

No warm inflation from a vanilla axion

Sebastian Zell^{✉*}

Arnold Sommerfeld Center, *Ludwig-Maximilians-Universität*,
Theresienstraße 37, 80333 München, Germany,
Max-Planck-Institut für Physik, Boltzmannstraße 8, 85748 Garching bei München, Germany,
and Centre for Cosmology, Particle Physics and Phenomenology—CP3,
Université catholique de Louvain, B-1348 Louvain-la-Neuve, Belgium



(Received 18 October 2024; accepted 24 September 2025; published 17 October 2025)

At finite temperature, the coupling of an axion to non-Abelian gauge fields causes dissipation due to sphaleron heating. This mechanism has been considered as ideal for realizing warm inflation, since it can lead to large thermal friction while preserving the flatness of the potential. We show, however, that requiring standard properties of an axion—in particular, a discrete shift symmetry—excludes the strong regime of warm slow-roll inflation, in which thermal friction dominates. The present argument, which does not rely on any phenomenological input, leaves room for the weak regime of warm axion inflation, but in this case a super-Planckian decay constant represents a well-known issue. Finally, we discuss nonminimal and axionlike models as a way out.

DOI: [10.1103/kp15-3s1c](https://doi.org/10.1103/kp15-3s1c)

Cold vs warm inflation. There is overwhelming evidence that a hot big bang (BB) took place about 14 billion years ago. At this point, our Universe was small, dense, and filled with a nearly homogeneous plasma. Subsequently, it expanded and cooled, and the initial inhomogeneities developed into the structures that we see today, such as galaxies, stars, and planets. This leads to a fundamental question about the cause of the BB. Arguably, the leading candidate to address it is inflation [1–4]—a phase of accelerated expansion preceding the BB. If it lasts long enough, corresponding to an amplification of space by a factor of at least $\sim e^{50}$, it can set the initial conditions for the BB while accounting for the observed homogeneity on macroscopic scales. The paradigm of inflation leads to predictions that are in excellent agreement with cosmological observations, in particular, of the cosmic microwave background [5,6].

The simplest way to realize inflation consists in endowing a scalar field φ —the so-called inflaton—with a large potential energy V (see [7–10] for reviews). Key for maintaining accelerated expansion over an extended period of time and for achieving at least 50 e-folds of spatial amplification is a source of friction that slows down the movement of the inflaton along its potential. The original

proposals [1–4] relied on Hubble friction, i.e., the fact that φ is decelerated in the background of an expanding Universe. This effect is efficient only if the potential is sufficiently flat (as measured in units of the Planck mass M_P). However, such potentials are regarded as problematic, since it has proven difficult to maintain flatness in the presence of quantum corrections (see [11–15]).

Addressing this criticism against inflation, a second possibility for achieving slow roll was developed: In *warm inflation*, the main source of friction is the interaction with a thermal bath [16,17], which allows for inflationary expansion on much steeper potentials. However, warm inflation comes with its own challenges. The same effect that generates a sufficiently large thermal friction generically also causes strong corrections to the potential, again spoiling inflation [18].

The leading solution to this issue is provided by using the axion of a non-Abelian gauge group as inflaton.¹ In this case, sphalerons [20,21] lead to an effect dubbed *sphaleron heating* that is able to induce a large thermal friction without causing relevant changes in the potential [22–26]. Therefore, sphaleron heating has come to be considered as ideal mechanism for driving warm inflation [22–29]. However, difficulties have already been observed in reconciling warm axion inflation with standard properties of an axion, especially its discrete shift symmetry [25]. *This motivates us to show that demanding friction to be dominated by sphaleron heating together with the*

*Contact author: sebastian.zell@lmu.de

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

¹As originally proposed [19], this idea of natural inflation can also be realized at zero temperature.

requirement of preserving the usual properties of a “vanilla” axion, in fact, excludes slow-roll inflation.

First, we shall review warm axion inflation driven by sphaleron heating, before the class of theories under consideration and our main statement about the incompatibility with slow-roll inflation will be made precise starting from Eq. (10). The expert reader may choose to skip forward to this point. After the subsequent simple proof of our claim culminating in Eq. (20), we will show that the present argument does not exclude the so-called weak regime of warm inflation, in which thermal friction does not dominate. In this case, however, a super-Planckian decay constant represents a well-known issue. Finally, we will comment on “nonvanilla” (axionlike) models and directions forward.

Vanilla scenario of warm axion inflation. Once effects of a finite temperature T are included, the equation of motion of the inflaton is (with dot and prime denoting derivatives with respect to time and φ , respectively, and recent reviews in [30,31])

$$V' + (3H + \Upsilon(T))\dot{\varphi} = -\ddot{\varphi}, \quad (1)$$

where H is the Hubble scale while $\Upsilon(T)$ corresponds to the coefficient of thermal dissipation. Defining

$$Q \equiv \frac{\Upsilon(T)}{3H}, \quad (2)$$

we see that the main source of friction is thermal if $Q > 1$ —this is the so-called *strong regime* of warm inflation [23] on which we shall focus. It is important to remark that a finite temperature can be relevant even if $Q < 1$: As long as $T \gtrsim H$, perturbations are strongly influenced by thermal effects, which corresponds to the *weak regime* of warm inflation [23]. For all temperatures, friction-dominated motion, leading to $\ddot{\varphi} \approx 0$ and accelerated cosmological expansion, is achieved as long as the first slow-roll parameter

$$\epsilon \equiv \frac{1}{1+Q} \frac{M_P^2 V'^2}{2V^2} \quad (3)$$

is smaller than 1. Clearly, thermal effects with $Q \gg 1$ allow for inflation on much steeper potentials as opposed to the cold regime.

The axion, which was originally introduced for a dynamical mechanism to address the strong CP problem of QCD [32–35] (see [36] for a review), is defined by its coupling to the topological density of a non-Abelian gauge group $SU(N_c)$:

$$\frac{\varphi}{f} \frac{\alpha \text{Tr} G_{\mu\nu} \tilde{G}^{\mu\nu}}{4\pi} \equiv \frac{\varphi}{f} q. \quad (4)$$

Here, f is the decay constant, α represents the gauge coupling, and $G_{\mu\nu}$ and $\tilde{G}^{\mu\nu}$ are the field strength of the gauge group and its dual, respectively. The vacuum structure of the $SU(N_c)$ gauge group is nontrivial [37,38], and, for sufficiently large T , sphalerons mediate unsuppressed transitions between different vacua [20]. In this background, the thermal average in the low-frequency regime yields [21,39]

$$-q = \frac{\Gamma_{\text{sph}}}{T} \dot{\varphi}, \quad \Gamma_{\text{sph}} = \tilde{c}_{\text{sph}} T^4. \quad (5)$$

Here, Γ_{sph} is the sphaleron rate, and its prefactor is suppressed as $\tilde{c}_{\text{sph}} \sim (\alpha N_c)^5$, where we did not display a factor of the order of 1 that depends weakly on other model parameters and dynamical quantities (see [23,25,40,41]). Plugging Eq. (5) into the equation of motion following from Eq. (4), we see that the resulting friction rate is [21,39]

$$\Upsilon(T) = \frac{\Gamma_{\text{sph}}}{f^2 T}, \quad (6)$$

corresponding to a process dubbed *sphaleron heating* [24]. Crucially, the fact that $\text{Tr} G_{\mu\nu} \tilde{G}^{\mu\nu}$ is a total derivative implies that the coupling (4) is shift symmetric, and so it cannot cause perturbative corrections to the flatness of the potential. Therefore, warm axion inflation is protected from dangerous quantum effects [22–26], which resolves the problem raised in [18].²

At the nonperturbative level (and independently of thermal effects), a coupling of the form (4) can induce a potential for the axion [32–35], thereby breaking the continuous shift invariance. However, the topological character of $G_{\mu\nu} \tilde{G}^{\mu\nu}$ ensures that a discrete shift symmetry is still preserved:

$$\varphi \rightarrow \varphi + 2\pi f. \quad (7)$$

This invariance represents a key property of the axion. First, it strongly constrains the form of the axion potential, e.g., by excluding simple polynomial contributions, thereby selecting the operator (4) as leading interaction. This is crucial for the predictivity of axion models and for distinguishing them from generic inflationary scenarios with *a priori* arbitrary potentials. Second, conventional axion models such as [44–47] identify φ/f as the phase of a complex Peccei-Quinn (PQ) field, and so the shift symmetry (7) must be obeyed by construction.

²In the absence of a shift symmetry, a large dissipation coefficient $\Upsilon(T)$ is almost inevitably accompanied by sizable finite-temperature contributions to the effective potential, since both effects come from the imaginary and real parts of the same diagram [42,43].

Below the confinement scale $\Lambda \equiv \sqrt{m\bar{f}}$, the nonperturbatively generated periodic potential can be approximated as³ [49]

$$V(\varphi) = 4m^2 f^2 \left(1 - \left| \cos\left(\frac{\varphi}{2f}\right) \right| \right), \quad (9)$$

where m is the mass of the axion. Evidently, inflation is impossible with simple QCD axion models, since $\Lambda_{\text{QCD}} \sim 100$ MeV certainly lies below inflationary energies and, moreover, the presence of light quarks is known to suppress sphaleron heating [21,22,50], although it is straightforward to construct models that feature sphaleron heating in the presence of light fermions [28]. Instead, Eq. (9) with parameters suitable for inflation can, e.g., arise from confinement above inflationary scales with an unbroken subgroup $SU(N_c)$ [25] or from heavy axion models coupled to an additional dark gauge group (see [51–60] and overview in [29]). Realizations of warm axion inflation involving dark gauge groups have been discussed in [22,61–66], with a concrete implementation of sphaleron heating in [22], while a model of a heavy QCD axion coupled to additional particles was proposed recently [29].

So far, we have not considered phenomenology. In the strong regime $Q > 1$, it is well known that periodic potentials of the form (9) tend to produce a blue scalar spectral index [22,67], which does not match the experimentally observed red tilt. In both [22,29], consistency with observations has been achieved in a hybrid scenario [68,69] of inflation.

Main statement. Leaving aside phenomenology, the above discussions imply that warm inflation driven by sphaleron heating with a vanilla axion is described the equation of motion

$$V' + \Upsilon(T)\dot{\varphi} \approx 0, \quad (10)$$

where the friction coefficient and potential obey the properties

$$\Upsilon(T) = \frac{\tilde{c}_{\text{sph}} T^3}{f^2}, \quad (11)$$

$$V(\varphi) \equiv m^2 f^2 \mathcal{V}(\varphi/f), \quad (12)$$

$$\mathcal{V}(\varphi/f + 2\pi) = \mathcal{V}(\varphi/f), \quad (13)$$

³An alternative, less accurate approximation is (see [19,48])

$$V(\varphi) = m^2 f^2 \left(1 - \cos\left(\frac{\varphi}{f}\right) \right), \quad (8)$$

which agrees with Eq. (9) up to the order of φ^2 . The difference between Eqs. (8) and (9) is inessential for the present discussion.

with $\mathcal{V}(\varphi/f) \lesssim 1$. Evidently, the above form of $\Upsilon(T)$ can be derived from sphaleron heating [cf. Eq. (5)], and Eq. (13) implements the crucial discrete shift symmetry (7). Matching the nonperturbative axion potential (9), we additionally assume $\mathcal{V} = 0$ and $\mathcal{V}'' \sim 1$ at the minimum of the potential so that the vacuum mass of φ is on the order of m , but this property is not essential for our argument. Although the requirements just described do not appear to be very constraining, we shall show that, in fact, they do not allow for slow-roll inflation; i.e., *sphaleron heating with a vanilla axion is unable to drive the strong regime of warm inflation*.

It is important to point out that the same decay constant f appears both in the friction rate (11) and the potential (12) (see also [25,27]). If we momentarily considered a different f_V in the potential, then $f_V > f$ is excluded, since a shift symmetry $\mathcal{V}(\varphi/f_V + 2\pi) = \mathcal{V}(\varphi/f_V)$ would not suffice for preserving invariance (7) under $\varphi \rightarrow \varphi + 2\pi f$. The opposite case $f_V < f$ may be admissible (provided f/f_V is integer), but this scenario is equivalent to a single decay constant f_V together with a smaller value of $\tilde{c}_{\text{sph}}$, and so our argument still applies.

We emphasize that our assumptions, as detailed in Eqs. (10)–(13), cover a wide range of scenarios. First, we do not assume that the gauge group is vanilla, thereby including among others scenarios of partial confinement [25], modified axion potentials [25], and heavy axion models [51–60]. Moreover, if the axion arises as phase of a complex PQ field, our conclusion still applies even if PQ symmetry is broken explicitly.⁴

For the following straightforward argument, we shall consider Eqs. (10)–(13) in the strong warm regime of slow-roll inflation, i.e., for $Q > 1$ and $\epsilon < 1$. We use the corresponding standard approximations, namely, slow roll in Eq. (10), a Hubble scale dominated by the potential, $H \approx \sqrt{V/3}/M_P$, and, moreover, neglect temperature-dependent contributions to the energy [22,24,28–30]. Our goal is to show this cannot lead to a sufficiently long period of inflation. As is well known, stationary sphaleron heating determines the energy density of the radiation bath to be $g_\star T^4 \pi^2/30 = \Upsilon(T)\dot{\varphi}^2/(4H)$, where $g_\star$ is the effective number of degrees of freedom, so that we can express

⁴For example, gravity is expected to violate PQ symmetry by operators of the form $(\exp(i\varphi/f))^n + \text{H.c.}$, which are suppressed by powers of M_P [70] (see, e.g., also [71,72]). These generate additional contributions to the axion potential [70–72]:

$$\Delta V \sim \tilde{\Lambda}^4 \left(1 - \cos\left(n \frac{\varphi}{f} + \delta\right) \right), \quad (14)$$

where $\tilde{\Lambda}$ is an energy scale and δ represents a phase. Evidently, the modified potential $V + \Delta V$ still obeys the discrete shift symmetry (7). The same is true if the explicit breaking of PQ symmetry happens through a nonminimal coupling to gravity [73,74].

temperature as (see [22,28])

$$T = \left(\frac{15\sqrt{3}f^2 M_p V'^2}{2\pi^2 g_\star \tilde{c}_{\text{sph}} \sqrt{V}} \right)^{1/7}. \quad (15)$$

Plugging this alongside $\Upsilon(T)$ into Eq. (2), we get

$$Q = \left(\frac{375\tilde{c}_{\text{sph}}^4 M_p^{10} V'^6}{8\pi^6 g_\star^3 f^8 V^5} \right)^{1/7}. \quad (16)$$

Now demanding $Q > 1$ leads to a lower bound on the derivative of the potential:

$$V' > \left(\frac{8\pi^6 g_\star^3 f^8 V^5}{375\tilde{c}_{\text{sph}}^4 M_p^{10}} \right)^{1/6}. \quad (17)$$

Only if the potential is sufficiently steep can it generate enough thermal friction to maintain the strong regime of warm inflation. On the other hand, inserting Eq. (16) into Eq. (3) shows that the slow-roll parameter scales as $\epsilon \sim V'^{8/7}$, and so requiring $\epsilon < 1$ yields an upper bound on V' . The potential must not be too steep, since otherwise slow roll is violated. It is the combination of these lower and upper bounds on V' that cannot be fulfilled for a sufficiently long period of time.

In order to see this, we compute the number of e-folds as usual (see, e.g., [29,30]):

$$N = \frac{1}{M_p^2} \int_{\varphi_{\text{end}}}^{\varphi_{\text{in}}} d\varphi \frac{V}{V'} (1 + Q) < \frac{2}{M_p^2} \int_{\varphi_{\text{end}}}^{\varphi_{\text{in}}} d\varphi \left(\frac{375\tilde{c}_{\text{sph}}^4 M_p^{10} V'^2}{8\pi^6 g_\star^3 f^8 V'} \right)^{1/7}, \quad (18)$$

where φ_{in} and φ_{end} are initial and final field values, respectively, and we used Eq. (16) in the second step (along with $Q + 1 < 2Q$). Now we come to the key assumptions about our model. First, the lower bound (17) on V' resulting from maintaining the strong regime of warm inflation together with the monotonicity of V implies

$$N < (\varphi_{\text{in}} - \varphi_{\text{end}}) \left(\frac{3000\tilde{c}_{\text{sph}}^4 V(\varphi_{\text{in}})}{\pi^6 g_\star^3 f^8 M_p^2} \right)^{1/6}. \quad (19)$$

Second, it follows from periodicity (13) that the field range is compact, $\varphi_{\text{in}} - \varphi_{\text{end}} < 2\pi f$, and so we arrive at

$$N < 2 \left(\frac{3000\tilde{c}_{\text{sph}}^4 V(\varphi_{\text{in}})}{g_\star^3 f^2 M_p^2} \right)^{1/6} < 7.6 \frac{\tilde{c}_{\text{sph}}^{2/3}}{g_\star^{1/2}} \left(\frac{m}{M_p} \right)^{1/3}, \quad (20)$$

where we additionally plugged in the form (12) of the potential in the last step. The ratio of dimensionless quantities in the first fraction works to suppress N , since

$\tilde{c}_{\text{sph}} \lesssim 1$ (see [23,41]) and $g_\star \geq 1$ by definition. Clearly, the mass of any particle must obey $m < M_p$, and so Eq. (20) shows that it is impossible to achieve a sufficient number of e-foldings. This proves the main statement of the paper. We emphasize that the bound (20) is very conservative—e.g., the first maximum of the axion potential generically occurs for $\varphi < \pi f$ [see Eq. (9)], which introduces another factor of $1/2$ on the rhs of Eq. (20)—and so it appears challenging to construct a viable model of a vanilla axion in which sphaleron heating maintains the strong regime of warm inflation for even a single e-fold.

Ways out. Searching for possible ways to still implement inflation driven by thermal friction, one might contemplate a short transient phase of $Q > 1$ during the generation of cosmic microwave background perturbations followed by different inflationary dynamics. However, the scaling $Q \sim V'^{6/7}$ of (16) implies that this generically requires a non-trivial potential where V' is not monotonic (see also [28]). In a similar spirit, the above argument relies on the slow-roll approximation, and so, e.g., a phase of ultraslow roll could, in principle, provide a way out. Although this has to be checked on a case-by-case basis, it seems unlikely that such an approach will be successful as ultraslow roll can at most last for a few e-foldings before invalidating inflationary perturbation theory [75] (see also [76]), and, moreover, it seems difficult to maintain a sufficiently high temperature of the thermal plasma [77].

In [22,29], the strong regime of warm inflation is implemented by abandoning the discrete shift symmetry (7).⁵ Another option consists in taking $\tilde{c}_{\text{sph}} \gg 1$ (see [67,78,79]), which, however, leads to a concern about strong coupling.⁶ In a PQ scenario, it is possible to include a nonminimal coupling of the radial field to gravity, which would lead to a decay constant that depends on the inflationary Hubble scale H [68,80–85]. Evidently, one can also replace sphaleron heating by another source of thermal friction with a different $\Upsilon(T)$ (see [30,86], and references therein), but in this case care must be taken to avoid the problem [18] of spoiling inflation because of large thermal corrections to the potential. Finally, it is possible to consider an axionlike particle (ALP) that does not necessarily obey a discrete shift symmetry (7) or arise as phase of a complex field (see review [48] and, e.g., [87] for an independent motivation to consider such an ALP).

⁵Both [22,29] rely on a hybrid setup, in which inflation can take place only for $\varphi > \varphi_c$, where φ_c is a critical field value. In [22], $\varphi_c > 10^8 f$ and [29] has $\varphi_c > 10^4 f$ in the strong regime, so in both cases the shift symmetry (7) is broken.

⁶One may try to absorb $\tilde{c}_{\text{sph}} \gg 1$ in terms of two different decay constants, defining a small $\tilde{f} = f/\sqrt{\tilde{c}_{\text{sph}}}$ in the friction rate (11) while still using f in the potential [Eqs. (12) and (13)].

Weak regime. As already mentioned, a finite temperature can have an important effect even if $Q < 1$. In this case, the main source of friction is Hubble dilution; i.e., the background evolution of the inflaton field is to leading order unchanged by thermal effects. As long as $T > H$, however, perturbations and, therefore, inflationary observables are crucially altered by the presence of a thermal plasma [24,25,30,86,88–105]—this is the *weak regime* of warm inflation. With an analogous argument as above, requiring $T \gtrsim H$ leads to the condition

$$V' \gtrsim \left(\frac{2\pi^2 g_\star \tilde{c}_{\text{sph}} V^4}{1215 f^2 M_P^8} \right)^{1/2}, \quad (21)$$

and so the number of e-foldings is bounded as

$$N < \frac{2}{M_P^2} \int_{\varphi_{\text{end}}}^{\varphi_{\text{in}}} d\varphi \frac{V}{V'} \lesssim \frac{99}{\sqrt{g_\star \tilde{c}_{\text{sph}}}} \left(\frac{M_P}{m} \right)^2. \quad (22)$$

Clearly, a restrictive bound on N can now be avoided, especially by choosing the prefactor $\tilde{c}_{\text{sph}}$ of thermal friction and the scale of the potential V (equivalently, m) sufficiently small. While inflation *from* sphaleron heating, corresponding to $Q > 1$, runs into trouble, our argument does not exclude inflation *with* sphaleron heating, i.e., $Q < 1$.

When thermal friction is not dominant, slow-roll parameters scale with $(M_P/f)^2$ [cf. Eq. (3)]. Therefore, warm axion inflation in the weak regime is possible only with a super-Planckian decay constant $f > M_P$; see [25,26,105] for concrete realizations. However, $f > M_P$ puts into question the validity of effective field theory (EFT) [14,106,107], and nonvanilla axion models, e.g., involving more than one axion field [108,109] or relying on nontrivial string compactifications [110,111], are required in order to evade this concern. As a second independent aspect, the simultaneous presence of sphaleron heating and a non-perturbatively generated potential has so far been realized only with a nontrivial gauge sector (see [25,26,105], and discussion above). Therefore, the weak regime of warm axion inflation cannot be viewed as vanilla, either.

Summary. Among the most compelling virtues of warm inflation is that it permits inflationary expansion on a wider class of potentials, which need not be as flat as in the absence of thermal friction. This addresses one of the central challenges of cold inflation: the difficulty [11–15] of maintaining a flat potential in the presence of quantum corrections. However, when thermal effects dominate, they also tend to introduce large corrections to the inflaton potential, thereby undermining slow-roll conditions [18]. Sphaleron heating—arising from coupling an axion to a non-Abelian gauge group at finite temperature—has been proposed as an elegant solution: The shift symmetry of the

axion protects the flatness of the potential even when thermal friction is strong [22–26].

The key insight added by the present work is that sphaleron heating with a vanilla axion, as defined in Eqs. (10)–(13), cannot support the strong regime of warm inflation. Crucially, this result does not rely on any phenomenological input but follows directly from the fundamental properties of an axion—most notably, its discrete shift symmetry. Combined with the established fact that axion inflation without strong thermal friction necessitates a super-Planckian decay constant, we conclude that *there is no warm inflation from a vanilla axion*.

The future of sphaleron heating. Sphaleron heating is still relevant in contexts beyond inflation—e.g., for a dynamical explanation of dark energy [50,112], where prolonged slow roll is not essential, or for the generation of dark matter [113–116] (see also [117]). Existing inflationary models employing this mechanism, such as [22,29], also remain valid as low-energy EFTs. Crucially, however, their ultra-violet completion must differ from earlier assumptions: The inflaton cannot be an axion, especially not one arising as the phase of a complex scalar field. This has a key implication: In the absence of a discrete shift symmetry, there is no longer any compelling reason to single out the specific nonrenormalizable operator (4). A much broader range of interactions becomes admissible, and a new principle is now required to account for their suppression or absence.

Paradoxically, this apparent limitation opens new avenues for model building. Once freed from the constraints of axion-based EFTs, two important advantages emerge.

- (i) The EFT of an axion is valid only up to the scale f . In contrast, the operator (4) remains under control up to the higher cutoff scale f/α .
- (ii) The axion potential contains a mass term—see Eq. (9)—whereas a generic inflaton may possess a different, potentially more favorable potential.

In particular, these two features—the higher effective cutoff and the freedom in potential choice—enable the construction of new models of warm inflation driven purely by Standard Model interactions, as already demonstrated in [118]. Such scenarios stand out not only for their minimal field content, but also for the possibility to yield observable signals in axion experiments.

Acknowledgments. This work was started at the Albert Einstein Center (AEC) at the University of Bern, and I thank the AEC and especially Mikko Laine for hospitality and support through their visitor program. I am very grateful to Marco Drewes and Mikko Laine for providing initial impulses for this project as well as insightful feedback, and I thank Kim Berghaus and Misha Shaposhnikov for many helpful comments and discussions. Moreover,

I am indebted to the participants of the CERN-TH institute “Particle Production in the Early Universe” for interesting questions and feedback. This work was supported by the European Research Council Gravites Horizon Grant AO No. 850 173-6 and the *Fonds de la Recherche Scientifique* (FNRS). Funded by the European Union.

Views and opinions expressed are, however, those of the authors only and do not necessarily reflect those of the European Union or European Research Council.

Data availability. No data were created or analyzed in this study.

-
- [1] A. A. Starobinsky, A new type of isotropic cosmological models without singularity, *Phys. Lett.* **91B**, 99 (1980).
 - [2] A. H. Guth, The inflationary universe: A possible solution to the horizon and flatness problems, *Phys. Rev. D* **23**, 347 (1981).
 - [3] A. D. Linde, A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems, *Phys. Lett.* **108B**, 389 (1982).
 - [4] V. F. Mukhanov and G. V. Chibisov, Quantum fluctuations and a nonsingular universe, *JETP Lett.* **33**, 532 (1981), http://jetpletters.ru/ps/1510/article_23079.pdf.
 - [5] Y. Akrami *et al.* (Planck Collaboration), Planck 2018 results. X. Constraints on inflation, *Astron. Astrophys.* **641**, A10 (2020).
 - [6] P. A. R. Ade *et al.* (BICEP, Keck Collaborations), Improved constraints on primordial gravitational waves using Planck, WMAP, and BICEP/Keck observations through the 2018 observing season, *Phys. Rev. Lett.* **127**, 151301 (2021).
 - [7] A. Riotto, Inflation and the theory of cosmological perturbations, *ICTP Lect. Notes Ser.* **14**, 317 (2003), [arXiv:hep-ph/0210162](https://arxiv.org/abs/hep-ph/0210162).
 - [8] V. Mukhanov, *Physical Foundations of Cosmology* (Cambridge University Press, Oxford, 2005).
 - [9] D. S. Gorbunov and V. A. Rubakov, *Introduction to the Theory of the Early Universe: Cosmological Perturbations and Inflationary Theory* (World Scientific, Singapore, 2011).
 - [10] D. Baumann, *Cosmology* (Cambridge University Press, Cambridge, England, 2022).
 - [11] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart, and D. Wands, False vacuum inflation with Einstein gravity, *Phys. Rev. D* **49**, 6410 (1994).
 - [12] P. Binetruy and G. R. Dvali, D term inflation, *Phys. Lett. B* **388**, 241 (1996).
 - [13] G. R. Dvali and S. H. H. Tye, Brane inflation, *Phys. Lett. B* **450**, 72 (1999).
 - [14] N. Arkani-Hamed, H.-C. Cheng, P. Creminelli, and L. Randall, Pseudonatural inflation, *J. Cosmol. Astropart. Phys.* **07** (2003) 003.
 - [15] D. Baumann and L. McAllister, *Inflation and String Theory*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, England, 2015).
 - [16] A. Berera and L.-Z. Fang, Thermally induced density perturbations in the inflation era, *Phys. Rev. Lett.* **74**, 1912 (1995).
 - [17] A. Berera, Warm inflation, *Phys. Rev. Lett.* **75**, 3218 (1995).
 - [18] J. Yokoyama and A. D. Linde, Is warm inflation possible?, *Phys. Rev. D* **60**, 083509 (1999).
 - [19] K. Freese, J. A. Frieman, and A. V. Olinto, Natural inflation with pseudo—Nambu-Goldstone bosons, *Phys. Rev. Lett.* **65**, 3233 (1990).
 - [20] V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov, On the anomalous electroweak baryon number nonconservation in the early universe, *Phys. Lett.* **155B**, 36 (1985).
 - [21] L. D. McLerran, E. Mottola, and M. E. Shaposhnikov, Sphalerons and axion dynamics in high temperature QCD, *Phys. Rev. D* **43**, 2027 (1991).
 - [22] K. V. Berghaus, P. W. Graham, and D. E. Kaplan, Minimal warm inflation, *J. Cosmol. Astropart. Phys.* **03** (2020) 034; **10** (2023) E02.
 - [23] M. Laine and S. Procacci, Minimal warm inflation with complete medium response, *J. Cosmol. Astropart. Phys.* **06** (2021) 031.
 - [24] M. Mirbabayi and A. Gruzinov, Shapes of non-Gaussianity in warm inflation, *J. Cosmol. Astropart. Phys.* **02** (2022) 012.
 - [25] P. Klose, M. Laine, and S. Procacci, Gravitational wave background from vacuum and thermal fluctuations during axion-like inflation, *J. Cosmol. Astropart. Phys.* **12** (2022) 020.
 - [26] H. Kolesova, M. Laine, and S. Procacci, Maximal temperature of strongly-coupled dark sectors, *J. High Energy Phys.* **05** (2023) 239.
 - [27] P. Klose, M. Laine, and S. Procacci, Gravitational wave background from non-Abelian reheating after axion-like inflation, *J. Cosmol. Astropart. Phys.* **05** (2022) 021.
 - [28] M. Drewes and S. Zell, On sphaleron heating in the presence of fermions, *J. Cosmol. Astropart. Phys.* **06** (2023) 038.
 - [29] K. V. Berghaus, M. Forslund, and M. V. Guevarra, Minimal warm inflation with a heavy QCD axion, *J. Cosmol. Astropart. Phys.* **10** (2024) 103.
 - [30] V. Kamali, M. Motaharfar, and R. O. Ramos, Recent developments in warm inflation, *Universe* **9**, 124 (2023).
 - [31] A. Berera, The warm inflation story, *Universe* **9**, 272 (2023).
 - [32] R. D. Peccei and H. R. Quinn, Constraints imposed by *CP* conservation in the presence of instantons, *Phys. Rev. D* **16**, 1791 (1977).

- [33] R. D. Peccei and H. R. Quinn, CP conservation in the presence of instantons, *Phys. Rev. Lett.* **38**, 1440 (1977).
- [34] F. Wilczek, Problem of strong P and T invariance in the presence of instantons, *Phys. Rev. Lett.* **40**, 279 (1978).
- [35] S. Weinberg, A new light boson?, *Phys. Rev. Lett.* **40**, 223 (1978).
- [36] L. Di Luzio, M. Giannotti, E. Nardi, and L. Visinelli, The landscape of QCD axion models, *Phys. Rep.* **870**, 1 (2020).
- [37] C. G. Callan, Jr., R. F. Dashen, and D. J. Gross, The structure of the gauge theory vacuum, *Phys. Lett. B* **63**, 334 (1976).
- [38] R. Jackiw and C. Rebbi, Vacuum periodicity in a Yang-Mills quantum theory, *Phys. Rev. Lett.* **37**, 172 (1976).
- [39] P. B. Arnold, D. Son, and L. G. Yaffe, The hot baryon violation rate is $O(\alpha_w^5 T^4)$, *Phys. Rev. D* **55**, 6264 (1997).
- [40] G. D. Moore and M. Tassler, The sphaleron rate in $SU(N)$ gauge theory, *J. High Energy Phys.* **02** (2010) 105.
- [41] M. Laine, L. Niemi, S. Procacci, and K. Rummukainen, Shape of the hot topological charge density spectral function, *J. High Energy Phys.* **11** (2022) 126.
- [42] G. Buldgen, M. Drewes, J. U. Kang, and U. R. Mun, General Markovian equation for scalar fields in a slowly evolving background, *J. Cosmol. Astropart. Phys.* **05** (2019) 039.
- [43] K. Kainulainen and O. Koskivaara, Non-equilibrium dynamics of a scalar field with quantum backreaction, *J. High Energy Phys.* **12** (2021) 190.
- [44] J. E. Kim, Weak interaction singlet and strong CP invariance, *Phys. Rev. Lett.* **43**, 103 (1979).
- [45] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Can confinement ensure natural CP invariance of strong interactions?, *Nucl. Phys.* **B166**, 493 (1980).
- [46] A. R. Zhitnitsky, On possible suppression of the axion hadron interactions. (In Russian), *Sov. J. Nucl. Phys.* **31**, 260 (1980).
- [47] M. Dine, W. Fischler, and M. Srednicki, A simple solution to the strong CP problem with a harmless axion, *Phys. Lett.* **104B**, 199 (1981).
- [48] D. J. E. Marsh, Axion cosmology, *Phys. Rep.* **643**, 1 (2016).
- [49] G. Grilli di Cortona, E. Hardy, J. Pardo Vega, and G. Villadoro, The QCD axion, precisely, *J. High Energy Phys.* **01** (2015) 034.
- [50] K. V. Berghaus, P. W. Graham, D. E. Kaplan, G. D. Moore, and S. Rajendran, Dark energy radiation, *Phys. Rev. D* **104**, 083520 (2021).
- [51] S. H. H. Tye, A superstrong force with a heavy axion, *Phys. Rev. Lett.* **47**, 1035 (1981).
- [52] V. A. Rubakov, Grand unification and heavy axion, *JETP Lett.* **65**, 621 (1997).
- [53] Z. Berezhiani, L. Gianfagna, and M. Giannotti, Strong CP problem and mirror world: The Weinberg-Wilczek axion revisited, *Phys. Lett. B* **500**, 286 (2001).
- [54] A. Hook, Anomalous solutions to the strong CP problem, *Phys. Rev. Lett.* **114**, 141801 (2015).
- [55] H. Fukuda, K. Harigaya, M. Ibe, and T. T. Yanagida, Model of visible QCD axion, *Phys. Rev. D* **92**, 015021 (2015).
- [56] S. Dimopoulos, A. Hook, J. Huang, and G. Marques-Tavares, A collider observable QCD axion, *J. High Energy Phys.* **11** (2016) 052.
- [57] A. Hook, S. Kumar, Z. Liu, and R. Sundrum, High quality QCD axion and the LHC, *Phys. Rev. Lett.* **124**, 221801 (2020).
- [58] A. Valenti, L. Vecchi, and L.-X. Xu, Grand color axion, *J. High Energy Phys.* **10** (2022) 025.
- [59] R. T. Co, S. Kumar, and Z. Liu, Searches for heavy QCD axions via dimuon final states, *J. High Energy Phys.* **02** (2023) 111.
- [60] D. I. Dunskey, L. J. Hall, and K. Harigaya, A heavy QCD axion and the mirror world, *J. High Energy Phys.* **02** (2023) 212.
- [61] S. Mohanty and A. Nautiyal, Natural inflation at the GUT scale, *Phys. Rev. D* **78**, 123515 (2008).
- [62] H. Mishra, S. Mohanty, and A. Nautiyal, Warm natural inflation, *Phys. Lett. B* **710**, 245 (2012).
- [63] L. Visinelli, Natural warm inflation, *J. Cosmol. Astropart. Phys.* **09** (2011) 013.
- [64] R. Z. Ferreira and A. Notari, Thermalized axion inflation, *J. Cosmol. Astropart. Phys.* **09** (2017) 007.
- [65] R. Z. Ferreira and A. Notari, Thermalized axion inflation: Natural and monomial inflation with small r , *Phys. Rev. D* **97**, 063528 (2018).
- [66] V. Kamali, Warm pseudoscalar inflation, *Phys. Rev. D* **100**, 043520 (2019).
- [67] G. Montefalcone, V. Aragam, L. Visinelli, and K. Freese, Observational constraints on warm natural inflation, *J. Cosmol. Astropart. Phys.* **03** (2022) 002.
- [68] A. D. Linde, Axions in inflationary cosmology, *Phys. Lett. B* **259**, 38 (1991).
- [69] A. D. Linde, Hybrid inflation, *Phys. Rev. D* **49**, 748 (1994).
- [70] M. Kamionkowski and J. March-Russell, Planck scale physics and the Peccei-Quinn mechanism, *Phys. Lett. B* **282**, 137 (1992).
- [71] J. Alvey and M. Escudero, The axion quality problem: Global symmetry breaking and wormholes, *J. High Energy Phys.* **01** (2021) 032; **11** (2023) 223(E).
- [72] P. G. Catinari and A. Urbano, Gravitational instantons and the quality problem of the QCD axion: Facts, speculations, and statements in between, *Phys. Rev. D* **111**, 125007 (2025).
- [73] F. Takahashi and M. Yamada, Strongly broken Peccei-Quinn symmetry in the early universe, *J. Cosmol. Astropart. Phys.* **10** (2015) 010.
- [74] M. Berbig, Minimal solution to the axion isocurvature problem from nonminimal coupling, *Phys. Rev. D* **110**, 095008 (2024).
- [75] C. Germani and T. Prokopec, On primordial black holes from an inflection point, *Phys. Dark Universe* **18**, 6 (2017).
- [76] A. Poinson, I. Timiryasov, and S. Zell, Critical points in Palatini Higgs inflation with small non-minimal coupling, *J. High Energy Phys.* **03** (2023) 130.
- [77] S. Biswas, K. Bhattacharya, and S. Das, Embedding ultraslow-roll inflaton dynamics in warm inflation, *Phys. Rev. D* **109**, 023501 (2024).
- [78] Y. Reyimuaji and X. Zhang, Warm-assisted natural inflation, *J. Cosmol. Astropart. Phys.* **04** (2020) 077.

- [79] M. Correa, M. R. Gangopadhyay, N. Jaman, and G. J. Mathews, Primordial black-hole dark matter via warm natural inflation, *Phys. Lett. B* **835**, 137510 (2022).
- [80] T. Higaki, K. S. Jeong, and F. Takahashi, Solving the tension between high-scale inflation and axion isocurvature perturbations, *Phys. Lett. B* **734**, 21 (2014).
- [81] K. Choi, K. S. Jeong, and M.-S. Seo, String theoretic QCD axions in the light of PLANCK and BICEP2, *J. High Energy Phys.* **07** (2014) 092.
- [82] E. J. Chun, Axion dark matter with high-scale inflation, *Phys. Lett. B* **735**, 164 (2014).
- [83] M. Fairbairn, R. Hogan, and D. J. E. Marsh, Unifying inflation and dark matter with the Peccei-Quinn field: Observable axions and observable tensors, *Phys. Rev. D* **91**, 023509 (2015).
- [84] G. Ballesteros, J. Redondo, A. Ringwald, and C. Tamarit, Standard Model—axion—seesaw—Higgs portal inflation. Five problems of particle physics and cosmology solved in one stroke, *J. Cosmol. Astropart. Phys.* **08** (2017) 001.
- [85] C. Rigouzzo and S. Zell, No dark matter axion during minimal Higgs inflation, [arXiv:2504.02952](https://arxiv.org/abs/2504.02952).
- [86] G. Ballesteros, A. Perez Rodriguez, and M. Pierre, Monomial warm inflation revisited, *J. Cosmol. Astropart. Phys.* **03** (2023) 003.
- [87] G. K. Karananas, M. Shaposhnikov, and S. Zell, Weyl-invariant Einstein-Cartan gravity: Unifying the strong CP and hierarchy puzzles, *J. High Energy Phys.* **11** (2024) 146.
- [88] S. Gupta, A. Berera, A. F. Heavens, and S. Matarrese, Non-Gaussian signatures in the cosmic background radiation from warm inflation, *Phys. Rev. D* **66**, 043510 (2002).
- [89] L. M. H. Hall, I. G. Moss, and A. Berera, Scalar perturbation spectra from warm inflation, *Phys. Rev. D* **69**, 083525 (2004).
- [90] S. Gupta, Dynamics and non-Gaussianity in the weak-dissipative warm inflation scenario, *Phys. Rev. D* **73**, 083514 (2006).
- [91] I. G. Moss and C. Xiong, Non-Gaussianity in fluctuations from warm inflation, *J. Cosmol. Astropart. Phys.* **04** (2007) 007.
- [92] C. Graham and I. G. Moss, Density fluctuations from warm inflation, *J. Cosmol. Astropart. Phys.* **07** (2009) 013.
- [93] I. G. Moss and T. Yeomans, Non-gaussianity in the strong regime of warm inflation, *J. Cosmol. Astropart. Phys.* **08** (2011) 009.
- [94] M. Bastero-Gil, A. Berera, and R. O. Ramos, Shear viscous effects on the primordial power spectrum from warm inflation, *J. Cosmol. Astropart. Phys.* **07** (2011) 030.
- [95] D. Lopez Nacir, R. A. Porto, L. Senatore, and M. Zaldarriaga, Dissipative effects in the effective field theory of inflation, *J. High Energy Phys.* **01** (2011) 075.
- [96] R. O. Ramos and L. A. da Silva, Power spectrum for inflation models with quantum and thermal noises, *J. Cosmol. Astropart. Phys.* **03** (2013) 032.
- [97] M. Bastero-Gil, A. Berera, I. G. Moss, and R. O. Ramos, Cosmological fluctuations of a random field and radiation fluid, *J. Cosmol. Astropart. Phys.* **05** (2014) 004.
- [98] M. Bastero-Gil, A. Berera, I. G. Moss, and R. O. Ramos, Theory of non-Gaussianity in warm inflation, *J. Cosmol. Astropart. Phys.* **12** (2014) 008.
- [99] M. Bastero-Gil, A. Berera, and J. R. Calderón, Re-examination of the warm inflation curvature perturbations spectrum, *J. Cosmol. Astropart. Phys.* **07** (2019) 019.
- [100] S. Das and R. O. Ramos, Runaway potentials in warm inflation satisfying the Swampland conjectures, *Phys. Rev. D* **102**, 103522 (2020).
- [101] Y. Qiu and L. Sorbo, Spectrum of tensor perturbations in warm inflation, *Phys. Rev. D* **104**, 083542 (2021).
- [102] M. Bastero-Gil, J. M. Gomes, and J. a. G. Rosa, Thermal curvature perturbations in thermal inflation, *J. Cosmol. Astropart. Phys.* **06** (2023) 060.
- [103] G. Montefalcone, V. Aragam, L. Visinelli, and K. Freese, WarmSPy: A numerical study of cosmological perturbations in warm inflation, *J. Cosmol. Astropart. Phys.* **01** (2023) 032.
- [104] M. Drewes, Y. Georis, J. Klaric, and P. Klose, Upper bound on thermal gravitational wave backgrounds from hidden sectors, *J. Cosmol. Astropart. Phys.* **06** (2023) 073.
- [105] M. Laine, S. Procacci, and A. Rogelj, Evolution of coupled scalar perturbations through smooth reheating. I. Dissipative regime, *J. Cosmol. Astropart. Phys.* **10** (2024) 040.
- [106] N. Arkani-Hamed, H.-C. Cheng, P. Creminelli, and L. Randall, Extra natural inflation, *Phys. Rev. Lett.* **90**, 221302 (2003).
- [107] T. Banks, M. Dine, P. J. Fox, and E. Gorbatov, On the possibility of large axion decay constants, *J. Cosmol. Astropart. Phys.* **06** (2003) 001.
- [108] J. E. Kim, H. P. Nilles, and M. Peloso, Completing natural inflation, *J. Cosmol. Astropart. Phys.* **01** (2005) 005.
- [109] S. Dimopoulos, S. Kachru, J. McGreevy, and J. G. Wacker, N-flation, *J. Cosmol. Astropart. Phys.* **08** (2005) 003.
- [110] E. Silverstein and A. Westphal, Monodromy in the CMB: Gravity waves and string inflation, *Phys. Rev. D* **78**, 106003 (2008).
- [111] L. McAllister, E. Silverstein, and A. Westphal, Gravity waves and linear inflation from axion monodromy, *Phys. Rev. D* **82**, 046003 (2010).
- [112] K. V. Berghaus and T. Karwal, Thermal friction as a solution to the Hubble tension, *Phys. Rev. D* **101**, 083537 (2020).
- [113] A. Papageorgiou, P. Quilez, and K. Schmitz, Axion dark matter from frictional misalignment, *J. High Energy Phys.* **01** (2022) 169.
- [114] K. Choi, S. H. Im, H. J. Kim, and H. Seong, Axion dark matter with thermal friction, *J. High Energy Phys.* **02** (2022) 180.
- [115] S. Biondini, H. Kolesova, and S. Procacci, Smooth reheating and dark matter via non-Abelian gauge theory, *Phys. Lett. B* **857**, 138995 (2024).
- [116] D. McKeen, R. Mizuta, D. E. Morrissey, and M. Shamma, Dark matter from dark glueball dominance, *Phys. Rev. D* **111**, 015044 (2025).
- [117] K. Freese, G. Montefalcone, and B. Shams Es Haghi, Dark matter production during warm inflation via freeze-in, *Phys. Rev. Lett.* **133**, 211001 (2024).
- [118] K. V. Berghaus, M. Drewes, and S. Zell, Warm inflation with the standard model, [arXiv:2503.18829](https://arxiv.org/abs/2503.18829).