Comment on "Undecidability in spacetime geometry via the AdS/CFT correspondence"

Agus Hasan Budiyanto*

School of Natural and Mathematical Sciences, University of Mount Saint Vincent, Riverdale, New York 10471, USA

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I. CENTRAL ERROR, SCOPE, AND THE BOUNDARY-VALUE PROBLEM

Mir *et al.* [1] propose to transfer spectral-gap undecidability to the choice between Poincaré anti-de Sitter (AdS_4) spacetime and the AdS_4 soliton. We identify a definite error in the nonhalting branch that invalidates the published saddle-selection argument. For $\vartheta = 0$, Ref. [1] sets the halting coupling and scalar source to zero and states that the scalar profile is trivial. On the common compact boundary required for the Poincaré/soliton comparison, its own Einstein actions then give $f_P = 0$ and $f_{\rm sol} = -A < 0$: the AdS soliton, not Poincaré AdS, dominates. This directly contradicts the zero/Poincaré row of the published action table. The remaining issues discussed below are secondary consistency checks. We also ask whether the broader idea could be repaired: we find no no-go theorem, but a repair would require a genuinely new gravitational mechanism, not a local correction of the displayed formulas.

This Comment has one central claim. In the nonhalting branch $\vartheta = 0$, Ref. [1] turns off the halting-dependent scalar deformation and states that the scalar profile is trivial. On the common compact boundary on which Poincaré AdS and the AdS soliton are compared, the source's own on-shell actions then make the soliton the lower-action saddle. The published zero/Poincaré row is therefore false. Section II gives this argument immediately below. The microscopic and one-loop observations in Secs. III and IV are secondary checks and are not needed for that conclusion.

We comment on Ref. [1]; Supplemental equation numbers refer to its published Supplemental Material [1]. Its two Theorems 1 are distinguished below as microscopic (Supplemental Sec. I) and holographic (Supplemental Sec. VI). We do not question the Cubitt–Pérez-García–Wolf (CPW) spectral-gap theorem itself: its halting and nonhalting branches are gapped and gapless, respectively; see Nature Theorem 1(ii)–(iii) [2], equivalently Theorem 3 (ii)–(iii) of the published full version [3]. Our criticism concerns the transfer from the microscopic construction to the gravitational saddle map written in Ref. [1].

The central issue already produces a boundary-data dilemma. The "Casimir Test" uses $S^1_\beta \times \mathbb{R}^2$, whereas the soliton of Letter Eq. (9) has boundary $S^1_\beta \times \mathbb{R}_x \times S^1_\theta$. If the spatial circle is absent in the $\vartheta = 0$ row, the soliton is inadmissible there, but the two rows then refer to different boundary theories; a bulk scalar source cannot create or remove the nondynamical boundary circle. If instead the circle is fixed so that Poincaré AdS and the soliton are compared on one compact boundary, the $\vartheta = 0$ scalar is trivial and the soliton is an admissible zero-scalar filling. An undecidability reduction may use a uniformly computable input-dependent family, but then the topology-changing boundary data themselves require an explicit computable map from the finite microscopic input; writing the choice through $\vartheta(u)$ presupposes rather than derives that map.

The temperature statements reinforce the fixed-boundary horn. Assumption (A3) fixes $\beta/L_\theta \gg 1$, yet in that same window the Letter says that $\vartheta = 0$ selects the Poincaré branch at $T > T_c$ with $T_c = O(L_\theta^{-1})$. These conditions are mutually incompatible: $\beta/L_\theta \gg 1$ means $T \ll L_\theta^{-1}$. Moreover, the source itself identifies the high-T saddle as the AdS_4 black brane, as in the standard Hawking-Page/AdS-soliton thermodynamics [4], not Poincaré AdS. In the stated low-T regime the Letter gives $F_{\rm bb} = -c_T T^3 L_\theta \to 0$, while $F_{\rm sol} = -c_s/L_\theta^2$ remains finite and negative. Thus neither the stated low-temperature window nor the high-temperature alternative supplies the claimed Poincaré-dominant $\vartheta = 0$ row.

For $\vartheta = 1$, by contrast, a mixed or multitrace scalar boundary condition may in general require more than the Einstein action at $\Phi = 0$. The stationary quantity can contain the renormalized scalar action, a finite boundary functional, and metric backreaction. We do not claim that

*Contact author: agus.budiyanto@umsv.edu

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the undeformed Einstein ordering must survive every such completion. This caveat does not apply to the definite $\vartheta = 0$ contradiction just stated, because Ref. [1] itself sets $g_{\mathrm{halt}} = J = 0$ there and states that the scalar profile is trivial.

II. THE NONHALTING ROW IS FALSE ON THE COMMON BOUNDARY

This is the central point. On the fixed compact boundary used for the soliton comparison, let $f = I_{\mathrm{ren}}/(\beta V_x)$. The cancellation following Supplemental Eq. (125) gives the Poincaré value $I_P = 0$, while Supplemental Eq. (131) gives

$$f_P^{(0)} = 0, \qquad f_{\mathrm{sol}}^{(0)} = -A, \qquad A \coloneqq \frac{4\pi^2 L_{\mathrm{AdS}}^2}{27 G_4 L_\theta^2} > 0, \quad (1)$$

consistent with the standard AdS-soliton result [5]. These are the zero-scalar Einstein values actually evaluated in the source.

For $\vartheta = 0$ they are not merely a truncation of some unspecified deformed problem. The Supplemental construction sets $g_{\mathrm{halt}} = 0$ and $J = \phi_{(0)} = 0$, and the text before Supplemental Eq. (98) states that this produces the trivial scalar profile. Because the mixed/multitrace coefficient is proportional to the same halting coupling, the scalar boundary condition is homogeneous in this branch; hence $\Phi = 0$ satisfies it under either admissible quantization. Therefore $\Phi = 0$ on the soliton is an admissible configuration of the source's nonhalting branch whenever the compact S_θ^1 boundary is fixed. Its action is lower than Poincaré AdS by

$$\Delta I_{\vartheta=0} = I_{\mathrm{sol}}^{(0)} - I_P^{(0)} = -\beta V_x A < 0. \quad (2)$$

The black brane is also subdominant under (A3). Thus Supplemental Eq. (134)'s $\Delta F/V_x = 0$ Poincaré row, and the proof's statement that $\Delta I = 0$ for nonhalting inputs as $\beta \to \infty$, are contradicted on the common compact boundary by the source's own saddle calculation. This conclusion is independent of whether ΔF in Eq. (134) is read as $F_{\mathrm{sol}} - F_P$ or as $F_{\mathrm{dominant}} - F_P$: Eq. (1) gives $-A$ in either reading because the soliton is the lower-action saddle. The assertion in the text before Supplemental Eq. (98) that $J = 0$ has Poincaré AdS as the "unique smooth solution with flat boundary" is likewise false under that boundary reading: the zero-scalar AdS soliton is smooth and has the same fixed compact conformal boundary.

The alternative reading does not rescue the advertised saddle map. If the $\vartheta = 0$ boundary is instead $S_\beta^1 \times \mathbb{R}^2$ with no S_θ^1, then the soliton is indeed inadmissible, but the two rows no longer compare saddles of the same boundary-value problem. In particular L_θ, used in (A3) and in the $\vartheta = 1$ action scale, is fixed boundary data; it is not generated by turning on a bulk scalar source. The construction would then need a uniform computable microscopic-to-boundary map that changes the boundary topology without using the halting predicate as an oracle. No such map is given.

For $\vartheta = 1$, a nonzero mixed-boundary term and scalar backreaction may require regular scalar-haired saddles and the full renormalized Einstein-scalar functional. Reference [1], however, does not compute those stationary actions before asserting Eq. (134). Moreover, its own approximation states around Supplemental Eq. (49) and Fig. 6 that scalar backreaction is negligible and that saddle selection is unaffected; the Letter similarly says large-N_c factorization leaves the saddle competition unaffected. This gives a source-internal dichotomy. If those approximations are used, the deformation cannot be invoked to reverse the leading ordering in Eq. (1). If the deformation instead contributes at leading order strongly enough to modify (c_s, c_T) or the saddle ordering, then the "unaffected" approximation must be abandoned and the deformed thermodynamic potentials and stationary actions recomputed. The printed one-loop term is in any case bounded by $9A/(64\pi^2 N_c^2)$ in magnitude [Supplemental Eq. (116); Sec. IV], so that specific subleading term cannot provide an $O(A)$ cancellation.

Nor is the missing halting-row calculation supplied merely by changing boundary conditions in prose. Before Supplemental Eq. (99), g_{halt} is used as a Dirichlet source; after Eq. (115) it enters a mixed condition. A consistent completion may choose an appropriate variational prescription, but its branch-dependent on-shell actions must then be evaluated. Our definite correction concerns the $\vartheta = 0$ row; for $\vartheta = 1$ we claim only that the published deformed action is not derived.

III. SECONDARY MICROSCOPIC CONSISTENCY CHECKS

This section is not part of the central $\vartheta = 0$ saddle argument. The full CPW theorem is not at issue here. The narrower point is that the abbreviated amplifier/uplift actually written in the Supplemental Material does not establish the claims subsequently assigned to it. In published Supplemental Eqs. (19) and (20),

$$H_{\mathrm{amp}} = \sum_k L_k^q s_k,$$
$$H'_{\mathrm{halt}} = -\delta \sum_k s_k \Pi_{\mathrm{halt}}^{(k)}, \quad q > 4, \quad (3)$$

where $\Pi_{\mathrm{halt}}^{(k)}$ is a projector and $\delta \ll L_k^q$. Supplemental Eq. (21) contains s_k only through these terms. Thus switching $s_k : 0 \to 1$ costs at least $L_k^q - \delta > 0$, so the displayed Hamiltonian minimizes at $s_k = 0$ in either branch. If Eqs. (19)–(21) are only illustrative, they cannot by themselves support the subsequent microscopic

biconditional; if the full CPW interaction table is intended instead, the mapping from that table into the continuum variables must be supplied separately.

The density paragraph immediately following these equations also does not follow from its own scalings. It writes a line shift $-\delta_T/(T+1) \sim -\alpha(T+1)^\kappa$ with $T(\ell) = \Theta(\ell)$ and $\Theta(L_{\rm box}^2/\ell^2)$ active lines. The resulting density scales as $-\alpha\ell^{\kappa-2}$, not $-\alpha\ell^{-2}$ unless $\kappa = 0$, which the source excludes; activating $s_k = 1$ simultaneously adds the positive amplifier density $+\Theta(\ell^{q-2})$. This is a defect of the printed amplifier argument, not a challenge to CPW. The published CPW overview in Sec. 1.4, Step 3, has the opposite relative energy ordering between the halting and nonhalting branches [3]; a common additive energy shift cannot reverse that ordering.

The finite-time Lemmas 1–2 also do not cover unrestricted halting as stated: Lemma 1 treats halting within $T = 2^{p(|u|)}$, while Lemma 2 treats never halting. Restricting to those two cases gives a promise decidable by T-step simulation. If the finite cutoff is only shorthand for the full hierarchy, the conclusion may be recoverable, but the printed proof does not establish it: among prehalting lines the smallest finite-volume gap is controlled by the largest nonhalting scale and therefore by the input's halting time, rather than by the fixed base scale invoked in the proof. Again, replacing this subsection by the full CPW construction would not address the independent holographic issue in Sec. II.

Finally, the incompatible printed forms $g_{\rm halt} = \vartheta\mu_h^2$, $\vartheta\Lambda_{\rm UV}^{p(L)-2}$ [Supplemental Eq. (39)], and $\mu_h^2 = \Lambda_{\rm UV} f[p(L)]$ [text after Eq. (72)] are dimensionally incomplete, but an explicit reference mass scale could repair this. We therefore treat the issue as normalization rather than an obstruction.

IV. FORMULA-LEVEL CORRECTIONS

Several further points are corrections to the published formulas, not independent no-go theorems.

First, the Brown-York tensor quoted before Letter Eq. (11) has the wrong soliton index pattern. For the soliton metric of Letter Eq. (9), the Fefferman-Graham coefficient is

$$g_{(3)ij} = \frac{r_0^3}{3L_{\rm AdS}^6}\,\mathrm{diag}(1,1,-2)_{(\tau,x,\theta)}. \tag{4}$$

For a flat $d = 3$ boundary, $\langle T_{ij}\rangle = (3L_{\rm AdS}^2/16\pi G_4) g_{(3)ij}$ [6]. An overall normal-sign change therefore cannot turn the soliton pattern $\pm a\,\mathrm{diag}(1,1,-2)$ into the Letter's spatially isotropic $a\,\mathrm{diag}(-1,1/2,1/2)$. The latter also has Euclidean trace zero, so Letter Eq. (11)'s value $-3a$ does not follow from its own components; the text after Supplemental Eq. (114) likewise says the trace vanishes. The text before Supplemental Eq. (123) further states $K = -3/L_{\rm AdS}$ while Eq. (124) evaluates the Gibbons-Hawking term with the opposite sign. These are corrections, not a determination of the deformed ordering.

Second, the geometric premise of the one-loop image calculation fails as written. The supplement calls the soliton a "freely acting $\mathbb{Z}$ quotient" along θ, but the S_θ^1 circle shrinks at the smooth tip, so the action is not free and the translational image construction does not compute the determinant on the actual soliton. Consequently we do not claim to compute the true soliton determinant here. We only note additional internal inconsistencies in the printed derivation. Supplemental Eq. (57) and published footnote 1 use the $D = 4$ short-time factor $(4\pi s)^{-2}$, while Eq. (59) inserts an additional $L_\theta/\sqrt{4\pi s}$. The literal proper-time power is then $s^{-7/2}$, which produces $K_{5/2}$ by Eq. (10.32.10) of the NIST Digital Library of Mathematical Functions (DLMF) [7]; enforcing the stated $D = 4$ power instead gives s^{-3} and K_2. Neither route yields the K_1 appearing in Supplemental Eq. (60). Moreover, Eqs. (56)–(59) contain the fixed bulk mass m_Φ, while Eq. (60) replaces it by the branch-dependent $m_{\rm eff}$ without deriving that identification.

Even granting Supplemental Eq. (60) merely as the intended quantity,

$$\delta f_\Phi^{(1)}(\mu) = -\frac{1}{2\pi L_\theta}\sum_{n\geq 1}\frac{\mu}{n}K_1(n\mu L_\theta). \tag{5}$$

Since $d[zK_1(z)]/dz = -zK_0(z) < 0$ and $zK_1(z) \to 1$ [DLMF 10.29.4 [7]], its maximal magnitude is $\pi/(12L_\theta^2)$. Supplemental Eq. (116), together with the source's dictionary $L_{\rm AdS}^2/G_4 = 4\pi N_c^2$, then gives the bound used in Sec. II. More generally, for a fixed finite set of bulk fields a genuine one-loop determinant is $O(G_4^0)$, whereas the classical saddle difference is $O(L_{\rm AdS}^2/G_4)$, i.e., $O(N_c^2)$ in the large-N_c counting; within the stated semiclassical/large-N_c expansion no one-loop determinant, printed or correctly recomputed, can overturn a leading $O(A)$ ordering without leaving that expansion. The reversed inequality in Supplemental Eq. (132) is repairable by the order monotonicity in DLMF Sec. 10.37 [7]. These observations still do not compute the determinant on the true mixed-boundary Einstein-scalar soliton; they show only that the printed determinant is not derived by the displayed image calculation.

Finally, the small-mass behavior of the printed Casimir formula itself is $\varepsilon_{\rm Cas} = -m_*/(12\beta^2) + o(m_*) \to 0$, because Supplemental Eq. (115) carries the prefactor $m_*^2/(2\pi^2\beta)$ and $K_1(z) \sim z^{-1}$. Thus the printed expression actually vanishes in the gapless $m_* \to 0$ limit rather than retaining a finite β^{-3} massless term, and in particular it does not approach the Supplemental text's claimed $-\pi^2/(90\beta^3)$. For comparison, the standard massless $2+1$-dimensional thermal free-energy density is $\beta^{-1}\int d^2k/(2\pi)^2 \ln(1-e^{-\beta|k|}) = -\zeta(3)/(2\pi\beta^3)$, so the

quoted $\pi^2/90$ coefficient is not the $2+1$-dimensional massless coefficient either. This is a correctable formula-level discrepancy.

V. CAN THE CONSTRUCTION BE REPAIRED?

The error above is a failure of the published construction, not a no-go theorem for holographic undecidability. A substantially different gravitational realization could conceivably map the CPW distinction to competing semiclassical phases. What the $\vartheta = 0$ branch shows is that the present mechanism cannot be repaired merely by correcting the Brown-York tensor, changing a Bessel index, or adding the printed subleading determinant: when the halting-dependent scalar deformation is turned off, the ordinary zero-scalar Einstein competition on the common compact boundary already selects the soliton.

A viable repair must therefore modify the leading saddle-selection mechanism itself. At minimum, it must specify a single consistent boundary-value problem (or an explicit uniformly computable map to an input-dependent family of boundary problems), generate a branch-sensitive leading contribution without inserting the halting predicate as an oracle, and compute the corresponding regular Einstein-matter stationary solutions and renormalized on-shell actions. For the $\vartheta = 1$ branch, a sufficiently strong mixed/multitrace deformation might in principle alter the ordering, but then the negligible-backreaction and "unaffected saddle competition" approximations used in Ref. [1] cannot simultaneously justify the result. For the $\vartheta = 0$ branch, any repair must introduce some additional mechanism that remains operative when the present halting-dependent scalar source vanishes.

There is also a structural burden that any repair must address. The CPW predicate is a property of the thermodynamic-limit family: no fixed finite-volume truncation decides the halting/gap branch. In Ref. [1], by contrast, the semiclassical saddle is controlled by a finite set of effective boundary/IR parameters such as m_{eff}, the scalar boundary coupling, β, and L_θ. A repair must therefore exhibit a uniform computable map from the finite microscopic interaction table to boundary data whose limiting behavior produces a leading $O(N_c^2)$ phase distinction, rather than insert the limit-only predicate $\vartheta(u)$ into those parameters. A singular or thermodynamic limit could in principle amplify such information into a phase choice, but that mechanism must be constructed and its saddle actions calculated; continuity of an ordinary finite-parameter deformation away from a phase boundary does not supply it automatically.

Possible routes could involve a different matter sector, a different pair of competing saddles, or a revised microscopic-to-boundary dictionary. We do not analyze such constructions here and therefore leave open whether the broader program can be realized. The limited conclusion is that such a repair would be a new gravitational construction rather than an erratum-level modification of the saddle table derived in Ref. [1].

VI. CONCLUSION

The central result is the $\vartheta = 0$ contradiction. On the common compact boundary required by the advertised Poincaré/soliton comparison, Ref. [1] sets $g_{\mathrm{halt}} = J = 0$ and the scalar profile is trivial. Its own actions then give Eq. (2), so the zero/Poincaré row of Supplemental Eq. (134) is a definite false conclusion, not merely an omitted deformed calculation. If instead the spatial circle is removed only from that row, the construction compares different boundary theories and requires an additional uniform computable topology-changing boundary map.

For the halting branch, a fully specified mixed-boundary Einstein-scalar functional could in principle alter the undeformed ordering, but the required stationary actions are not evaluated while the source calls scalar backreaction negligible and saddle selection unaffected. The Brown-York tensor, abbreviated amplifier, dimensional normalization, and one-loop image formula are secondary correctable issues; none is needed for the $\vartheta = 0$ conclusion, and ordinary one-loop effects are parametrically subleading to the classical $O(N_c^2)$ saddle difference in the stated semiclassical regime. As discussed in Sec. V, our result does not exclude a substantially different holographic construction. It shows that the published mechanism is wrong as written and that repairing it requires a new leading saddle-selection mechanism, together with an explicit consistent variational problem and on-shell action calculation. This does not follow from Supplemental Eqs. (125), (131), and (134) as presently written.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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