

Investigation of the photoproduction of bottom-charmed baryons within NRQCD

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We present a further theoretical study of the orbital P -wave bottom-charmed baryon within the framework of nonrelativistic QCD, considering both the direct photoproduction channel $\gamma + \gamma \rightarrow \Xi_{bc} + \bar{c} + \bar{b}$ and the resolved photoproduction channel $\gamma + g \rightarrow \Xi_{bc} + \bar{c} + \bar{b}$. At future linear colliders, ILC and CLIC, the initial photons can be emitted from the laser backscattering and then the parton gluon can be emitted from the photon. The formation of Ξ_{bc} can be modeled in two steps: a compact diquark state $\langle bc \rangle [n]$ is formed first and subsequently captures a light quark from the vacuum to hadronize into the baryon Ξ_{bc} . The color and spin quantum number $[n]$ of the $\langle bc \rangle$ diquark can be $[^3S_1]_{\bar{3}/6}$, $[^1S_0]_{\bar{3}/6}$, $[^1P_1]_{\bar{3}/6}$, or $[^3P_J]_{\bar{3}/6}$ with $J = 0, 1, 2$. Using three benchmark collision energies and an illustrative integrated luminosity, we analyze the cross sections, differential distributions, and the estimated produced events of the Ξ_{bc} baryon. For the conservative $h_6 = 0$ baseline, the combined P -wave contribution amounts to approximately 8% of the combined S -wave rate at three considered collision energies and is therefore non-negligible.

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I. INTRODUCTION

Since the discovery of the doubly charmed baryon Ξ_{cc}^{++} by LHCb Collaboration in 2017 [1–3], the quark model [4–6] has achieved a successful verification, while also presenting new challenges. It predicted the existence of bottom-charmed baryons, which consist of two different heavy quarks (b and c) and a light quark (u , d , or s) in multiplets. Because of its heavy mass and the need for both b and c component quarks, compared to the doubly charmed baryons, its cross section is smaller. The LHCb Collaboration using proton-proton collision data performed the first search for the $\Xi_{bc}^+(bcu)$, $\Xi_{bc}^0(bcd)$, and $\Omega_{bc}^0(bcs)$ baryons through the decay channels $\Xi_{bc}^+ \rightarrow J/\psi \Xi_c^+$, $\Xi_{bc}^0 \rightarrow D^0 p K^-$, and $\Xi_{bc}^0/\Omega_{bc}^0 \rightarrow \Xi_c^+/\Lambda_c^+ \pi^-$ [7–9]. However, these bottom-charmed baryons have not yet had their existence confirmed through experiments to date. Meanwhile, a series of in-depth theoretical studies were carried out using theoretical potential models and phenomenology to focus on

the mass, lifetime, production mechanisms, and decay properties of Ξ_{bc} (here, the light quark is not explicitly identified) and their excited states [10–19]. The study of these hadrons plays an indispensable role in understanding the quantum chromodynamics (QCD) theory, revealing the interaction between quarks and gluons and exploring the structure of baryons.

With the improvement of the collision energy and luminosity in high-energy physics experiments, the feasibility of discovering the bottom-charmed baryon has been greatly enhanced. At the same time, more precise theoretical calculation results are required for the guidance of experimental exploration. In the early stages, a large number of theoretical predictions focused on the production mechanisms of doubly heavy baryons Ξ_{cc} , Ξ_{bc} , and Ξ_{bb} in S waves on multiple platforms, including direct production at hadron colliders [20–31], electron-positron colliders [32–35], $\gamma\gamma$ [20,36–38] collision, ep collision [20,39,40], and heavy-ion colliders [26,41], as well as the indirect production through top quark decays [42], Higgs [43], W^+ [44], and Z^0 [45] boson decay. Subsequently, research on the production mechanism of excited P -wave doubly heavy baryons has been carried out successively [44,46–49]. The results show that the contributions of doubly heavy baryons in P waves cannot be negligible, reaching several percent, and it is worthy of further in-depth study. Therefore, at e^+e^- colliders, the photoproduction of doubly heavy baryons Ξ_{bc} in P waves can be conducted.

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Regarding the photoproduction of doubly heavy baryons Ξ_{bc} at e^+e^- colliders, it is typically a direct process $\gamma + \gamma \rightarrow \Xi_{bc} + \bar{c} + \bar{b}$, as well as the resolved channel $\gamma + g \rightarrow \Xi_{bc} + \bar{c} + \bar{b}$ [50]. The initial γ can be emitted either from the bremsstrahlung process and its energy distribution can be well delineated within the Weizsäcker-Williams approximation (WWA) [51] or from the process of laser backscattering (LBS) of e^+e^- at future linear colliders, ILC and CLIC [52,53]. However, the photons from these two sources have different trends. The photon spectrum of LBS shows an upward trend, while that of WWA shows a logarithmic decline trend [38,54]. Future e^+e^- colliders, such as FCC-ee [55], CEPC [56,57], ILC [52,58], and CLIC [53] will provide a very favorable experimental platform for the search for bottom-charmed baryons. Especially at ILC and CLIC, the contribution of resolved photoproduction channel ($\gamma + g$) of Ξ_{bc} is significant, which is attributed to the LBS photon [59–63]. The contribution from a double resolved photoproduction channel is usually very small and can be disregarded. After the LBS photon undergoes a process of resolution, its partons (γ, g, u, d, s) will participate in the subsequent strong interaction, resulting in a high-energy process that must involve final-state particle b and c quarks. However, for the production of bottom-charmed Ξ_{bc} baryons, the key lies in the nonperturbative hadronization process.

Nonrelativistic quantum chromodynamics (NRQCD) [64] provides a good theoretical framework to effectively solve such production problems [65]. Within the framework of NRQCD, the production of hadrons can be factorized into two parts: the perturbative region and the nonperturbative region. First, in the hard process, the free final-state particle b and c quarks are perturbatively produced into a diquark with a distinct quantum state $\langle bc \rangle[n]$, and then this intermediate diquark state nonperturbatively transitions to the bottom-charmed baryon Ξ_{bc} . This transition process is quantified by long-distance matrix elements (LDMEs) and can be approximated by the wave function at the origin (or its spatial derivative for orbital excited states) within potential models. The spin and color quantum number $[n]$ of $\langle bc \rangle$ diquark can be $[^1S_0]_{\bar{3}/6}$, $[^3S_1]_{\bar{3}/6}$, $[^1P_1]_{\bar{3}/6}$, and $[^3P_J]_{\bar{3}/6}$ ($J = 0, 1, 2$). These diquark states, respectively, correspond to 1S_0 and 3S_1 configurations of S waves and 1P_1 and 3P_J configurations for excited P waves. $\bar{3}$ and 6 are the color quantum numbers of $\langle bc \rangle$ diquarks, representing the color antitriplet state and sextet state for the decomposition of $SU(3)_c$ color group $3 \otimes 3 = \bar{3} \oplus 6$. According to the velocity scaling rule in NRQCD, the orbital angular momentum L yields a suppression v^{2L} in the LDMEs, where v represents the relative velocity between two constituent quarks in the rest frame of the heavy diquark state. So the contribution of P waves ($L = 1$) gain a v^2 suppression against that of S waves ($L = 0$). Moreover, the higher angular momentum D -wave

states carry v^4 suppression compared to S waves. Hence, the contributions of D waves can be safely neglected in our calculations. In the perturbative hard process, that is, the short-distance coefficient, the energy dependence of the cross section for the photoproduction of Ξ_{bc} in S waves initially increases rapidly, reaches its maximum at around 90 GeV, and then decreases steadily [59]. Here, in order to give a comprehensive comparison and analysis, the cross section for the photoproduction of the ground and excited state Ξ_{bc} baryons is investigated at ILC and CLIC with collision energy $\sqrt{s} = 250, 500, 1000$ GeV, respectively.

In this work, the photoproduction of bottom-charmed baryons is analyzed through the direct channel $\gamma + \gamma \rightarrow \Xi_{bc} + \bar{c} + \bar{b}$ and the resolved channel $\gamma + g \rightarrow \Xi_{bc} + \bar{c} + \bar{b}$ within the NRQCD factorization framework. The rest of the paper is arranged as follows. In Sec. II, the necessary calculation formulas and technology are provided. The numerical results are presented in Sec. III, involving cross sections, transverse momentum, rapidity, and angular and invariant mass distributions. Section IV is reserved for a brief summary.

II. CALCULATION TECHNOLOGY

In the factorization framework of NRQCD, the differential cross section for the photoproduction of Ξ_{bc} can be factorized into

$$\begin{aligned} d\sigma(e^+e^- \rightarrow e^+e^- + \Xi_{bc} + \bar{c} + \bar{b}) \\ = \int dx_1 f_{\gamma/e}(x_1) \int dx_2 f_{\gamma/e}(x_2) \\ \times \sum_{i,j} \int dx_i f_{i/\gamma}(x_i; \mu_F) \int dx_j f_{j/\gamma}(x_j; \mu_F) \\ \times \sum_n d\hat{\sigma}(ij \rightarrow \langle bc \rangle[n] + \bar{c} + \bar{b}; s_{ij}, \mu_F, \mu_R) \langle \mathcal{O}^{\Xi_{bc}}[n] \rangle, \end{aligned} \quad (1)$$

where the partonic center-of-mass energy squared is defined as $s_{ij} = x_i x_j s_{\gamma\gamma} = x_i x_j x_1 x_2 s$, with $\sqrt{s}$ being the e^+e^- collision energy. μ_R and μ_F are the renormalization and factorization scales, respectively. The energy spectrum of the LBS photon $f_{\gamma/e}(x)$ can be described by Ginzburg *et al.* [66],

$$f_{\gamma/e}(x) = \frac{1}{N} \left(1 - x + \frac{1}{1-x} - \frac{4x}{x_m(1-x)} + \frac{4x^2}{x_m^2(1-x)^2} \right), \quad (2)$$

where $x = E_\gamma/E_e$ represents the energy fraction of the LBS photon emitted from the initial electron or positron, and the normalization factor N is

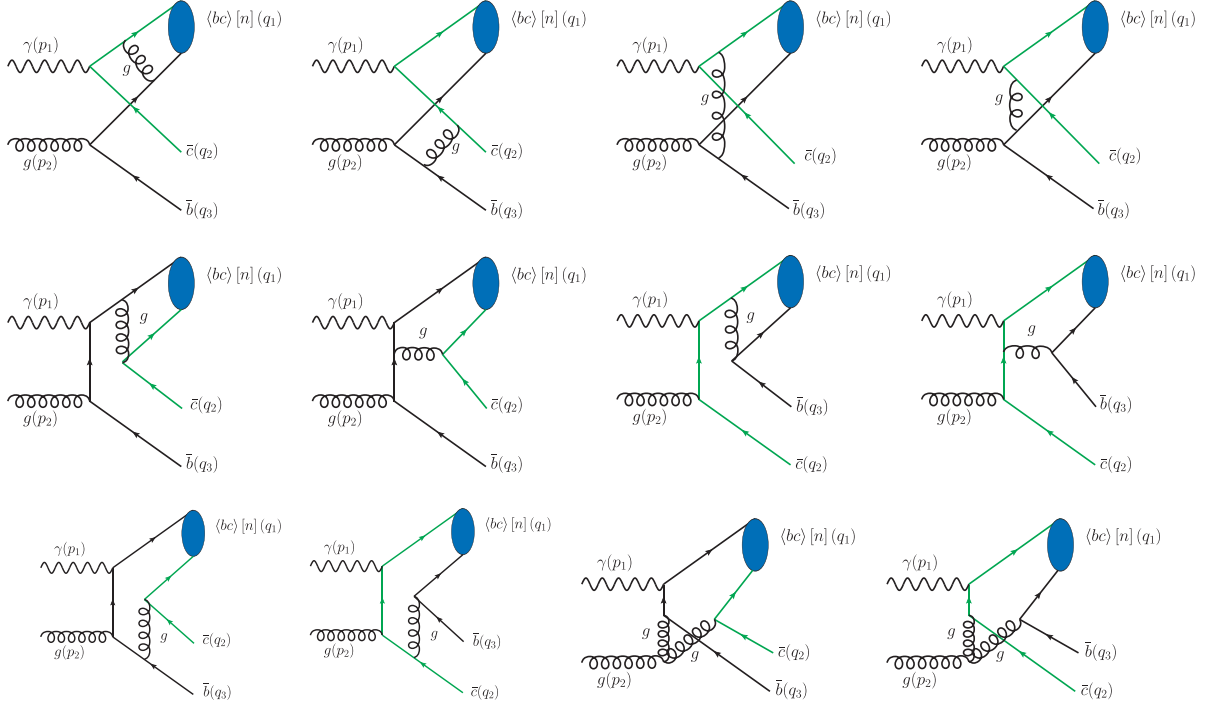


FIG. 1. Twelve typical Feynman diagrams for the photoproduction of Ξ_{bc} through the intermediate subprocess $\gamma + g \rightarrow \langle bc \rangle[n] + \bar{c} + \bar{b}$; another 12 can be obtained by interchanging the initial photon and gluon lines.

$$N = \left(1 - \frac{4}{x_m} - \frac{8}{x_m^2}\right) \log(1 + x_m) + \frac{1}{2} + \frac{8}{x_m} - \frac{1}{2(1 + x_m)^2}, \quad (3)$$

with $x_m = 4E_e E_l \cos^2 \frac{\theta}{2}$ (E_e and E_l represent the energies of the incident electron and laser beams, respectively, and the angle between them is denoted as θ). The range of energy fraction x for the LBS photon is constrained by [67]

$$0 \leq x \leq \frac{x_m}{1 + x_m} (x_m \approx 4.83). \quad (4)$$

For two different initial subprocesses, $\gamma + \gamma$ and $\gamma + g$, the Glück-Reya-Schienbein (GRS) distribution function $f_{i(j)/\gamma}$ [$i(j) = \gamma, g, u, d, s$] of parton $i(j)$ in the photon [68] is needed, and $f_{\gamma/\gamma}(x) = \delta(1 - x)$ is for the direct photoproduction process $\gamma + \gamma$. The differential partonic cross section $d\hat{\sigma}(ij \rightarrow \langle bc \rangle[n] + \bar{c} + \bar{b})$ can be evaluated perturbatively for $\gamma + \gamma$ and $\gamma + g$ subprocesses. For the partonic processes at leading order, there are 20 Feynman diagrams for $\gamma + \gamma \rightarrow \Xi_{bc} + \bar{c} + \bar{b}$ in $\mathcal{O}(\alpha^2 \alpha_s^2)$ and 24 diagrams for $\gamma + g \rightarrow \Xi_{bc} + \bar{c} + \bar{b}$ in $\mathcal{O}(\alpha \alpha_s^3)$. For $\gamma(p_1) + g(p_2) \rightarrow \langle bc \rangle[n](q_1) + \bar{c}(q_2) + \bar{b}(q_3)$, 12 typical Feynman diagrams are shown in Fig. 1, and another 12 can be obtained by interchanging the initial photon and gluon lines. The Feynman diagrams for $\gamma(p_1) + \gamma(p_2) \rightarrow \langle bc \rangle[n](q_1) + \bar{c}(q_2) + \bar{b}(q_3)$ can be found in an earlier study [59], and they will not be shown here. The additional

couplings associated with the partonic content of the resolved photon and with the LBS photon are included in $f_{i/\gamma}$ and $f_{\gamma/e}$, respectively. The GRS parametrization is used for the central predictions. Different photon-parton distribution function (PDF) parametrizations [69,70] may introduce an additional normalization and kinematic-shape dependence, especially for the resolved $\gamma + g$ channel through the gluon distribution in the photon.

The differential cross section $d\hat{\sigma}(ij \rightarrow \langle bc \rangle[n] + \bar{c} + \bar{b})$ can be rewritten as

$$d\hat{\sigma}(ij \rightarrow \langle bc \rangle[n] + \bar{c} + \bar{b}) = \frac{1}{4\sqrt{(p_1 \cdot p_2)^2 - m_e^4}} \bar{\sum} |\mathcal{M}[n]|^2 d\Phi_3, \quad (5)$$

in which $\bar{\sum}$ is the average over the spins of the initial states and the sum over colors and spins of the final states, and $d\Phi_3$ is the three-body phase space. For the production of bottom-charmed baryons Ξ_{bc} , the spin and color quantum number $[n]$ of $\langle bc \rangle$ -diquark configurations can be $\langle bc \rangle[{}^1S_0]_{\bar{3}/6}$, $\langle bc \rangle[{}^3S_1]_{\bar{3}/6}$, $\langle bc \rangle[{}^1P_1]_{\bar{3}/6}$, or $\langle bc \rangle[{}^3P_J]_{\bar{3}/6}$. To obtain the amplitude $\mathcal{M}[n]$, two necessary actions need to be applied. First, the charge conjugation matrix $C = -i\gamma^2\gamma^0$ needs to be applied to reverse the $c \sim \bar{c}$ fermion chain (green line in Fig. 1). It has been proven that, with the charge conjugation matrix C , the amplitude for the production of $b(q_{11})c(q_{12}) + \bar{c}(q_2) + \bar{b}(q_3)$ can be related to the amplitude of $b(q_{11})\bar{c}(q_{12}) + c(q_2) + \bar{b}(q_3)$ with an

additional factor $(-1)^{(n+1)}$ (n is the number of vector vertices in the fermion chain) [35,38,59]. Here $q_{11} = \frac{m_b}{M_{bc}}q_1 + q$ and $q_{12} = \frac{m_c}{M_{bc}}q_1 - q$ are the momenta of b and c constituent quarks with q_1 being the momentum of the $\langle bc \rangle$ diquark. The mass of the $\langle bc \rangle$ diquark is set as $M_{bc} = m_b + m_c$ to ensure that the constituent quarks are strictly on shell, which guarantees the gauge invariance of the amplitude. At the leading order in v , the small relative momentum q (with $|\vec{q}| \sim M_{bc}v$) can be set to zero directly in the short-distance amplitude for the production of S -wave diquarks. The second step is to introduce the spin projector $\Pi_{q_1}(q)[n]$ to transform the $b(q_{11})\bar{c}(q_{12})$ constituent quarks into bound states $\langle b\bar{c} \rangle(q_1)$ and it takes the form of [71]

$$\Pi_{q_1}(q)[n] = \frac{1}{2\sqrt{M_{bc}}} \varepsilon[n](\not{q}_1 + M_{bc}), \quad (6)$$

for $\varepsilon[{}^1S_0] = \gamma^5$ and $\varepsilon[{}^3S_1] = \not{\epsilon}$ with the polarization vector ϵ^β of 3S_1 diquark state. The amplitudes of P -wave Ξ_{bc} can be obtained by the first derivation of the relative momentum q between two constituent quarks in S -wave amplitude,

$$\mathcal{M}[{}^1P_1] = \varepsilon_\alpha^l(q_1) \frac{d}{dq_\alpha} \mathcal{M}[{}^1S_0]|_{q=0}, \quad (7)$$

$$\mathcal{M}[{}^3P_J] = \varepsilon_{\alpha\beta}^J(q_1) \frac{d}{dq_\alpha} \mathcal{M}[{}^3S_1]|_{q=0}, \quad (8)$$

in which q is always included in the momenta of the constituent quark of the diquark, q_{11} and q_{12} , and appears in the spin projector Π_{q_1} and propagators. $\varepsilon_\alpha^l(q_1)$ and $\varepsilon_{\alpha\beta}^J(q_1)$ correspond to the polarization vector and polarization tensor of the spin singlet and spin triplet P -wave states, where J can be 0, 1, or 2. The summations over polarization vector and tensor satisfy [71]

$$\sum_{l_z} \varepsilon_\alpha^l \varepsilon_\alpha^{l*} = \Pi_{\alpha\alpha'}, \quad (9)$$

$$\varepsilon_{\alpha\beta}^0 \varepsilon_{\alpha'\beta'}^{0*} = \frac{1}{3} \Pi_{\alpha\beta} \Pi_{\alpha'\beta'}, \quad (10)$$

$$\sum_{J_z} \varepsilon_{\alpha\beta}^1 \varepsilon_{\alpha'\beta'}^{1*} = \frac{1}{2} (\Pi_{\alpha\alpha'} \Pi_{\beta\beta'} - \Pi_{\alpha\beta'} \Pi_{\alpha'\beta}), \quad (11)$$

$$\sum_{J_z} \varepsilon_{\alpha\beta}^2 \varepsilon_{\alpha'\beta'}^{2*} = \frac{1}{2} (\Pi_{\alpha\alpha'} \Pi_{\beta\beta'} + \Pi_{\alpha\beta'} \Pi_{\alpha'\beta}) - \frac{1}{3} \Pi_{\alpha\beta} \Pi_{\alpha'\beta'}, \quad (12)$$

with the definition $\Pi_{\alpha\beta} = -g_{\alpha\beta} + \frac{q_{1\alpha}q_{1\beta}}{M_{bc}^2}$. The color factor $\mathcal{C}_{ij,k}$ for all the Feynman diagrams has been extracted from the amplitude and it satisfies $\mathcal{C}_{ij,k} = \mathcal{N} \times \sum_{a,m,n} (T^a)_{mi} (T^a)_{nj} \times G_{mnk}$, where the normalization

constant $\mathcal{N} = \sqrt{1/2}$, i, j, m, n are the color indices of four heavy quarks, k denotes the color index of the diquark, and G_{mnk} corresponds to either the antisymmetric function ε_{mnk} for the color antitriplet state or the symmetric function f_{mnk} for the sextet state; they satisfy

$$\varepsilon_{mnk} \varepsilon_{m'n'k} = \delta_{mm'} \delta_{nn'} - \delta_{mn'} \delta_{nm'}, \quad (13)$$

$$f_{mnk} f_{m'n'k} = \delta_{mm'} \delta_{nn'} + \delta_{mn'} \delta_{nm'}. \quad (14)$$

Through the above perturbative QCD calculations, we can obtain the short-distance coefficients for the production of intermediate $\langle bc \rangle$ -diquark states. The next step is the crucial hadronization process of the diquark state transitioning to the Ξ_{bc} baryons through the strong interaction. The LDME $\langle \mathcal{O}_{\Xi_{bc}}[n] \rangle$ in Eq. (1) can be described by $h_{\bar{3}}$ and h_6 for antitriplet and sextet states, which is nonperturbative and accompanied by significant theoretical uncertainty. Typically, $h_{\bar{3}}$ can be obtained by the potential model and approximately correlated to the Schrödinger wave function at the origin for the S -wave diquark and the first-derivative wave function at the origin for P -wave states [20,72–75], which can be naturally connected to the (derivative) radial wave function at the origin, i.e.,

$$h_{\bar{3}}[S] \simeq |\Psi_{bc}(0)|^2 = \frac{1}{4\pi} |R_{bc}(0)|^2, \quad (15)$$

$$h_{\bar{3}}[P] \simeq |\Psi'_{bc}(0)|^2 = \frac{3}{4\pi} |R'_{bc}(0)|^2. \quad (16)$$

As for h_6 , there is no such clear correlation; however, according to NRQCD theory, the doubly heavy baryon Ξ_{bc} can be expanded into a series of Fock states,

$$|\Xi_{bc}\rangle = c_1(v)|\langle bc \rangle q\rangle + c_2(v)|\langle bc \rangle qg\rangle + c_3(v)|\langle bc \rangle qgg\rangle + \dots, \quad (17)$$

in which v is the relative velocity of the constituent heavy quarks in the baryon rest frame. In the early literature, there are two points of view for accounting for the contributions of each Fock state. One power counting rule indicates that since a light quark can readily emit gluons, those Fock states in Eq. (17) are of same importance, specifically, $c_1(v) \sim c_2(v) \sim c_3(v)$ [65]. Thus, both h_6 and $h_{\bar{3}}$ are assigned equivalent orders, i.e., $h_6 = h_{\bar{3}}$. While in “one-gluon-exchange” models, the interaction in the antitriplet state is attractive, which changes to be repulsive for the sextet channel, suggesting suppression at least v^2 relative to $h_{\bar{3}}$, or too small to be ignored [35,65]. Therefore, we adopt $h_6 = 0$ as the conservative baseline in the subsequent calculations and treat other choices explicitly as speculative to estimate the contribution of the color sextet.

III. NUMERICAL RESULTS

In the numerical calculation, the input parameters are listed as follows [20,76]:

$$\begin{aligned}
 |R_{bc}(0)| &= 0.722 \text{ GeV}^{3/2}, \quad |R'_{bc}(0)| = 0.200 \text{ GeV}^{5/2}, \\
 m_b &= 5.1 \text{ GeV}, \quad m_c = 1.8 \text{ GeV}, \quad M_{bc} = m_b + m_c, \\
 \alpha &= 1/132.23, \quad \mu_R = \mu_F \equiv \mu_0 = \sqrt{M_{\Xi_{bc}}^2 + p_T^2},
 \end{aligned}
 \tag{18}$$

where $|R_{bc}(0)|$ and $|R'_{bc}(0)|$ are evaluated under the K^2O potential motivated by QCD with a three-loop function [77]. For the central prediction, the renormalization and factorization scales are identified and chosen as the transverse mass of Ξ_{bc} , $\mu_R = \mu_F = \mu_0 = \sqrt{M_{\Xi_{bc}}^2 + p_T^2}$. The one-loop running coupling $\alpha_s(\mu_R)$ and the GRS photon PDF evaluated at μ_F are then used consistently.

At ILC and CLIC with the collision energy $\sqrt{s} = 250, 500, 1000$ GeV, the cross sections of all considered antitriplet intermediate diquark states are conducted via both photon-photon fusion and photon-gluon fusion, and the results are provided in Tables I and II. By summing up all the considered antitriplet states, we obtained the combined contributions of S and P waves, and the results are listed in Table III. From these three tables, we can see that

TABLE I. The cross sections σ (in units of femtobarn) of all considered antitriplet intermediate diquark states for Ξ_{bc} photoproduction via photon-photon fusion using LBS spectrum at ILC and CLIC with different collision energy $\sqrt{s}$ (GeV).

$\sigma(n)$						
$\sqrt{s}$	$[^1S_0]_{\bar{3}}$	$[^3S_1]_{\bar{3}}$	$[^1P_1]_{\bar{3}}$	$[^3P_0]_{\bar{3}}$	$[^3P_1]_{\bar{3}}$	$[^3P_2]_{\bar{3}}$
250	7.90	17.47	0.40	0.33	0.33	1.08
500	3.80	8.57	0.21	0.17	0.16	0.55
1000	1.61	3.69	0.09	0.07	0.07	0.24

TABLE II. The cross sections σ (in units of femtobarn) of all considered antitriplet intermediate diquark states for Ξ_{bc} photoproduction via photon-gluon fusion using LBS spectrum at ILC and CLIC with different collision energy $\sqrt{s}$ (GeV).

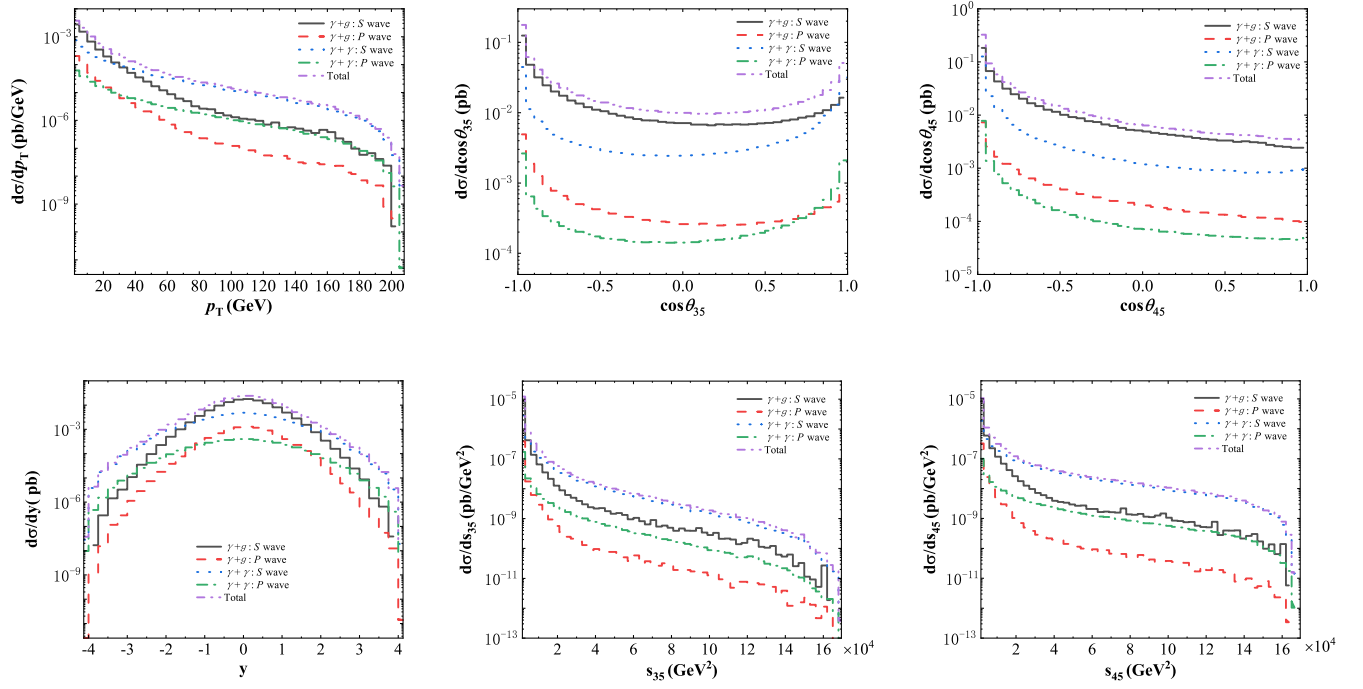
$\sigma(n)$						
$\sqrt{s}$	$[^1S_0]_{\bar{3}}$	$[^3S_1]_{\bar{3}}$	$[^1P_1]_{\bar{3}}$	$[^3P_0]_{\bar{3}}$	$[^3P_1]_{\bar{3}}$	$[^3P_2]_{\bar{3}}$
250	3.27	10.90	0.28	0.08	0.16	0.53
500	6.87	22.37	0.57	0.15	0.35	1.11
1000	12.44	40.08	1.03	0.27	0.62	1.98

TABLE III. The total cross sections σ (in units of femtobarn) for Ξ_{bc} photoproduction via photon-photon and photon-gluon fusion using LBS spectrum at ILC and CLIC with different collision energy $\sqrt{s}$ (GeV).

$\sqrt{s}$	$\gamma + \gamma$			$\gamma + g$			Total
	S wave	P wave	Total	S wave	P wave	Total	
250	25.37	2.14	27.51	14.17	1.05	15.22	42.73
500	12.37	1.09	13.46	29.24	2.18	31.42	44.88
1000	5.30	0.47	5.77	52.52	3.90	56.42	62.19

- (i) With the increasing of $\sqrt{s}$, the contribution of each $\langle bc \rangle$ -diquark state decreases via $\gamma + \gamma$ fusion. However, for $\gamma + g$ fusion, the contribution of each $\langle bc \rangle$ -diquark state increases as the energy increases. Based on the comprehensive analysis of the two subprocesses of photoproduction, the total cross section increases with the increase of collision energy. At $\sqrt{s} = 250$ GeV, the contribution of $\gamma + \gamma$ fusion is dominant in the photoproduction process. When the collision energy reaches 500 GeV or higher, the contribution of photon-gluon fusion shows a clearly overwhelming trend.
- (ii) Among these diquark states, the contribution of $[^3S_1]_{\bar{3}}$ is the largest, accounting for 63.50%, 63.67%, and 63.95% (71.62%, 71.20%, and 71.04%) of the total in $\gamma + \gamma$ ($\gamma + g$) fusion when $\sqrt{s} = 250, 500, 1000$ GeV. The $[^3P_2]_{\bar{3}}$ state dominates the total P -wave state, accounting for 50.47% (50.46% and 51.06%) and 50.48% (50.92% and 50.77%) in $\gamma + \gamma$ and $\gamma + g$ fusion when $\sqrt{s} = 250$ (500 and 1000) GeV, respectively.
- (iii) The contribution of S -wave diquark states is significantly larger than that of the P wave. For the subprocess $\gamma + \gamma$ ($\gamma + g$) fusion when $\sqrt{s} = 250, 500$, and 1000 GeV, the contribution of the P -wave state is 8.44%, 8.81%, and 8.87% (7.41%, 7.46%, and 7.43%) of that of the S wave.
- (iv) The combined (S -wave, P -wave, total) cross sections are (39.54, 3.19, 42.73), (41.61, 3.27, 44.88), and (57.82, 4.37, 62.19) fb for $\sqrt{s} = 250, 500$, and 1000 GeV, respectively. The corresponding ratios $\sigma_{P\text{-wave}}/\sigma_{S\text{-wave}}$ are 8.07%, 7.86%, and 7.56%, while $\sigma_{P\text{-wave}}/\sigma_{\text{tot}}$ are 7.47%, 7.29%, and 7.03%.

Based on the analysis of two subprocesses of photoproduction, the contributions of the P -wave state is approximately 8% of the S -wave production. Assuming that at ILC and CLIC, with an illustrative integrated luminosity of $\mathcal{O}(10^4)\text{fb}^{-1}$ [37,38,48], the produced events of P -wave excited baryons Ξ_{bc} from two subprocesses of photoproduction can be approximately 3.19×10^4 ($3.27 \times 10^4, 4.37 \times 10^4$) at $\sqrt{s} = 250$ (500, 1000) GeV. Subsequently, if the P -wave states can decay to the ground state with a 100% probability, approximately 4.27×10^5

FIG. 2. Kinematic distributions for the photoproduction of Ξ_{bc} at ILC and CLIC with $\sqrt{s} = 500$ GeV.

(4.49×10^5 , 6.22×10^5) of baryons could be produced at $\sqrt{s} = 250$ (500, 1000) GeV, providing an upper-bound estimation for the feed-down contributions. Note that these estimated events are just parton-level inclusive rates. To evaluate the actual experimental discovery potential, one must further consider the realistic detector acceptance, hadronization efficiencies, and physical background subtractions.

To reveal more kinematic characteristics for the photoproduction of Ξ_{bc} at ILC and CLIC ($\sqrt{s} = 500$ GeV is selected as a representative), the differential distributions are presented in Fig. 2, including the transverse momentum (p_T) and the rapidity (y) of Ξ_{bc} in diquark multistates, the

angular ($\cos \theta_{35}$ and $\cos \theta_{45}$) and the invariant mass (s_{35} and s_{45}) distributions. Here the definitions of the invariant mass are $s_{35} = (q_1 + q_3)^2$ and $s_{45} = (q_2 + q_3)^2$, and θ_{35} (θ_{45}) is the angle between the momenta q_1 (q_2) and q_3 . Because of the excessive intermediate $\langle bc \rangle$ -diquark states in the two subprocesses of photoproduction and the similar trends of each diquark state in S and P waves, the contribution of each diquark state in S and P waves is summed when presenting the differential distributions. To analyze the overall distribution, the contributions of S and P waves are summed up and labeled as “Total.” Six different distributions all show that the contribution of the S wave is clearly dominant throughout the whole region compared with the

TABLE IV. Heavy-quark-mass dependence of the color antitriplet contribution for the direct photoproduction channel $\gamma + \gamma \rightarrow \Xi_{bc} + \bar{c} + \bar{b}$ at $\sqrt{s} = 500$ GeV. The cross sections are given in units of femtobarn.

State	$m_c = 1.5$	$(m_c, m_b) = (1.8, 5.1)$	$m_c = 2.1$	$m_b = 4.8$	$m_b = 5.4$
$[^1S_0]_{\bar{3}}$	5.62	3.80	2.74	4.32	3.35
$[^3S_1]_{\bar{3}}$	12.97	8.57	6.05	9.73	7.59
$[^1P_1]_{\bar{3}}$	0.44	0.21	0.11	0.24	0.18
$[^3P_0]_{\bar{3}}$	0.31	0.17	0.10	0.20	0.14
$[^3P_1]_{\bar{3}}$	0.33	0.16	0.09	0.19	0.14
$[^3P_2]_{\bar{3}}$	1.15	0.55	0.30	0.63	0.48
σ_S	18.59	12.37	8.79	14.05	10.94
σ_P	2.23	1.09	0.60	1.26	0.94
σ_{tot}	20.82	13.46	9.39	15.31	11.88
$\sigma_P/\sigma_S(\%)$	12.00	8.81	6.83	8.97	8.59
$\sigma_P/\sigma_{\text{tot}}(\%)$	10.71	8.10	6.39	8.23	7.91

TABLE V. Heavy-quark-mass dependence of the color antitriplet contribution for the resolved photoproduction channel $\gamma + g \rightarrow \Xi_{bc} + \bar{c} + \bar{b}$ at $\sqrt{s} = 500$ GeV. The cross sections are given in units of femtobarn.

State	$m_c = 1.5$	$(m_c, m_b) = (1.8, 5.1)$	$m_c = 2.1$	$m_b = 4.8$	$m_b = 5.4$
$[^1S_0]_{\bar{3}}$	11.75	6.87	4.36	8.35	5.72
$[^3S_1]_{\bar{3}}$	37.69	22.37	14.41	27.28	18.59
$[^1P_1]_{\bar{3}}$	1.32	0.57	0.28	0.71	0.46
$[^3P_0]_{\bar{3}}$	0.32	0.15	0.08	0.19	0.12
$[^3P_1]_{\bar{3}}$	0.80	0.35	0.17	0.42	0.28
$[^3P_2]_{\bar{3}}$	2.55	1.11	0.54	1.37	0.89
σ_S	49.44	29.24	18.77	35.63	24.31
σ_P	4.99	2.18	1.07	2.69	1.75
σ_{tot}	54.43	31.42	19.84	38.32	26.06
$\sigma_P/\sigma_S(\%)$	10.09	7.46	5.70	7.55	7.20
$\sigma_P/\sigma_{\text{tot}}(\%)$	9.17	6.94	5.39	7.02	6.72

P wave of each subprocess. The distribution of transverse momentum p_T in Fig. 2 reveals that, both in S and P waves, the photoproduction of Ξ_{bc} through $\gamma + g$ fusion is larger than that via the photon-photon fusion channel in the region of small p_T . Such behavior is mainly due to the rapid growth of the gluon distribution $f_{g/\gamma}(x)$ in the small- x region, combined with the collinear enhancement from the triple-gluon vertex diagrams at small p_T . However, when p_T is greater than about 25 GeV, a significant reverse increase occurs. The overall trend of total S and P waves shows a downward trend, and there are significantly more events in the small- p_T region. For the rapidity distribution for the photoproduction, two subprocesses are concentrated within the range of $[-4.2, +4.2]$, and the curve of $\gamma + g$ has a steeper slope, while that of $\gamma + \gamma$ is more gradual. It can be seen from the angular distributions $\cos\theta_{45}$ that when the angle is π , that is, when the two heavy antiquarks move back to back, the contribution is the greatest. $\cos\theta_{35}$ indicates that the contribution is significant when the angle is 0 or π , meaning that the bottom-charmed baryon is most likely to move parallel to the two heavy antiquarks. The same conclusion has also been evident in the distributions of the invariant masses s_{35} and s_{45} .

Subsequently, the heavy-quark-mass dependence and the renormalization-scale dependence of the cross sections are quantified. The color sextet LDME h_6 is treated separately as a hadronization model benchmark rather than as a statistically defined perturbative uncertainty. In addition, the choice of photon PDF constitutes an unquantified systematic dependence of the resolved channel, as noted in Sec. II. As constituent quarks, the masses m_b and m_c of heavy quarks play a crucial role for the search of doubly heavy baryon Ξ_{bc} . The choice of heavy quark masses introduces significant theoretical uncertainty. We analyze the theoretical uncertainty of heavy quark masses by varying $m_c = 1.8 \pm 0.3$ and $m_b = 5.1 \pm 0.3$ GeV, and the results are shown in Tables IV and V, corresponding to the direct and resolved photoproduction of Ξ_{bc} baryons.

When varying m_c (m_b), the other heavy quark mass is fixed at $m_b = 5.1$ GeV ($m_c = 1.8$ GeV). In Tables IV and V, σ_S is the contribution of S -wave states, $[^1S_0]_{\bar{3}}$ and $[^3S_1]_{\bar{3}}$, and σ_P is the contribution of P -wave states, $[^1P_1]_{\bar{3}}$ and $[^3P_J]_{\bar{3}}$ ($J = 0, 1, 2$). σ_{tot} is the total contribution of S - and P -wave states for the photoproduction of Ξ_{bc} . One can find that the cross section of each intermediate diquark state has a strong dependence on the heavy quark masses m_c and m_b , and it shows a monotonically decreasing trend as the heavy quark mass increases, mainly due to the inhibition of phase space. Regarding the photoproduction channel $\gamma + \gamma$ ($\gamma + g$), the total cross section σ_{tot} can vary by $-30.24\% \sim +54.68\%$ ($-36.86\% \sim +73.23\%$) for $m_c = 1.8 \pm 0.3$ GeV and $-11.74\% \sim +13.74\%$ ($-17.06\% \sim +21.96\%$) for $m_b = 5.1 \pm 0.3$ GeV. The contribution of P -wave states can be $6.83\% \sim 12.00\%$ ($5.70\% \sim 10.09\%$) of that of the S -wave states and $6.39\% \sim 10.71\%$ ($5.39\% \sim 9.17\%$) of the total cross section with the uncertainty of heavy quark mass.

TABLE VI. Common renormalization-/factorization-scale dependence of the color antitriplet contribution for the direct photoproduction channel $\gamma + \gamma \rightarrow \Xi_{bc} + \bar{c} + \bar{b}$ at $\sqrt{s} = 500$ GeV with $(m_c, m_b) = (1.8, 5.1)$ GeV.

State	$\mu = \mu_0/2$	$\mu = \mu_0$	$\mu = 2\mu_0$
$[^1S_0]_{\bar{3}}$	5.57	3.80	2.79
$[^3S_1]_{\bar{3}}$	12.58	8.57	6.29
$[^1P_1]_{\bar{3}}$	0.30	0.21	0.15
$[^3P_0]_{\bar{3}}$	0.25	0.17	0.12
$[^3P_1]_{\bar{3}}$	0.24	0.16	0.12
$[^3P_2]_{\bar{3}}$	0.81	0.55	0.40
σ_S	18.15	12.37	9.08
σ_P	1.60	1.09	0.79
σ_{tot}	19.75	13.46	9.87
$\sigma_P/\sigma_S(\%)$	8.82	8.81	8.70
$\sigma_P/\sigma_{\text{tot}}(\%)$	8.10	8.10	8.00

TABLE VII. Common renormalization-/factorization-scale dependence of the color antitriplet contribution for the resolved photoproduction channel $\gamma + g \rightarrow \Xi_{bc} + \bar{c} + \bar{b}$ at $\sqrt{s} = 500$ GeV with $(m_c, m_b) = (1.8, 5.1)$ GeV.

State	$\mu = \mu_0/2$	$\mu = \mu_0$	$\mu = 2\mu_0$
$[^1S_0]_{\bar{3}}$	7.99	6.87	5.94
$[^3S_1]_{\bar{3}}$	25.94	22.37	19.42
$[^1P_1]_{\bar{3}}$	0.66	0.57	0.50
$[^3P_0]_{\bar{3}}$	0.18	0.15	0.13
$[^3P_1]_{\bar{3}}$	0.40	0.35	0.30
$[^3P_2]_{\bar{3}}$	1.28	1.11	0.95
σ_S	33.93	29.24	25.36
σ_P	2.52	2.18	1.88
σ_{tot}	36.45	31.42	27.24
$\sigma_P/\sigma_S(\%)$	7.43	7.46	7.41
$\sigma_P/\sigma_{\text{tot}}(\%)$	6.91	6.94	6.90

For the partonic processes at leading order, the direct photoproduction channel $\gamma + \gamma \rightarrow \Xi_{bc} + \bar{c} + \bar{b}$ is in $\mathcal{O}(\alpha^2\alpha_s^2)$ and the resolved photoproduction channel $\gamma + g \rightarrow \Xi_{bc} + \bar{c} + \bar{b}$ is in $\mathcal{O}(\alpha\alpha_s^3)$. To assess the residual scale dependence, the common scale $\mu_R = \mu_F \equiv \mu$ is varied from $\mu_0/2$ to $2\mu_0$ with $\mu_0 = \sqrt{M_{\Xi_{bc}}^2 + p_T^2}$ at $\sqrt{s} = 500$ GeV. The corresponding results for the direct and resolved channels are presented in Tables VI and VII, respectively. The cross sections are given in units of femtobarn. Clearly, due to the running behavior of the strong coupling constant with the energy scale, each state and the overall result are decreasing as the renormalization scale increases; the theoretical uncertainty of the total contribution σ_{tot} is approximately -26.67% to $+46.73\%$ (-13.30% to $+16.01\%$) for $\gamma + \gamma$ ($\gamma + g$) with $\mu \in [\mu_0/2, 2\mu_0]$. However, the contribution of the P -wave states relative to that of the S -wave states and the total cross section has a relatively stable proportion.

It should be mentioned that, in the hadronization of the bottom-charmed baryons, it will be accompanied by a large nonperturbative effect, which brings a large theoretical uncertainty. Especially for the color sextet intermediate diquark, there is no clear correlation with the wave functions. According to the one-gluon-exchange interaction in the diquark state, the contribution of the color sextet shall be suppressed at least by v^2 than the color antitriplet,

or too small to be ignored [35,65], and the total events of the Ξ_{bc} baryon are decreased accordingly. We therefore use $h_6 = 0$ as the conservative baseline and regard $h_6 = v^2 h_3$ and $h_6 = h_3$ as phenomenological hadronization benchmarks. Within the NRQCD framework, the average relative velocity squared v^2 of the constituent heavy quarks is a crucial parameter determining the validity of the non-relativistic expansion. Generally, the velocity scaling rules for heavy quarkonium mesons yield $\langle v^2 \rangle_{c\bar{c}} \approx 0.3$ and $\langle v^2 \rangle_{b\bar{b}} \approx 0.1$ [64,78,79]. The $[bc]$ -diquark system in the color antitriplet state is more loosely bound than heavy quarkonium mesons, leading to a smaller relative velocity. To perform a conservative estimation of the theoretical uncertainty, $\langle v^2 \rangle_{bc}$ is set within the envelope range of $[0.1, 0.3]$. It is indicated that $\mathcal{O}(v^2)$ relativistic corrections may introduce a systematic uncertainty of 10%–30%. As presented in Table VIII, although the different phenomenological treatments of the color sextet h_6 shift the hadronization benchmark, the total predicted events of Ξ_{bc} are enhanced by up to 61.60%, 71.79%, and 77.60% for $\sqrt{s} = 250, 500$, and 1000 GeV, respectively, when $h_6 = h_3$. Finally, it is worth emphasizing that the radial wave functions at the origin describing the transition probability h_3 , namely, $|R_{bc}(0)|^2$ and $|R'_{bc}(0)|^2$ corresponding to S and P waves, are overall parameters of the cross sections. Therefore, this source of uncertainty acts merely as a global normalization factor; if more precise calculations (e.g., from lattice QCD or improved potential models) of these wave functions become available in the future, our baseline cross section predictions can be directly and easily adjusted.

IV. SUMMARY

Within the factorization framework of NRQCD, the photoproduction of ground and excited Ξ_{bc} are investigated at ILC and CLIC with three different collision energies $\sqrt{s} = 250, 500$, and 1000 GeV, respectively. The direct and resolved photoproduction channels are both taken into account with intermediate $\langle bc \rangle$ -diquark states, i.e., $\gamma + \gamma \rightarrow \langle bc \rangle[n] + \bar{c} + \bar{b} \rightarrow \Xi_{bc} + \bar{c} + \bar{b}$ and $\gamma + g \rightarrow \langle bc \rangle[n] + \bar{c} + \bar{b} \rightarrow \Xi_{bc} + \bar{c} + \bar{b}$. Twelve configurations of $\langle bc \rangle$ -diquark states can be $(bc)[^1S_0]_{\bar{3}/6}$, $(bc)[^3S_1]_{\bar{3}/6}$, $(bc)[^1P_1]_{\bar{3}/6}$, $(bc)[^3P_0]_{\bar{3}/6}$, $(bc)[^3P_1]_{\bar{3}/6}$, and $(bc)[^3P_2]_{\bar{3}/6}$. For the

TABLE VIII. Hadronization-benchmark dependence of the total cross sections σ (in femtobarn) for Ξ_{bc} photoproduction, obtained with $h_6 = 0$, $h_6 = v^2 h_3$ ($v^2 = 0.1$ – 0.3) and $h_6 = h_3$ at three collision energies (in GeV).

$\sqrt{s}$	$h_6 = 0$			$h_6 = v^2 h_3$			$h_6 = h_3$		
	$\gamma + \gamma$	$\gamma + g$	Total	$\gamma + \gamma$	$\gamma + g$	Total	$\gamma + \gamma$	$\gamma + g$	Total
250	27.51	15.22	42.73	28.89–31.64	16.48–18.99	45.36–50.63	41.26	27.79	69.05
500	13.46	31.42	44.88	14.13–15.48	33.97–39.07	48.10–54.55	20.18	56.92	77.10
1000	5.77	56.42	62.19	6.06–6.64	60.96–70.03	67.02–76.67	8.66	101.79	110.45

conservative $h_6 = 0$ baseline, the two-channel combined P -wave contribution is 8.07%, 7.86%, and 7.56% of that of the S wave at $\sqrt{s} = 250, 500$, and 1000 GeV, respectively. Assuming that a 100% decay probability of P -wave states into the ground state as an upper-bound estimation, the total produced events of ground-state Ξ_{bc} from two subprocesses can reach 4.27×10^5 , 4.49×10^5 , and 6.22×10^5 at $\sqrt{s} = 250, 500$, and 1000 GeV, respectively, with an integrated luminosity of 10^4fb^{-1} . Note that these yields only represent the parton-level inclusive rates without considering the detector acceptance and backgrounds.

Then the related differential distributions are presented, including p_T , y , s_{35} , s_{45} , $\cos \theta_{35}$, and $\cos \theta_{45}$ distributions. The differential distributions all emphasize that the contribution of the S wave is dominant throughout the whole region compared with the P wave of each subprocess. The transverse momentum distribution reveals that both in S and P waves, the photoproduction of Ξ_{bc} through the resolved $\gamma + g$ channel is larger than that via photon-photon fusion in the region of small p_T . The overall trend of total S and P waves shows a downward trend, and there are significantly more events in the small- p_T region. For the rapidity distribution for the photoproduction, the curve of $\gamma + g$ has a steeper slope, while that of $\gamma + \gamma$ is more gradual. It can be seen from the angular distributions $\cos \theta_{35}$ and $\cos \theta_{45}$ that the bottom-charmed baryon is most likely to move parallel to the two heavy antiquarks. The same conclusion has also been evident in the distributions of the invariant masses s_{35} and s_{45} .

Finally, the heavy-quark-mass and common-scale dependences have been quantified. The choices $h_6 = 0$, $v^2 h_3$ ($v^2 = 0.1\text{--}0.3$), and h_3 are presented as hadronization benchmarks. The quoted resolved-channel results are GRS-based central predictions, and different photon-PDF parametrizations may introduce an additional systematic dependence. The cross section of each intermediate diquark state has a strong dependence on the heavy quark masses

m_c and m_b , and it shows a monotonically decreasing trend as the heavy quark mass increases. Regarding the photoproduction channel $\gamma + \gamma$ ($\gamma + g$), the total cross section σ_{tot} can vary by $-30.24\% \sim +54.68\%$ ($-36.86\% \sim +73.23\%$) for $m_c = 1.8 \pm 0.3$ GeV and $-11.74\% \sim +13.74\%$ ($-17.06\% \sim +21.96\%$) for $m_b = 5.1 \pm 0.3$ GeV. The contribution of P -wave states can be 6.83% $\sim$ 12.00% (5.70% $\sim$ 10.09%) of that of the S -wave states, 6.39% $\sim$ 10.71% (5.39% $\sim$ 9.17%) of the total cross section with the uncertainty of heavy quark mass. The theoretical uncertainty of σ_{tot} caused by the common scale is approximately $-26.67\% \sim +46.73\%$ ($-13.30\% \sim +16.01\%$) for $\gamma + \gamma$ ($\gamma + g$) with $\mu \in [\mu_0/2, 2\mu_0]$. However, the contribution of the P -wave states relative to that of the S -wave states and the total cross section has a relatively stable proportion. Relative to the conservative $h_6 = 0$ baseline, the extreme benchmark $h_6 = h_3$ increases the total rate by 61.60%, 71.79%, and 77.60% at $\sqrt{s} = 250, 500$, and 1000 GeV, respectively. The overall order of magnitude of the predicted inclusive rates remains unchanged.

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DATA AVAILABILITY

The corresponding results or data are already provided in our manuscript, and they can be obtained by using the parameters and relevant tools given in our manuscript.

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