

Implications of a portal vectorlike lepton on associated Higgs boson production at a multi-TeV muon collider

Krishna Tewary^{1,*}, Sanjoy Biswas^{1,†} and Shivam Verma^{2,‡}

¹*Department of Physics, Ramakrishna Mission Vivekananda Educational and Research Institute, Belur Math, Howrah 711202, India*

²*Theoretical Physics Division, Physical Research Laboratory, Shree Pannalal Patel Marg, Ahmedabad 380009, Gujarat, India*



(Received 7 December 2025; accepted 13 July 2026; published 21 August 2026)

We explore a portal vectorlike lepton (pVLL) extension of the Standard Model (SM) and study its implications for Higgs and vector-boson associated production (hV , with $V = Z$ -boson or dark photon) at a future muon collider facility. We show that while the $\mu^+\mu^- \rightarrow hZ$ production rate remains close to its SM prediction in a wide range of parameter space, the rate for $\mu^+\mu^- \rightarrow h\gamma_d$ can be substantially enhanced owing to the nondecoupling nature of the interaction involving the heavy lepton, the muon and the dark photon. We demonstrate that the $h\gamma_d$ production rate can exceed the corresponding hZ rate by a factor of 1–100 within the perturbative unitarity limit, making it a promising channel for probing Higgs interactions and potential new physics effects. We also examine the role of the pVLL state in the context of dark matter (DM) phenomenology and identify regions of parameter space consistent with the observed relic abundance by extending the simplified setup with a viable DM candidate. The $h\gamma_d$ production can also be used to constrain the dark photon mass (m_{γ_d}) and/or the dark gauge coupling (g_d) consistent with various constraints including the current muon $g-2$ measurements within the pVLL framework. We perform a detailed collider analysis of the $h\gamma_d$ process in the $b\bar{b} +$ missing energy final state. A 2σ exclusion limit for m_{γ_d} up to 80 GeV is obtained assuming $g_d = 0.05$, $\sin\theta_L = 4 \times 10^{-5}$, $\sin\theta_S = 0.05$, for a heavy lepton mass ~ 3 TeV at a 3 TeV muon collider with an integrated luminosity of 1 ab^{-1} .

DOI: [10.1103/kpdt-4zm2](https://doi.org/10.1103/kpdt-4zm2)

I. INTRODUCTION

The discovery of the Higgs boson [1,2] marks a remarkable triumph of theoretical and experimental particle physics. To probe its properties with greater precision, however, we require an abundant production of Higgs bosons at high-energy (HE) and high-luminosity (HL) facilities such as the Large Hadron Collider (LHC) and future collider experiments. Higgs production in association with a gauge boson is a well-studied process at lepton colliders [3–7], both in past experiments [8,9] and in

proposed future facilities [10–13]. The presence of new physics, particularly, scenarios predicting vectorlike leptons (VLLs), can significantly alter the associated Higgs production cross section at lepton colliders—either through modified Higgs couplings, the appearance of new diagrams, or both. Moreover, such scenarios may open up additional Higgs production channels with enhanced rates. In this work, we explore the implications of VLLs acting as portal matter on the associated Higgs production cross section at a future muon collider.

VLLs constitute a simple yet well-motivated extension of the Standard Model (SM) of particle physics. VLL extension of the SM has been studied extensively in the literature in the context of various anomalies, including the muon $g-2$, the W -boson mass, the Cabibbo angle anomaly, and dark matter phenomenology [14–27]. They naturally arise in a variety of theoretical frameworks [28–33], including grand unified theories, models addressing the Higgs mass hierarchy problem, and constructions inspired by string theory. The search for VLLs at collider experiments remains an active area of investigation, including both prompt and long-lived signatures, primarily in SM final states [34–36]. VLL is also a potential candidate to

*Contact author: krishna.tewary.phy22@gm.rkmvu.ac.in, krishnatewary3@gmail.com

†Contact author: sanjoy.phy@gm.rkmvu.ac.in, sanjoyhri@gmail.com

‡Contact author: shivamverma@prl.res.in, shivam.59910103@gmail.com

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

search for at the future colliders such as future pp , e^+e^- , muon collider [37–41].

In recent years, the idea of VLLs as portal matter [18,42–44] or portal VLLs (pVLLs) has gained significant interest owing to their rich phenomenology, their ability to evade existing collider constraints, and their potential connection to the dark sector, particularly dark matter phenomenology [45,46].

In this work, we consider an extension of the Standard Model (SM) with an $SU(2)$ -singlet VLL that transforms under both the SM hypercharge group $U(1)_Y$ and an additional Abelian dark gauge group $U(1)_D$ [18,42–44]. The gauge boson associated with $U(1)_D$ is referred to as the dark photon (γ_d). The model also contains an additional complex scalar field, singlet under the SM gauge group but carrying nonzero $U(1)_D$ charge. The full gauge symmetry, $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_D$, is spontaneously broken to $SU(3)_C \times U(1)_{EM}$ through the nonvanishing vacuum expectation values (VEVs) of the SM Higgs doublet and the dark scalar. The VEV of the dark scalar not only generates the dark photon mass but also induces mixing between the pVLL and SM leptons through a Yukawa-type portal interaction. The $U(1)_Y$ hypercharge assignment of the pVLL considered here corresponds to an electric charge of -1 , and it is allowed to mix only with one generation of SM charged leptons in order to evade stringent bounds from lepton-flavor violation. In this work, we focus on pVLL mixing with the second-generation charged lepton.

An important feature of this model is that the right-handed second generation charged lepton (μ'_R) and the pVLL (μ'_p) carry identical $SU(2)_L \times U(1)_Y$ charges. Consequently, any rotation between μ'_R and μ'_{pR} preserves the diagonal structure of the Z -boson couplings, ensuring that no $\mu_R - \mu_{pR} - Z$ interaction arises after transforming to the mass basis. In contrast, since the muon is uncharged under $U(1)_D$, such a rotation induces an off-diagonal $\mu_R - \mu_{pR} - \gamma_d$ coupling, suppressed by $\sin\theta_R$, the sine of the corresponding mixing angle. One can have a sizable value of $\sin\theta_R \sim 0.1$ for heavy muon mass $\sim$ a few TeV even when the mixing angle ($\sin\theta_L$) involving the corresponding left-chiral fermions is kept at a tiny value $\sim 10^{-5}$. The nondecoupling nature of the interaction is central to both the dark matter phenomenology and the collider signatures of the model.

After examining the various constraints on the model parameter space arising from Higgs signal strength measurements, heavy Higgs and di-Higgs searches at the LHC, limits from lepton flavor universality and other electroweak precision observables, searches for vectorlike leptons at the LHC, as well as consistency with current experimental results on the muon $g-2$, we proceed to explore the implications for dark matter relic abundance and associated Higgs production at a proposed future lepton collider, utilizing the nondecoupling effect within the perturbative unitarity limit.

We investigate the role of the pVLL in the context of DM relic density calculations, assuming that the dark sector contains a Dirac DM particle (χ) charged under $U(1)_D$ as a motivation behind the consideration of pVLL extension of the SM. In the absence of the pVLL state, the annihilation rate in the $\chi\bar{\chi} \rightarrow f\bar{f}$ channel is typically suppressed due to the small gauge kinetic mixing (KM), often resulting in an overabundant relic density. The indirect effect of the pVLL is to facilitate DM annihilation into a pair of muons by modifying the $\mu^+\mu^-\gamma_d$ vertex, without introducing any additional Feynman diagrams. Furthermore, its presence introduces a nontrivial interplay between the gauge kinetic mixing and fermion mixing parameters.

The nondecoupling effect also show up in the Higgs production in association with a vector boson at the proposed future muon collider ($\mu^+\mu^- \rightarrow hV$ with $V = Z, \gamma_d$). The muon collider [47–49] is envisioned as a high-energy circular lepton collider that combines the clean experimental environment of lepton collisions with access to multi-TeV center-of-mass energies ($\sqrt{s}$), in contrast to circular e^+e^- machines. It is designed to operate at energy stages from an initial 3 TeV to 10 TeV (or higher), corresponding to circumferences of about 4.5 km and 10 km, respectively, with expected peak integrated luminosities of 1 ab^{-1} at $\sqrt{s} = 3 \text{ TeV}$ and 10 ab^{-1} at $\sqrt{s} = 10 \text{ TeV}$ [50].

We have shown that while the hZ production cross section remains essentially unchanged from the SM prediction, the $h\gamma_d$ production rate can be significant even for very small $\sin\theta_L$, owing to the nondecoupling effect. In this scenario, the dark photon is treated as an invisible particle.¹ We analyze this process in the $b\bar{b}$ plus missing energy final state at $\sqrt{s} = 3$ and 10 TeV muon colliders, where the $b\bar{b}$ system is highly boosted due to the $2 \rightarrow 2$ kinematics and large center-of-mass energy. The dominant SM backgrounds to this channel are processes leading to $b\bar{b}\nu\bar{\nu}$ final state, with $h\nu\bar{\nu}$ and $Z\nu\bar{\nu}$ accounting for more than 90% of the total cross section. We demonstrate that the requirement of jet substructure, together with additional kinematic selections such as missing energy and the $b\bar{b}$ invariant mass, can efficiently suppress these backgrounds.

The main highlight of this work is that the $h\gamma_d$ process not only yields a higher Higgs production rate than hZ production at a muon collider, but also offers a noble way to explore dark photons in the mass range 10–100 GeV, independent of the gauge kinetic mixing parameter (ϵ) [51–54]. This is because of the fact that the muon-heavy muon-Higgs vertex is sensitive to the dark photon mass (m_{γ_d}) and the dark gauge coupling (g_d). Consequently, the absence of any observable enhancement in Higgs production along with missing energy at a muon collider would imply a

¹This will be the case for light DM with $m_\chi < m_{\gamma_d}/2$, where the dark photon predominantly decays to an invisible final state.

stringent limit on m_{γ_d} or g_d , for fixed values of other relevant model parameters. We illustrate this using several representative benchmark points and show that, even for fermion mixing angle as small as $\sim 10^{-6}$ and heavy muon masses in the range 1–3 TeV, a wide range of dark photon masses can be excluded. We illustrate that a substantial region of the parameter space consistent with current measurement of muon $g-2$ and DM relic abundance can be probed at the future muon collider using our analysis.

The remainder of the manuscript is organized as follows. In Sec. II, we present the theoretical framework considered in this work to illustrate the impact of portal VLLs on associated Higgs production at muon collider, along with a discussion of relevant constraints on the model parameters. Section III contains the parameter space consistent with observed DM relic abundance. In Sec. IV, we describe the collider analysis of Higgs production in association with a vector boson. The results of our analysis are presented in Sec. V, and finally we conclude in Sec. VI.

II. THEORETICAL FRAMEWORK

We consider a framework in which the Standard Model is extended by a vectorlike lepton, singlet under $SU(3)_C \times SU(2)_L$ with $U(1)_Y$ hypercharge -1 . The VLL plays the role of a portal matter, linking the visible SM sector with a dark sector. A simplified realization of such a scenario is to have a VLL transforming nontrivially under an additional local $U(1)_D$ symmetry of the dark sector with corresponding gauge boson referred to as the dark photon (γ_d). The dark photon acquires mass through spontaneous breaking of $U(1)_D$, achieved by introducing a complex scalar field Φ_d which is a singlet under the SM gauge group but charged under $U(1)_D$. Furthermore, if Φ_d carries the same $U(1)_D$ charge as the pVLL, a Yukawa-type portal interaction $\omega_f \Phi_d \bar{\mu}'_L \mu'_R$ is permitted. The Yukawa interaction reduces to a bilinear term between the pVLL and the SM lepton once Φ_d gets a nonzero VEV. In this work, we focus on the case where the pVLL mixes with the second-generation charged lepton only to avoid constraints coming from flavor violating observables. The assignment of charge of the relevant field content is presented in Table I.

In Table I, μ'_R and L_L represent the second-generation SM right-handed charged lepton and left-handed lepton

doublet, respectively, and Φ corresponds to the SM complex scalar field doublet. The additional VLL singlet under $SU(2)_L$ is represented by μ'_p . Φ_d is the complex scalar field that does not carry charges under the SM gauge group. All fields are expressed in their interaction basis. The complete Lagrangian of the model invariant under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_D$ can be expressed as

$$\mathcal{L} = \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Scalar}} + \mathcal{L}_{\text{Fermions}} + \mathcal{L}_{\text{Yukawa}} \quad (1)$$

where $\mathcal{L}_{\text{Gauge}}$, $\mathcal{L}_{\text{Scalar}}$, $\mathcal{L}_{\text{Fermions}}$ are part of the full Lagrangian containing gauge invariant kinetic terms involving the gauge fields, scalar fields including corresponding scalar potential, and fermion fields including the mass term of the pVLL, respectively. Finally, $\mathcal{L}_{\text{Yukawa}}$ represents the trilinear Yukawa interactions involving scalar and fermions.

In particular, the gauge sector of the theory is given by

$$\mathcal{L}_{\text{Gauge}} = -\frac{1}{4} G_{\mu\nu}^a G^{a,\mu\nu} - \frac{1}{4} W_{\mu\nu}^i W^{i,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} B_{d,\mu\nu} B_d^{\mu\nu} - \frac{\epsilon}{2} B_{d,\mu\nu} B^{\mu\nu}, \quad (2)$$

where $G_{\mu\nu}^a$ denotes the QCD field strength tensor with $a = 1, \dots, 8$ labeling the adjoint of $SU(3)_C$, $W_{\mu\nu}^i$ ($i = 1, 2, 3$) are the field strength tensors of $SU(2)_L$, $B_{\mu\nu}$ corresponds to the $U(1)_Y$ hypercharge, and $B_{d,\mu\nu}$ is the field strength tensor of the dark $U(1)_D$. The noncanonical form of the gauge boson kinetic terms is due to the presence of kinetic mixing involving the field strength tensors corresponding to two Abelian gauge groups, $U(1)_Y$ and $U(1)_D$, respectively. The strength of this gauge kinetic mixing is parametrized by the parameter ϵ . In order to bring the gauge kinetic terms into canonical form, we perform the following nonorthogonal transformation between the hypercharge gauge boson B_μ and corresponding $U(1)_D$ gauge boson:

$$\begin{aligned} B'_\mu &= B_\mu + \epsilon B_{d,\mu} \\ B'_{d,\mu} &= \sqrt{1 - \epsilon^2} B_{d,\mu} \end{aligned} \quad (3)$$

In the primed basis, the gauge kinetic terms have canonical form. However, after the breaking of $SU(2) \times U(1)_Y \times U(1)_D \rightarrow U(1)_{\text{EM}}$, the mass matrix involving the neutral gauge boson in the basis $(B'_\mu, W_\mu^3, B'_{d,\mu})$ has off-diagonal terms. The physical neutral gauge boson mass eigenstates (A_μ , Z_μ , and $A_{d,\mu}$) are related to this by the following orthogonal transformation that involves two mixing angles, the weak mixing angle (θ_W) and θ_d [55].

TABLE I. Representation of relevant fields and their $U(1)$ charge assignments under the extended gauge group.

Fields	$SU(3)_C$	$SU(2)_L$	Y	Y_d
μ'_R	1	1	-1	0
$L_L = \begin{pmatrix} \nu_{\mu_L} \\ \mu'_L \end{pmatrix}$	1	2	-1/2	0
Φ	1	2	1/2	0
μ'_p	1	1	-1	1
Φ_d	1	1	0	1

$$\begin{pmatrix} A_\mu \\ Z_\mu \\ A_{d,\mu} \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W & 0 \\ -\cos \theta_d \sin \theta_W & \cos \theta_d \cos \theta_W & \sin \theta_d \\ \sin \theta_d \sin \theta_W & -\sin \theta_d \cos \theta_W & \cos \theta_d \end{pmatrix} \times \begin{pmatrix} B'_\mu \\ W_\mu^3 \\ B'_{d,\mu} \end{pmatrix}. \quad (4)$$

The angle of rotation θ_d is given by

$$\tan 2\theta_d = \left(\frac{-2\varepsilon\sqrt{1-\varepsilon^2}\sin\theta_W M_{Z_0}^2}{M_{A_{d_0}}^2 - M_{Z_0}^2(1-\varepsilon^2(1+\sin^2\theta_W))} \right)$$

where, $M_{Z_0} = v_{EW}(g^2 + g'^2)/2$ and $M_{A_{d_0}} = v_d g_d$ are the Z boson mass and dark photon mass, respectively, without the kinetic mixing.

The scalar-sector Lagrangian is

$$\mathcal{L}_{\text{Scalar}} = |D_\mu \Phi|^2 + |D_\mu \Phi_d|^2 - V(\Phi, \Phi_d), \quad (5)$$

with the scalar potential

$$V(\Phi, \Phi_d) = -\mu_h^2 |\Phi|^2 - \mu_{h_d}^2 |\Phi_d|^2 + \lambda_h |\Phi|^4 + \lambda_{h_d} |\Phi_d|^4 + \lambda_{hh_d} |\Phi|^2 |\Phi_d|^2. \quad (6)$$

The symmetry breaking condition and boundedness of the potential require $\mu_h^2, \mu_{h_d}^2 > 0$ and $(\lambda_h \lambda_{h_d} - \lambda_{hh_d}^2), \lambda_h, \lambda_{h_d} > 0$. The field configurations of Φ and Φ_d at the minima of the scalar potential in unitary gauge is given by

$$\Phi = \begin{pmatrix} 0 \\ \frac{v_{EW} + h_1}{\sqrt{2}} \end{pmatrix}, \quad \Phi_d = \frac{1}{\sqrt{2}}(v_d + h_2). \quad (7)$$

where h_1 and h_2 denote the fluctuation around the minimum field configurations and v_{EW} and v_d represent the VEV of the respective scalar fields. We denote the fields in the mass basis by h and h_d which correspond to the observed Higgs and the hypothetical dark Higgs fields, respectively. The physical scalar fields (h, h_d) are related to the (h_1, h_2) by the following unitary transformation

$$\begin{pmatrix} h \\ h_d \end{pmatrix} = \begin{pmatrix} \cos \theta_s & -\sin \theta_s \\ \sin \theta_s & \cos \theta_s \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \quad (8)$$

Here, θ_s is the scalar mixing angle.

The expression for $\mathcal{L}_{\text{Fermion}}$ and $\mathcal{L}_{\text{Yukawa}}$ appearing in Eq. (1) involving second generation leptons and pVLL

consistent with gauge symmetry are given by

$$\mathcal{L}_{\text{Fermions}} \supset i[\bar{L}_L \not{D} L_L + \bar{\mu}'_R \not{D} \mu'_R + \bar{\mu}'_p \not{D} \mu'_p] - m_{\mu'_p} \bar{\mu}'_p \mu'_p. \quad (9)$$

$$\mathcal{L}_{\text{Yukawa}} \supset -y_m \bar{L}_\mu \Phi \mu'_R - \omega_f \Phi_d \bar{\mu}'_{pL} \mu'_R + \text{H.c.} \quad (10)$$

The mixing between μ' and μ'_p is generated once the dark Higgs assumes nonzero VEV. After symmetry breaking, the mass matrix in the interaction basis μ' and μ'_p takes the form

$$\mathcal{M} = \begin{pmatrix} \frac{y_m v_{EW}}{\sqrt{2}} & 0 \\ \frac{\omega_f v_d}{\sqrt{2}} & m_{\mu'_p} \end{pmatrix} \quad (11)$$

The mass eigenbasis μ and μ_p in which the mass matrix is diagonal, represent the physical muon and the corresponding heavy muon partner, respectively. The mass eigenbasis and the interaction basis are connected by the following biunitary transformation,

$$\begin{pmatrix} \mu \\ \mu_p \end{pmatrix}_{L,R} = \begin{pmatrix} \cos \theta_{L,R} & -\sin \theta_{L,R} \\ \sin \theta_{L,R} & \cos \theta_{L,R} \end{pmatrix} \begin{pmatrix} \mu' \\ \mu'_p \end{pmatrix}_{L,R} \quad (12)$$

where, $\theta_{L,R}$ is the mixing angle between the left (right) chiral muon and the corresponding heavy muon counter part with the same chirality.

The model parameters $y_m, \omega_f, m_{\mu'_p}$ appearing in the Lagrangian of Eq. (9) and (10) can be expressed in terms of the physical parameters m_μ, m_{μ_p} and $\sin \theta_L$ as

$$y_m = \frac{\sqrt{2}}{v_{EW}} m_{\mu_p} D \quad (13a)$$

$$\omega_f = \frac{m_{\mu_p} (1 - \frac{m_\mu^2}{m_{\mu_p}^2})}{\sqrt{2} v_d D} \sin 2\theta_L \quad (13b)$$

$$m_{\mu'_p} = \frac{m_\mu}{D} \quad (13c)$$

where D is a dimensionless quantity given by $D = \sqrt{\frac{m_\mu^2}{m_{\mu_p}^2} \cos^2 \theta_L + \sin^2 \theta_L}$.

The absolute value of the sine of the mixing angle between the left chiral second generation charged lepton and the pVLL is expressed as

$$|\sin \theta_L| = \frac{1}{2} \sqrt{\frac{2m_{\mu_p}^2 - 2m_\mu^2 - \omega_f^2 v_d^2}{m_{\mu_p}^2 - m_\mu^2} \left(1 - \sqrt{1 - \frac{8\omega_f^2 v_d^2 m_\mu^2}{(2m_{\mu_p}^2 - 2m_\mu^2 - \omega_f^2 v_d^2)^2}} \right)} \quad (14)$$

The reality of $\sin \theta_L$ requires

$$|\omega_f| < \sqrt{2} \frac{(m_{\mu_p} - m_\mu)}{v_d} \quad (15)$$

On the other hand, the perturbative unitarity [56] demands

$$|\omega_f| < 4\sqrt{2}\pi \quad (16)$$

These two conditions can be integrated as

$$|\omega_f| < \sqrt{2} \min \left(\frac{m_{\mu_p} - m_\mu}{v_d}, 4\sqrt{\pi} \right) \quad (17)$$

The corresponding mixing angle ($\sin \theta_R$) between the right chiral muon and heavy muon is given by

$$\sin \theta_R = \frac{\sin \theta_L}{\sqrt{\frac{m_\mu^2}{m_{\mu_p}^2} \cos^2 \theta_L + \sin^2 \theta_L}} \quad (18)$$

It turns out that $\sin \theta_R \sim \mathcal{O}(0.1)$ for m_{μ_p} in the TeV range even for a tiny $\sin \theta_L$ ($\sim 10^{-5}$) and this nondecoupling effect plays a crucial role both in the context of dark matter and collider analysis.

This framework naturally gives rise to three portal interactions connecting the visible and dark sectors: the gauge kinetic mixing, Yukawa, and scalar portals. In particular, gauge kinetic mixing between the SM $U(1)_Y$ and dark $U(1)_D$ arises at one loop and is strongly constrained by low-energy precision measurements [53,57,58], with allowed values sensitive to the dark photon mass. The dark photon couples to the SM fermions exclusively via gauge kinetic mixing, except for muons, which also couple through the muon–heavy muon mixing. While the kinetic-mixing portal does not play any crucial role, the scalar and Yukawa portals significantly affect Higgs-associated production at a muon collider in this scenario.

The list of free parameters relevant for the discussions in the later sections are mass of the heavy muon (m_{μ_p}), mixing angle between the muon and its heavier counter part ($\sin \theta_L$), mass of the dark photon (m_{γ_d}), the dark gauge coupling (g_d), and the scalar mixing angle ($\sin \theta_s$) along with the SM parameters. Mass of the additional scalar (m_{h_d}) is an independent parameter of the model, however, its impact on our analysis is less significant unlike the scalar mixing angle ($\sin \theta_s$). The kinetic mixing parameter ϵ which will play a crucial role in the DM phenomenology along with the fermion mixing does not have significant impact on the collider analysis as long as its value is small enough to evade the existing constraints coming from low energy observations in the relevant dark photon mass range considered in our work.

A. Constraints

In this subsection we discuss various observational constraints on the allowed values of the relevant model parameters discussed above.

1. Collider constraints

Scalar search: The scalar mixing angle which simply plays a role of overall coupling rescaling factor in the analysis presented in this paper is constrained by various observations across the mass of the heavy scalar [59–65]. For scalar mass below 100 GeV there are constraints from the LEP experiment and invisible decay of the Higgs boson. For example, if $m_{h_d} < \frac{m_h}{2}$ the corresponding bound on the scalar mixing angle coming from the measurement of the Higgs to invisible final state at LHC is of the order of 0.01 for $v_d \sim 100$ GeV [66,67]. The LEP experiment suggests $\sin \theta_s$ in the range (0.9–0.998) is ruled out for m_{h_d} in the range 10–100 GeV and $v_d \sim v_{EW}$ [59–62,64]. For masses beyond 100 GeV the constraint coming from LHC experiment is more relevant. The constraint on $\sin \theta_s$ in the range 100 GeV $< m_{h_d} < 200$ GeV is primarily due to the Higgs signal strength measurement and $|\sin \theta_s| > 0.1$ is ruled out in this mass range [64] with $v_d \sim 0.1 \times v_{EW}$. For $m_{h_d} > 200$ GeV, $\sin \theta_s > 0.2$ is disfavored by the LHC data [62,68].

The search for resonant and nonresonant di-Higgs production at the LHC can give indirect constraint on the scalar sector parameter as well. The new physics effect can alter the di-Higgs cross section at the LHC either via modification of the trilinear Higgs coupling or via new resonances or both. The CMS observation of nonresonant di-Higgs process at 13 TeV LHC center of mass energy at an integrated luminosity of 138 fb⁻¹ sets 95% confidence level (CL) limit on modified trilinear Higgs coupling (λ_{hhh}). The modification to this coupling with respect to its SM value is parametrized by κ_λ and defined as

$$\kappa_\lambda = \frac{\lambda_{hhh}}{\lambda_{hhh}^{\text{SM}}} \quad (19)$$

where λ_{hhh} has the following form

$$\lambda_{hhh} = \frac{3m_h^2}{v_{EW}} \left(\cos^3 \theta_s - \frac{v_{EW}}{v_d} \sin^3 \theta_s \right). \quad (20)$$

The current 95% CL limit on κ_λ is $-1.39 < \kappa_\lambda < 7.02$ [69], where $\kappa_\lambda = 1$ corresponds to the SM prediction. This constraint on κ_λ implies a restriction on the choice of $\sin \theta_s$ as indicated by Eq. (20). In Fig. 1 we plot κ_λ vs $\sin \theta_s$ which suggests For $v_d > 100$ GeV, $-1 < \sin \theta_s < 0.85$ is consistent with the current LHC limit on κ_λ . The search for resonant di-Higgs production at the LHC at $\sqrt{s} = 13$ TeV by ATLAS collaboration [70] sets upper limit on the resonant di-Higgs cross section as a function of the mass

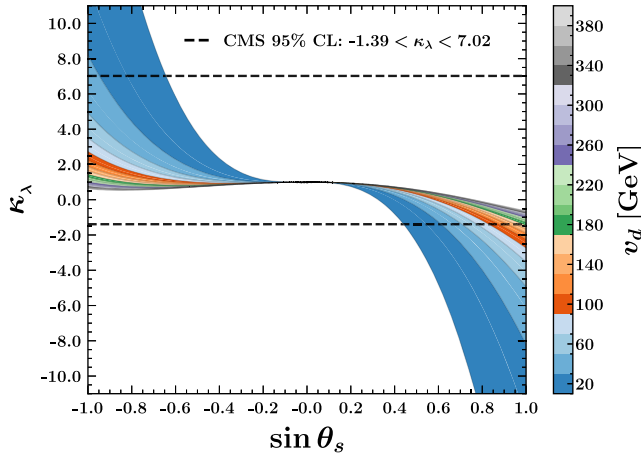


FIG. 1. Deviation of the triple Higgs coupling compared to its SM value as a function of $\sin \theta_s$ in the present framework. The color bar represents the range of the dark Higgs VEV considered above.

of the heavy resonance in the range 250 GeV–600 GeV at 95% CL limit. In Fig. 2 we plot both the observed 95% CL limit on the resonant di-Higgs cross section obtained by ATLAS collaboration [70] and that predicted by the present model as a function of the mass of the dark Higgs (m_{h_d}) for different choices of dark Higgs VEV. For $v_d = 100$ GeV and $\sin \theta_s = 0.2$, the allowed upper limit on the dark Higgs mass is around 325 GeV. This constraint will be relaxed further for higher v_d values and/or smaller $\sin \theta_s$.

Additionally, the scalar mixing angle is also sensitive to studies at the future collider experiments such as HL-LHC, ILC, CLIC, CEPC etc. The stringent bound on $\sin \theta_s$ comes from FCC-ee which suggests $\sin \theta_s < 0.032$ can be ruled out. The projected sensitivity on the scalar mixing angle in the context of rest of the future collider experiments listed above are all consistent with $\sin \theta_s < 0.045$ [71]. However, these projections assume 1σ deviations from SM predictions.

VLL search: Various extensions of the SM containing the VLLs have been probed at the LHC as well as at the LEP experiment. Due to the limited center-of-mass energy available at the LEP collider, VLLs with mass above 102.6 GeV [72] are not excluded by the LEP experiment. VLL decaying in the SM mode i.e., $\nu W, \ell Z$ has been probed at the LHC in the multilepton [34,73] and hadronically decaying tau lepton with additional jets and/or missing transverse energy channels [74,75]. The ATLAS experiment has searched for VLL mixing with first two generations of charged leptons and has obtained an exclusion limit up to 320 GeV (1220 GeV) on the allowed values of the singlet (doublet) like electron leptons [34]. The corresponding bound for VLL mixing with muons is 400 GeV (1270 GeV) obtained by same experiment if it is an $SU(2)_L$ singlet (doublet). The CMS experiment at the LHC sets a lower limit on the allowed value of the doublet (singlet)

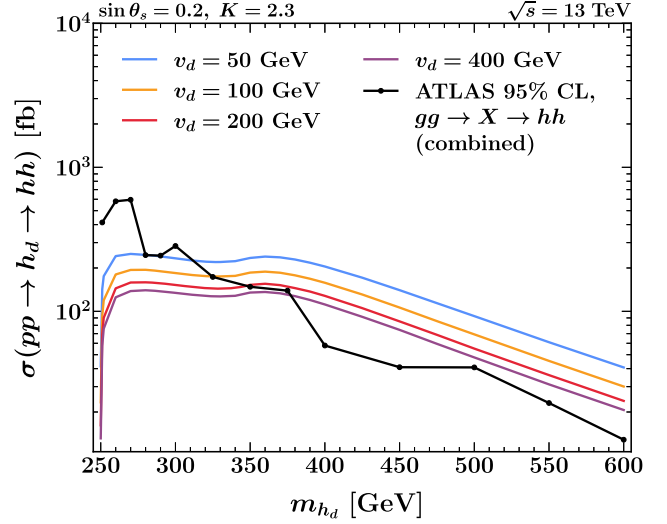


FIG. 2. Resonant di-Higgs cross section (in fb) as a function of the mass of the dark Higgs for various choices of its vacuum expectation value at $\sqrt{s} = 13$ TeV and $\sin \theta_s = 0.2$. The estimated cross section also assumes a K -factor of 2.3. The solid black line represents observed 95% CL limit on the resonant di-Higgs cross section by ATLAS experiment [76].

vectorlike tau lepton mass to be $\gtrsim 1045$ GeV (150 GeV) with 95% CL [73] using 138 fb^{-1} data collected during 2016–2018. The corresponding lower limit obtained by ATLAS experiment on the allowed value of mass of the tau VLL transforming as $SU(2)_L$ doublet is 900 GeV [74]. Moreover, these limits can be relaxed further if the VLLs are allowed to decay in the nonstandard modes such as pVLL decaying to a lepton and a dark photon as considered in the present model.

2. Lepton flavor universality constraint

In the SM, the universality of the gauge coupling ensures that the Z boson decays in the leptonic final states, in particular, in the final state with electron or muon with equal rate and the ratio of these two rates is predicted to be close to unity in the SM [19]. This has been verified in the LEP experiment at CERN and the observed value of the ratio of these two decay rates (R) is found to be [77]

$$R = \Gamma(Z \rightarrow \mu^+ \mu^-) / \Gamma(Z \rightarrow e^+ e^-) = 1.0009 \pm 0.0028 \quad (21)$$

The model considered in this work introduces a mixing between the second-generation charged lepton and the pVLL. As a consequence, the $Z - \mu - \mu$ vertex, in particular, the $Z - \mu_L^- - \mu_L^-$ interaction vertex is modified to $\frac{g_z}{2} (2 \sin^2 \theta_W - \cos^2 \theta_L)$ (See Eq. (A7) in Appendix) compared to that in the SM² while corresponding right-handed vertex factor remains the same due to the choice of our charge assignment.

²The corresponding factor for the SM $Z - \mu_L^- - \mu_L^-$ vertex is $\frac{g_z}{2} (2 \sin^2 \theta_W - 1)$.

Therefore, for large mixing one expects deviation in the predicted value of R inconsistent with the above observation. It turns out that, the lepton flavor universality constraint strongly disfavor $[\sin\theta_L > 0.030(0.047)]$ at 1σ (2σ) level [27].

3. Electroweak precision constraints

The presence of VLL has various observable consequences and can potentially modify the predictions of the SM. It can contribute to the modification of muon lifetime, the branching ratio of $Z \rightarrow \mu^+\mu^-$, total width of the Z and

W bosons and correction to the oblique parameters etc. Since the VLL carries electromagnetic charge it can modify the Higgs to diphoton decay rate as well. In our model, the pVLL can couple to the Higgs via muon-heavy muon mixing and/or Higgs-dark Higgs mixing. Therefore, the corresponding interaction vertices will be proportional to $\sim(y_m \cos\theta_R \cos\theta_L \cos\theta_s + \omega_f \cos\theta_R \sin\theta_L \sin\theta_s)$. Hence, for small $\sin\theta_L$ and $\sin\theta_s$ the deviation from the SM $h - \mu - \mu$ vertex factor is less significant. The correction to the oblique parameters, in particular, S and T due to the singlet VLL is expressed as [78]

$$\Delta T = \frac{m_\mu^2}{16\pi\sin_W^2 m_W^2} \sin^2\theta_L \left[-(1 + \cos^2\theta_L) + 2\cos^2\theta_L \frac{r_\mu}{r_\mu - 1} \log(r_\mu) + r_\mu \sin^2\theta_L \right] \quad (22)$$

$$\Delta S = \frac{1}{18\pi} \sin^2\theta_L \left[\log(r_\mu) + \cos^2\theta_L \left(\frac{5(r_\mu^2 + 1) - 22r_\mu}{(1 - r_\mu)^2} + \frac{3(r_\mu + 1)(r_\mu^2 - 4r_\mu + 1)}{(1 - r_\mu)^3} \log(r_\mu) \right) \right] \quad (23)$$

where, $r_\mu = m_{\mu_p}/m_\mu$.

The updated global fit to the electroweak precision observables tells that these corrections are constrained as follows [79]:

$$S = -0.05 \pm 0.07 \quad (24)$$

$$T = 0.00 \pm 0.06 \quad (25)$$

4. Muon $g-2$

The presence of muon-heavy muon-dark photon vertex can give leading contribution to the anomalous muon $g-2$ at one-loop in this pVLL framework. In the limit $m_{\mu_p} \approx m_{\mu'}$ and $m_\mu \approx m_{\mu'} = \frac{1}{\sqrt{2}} y_m v_{EW}$, this contribution is given by [21,27]

$$\Delta a_\mu \approx \begin{cases} \frac{g_d^2 m_{\mu_p} m_\mu}{16\pi^2 m_{\gamma_d}^2} \sin 2\theta_R \sin 2\theta_L, & m_{\mu_p} \gg m_{\gamma_d} \\ \frac{g_d^2 m_{\mu'} m_\mu}{4\pi^2 m_{\gamma_d}^2} \sin 2\theta_R \sin 2\theta_L, & m_\mu \ll m_{\mu_p} \ll m_{\gamma_d}. \end{cases} \quad (26)$$

The current world average value of $\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$, combining Fermilab Run 1–6 data [80] with the results of Brookhaven experiment [81] is

$$\Delta a_\mu = 38(63) \times 10^{-11} \quad (27)$$

which corresponds to 0.6σ deviation compared to the SM prediction ($a_\mu^{\text{SM}} = 0.00116592033(62)$ [82]).

In Fig. 3, we show the muon $g-2$ allowed region of parameter space both in the $m_{\gamma_d} - g_d$ and $m_{\mu_p} - m_{\gamma_d}$ planes

consistent with Δa_μ mentioned in Eq. (26) at 1σ level. Figure 3(a) corresponds to $\sin\theta_L = 4 \times 10^{-5}$ and four different heavy muon masses (1 TeV, 1.5 TeV, 2 TeV and 3 TeV), whereas Fig. 3(b) corresponds to two distinct sets of $(g_d, \sin\theta_L)$ with values $(0.05, 4 \times 10^{-5})$ and $(0.5, 1.66 \times 10^{-6})$.

In the remainder of this paper, we work with benchmark points consistent with all the constraints mentioned above.

III. DM RELIC DENSITY

In this section, we present the estimation of DM relic density assuming a viable DM candidate to demonstrate that the simplified pVLL scenario can naturally accommodate the observed DM relic density even when the kinetic mixing parameter (ϵ) assumes tiny values or essentially absent due to strong constraints coming from low-energy observations and DM direct-detection experiments.

The observed dark matter relic density reported by the Planck 2018 collaboration suggests [83]

$$\Omega h^2 = 0.120 \pm 0.001$$

We augment the simplified model introduced earlier with a Dirac fermion (χ) charged under the local $U(1)_D$ and neutral under SM gauge interaction as a potential DM candidate to account for the observed DM relic abundance. The Lagrangian presented in Eq. (1) is extended by a DM sector ($\mathcal{L}_{DM}$), and is expressed as

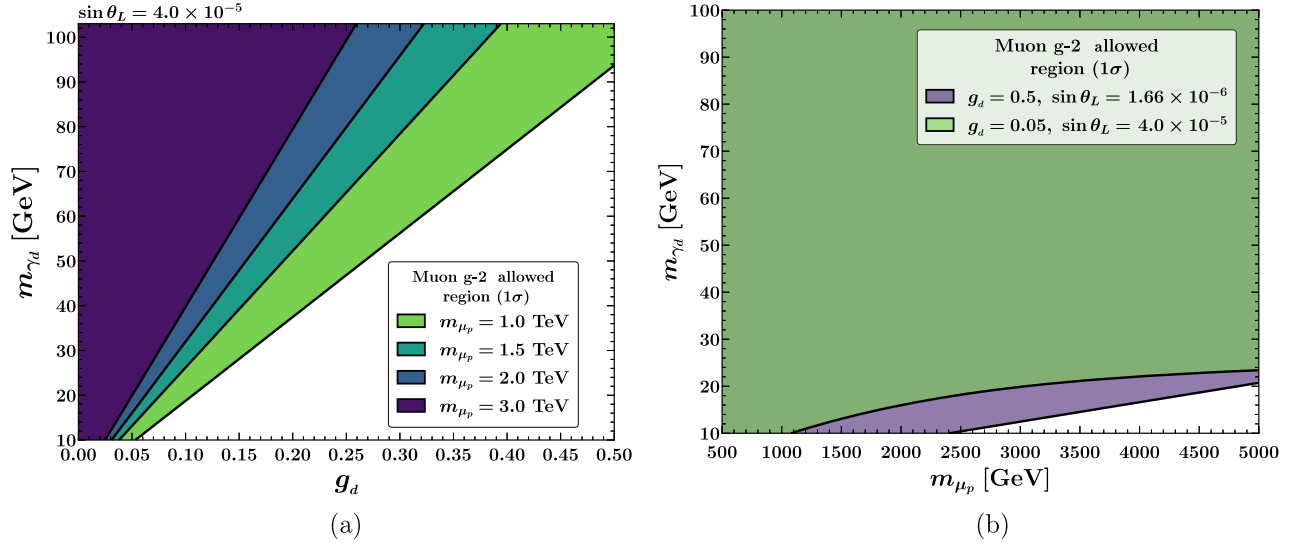


FIG. 3. Region (colored) of parameter space allowed by muon $g-2$ measurement at 1σ level [82] in the (a) $m_{\gamma_d} - g_d$ plane for $\sin \theta_L = 4 \times 10^{-5}$ and four different m_{μ_p} values (1 TeV, 1.5 TeV, 2 TeV and 3 TeV) and (b) $m_{\mu_p} - m_{\gamma_d}$ plane for two sets of $(g_d, \sin \theta_L)$.

$$\mathcal{L}_{\text{DM}} = i\bar{\chi}\gamma^\mu D_\mu\chi - m_\chi\bar{\chi}\chi \quad (28)$$

In absence of the pVLL state, the DM annihilation mainly proceeds via the following annihilation channels:

$$\chi\bar{\chi} \rightarrow f\bar{f}, \quad \chi\bar{\chi} \rightarrow VV, \quad \chi\bar{\chi} \rightarrow VS$$

where f represent the SM fermions, $V = Z, \gamma_d$ and $S = h, h_d$. The corresponding Feynman diagrams are depicted in Fig. 4. For light DM in the mass regime $m_f < m_\chi < m_{\gamma_d}$, the dominant annihilation channel is $\chi\bar{\chi} \rightarrow f\bar{f}$ while for $m_\chi > m_{\gamma_d}$ or $2m_\chi > m_{\gamma_d} + m_S$ either or both of the remaining two channels dominate. The annihilation rate for $\chi\bar{\chi} \rightarrow f\bar{f}$ via s-channel Z/γ_d mediation scales as $g_d^2 \epsilon^2 m_\chi^2 / m_V^4$ away from resonance, and are typically suppressed due to the small gauge kinetic mixing, resulting in an overabundance of DM. The presence of a

strong first order phase transition (SFOPT) in the dark sector may overcome this issue, as suggested in recent studies [84,85].

In the present model, the issue of an overabundant DM relic density for light DM scenario, can be addressed through the portal vectorlike lepton state without invoking the requirement of a SFOPT. Moreover, this framework can evade the direct detection bound, as long as the gauge kinetic mixing is small. The role of pVLL state exclusively mixing with the second generation SM charged lepton is to *enhance* the annihilation rate $\chi\bar{\chi} \rightarrow \mu^+\mu^-$ for light DM away from resonance, through the modification of the $\mu^+ - \mu^- - \gamma_d$ vertex without introducing additional Feynman diagrams. The other two channels ($\chi\bar{\chi} \rightarrow \gamma_d\gamma_d, \gamma_d h_d$) which are dominant in the mass region $m_\chi > m_{\gamma_d}$ or $2m_\chi > m_{\gamma_d} + m_{h_d}$, on the other hand are controlled by the dark gauge coupling and remain unaffected due to the presence of the pVLL.

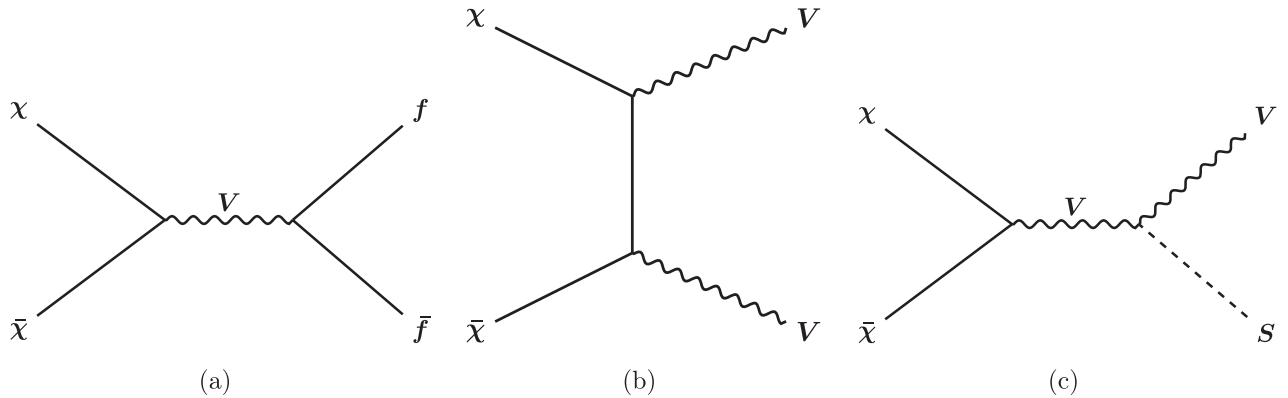


FIG. 4. Feynman diagrams for DM-DM annihilation processes. Here, $V = Z, \gamma_d$ and $S = h, h_d$.

In presence of the portal matter state, the dark photon coupling to the muon receives an additional contribution proportional to $g_d \sin^2 \theta_{R/L}$ beyond the usual vertex factor arising from the KM alone, while the couplings to all other SM fermions remain unaltered. The modified $\mu^+ - \mu^- - \gamma_d$ vertex has the following structure:

$$\begin{aligned} \mu_R^+ - \mu_L^- - \gamma_d &\sim \frac{i \cos \theta_d}{\sqrt{1 - \varepsilon^2}} \left[\frac{e\varepsilon}{\cos \theta_w} - \frac{e\varepsilon \cos^2 \theta_L}{2 \cos \theta_w} + g_d \sin^2 \theta_L \right] + ie \sin \theta_d \left[\frac{\cos^2 \theta_L}{2 \sin \theta_w \cos \theta_w} - \frac{\sin \theta_w}{\cos \theta_w} \right] \\ \mu_L^+ - \mu_R^- - \gamma_d &\sim \frac{i \cos \theta_d}{\sqrt{1 - \varepsilon^2}} \left[\frac{e\varepsilon}{\cos \theta_w} + g_d \sin^2 \theta_R \right] - ie \sin \theta_d \frac{\sin \theta_w}{\cos \theta_w} \end{aligned}$$

As a result, in this scenario $\chi\bar{\chi} \rightarrow \mu^+\mu^-$ can be a dominant annihilation channel for light DM ($m_\mu m_\chi < m_{\gamma_d}$) even for a small KM ($\varepsilon < 10^{-3}$) away from resonance. Importantly, this channel does not suffer from strong suppression factors, since $\sin \theta_R$ can be sizable (~ 0.1 to 0.3) even when $\sin \theta_L$ is small as already pointed out—thanks to the same nondecoupling effect.

We have implemented the full Lagrangian ($\mathcal{L} + \mathcal{L}_{\text{DM}}$) in the FeynRules2.0 [86] incorporating gauge kinetic, scalar and fermion mixings and interfaced the resulting model files with the MICRomegas [87] to estimate the DM relic density. In Fig. 5, we plot the relic density as a function of the DM mass (m_χ) in the range 10–130 GeV for two different choices of dark photon masses ($m_{\gamma_d} = 50$ GeV and 90 GeV) and three

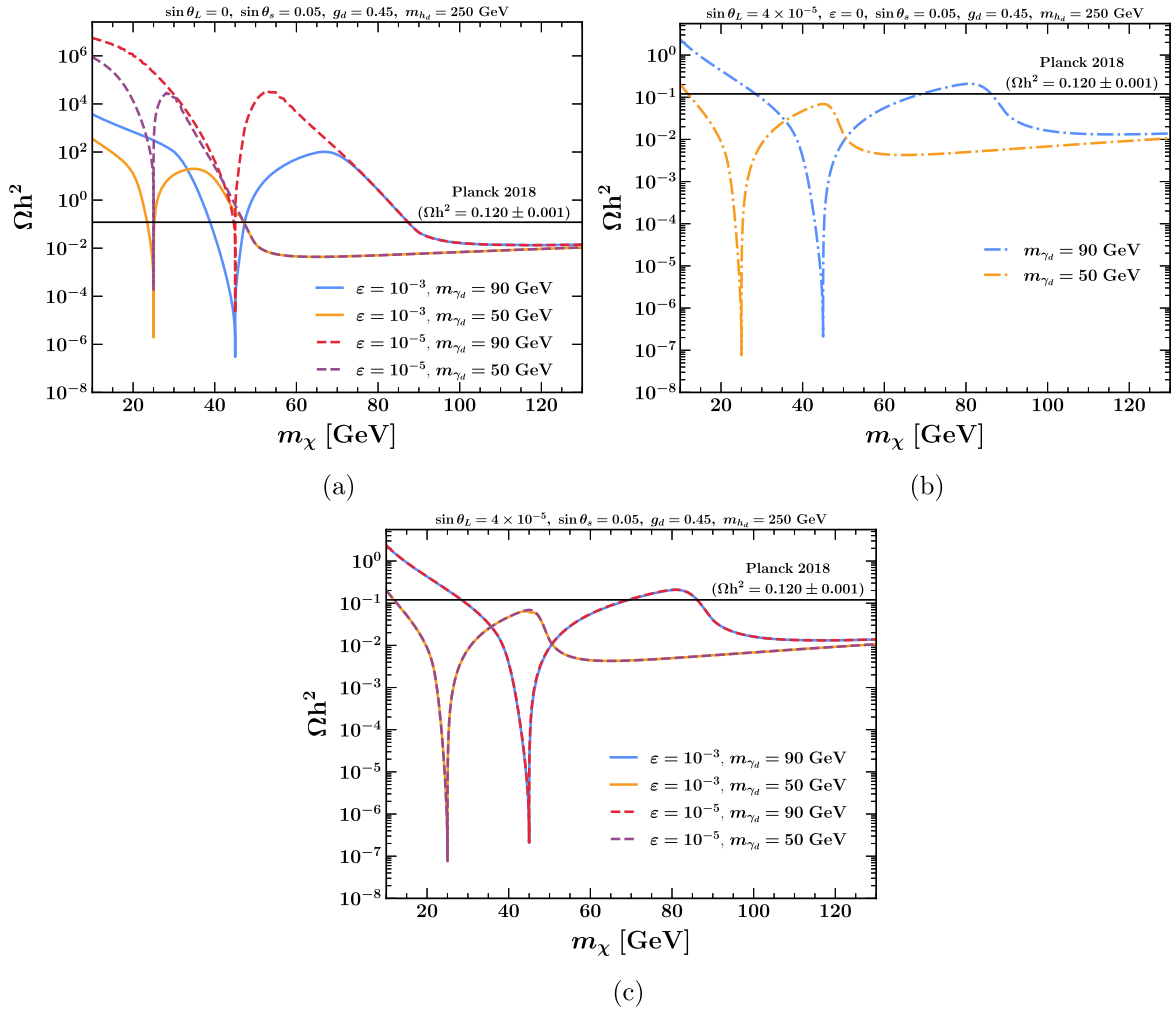


FIG. 5. Dark matter relic abundance as a function of dark matter mass for (a) $\varepsilon = 10^{-3}, 10^{-5}$, $\sin \theta_L = 0$, (b) $\varepsilon = 0$, $\sin \theta_L = 4 \times 10^{-5}$, and (c) $\varepsilon = 10^{-3}, 10^{-5}$, $\sin \theta_L = 4 \times 10^{-5}$ for two different dark photon masses (50 GeV and 90 GeV) assuming $g_d = 0.45$, $\sin \theta_s = 0.05$ and $m_{\mu_p} = 1$ TeV.

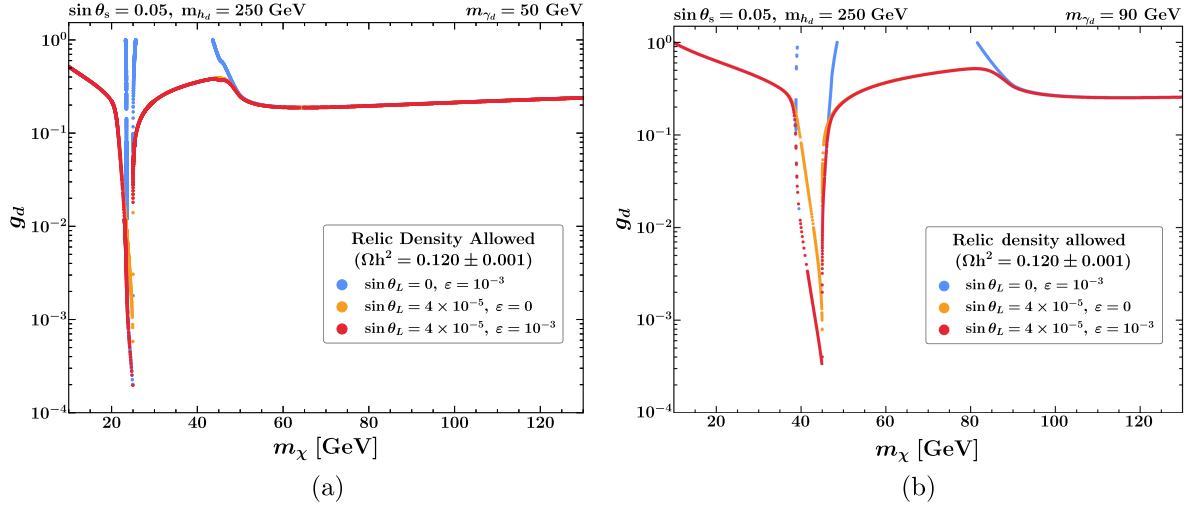


FIG. 6. Dark matter relic abundance allowed parameter space in the g_d vs m_χ plane for various combinations of $\sin \theta_L$ and ϵ assuming $\sin \theta_s = 0.05$ and $m_{\mu_p} = 1$ TeV. The figure on the left panel corresponds to $m_{\gamma_d} = 50$ GeV and that on the right panel considers $m_{\gamma_d} = 90$ GeV.

different sets of ϵ and $\sin \theta_L$: (a) $\epsilon = 10^{-3}, 10^{-5}$; $\sin \theta_L = 0$, (b) $\epsilon = 0$; $\sin \theta_L = 4 \times 10^{-5}$, and (c) $\epsilon = 10^{-3}, 10^{-5}$; $\sin \theta_L = 4 \times 10^{-5}$.

In absence of the pVLL state, for light dark matter ($m_\chi < m_{\gamma_d}$ or $2m_\chi < m_{\gamma_d} + m_{h/h_d}$), the relative annihilation rate to individual SM fermion modes ($f = e, \mu, \tau, \nu, d, u, s, c, b$) are predicted by their respective SM gauge charge and color factor. One needs $\epsilon \sim 10^{-2}$ or higher for $g_d \sim 0.45$ to achieve underabundant DM relic density away from resonance. The value of the KM parameter in this range is not only strongly constrained by the various low energy experiments in the context of dark photon searches depending on the mass of the dark photon but is also extremely sensitive to the DM direct detection experiments, as we will discuss later. For $m_\chi > m_{\gamma_d}$ the predicted DM relic abundance is largely independent of the KM and a dark gauge coupling of the order of 0.45 typically results in an underabundant DM relic density [Fig. 5(a)]. In presence of the pVLL, assuming no [Fig. 5(b)] or small [Fig. 5(c)] KM, the relic abundance in the light DM regime away from resonance are primarily controlled by the fermion mixing and dark gauge coupling. At the resonance point the relic abundance depends on the interplay among KM, fermion mixing and dark gauge coupling. In the mass regime where $\chi\bar{\chi} \rightarrow \gamma_d \gamma_d$, or $\gamma_d h_d$ is dominant DM relic abundance remains essentially unaltered in the presence of pVLL.

To identify the DM relic abundance compatible parameter space in the DM mass range 10–130 GeV, we perform a parameter space scan in the g_d vs m_χ plane for various combinations of fermion and gauge kinetic mixings. This is depicted in Fig. 6 for two representative values of dark photon mass ($m_{\gamma_d} = 50$ GeV and 90 GeV), kinetic mixing parameter ($\epsilon = 0, 10^{-3}$) and fermion mixing angle

($\sin \theta_L = 0, 4 \times 10^{-5}$). We keep the scalar mixing angle ($\sin \theta_s$), masses of the heavy scalar (m_{h_d}) and the heavy muon (m_{μ_p}) fixed at 0.05, 250 GeV and 1 TeV, respectively, throughout the DM analysis.

Direct detection: The value of the KM parameter is strongly constrained by several low-energy observations related to dark-photon searches [53,57]. These observations typically set an upper bound, $\epsilon \lesssim 10^{-3}$, on the allowed values of the KM parameter for dark-photon masses in the range 10–100 GeV. However, if the dark photon couples to a DM particle, the most stringent constraints on the KM parameter arise from direct-detection experiments. In Fig. 7, we display the observed upper limits on the DM–nucleon scattering cross section as a function of the DM mass in the range 10 GeV–10 TeV, obtained from several direct-detection experiments, such as XENON1T (2018) [88], XENONnT (2025) [89], and LZ (2025) [90]. We also plot the theoretical prediction for the DM–nucleon scattering cross section in the same mass range for different choices of the KM parameter and the dark-photon mass, in the absence of any pVLF state.

The strongest bound on the KM parameter comes from the LZ (2025) experiment which suggests for $g_d = 0.45$ and dark photon mass, $m_{\gamma_d} > 90$ GeV, $\epsilon \lesssim 10^{-5}$ can remain allowed over a wide range of DM masses. This provides a strong motivation to consider DM models with portal vectorlike leptons. Their effect in the context of DM relic abundance can enter at tree-level if mixing with one or more SM fermion is assumed, whereas in the context of direct detection their contribution arises via vacuum polarization diagrams at one loop. Consequently, the direct-detection cross section becomes highly model dependent and sensitive to the presence of pVLF states, especially their charge assignments and the number of such portal

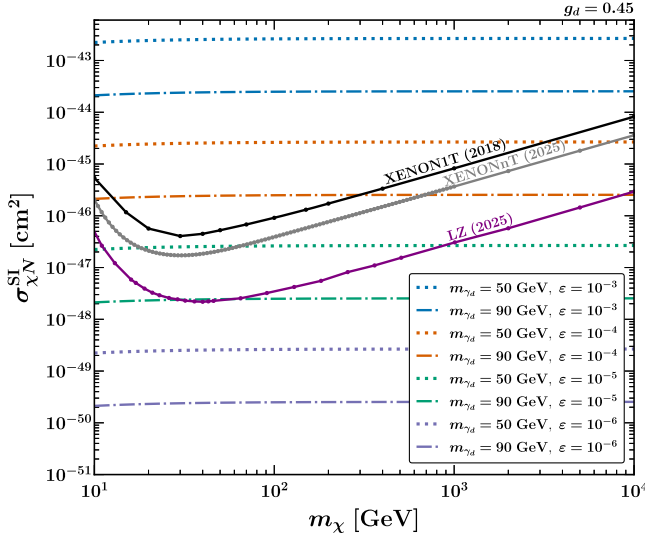


FIG. 7. Observed or -1σ power constrained upper limits (solid lines), as applicable, on the spin-independent direct detection cross sections as a function of the dark matter mass from XENON1T 2018 (black) [88], XENONnT 2025 (gray) [89] and LZ 2025 (purple) [90] experiments. The dotted and dot-dashed lines represent the theoretical prediction of spin-independent DM-nucleon scattering cross sections as a function of the dark matter mass in presence of the kinetic mixing alone ($\epsilon = 10^{-3}, 10^{-4}, 10^{-5}$ and 10^{-6}) for two different choices of the dark photon masses $m_{\gamma_d} = 50$ GeV and 90 GeV assuming $g_d = 0.45$.

VLF states [44], and therefore on the detailed structure of the dark-sector (DS) physics.

However, in this work we restrict ourselves to a simplified framework, focusing primarily on the implications of a single pVLF state mixing with the SM muon for associated Higgs production at a muon collider. As long as only one of the pVLF states is allowed to mix with the SM second generation charged lepton, the collider consequences presented in the latter section remain unaltered. In the remaining part of this work we also ignore the KM as its effect in the collider analysis is essentially insignificant.

IV. COLLIDER ANALYSIS

In this section, we demonstrate the effect of the portal vectorlike lepton on the Higgs production in association with a vector boson ($V = Z, \gamma_d$) at future muon collider. We first estimate the effect of pVLL on the production rate of $\mu^+\mu^- \rightarrow hV$ and compare this with respect to the SM prediction of $\mu^+\mu^- \rightarrow hZ$. The Feynman diagrams for $\mu^+\mu^- \rightarrow hV$ process are depicted in Fig. 8. In order to generate the events and estimate the cross section, we use the universal FeynRules output [91], obtained from the implementation mentioned earlier in Sec. III, and interface it with the event generator MadGraph5_aMC@NLO (version 3.5.5) [92]. We then compute the leading order (LO) cross

TABLE II. Representative benchmark points for collider analysis.

BPs	m_{μ_p} [TeV]	$\sin \theta_L$	g_d	$\sin \theta_s$	m_{γ_d} [GeV]
BP1	1	4.0×10^{-5}	0.45	0.05	90
BP2	2	2.0×10^{-5}	0.36	0.05	90
BP3	3	1.7×10^{-6}	0.50	0.05	90
BP4	1	4.0×10^{-5}	0.36	0.05	50
BP5	2	2.0×10^{-5}	0.30	0.05	50
BP6	3	1.7×10^{-6}	0.45	0.05	50

section in the context of future muon collider assuming center-of-mass energies 3 TeV and 10 TeV.

We have chosen six benchmark points (BPs) with parameter values³ consistent with the constraints mentioned in Sec. II A, as shown in Table II.

A. hZ vs $h\gamma_d$ at muon collider

Here we present a comparison between $\mu^+\mu^- \rightarrow h\gamma_d$ and $\mu^+\mu^- \rightarrow hZ$ production rates in the context of the model described in Sec. II with respect to the SM hZ production rate at future muon collider. In Fig. 9, we depict various ratios of cross sections defined in Eq. (29) for three different benchmark points.

$$R_1 = \frac{\sigma_{\text{BSM}}(\mu^+\mu^- \rightarrow hZ)}{\sigma_{\text{SM}}(\mu^+\mu^- \rightarrow hZ)} \quad (29a)$$

$$R_2 = \frac{\sigma_{\text{BSM}}(\mu^+\mu^- \rightarrow h\gamma_d)}{\sigma_{\text{SM}}(\mu^+\mu^- \rightarrow hZ)} \quad (29b)$$

$$R_3 = \frac{\sigma_{\text{BSM}}(\mu_R^+\mu_L^- \rightarrow h\gamma_d)}{\sigma_{\text{SM}}(\mu^+\mu^- \rightarrow hZ)} \quad (29c)$$

$$R_4 = \frac{\sigma_{\text{BSM}}(\mu_L^+\mu_R^- \rightarrow h\gamma_d)}{\sigma_{\text{SM}}(\mu^+\mu^- \rightarrow hZ)} \quad (29d)$$

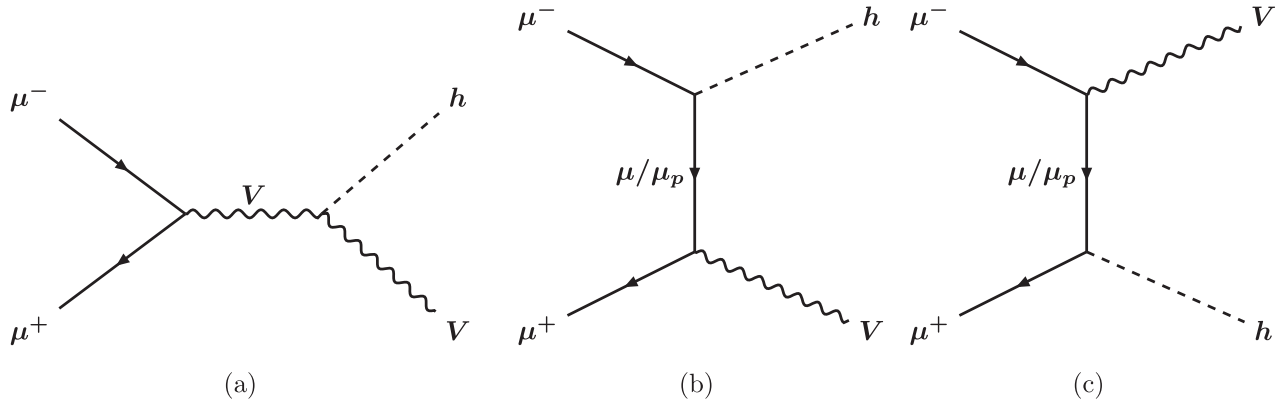
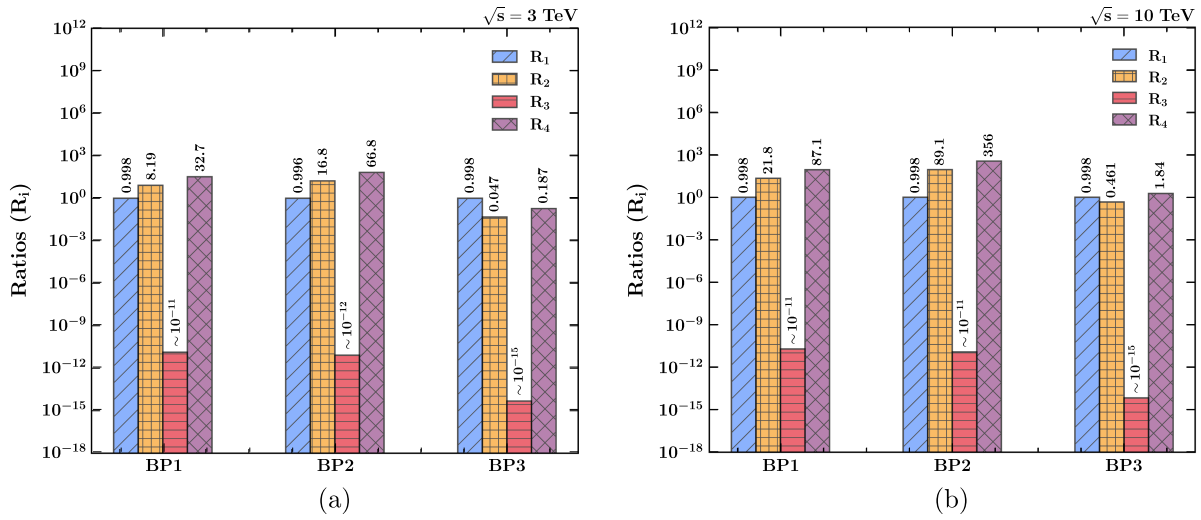
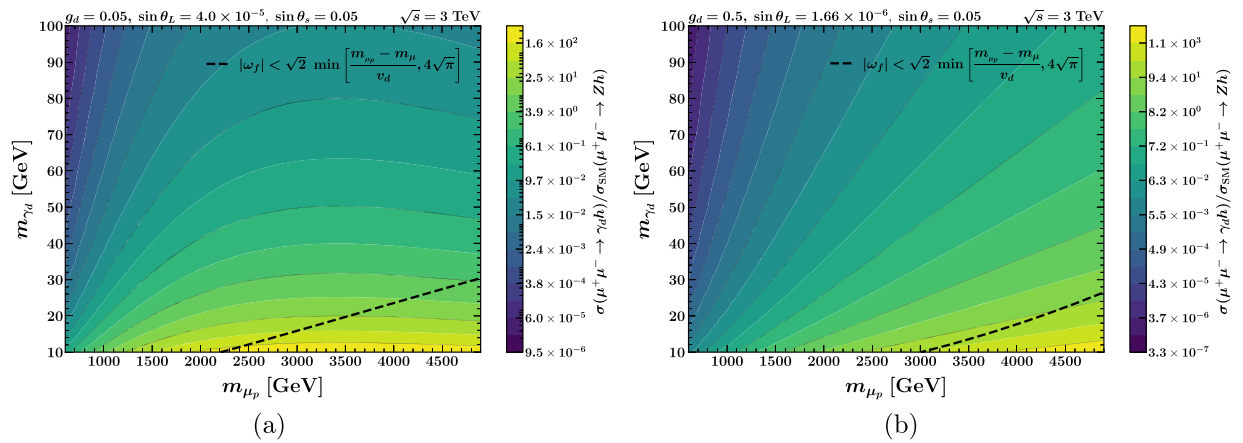
where,

$$\sigma_{\text{SM}}(\mu^+\mu^- \rightarrow hZ) = \begin{cases} 1.36 \text{ fb,} & \text{for } \sqrt{s} = 3 \text{ TeV} \\ 0.12 \text{ fb,} & \text{for } \sqrt{s} = 10 \text{ TeV} \end{cases} \quad (30)$$

One can see that while the cross section for $\mu^+\mu^- \rightarrow hZ$ production in this scenario remains the same as that in the SM, a higher Higgs production rate is predicted in the $\mu^+\mu^- \rightarrow h\gamma_d$ channel. This is true even for a $\sin \theta_L$ as small⁴

³For the collider analysis, the kinetic mixing parameter (ϵ) is set to zero without loss of generality, as it does not affect the estimated cross sections within its allowed range of values for the range of dark photon masses considered in this work.

⁴Small $\sin \theta_L$ ($\sim 10^{-4}$ or less) is favored by perturbative unitarity requirements for heavy muon mass beyond TeV scale

FIG. 8. Feynman diagrams for associated production of Higgs and a vector boson ($V = Z, \gamma_d$) at muon collider ($\mu^+\mu^- \rightarrow hV$).FIG. 9. Ratios of cross sections [R_i 's defined in Eq. (29)] for different benchmark points (BP1–BP3) corresponding to dark photon mass 90 GeV at (a) $\sqrt{s} = 3$ TeV and (b) $\sqrt{s} = 10$ TeV.FIG. 10. Variation of ratio of cross sections (R_2) for the process $\mu^+\mu^- \rightarrow h\gamma_d$ with respect to the SM hZ production rate in $m_{\gamma_d} - m_{\mu_p}$ plane assuming $\sqrt{s} = 3$ TeV for two different set of $(g_d, \sin \theta_L)$ and a fixed value of $\sin \theta_s$. (a) The left plot corresponds to $g_d = 0.05$, $\sin \theta_s = 0.05$, and $\sin \theta_L = 4 \times 10^{-5}$ and (b) the right plot corresponds to $g_d = 0.5$, $\sin \theta_s = 0.05$, and $\sin \theta_L = 1.66 \times 10^{-6}$. The region above the dashed (black) line is allowed by the constraint on ω_f described in Eq. (17).

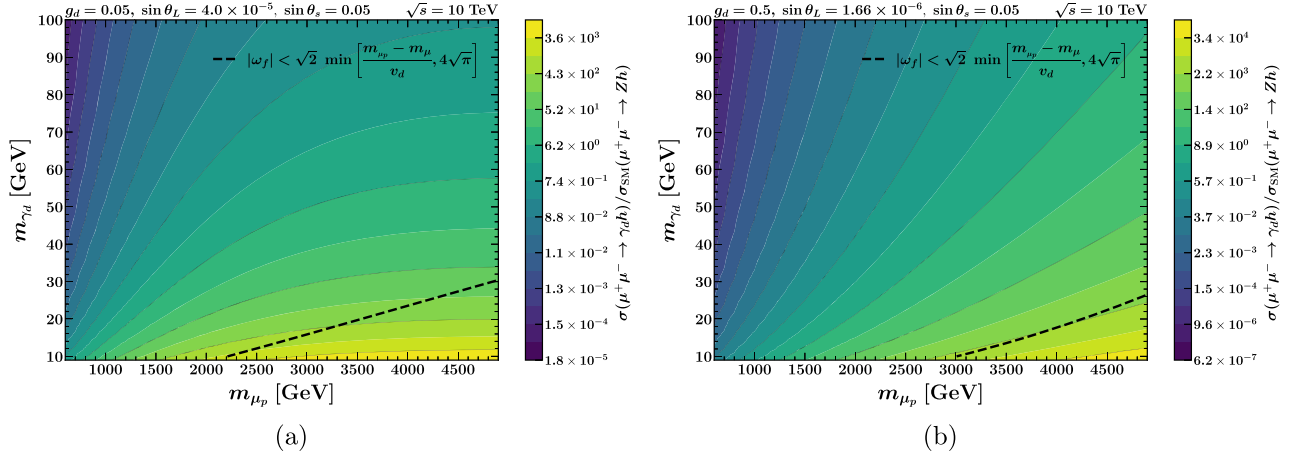


FIG. 11. Variation of ratio of cross sections (R_2) for the process $\mu^+\mu^- \rightarrow h\gamma_d$ with respect to the SM hZ production rate in $m_{\gamma_d}-m_{\mu_p}$ plane assuming $\sqrt{s} = 10$ TeV for two different set of $(g_d, \sin\theta_L)$ and a fixed value of $\sin\theta_s$. (a) The left plot corresponds to $g_d = 0.05$, $\sin\theta_s = 0.05$, and $\sin\theta_L = 4 \times 10^{-5}$ and (b) the right plot corresponds to $g_d = 0.5$, $\sin\theta_s = 0.05$, and $\sin\theta_L = 1.66 \times 10^{-6}$. The region above the dashed (black) line is allowed by the constraint on ω_f described in Eq. (17).

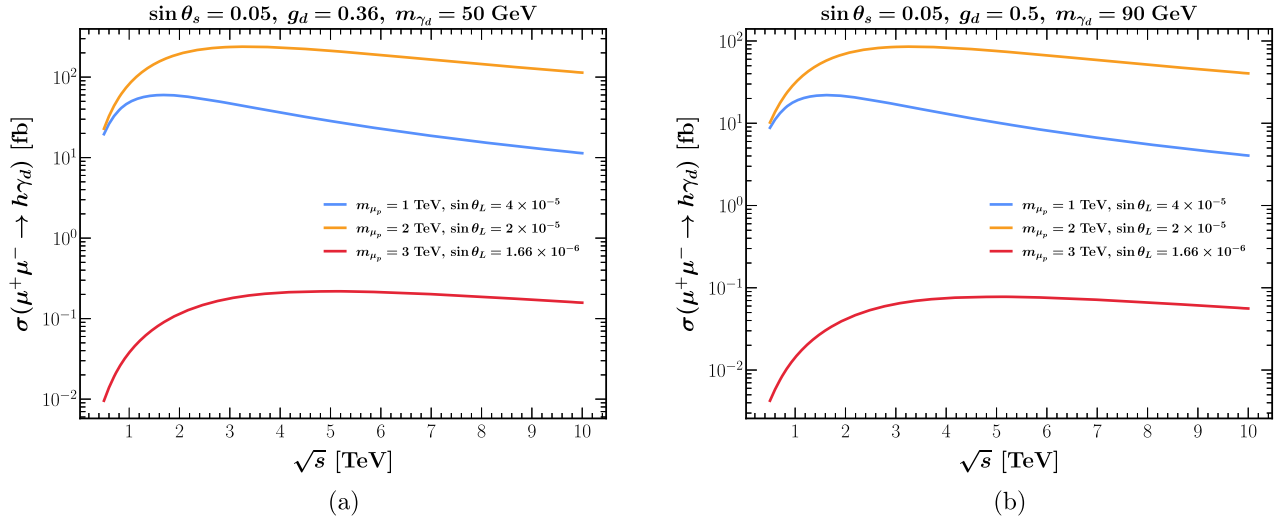


FIG. 12. LO cross sections (in fb) for $\mu^+\mu^- \rightarrow h\gamma_d$ as a function of the muon collider center-of-mass energy for various choices of the m_{μ_p} and $\sin\theta_L$ assuming (a) $\sin\theta_s = 0.05$, $g_d = 0.36$, $m_{\gamma_d} = 50$ GeV and (b) $\sin\theta_s = 0.05$, $g_d = 0.50$, $m_{\gamma_d} = 90$ GeV.

as $\sim 10^{-5}$ with reasonable choices of other relevant parameters (see Table II for specification of benchmark points). The underlying reasons behind this is that μ'_R and μ'_{p_R} share identical $SU(2)_L \times U(1)_Y$ charges. Hence, after transforming them into mass eigenbasis using Eq. (12) no off-diagonal $\mu_R^- - \mu_{p_R}^+ - Z$ interaction arises.⁵ However, the absence of $U(1)_D$ charge for the muon induces an off-diagonal $\mu_R^- - \mu_{p_R}^+ - \gamma_d$ interaction in the mass basis. It turns out that the corresponding interaction vertex $\mu_L^+ - \mu_{p_R}^- - \gamma_d$ (or $\mu_R^- - \mu_{p_R}^+ - \gamma_d$) is proportional to $\sim g_d \sin\theta_R \cos\theta_R$ [See Eq. (A2)] which therefore controls

the $h\gamma_d$ production rate due to sizable values of $\sin\theta_R$ as mentioned earlier. This can also be independently understood by the result that the polarized cross section $\mu_L^+ \mu_R^- \rightarrow h\gamma_d$ constitutes the dominant part of the average unpolarized cross section for $\mu^+\mu^- \rightarrow h\gamma_d$. Note that $\mu_L^+ \mu_R^- \rightarrow h\gamma_d$ gets contributions from t - and u - channel heavy muon mediated diagrams as well as that from s -channel dark photon mediation. The latter contribution is *suppressed* due to the fact that $\mu_L^+ - \mu_R^- - \gamma_d$ vertex is $\sim g_d \sin^2\theta_R$ in addition to the usual s -channel suppression at high center-of-mass energy.

In Figs. 10 and 11, we plot R_2 [defined in Eq. (29)] which represents the ratio of average unpolarized cross section for $\mu^+\mu^- \rightarrow h\gamma_d$ to that of the SM $\mu^+\mu^- \rightarrow hZ$ in

⁵This is actually true in the limit of zero kinetic mixing.

TABLE III. LO cross sections (in fb) for $\mu^+\mu^- \rightarrow h\gamma_d$ at $\sqrt{s} = 3$ TeV and 10 TeV for various choices of our benchmark points.

$\sqrt{s}$ [TeV]	$\sigma(\mu^+\mu^- \rightarrow h\gamma_d)$ [fb]					
	BP1	BP2	BP3	BP4	BP5	BP6
3	11.1	22.8	0.064	47.2	114	0.436
10	2.66	10.9	0.056	11.4	55.0	0.388

the $m_{\gamma_d}-m_{\mu_p}$ plane at $\sqrt{s} = 3$ TeV and $\sqrt{s} = 10$ TeV, respectively, to illustrate the effect of these two parameters on R_2 and hence the $h\gamma_d$ cross section for two different g_d and $\sin\theta_L$ and fixed $\sin\theta_s$. The perturbative unitarity limit obtained using Eq. (17) is also displayed on the same plot. Since the requirement of perturbative unitarity disfavors low values of v_d , one cannot achieve an arbitrary high values of R_2 by going to lower values of dark photon mass or higher values of g_d and a trade off is maintained in the choice of the two independent parameters m_{γ_d} and g_d .

Therefore, depending on the parameter space, Higgs production in association with a dark photon at muon collider can provide us a Higgs production rate higher than hZ production by a factor ranging from 1 to 100.

B. Analysis of $h\gamma_d$ process

In the remaining part of this article, we focus on the $h\gamma_d$ production at the future muon collider. As discussed above, the rate of this process can be significantly large compared to the hZ production in a wide range of dark photon mass. Hence, it can contribute as an additional channel to the Higgs production at muon collider.

In Fig. 12, we present the LO cross sections for $\mu^+\mu^- \rightarrow h\gamma_d$ as a function of the center-of-mass energy. The LO cross sections for the process $\mu^+\mu^- \rightarrow h\gamma_d$ at the muon collider center-of-mass energies $\sqrt{s} = 3$ TeV and 10 TeV for various benchmark points mentioned in Table II are presented in Table III.

The cross section for $h\gamma_d$ production strongly depends on dark photon mass and dark gauge coupling as well as on other model parameters (see Figs. 10 and 11, as well as the corresponding vertex factors in the Appendix A and the squared matrix elements in the Appendix B).

The dark photon in this scenario is assumed to be coupled to a light dark matter particle and therefore gives rise to invisible signature in terms of missing energy at the collider detector. The Higgs on the other hand predominantly decays to a pair of bottom quarks. Therefore, the final state consists of $b\bar{b}$ along with missing energy at muon collider (see Fig. 13).

The dominant SM backgrounds that contribute to the $b\bar{b}$ plus missing energy final state, namely, $\mu^+\mu^- \rightarrow b\bar{b}\nu\bar{\nu}$ process comes from all those subprocesses that lead to $b\bar{b}\nu\bar{\nu}$ final state at muon collider. The various Feynman diagrams representing these subprocesses can be subdivided into $2 \rightarrow 2$, $2 \rightarrow 3$ and $2 \rightarrow 4$ categories. Some representative Feynman diagrams corresponding to these categories are shown in Fig. 14.

We separately generate and analyze Zh , ZZ , $h\nu\bar{\nu}$, and $Z\nu\bar{\nu}$ processes along with the total $b\bar{b}\nu\bar{\nu}$ process for comparative study of the SM background contribution. In Table IV, we tabulate the cross sections for individual SM backgrounds. The total cross section for the $b\bar{b}\nu\bar{\nu}$ background is obtained summing over all contributing Feynman diagrams leading to this final state. We would like to emphasize that the cross section quoted for $b\bar{b}\nu\bar{\nu}$ (total) is *not equal* to the algebraic sum of the cross sections of the individual processes listed in the same table.

Events generated using the event generator MadGraph5_aMC@NLO for both the signal and the SM background processes are then interfaced with Pythia8 (version 8.3) [93] for showering, hadronization, decay and further collider analysis. The detector response in the context of future muon collider is parametrized using a Gaussian smearing function as given in Eq. (31) following the Refs. [94,95].

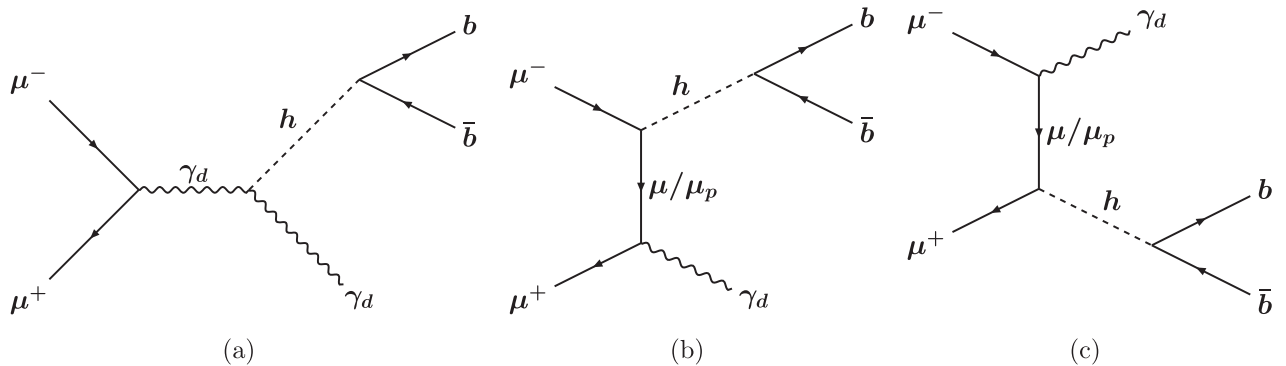


FIG. 13. Feynman diagrams for various subprocesses representing $\mu^+\mu^- \rightarrow h(b\bar{b}) + \gamma_d$ leading to $b\bar{b}$ plus missing energy final state, in absence of gauge kinetic mixing.

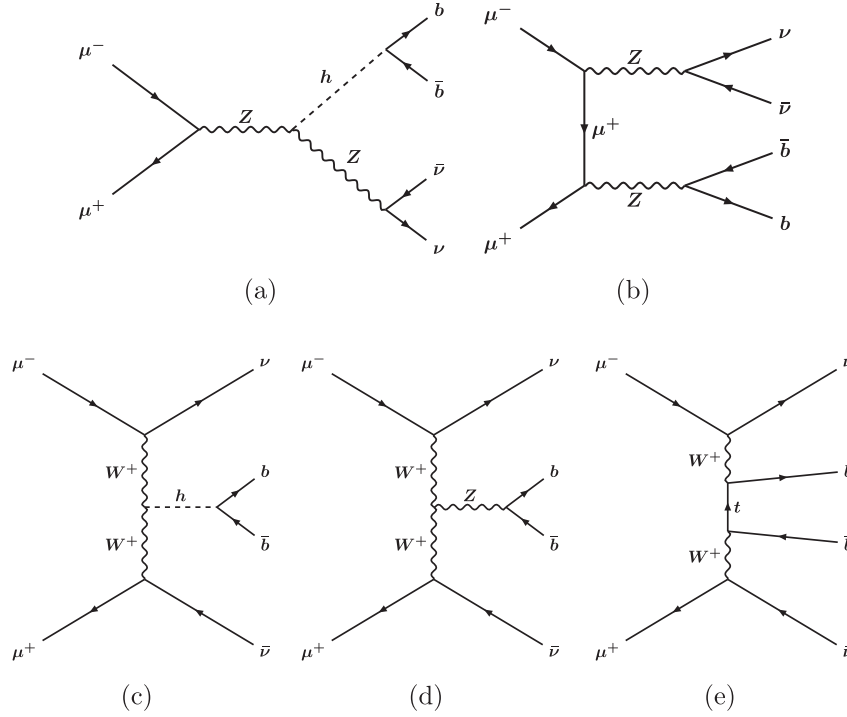


FIG. 14. Some representative Feynman diagrams leading to $b\bar{b}\nu\bar{\nu}$ final state in $\mu^+\mu^-$ collisions within the SM.

$$\frac{\sigma_E}{E} = \sqrt{\left(\frac{N}{E}\right)^2 + \left(\frac{S}{\sqrt{E}}\right)^2 + C^2} \quad (31)$$

where E is the energy associated with the final state object and σ_E is the standard deviation in the measurement of E . The parameters N , S and C represents respectively the

noise, stochastic and the constant terms. Depending on the detector these parameters have different values for specific final state objects. In the context of future muon collider for jets in the central region ($|\eta| < 2.5$), we have used the following set of values for these parameters [96]

$$N = 0, \quad S = 0.3, \quad C = 0.05$$

TABLE IV. LO cross section (in fb) at $\sqrt{s} = 3$ TeV and 10 TeV muon collider for various SM background processes, Zh , ZZ , $h\nu\bar{\nu}$, $Z\nu\bar{\nu}$ leading to $b\bar{b}\nu\bar{\nu}$ final state. The cross sections quoted in the last row (boldface) represent the total cross sections for the $\mu^+\mu^- \rightarrow b\bar{b}\nu\bar{\nu}$ process obtained by summing over all contributing Feynman diagrams (see Fig. 14) that leads to this final state and *not equal to* the algebraic sum of the cross sections of individual Zh , ZZ , $h\nu\bar{\nu}$, $Z\nu\bar{\nu}$ processes mentioned above.

Background processes ^a	σ [fb] at $\sqrt{s} = 3$ TeV	σ [fb] at $\sqrt{s} = 10$ TeV
$b\bar{b}\nu\bar{\nu}$ (Zh)	0.18	0.02
$b\bar{b}\nu\bar{\nu}$ (ZZ)	1.45	0.18
$b\bar{b}\nu\bar{\nu}$ ($h\nu\bar{\nu}$)	318	540
$b\bar{b}\nu\bar{\nu}$ ($Z\nu\bar{\nu}$)	305	526
$b\bar{b}\nu\bar{\nu}$ (total)	641	1101

^aIn Table IV, the cross section quoted for $h\nu\bar{\nu}$ includes contributions from both $\mu^+\mu^- \rightarrow Zh \rightarrow b\bar{b}\nu\bar{\nu}$ ($2 \rightarrow 2$) and $\mu^+\mu^- \rightarrow h\nu\bar{\nu} \rightarrow b\bar{b}\nu\bar{\nu}$ ($2 \rightarrow 3$) subprocesses with an on-shell Higgs and on-shell/off-shell Z boson. Similarly, for $Z\nu\bar{\nu}$ contributions from all relevant Feynman diagrams with one on-shell Z are considered.

We have used anti-kt jet clustering algorithm [97] with clustering radius $R = 0.4$ using FastJet (version 3.5.0) [98]. We also work with jets in the central region (central jet), defined as

$$p_T > 20 \text{ GeV} \quad \text{and} \quad |\eta| < 2.5$$

In Fig. 15, we plot the central-jet multiplicity distributions for various signal benchmark points. This has been done keeping in mind the fact that at the muon collider center-of-mass energies ($\sqrt{s} = 3$ TeV and 10 TeV), both the Higgs and the dark photon are always highly boosted, each sharing almost half of the available center-of-mass energies. Therefore, most of the signal events are expected to have one collimated $b\bar{b}$ pair originating from the Higgs decays resulting into a single narrow jet. Only a fraction of signal events due to final state radiation, showering and hadronization effect result into events with more than one central jet.

It turns out at 3 TeV center-of-mass energy a large fraction ($\sim 87\% - 90\%$) of the signal events consists of exactly one central jet and remaining 10%–13% corresponds to exactly

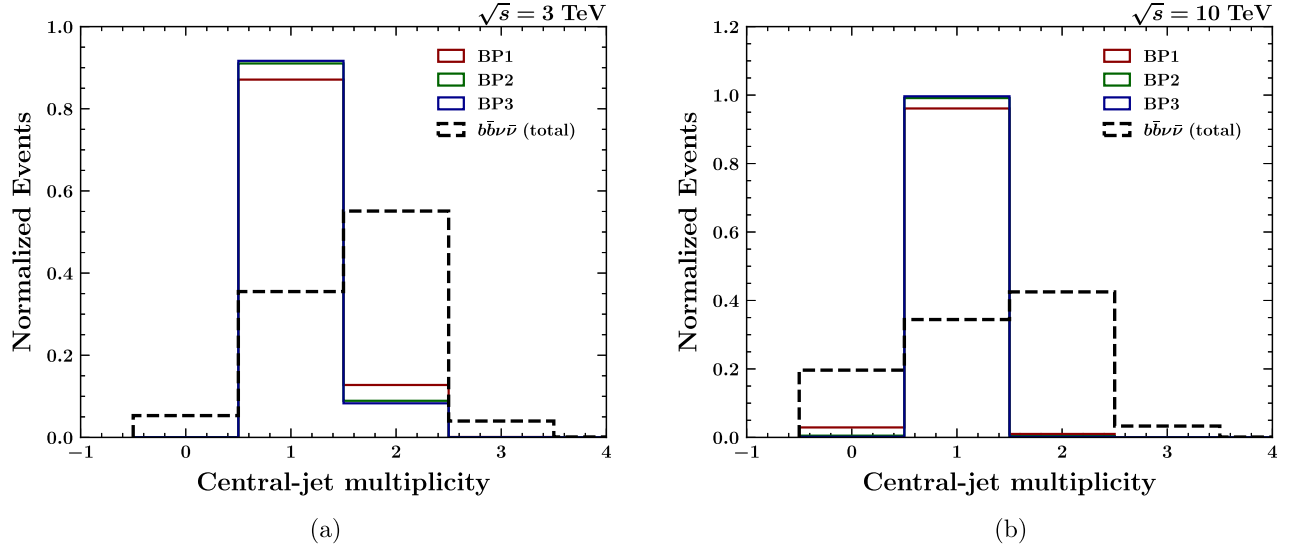


FIG. 15. Central-jet multiplicity distribution at $\sqrt{s} = 3$ TeV and 10 TeV for three different signal benchmark points and the total SM background.

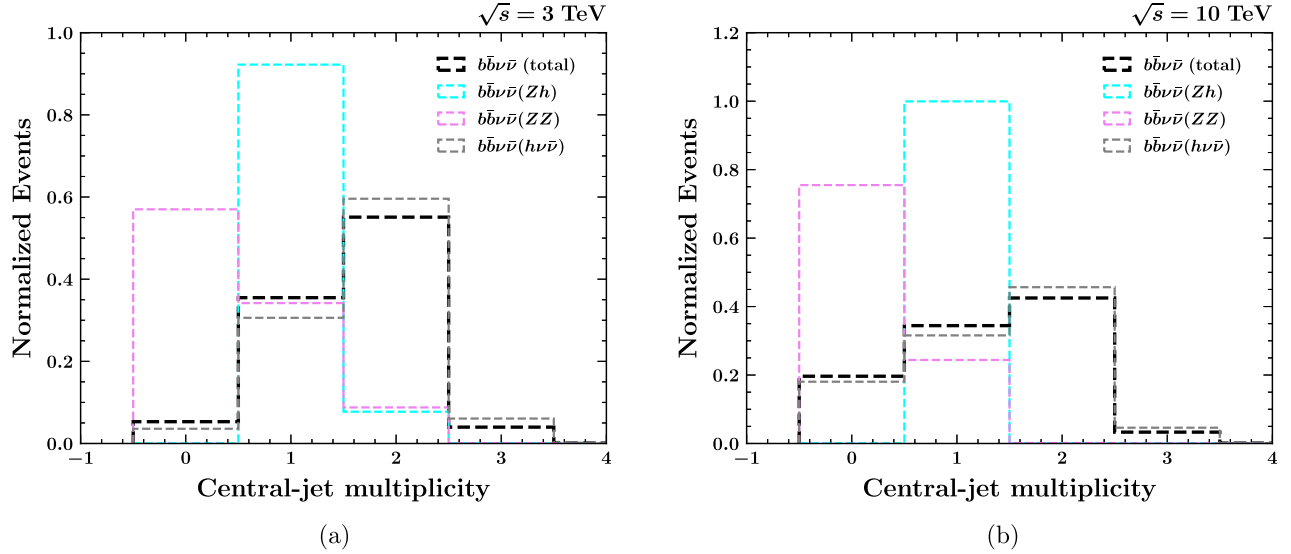


FIG. 16. Central-jet multiplicity distribution at $\sqrt{s} = 3$ TeV and 10 TeV for various SM backgrounds and the total $b\bar{b}\nu\bar{\nu}$ background.

two central-jet events. This pattern changes at 10 TeV center-of-mass energy. In this case, almost all the signal events ($\gtrsim 96\%$) correspond to one central jet, as expected. The central-jet multiplicity distribution corresponding to the total SM background in the $b\bar{b}\nu\bar{\nu}$ final state is also displayed on the same figure. One can see that the SM background events are distributed among 0, 1, 2 or 3 central-jet events categories.⁶ At $\sqrt{s} = 3$ TeV the central-jet multiplicity distribution has a peak around two containing almost 54% of the total background events. At 10 TeV center-of-mass energy, there is a

significant decrease (increase) in number of events in exactly two (zero) central-jet event categories.

The behavior of the central-jet multiplicity distribution for the total background shown in Fig. 16 can be understood as follows: The Higgs or Z boson produced in association with a nonresonant neutrino pair in a $2 \rightarrow 3$ process is generally not boosted, often resulting in multijet final states. Similar arguments apply to the continuum $b\bar{b}\nu\bar{\nu}$ background. In contrast, the SM Zh background, with both Z and h on-shell, being an s -channel $2 \rightarrow 2$ process, displays a different central-jet number distribution, mostly peaking around one. The SM ZZ background with both Z on-shell, despite being a $2 \rightarrow 2$ process, is expected to give more zero central-jet events than Zh as it is produced via t/u -channel muon mediation. These features are also

⁶Here, the zero central-jet event category represents all events with no jets in the central region (i.e., $|\eta| < 2.5$) having $p_T > 20$ GeV and is not considered for any further analysis.

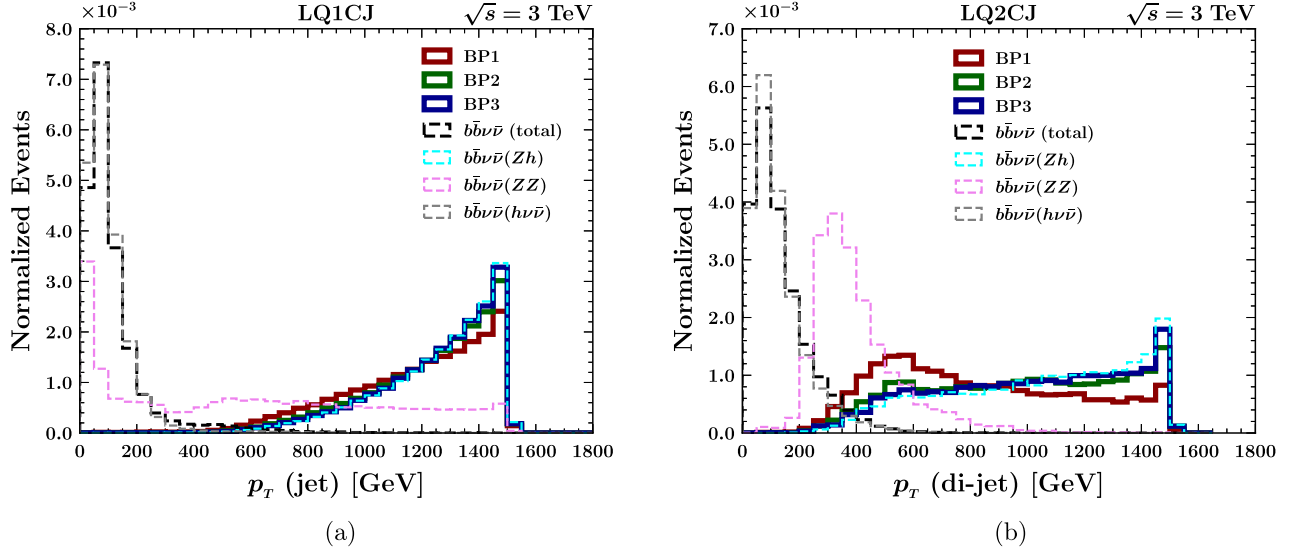


FIG. 17. Transverse momentum (p_T) distributions of the (a) jet and (b) the dijet in the LQ1CJ and LQ2CJ final states, respectively, at 3 TeV muon collider center-of-mass energy for both signal and the SM background events.

reflected in Fig. 16. Since the cross sections for SM Zh and ZZ processes are very small compared to the total background cross section, the normalized central-jet multiplicity distribution mostly follows that of the $h\nu\bar{\nu}$ (and $Z\nu\bar{\nu}$).

This motivates us to focus on two different final states, namely, leptonically quiet exactly one central jet (LQ1CJ) and exactly two central jets (LQ2CJ), both in conjunction with missing energy at two different $\mu^+\mu^-$ collision energies ($\sqrt{s} = 3$ TeV and 10 TeV).

As already mentioned, Higgs boson produced at the future muon collider center-of-mass energies in a $2 \rightarrow 2$ process are typically highly boosted. The Higgs subsequently decays into a pair of bottom quarks which due to their small angular separation, manifest as a single collimated jet in the detector. In Figs. 17 and 18, we plot the transverse momentum distributions of a jet or dijet system, as applicable, for two different muon collider center-of-mass energies. The purpose of these distributions are two-fold. They provide an explanation of Figs. 15 and 16 and also give a motivation for the following discussion.

To identify and analyze the boosted Higgs jets, we employ jet substructure analysis and dedicated Higgs tagging techniques. For this purpose we have used *MassDropTagger* [99] to identify a boosted Higgs structure inside jets. These jets are obtained after reclustering all the hadrons in a particular final state with Cambridge-Aachen (CA) jet clustering algorithm [100] and energy-scheme recombination assuming jet-clustering radius⁷ $R = 1.2$. The CA algorithm is chosen because it clusters particles

based solely on angular distance, making it well-suited for subsequent substructure analyses.

Below we briefly summarize the algorithm behind the jet substructure analysis implemented in *MassDropTagger*:

- (1) The leading- p_T jet (with transverse momentum $p_T > 200$ GeV) is first passed through the *MassDropTagger* procedure. The algorithm iteratively declusters the jet to find an imprint of a heavy particle by applying the following two conditions characterized by the dimensionless parameters μ and y_{cut} :

- (a) Mass-drop condition: $\frac{m_{j_1}}{m_j} < \mu$
- (b) Symmetry condition: $y = \frac{\min(p_{Tj_1}^2, p_{Tj_2}^2)}{m_j^2} \Delta R_{j_1 j_2}^2 > y_{\text{cut}}$.

where j_1 and j_2 (with $m_{j_1} > m_{j_2}$) denote the two subjets obtained by undoing the final stage of clustering of a parent jet j . Here, $\Delta R_{j_1 j_2}$ (with $\Delta R = \sqrt{\Delta\eta^2 + \Delta\phi^2}$) is the angular separation between the two subjets in the $\eta - \phi$ plane. The values of the dimensionless parameters μ and the y_{cut} are 0.67 and 0.09, respectively [99].

- (2) If both the above criteria are satisfied, the jet is tagged as a boosted heavy-particle candidate consisting of two subjets j_1 and j_2 . Otherwise, the lighter subjet is discarded, and the procedure is repeated by treating j_1 as the parent jet until a valid substructure is identified. We select an event for further analysis if it contains at least one $\text{CA}_{1.2}$ jet with a substructure.

The application of jet-substructure techniques or the use of the transverse momentum of the $b\bar{b}$ system, in combination with missing energy, prove highly effective in suppressing a large fraction of the total SM background and only the $2 \rightarrow 2$ processes with on-shell Zh and ZZ

⁷We call these jets as $\text{CA}_{1.2}$ jets. Also the above analysis is not too sensitive to the choice of the jet clustering radius values. The algorithm works fine in the range $0.8 \leq R \leq 1.4$.

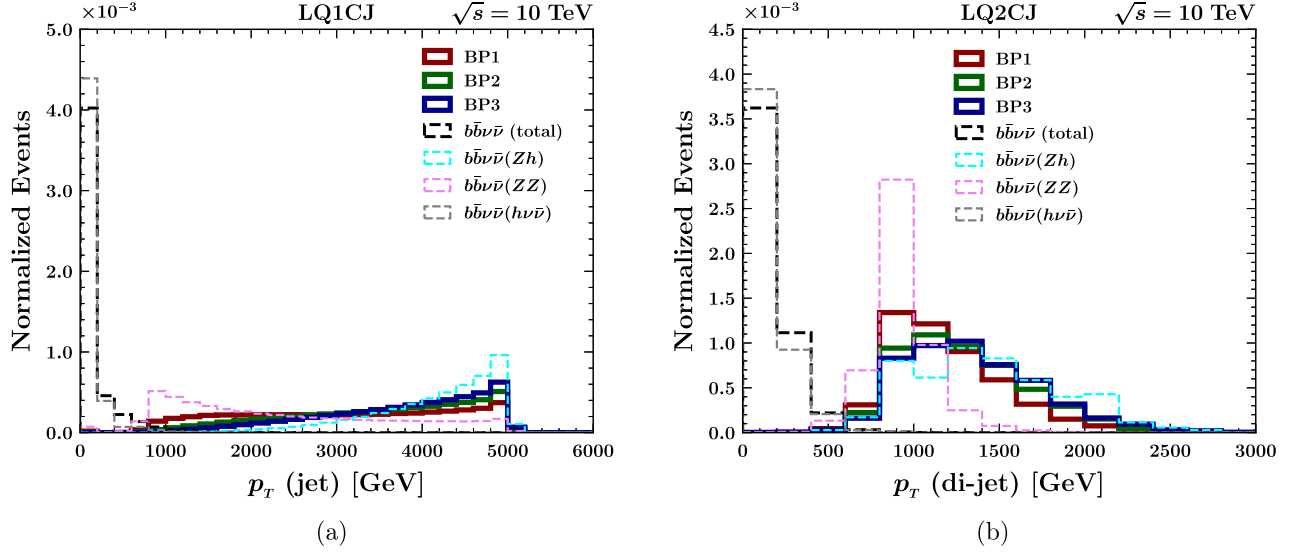


FIG. 18. Transverse momentum (p_T) distributions of the (a) jet and (b) the dijet in the LQ1CJ and LQ2CJ final states, respectively, at 10 TeV muon collider center-of-mass energy for both signal and the SM background events.

backgrounds surpass the substructure requirement. However, the cross sections for these processes are significantly small compared to the total SM background cross section (see Table IV).

C. Kinematic variables

In this subsection we discuss few useful kinematic variables and choice of selection cuts. The next stage of our cut based analysis utilizes both the missing energy and invariant mass of a jet (or dijet system) to reduce/eliminate the SM backgrounds.

- (i) Missing energy ($\cancel{E}$): In collider experiments, missing energy corresponds to the total energy carried by the invisible particles which remain unobserved at the detector level. In a particular final state consisting of

both visible and invisible particles, using the energy conservation we can find the missing energy of the system as

$$\cancel{E} = \sqrt{s} - E_{\text{vis}} \quad (32)$$

where, $\sqrt{s}$ is center-of-mass energy of the collisions and E_{vis} is the total energy of all the visible final state particles.

In collider investigations, the visible particles in general consist of the hadrons clustered as jets, isolated photons and leptons, and soft unclustered components. Thus, the expression of missing energy at the detector level can be written as

$$\cancel{E} = \sqrt{s} - \left(\sum_{i \in \text{iso leps}} E(i) + \sum_{i \in \text{iso phos}} E(i) + \sum_{i \in \text{jets}} E(i) + \sum_{i \in \text{soft}} E(i) \right) \quad (33)$$

In Fig. 19, we plot the missing energy distributions for the signal events for various benchmark scenarios corresponding to two different final states at center-of-mass energy $\sqrt{s} = 3$ TeV. The corresponding distributions at 10 TeV center-of-mass energy are shown in Fig. 20. The contribution to the missing energy for the signal events is due to the invisible dark photon and the distribution is expected to peak $\sim \sqrt{s}/2$ (neglecting the finite mass effect) in a $2 \rightarrow 2$ process. However, due to the presence of additional neutrinos coming from B meson decays the missing energy distribution extends beyond $\sqrt{s}/2$. Since the SM Zh and ZZ form a very small fraction of the total background cross section, the normalized missing energy distribution for the

total SM background follows that of the $h\nu\bar{\nu}/Z\nu\bar{\nu}$ process. The pair of neutrinos in the $h\nu\bar{\nu}$ or $Z\nu\bar{\nu}$ processes carries maximum share of the available center-of-mass energy leading to a peak around $\sqrt{s}$ in the missing energy distribution. This is reflected in the missing energy distributions for individual SM backgrounds and the total SM background as depicted in the same figures. Therefore, missing energy variable is an extremely useful discriminator between the signal and the background.

- (ii) $b\bar{b}$ invariant mass ($M_{b\bar{b}}$): In our analysis the invariant mass associated with the $b\bar{b}$ system is extracted in two different ways using the following formula

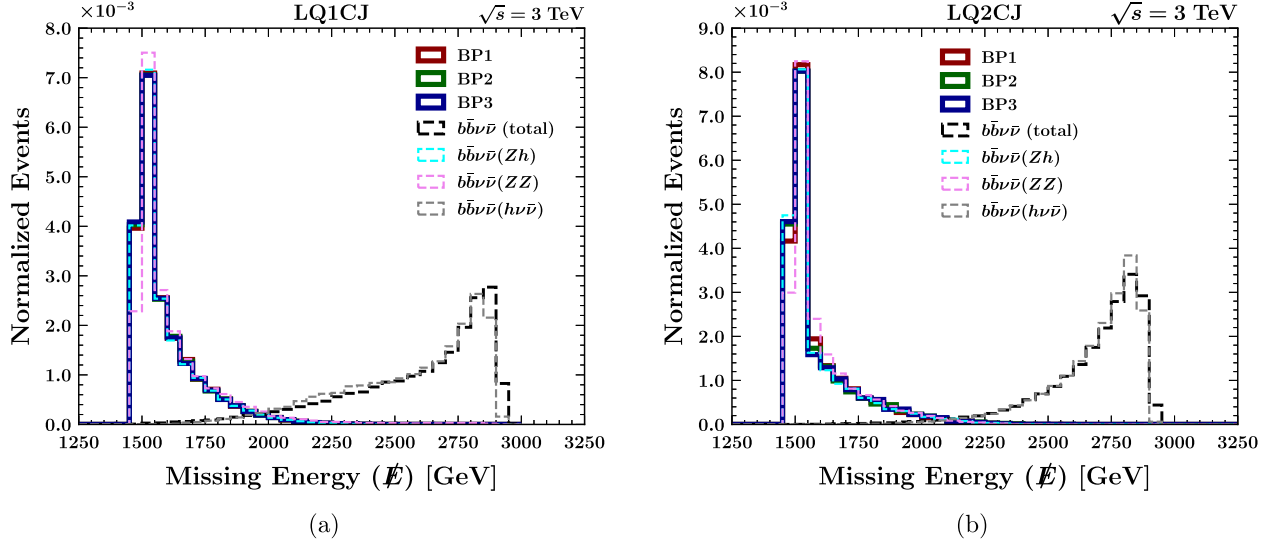


FIG. 19. Missing energy (E) distributions at $\sqrt{s} = 3$ TeV in the (a) LQ1CJ and (b) LQ2CJ final states for both signal and SM background events.

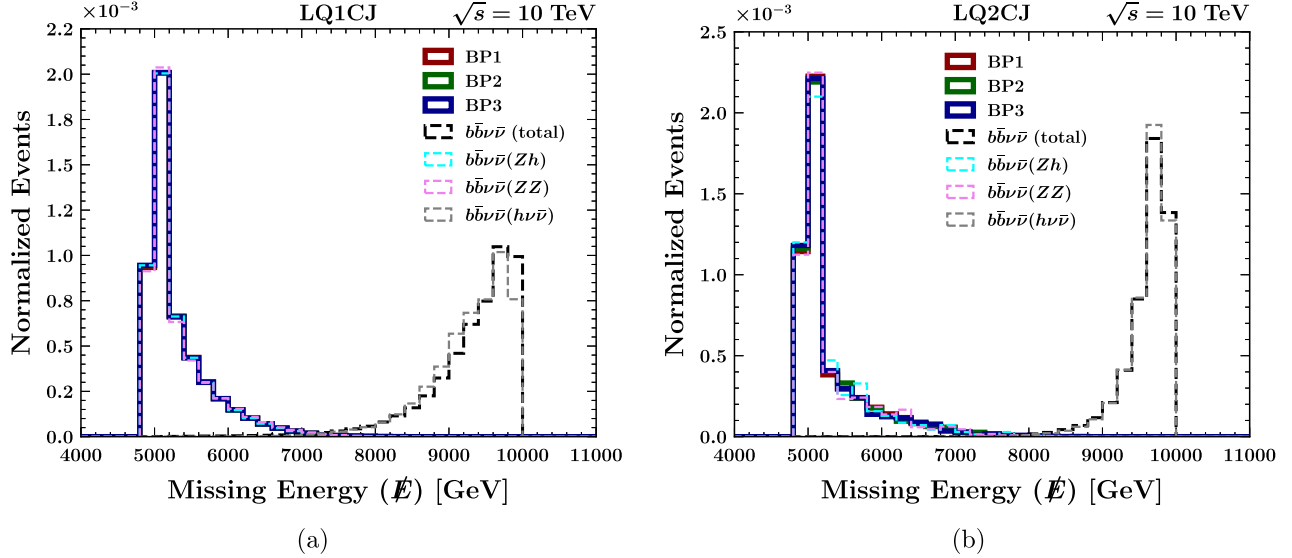


FIG. 20. Missing energy (E) distributions at $\sqrt{s} = 10$ TeV in the (a) LQ1CJ and (b) LQ2CJ final states for both signal and SM background events.

$$M_{12}^2 = (p_1 + p_2)^2 = (E_1 + E_2)^2 - |\vec{p}_1 + \vec{p}_2|^2 \quad (34)$$

where $p_1 = (E_1, |\vec{p}_1|)$ and $p_2 = (E_2, |\vec{p}_2|)$ are the four momentum associated with the two subjets (jets) in the final state comprising one (two) central jet. In events with exactly one central jet we combine the four momentum of the two subjets inside a boosted $CA_{1,2}$ jet to get the invariant mass distribution. Whereas in the case of exactly two central-jet events, we combine the four momentum of two AK4 jets in order to get the invariant mass associated with

the dijet system. The corresponding distributions in these two final states are plotted in Figs. 21 and 22. One can see that for signal events this distribution has a peak near the Higgs mass. However, the same for the SM backgrounds peak at varying locations, namely, at the mass of the Higgs and Z boson as expected. One also sees a peak at a very low invariant mass which has origin in the final state radiation and hadronization effects from one of the bottom quarks only, and may give rise to jet substructure. The $b\bar{b}$ invariant mass distribution in the LQ2CJ final state has two clear peaks at the values of Z and Higgs mass. This is due to the

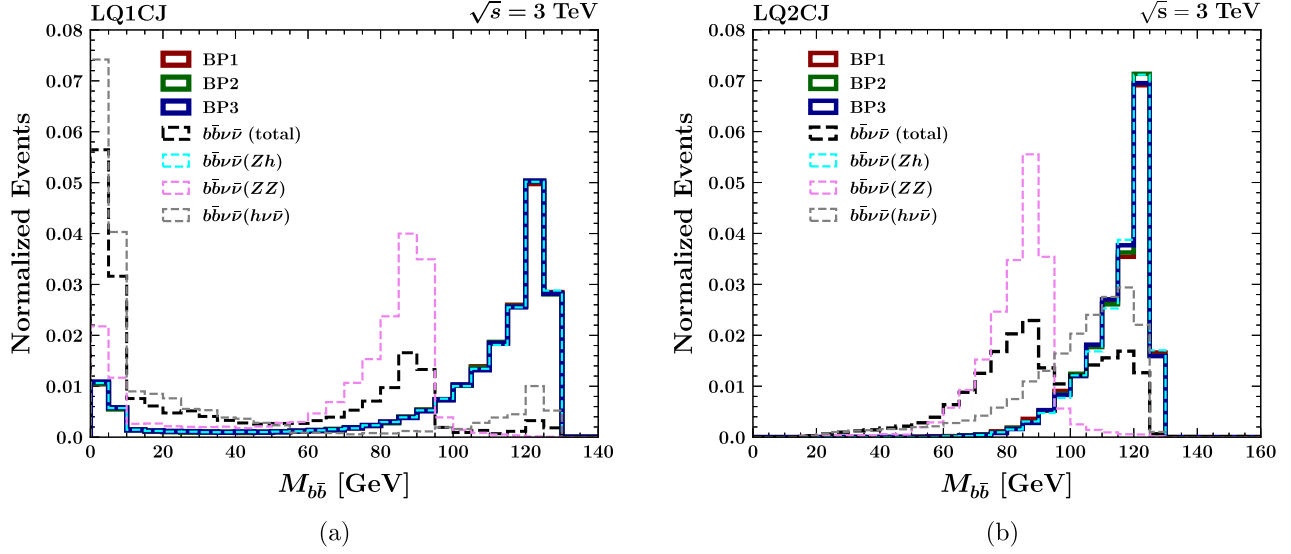


FIG. 21. Invariant mass ($M_{b\bar{b}}$) distribution at $\sqrt{s} = 3$ TeV in the (a) LQ1CJ and (b) LQ2CJ final state for both signal and background events.

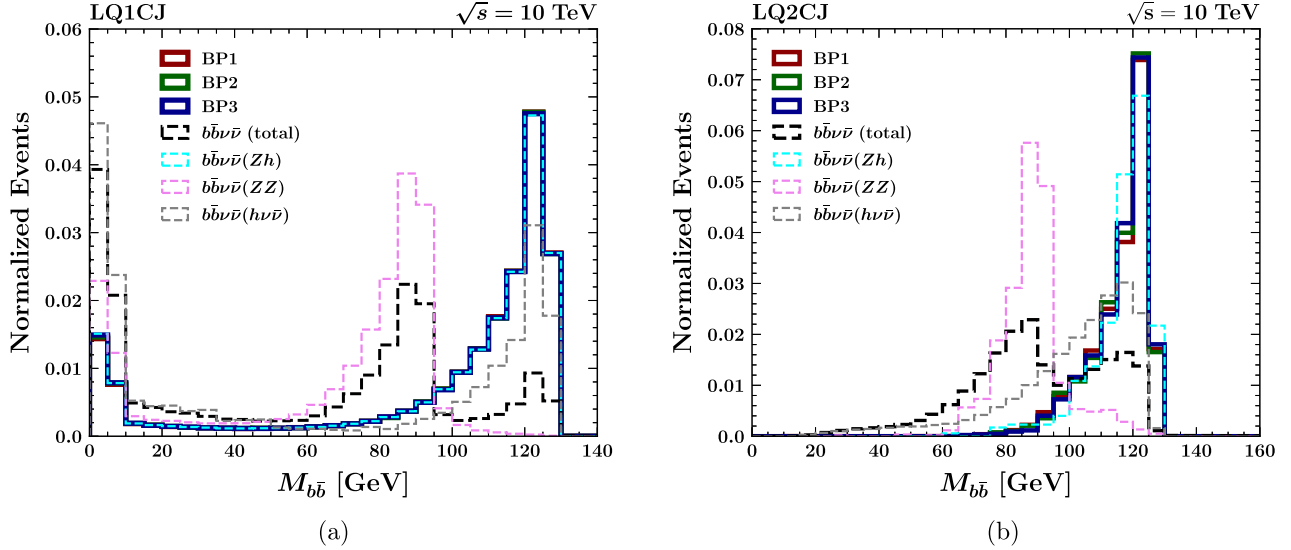


FIG. 22. Invariant mass ($M_{b\bar{b}}$) distribution at $\sqrt{s} = 10$ TeV in the (a) LQ1CJ and (b) LQ2CJ final state for both signal and background events.

presence of an on-shell Z and an on-shell Higgs, decaying to a bottom quark pair, in the total SM background. The tail in all these invariant mass distributions at values lower than the peak values is due to the missing momentum component associated with the invisible neutrinos produced in B decays.

One can see that the normalized signal distributions are mostly insensitive to the intermediate heavy muon mass. While presenting the kinematic distributions for the signal benchmark points (BPs) we have chosen (BP1, BP2, and BP3) as our representative BPs corresponding to three different heavy muon masses (1, 2, and 3 TeV). Similar

distributions exist for other signal benchmark points {BP4–BP6} corresponding to $m_{\gamma_d} = 50$ GeV as well. However, the effect of the dark photon mass on the normalized distributions are not significant enough. Hence, we choose not to display them here as they closely resemble the distributions already presented.

The study of SM $h\nu\bar{\nu}/Z\nu\bar{\nu}$ is useful to understand the behavior of the total background and its rates at various stages of our kinematic selections. However, to get an idea about the behavior of the resonant $\nu\bar{\nu}$ component of the total background, we separately analyze the SM Zh/ZZ processes. For significance estimate, we only use the total $b\bar{b}\nu\bar{\nu}$ background which is obtained by summing over all possible

TABLE V. The effect of cut flow on the signal cross section for various representative benchmark points in two different final states (LQ1CJ and LQ2CJ) at $\sqrt{s} = 3$ TeV assuming $\text{BR}(h \rightarrow b\bar{b}) = 1$.

BPs	Production cross section (fb)	Cross sections (fb): LQ1CJ final state				Cross sections (fb): LQ2CJ final state			
		Basic cuts	E cut ($E < E_0$)	Jet substructure requirement	$M_{b\bar{b}}$ cut ($M_{b\bar{b}} > M_0$)	Basic cuts	E cut ($E < E_0$)	Jet substructure requirement	$M_{b\bar{b}}$ cut ($M_{b\bar{b}} > M_0$)
BP1	11.1	9.70	9.51	9.39	7.36	1.42	1.38	1.30	1.24
BP2	22.8	20.7	20.3	20.1	15.7	2.03	1.97	1.84	1.76
BP3	0.064	0.059	0.058	0.057	0.044	0.005	0.005	0.005	0.005
BP4	47.2	41.1	40.3	39.8	31.3	5.97	5.79	5.49	5.24
BP5	114	104	102	101	78.8	9.95	9.63	8.97	8.65
BP6	0.436	0.401	0.393	0.388	0.303	0.035	0.034	0.031	0.030

TABLE VI. The effect of cut flow on the cross sections for various SM background processes and the total $b\bar{b}\nu\bar{\nu}$ background in two different final states (LQ1CJ and LQ2CJ) at $\sqrt{s} = 3$ TeV. Here the total background cross sections (in boldface) quoted after each selection cut are not equal to the algebraic sums of the corresponding cross sections of the individual background processes mentioned above.

Background processes	Production cross section (fb)	Cross sections (fb): LQ1CJ final state				Cross sections (fb): LQ2CJ final state			
		Basic cuts	E cut ($E < E_0$)	Jet substructure requirement	$M_{b\bar{b}}$ cut ($M_{b\bar{b}} > M_0$)	Basic cuts	E cut ($E < E_0$)	Jet substructure requirement	$M_{b\bar{b}}$ cut ($M_{b\bar{b}} > M_0$)
$b\bar{b}\nu\bar{\nu}(Zh)$	0.178	0.164	0.161	0.160	0.124	0.014	0.013	0.012	0.012
$b\bar{b}\nu\bar{\nu}(ZZ)$	1.45	0.495	0.473	0.368	0.014	0.127	0.124	0.120	0.007
$b\bar{b}\nu\bar{\nu}(h\nu\bar{\nu})$	318	96.2	4.44	0.745	0.420	187	1.38	1.03	0.914
$b\bar{b}\nu\bar{\nu}(Z\nu\bar{\nu})$	305	121	5.81	3.07	0.113	152	2.73	2.47	0.168
$b\bar{b}\nu\bar{\nu}(\text{tot})$	641	222	10.9	4.00	0.538	351	4.75	3.97	1.44

Feynman diagrams leading to $b\bar{b}\nu\bar{\nu}$ final state and is not equal to the algebraic sum of individual SM background processes used to illustrate various kinematic distributions and rates.

D. Cuts

To separate the signal from the corresponding SM background, we have used the following selection cuts on the observables associated with the individual final states.

- (i) $\cancel{E} < \cancel{E}_0$, where $\cancel{E}_0 = 2(7)$ TeV for $\sqrt{s} = 3(10)$ TeV.
- (ii) Jet substructure requirement
- (iii) $M_{b\bar{b}} > M_0$, where $M_0 = 95$ GeV.

V. RESULTS

In this section, we present the results of our analysis of the process Higgs production in association with a dark photon at the future muon collider. In the previous section, we have discussed the production rate of $\mu^+\mu^- \rightarrow h\gamma_d$ and its comparison with the SM Zh production in presence of the portal vectorlike lepton. It also includes the choice of various kinematic variables and selection criteria. In Table V and VII, we tabulate the production cross sections for the signal process as well as the effects of our selection criteria on it for various choices of our benchmark points (BP1-BP6) in the leptonically quiet exactly one and two

TABLE VII. The effect of cut flow on the signal cross section for various representative benchmark points in two different final states (LQ1CJ and LQ2CJ) at $\sqrt{s} = 10$ TeV assuming $\text{BR}(h \rightarrow b\bar{b}) = 1$.

BPs	Production cross section (fb)	Cross sections (fb): LQ1CJ final state				Cross sections (fb): LQ2CJ final state			
		Basic cuts	E cut ($E < E_0$)	Jet substructure requirement	$M_{b\bar{b}}$ cut ($M_{b\bar{b}} > M_0$)	Basic cuts	E cut ($E < E_0$)	Jet substructure requirement	$M_{b\bar{b}}$ cut ($M_{b\bar{b}} > M_0$)
BP1	2.66	2.55	2.53	2.49	1.84	0.027	0.026	0.024	0.023
BP2	10.9	10.8	10.7	10.5	7.72	0.040	0.039	0.036	0.035
BP3	0.056	0.056	0.056	0.055	0.040	$\sim 10^{-4}$	$\sim 10^{-4}$	$\sim 10^{-4}$	$\sim 10^{-4}$
BP4	11.4	10.9	10.8	10.6	7.85	0.114	0.112	0.102	0.099
BP5	55.0	54.5	54.0	53.1	38.9	0.207	0.203	0.184	0.180
BP6	0.388	0.386	0.383	0.376	0.276	$\sim 6 \times 10^{-4}$	$\sim 6 \times 10^{-4}$	$\sim 6 \times 10^{-4}$	$\sim 6 \times 10^{-4}$

TABLE VIII. The effect of cut flow on the cross sections for various SM background processes and the total $b\bar{b}\nu\bar{\nu}$ background in two different final states (LQ1CJ and LQ2CJ) at $\sqrt{s} = 10$ TeV. Here the total background cross sections (in boldface) quoted after each selection cut are not equal to the algebraic sums of the corresponding cross sections of the individual background processes mentioned above.

Background processes	Production cross section (fb)	Cross sections (fb): LQ1CJ final state				Cross sections (fb): LQ2CJ final state			
		Basic cuts	E cut ($E < E_0$)	Jet substructure requirement	$M_{b\bar{b}}$ cut ($M_{b\bar{b}} > M_0$)	Basic cuts ($E < E_0$)	E cut	Jet substructure requirement	$M_{b\bar{b}}$ cut ($M_{b\bar{b}} > M_0$)
$b\bar{b}\nu\bar{\nu}(Zh)$	0.016	0.016	0.016	0.015	0.011	$<10^{-5}$	$<10^{-5}$	$<10^{-5}$	$<10^{-5}$
$b\bar{b}\nu\bar{\nu}(ZZ)$	0.183	0.045	0.044	0.043	0.002	$\sim 2 \times 10^{-4}$	$\sim 2 \times 10^{-4}$	$\sim 2 \times 10^{-4}$	$<10^{-4}$
$b\bar{b}\nu\bar{\nu}(h\nu\bar{\nu})$	540	167	0.718	0.583	0.470	245	0.086	0.086	0.081
$b\bar{b}\nu\bar{\nu}(Z\nu\bar{\nu})$	526	197	2.80	2.74	0.158	204	0.111	0.100	0.016
$b\bar{b}\nu\bar{\nu}(\text{tot})$	1101	373	4.03	3.53	0.673	466	0.488	0.427	0.310

central-jet(s) final states at $\sqrt{s} = 3$ TeV and 10 TeV, respectively. The signal cross sections mentioned in these tables assume $h \rightarrow b\bar{b}$ decay branching ratio (BR) to be 100%. The effect of cut-flow on the individual SM background processes ($Zh, ZZ, h\nu\bar{\nu}, Z\nu\bar{\nu}$) and the total SM background in the $b\bar{b}\nu\bar{\nu}$ final state are presented in Table VI and Table VIII at muon collider center-of-mass energies 3 and 10 TeV, respectively. A significant fraction of the $\mu^+\mu^- \rightarrow h\nu\bar{\nu}$ background can be effectively reduced by rejecting events beyond a certain missing energy value (E_0) as can be seen from Figs. 19 and 20. These cross sections for the SM backgrounds are quoted taking into account appropriate decay BRs, as applicable.

It is clear from the Tables V and VII that at muon collider center-of-mass energies the signal events dominantly contribute in the LQ1CJ final state than the other one because of the fact that it is a $2 \rightarrow 2$ process. Moreover, the contribution of signal events in this final state increases with increasing mass of the heavy muon partner in the t/u channel. This is evident if we compare the event rate after basic selection criteria for various benchmark points ({BP1-BP3} or {BP4-BP6}) which corresponds to heavy muon mass in the range {1–3} TeV. This feature is also visible in Fig. 15.

The effect of our selection criteria on the signal events is less severe in both the final states as can be seen from the cut-flow tables. Whereas one has a factor of $\sim \frac{1}{400}$ and $\frac{1}{550}$ suppression in the SM background rate in the exactly one central-jet final state after applying the combination of cuts mentioned in Sec. IV D at $\sqrt{s} = 3$ TeV and 10 TeV, respectively. This factor for the exactly two central-jet final state is $\sim \frac{1}{250}$ for $\sqrt{s} = 3$ TeV and even larger at 10 TeV center-of-mass energy. The significant effect on the background comes from the missing energy cut as already explained in Sec. IV.

The cut-flow tables corresponding to the signal events also suggest that the effect of the dark photon mass on the kinematics, in particular, on the cut efficiencies is very small as long as $m_{\gamma_d} \ll \sqrt{s}$. However, it plays a significant role in the estimation of the cross section as it depends on

values the of dark photon mass as well as the dark gauge coupling.

In Table IX, we quote the signal significances for 3 TeV and 10 TeV muon collider center-of-mass energies for various choices of our benchmark points for two different final states assuming an integrated luminosity of 100 fb^{-1} . To estimate the signal significance (σ) and set a 2σ exclusion limit we have used the following formula [101–103]

$$\sigma = \sqrt{2 \left(S - B \ln \left(1 + \frac{S}{B} \right) \right)}. \quad (35)$$

where S and B refer to the number of signal and background events obtained after applying the selection criteria mentioned earlier at a given integrated luminosity.

While estimating the significance we have also taken into account the appropriate branching ratio (57%) for $h \rightarrow b\bar{b}$ decay to get the actual number of signal events. The number of background events B is estimated using the total $b\bar{b}\nu\bar{\nu}$ event sample, and not adding the contributions of the individual background processes.

TABLE IX. Estimated signal significance in the context of $\mu^+\mu^- \rightarrow h\gamma_d$ process in two different final states (LQ1CJ and LQ2CJ) under consideration at two different muon collider center-of-mass energies ($\sqrt{s} = 3$ and 10 TeV) assuming an integrated luminosity of 100 fb^{-1} .

BPs	Signal significance (σ)			
	$\sqrt{s} = 3 \text{ TeV}$		$\sqrt{s} = 10 \text{ TeV}$	
	LQ1CJ	LQ2CJ	LQ1CJ	LQ2CJ
BP1	24.6	5.15	9.12	0.23
BP2	38.5	6.97	24.6	0.35
BP3	0.34	0.02	0.27	...
BP4	56.4	16.6	24.9	0.96
BP5	92.2	23.6	62.9	1.67
BP6	2.14	0.14	1.78	...

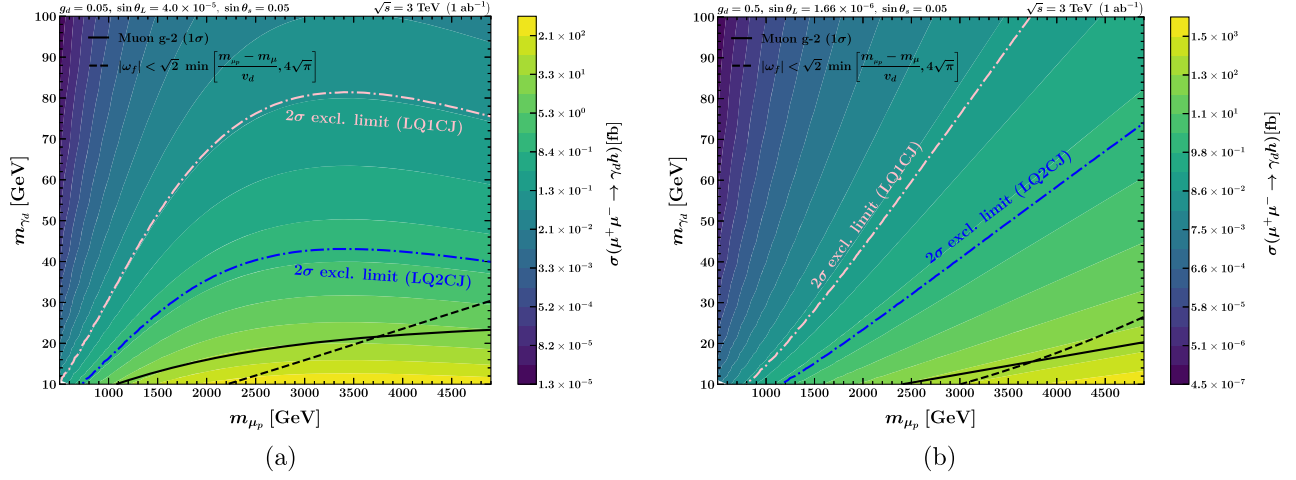


FIG. 23. 2σ exclusion limit in the $m_{\gamma_d} - m_{\mu_p}$ plane for two different final states at 3 TeV muon collider center-of-mass energy for (a) $g_d = 0.05, \sin \theta_L = 4.0 \times 10^{-5}, \sin \theta_s = 0.05$ and (b) $g_d = 0.5, \sin \theta_L = 1.66 \times 10^{-6}, \sin \theta_s = 0.05$ assuming an integrated luminosity of 1 ab^{-1} . The exclusion limits in LQ1CJ (pink) and LQ2CJ (blue) final states are represented by the region below the dash-dot lines. The region above the dashed (black) line is allowed by the constraint represented in Eq. (17) while the muon $g-2$ allowed region is represented by the one above solid (black) line.

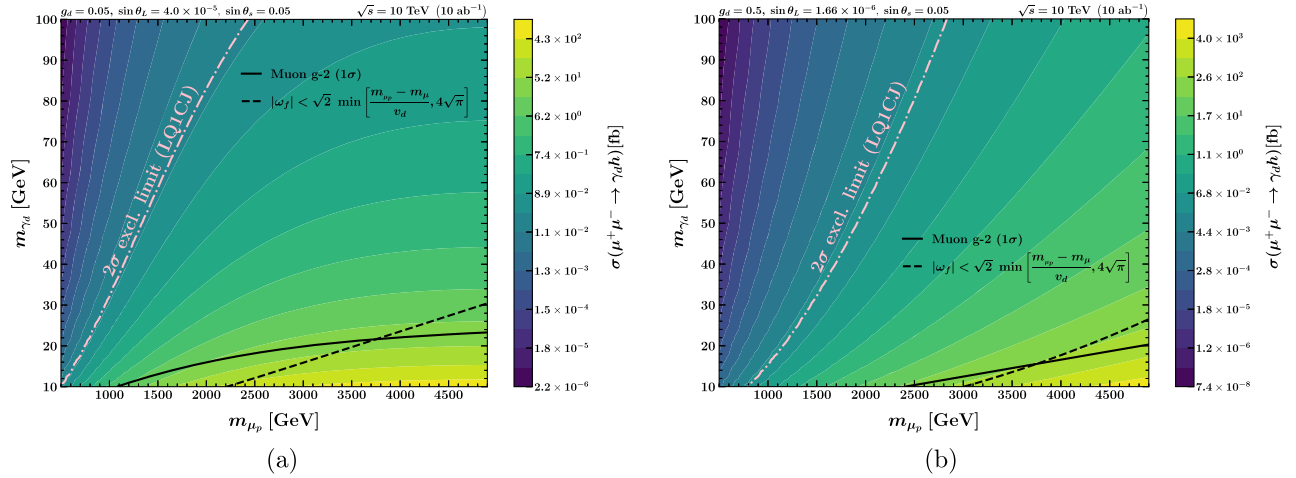


FIG. 24. 2σ exclusion limit in the $m_{\gamma_d} - m_{\mu_p}$ plane in LQ1CJ final state at 10 TeV muon collider center-of-mass energy for (a) $g_d = 0.05, \sin \theta_L = 4.0 \times 10^{-5}, \sin \theta_s = 0.05$ and (b) $g_d = 0.5, \sin \theta_L = 1.66 \times 10^{-6}, \sin \theta_s = 0.05$ assuming an integrated luminosity of 10 ab^{-1} . The 2σ exclusion limit is represented by dash-dot line (pink). The region above the dashed (black) line is allowed by the constraint represented in Eq. (17) while the muon $g-2$ allowed region is represented by the one above solid (black) line.

The increase in significance for BP2 compared to BP1 is due to higher signal cross section at this benchmark point even though the mass of the pVLL in the t/u -channel propagator is higher. Also one has a better estimate of the signal significance for the set of BPs corresponding to dark photon mass 50 GeV compared to that for dark photon mass 90 GeV as the cross section for $h\gamma_d$ production is expected to increase with decreasing dark photon mass.

In Fig. 23, we present the 2σ exclusion limit in the $m_{\gamma_d} - m_{\mu_p}$ plane at $\sqrt{s} = 3 \text{ TeV}$ assuming an integrated

luminosity of 1 ab^{-1} in two different final states for two sets of g_d and $\sin \theta_L$ keeping $\sin \theta_s$ fixed at 0.05. The same at $\sqrt{s} = 10 \text{ TeV}$ and an integrated luminosity of 10 ab^{-1} is presented in Fig. 24. The Fig. 25 represents the 2σ exclusion limit in the $g_d - m_{\gamma_d}$ plane corresponding to $\sqrt{s} = 3 \text{ TeV}$ and an integrated luminosity of 1 ab^{-1} . The regions allowed by the constraint presented in Eq. (17) and the current muon $g-2$ measurements at 1σ level are represented by the one above the dashed (black) and the solid (black) lines, respectively, in all these figures.

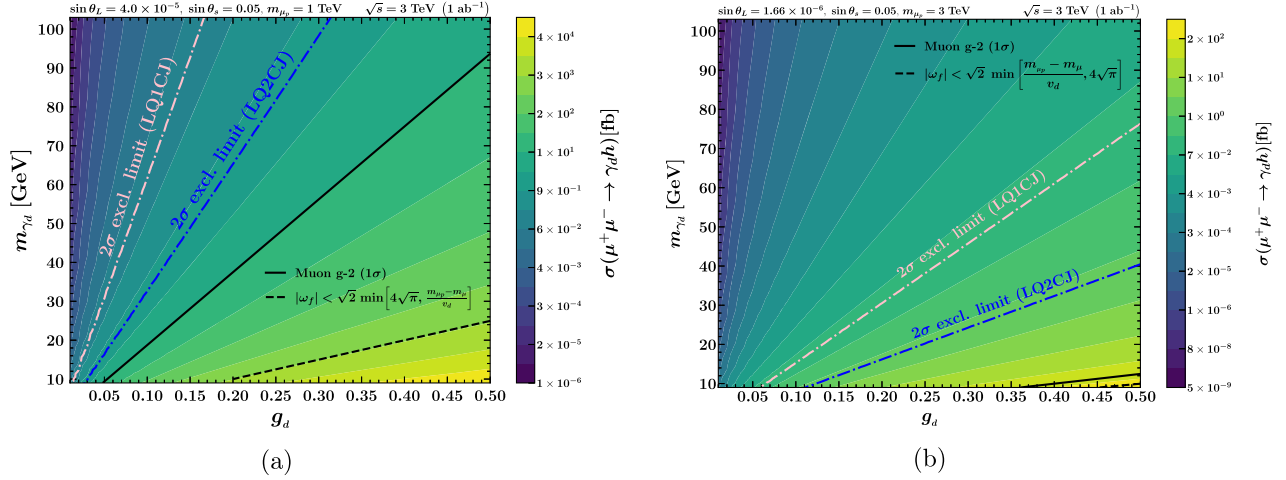


FIG. 25. 2σ exclusion limit (represented by dashed-dot line) in the $g_d - m_{\gamma_d}$ plane using the data corresponding to $\sqrt{s} = 3$ TeV and an integrated luminosity of 1 ab^{-1} in the LQ1CJ (pink) and LQ2CJ (blue) final states for two different sets of $\sin\theta_L$ and m_{μ_p} : (a) $\sin\theta_L = 4 \times 10^{-5}$, $m_{\mu_p} = 1 \text{ TeV}$ and (b) $\sin\theta_L = 1.66 \times 10^{-6}$, $m_{\mu_p} = 3 \text{ TeV}$. The region above the dashed (black) line is allowed by the constraint represented in Eq. (17) while the muon $g - 2$ allowed region is represented by the one above solid (black) line.

VI. CONCLUSION

Motivated by the prospects of Higgs physics at proposed future lepton colliders and precise measurements of the Higgs properties, we have investigated the implications of a portal vectorlike lepton on the Higgs production in association with a neutral gauge boson (Z/γ_d) at the proposed future muon collider experiment and its consequence on dark photon. In particular, we focus on the Higgs and dark photon associated production via t and u channel pVLL exchange in addition to the dark photon mediated s -channel contribution. To avoid constraints coming from the low energy flavor changing observables we consider pVLL mixing only with one generation of lepton family, in particular, the second generation charged lepton.

We have investigated the role of the pVLL in the context of the muon $g - 2$ anomaly. Compatibility with the current value of muon $g - 2$ suggests that one can have a sizable value of $\sin\theta_R$ ($\sim 10^{-1}$), while $\sin\theta_L$ can remain small ($\sim 10^{-5}$). We have shown that there exists a wide region of parameter space in the $g_d - m_{\gamma_d}$ plane that is consistent with the observed value of muon $g - 2$ as well as other existing constraints coming from searches for additional scalar and fermions. The presence of the pVLL also plays a crucial role in the context of DM relic abundance calculation. In the absence of the pVLL state, one requires large kinetic mixing ($\epsilon \sim 10^{-2}$) away from resonance to obtain an underabundant DM relic density for low mass dark photon ($m_\chi < m_{\gamma_d}$). Such a large kinetic mixing is disfavored by direct detection experiments. However, the presence of the pVLL state not only enhances dark matter annihilation into a pair of muons—if the pVLL mixes with the Standard Model second-generation charged lepton—even for small kinetic mixing, but also helps in evading direct detection bounds.

Next we investigate the implication of the pVLL on the associated Higgs production at muon collider and present a detailed comparison of the $h\gamma_d$ production rate with that of the hZ production at muon collider. The later process is considered both in the context of SM and in presence of the portal VLL extension of the SM. The $\mu^+\mu^- \rightarrow h\gamma_d$ process is highly sensitive to the following independent model parameters: the dark photon mass (m_{γ_d}), dark gauge coupling (g_d), and both fermion ($\sin\theta_L$) and scalar ($\sin\theta_s$) mixing angles. We explore the nondecoupling effect owing to the presence of the muon-heavy muon-dark photon vertex and show that a significant region of parameter space consistent with the perturbative unitarity, electroweak precision measurements, lepton flavor universality, allowed ranges of scalar mixing angle and other relevant constraints can be found where the $h\gamma_d$ cross section is $\sim \text{few fb to } 10^2 \text{ fb}$. Consequently, one can achieve up to 10% better precision in the measurement of Higgs properties with respect to that achievable in the hZ channel.

A crucial outcome of our study is that this process not only provides an enhanced Higgs production rate at the muon collider but also helps to set a strong limit on the dark photon mass even for a fermion mixing angle as low as 10^{-6} , independent of the gauge kinetic mixing parameter.

We have proposed two final states, i.e., leptonically quiet exactly one and exactly two central jet(s) along with missing energy. These final states originate from the $h\gamma_d$ process when the Higgs boson decays to a pair of bottom quarks and the dark photon is treated as an invisible particle at the detector. In order to set a 2σ exclusion limit in the $m_{\gamma_d} - m_{\mu_p}$ plane using the $h\gamma_d$ process in the aforementioned final states, we have also considered an exhaustive study of the SM background processes which contribute to

these final states. With the help of certain kinematic variables such as missing energy, requirement of the presence of substructure in the leading central $CA_{1.2}$ jet, and the invariant mass associated with the $b\bar{b}$ system to lie around the Higgs mass it is possible to substantially reduce the SM background contributions and efficiently isolate the signal of $h\gamma_d$ production at the future muon collider.

A 2σ exclusion limit upto $m_{\gamma_d} = 80(76)$ GeV can be obtained using our analysis in the exactly one central-jet plus missing energy final state at 3 TeV muon collider center-of-mass energy for $g_d = 0.05(0.5)$, $\sin\theta_L = 4 \times 10^{-5}(1.66 \times 10^{-6})$, $\sin\theta_s = 0.05$ for a heavy muon mass $m_{\mu_p} \sim 3$ TeV. The corresponding bounds in the exactly two central jets plus missing energy final state are $m_{\gamma_d} = 43$ GeV and 46 GeV for a heavy muon mass $m_{\mu_p} \sim 3.3$ TeV using the same set of choices of the other parameters as mentioned above. We also present the 2σ exclusion limit for the exactly one central-jet plus missing energy final state in the $m_{\mu_p}-m_{\gamma_d}$ plane for a higher-energy muon collider scenario with $\sqrt{s} = 10$ TeV and corresponding integrated luminosity of 10 ab^{-1} . Assuming $m_{\mu_p} = 1$ TeV, $\sin\theta_L = 4 \times 10^{-5}$ and $\sin\theta_s = 0.05$ one can exclude the value of the dark gauge coupling above 0.15 (0.28) for $m_{\gamma_d} \leq 90$ GeV at $\sqrt{s} = 3$ TeV with an integrated luminosity of 1 ab^{-1} in the exactly one (two) central jet(s) plus missing energy final state. The corresponding limits on g_d for $m_{\mu_p} = 3$ TeV, $\sin\theta_L = 1.66 \times 10^{-6}$, $\sin\theta_s = 0.05$ and $m_{\gamma_d} < 40$ GeV are respectively 0.26 and 0.5 for these two final states assuming the same muon collider collision energy and integrated luminosity. Thus, the Higgs production in association with a dark photon at future muon collider facilities provide a sensitive probe of a large region of parameter space in the $g_d-m_{\gamma_d}$ and $m_{\mu_p}-m_{\gamma_d}$ planes while addressing the muon $g-2$ and DM relic density.

ACKNOWLEDGMENTS

We gratefully acknowledge Satyanarayan Mukhopadhyay for his valuable insights and useful comments. K.T. thanks the Department of Science and Technology (DST), Government of India, for financial support under the INSPIRE Fellowship Programme (IF220170). The work of S.V. at Physical Research Laboratory was supported by Department of Space, Govt. of India. K.T. is also grateful to Sujay Mondal for providing access to high-performance computational facilities which significantly aided the progress of this work. K.T. further extends her thanks to the organizers and instructors of the Iwate Collider School 2025 for their excellent lectures and tutorial sessions.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: VERTEX FACTORS

In this Appendix, we present Feynman rules for the relevant vertices assuming all the associated four momenta are coming out of the vertex in the zero kinetic mixing limit ($\epsilon \rightarrow 0$). Below the L and R represent the left- and right-chiral fermions, respectively.

1. Vertices with dark photon

$$(1) \mu_{pR}^+ - \mu_L^- - \gamma_d / \mu_R^+ - \mu_{pL}^- - \gamma_d$$

$$VF_1 = -ig_d \sin\theta_L \cos\theta_L \gamma^\mu \quad (A1)$$

$$(2) \mu_{pL}^+ - \mu_R^- - \gamma_d / \mu_L^+ - \mu_{pR}^- - \gamma_d$$

$$VF_2 = -ig_d \sin\theta_R \cos\theta_R \gamma^\mu \quad (A2)$$

$$(3) \mu_L^+ - \mu_R^- - \gamma_d$$

$$VF_3 = -ig_d \sin^2\theta_R \gamma^\mu \quad (A3)$$

$$(4) \mu_R^+ - \mu_L^- - \gamma_d$$

$$VF_4 = -ig_d \sin^2\theta_L \gamma^\mu \quad (A4)$$

2. Vertices with Z boson

$$(1) \mu_{pR}^+ - \mu_L^- - Z / \mu_R^+ - \mu_{pL}^- - Z$$

$$VF_5 = -\frac{i}{2} g_z \sin\theta_L \cos\theta_L \gamma^\mu \quad (A5)$$

$$(2) \mu_L^+ - \mu_R^- - Z$$

$$VF_6 = ig_z \sin^2\theta_W \gamma^\mu \quad (A6)$$

$$(3) \mu_R^+ - \mu_L^- - Z$$

$$VF_7 = i \frac{g_z}{2} (2 \sin^2\theta_W - \cos^2\theta_L) \gamma^\mu \quad (A7)$$

3. Vertices with Higgs

$$(1) \mu_{pL}^+ - \mu_L^- - h / \mu_R^+ - \mu_{pR}^- - h$$

$$VF_8 = -\frac{i}{\sqrt{2}} \sin\theta_R (\sin\theta_L \sin\theta_s \omega_f + \cos\theta_L \cos\theta_s y_m) \quad (A8)$$

$$(2) \mu_{p_R}^+ - \mu_R^- - h / \mu_L^+ - \mu_{p_L}^- - h$$

$$(4) Z - Z - h$$

$$VF_9 = \frac{i}{\sqrt{2}} \cos \theta_R (\cos \theta_L \sin \theta_s \omega_f - \sin \theta_L \cos \theta_s y_m) \quad (A9)$$

$$VF_{11} = ig_z m_z \cos \theta_s \quad (A11)$$

$$(3) \gamma_d - \gamma_d - h$$

$$VF_{10} = -2ig_d m_{\gamma_d} \sin \theta_s \quad (A10)$$

APPENDIX B: SQUARED MATRIX ELEMENT FOR $\mu^+ \mu^- \rightarrow h \gamma_d$

The t - and u - channel diagrams for the $\mu^+ \mu^- \rightarrow h \gamma_d$ process via portal vectorlike lepton mediation (μ_p) are presented in Fig 26.

The amplitude for the t -channel diagram is given by

$$i\mathcal{M}_t = -i \left[\bar{v}_{s_2}(k_2) \gamma^\mu (c_V^t + c_A^t \gamma^5) \epsilon_\mu^{*\lambda}(\vec{p}_2) \frac{(\not{k}_1 - \not{p}_1 + m_{\mu_p})}{(k_1 - p_1)^2 - m_{\mu_p}^2} (c_s^t + c_p^t \gamma^5) u_{s_1}(k_1) \right] \quad (B1)$$

where c_s^t, c_p^t, c_V^t , and c_A^t are given by

$$\begin{aligned} c_s^t &= \frac{m_{\mu_p}}{4} \sin 2\theta_L \left[\frac{1}{D^2} \frac{g_d}{m_{\gamma_d}} \sin \theta_s \left(1 - \frac{m_\mu^2}{m_{\mu_p}^2} \right) \left(\sin^2 \theta_L - \frac{m_\mu}{m_{\mu_p}} \cos^2 \theta_L \right) + \frac{1}{v_{EW}} \cos \theta_s \left(1 + \frac{m_\mu}{m_{\mu_p}} \right) \right] \\ c_p^t &= -\frac{m_{\mu_p}}{4} \sin 2\theta_L \left[\frac{1}{D^2} \frac{g_d}{m_{\gamma_d}} \sin \theta_s \left(1 - \frac{m_\mu^2}{m_{\mu_p}^2} \right) \left(\sin^2 \theta_L + \frac{m_\mu}{m_{\mu_p}} \cos^2 \theta_L \right) + \frac{1}{v_{EW}} \cos \theta_s \left(1 - \frac{m_\mu}{m_{\mu_p}} \right) \right] \\ c_V^t &= \frac{g_d}{2} \cos \theta_L \sin \theta_L \left(\frac{m_\mu}{m_{\mu_p}} \frac{1}{D^2} + 1 \right) \\ c_A^t &= \frac{g_d}{2} \cos \theta_L \sin \theta_L \left(\frac{m_\mu}{m_{\mu_p}} \frac{1}{D^2} - 1 \right) \end{aligned}$$

The amplitude of the u -channel diagram is given by

$$i\mathcal{M}_u = -i \left[\bar{v}_{s_2}(k_2) (c_s^u + c_p^u \gamma^5) \frac{(\not{p}_1 - \not{k}_2 + m_{\mu_p})}{(p_1 - k_2)^2 - m_{\mu_p}^2} \gamma^\mu (c_V^u + c_A^u \gamma^5) \epsilon_\mu^{*\lambda}(\vec{p}_2) u_{s_1}(k_1) \right] \quad (B2)$$

where c_s^u, c_p^u, c_V^u , and c_A^u are given by

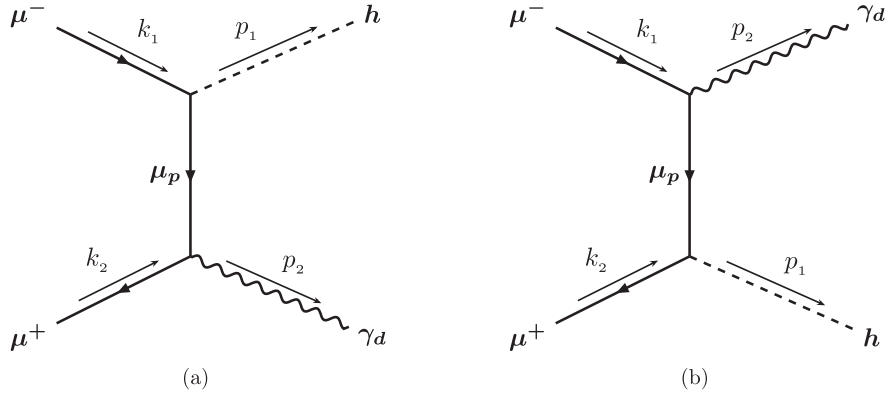


FIG. 26. Feynman diagrams for the process $\mu^+ \mu^- \rightarrow h \gamma_d$ mediated by the portal matter m_{μ_p} through the (a) t -channel and (b) u -channel.

$$\begin{aligned}
c_s^u &= \frac{m_{\mu_p}}{4} \sin 2\theta_L \left[\frac{1}{D^2} \frac{g_d}{m_{\gamma_d}} \sin \theta_s \left(1 - \frac{m_\mu^2}{m_{\mu_p}^2} \right) \left(\sin^2 \theta_L - \frac{m_\mu}{m_{\mu_p}} \cos^2 \theta_L \right) + \frac{1}{v_{\text{EW}}} \cos \theta_s \left(1 + \frac{m_\mu}{m_{\mu_p}} \right) \right] \\
c_p^u &= \frac{m_{\mu_p}}{4} \sin 2\theta_L \left[\frac{1}{D^2} \frac{g_d}{m_{\gamma_d}} \sin \theta_s \left(1 - \frac{m_\mu^2}{m_{\mu_p}^2} \right) \left(\sin^2 \theta_L + \frac{m_\mu}{m_{\mu_p}} \cos^2 \theta_L \right) + \frac{1}{v_{\text{EW}}} \cos \theta_s \left(1 - \frac{m_\mu}{m_{\mu_p}} \right) \right] \\
c_V^u &= \frac{g_d}{2} \cos \theta_L \sin \theta_L \left(\frac{m_\mu}{m_{\mu_p}} \frac{1}{D^2} + 1 \right) \\
c_A^u &= \frac{g_d}{2} \cos \theta_L \sin \theta_L \left(\frac{m_\mu}{m_{\mu_p}} \frac{1}{D^2} - 1 \right).
\end{aligned}$$

The spin-averaged squared amplitude, neglecting the s channel dark photon mediated and t/u channel muon mediated contribution, is

$$|\overline{\mathcal{M}}|^2 = |\overline{\mathcal{M}_t}|^2 + |\overline{\mathcal{M}_u}|^2 + 2\text{Re}(\overline{\mathcal{M}_t} \overline{\mathcal{M}_u}^*) \quad (\text{B3})$$

where the t -channel contribution in the limit $m_\mu \rightarrow 0$ is

$$\begin{aligned}
|\overline{\mathcal{M}_t}|^2 &= \frac{2}{[(k_1 - p_1)^2 - m_{\mu_p}^2]^2} \left[A_1 \{ 2(k_1 \cdot p_1)(k_2 \cdot p_1) - p_1^2(k_1 \cdot k_2) \} + A_2 m_{\mu_p}^2 (k_1 \cdot k_2) \right. \\
&\quad + \frac{1}{m_{\gamma_d}^2} \left(A_1 \left\{ 2(k_1 \cdot p_1)(k_2 \cdot p_2)(p_1 \cdot p_2) - p_1^2(k_1 \cdot p_2)(k_2 \cdot p_2) - p_2^2(k_1 \cdot p_1)(k_2 \cdot p_1) \right. \right. \\
&\quad \left. \left. + \frac{p_1^2 p_2^2}{2} (k_1 \cdot k_2) \right\} + A_2 m_{\mu_p}^2 \left\{ (k_1 \cdot p_2)(k_2 \cdot p_2) - \frac{p_2^2}{2} (k_1 \cdot k_2) \right\} \right) \left. \right] \quad (\text{B4})
\end{aligned}$$

Similarly, the u -channel contribution in the same limit reads

$$\begin{aligned}
|\overline{\mathcal{M}_u}|^2 &= \frac{2}{[(p_1 - k_2)^2 - m_{\mu_p}^2]^2} \left[A_1 \{ 2(k_1 \cdot p_1)(k_2 \cdot p_1) - p_1^2(k_1 \cdot k_2) \} + A_2 m_{\mu_p}^2 (k_1 \cdot k_2) \right. \\
&\quad + \frac{1}{m_{\gamma_d}^2} \left(A_1 \left\{ 2(k_1 \cdot p_2)(k_2 \cdot p_1)(p_1 \cdot p_2) - p_1^2(k_1 \cdot p_2)(k_2 \cdot p_2) - p_2^2(k_1 \cdot p_1)(k_2 \cdot p_1) \right. \right. \\
&\quad \left. \left. + \frac{p_1^2 p_2^2}{2} (k_1 \cdot k_2) \right\} + A_2 m_{\mu_p}^2 \left\{ (k_1 \cdot p_2)(k_2 \cdot p_2) - \frac{p_2^2}{2} (k_1 \cdot k_2) \right\} \right) \left. \right] \quad (\text{B5})
\end{aligned}$$

Here, the coefficients A_1 and A_2 are as follows

$$\begin{aligned}
A_1 &= (c_V^2 + c_A^2)(c_s^2 + c_p^2) - 4c_s c_p c_V c_A \\
A_2 &= (c_V^2 + c_A^2)(c_s^2 + c_p^2) + 4c_s c_p c_V c_A
\end{aligned} \quad (\text{B6})$$

with the coupling identifications

$$c_s = c_s^t = c_s^u, \quad c_p = c_p^t = -c_p^u, \quad c_V = c_V^t = c_V^u, \quad c_A = c_A^t = c_A^u.$$

The corresponding interference term is

$$\begin{aligned} \overline{\mathcal{M}_t \mathcal{M}_u^*} = & \frac{2}{[(k_1 - p_1)^2 - m_{\mu_p}^2][(p_1 - k_2)^2 - m_{\mu_p}^2]} \left[2A_3(k_1 \cdot p_1)(k_2 \cdot p_1) + A_2 m_{\mu_p}^2 (k_1 \cdot k_2) \right. \\ & - \frac{1}{m_{\gamma_d}^2} \left(A_3 \left\{ p_1^2(k_1 \cdot p_2)(k_2 \cdot p_2) + p_2^2(k_1 \cdot p_1)(k_2 \cdot p_1) - (p_1 \cdot p_2)(k_1 \cdot p_2)(k_2 \cdot p_1) \right. \right. \\ & \left. \left. - (p_1 \cdot p_2)(k_1 \cdot p_1)(k_2 \cdot p_2) + (p_1 \cdot p_2)^2(k_1 \cdot k_2) - \frac{p_1^2 p_2^2}{2}(k_1 \cdot k_2) \right\} \right. \\ & \left. \left. + \frac{A_2}{2} m_{\mu_p}^2 \{ p_2^2(k_1 \cdot k_2) - (k_1 \cdot p_2)(k_2 \cdot p_2) \} \right) \right] \end{aligned} \quad (\text{B7})$$

where the coefficient A_3 takes the form

$$A_3 = (c_s^2 - c_p^2)(c_V^2 - c_A^2) \quad (\text{B8})$$

In the center-of-mass frame and in the limit $m_h, m_{\gamma_d} \ll E_{\text{CM}}$, the $|\overline{\mathcal{M}}|^2$ for the pure t - and u - channels and the corresponding interference contribution take the form

$$\begin{aligned} |\overline{\mathcal{M}_t}|^2 = & \frac{E_{\text{CM}}^2}{(a - b \cos \theta)^2} \left[A_1 \left(\frac{E_{\text{CM}}^2}{4} \sin^2 \theta - m_h^2 \right) + A_2 m_{\mu_p}^2 + \frac{1}{2m_{\gamma_d}^2} \left(A_1 \left\{ \frac{E_{\text{CM}}^4}{4} (1 - \cos \theta)^2 \right. \right. \right. \\ & \left. \left. - \frac{(m_h^2 + m_{\gamma_d}^2) E_{\text{CM}}^2}{4} \sin^2 \theta + m_h^2 m_{\gamma_d}^2 \right\} + A_2 m_{\mu_p}^2 \left\{ \frac{E_{\text{CM}}^2}{4} \sin^2 \theta - m_{\gamma_d}^2 \right\} \right) \right] \end{aligned} \quad (\text{B9})$$

$$\begin{aligned} |\overline{\mathcal{M}_u}|^2 = & \frac{E_{\text{CM}}^2}{(a + b \cos \theta)^2} \left[A_1 \left(\frac{E_{\text{CM}}^2}{4} \sin^2 \theta - m_h^2 \right) + A_2 m_{\mu_p}^2 + \frac{1}{2m_{\gamma_d}^2} \left(A_1 \left\{ \frac{E_{\text{CM}}^4}{4} (1 + \cos \theta)^2 \right. \right. \right. \\ & \left. \left. - \frac{(m_h^2 + m_{\gamma_d}^2) E_{\text{CM}}^2}{4} \sin^2 \theta + m_h^2 m_{\gamma_d}^2 \right\} + A_2 m_{\mu_p}^2 \left\{ \frac{E_{\text{CM}}^2}{4} \sin^2 \theta - m_{\gamma_d}^2 \right\} \right) \right] \end{aligned} \quad (\text{B10})$$

$$\begin{aligned} \overline{\mathcal{M}_t \mathcal{M}_u^*} = & \frac{E_{\text{CM}}^2}{(a^2 - b^2 \cos^2 \theta)} \left[\left(A_3 \frac{E_{\text{CM}}^2}{4} \sin^2 \theta + A_2 m_{\mu_p}^2 \right) + \frac{1}{2m_{\gamma_d}^2} \left(A_3 \left\{ m_h^2 m_{\gamma_d}^2 \right. \right. \right. \\ & \left. \left. - \frac{(E_{\text{CM}}^2 + m_h^2 + m_{\gamma_d}^2) E_{\text{CM}}^2}{4} \sin^2 \theta \right\} + A_2 m_{\mu_p}^2 \left\{ \frac{E_{\text{CM}}^2}{8} \sin^2 \theta - m_{\gamma_d}^2 \right\} \right) \right] \end{aligned} \quad (\text{B11})$$

Here,

$$a = \frac{E_{\text{CM}}^2}{2} + m_{\mu_p}^2, \quad b = \frac{E_{\text{CM}}^2}{2}.$$

-
- [1] G. Aad *et al.* (ATLAS Collaboration), Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC, *Phys. Lett. B* **716**, 1 (2012).
- [2] S. Chatrchyan *et al.* (CMS Collaboration), Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC, *Phys. Lett. B* **716**, 30 (2012).
- [3] W. Kilian, M. Kramer, and P. M. Zerwas, Higgsstrahlung and W W fusion in e+ e- collisions, *Phys. Lett. B* **373**, 135 (1996).
- [4] Z. Liu, L.-T. Wang, and H. Zhang, Exotic decays of the 125 GeV Higgs boson at future e^+e^- lepton colliders, *Chin. Phys. C* **41**, 063102 (2017).
- [5] A. G. Akeroyd, A. Arhrib, and M. Capdequi Peyranere, CP odd Higgs boson production in association with neutral

- gauge boson in high-energy e^+e^- collisions, *Mod. Phys. Lett. A* **14**, 2093 (1999).
- [6] S. Kanemura, Possible enhancement of the $e^+e^- \rightarrow H^+W^-$ cross-section in the two Higgs doublet model, *Eur. Phys. J. C* **17**, 473 (2000).
- [7] B. A. Ouazghour, A. Arhrib, K. Cheung, E.-s. Ghourmin, and L. Rahili, Associated charged Higgs boson production within the 2HDM: e^+e^- versus $\mu^+\mu^-$ colliders, *Phys. Rev. D* **110**, 095026 (2024).
- [8] R. Barate *et al.* (LEP Working Group for Higgs boson searches Collaboration, ALEPH Collaboration, DELPHI Collaboration, L3 Collaboration, and OPAL Collaboration), Search for the standard model Higgs boson at LEP, *Phys. Lett. B* **565**, 61 (2003).
- [9] G. Abbiendi *et al.* (OPAL Collaboration), Search for the standard model Higgs boson with the OPAL detector at LEP, *Eur. Phys. J. C* **26**, 479 (2003).
- [10] A. Abada *et al.* (FCC Collaboration), FCC-ee: The Lepton Collider: Future circular collider conceptual design report volume 2, *Eur. Phys. J. Special Topics* **228**, 261 (2019).
- [11] H. Abramowicz *et al.*, Higgs physics at the CLIC electron-positron linear collider, *Eur. Phys. J. C* **77**, 475 (2017).
- [12] R. Franceschini and M. Greco, Higgs and BSM physics at the future muon collider, *Symmetry* **13**, 851 (2021).
- [13] Y. Hamada, R. Kitano, R. Matsudo, S. Okawa, R. Takai, H. Takaura, and L. Treuer, Higgs boson production at $\mu^+\mu^-$ colliders, *Phys. Rev. D* **110**, 113011 (2024).
- [14] E. J. Chun and T. Mondal, Explaining $g-2$ anomalies in two Higgs doublet model with vector-like leptons, *J. High Energy Phys.* **11** (2020) 077.
- [15] P. Asadi, S. Homiller, A. Radick, and T.-T. Yu, Fermion-portal dark matter at a high-energy muon collider, *Phys. Rev. D* **112**, 055038 (2025).
- [16] A. Crivellin, F. Kirk, C. A. Manzari, and M. Montull, Global electroweak fit and vector-like leptons in light of the Cabibbo angle anomaly, *J. High Energy Phys.* **12** (2020) 166.
- [17] M. Kirk, Cabibbo anomaly versus electroweak precision tests: An exploration of extensions of the Standard Model, *Phys. Rev. D* **103**, 035004 (2021).
- [18] G. N. Wojcik, L. L. Everett, S. T. Eu, and R. Ximenes, Lepton flavor portal matter, *Phys. Rev. D* **108**, 055033 (2023).
- [19] Z. Poh and S. Raby, Vectorlike leptons: Muon $g-2$ anomaly, lepton flavor violation, Higgs boson decays, and lepton nonuniversality, *Phys. Rev. D* **96**, 015032 (2017).
- [20] R. Dermisek, K. Hermanek, and N. McGinnis, Muon $g-2$ in two-Higgs-doublet models with vectorlike leptons, *Phys. Rev. D* **104**, 055033 (2021).
- [21] H. M. Lee, J. Song, and K. Yamashita, Seesaw lepton masses and muon $g-2$ from heavy vector-like leptons, *J. Korean Phys. Soc.* **79**, 1121 (2021).
- [22] J. Kawamura, S. Raby, and A. Trautner, Complete vectorlike fourth family with $U(1)'$: A global analysis, *Phys. Rev. D* **101**, 035026 (2020).
- [23] M. Endo and S. Mishima, Muon $g-2$ and CKM unitarity in extra lepton models, *J. High Energy Phys.* **08** (2020) 004.
- [24] C. A. Manzari, Vector-like leptons in light of the Cabibbo-angle anomaly, [arXiv:2105.03399](https://arxiv.org/abs/2105.03399).
- [25] B. Capdevila, A. Crivellin, C. A. Manzari, and M. Montull, Explaining $b \rightarrow s\ell^+\ell^-$ and the Cabibbo angle anomaly with a vector triplet, *Phys. Rev. D* **103**, 015032 (2021).
- [26] W. Abdallah, M. Ashry, J. Kawamura, and A. Moursy, Semivisible dark photon in a model with vectorlike leptons for the $(g-2)_e$, μ and W-boson mass anomalies, *Phys. Rev. D* **109**, 015031 (2024).
- [27] H. M. Lee and K. Yamashita, A model of vector-like leptons for the muon $g-2$ and the W boson mass, *Eur. Phys. J. C* **82**, 661 (2022).
- [28] C.-H. Lee and R. N. Mohapatra, Vector-like quarks and leptons, $SU(5) \otimes SU(5)$ grand unification, and proton decay, *J. High Energy Phys.* **02** (2017) 080.
- [29] F. Gursev, P. Ramond, and P. Sikivie, A universal gauge theory model based on E_6 , *Phys. Lett. B* **60**, 177 (1976).
- [30] B. Kyae and C. S. Shin, Vector-like leptons and extra gauge symmetry for the natural Higgs boson, *J. High Energy Phys.* **06** (2013) 102.
- [31] M. Cvetič, G. Shiu, and A. M. Uranga, Three family supersymmetric standard-like models from intersecting brane worlds, *Phys. Rev. Lett.* **87**, 201801 (2001).
- [32] G. B. Cleaver, A. E. Faraggi, D. V. Nanopoulos, and J. W. Walker, Phenomenological study of a minimal superstring standard model, *Nucl. Phys. B* **593**, 471 (2001).
- [33] F. Marchesano, D. Regalado, and L. Vazquez-Mercado, Discrete flavor symmetries in D-brane models, *J. High Energy Phys.* **09** (2013) 028.
- [34] G. Aad *et al.* (ATLAS Collaboration), Search for vector-like leptons coupling to first- and second-generation Standard Model leptons in pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector, *J. High Energy Phys.* **05** (2025) 075.
- [35] A. Tumasyan *et al.* (CMS Collaboration), Search for pair-produced vector-like leptons in final states with third-generation leptons and at least three b quark jets in proton-proton collisions at $\sqrt{s} = 13$ TeV, *Phys. Lett. B* **846**, 137713 (2023).
- [36] A. M. Sirunyan *et al.* (CMS Collaboration), Search for vector-like leptons in multilepton final states in proton-proton collisions at $\sqrt{s} = 13$ TeV, *Phys. Rev. D* **100**, 052003 (2019).
- [37] C.-X. Yue, Y.-Q. Wang, H. Wang, Y.-H. Wang, and S. Li, Searching for singlet vector-like leptons via pair production at ILC, *Nucl. Phys. B* **1000**, 116482 (2024).
- [38] L. Shang, M. Wang, Z. Heng, and B. Yang, Search for the singlet vector-like lepton at future e^+e^- colliders, *Eur. Phys. J. C* **81**, 415 (2021).
- [39] Q. Guo, L. Gao, Y. Mao, and Q. Li, Vector-like lepton searches at a muon collider in the context of the 4321 model, *Chin. Phys. C* **47**, 103106 (2023).
- [40] N. Ghosh, S. K. Rai, and T. Samui, Search for a leptoquark and vector-like lepton in a muon collider, *Nucl. Phys. B* **1004**, 116564 (2024).
- [41] P. N. Bhattiprolu and S. P. Martin, Prospects for vectorlike leptons at future proton-proton colliders, *Phys. Rev. D* **100**, 015033 (2019).

- [42] T. G. Rizzo, Portal matter and dark sector phenomenology at colliders, Report No. SLAC-PUB-17633, 2022.
- [43] T. D. Rueter and T. G. Rizzo, Towards a UV-model of kinetic mixing and portal matter, *Phys. Rev. D* **101**, 015014 (2020).
- [44] T. G. Rizzo, Kinetic mixing and portal matter phenomenology, *Phys. Rev. D* **99**, 115024 (2019).
- [45] A. Belyaev, L. Panizzi, N. Thongyoi, and F. Wilhelm, The muonic portal to vector dark matter: Connecting precision muon physics, cosmology, and colliders, *J. High Energy Phys.* **06** (2026) 240.
- [46] J. Lahiri, D. Pradhan, and A. Sarkar, The influence of lepton portal on the WIMP-pFIMP framework, *J. High Energy Phys.* **08** (2025) 019.
- [47] J. P. Delahaye, M. Diemoz, K. Long, B. Mansoulié, N. Pastrone, L. Rivkin *et al.*, Muon Colliders, [arXiv:1901.06150](#).
- [48] D. Schulte (International Muon Collider Collaboration), The muon collider, JACoW IPAC2022 (2022) 821, [10.18429/JACoW-IPAC2022-TUIZSP2](#).
- [49] K. M. Black *et al.*, Muon Collider Forum report, *J. Instrum.* **19**, T02015 (2024).
- [50] C. Accettura *et al.* (International Muon Collider Collaboration), Interim report for the International Muon Collider Collaboration (IMCC), *CERN Yellow Rep. Monogr.* **2/2024**, 176 (2024).
- [51] B. Holdom, Searching for ϵ charges and a new U(1), *Phys. Lett. B* **178**, 65 (1986).
- [52] P. Fayet, Extra U(1)'s and new forces, *Nucl. Phys.* **B347**, 743 (1990).
- [53] M. Fabbrichesi, E. Gabrielli, and G. Lanfranchi, *The Dark Photon* (Springer, Cham, Switzerland, 2021).
- [54] J. Alexander *et al.*, Dark sectors 2016 workshop: Community report, tech. rep., Fermi National Accelerator Laboratory (FNAL), Batavia, IL (United States), 2016.
- [55] K. S. Babu, C. F. Kolda, and J. March-Russell, Implications of generalized Z-Z-prime mixing, *Phys. Rev. D* **57**, 6788 (1998).
- [56] J. H. Kim, S. D. Lane, H.-S. Lee, I. M. Lewis, and M. Sullivan, Searching for dark photons with maverick top partners, *Phys. Rev. D* **101**, 035041 (2020).
- [57] M. Bauer, P. Foldenauer, and J. Jaeckel, Hunting all the hidden Photons, *J. High Energy Phys.* **07** (2018) 094.
- [58] D. He *et al.* (APEX Collaboration), Dark photon constraints from a 7.139 GHz cavity haloscope experiment, *Phys. Rev. D* **110** (2024) L021101.
- [59] T. Robens and T. Stefaniak, Status of the Higgs singlet extension of the Standard Model after LHC run 1, *Eur. Phys. J. C* **75**, 104 (2015).
- [60] T. Robens, Investigating extended scalar sectors at current and future colliders, *Proc. Sci. LHCP2019* 138 (2019).
- [61] T. Robens, More Doublets and Singlets, Report No. RBI-ThPhys-2022-16, 2022.
- [62] S. Adhikari, S. D. Lane, I. M. Lewis, and M. Sullivan, Complex scalar singlet model benchmarks for Snowmass, [arXiv:2203.07455](#).
- [63] S. D. Lane, I. M. Lewis, and M. Sullivan, Resonant multiscalar production in the generic complex singlet model in the multi-TeV region, *Phys. Rev. D* **110**, 055017 (2024).
- [64] T. Robens, Constraining extended scalar sectors at current and future colliders—an update, *Springer Proc. Phys.* **292**, 141 (2023).
- [65] T. Robens and R. Santos, BSM: Extended scalar sectors, tech. rep., Boskovic Inst., Zagreb, CERN, 2025.
- [66] G. Aad *et al.* (ATLAS Collaboration), Combination of searches for invisible decays of the Higgs boson using 139 fb⁻¹ of proton-proton collision data at $\sqrt{s}=13$ TeV collected with the ATLAS experiment, *Phys. Lett. B* **842**, 137963 (2023).
- [67] A. Tumasyan *et al.* (CMS Collaboration), A search for decays of the Higgs boson to invisible particles in events with a top-antitop quark pair or a vector boson in proton-proton collisions at $\sqrt{s}=13$ TeV, *Eur. Phys. J. C* **83**, 933 (2023).
- [68] A. Papaefstathiou, T. Robens, and G. White, Signal strength and W-boson mass measurements as a probe of the electro-weak phase transition at colliders—Snowmass White Paper, [arXiv:2205.14379](#).
- [69] A. Hayrapetyan *et al.* (CMS Collaboration), Combination of searches for nonresonant Higgs boson pair production in proton-proton collisions at $\sqrt{s}=13$ TeV, [arXiv:2510.07527](#).
- [70] A. Cheng (ATLAS Collaboration), Combination of searches for resonant Higgs boson pair production using pp collisions at $\sqrt{s}=13$ TeV with the ATLAS detector, *Proc. Sci. LHCP2024* (2025) 248.
- [71] D. Buttazzo, F. Sala, and A. Tesi, Singlet-like Higgs bosons at present and future colliders, *J. High Energy Phys.* **11** (2015) 158.
- [72] A. Sopczak, Search for new particles at LEP, *Acta Phys. Pol. B* **30**, 3229 (1999).
- [73] A. Tumasyan *et al.* (CMS Collaboration), Inclusive non-resonant multilepton probes of new phenomena at $\sqrt{s}=13$ TeV, *Phys. Rev. D* **105**, 112007 (2022).
- [74] G. Aad *et al.* (ATLAS Collaboration), Search for third-generation vector-like leptons in pp collisions at $\sqrt{s}=13$ TeV with the ATLAS detector, *J. High Energy Phys.* **07** (2023) 118.
- [75] V. Chekhovsky *et al.* (CMS Collaboration), Search for vector-like leptons with long-lived particle decays in the CMS muon system in proton-proton collisions at $\sqrt{s}=13$ TeV, *J. High Energy Phys.* **08** (2025) 156.
- [76] G. Aad *et al.* (ATLAS Collaboration), Combination of searches for resonant Higgs boson pair production using pp collisions at $\sqrt{s}=13$ TeV with the ATLAS detector, *Phys. Rev. Lett.* **132**, 231801 (2024).
- [77] S. Schael *et al.* (ALEPH Collaboration, DELPHI Collaboration, L3 Collaboration, OPAL Collaboration, SLD Collaboration, LEP Electroweak Working Group, SLD Electroweak Group, and SLD Heavy Flavour Group), Precision electroweak measurements on the Z resonance, *Phys. Rep.* **427**, 257 (2006).
- [78] C.-Y. Chen, S. Dawson, and E. Furlan, Vectorlike fermions and Higgs effective field theory revisited, *Phys. Rev. D* **96**, 015006 (2017).
- [79] S. Navas *et al.* (Particle Data Group), Review of particle physics, *Phys. Rev. D* **110**, 030001 (2024).

- [80] D. P. Aguillard *et al.* (Muon g-2 Collaboration), Measurement of the positive muon anomalous magnetic moment to 127 ppb, *Phys. Rev. Lett.* **135**, 101802 (2025).
- [81] G. W. Bennett *et al.* (Muon g-2 Collaboration), Final report of the muon E821 anomalous magnetic moment measurement at BNL, *Phys. Rev. D* **73**, 072003 (2006).
- [82] R. Aliberti *et al.*, The anomalous magnetic moment of the muon in the Standard Model: An update, *Phys. Rep.* **1143**, 1 (2025).
- [83] N. Aghanim *et al.* (Planck Collaboration), Planck 2018 results. VI. Cosmological parameters, *Astron. Astrophys.* **641**, A6 (2020).
- [84] S. Mahapatra, P. K. Paul, and N. Sahu, Forbidden dark matter assisted by first-order phase transition and associated gravitational waves, [arXiv:2601.12319](https://arxiv.org/abs/2601.12319).
- [85] P. Huang, A. D. Medina, and C. E. M. Wagner, Rescuing overabundant dark matter with a strongly first order phase transition in the dark sector, [arXiv:2602.16822](https://arxiv.org/abs/2602.16822).
- [86] A. Alloul, N. D. Christensen, C. Degrande, C. Duhr, and B. Fuks, Feynrules 2.0—a complete toolbox for tree-level phenomenology, *Comput. Phys. Commun.* **185**, 2250 (2014).
- [87] G. Alguero, G. Belanger, F. Boudjema, S. Chakraborti, A. Goudelis, S. Kraml, A. Mjallal, and A. Pukhov, micrOMEGAs 6.0: N-component dark matter, *Comput. Phys. Commun.* **299**, 109133 (2024).
- [88] E. Aprile *et al.* (XENON Collaboration), Dark matter search results from a one Ton-Year exposure of XENON1T, *Phys. Rev. Lett.* **121**, 111302 (2018).
- [89] E. Aprile *et al.* (XENON Collaboration), WIMP dark matter search using a 3.1 Tonne-Year exposure of the XENONnT experiment, *Phys. Rev. Lett.* **135**, 221003 (2025).
- [90] J. Aalbers *et al.* (LZ Collaboration), Dark matter search results from 4.2 Tonne-Years of exposure of the LUX-ZEPLIN (LZ) experiment, *Phys. Rev. Lett.* **135**, 011802 (2025).
- [91] C. Degrande, C. Duhr, B. Fuks, D. Grellscheid, O. Mattelaer, and T. Reiter, UFO-The universal FeynRules output, *Comput. Phys. Commun.* **183**, 1201 (2012).
- [92] J. Alwall, R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer, H.-S. Shao, T. Stelzer, P. Torrielli, and M. Zaro, The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations, *J. High Energy Phys.* **07** (2014) 079.
- [93] C. Bierlich *et al.*, A comprehensive guide to the physics and usage of Pythia8.3, *SciPost Phys. Codebases* **2022**, 8 (2022).
- [94] T. H. Park, Jet energy resolution measurement of the atlas detector using momentum balance, 2018.
- [95] V. Khachatryan *et al.* (CMS Collaboration), Jet energy scale and resolution in the CMS experiment in pp collisions at 8 TeV, *J. Instrum.* **12**, P02014 (2017).
- [96] I. Pezzotti *et al.*, Dual-readout calorimetry for future experiments probing fundamental physics, [arXiv:2203.04312](https://arxiv.org/abs/2203.04312).
- [97] M. Cacciari, G. P. Salam, and G. Soyez, The anti- k_t jet clustering algorithm, *J. High Energy Phys.* **04** (2008) 063.
- [98] M. Cacciari, G. P. Salam, and G. Soyez, FastJet user manual, *Eur. Phys. J. C* **72**, 1896 (2012).
- [99] J. M. Butterworth, A. R. Davison, M. Rubin, and G. P. Salam, Jet substructure as a new Higgs search channel at the LHC, *Phys. Rev. Lett.* **100**, 242001 (2008).
- [100] S. Bentvelsen and I. Meyer, The Cambridge jet algorithm: Features and applications, *Eur. Phys. J. C* **4**, 623 (1998).
- [101] G. Cowan, K. Cranmer, E. Gross, and O. Vitells, Asymptotic formulae for likelihood-based tests of new physics, *Eur. Phys. J. C* **71**, 1554 (2011).
- [102] N. Kumar and S. P. Martin, Vectorlike leptons at the large hadron collider, *Phys. Rev. D* **92**, 115018 (2015).
- [103] P. N. Bhattiprolu, S. P. Martin, and J. D. Wells, Criteria for projected discovery and exclusion sensitivities of counting experiments, *Eur. Phys. J. C* **81**, 123 (2021).