

Self-consistent model of cosmic ray penetration into molecular clouds: Effect of energy losses

D. O. Chernyshov^{1,*}, A. V. Ivlev^{2,†} and V. A. Dogiel¹

¹*I. E. Tamm Theoretical Physics Division of P. N. Lebedev Institute of Physics, 119991 Moscow, Russia*

²*Max-Planck-Institut für extraterrestrische Physik, 85748 Garching, Germany*



(Received 13 June 2025; accepted 2 October 2025; published 17 October 2025)

The theory of cosmic-ray (CR) penetration into dense molecular clouds developed recently for relativistic particles by Chernyshov *et al.* (2024) is extended to nonrelativistic CRs. Interstellar CRs propagating into the clouds are able to resonantly excite magnetohydrodynamic waves in diffuse cloud envelopes. This leads to the self-modulation, such that particles are scattered off of the self-generated waves. In contrast to relativistic CRs, transport of lower-energy particles in the envelopes is generally heavily affected by ionization losses; furthermore, both CR protons and electrons contribute to wave excitation. We show that these effects have profound impact on the self-modulation, and can dramatically reduce CR spectra even for clouds with moderate column densities of a few times 10^{21} cm^{-2} .

DOI: [10.1103/kkmh-5by1](https://doi.org/10.1103/kkmh-5by1)

I. INTRODUCTION

Interstellar (ISM) cosmic rays (CRs) are able to resonantly excite magnetohydrodynamic (MHD) waves in diffuse gas surrounding dense molecular clouds while propagating into the clouds [1–3]. This phenomenon can lead to *self-modulation* of penetrating CRs: particles are efficiently scattered off of the self-generated waves and, as a result, the CR spectrum in the cloud is reduced compared to the ISM spectrum.

An overall concept of CR self-modulation, discussed in detail in Refs. [2,4], is sketched in Fig. 1. Consider a molecular cloud with a dense subparsec clump in the center (completely dominating the total column density of the cloud) surrounded by a diffuse parsec-scale envelope. Interstellar CRs entering on either side of the cloud along the local magnetic field lines lose a certain fraction of their energy while crossing the clump, which creates a net CR flux directed into the cloud. The flux velocity, an increasing function of the cloud column density [3], may exceed the local velocity of Alfvén waves—which is the necessary condition for the resonant streaming instability [5]. If the excitation rate due to free-streaming CRs exceeds the wave damping rate, the locally generated turbulence starts

scattering penetrating CRs, such that some particles are reflected back to the ISM. The resulting CR flux is then self-regulated by the condition that the wave excitation is exactly balanced by damping. As the damping rate is proportional to the local gas density, the wave excitation occurs in outer regions of diffuse envelope, near the transition to the ISM.

In our recent paper by Chernyshov *et al.* [4] we studied self-modulation of relativistic CRs. We generalized our earlier model by Ivlev *et al.* [2] and Dogiel *et al.* [3] (in which the value for the envelope’s gas density where CRs are able to excite waves was treated as a free parameter) and obtained a universal analytical solution, applicable for arbitrary density distribution monotonically increasing in the envelope toward the dense clump. In Ref. [4], we self-consistently derived the CR diffusion coefficient as well as the outer and inner boundaries of the *turbulent zone* (or *diffusion zone*), the region where the wave excitation takes place. Location of these boundaries depends on the particle momentum (energy), as depicted in the inset of Fig. 1. Waves can only be excited within the envelope, since the damping rate is too high in the warm neutral ISM [4].

The model by Chernyshov *et al.* [4] was tested by computing the impact of CR self-modulation on the gamma-ray emission. We obtained excellent quantitative agreement with the recent Fermi LAT observations of nearby giant molecular clouds [6], showing characteristic deficits in the emission at energies below a few GeV, as predicted by the theory.

The aim of the present paper is to extend the results of Chernyshov *et al.* [4] to nonrelativistic CRs. This work is motivated by the fact that the low-energy part of the

*Contact author: chernyshov@lpi.ru

†Contact author: ivlev@mpe.mpg.de

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article’s title, journal citation, and DOI. Open access publication funded by the Max Planck Society.

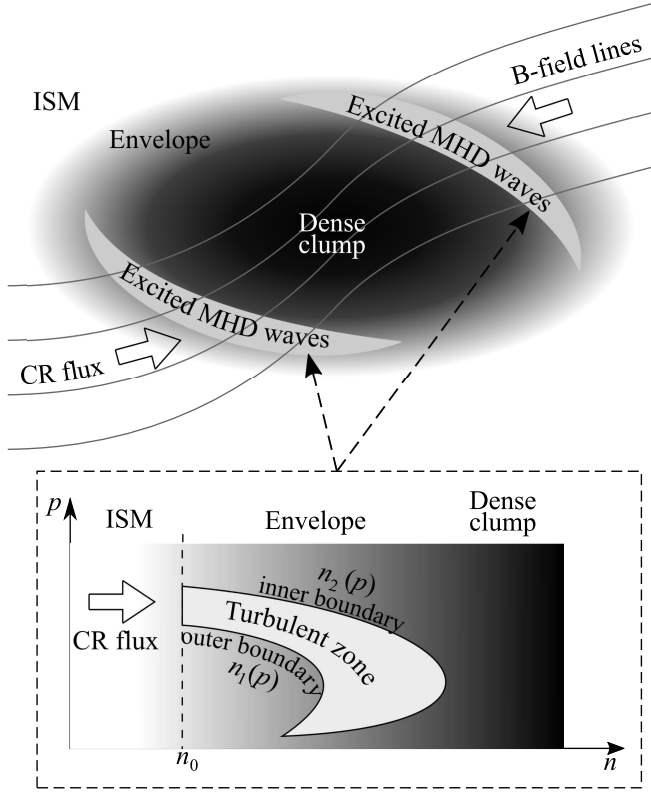


FIG. 1. Sketch illustrating self-modulation of CRs penetrating a dense molecular cloud. Interstellar CRs, being isotropic in the absence of the cloud, propagate along the local magnetic field lines in both directions. The net flux of CRs into the cloud is formed due to their attenuation in the central dense clump. This triggers the resonant streaming instability in the diffuse envelope surrounding the clump, generating the turbulent zones in outer envelope regions. The gas density n in the envelope increases monotonically toward the dense clump; the inset highlights the fact that the outer and inner boundaries of the turbulent zone, $n_1(p)$ and $n_2(p)$, are functions of the particle momentum p .

interstellar CR spectrum determines ionization and thus plays a crucial role in the evolution of molecular clouds, controlling multiple physical and chemical processes that accompany practically all stages of star formation (see, e.g., reviews by Padovani *et al.* [7], Gabici [8], and references therein). Our research will help in providing more insights into the potential sources and transport models of low-energy CRs, since such particles also determine the 6.4 keV Fe K α line emission as well as the nuclear deexcitation line emission from molecular gas [9–15].

We show that the major difference with respect to relativistic particles, whose propagation in diffuse envelopes is loss free [4], is that (i) ionization losses now become generally important, and (ii) both CR protons and electrons contribute to wave excitation. Our analysis suggests that these effects can dramatically reduce spectra of penetrating CRs even for clouds with moderate column densities.

II. GOVERNING EQUATIONS AND THE SOLUTION

According to Chernyshov *et al.* [4] and Ivlev *et al.* [2], the CR streaming instability produces a turbulent zone, the region where penetrating particles are efficiently scattered at the self-generated waves. The turbulent zone has a crescent shape in the plane spanned by the CR energy/momentum and the gas density (see the inset in Fig. 1), with the upper tip set by the *excitation threshold*—the maximum energy at which the CR flux is able to trigger waves. Depending on the interstellar spectrum and the cloud column density, the CR spectrum in the cloud interior may be significantly modulated at energies below the excitation threshold [4].

Outer regions of diffuse envelopes have negligible contribution to the total column density of molecular clouds. This implies that the net flux velocity of penetrating CRs is completely determined by their attenuation in the dense clump [4]. At the same time, generation of waves is only possible in diffuse envelopes, as the waves are efficiently damped at higher densities. Thus, the solution for the turbulent zone is reduced to analysis of the processes occurring in the envelope, while the dense clump only sets the value of the flux velocity (see Sec. III).

Propagation of CRs within the turbulent zone is generally described by the advection-diffusion transport equation,

$$\frac{\partial}{\partial z} \left(v_A f - D \frac{\partial f}{\partial z} \right) - \frac{\partial}{\partial p} (\dot{p} f) = 0, \quad (1)$$

where $f(p, z)$ is the CR distribution function (spectrum) in the momentum space, normalized such that the local number density of CRs is $\int f(p, z) dp$, and z is the coordinate along the magnetic field line. The flux, $v_A f - D \partial f / \partial z \equiv S(p, z)$, is a sum of the advection and diffusion components: the latter is proportional to the diffusion coefficient $D(p)$, the former is set by the velocity of self-excited waves, equal to the Alfvén velocity $v_A(z) = B / \sqrt{4\pi m_i \xi_i n}$, which is determined by the longitudinal magnetic field $B(z)$, the gas density $n(z)$, and the mass m_i of the dominant ions with the abundance ξ_i .

Attenuation of CRs, controlled by continuous losses due to interaction with gas, is described by the momentum loss function $\dot{p} > 0$. The dominant loss mechanism for non-relativistic particles is ionization. At higher energies, losses become catastrophic: they are determined by the pion production (fragmentation) for protons (nuclei), and by bremsstrahlung for electrons. As shown in Ref. [16] (see their Fig. 1 and Appendix A.2), synchrotron losses are typically negligible for electron energies well below ~ 1 TeV. Finally, contribution of adiabatic losses, representing conversion of the CR energy into the energy of MHD waves [2, 17], can also be neglected in our problem (see Sec. IV A).

The rate of resonant wave excitation by streaming CRs is proportional to the diffusion component of the flux [18]. Waves are damped due to ion-neutral collisions, and therefore the diffusion term in Eq. (1) is directly obtained from the excitation-damping balance [2,4],

$$-D \frac{\partial f}{\partial z} \equiv S_D(p, z) = \frac{B c \nu_{\text{in}}}{\pi^2 e v_A p} \propto \frac{n^{3/2}}{p}, \quad (2)$$

where $\nu_{\text{in}} = \nu_0 n / n_0$ is the rate of wave damping, scaling linearly with gas density. Thus,

$$-\frac{\partial}{\partial z} \left(D \frac{\partial f}{\partial z} \right) = \frac{3}{2} \frac{S_D}{n} \frac{dn}{dz}. \quad (3)$$

Since S_D does not depend on B , here we do not need to make assumptions about $B(z)$.

Unlike the loss-free case of relativistic CRs studied in Chernyshov *et al.* [4], the flux of lower-energy CRs is generally not conserved, i.e., we need to find a solution of Eq. (1). We introduce a new function,

$$\tilde{f} = \frac{\dot{p}}{n} v_A f, \quad (4)$$

and take into account that the ratio $\dot{p}/n \equiv L(p)$ does not depend on density (it is identical to the energy loss function per unit column density; see, e.g., Refs. [7,19]). Then, after rearrangement the transport equation can be transformed to

$$\frac{v_A}{n} \frac{\partial \tilde{f}}{\partial z} - L \frac{\partial \tilde{f}}{\partial p} = -\frac{3 v_A L S_D}{2 n^2} \frac{dn}{dz}, \quad (5)$$

suggesting the following new variables:

$$l(z) = \frac{v_{A0}}{n_0} \int \frac{n}{v_A} dz, \quad \lambda_0(p) = \frac{v_{A0}}{n_0} \int \frac{dp}{L}. \quad (6)$$

The choice of the common normalization factor $v_{A0}/n_0 \propto n_0^{-3/2}$ for both new variables is such that l would reduce to z in a homogeneous case.¹

As discussed in Chernyshov *et al.* [4], the density n_0 identifies a border through which CRs enter the envelope from the ISM: the border is set at a local minimum of ν_{in} , which is associated with a transition between different dominant ions in the envelope (C^+) and the ISM (H^+), such that penetrating CRs excite waves only at $n \geq n_0$.

With the new variables, the transport equation takes the following form:

$$\frac{\partial \tilde{f}}{\partial l} - \frac{\partial \tilde{f}}{\partial \lambda_0} = -\frac{3 n_0 v_A L S_D}{2 v_{A0} n^2} \frac{dn}{dz}. \quad (7)$$

¹As in Chernyshov *et al.* [4], we denote the values of parameters at $n = n_i$ by the corresponding indices, e.g., $v_A|_{n_0} \equiv v_{A0}$, $S_D|_{n_1} \equiv S_{D1}$, etc.

For $f_0(p)$ being the spectrum of interstellar CRs, the solution is obtained by the method of characteristics,

$$\tilde{f}(\lambda_0, l) = \int_{l_1}^l G(\lambda_0 + l - l', l') dl' + \tilde{f}_0(\lambda_0 + l - l_1), \quad (8)$$

assuming the condition $\tilde{f}(\lambda_0, l_1) = \tilde{f}_0(\lambda_0)$ at the outer boundary $l_1 = l(z_1)$ (where $n = n_1$ does not necessarily coincide with n_0 , see Sec. III A), and

$$G(\lambda_0, l) = -\frac{3 c \nu_0 B L}{2 \pi^2 e v_{A0} p n} \frac{dn}{dz}, \quad (9)$$

where the explicit expression for S_D is substituted from Eq. (2), and Eq. (6) provides the relation to λ_0 and l . Hereafter, $\tilde{f}_0(\lambda_0 + l - l_1)$ should be interpreted as

$$\tilde{f}_0(\lambda_0 + l - l_1) \equiv v_A L(p_i) f_0(p_i), \quad (10)$$

where the “initial” momentum p_i is derived from

$$\frac{v_{A0}}{n_0} \int_p^{p_i} \frac{dp}{L} = l - l_1. \quad (11)$$

In analogy to the free-streaming regime [16], Eq. (11) defines the “Alfvénic” stopping range of trapped CRs moving with the Alfvén velocity.

The ionization loss function can be closely approximated by a power-law dependence (see, e.g., Ref. [20]),

$$L(p) = \text{const } p^{-\beta}, \quad (12)$$

with $\beta \equiv -d \log L / d \log p \approx 1.6$. For protons with the kinetic energy between 0.1 MeV and 0.2 GeV (or for 15 MeV $\lesssim pc \lesssim 0.7$ GeV, and analogous for heavier nuclei), this deviates by less than 10% from the exact expression [16]. Such energies correspond to the values of stopping range between $\sim 10^{19} \text{ cm}^{-2}$ and $\sim 10^{25} \text{ cm}^{-2}$, so that Eq. (12) is appropriate to describe continuous losses for all relevant column densities of molecular clouds. We then obtain the loss scale,

$$\lambda_0(p) \approx \frac{v_{A0}}{(1 + \beta) n_0} \frac{p}{L}, \quad (13)$$

and

$$G(\lambda_0, l) \approx -\frac{3 c \nu_0 B}{2 (1 + \beta) \pi^2 e n_0 \lambda_0 n} \frac{dn}{dz}. \quad (14)$$

This allows us to transform Eq. (8) to the following solution for the CR spectrum:

$$f = \frac{\tilde{f}_0(\lambda_0 + l - l_1)}{L v_A} - \frac{3 S_{D0} \lambda_0}{2 B_0 v_A} \int_{l_1}^l \left(\frac{B}{n} \frac{dn}{dz} \right) \frac{dl'}{\lambda_0 + l - l'}. \quad (15)$$

We assume that $B \approx \text{const}$ in diffuse envelopes of molecular clouds [21], and that the length scale of density inhomogeneity Λ , defined as

$$\Lambda \equiv \frac{2n}{3} \left(\frac{dn}{dz} \right)^{-1}, \quad (16)$$

varies weakly in the envelopes, i.e., $\Lambda \approx \text{const}$. In this case we have $l(n) = \Lambda(n/n_0)^{3/2} \equiv \Lambda(\lambda_0/\lambda)$, which is expressed in terms of the renormalized loss scale,

$$\lambda(p, n) = \lambda_0(p) \left(\frac{n_0}{n} \right)^{3/2}. \quad (17)$$

This gives us the following analytic solution for the CR spectrum as a function of density n (replacing coordinate z):

$$f(p, n) \approx \frac{\tilde{f}_0(\lambda_0 \xi)}{L v_A} - \frac{S_{D0} \lambda_0}{v_A \Lambda} \ln \xi, \quad (18)$$

where for brevity we introduced

$$\xi(p, n) \equiv 1 + \Lambda \left(\frac{1}{\lambda} - \frac{1}{\lambda_1} \right), \quad (19)$$

and $\lambda_1(p) \equiv \lambda(p, n_1)$. Since the product $S_{D0} \lambda_0$ does not depend on n_0 , the solution does not depend on it, either.

The behavior of the obtained solution is determined by the ratio Λ/λ : for $\lambda \gg \Lambda$ (the local value of the loss scale is much larger than the inhomogeneity length scale) it is a small parameter, so that Eq. (18) can be expanded leading to

$$f(p, n) \approx f_0(p) \frac{v_{A1}}{v_A} - \frac{S_D - S_{D1}}{v_A}. \quad (20)$$

In this case, the CR flux $S(p, n) = S_D + v_A f(p, n)$ is approximately conserved,

$$S(p, n) \approx S(p, n_1) \equiv S_1(p), \quad (21)$$

i.e., the solution is reduced to that in the loss-free case [4].

A. Fiducial parameters

For numerical results and examples presented below, we assume the following fiducial values of the model parameters:

Length scale of density inhomogeneity:	$\Lambda = 5 \text{ pc}$
Magnetic field strength:	$B = 3 \text{ } \mu\text{G}$
Mass of the dominant ions:	$m_i = 12 m_p$
Abundance of the dominant ions:	$\xi_i = 1.5 \times 10^{-4}$
Density at the envelope border:	$n_0 = 1 \text{ cm}^{-3}$
Wave damping rate at $n = n_0$:	$\nu_0 = 9 \times 10^{-11} \text{ s}^{-1}$

The value of Λ is chosen based on a typical slope of density variations in nearby molecular clouds, as predicted by the 3D dust extinction maps [22] and illustrated in, e.g., Fig. 1 of Obolentseva *et al.* [23]. The selected values for other parameters are discussed in Chernyshov *et al.* [4].

III. BOUNDARY CONDITIONS

Similar to the loss-free case studied in Chernyshov *et al.* [4], there are two conditions regulating the *outer* and *inner* boundaries of the turbulent zone. We neglect heavier CR nuclei and only study the effect of protons: as was shown in Ref. [4] (see their Fig. 2), heavier nuclei significantly contribute to the wave excitation at energies well above $\sim 1 \text{ GeV}$ per nucleon, whereas in the present paper our focus is on the analysis of lower-energy CRs.

A. First boundary condition: Outer

The first condition is set on the outer boundary of the proton diffusion zone, denoted by $n_1(p)$. The condition follows from the requirement that $-D \partial f / \partial z \geq 0$ within the diffusion zone, and therefore $\partial f / \partial z \leq 0$. Using Eq. (1), we have for $n > n_0$,

$$\frac{\partial f}{\partial z} = \frac{n}{v_A} \frac{\partial}{\partial p} (L f) + \frac{1}{2} \left(f - \frac{3 S_D}{v_A} \right) \left(\frac{1}{n} \frac{dn}{dz} \right). \quad (22)$$

The outer boundary is formed at the “critical point” $n = n_{\text{cr}}^*$ where $\partial f / \partial z = 0$. Assuming that $\Lambda = \text{const}$ and using Eq. (13) for λ_0 , we reduce this condition to

$$S_D|_{n_{\text{cr}}^*} = \frac{f_0 v_A|_{n_{\text{cr}}^*}}{3} \left(1 - \frac{3(\alpha_0 + \beta)}{1 + \beta} \frac{\Lambda}{\lambda|_{n_{\text{cr}}^*}} \right), \quad (23)$$

where

$$\alpha_0(p) = - \frac{d \log f_0}{d \log p}, \quad (24)$$

is the (momentum-dependent) spectral index of interstellar protons. Substituting expressions for S_D and v_A , we obtain

$$\left(\frac{n_{\text{cr}}^*}{n_0} \right)^2 = 1 - \frac{3(\alpha_0 + \beta)}{1 + \beta} \frac{\Lambda}{\lambda_0} \left(\frac{n_{\text{cr}}^*}{n_0} \right)^{3/2}, \quad (25)$$

where $n_{\text{cr}}(p)$ is the critical point derived by Chernyshov *et al.* [4] for the loss-free case of relativistic CRs; its value is given by Eq. (15) therein,

$$n_{\text{cr}}(p) = \sqrt{\frac{\pi p f_0(p) n_0 \Omega_i}{12 \xi_i \nu_0}}, \quad (26)$$

where $\Omega_i = eB/m_i c$ is the cyclotron frequency of ions. Equation (25) is a fourth-order polynomial for $\sqrt{n_{\text{cr}}^*}$.

It always has a single positive root $n_{\text{cr}}^* \leq n_{\text{cr}}$, i.e., $n_{\text{cr}}^*/n_{\text{cr}}$ decreases in the presence of losses, and the effect is stronger for higher n_{cr} .

Following Chernyshov *et al.* [4], we set the outer boundary at $n_1 = n_0$ if n_{cr}^* occurs to be smaller than n_0 . Thus, the outer boundary of the diffusion zone is

$$n_1(p) = \max \{n_{\text{cr}}^*(p), n_0\}. \quad (27)$$

The total flux of interstellar CRs entering the diffusion zone is a combination of the diffusion and advection components, $S_1(p) = S_{D1} + v_{A1}f_0$. From Eqs. (23) and (25) we infer

$$\frac{S_{D1}}{v_{A1}f_0} = \frac{1}{3} \left(\frac{n_1(p)}{n_{\text{cr}}(p)} \right)^2, \quad (28)$$

and hence

$$\begin{aligned} S_1(p) &= \left[1 + \frac{1}{3} \left(\frac{n_1}{n_{\text{cr}}} \right)^2 \right] v_{A1}f_0 \\ &= \left[1 + 3 \left(\frac{n_{\text{cr}}}{n_1} \right)^2 \right] S_{D1} \equiv \mathcal{K}S_{D1}, \end{aligned} \quad (29)$$

where the factor $\mathcal{K}(p)$ coincides with the expression given by Eq. (19) of Chernyshov *et al.* [4].

B. Second boundary condition: Inner

The second condition, on the inner boundary $n_2(p)$ of the proton diffusion zone, follows from continuity of their net flux S . To derive the condition, let us consider the inner cloud region (beyond the inner boundary) where protons stream freely; we assume that no preexisting resonant turbulence can be present in such dense regions (see, e.g., Ref. [20]). The corresponding transport equation is

$$\bar{\mu}v \frac{\partial f}{\partial z} - n \frac{\partial}{\partial p}(Lf) + n\sigma v f = 0, \quad (30)$$

where $v(p)$ is the proton physical velocity and σ is the cross section of catastrophic processes associated with the pion production in proton-proton collisions. Isotropic CRs leave the diffusion zone with the mean value of pitch-angle cosine of $\bar{\mu} \approx 1/2$, and for the sake of simplicity we assume this value also for the free-streaming propagation. Below we show that, in case of weak losses in the inner cloud region, the resulting flux does not practically depend on $\bar{\mu}$.

It is convenient to consider a “reduced” Eq. (30) (without the last term): assuming σ to be constant (which is only violated close to the threshold), its solution should then be multiplied by $\exp(-\bar{\mu}^{-1}\mathcal{N}\sigma)$, where

$$\mathcal{N}(z) = \int ndz, \quad (31)$$

is the gas column density. In order to solve the reduced equation, we introduce a new variable,

$$R(p) = \int \frac{v}{L} dp, \quad (32)$$

which defines the stopping range (column density) in the free-streaming regime, and a new function $\tilde{f} = L(p)f(p, \mathcal{N})$. This leads to equation

$$\bar{\mu} \frac{\partial \tilde{f}}{\partial \mathcal{N}} - \frac{\partial \tilde{f}}{\partial R} = 0. \quad (33)$$

For spectrum $f_{\text{in}}(p)$ of CRs that stream into the dense clump after leaving the diffusion zone on one side of the cloud, we obtain the solution $f_{\text{out}}(p)$ on the other side,

$$f_{\text{out}}(p, \mathcal{N}) = \frac{\tilde{f}_{\text{in}}(R + \bar{\mu}^{-1}\mathcal{N})}{L} \exp(-\bar{\mu}^{-1}\mathcal{N}\sigma), \quad (34)$$

where $\tilde{f}_{\text{in}}(R + \bar{\mu}^{-1}\mathcal{N})$ should be interpreted similarly to Eq. (10). From continuity of the spectrum at the inner boundaries it follows $f_{\text{in}} + f_{\text{out}} = f_2$, where $f_2(p)$ is given by Eq. (18).

We express the net flux in terms of the derived solution,

$$S_2(p) = \bar{\mu}v(f_{\text{in}} - f_{\text{out}}) \equiv uf_2, \quad (35)$$

where $u(p, \mathcal{N})$ is the flux velocity. For weak losses, one can write $f_{\text{in}} \approx \frac{1}{2}f_2$ and

$$\begin{aligned} f_{\text{out}} - f_{\text{in}} &\approx \frac{\mathcal{N}}{\bar{\mu}} \left(\frac{1}{L} \frac{d\tilde{f}_{\text{in}}}{dR} - \sigma f_{\text{in}} \right) \\ &= \frac{\mathcal{N}}{\bar{\mu}} \left(\frac{1}{v} \frac{d(Lf_{\text{in}})}{dp} - \sigma f_{\text{in}} \right). \end{aligned} \quad (36)$$

Using this result, from Eq. (35) we readily obtain u for small $\mathcal{N}$. Given that in the opposite limit of large $\mathcal{N}$ the flux velocity tends to $u \approx \bar{\mu}v$, we interpolate it by

$$\frac{1}{u} \approx \frac{1}{\bar{\mu}v} + \frac{2}{\mathcal{N}} \left((\alpha_2 + \beta) \frac{L}{p} + \sigma v \right)^{-1}, \quad (37)$$

where

$$\alpha_2(p) = -\frac{d \log f_2}{d \log p}, \quad (38)$$

is the spectral index at the inner boundary.

Now we can write the condition for the inner boundary,

$$S_2(p) = S_{D2} + v_{A2}f_2 = uf_2. \quad (39)$$

Since u is a function of $\alpha_2 \propto \partial f_2 / \partial p + (\partial f_2 / \partial n_2) \times (dn_2 / dp)$, we conclude that Eq. (39) is a differential

equation for $n_2(p)$. The “initial” condition for it is set at the excitation threshold p_{ex} —the maximum momentum at which the proton flux is able to generate waves. The inner and outer boundaries merge at $p = p_{\text{ex}}$, so that $f_2 = f_0$ and $\alpha_2 = \alpha_0$. Hence, the value of the threshold is derived from

$$p_{\text{ex}}: S_{D1} + v_{A1}f_0(p_{\text{ex}}) = u(p_{\text{ex}})f_0(p_{\text{ex}}), \quad (40)$$

where S_{D1} and v_{A1} are taken at $n_1(p_{\text{ex}})$. Then, $n_2(p_{\text{ex}}) = n_1(p_{\text{ex}})$ yields the sought initial condition for Eq. (39).

IV. SPECTRUM AND FLUX OF SELF-MODULATED PROTONS

Model spectra of interstellar CRs may be chosen based on available observational data, or they can be approximated by power-law momentum distributions produced in diffusive shock acceleration. The former class is constrained by the AMS-02 measurements [24] with $f_0(p) \propto p^{-2.7}$ for relativistic protons, and by Voyager measurements for nonrelativistic protons [25,26], suggesting $f_0(p) \propto p^{0.2}$ for energies around several MeV. Following Padovani *et al.* [16], one can introduce a broken power-law distribution,

$$f_0(p) = \frac{4\pi C E^{-\alpha_0/2}}{(E_{\text{tr}} + E)^{2.7-\alpha_0/2}} \text{ eV}^{-1} \text{ s}^{-1} \text{ cm}^{-2}. \quad (41)$$

For $C \approx 2.4 \times 10^{15}$, $E_{\text{tr}} \approx 650$ MeV, and $\alpha_0 \approx -0.2$, this corresponds to the proton spectrum measured by the Voyager spacecraft and extrapolated to energies below 3 MeV; it is commonly referred to as “model $\mathcal{L}$ ” [16]. The latter class of distributions with a constant spectral index can be presented as

$$f_0(p) = \frac{4\pi C_{\text{PL}}(pc)^{-\alpha_0}}{(2m_p c^2)^{2.7-\alpha_0}}, \quad (42)$$

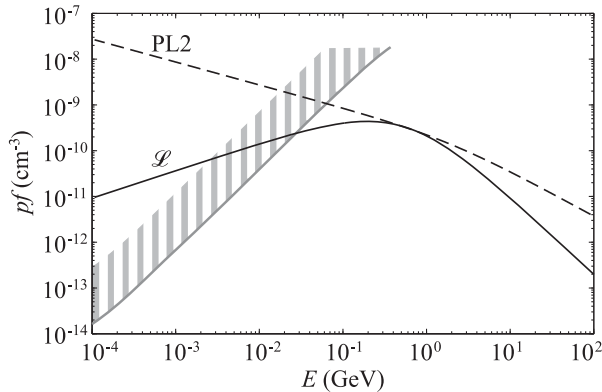


FIG. 2. Characteristic spectra of interstellar protons adopted in the present paper: “model $\mathcal{L}$ ” given by Eq. (41) with $\alpha_0 = -0.2$ (black solid line), and a power-law spectrum given by Eq. (42) with $\alpha_0 = 2$ (“PL2”, black dashed line). The gray solid line with hatching shows where inequality (44) is violated.

where the normalization is chosen such that C_{PL} and C have the same dimensions; for $C_{\text{PL}} = C(2m_p c^2/E_{\text{tr}})^{2.7-\alpha_0/2}$, the power-law spectrum (42) coincides with the low-energy part of spectrum (41).

Figure 2 displays two characteristic realizations of the proton spectra adopted in our paper: model $\mathcal{L}$ and a power-law model with $\alpha_0 = 2$, hereafter referred to as “PL2” (where constant C_{PL} is adjusted such that the two curves touch).

Figure 3 shows the proton diffusion zones computed for the two model spectra. The outer and inner boundaries are derived from Eq. (27) and Eq. (39), respectively. It is practical to solve Eq. (39) by employing the following iterative procedure. Knowing f_2 (and thus u) at a previous iteration step, we compute n_2 from Eq. (39), which allows us to derive f_2 for the next iteration from Eq. (18). For the first step, we set $f_2 = f_0$. We point out that utilizing interpolation (37) for u may lead to instability of the procedure, and therefore it is preferable to use exact Eqs. (34) and (35) for it.

A. Effect of losses

Equation (25) shows that the critical gas density tends to the loss-free value, $n_{\text{cr}}^* \rightarrow n_{\text{cr}}$, if

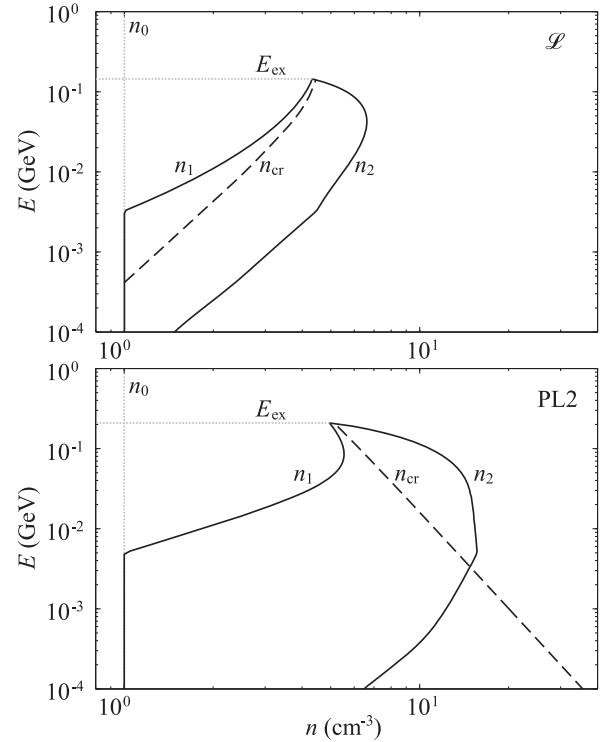


FIG. 3. Diffusion zone of CR protons, computed for interstellar spectrum $\mathcal{L}$ (top panel) and PL2 (bottom panel). For clarity, the results are plotted in the plane of proton kinetic energy E (instead of momentum) and gas density n . The solid lines show the outer, $n_1(E)$, and inner, $n_2(E)$, boundaries merging at the excitation threshold E_{ex} . The dashed lines indicate the (outer boundary) critical density $n_{\text{cr}}(E)$ in the loss-free case, given by Eq. (26). The results are for the cloud column density of $\mathcal{N} = 10^{22} \text{ cm}^{-2}$.

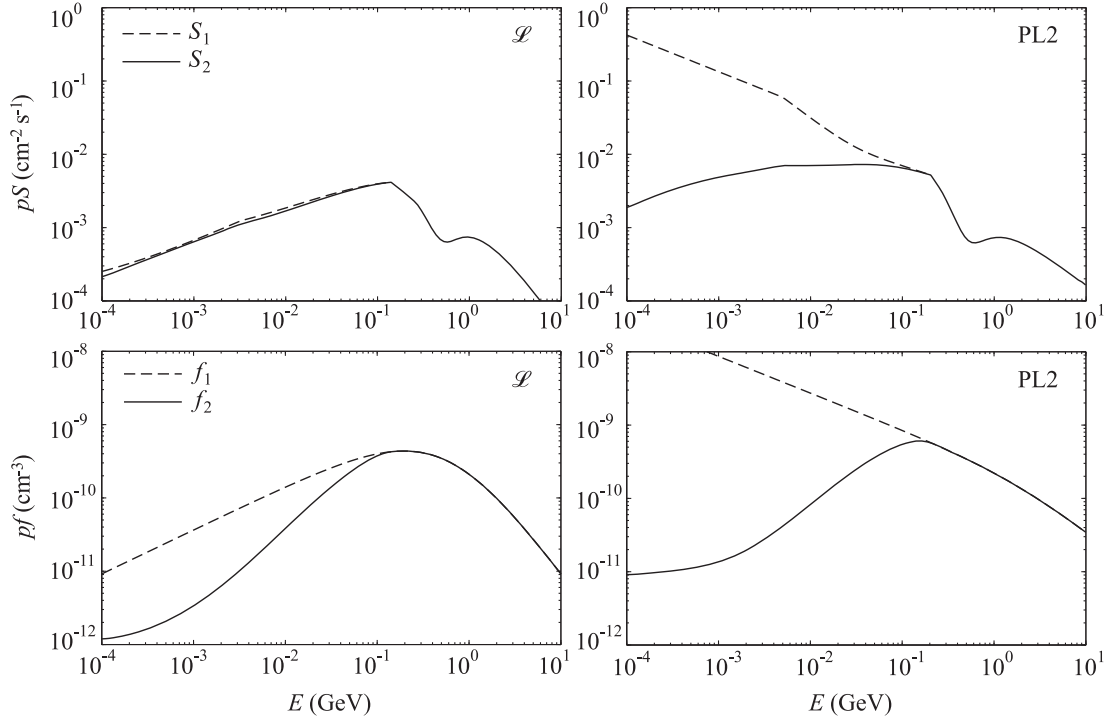


FIG. 4. Net flux S (top panels) and spectrum f (bottom panels) of protons at the outer boundary n_1 (where $f_1 = f_0$, dashed lines) and inner boundary n_2 (solid lines) of the diffusion zone. The results are for the interstellar spectrum $\mathcal{L}$ (left) and PL2 (right), assuming the cloud column density of $\mathcal{N} = 10^{22} \text{ cm}^{-2}$.

$$\frac{\Lambda}{\lambda_{\text{cr}}} \ll \frac{1}{3}, \quad (43)$$

where $\lambda_{\text{cr}} \equiv \lambda_0(n_0/n_{\text{cr}})^{3/2}$. This inequality can be rewritten in terms of the interstellar spectrum. By substituting Eq. (26), we obtain the following condition to neglect losses:

$$pf_0(p) \ll \frac{4\xi_i n_0 \nu_0}{3^{1/3} \pi \Omega_i} \left(\frac{\lambda_0}{\Lambda} \right)^{4/3}, \quad (44)$$

where the rhs scales as $\propto p^{4/3(1+\beta)}$ and does not depend on n_0 . This condition is shown in Fig. 2: we see the impact is stronger for softer spectra of nonrelativistic protons.

This effect of losses on the outer boundary of the diffusion zone is evident in Fig. 3: it is moderate for a relatively hard spectrum $\mathcal{L}$, while for soft spectra represented by model PL2 the deviation from the loss-free boundary becomes dramatic. Hence, losses can have a major impact on the diffusion zone, even for relatively high energies.

Let us now analyze how losses modify the proton flux S . Equation (29) shows that in the regime $n_1 = n_{\text{cr}}^*(p)$ (corresponding to higher energies in Fig. 3) the proportion between advection and diffusion components exceeds 3/1 in the presence of losses, since $n_{\text{cr}}^*/n_{\text{cr}} < 1$ in this case. At lower energies, where $n_1 = n_0$, the proportion depends on the slope of interstellar spectrum: for $\alpha_0 < 1$ (>1), $n_{\text{cr}}(p)$

decreases (increases) with decreasing p and, thus, the advection (diffusion) component asymptotically vanishes.

Using Eq. (29) along with Eqs. (25)–(27) for the outer boundary and Eqs. (39) and (40) for the inner boundary, we compute the respective fluxes S_1 and S_2 . The modulated spectrum f_2 penetrating the cloud is derived from Eq. (18). The results for two model interstellar spectra are presented in Fig. 4. We see that S is practically conserved for model $\mathcal{L}$, while for PL2 the flux reduction is huge at lower energies. In Appendix A we present a detailed discussion of this behavior.

We note that in the regimes where the flux remains approximately conserved within the diffusion zone, $S_1 \approx S_2$, one can directly evaluate the modulated spectrum from $f_2 \approx S_1/u$, using Eq. (29) with u from Eq. (37).

Finally, we point out that adiabatic losses can be completely neglected in our problem. In Appendix B we show that their contribution may only exceed that of regular losses if condition (43) is already satisfied, i.e., adiabatic losses do not affect the diffusion zone at any energy. Similarly, we show that adiabatic losses have no effect on the flux conservation.

V. TRANSPORT OF ELECTRONS AND THEIR IMPACT ON PROTONS

The resonance between waves having a given wave number and CRs occurs for the same value of p for protons

and electrons, i.e., the resonant electrons have a much higher velocity than the resonant (nonrelativistic) protons. For example, protons with the kinetic energy of 100 keV (and velocity $v_p \approx 0.015c$) are resonant with the same waves as ultrarelativistic electrons with the energy of 15 MeV. As a result, such electrons experience much lower energy losses (which are approximately inversely proportional to velocity), and also have a much higher diffusion coefficient (since $D_p/v_p \approx D_e/c$).

Therefore, if a turbulent zone is generated by CR electrons, one can neglect the diffusion component of proton flux within this zone and assume their advective propagation with the velocity $u \approx v_A$. The local Alfvén velocity $v_A(n)$ decreases toward the cloud center. Hence, if conditions are met for electrons with a given momentum to excite a turbulent zone, then protons with the same momentum may excite their zone only at higher densities, after leaving the electron zone—where their flux velocity (whose value is set at the inner boundary of the electron zone) exceeds $v_A(n)$.

Thus, in regimes where both protons and electrons may generate their own turbulent zones, we need to take into account the following aspects. If the electron turbulent zone is present, the outer boundary of the proton zone may only be located deeper than the inner electron boundary. Therefore, the condition for protons at the outer boundary of their turbulent zone is either set by the advective proton flux leaving the electron zone, or by the interstellar spectrum f_0 if no electron zone is present. The condition at the inner boundary of the proton zone remains unchanged.

For electrons, the condition at the outer boundary of their diffusion zone is $f_e = f_{e0}$, whereas at the inner boundary we need to include backward scattering of electrons by the proton diffusion zone (if present).

In this paper we assume the interstellar electron spectrum of the form [16]

$$f_{e0}(p) = \frac{4\pi C_e (pc)^{-1.3}}{(E_{tr,e} + pc)^{1.9}} \text{ eV}^{-1} \text{ s}^{-1} \text{ cm}^{-2}, \quad (45)$$

where $C_e \approx 2.1 \times 10^{18}$ and $E_{tr,e} \approx 710$ MeV. Similar to the proton model spectrum (41), Eq. (45) provides a fit to the AMS-02 data for $pc \gtrsim 1$ GeV, with the slope of -3.2 [24], and describes a transition to the Voyager data for energies down to ≈ 3 MeV, with the slope of -1.3 [26]; the extrapolation to lower energies assumes a constant slope for the momentum distribution.

A. High energies: Passive electrons

For sufficiently high energies electrons are unable to excite turbulence, and thus remain passive CR species. One can also assume that such electrons do not experience substantial losses in the proton diffusion zone.

The key parameter characterizing the ratio of the advection and diffusion components of the proton flux in the loss-free case is [2]

$$\begin{aligned} \eta(p) &= \int_{z_1}^{z_2} \frac{v_A}{D} dz \equiv - \int_{z_1}^{z_2} \frac{v_A}{S_D} \frac{\partial f}{\partial z} dz \\ &= - \frac{v_A}{S_D} f \Big|_{z_1}^{z_2} - 2 \int_{n_1}^{n_2} \frac{S}{S_D} \frac{dn}{n} + 2 \ln \frac{n_2}{n_1}, \end{aligned} \quad (46)$$

where the last equality is obtained by integrating in parts, keeping in mind that $v_A/S_D \propto n^{-2}$, and substituting $v_A f = S - S_D$. If $S \approx \text{const}$, which is typical for hard proton spectra (see Appendix A), the above expression can be further simplified,

$$\begin{aligned} \eta &\approx \frac{S}{3S_{D2}} - \frac{S}{3S_{D1}} + 2 \ln \frac{n_2}{n_1} \\ &\equiv - \frac{\mathcal{K}}{3} \left[1 - \left(\frac{n_1}{n_2} \right)^{3/2} \right] + 2 \ln \frac{n_2}{n_1}, \end{aligned} \quad (47)$$

which coincides with the loss-free result derived in Chernyshov *et al.* [4], see Eq. (A3) therein. In our case, the effect of losses is implicitly included here via the value of n_1 .

The diffusion parameter $\eta_e(p)$ for electrons is different from that for protons by the velocity ratio,²

$$\eta_e = \frac{v}{v_e} \eta. \quad (48)$$

The spectrum of passive electrons at the inner boundary of the proton diffusion zone is then obtained from the (loss-free) solution of Eq. (1),

$$f_e(p, n_2) = e^{\eta_e} \left(f_{e0} - S_{e0} \int_0^{\eta_e} \frac{e^{-\eta'_e}}{v_A} d\eta'_e \right), \quad (49)$$

where $S_{e0}(p) = u_e f_e(p, n_2)$ is the electron flux into the cloud determined by the net velocity $u_e(p, \mathcal{N})$ (see Sec. VA 1 below).

Typical values of η do not deviate substantially from unity if the flux of protons is conserved in their diffusion zone. At the same time, velocities of nonrelativistic protons are much smaller than velocities of electrons with the same momentum. According to Eq. (48), in this case we have $\eta_e \ll 1$ for electrons with energies well below ~ 1 GeV. Then, expanding the integral in Eq. (49) over η_e we readily infer that the electron modulation is negligible if $u_e/v_{A2} \ll v_e/v$.

A strong electron modulation can be observed for soft spectra of interstellar protons. The role of losses is significant in this case, so that $S \gg S_{D1}$ [see Eq. (29) and Fig. 4], i.e., the advection component of the flux

²We use the subscript e to identify parameters attributed to CR electrons, and omit the subscript for protons.

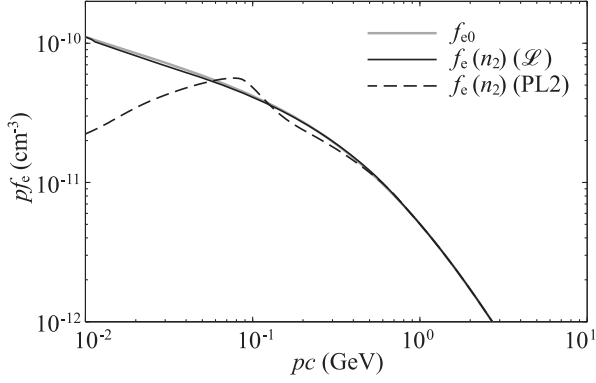


FIG. 5. Modulation of CR electrons in the proton diffusion zone, computed from Eq. (49). The interstellar electron spectrum, Eq. (45), is plotted by the gray line, the modulated spectra at the inner boundary of the diffusion zone are depicted by the black lines, representing model $\mathcal{L}$ (solid line) and PL2 (dashed line) of interstellar protons. The results are for the cloud column density of $\mathcal{N} = 10^{22} \text{ cm}^{-2}$.

dominates and, hence, $\eta \gg 1$. As a result, values of $\eta_e \sim 1$ can be reached, implying a significant modulation.

Figure 5 illustrates passive modulation of CR electrons in the proton diffusion zone, computed for the two characteristic models of interstellar protons. It is evident that a significant electron modulation is only observed for a soft proton spectrum PL2, whose flux is not conserved. We point out that for sufficiently small values of the flux velocity ($u_e < v_A$) it follows from Eq. (49) that the spectrum at the inner boundary can exceed the interstellar spectrum.

1. Effect of secondary particles

The electron flux velocity $u_e(p, \mathcal{N})$ is computed in a similar way to protons, but with a notable difference associated with the effect of secondary electrons and positrons produced in the cloud core. The electron transport equation for the inner cloud region reads

$$\bar{\mu} v_e \frac{\partial f_e}{\partial z} - n \frac{\partial}{\partial p} (L_e f_e) = n q_e, \quad (50)$$

where $q_e(p, \mathcal{N})$ is the source term of secondary particles produced by protons per hydrogen atom. Unlike Eq. (30) for the proton transport, catastrophic (bremsstrahlung) losses in Eq. (50) are included for convenience in the electron loss function $L_e(p)$ [16,27].

By introducing the same variables as for Eq. (30), we derive the following general solution for $\tilde{f}_e \equiv L_e(p) \times f_e(p, \mathcal{N})$:

$$f_{e,\text{out}}(p, \mathcal{N}) = \frac{\tilde{f}_{e,\text{in}}(R_e + \bar{\mu}^{-1} \mathcal{N})}{L_e} + f_{e,\text{sec}}(p, \mathcal{N}), \quad (51)$$

where

$$f_{e,\text{sec}}(p, \mathcal{N}) = \frac{1}{\bar{\mu} L_e} \int_0^{\mathcal{N}} d\mathcal{N}' \frac{L_e(p') q_e(p', \mathcal{N}')}{v_e(p')} \Big|_{R_e(p')=R_e(p)+\bar{\mu}^{-1} \mathcal{N}'}, \quad (52)$$

and $f_{e,\text{in}} + f_{e,\text{out}} = f_e(n_2)$.

Generally speaking, we need to take into account attenuation of protons for computing q_e . However, the problem is substantially simplified for hard proton spectra (such as, e.g., model $\mathcal{L}$), as the generation of secondary particles in this case is dominated by protons with energies over hundreds of MeV. Clouds with column densities of $\mathcal{N} \lesssim 10^{24} \text{ cm}^{-2}$ are transparent for such protons, and therefore their spectra remain isotropic and equal to $f_2(p)$. Consequently, we can write

$$q_e(p) \approx \frac{1}{2} \int_{p'_{\min}}^{\infty} v(p') f_2(p') \frac{\partial \sigma}{\partial p}(p', p) dp', \quad (53)$$

where the integration is over the proton momentum. The differential cross section of proton-impact collisions, $\partial \sigma / \partial p$, includes both the ionization and generation of secondary electrons and positrons upon decay of the charged pions, with $p'_{\min}$ being the minimum proton momentum needed to eject an electron with momentum p . We note that the result obtained in this approximation does not depend on a particular gas distribution in the cloud.

Assuming weak electron losses in the core, i.e., $f_{e,\text{in}} \approx f_{e,\text{out}}$ we then derive $u_e(p, \mathcal{N})$ following the approach described in Sec. III B.

B. Onset of electron-driven turbulence

Knowing how CR electrons propagate in the passive regime, we now can determine the electron excitation threshold $p_{\text{ex},e}$ —the maximum momentum at which the electron flux is able to excite turbulence.

Consider the regime with $p < p_{\text{ex},e}$. Irrespective of whether the proton diffusion zone exists (see discussion in the beginning of Sec. V), the outer boundary of the electron diffusion zone, $n_{1,e}(p)$, is obtained by applying Eq. (26) for the interstellar electron spectrum $f_{e0}(p)$. The electron flux formed at that boundary,³

$$S_{e0}(p) = S_{D1,e} + v_{A1,e} f_{e0}, \quad (54)$$

is then conserved.

For solving a problem with coexisting electron and proton diffusion zones, it is more convenient to directly compute the electron spectrum at the inner boundary of the proton zone, instead of deriving the value of u_e . As we

³To distinguish between parameters at the boundaries of proton and electron diffusion zones, their values at the electron zone boundary are denoted by, e.g., $S_{D1,e}$ or $v_{A1,e}$ at the boundary $n_{1,e}$.

consider relativistic electrons and assume moderate column densities, so that $f_{e,\text{in}} \approx f_{e,\text{out}} = \frac{1}{2}f_e|_{n_2}$, one can integrate Eq. (50) over z across the cloud and over p to infinity.⁴ Using the relation $S_{e0} = \bar{\mu}v_e(f_{e,\text{in}} - f_{e,\text{out}})$, this allows us to write the electron spectrum at the inner boundary of the proton diffusion zone in the following form:

$$f_e|_{n_2} = \frac{2}{\mathcal{N}L_e} \int_p^\infty S_{e0} dp' + \frac{2}{L_e} \int_p^\infty q_e dp'. \quad (55)$$

It is worth noting that the momentum of protons generating secondary particles is always much higher than the resulting momentum of those particles. Therefore, the electron diffusion zone has no effect on the spectrum of such protons and thus on $q_e(p)$.

If the problem of proton propagation is solved and the value of η_e is known, we can now utilize Eq. (49) to infer the electron spectrum at the outer boundary of the proton diffusion zone,

$$f_e|_{n_1} = e^{-\eta_e} f_e|_{n_2} + S_{e0} \int_0^{\eta_e} \frac{e^{-\eta'_e}}{v_A} d\eta'_e. \quad (56)$$

This spectrum should be formed in the electron diffusion zone. Hence, the inner boundary of this zone, $n_{2,e}(p)$, is derived from the following matching condition:

$$f_e|_{n_1} = f_{e2}, \quad (57)$$

where the electron spectrum $f_{e2}(p)$ at the boundary is obtained from the relation $S_{e0}(p) = S_{D2,e} + v_{A2,e}f_{e2}$. The electron diffusion zone exists if $n_{2,e} \geq n_{1,e}$, and therefore the electron excitation threshold is given by the following condition:

$$p_{\text{ex},e}: n_{2,e}(p_{\text{ex},e}) = n_{1,e}(p_{\text{ex},e}). \quad (58)$$

Figure 6 shows the electron excitation threshold plotted versus the cloud column density. We consider two cases: with and without generation of secondary particles. In the latter case the net electron flux is naturally decreased and, therefore, the value of $p_{\text{ex},e}$ is reduced. For reference, the right axis displays the corresponding resonant energy of protons.

C. Low energies: Passive protons

Given that the ratio of proton and electron diffusion coefficients is equal to their velocity ratio, the diffusion component of the proton flux in the electron zone can be reasonably neglected, i.e., we can assume $S \approx v_A f$ within

⁴Electron spectra at the proton zone boundaries are denoted by, e.g., $f_e|_{n_1}$ at the boundary n_1 ; similarly, proton spectra (and also fluxes) at the electron zone boundaries are, e.g., $f|_{n_{1,e}}$ at the boundary $n_{1,e}$.

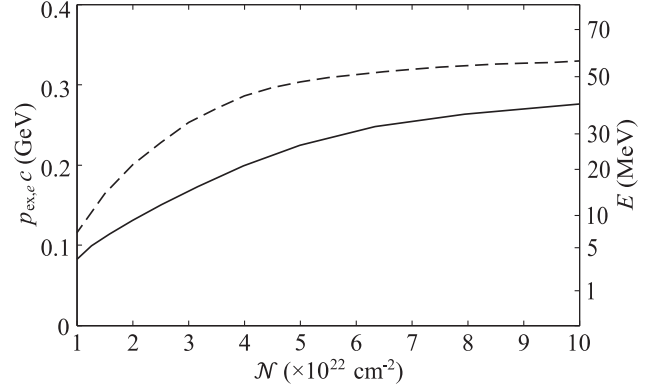


FIG. 6. Electron excitation threshold, $p_{\text{ex},e}$, derived for the interstellar spectrum (45) and plotted versus the cloud column density $\mathcal{N}$. To demonstrate the effect of secondary particles, the solid and dashed lines show the results computed, respectively, with and without taking into account the source term in Eq. (55). The right axis displays the corresponding resonant energy of protons.

the zone. Then, in order to describe propagation of passive protons with $p < p_{\text{ex},e}$, we only need to know boundaries of the electron diffusion zone, $n_{1,e}(p)$ and $n_{2,e}(p)$.

The proton flux in the advection-dominated regime can be directly deduced from the second term of Eq. (8). The flux leaving the electron diffusion zone is

$$S|_{n_{2,e}} = v_{A1,e} \frac{\tilde{f}_0(\lambda_0 + l_{2,e} - l_{1,e})}{L}, \quad (59)$$

(here $\tilde{f}_0 = Lf_0$). Equation (37) suggests that the velocity of proton flux formed in a sufficiently dense cloud could exceed the Alfvén velocity in the cloud envelope, which would lead to the excitation of an additional diffusion zone. In this case, the problem is reduced to that discussed in Sec. III, where we only need to replace f_0 at the outer boundary n_1 of the proton zone with

$$f_1 = \frac{S|_{n_{2,e}} - S_{D1}}{v_{A1}}. \quad (60)$$

Substituting this in Eq. (22), we derive n_1 from the condition $\partial f / \partial z|_{n_1} = 0$. Obviously, the necessary condition for the proton diffusion zone to form in this case is that the resulting f_1 is positive. If the proton diffusion zone does not form, the modulated spectrum of protons is simply $f|_{n_{2,e}} = S|_{n_{2,e}}/u$, where $u(p, \mathcal{N})$ is given by Eq. (37).

The outer boundary of the proton zone is detached from the electron zone if $n_1 > n_{2,e}$, otherwise it starts immediately at the inner boundary of the electron zone. The inner proton boundary $n_2(p)$ is derived from the same boundary condition as in the regular case described by Eq. (39).

In order to self-consistently include mutual effects of coexisting proton and electron diffusion zones, we use an

iterative procedure. We start with the proton zone boundaries computed without taking into account the possible existence of the electron zone—whose boundaries are derived as described in Sec. VB. Then we recompute the proton zone boundaries following the procedure discussed above in this section. If the proton diffusion zone is formed, we repeat the iterative procedure until it converges; if not, we rederive the electron zone boundaries without the effect of the proton zone.

The modulated spectra of CR protons and electrons as well as the corresponding diffusion zones, computed for model $\mathcal{L}$ of interstellar protons, are presented in Fig. 7 for different $\mathcal{N}$. For lower values of $\mathcal{N}$ the proton zone is “replaced” by the electron zone at $p < p_{\text{ex},e}$, whereas for higher $\mathcal{N}$ the two zones coexist within a certain momentum range.

We point out that for sufficiently soft spectra of interstellar protons, such as model PL2, the values of $p_{\text{ex},e}$ are very small, i.e., CR electrons remain passive in the considered range of energies.

D. Transition from Alfvén waves to whistlers

The model of electron diffusion zone discussed above is applicable for energies where excited turbulence is Alfvénic. This assumption is valid as long as the Alfvén frequency $v_A k$ is smaller than the cyclotron frequency of carbon ions Ω_i . Utilizing the resonant relation $k = eB/pc$ we obtain the following condition:

$$pc \gg m_i v_A c \approx 5 \times 10^{-3} \left(\frac{B}{3 \mu\text{G}} \right) \left(\frac{n}{1 \text{ cm}^{-3}} \right)^{-1/2} \text{ GeV}. \quad (61)$$

For smaller p we need to keep in mind that the resonant waves change from the Alfvén to the whistler branch, whose dispersion relation is given by (see, e.g., Ref. [28])

$$\omega \approx \frac{\Omega_e + i\nu_e}{\omega_{pe}^2} c^2 k^2, \quad (62)$$

where $\Omega_e = eB/m_e c$ is the electron cyclotron frequency, ω_{pe} is the electron plasma frequency, and ν_e is the momentum transfer rate for electron collisions (which generally includes collisions with neutrals and ions). Using the identity $\Omega_e \Omega_i c^2 \equiv \omega_{pe}^2 v_A^2$, we derive the phase velocity of whistlers,

$$v_w = \frac{v_A^2 k}{\Omega_i} \equiv v_A^2 \frac{m_i}{p}, \quad (63)$$

and their damping rate,

$$\nu_w = \nu_e \frac{m_e m_i v_A^2}{p^2} \equiv \nu_e \frac{m_e v_w}{p}. \quad (64)$$

The rate of resonant wave excitation by streaming CRs is proportional to the phase velocity of the waves [29], i.e., in the regular expression for the excitation rate [4,18] we now

need to replace v_A with v_w ,

$$\gamma_w \approx -\frac{\pi^2 e v_w p}{m_e c^2 \Omega_e} D \frac{\partial f}{\partial p}. \quad (65)$$

Hence, the excitation-damping balance reads

$$\gamma_w = \nu_w. \quad (66)$$

We note that both terms in Eq. (66) are proportional to v_w and thus the latter cancels out. However, one can see that the relative magnitude of the damping rate for whistlers is changed, compared to that for Alfvén waves, by a factor of $(\nu_e/\nu_{\text{in}})(m_e v_A/p)$. Given that the ratio v_A/v_e is very small for relevant electron energies, we conclude that the relative damping rate is reduced in the regime of whistler excitation.

Equation (66) yields the diffusion flux in the whistler regime,

$$S_{Dw} = \frac{B c \nu_e m_e}{\pi^2 e p^2} \propto \frac{n}{p^2}. \quad (67)$$

Comparing this to Eq. (2), we see a different scaling dependence and also a different magnitude of the flux: the Alfvén velocity in the denominator of S_D is now replaced with a much larger value of p/m_e .

The electron transport equation in the whistler regime is

$$\frac{\partial}{\partial z} (v_w f_e + S_{Dw}) - n \frac{\partial}{\partial p} (L_e f_e) = 0. \quad (68)$$

Since $v_w \propto (np)^{-1}$, we need to introduce new variables that are different from those used to solve the problem in Sec. II. Now, for $\tilde{f}_e = L_e v_A^2 f_e$ we use

$$l_w(z) = \frac{v_{A0}^2}{n_0} \int \frac{n}{v_A^2} dz, \quad \lambda_w(p) = \frac{m_i v_{A0}^2}{n_0} \int \frac{dp}{p L_e}, \quad (69)$$

which allows us to reduce the equation to

$$\frac{\partial \tilde{f}_e}{\partial l_w} - \frac{\partial \tilde{f}_e}{\partial \lambda_w} = -\frac{2 p L_e S_{Dw}}{3 m_i \Lambda} \left(\frac{n_0}{n} \right)^2. \quad (70)$$

Its general solution is then obtained from Eq. (8).

The outer boundary of the diffusion zone, $n_{1,e}(p)$, is derived from the following equation similar to Eq. (22):

$$\frac{\partial f_e}{\partial z} = \frac{n}{v_w} \frac{\partial}{\partial p} (L_e f_e) + \frac{2}{3\Lambda} \left(f_e - \frac{S_{Dw}}{v_w} \right), \quad (71)$$

with the conditions $\partial f_e / \partial z = 0$ and $f_e = f_{e0}$. The condition at the inner boundary, $n_{2,e}$, also has a similar form,

$$S_{Dw2,e} + v_{w2,e} f_{e0} = \bar{\mu} v_e (f_{e,\text{in}} - f_{e,\text{out}}), \quad (72)$$

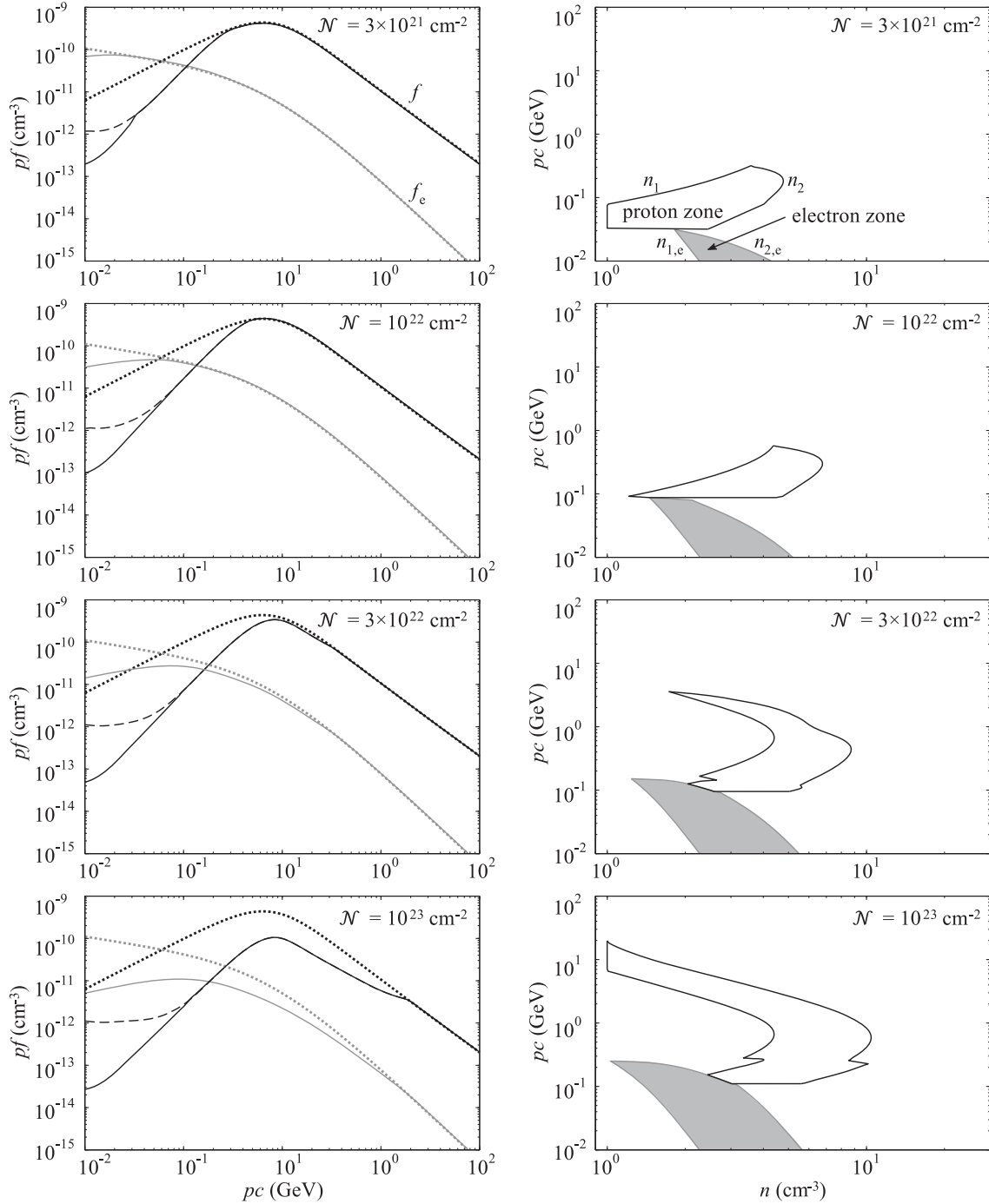


FIG. 7. CR spectra (left panels) and diffusion zones (right panels) for different values of the cloud column density $\mathcal{N}$. Proton and electron spectra (f and f_e) are depicted by the black and gray lines, respectively. The solid lines show the modulated spectra, computed at the inner(most) boundary of the diffusion zone, the dotted lines are the interstellar spectra, given by Eq. (41) with $\alpha_0 = -0.2$ (model $\mathcal{L}$) for protons and by Eq. (45) for electrons. For comparison, the black dashed lines show the modulated proton spectra derived neglecting the electron-driven turbulence. In the right panels, the proton and electron diffusion zones are depicted by the unshaded and shaded contours, respectively.

where the rhs is computed from Eq. (51). The resulting velocity of the net electron flux is always small, $u_e \ll v_e$, due to the effect of secondary particles, as discussed in Sec. VA 1.

The computed electron spectra are plotted in Fig. 8 for different values of gas temperature T in diffuse envelopes

of molecular clouds and for different cloud column densities $\mathcal{N}$. We solve Eq. (70) for $pc < 6 \times 10^{-3}$ GeV, and use the Alfvénic solution (18) for higher p ; for comparison, we also present the results obtained from Eq. (18) only. The growth seen in the spectra toward smaller

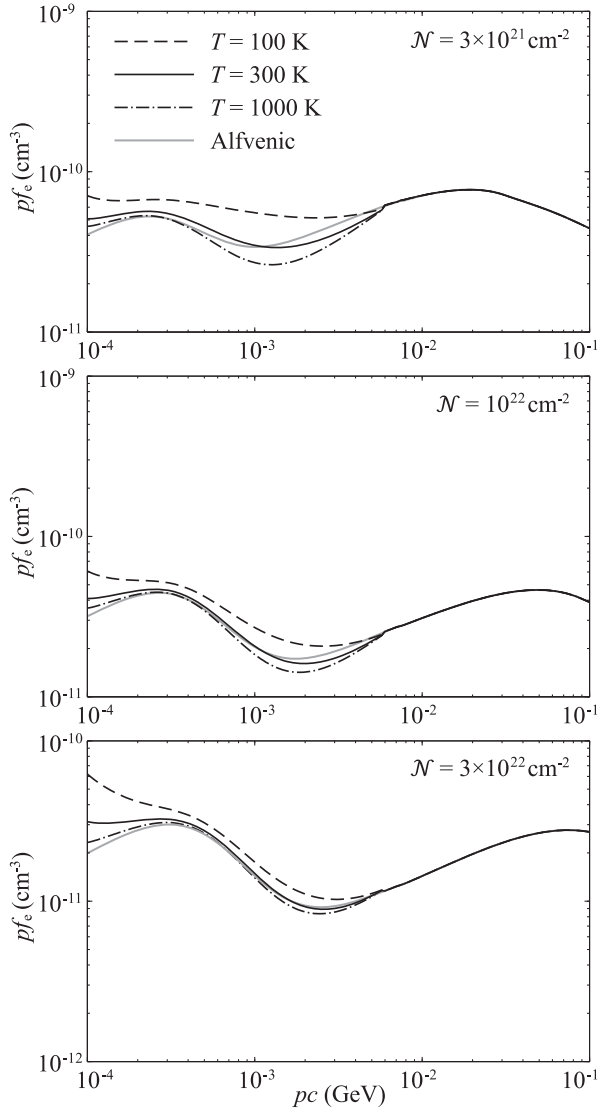


FIG. 8. Spectrum of modulated CR electrons at the inner boundary of the electron diffusion zone, derived for the interstellar spectrum (45) and plotted for different values of the gas temperature T . Transition between Alfvénic and whistler regimes occurs at $pc = 6 \times 10^{-3}$ GeV. Different panels show the results for different values of the cloud column density $\mathcal{N}$.

p is due to increasing contribution of secondary particles. Note that $pc = 10^{-4}$ GeV correspond to nonrelativistic electrons with the kinetic energy $\approx (pc)^2 / 2m_e c^2 \sim 10$ keV.

The gas temperature enters the results via the electron collision rate ν_e , which is dominated by the Coulomb collisions with ions for all temperatures expected in the WNM and CNM [30], scaling as $\nu_e \propto T^{-3/2}$. However, Fig. 8 shows that for $T \gtrsim 300$ K (relevant for diffuse envelopes) the derived electron spectra are practically insensitive to the temperature value. The reason behind this is that at such temperatures $S_{Dw2,e} \propto \nu_e$ rapidly decreases and becomes smaller than the contribution of secondary electrons in the rhs of Eq. (72) [see Eq. (51)]. In Fig. 9 we

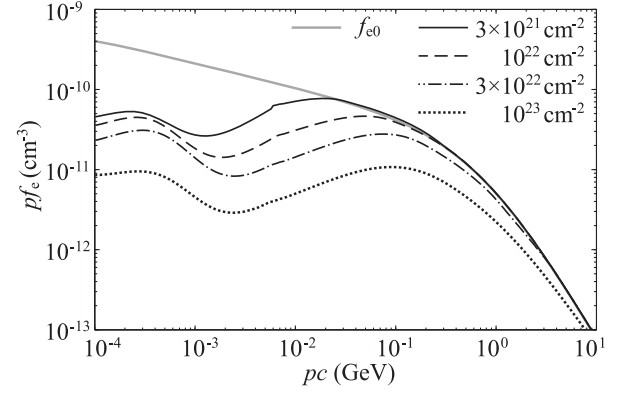


FIG. 9. Modulated electron spectrum for molecular clouds with different column densities (see Fig. 8), assuming the gas temperature in the envelopes of $T = 1000$ K.

illustrate dependence of the modulated electron spectrum on the gas column density, assuming $T = 1000$ K.

VI. CONCLUSIONS

The theory of CR penetration into dense molecular clouds developed for relativistic particles by Chernyshov *et al.* [4] has been extended to nonrelativistic CRs. We showed that, while such CRs undergo significant ionization losses in diffuse envelopes surrounding the clouds, the mechanism of self-modulation operating in nonuniform envelopes remains essentially unchanged. At the same time, the self-modulation of nonrelativistic CRs is much more efficient, and can be substantial even for clouds with moderate column density of $\mathcal{N} \gtrsim 10^{21} \text{ cm}^{-2}$. Our main finding can be summarized as follows:

- (1) Boundaries of a turbulent zone depend on the dimensionless ratio of the density inhomogeneity length scale in the envelope Λ to the local loss scale λ_{cr} [Eq. (43)]: for large $\Lambda/\lambda_{\text{cr}}$ losses dramatically modify the outer boundary $n_1(p)$ [Eqs. (25) and (27)], shifting it to a lower density, while in the opposite limit the outer boundary tends to the loss-free value [Eqs. (26) and (27)] derived in Chernyshov *et al.* [4]. The effect depends on the interstellar spectrum of CRs and is stronger for softer spectra, as illustrated in Fig. 3.
- (2) Even if $\Lambda/\lambda_{\text{cr}}$ is large and a turbulent zone is strongly modified by losses, the net flux of penetrating CRs is practically conserved for a fairly broad class of sufficiently hard interstellar spectra, such as, e.g., the proton spectrum measured by the Voyager spacecraft (see Fig. 4). The modulated spectrum in this case can be computed using formulas derived in Chernyshov *et al.* [4] for the loss-free case (but taking into account the shift of the outer boundary).
- (3) A turbulent zone can be excited both by CR protons and electrons. Protons generally dominate at higher momentum values, where electrons play a role of a

passive component modulated by proton-generated waves. As soon as the threshold condition for electron-driven turbulence is met at a lower momentum, protons become a passive component. For sufficiently high $\mathcal{N}$, a regime where both protons and electrons generate their own turbulent zones is realized within a certain momentum range; in this case, the outer boundary of the proton zone is always located deeper than the inner boundary of the electron zone, as depicted in Fig. 7.

- (4) At lower momentum values, where the turbulent zone is generated by electrons only, their net inward flux becomes gradually limited due to production of secondary electrons and positrons in the cloud interior, which correspondingly reduces the modulation depth (see Fig. 9).

We conclude that the self-modulation can be considered as an efficient mechanism of “unification” of CR spectra penetrating molecular clouds, making (potentially) diverse interstellar spectra similarly hard. This phenomenon is illustrated in Fig. 4, showing that very different spectra assumed for interstellar CRs become much more alike inside the clouds. Given that turbulent zones are generated in outer regions of diffuse envelopes, corresponding to gas densities of $\sim 1\text{--}10\text{ cm}^{-3}$, we expect that our results may be relevant for understanding nearly all available measurements of the CR ionization rate in molecular gas. Furthermore, combining the results of the present paper with our earlier results reported in Chernyshov *et al.* [4] may in the future help us to formulate a consistent view on the impact of Galactic CRs on ionization and gamma-ray emission produced in molecular clouds [7,8,12,13,31–36].

ACKNOWLEDGMENTS

A. V. I. acknowledges the support of the Max Planck Society, and thanks Kedron Silsbee for valuable discussions.

DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: VARIATION OF FLUX ACROSS THE DIFFUSION ZONE

Let us assume that $\Lambda \approx \text{const}$ in the diffusion zone, and try to evaluate conditions where $S(z) \approx \text{const}$. For this purpose, we substitute $f \approx (S - S_D)/v_A$ in the loss term of Eq. (1) and integrate the result over z between z_1 and z_2 . We obtain

$$S_2 - S_1 = \frac{\Lambda n_1}{v_{A1}} \left[\left(\frac{n_2}{n_1} \right)^{3/2} - 1 \right] \frac{\partial(LS)}{\partial p} - \frac{\Lambda n_1}{2v_{A1}} \left[\left(\frac{n_2}{n_1} \right)^3 - 1 \right] \frac{\partial(LS_D)}{\partial p} \Big|_{n_1(p)}, \quad (\text{A1})$$

where $\partial(LS_D)/\partial p = -(1 + \beta)LS_D/p$. On the other hand, $S(p) \approx S_{D1} + v_{A1}f_0$ depends on p explicitly, via $f_0(p)$ and $S_{D1}(p)$, and also implicitly, via $n_1(p)$ if $n_1 > n_0$. However, n_1 is the critical point for S_0 , i.e., $\partial S/\partial n|_{n_1} = 0$ if losses do not play a substantial role. Hence,

$$\frac{\partial(LS)}{\partial p} \approx -(1 + \beta) \frac{LS_D}{p} - (\alpha_0 + \beta) \frac{Lv_A f_0}{p}. \quad (\text{A2})$$

Introducing $X \equiv S_{D1}/(v_{A1}f_0)$ for brevity and utilizing Eqs. (28) and (29) as well as Eq. (22) from Chernyshov *et al.* [4], one can estimate the density ratio as $(n_2/n_1)^{3/2} \approx 1 + X^{-1}$. We finally arrive to

$$\frac{S_1 - S_2}{S} = \frac{\Lambda}{\lambda_1} \frac{\alpha_0 + \beta/2 - 1/2}{X(1 + \beta)(1 + X)}, \quad (\text{A3})$$

where $X = \frac{1}{3}(n_1/n_{\text{cr}})^2$ according to Eq. (28). We see that the flux is conserved if $\alpha_0 = (1 - \beta)/2 \approx -0.3$. Remarkably, this value is quite different from $\alpha_0 = -1.6$ which would follow from a steady-state solution $f(p) \propto L^{-1}$.

We see that the relative flux variation is determined by the magnitude of the factor $(\Lambda/\lambda_1)/X$. From the positiveness of the rhs of Eq. (25) we readily derive that

$$\frac{\Lambda}{\lambda_1} \frac{1}{X} \lesssim \frac{(n_{\text{cr}}/n_1)^{1/2}}{(n_{\text{cr}}^*/n_{\text{cr}})^{3/2}}. \quad (\text{A4})$$

We note that the rhs of Eq. (A4) is reduced to $(n_{\text{cr}}^*/n_{\text{cr}})^{-2}$ in the regime $n_1 = n_{\text{cr}}^*(> n_0)$, which is precisely the inverse of the ratio in Eq. (25). As shown in Fig. 3, the values of n_{cr}^* remain sufficiently close to n_{cr} for hard spectra (illustrated for $\alpha_0 = -0.2$), in which case the above factor is of the order of unity, and therefore $S \approx \text{const}$ is expected from Eq. (A3)—which is indeed seen in the top left panel of Fig. 4. On the other hand, for soft spectra ($\alpha_0 = 2$ in Fig. 3) we have $n_{\text{cr}}^* \ll n_{\text{cr}}$ due to increasing effect of losses, and hence Eq. (A3) predicts significant variation of the flux—which is evident in the top right panel of Fig. 4.

Figure 10 shows contour lines of the relative flux variation across the diffusion zone, derived from the exact solution for a pure power-law spectrum of interstellar protons, Eq. (42). The results are plotted in the plane of the spectral index and the proton energy. The top panel presents the results for $C_{\text{PL}} = C(2m_p c^2/E_{\text{tr}})^{2.7-\alpha_0/2}$, where the power-law spectrum matches Eq. (41) at lower energies (in particular, coincides with model $\mathcal{L}$ for $\alpha_0 = -0.2$), the bottom panel shows the case where C_{PL} is reduced by a factor of 10. We see that the flux is approximately conserved for a fairly broad range of spectral indices around $\alpha_0 = -0.3$, corresponding to relatively hard proton spectra (such as model $\mathcal{L}$). This behavior can be qualitatively understood as an approximate balance between CR attenuation operating at a given energy, and replenishment due to losses at higher

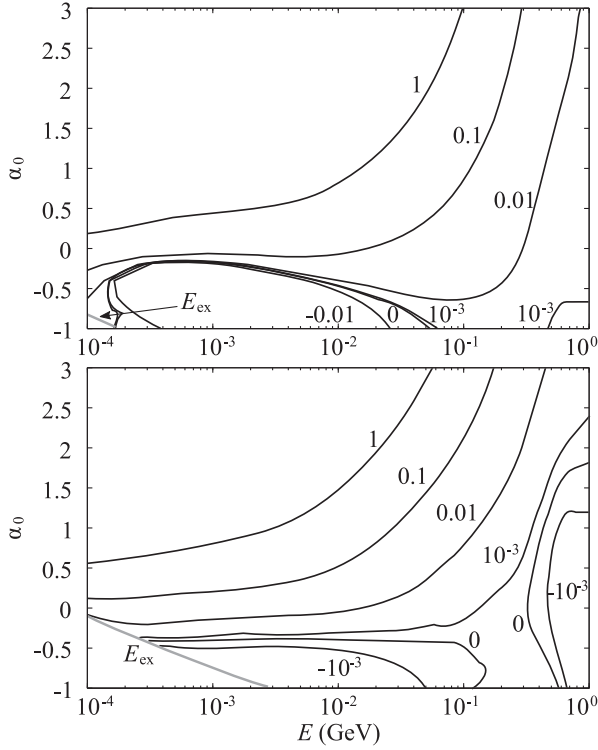


FIG. 10. Contour plot showing the relative variation of proton flux across the diffusion zone, $(S_1 - S_2)/S_2$, depending on the kinetic energy E and the spectral index α_0 of the power-law distribution (42). The top and bottom panels illustrate the effect of magnitude of CR distribution (see text). The thick gray line indicates the excitation threshold E_{ex} , Eq. (40), below which waves cannot be excited.

energies. For very hard spectra, this may even lead to $S_2 > S_1$ within a certain range of energies.

APPENDIX B: ROLE OF ADIABATIC LOSSES

In order to take into account adiabatic losses, we need to add them to regular losses in Eq. (1),

$$\dot{p} \rightarrow \dot{p} + \frac{p}{3} \frac{dv_A}{dz}. \quad (\text{B1})$$

Adiabatic losses become dominant if $\dot{p} \lesssim \frac{1}{6} v_A p (dn/dz)/n$, or

$$\frac{\Lambda}{\lambda} \lesssim \frac{1 + \beta}{9}. \quad (\text{B2})$$

This condition is stronger than that of Eq. (43), i.e., adiabatic losses may only dominate in the regime where the effect of regular losses on the diffusion zone is already negligible.

Similar to Appendix A, now we integrate Eq. (1) over z between z_1 and z_2 neglecting regular losses. Substituting $f \approx (S - S_D)/v_A$ into the adiabatic loss term gives

$$S_1 - S_2 \approx \frac{1}{6} \int_{n_1}^{n_2} \frac{\partial}{\partial p} [p(S - S_D)] \frac{dn}{n} = \frac{\ln(n_2/n_1)}{6} \frac{\partial(pS)}{\partial p}. \quad (\text{B3})$$

As $n_2/n_1 \lesssim 2.5$ and $\partial(pS)/\partial p = \frac{3}{4}(1 - \alpha_0)S$ in the loss-free case [see Eqs. (22) and (20), respectively, in Ref. [4]], we conclude that $|S_1 - S_2| \ll S$ for all reasonable values of the interstellar spectral index, i.e., adiabatic losses have no impact on the flux conservation.

Another effect is associated with the fact that turbulence disappears at the inner boundary, leading to an abrupt drop in the advection velocity from v_{A2} to zero. As a result, adiabatic losses cause a jump in the flux,

$$S_{2+} - S_{2-} = -\frac{v_{A2}}{3} \frac{\partial(pf_2)}{\partial p}, \quad (\text{B4})$$

so that the flux velocity changes as

$$u \rightarrow u + \frac{\alpha_2 - 1}{3} v_{A2}. \quad (\text{B5})$$

Using Eq. (37) for $u(\mathcal{N})$ we infer that the effect is unimportant if $\mathcal{N} \gtrsim n\lambda(\alpha p^{1+\beta}/\sqrt{n})$. Analysis shows that this condition is always satisfied within the diffusion zones.

- [1] J. Skilling and A. W. Strong, *Astron. Astrophys.* **53**, 253 (1976).
- [2] A. V. Ivlev, V. A. Dogiel, D. O. Chernyshov, P. Caselli, C. M. Ko, and K. S. Cheng, *Astrophys. J.* **855**, 23 (2018).
- [3] V. A. Dogiel, D. O. Chernyshov, A. V. Ivlev, D. Malyshev, A. W. Strong, and K. S. Cheng, *Astrophys. J.* **868**, 114 (2018).
- [4] D. O. Chernyshov, A. V. Ivlev, and A. M. Kiselev, *Phys. Rev. D* **110**, 043012 (2024).

- [5] R. Kulsrud and W. P. Pearce, *Astrophys. J.* **156**, 445 (1969).
- [6] R.-z. Yang, G.-X. Li, E. d. O. Wilhelmi, Y.-D. Cui, B. Liu, and F. Aharonian, *Nat. Astron.* **7**, 351 (2023).
- [7] M. Padovani, A. V. Ivlev, D. Galli, S. S. R. Offner, N. Indriolo, D. Rodgers-Lee, A. Marcowith, P. Girichidis, A. M. Bykov, and J. M. D. Kruijssen, *Space Sci. Rev.* **216**, 29 (2020).
- [8] S. Gabici, *Astron. Astrophys. Rev.* **30**, 4 (2022).

- [9] R. Ramaty, B. Kozlovsky, and R. E. Lingenfelter, *Astrophys. J. Suppl. Ser.* **40**, 487 (1979).
- [10] V. Tatischeff, *EAS Publ. Ser.* **7**, 79 (2003).
- [11] H. Benhabiles-Mezhoud, J. Kiener, V. Tatischeff, and A. W. Strong, *Astrophys. J.* **766**, 139 (2013).
- [12] V. H. M. Phan, S. Gabici, G. Morlino, R. Terrier, J. Vink, J. Krause, and M. Menu, *Astron. Astrophys.* **635**, A40 (2020).
- [13] Y. Fujita, K. K. Nobukawa, and H. Sano, *Astrophys. J.* **908**, 136 (2021).
- [14] B. Liu, R.-z. Yang, and F. Aharonian, *Astron. Astrophys.* **646**, A149 (2021).
- [15] Z. Shi, B. Liu, and R.-z. Yang, *Astron. Astrophys.* **692**, A128 (2024).
- [16] M. Padovani, A. V. Ivlev, D. Galli, and P. Caselli, *Astron. Astrophys.* **614**, A111 (2018).
- [17] V. S. Berezhinskii, S. V. Bulanov, V. A. Dogiel, V. L. Ginzburg, and V. S. Ptuskin, *Astrophysics of Cosmic Rays* (North-Holland, Amsterdam, 1990).
- [18] J. Skilling, *Mon. Not. R. Astron. Soc.* **173**, 255 (1975).
- [19] M. Padovani, D. Galli, and A. E. Glassgold, *Astron. Astrophys.* **501**, 619 (2009).
- [20] K. Silsbee and A. V. Ivlev, *Astrophys. J.* **879**, 14 (2019).
- [21] R. M. Crutcher, *Annu. Rev. Astron. Astrophys.* **50**, 29 (2012).
- [22] G. Edenhofer, C. Zucker, P. Frank, A. K. Saydjari, J. S. Speagle, D. Finkbeiner, and T. A. Enßlin, *Astron. Astrophys.* **685**, A82 (2024).
- [23] M. Obolentseva, A. V. Ivlev, K. Silsbee, D. A. Neufeld, P. Caselli, G. Edenhofer, N. Indriolo, T. G. Bisbas, and D. Lomeli, *Astrophys. J.* **973**, 142 (2024).
- [24] M. Aguilar, L. Ali Cavazonza, G. Ambrosi, L. Arruda, N. Attig, F. Barao, L. Barrin, A. Bartoloni, S. Başeğmez-du Pree, J. Bates *et al.*, *Phys. Rep.* **894**, 1 (2021).
- [25] A. C. Cummings, E. C. Stone, B. C. Heikkilä, N. Lal, W. R. Webber, G. Jóhannesson, I. V. Moskalenko, E. Orlando, and T. A. Porter, *Astrophys. J.* **831**, 18 (2016).
- [26] E. C. Stone, A. C. Cummings, B. C. Heikkilä, and N. Lal, *Nat. Astron.* **3**, 1013 (2019).
- [27] V. L. Ginzburg, *Theoretical Physics and Astrophysics* (Pergamon, Oxford, 1979).
- [28] V. L. Ginzburg, *The Propagation of Electromagnetic Waves in Plasmas* (Pergamon, Oxford, 1970).
- [29] R. M. Kulsrud, *Plasma Physics for Astrophysics* (Princeton University Press, Princeton, NJ, 2005).
- [30] B. T. Draine, *Physics of the Interstellar and Intergalactic Medium* (Princeton University Press, Princeton, NJ, 2011).
- [31] J. K. Becker, J. H. Black, M. Safarzadeh, and F. Schuppan, *Astrophys. J. Lett.* **739**, L43 (2011).
- [32] S. Vaupré, P. Hily-Blant, C. Ceccarelli, G. Dubus, S. Gabici, and T. Montmerle, *Astron. Astrophys.* **568**, A50 (2014).
- [33] E. R. Owen, A. Y. L. On, S.-P. Lai, and K. Wu, *Astrophys. J.* **913**, 52 (2021).
- [34] V. A. Dogiel, D. O. Chernyshov, A. M. Kiselev, M. Nobukawa, K. S. Cheng, C. Y. Hui, C. M. Ko, K. K. Nobukawa, and T. G. Tsuru, *Astrophys. J.* **809**, 48 (2015).
- [35] S. Ravikularaman, S. Recchia, V. H. M. Phan, and S. Gabici, *Astron. Astrophys.* **694**, A114 (2025).
- [36] N. Indriolo and B. J. McCall, *Chem. Soc. Rev.* **42**, 7763 (2013).