \hat{O}^{\text{Phys}}(x)] = 0. \quad (32)
\end{aligned}$$

Since the above operators do not involve the graviton field, we in fact have

$$[\hat{\mathcal{H}}, \hat{O}^{\text{Phys}}(x)] = 0, \quad [\hat{\mathcal{H}}_i, \hat{O}^{\text{Phys}}(x)] = 0, \quad (33)$$

where $\mathcal{H}$ and $\mathcal{H}_i$ are given in Eq. (30). As a result, the operators we constructed in Eq. (30) indeed satisfy the diffeomorphism constraints and qualify as local operators. Moreover, we can see that the existence of the nontrivial background configurations $\phi_0^\mu(x)$ is essential as otherwise $\phi_0^{\mu'}(x^\mu) = 0$ and the quantum rod and clock fields $\delta\phi^\mu(x)$ in Eq. (28) are not well defined. As a physical example of this construction, one can think of the quantum fluctuation of the inflaton as a quantum clock field during the inflationary period of the Universe. During the inflation, the inflaton field is rolling down the potential in time, and the Universe is described by an FRW metric. In the study of inflation, this rolling profile is treated as a background [13], and our construction implies that the fluctuation of the inflaton around this rolling background is in fact a quantum clock.

V. CONCLUSIONS AND DISCUSSIONS

In this paper, we provide an explicit construction of local operators in a gravitational universe with nontrivial background matter distributions. Our construction follows the original insight from Einstein in the early days of general relativity that physical observables are relatively defined in a gravitational theory. We found that this is the case even

⁵For example, this can be worked out for a probe scalar field using canonical quantization.

quantum mechanically. More precisely, matter sources curved the spacetime and are classicalized as background configurations, which can then be thought of as a reference frame. Hence probe operators can be dressed to these background configurations, i.e., defined with respect to the corresponding reference frame. Moreover, sensible local operators can exist only if the background configurations have strong enough features to serve as a good reference frame, for example in our case $\phi_0'' \neq 0$. In our construction, we make use of the quantum fluctuations of the reference frame to compensate for the change of local operators under diffeomorphism transforms. We should think of these fluctuations as components of a vector field $V^\mu(x)$ which however transforms as a Goldstone vector field under the diffeomorphism. The existence of this vector field is consistent with the fact that diffeomorphism symmetries are spontaneously broken by the background configurations $\phi_0''(x)$. Therefore, the construction in this paper is in the same spirit as the constructions in [8–10] in a specific theory of massive gravity. In that case, the Goldstone vector field $V^\mu(x)$ is a composite operator [9,10,14]. Moreover, correlators of the physical operators like $\hat{O}^{\text{Phys}}(x)$ can be straightforwardly computed in the unitary gauge, i.e., $V^\mu(x) = 0$, which reduces the computation to quantum field theory in the fixed background.⁶ It deserves to be emphasized that the construction in this paper works only to the first nontrivial order in the linearization of Eqs. (14) and (21), where the graviton field and the matter fields are decoupled. Based on the above discussion, we expect the construction can be extended to all orders in the

expansion which is in fact counted by the powers of κ , i.e., the Newton’s constant. We leave this construction for future work.⁷

We hope that the current paper clarifies some open questions in the recent literature [7,16–21] regarding subregion algebras in a gravitational universe (see also [22–41] for relevant discussions). For example, to define subregion algebras in gravity, one does not need the existence of an “observer” with a nonphysical Hamiltonian, and that “observer” can be taken as a simplified model of a nontrivial background configuration which defines the subregion. Such background configurations universally exist in cosmological scenarios; for example during the inflation the classical inflaton field is time dependent, and its quantum fluctuation can be used as a clock field according to our construction. Interestingly, this consideration is in fact implicit in the construction of effective actions for perturbations in an inflationary universe [13,42,43].

ACKNOWLEDGMENTS

We would like to thank Steve Giddings, Daniel Harlow, Åsmund Folkstad, Jonah Kudler-Flam, Juan Maldacena, Henry Maxfield, Massimo Porrati, Lisa Randall, Edward Witten, and Jiuci Xu for discussions. We would like to thank Daniel Jafferis, Pushkal Shrivastava, and Neeraj Tata for relevant collaborations. H. G. is supported by the Gravity, Spacetime, and Particle Physics (GRASP) Initiative from Harvard University.

DATA AVAILABILITY

No data were created or analyzed in this study.

⁶A caveat is that the state under which we are computing the correlator also has to satisfy the diffeomorphism constraints, i.e., it must obey the Wheeler-de Witt equations.

⁷A good starting point is low dimensional models of gravity coupled with matter [15].

-
- [1] W. Donnelly and S. B. Giddings, *Phys. Rev. D* **93**, 024030 (2016); **94**, 029903(E) (2016).
 - [2] W. Donnelly and S. B. Giddings, *Phys. Rev. D* **94**, 104038 (2016).
 - [3] W. Donnelly and S. B. Giddings, *Phys. Rev. D* **96**, 086013 (2017).
 - [4] W. Donnelly and S. B. Giddings, *Phys. Rev. D* **98**, 086006 (2018).
 - [5] S. B. Giddings and A. Kinsella, *J. High Energy Phys.* **11** (2018) 074.
 - [6] S. B. Giddings, D. Marolf, and J. B. Hartle, *Phys. Rev. D* **74**, 064018 (2006).
 - [7] Å. Folkstad, *J. High Energy Phys.* **05** (2024) 300.
 - [8] H. Geng, [arXiv:2312.13336](https://arxiv.org/abs/2312.13336).
 - [9] H. Geng, [arXiv:2502.08703](https://arxiv.org/abs/2502.08703).
 - [10] H. Geng, D. Jafferis, P. Shrivastava, and N. Tata, The fate of locality and holography in Quantum Gravity (to be published).
 - [11] R. L. Arnowitt, S. Deser, and C. W. Misner, *Gen. Relativ. Gravit.* **40**, 1997 (2008).
 - [12] P. Dirac, *Proc. R. Soc. A* **167**, 148 (1938).
 - [13] C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan, and L. Senatore, *J. High Energy Phys.* **03** (2008) 014.
 - [14] H. Geng, *J. High Energy Phys.* **09** (2024) 012.
 - [15] C.-H. Wu and J. Xu, *J. High Energy Phys.* **10** (2023) 094.
 - [16] V. Chandrasekaran, R. Longo, G. Penington, and E. Witten, *J. High Energy Phys.* **02** (2023) 082.
 - [17] E. Witten, *Proc. Symp. Pure Math.* **107**, 247 (2024).

- [18] E. Witten, *J. High Energy Phys.* **03** (2024) 077.
- [19] K. Jensen, J. Sorce, and A. J. Speranza, *J. High Energy Phys.* **12** (2023) 020.
- [20] C.-H. Chen and G. Penington, [arXiv:2406.02116](#).
- [21] J. Kudler-Flam, S. Leutheusser, and G. Satishchandran, [arXiv:2406.01669](#).
- [22] P. G. Bergmann, *Rev. Mod. Phys.* **33**, 510 (1961).
- [23] P. G. Bergmann, *Phys. Rev.* **124**, 274 (1961).
- [24] C. Rovelli, *Classical Quantum Gravity* **8**, 297 (1991).
- [25] C. Rovelli, *Classical Quantum Gravity* **8**, 317 (1991).
- [26] A. Nicolis, R. Penco, F. Piazza, and R. Rattazzi, *J. High Energy Phys.* **06** (2015) 155.
- [27] S. Bose, A. Mazumdar, G. W. Morley, H. Ulbricht, M. Toroš, M. Paternostro, A. Geraci, P. Barker, M. S. Kim, and G. Milburn, *Phys. Rev. Lett.* **119**, 240401 (2017).
- [28] F. Piazza, *Classical Quantum Gravity* **40**, 165014 (2023).
- [29] F. Piazza and A. J. Tolley, *Classical Quantum Gravity* **40**, 165013 (2023).
- [30] S. Carrozza, S. Eccles, and P. A. Hoehn, *SciPost Phys.* **17**, 048 (2024).
- [31] C. Goeller, P. A. Hoehn, and J. Kirklin, [arXiv:2206.01193](#).
- [32] P. A. Hoehn, A. Russo, and A. R. H. Smith, *Phys. Rev. D* **109**, 105011 (2024).
- [33] S. Ali Ahmad and R. Jefferson, *SciPost Phys. Core* **7**, 020 (2024).
- [34] N. Bamonti, [arXiv:2307.09338](#).
- [35] G. Torrieri, *Int. J. Mod. Phys. D* **34**, 2550016 (2025).
- [36] J. De Vuyst, S. Eccles, P. A. Hoehn, and J. Kirklin, [arXiv:2405.00114](#).
- [37] S. Ali Ahmad, M. S. Klinger, and S. Lin, *Phys. Rev. D* **111**, 045006 (2025).
- [38] S. Ali Ahmad and M. S. Klinger, *Phys. Rev. D* **111**, 105015 (2025).
- [39] S. Ali Ahmad, W. Chemissany, M. S. Klinger, and R. G. Leigh, *Nucl. Phys. B* **1015**, 116911 (2025).
- [40] S. Ali Ahmad, W. Chemissany, M. S. Klinger, and R. G. Leigh, *Phys. Rev. D* **110**, 065003 (2024).
- [41] P. A. Grassi and M. Porrati, *J. High Energy Phys.* **03** (2025) 175.
- [42] G. Gubitosi, F. Piazza, and F. Vernizzi, *J. Cosmol. Astropart. Phys.* **02** (2013) 032.
- [43] F. Piazza and F. Vernizzi, *Classical Quantum Gravity* **30**, 214007 (2013).