

Nonleptonic decays of the double-charmed baryon Ω_{cc}^+ within the nonrelativistic quark model

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In this work, we investigate the two-body nonleptonic decays of double-charmed baryon Ω_{cc}^+ within a nonrelativistic quark model, in which the nonfactorizable amplitudes contributed by W -exchange diagrams are evaluated under the pole model assumption. To reduce the sensitivity of decay amplitudes to an arbitrary choice of baryon wave functions, we adopt charmed baryon wave functions obtained by solving the Schrödinger equation with a nonrelativistic quark potential model. Our results show that the branching fractions of Cabibbo-favored decays $\Omega_{cc}^+ \rightarrow \Omega_c^0 \pi^+$ and $\Xi_c^{(\prime)+} \pi^+$, as well as the single Cabibbo-suppressed decays $\Omega_{cc}^+ \rightarrow \Omega_c^0 K^+$, $\Sigma_{cc}^{++} K^-$, $\Xi_c^{'+} \eta$, and $\Lambda_c^+ \bar{K}^0$ can reach up to a few percent. Particularly for the $\Omega_{cc}^+ \rightarrow \Omega_c^0 K^+$ mode, owing to the large factorizable amplitude and constructive pole contributions, the branching fraction is comparable to Cabibbo-favored decays. These decay modes can serve as the discovery channels for the double-charmed baryon Ω_{cc}^+ in future experiments, like LHCb and Belle II.

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I. INTRODUCTION

To complete the charmed-baryon family, the establishment of double-charmed baryons is particularly important. As predicted by the quark model, the double-charmed baryons Ξ_{cc}^{++} and Ξ_{cc}^+ with quark content ccu and ccd , respectively, form an isodoublet ($I = 1/2$), while the Ω_{cc}^+ forms an isosinglet ($I = 0$) with quark content ccs . In contrast to the well-established state Ξ_{cc}^{++} , which has been observed in the decay modes $\Lambda_c^+ K^- \pi^+ \pi^+$ [1], $\Xi_c^+ \pi^{(\prime)+}$ [2,3], and $\Xi_{cc}^{++} \rightarrow \Xi_c^0 \pi^+ \pi^+$ [4], evidence for Ξ_{cc}^+ was only reported by SELEX in the decay modes $\Lambda_c^+ K^- \pi^+$ [5] and $p D^+ K^-$ [6]. None of them were confirmed by the FOCUS [7], BABAR [8], Belle [9], and LHCb [10,11] collaborations. Moreover, the mass of Ξ_{cc}^+ was measured to be about 3519 MeV by SELEX, which differs from $m_{\Xi_{cc}^{++}} = (3621.55 \pm 0.23 \pm 0.30)$ MeV [12] by about 100 MeV. More experimental measurements of the Ξ_{cc}^+ are needed to establish its existence and precisely measure its properties. The Ω_{cc}^+ has been searched for in the $\Xi_c^+ K^- \pi^+$ final state at the LHCb experiment [13], but no significant signal was observed. Future experimental measurements are of key importance in completing the double-charmed baryon family. The ground double-charmed baryons with

$J^P = 1/2^+$ can decay only via the weak interaction; therefore, the study of their weak decays is essential to facilitate experimental discovery [14]. In this work, we focus on the two-body nonleptonic decays of the double-charmed baryon Ω_{cc}^+ , aiming to identify the highest-potential processes for probing the Ω_{cc}^+ state.

The study of double-charmed baryon weak decays also provides valuable insights into weak decay mechanisms. Based on the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, the concerned nonleptonic decays of the double-charmed baryon Ω_{cc}^+ can be divided into three groups: (i) the Cabibbo-favored (CF) decays induced by $c \rightarrow s \bar{u} d$ and $cd \rightarrow su$, (ii) the single Cabibbo-suppressed (SCS) decays induced by $c \rightarrow d \bar{u} d$ and $cd \rightarrow du$, as well as $c \rightarrow s \bar{u} s$ and $cs \rightarrow su$, and (iii) the doubly Cabibbo-suppressed (DCS) decays induced by $c \rightarrow d \bar{u} s$ and $cs \rightarrow du$. From another perspective, in theoretical studies of charmed-baryon nonleptonic decay, the decay amplitudes are dominated by the following Feynman diagrams: the external W -emission diagrams (T) and internal W -emission diagram (C), which are factorizable, and the inner W -emission diagram (C') and W -exchange diagram ($E_{1,2}, E'$), which are nonfactorizable [15]. The schematic Feynman diagrams are displayed in Fig. 1 and the corresponding Feynman diagrams to relevant decay channels are summarized in Table I.

In theoretical studies of charmed-baryon nonleptonic decay, the factorizable amplitudes T and C are often evaluated as the product of a baryon transition matrix element and meson decay constant under the naive factorization assumption, whereas the nonfactorizable

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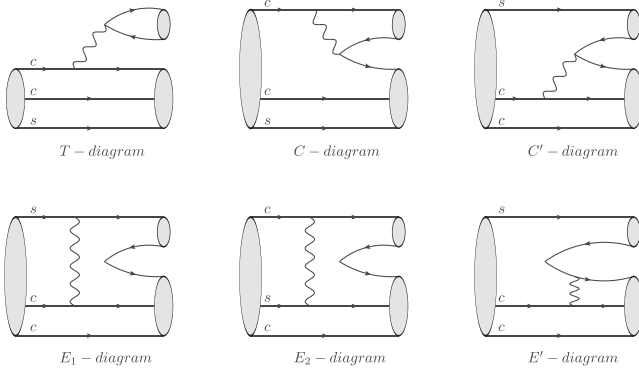


FIG. 1. Feynman diagrams for nonleptonic decays of the double-charmed baryon Ω_{cc}^+ .

amplitudes C' , $E_{1,2}$, and E' are more challenging to quantify. In previous experiments of heavy meson nonleptonic decay, the factorizable assumption worked well and the nonfactorizable contributions were usually neglected. However, this is no longer the case in charmed-baryon decay. The experimental measurement of pure W -exchange decay $\Lambda_c^+ \rightarrow \Xi^0 K^+$ [16] shows that W -exchange diagrams are important and cannot be ignored. To access the nonfactorizable contributions from W -exchange diagrams, a pole model was widely employed in the study of charmed-baryon nonleptonic decays [17–31]. Within the pole model, parity-violating (PV) amplitudes mainly arise from low-lying $J^P = 1/2^-$ poles, while parity-conserving (PC) amplitudes are dominated by $J^P = 1/2^+$ ground-state poles. To extend the framework to double-charmed baryon decay, the theoretical predictions of Ω_{cc}^+ nonleptonic decays were performed in Refs. [32–34], in which the nonperturbative parameters were evaluated using a bag model [33] or flavor symmetry [32,34]. The covariant confined quark model has also been adopted to estimate the W -exchange diagram [35,36]. Other than these, the irreducible SU(3) approach [37,38], which is based on flavor symmetry, while not addressed in dynamics, was also used to analyzed the double-charmed baryon two-body [37] and multibody nonleptonic decays [38].

In this work, we investigate the two-body nonleptonic decays $\Omega_{cc}^+ \rightarrow \mathcal{B}_c + P$ within the framework of the nonrelativistic quark model (NRQM), where P denotes a light-flavor pseudoscalar meson, such as π , K , or $\eta^{(\prime)}$. The nonfactorizable contributions from W -exchange diagrams are evaluated within the pole model framework. It is well known that the dominant source of uncertainty in theoretical calculations of nonleptonic decays lies in the hadron transition matrix element. Addressing this challenge, we propose using exact baryon wave functions obtained by solving the Schrödinger equation with a nonrelativistic potential model [18,39], rather than relying on an oversimplified Gaussian-type one. With the support of the charmed baryon spectrum, our study reduces the model dependence arising from baryon wave functions in the decay amplitudes.

This paper is organized as follows. After the Introduction, we derive the weak decay amplitudes within the NRQM in Sec. II. To obtain the involved baryon spatial wave functions, a nonrelativistic potential is also introduced. In Sec. III, we present our numerical results of the decay amplitudes and discuss future investigations the branching fraction and asymmetry parameter. Finally, this paper ends with a short summary in Sec. IV.

II. THEORETICAL FRAMEWORK

A. Decay amplitudes of W -emission diagrams

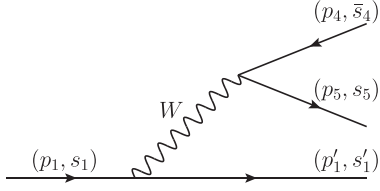
According to the conventions in Fig. 2, the effective Hamiltonian of the W -emission process can be written as

$$\mathcal{H}_{1 \rightarrow 3} = \frac{G_F}{\sqrt{2}} V_{\text{CKM}} V'_{\text{CKM}} \frac{\beta}{(2\pi)^3} \delta^3(\mathbf{p}_1 - \mathbf{p}'_1 - \mathbf{p}_4 - \mathbf{p}_5) \times \hat{\alpha}_1^{(-)} \hat{I}_P [c_1(\mu) \hat{O}_1 + c_2(\mu) \hat{O}_2], \quad (2.1)$$

where $G_F = 1.16637 \times 10^{-5} \text{ GeV}^2$ is the Fermi coupling constant [40] and $V_{\text{CKM}}^{(i)}$ are CKM matrix elements. $\mathbf{p}_i$ is the three-momentum of the i th quark. $\hat{\alpha}_1^{(-)} c = s$ (or d) is the flavor-changing operator, which acts on the first quark, and the factor β is assigned a value of 2. The $\hat{O}_{1,2}$ are

TABLE I. Feynman diagrams of two-body nonleptonic decays of the double-charmed baryon Ω_{cc}^+ .

CF	Diagrams	SCS	Diagrams	DCS	Diagrams
$\Omega_{cc}^+ \rightarrow \Omega_c^0 \pi^+$	T	$\Omega_{cc}^+ \rightarrow \Xi_c^{(i)0} \pi^+$	T, E_1	$\Omega_{cc}^+ \rightarrow \Xi_c^{(i)0} K^+$	T, E_1
$\Omega_{cc}^+ \rightarrow \Xi_c^{(i)+} \bar{K}^0$	C, C', E'	$\Omega_{cc}^+ \rightarrow \Omega_c^0 K^+$	T, E_1	$\Omega_{cc}^+ \rightarrow \Xi_c^{(i)+} K^0$	C, E_2
		$\Omega_{cc}^+ \rightarrow \Xi_c^{(i)+} \eta^{(i)}$	C, C', E_1, E_2, E'	$\Omega_{cc}^+ \rightarrow \Sigma_c^+ \pi^0$	E_1, E_2
		$\Omega_{cc}^+ \rightarrow \Xi_c^{(i)+} \pi^0$	C, E_1	$\Omega_{cc}^+ \rightarrow \Lambda_c^+ \pi^0$	E_1, E_2
		$\Omega_{cc}^+ \rightarrow \Sigma_c^{++} K^-$	E_2	$\Omega_{cc}^+ \rightarrow \Sigma_c^0 \pi^+$	E_1
		$\Omega_{cc}^+ \rightarrow \Sigma_c^+ \bar{K}^0$	C', E_2, E'	$\Omega_{cc}^+ \rightarrow \Sigma_c^{++} \pi^-$	E_2
		$\Omega_{cc}^+ \rightarrow \Lambda_c^+ \bar{K}^0$	C', E_2, E'	$\Omega_{cc}^+ \rightarrow \Sigma_c^+ \eta^{(i)}$	C', E_1, E_2, E'
				$\Omega_{cc}^+ \rightarrow \Lambda_c^+ \eta^{(i)}$	C', E_1, E_2, E'

FIG. 2. Kinematic conventions of the W -emission process.

abbreviations of the four-fermion interaction operators $\hat{O}_1 \equiv (\bar{q}_5 q_4)(\bar{q}'_1 q_1)$ and $\hat{O}_2 \equiv (\bar{q}'_1 q_4)(\bar{q}_5 q_1)$, and $c_{1,2}$ are corresponding Wilson coefficients. We adopt the Wilson coefficients as $c_1 = 1.346$ and $c_2 = -0.636$ at the energy scale $\mu = 1.25$ GeV [33].

In the nonrelativistic limit, the effective Hamiltonian associated with $c_1(\mu)$ can be expanded as [17,30,31]

$$\begin{aligned} \mathcal{H}_{1,1 \rightarrow 3}^{\text{PC}} &= \frac{G_F}{\sqrt{2}} V_{\text{CKM}} V'_{\text{CKM}} \frac{\beta}{(2\pi)^3} \delta^3(\mathbf{p}_1 - \mathbf{p}'_1 - \mathbf{p}_4 - \mathbf{p}_5) \left\{ \langle s'_{z,1} | I | s_{z,1} \rangle \langle s_{z,5} \bar{s}_{z,4} | \sigma | 0 \rangle \left(\frac{\mathbf{p}_5}{2m_5} + \frac{\mathbf{p}_4}{2m_4} \right) \right. \\ &\quad - \left[\left(\frac{\mathbf{p}'_1}{2m'_1} + \frac{\mathbf{p}_1}{2m_1} \right) \langle s'_{z,1} | I | s_{z,1} \rangle - i \langle s'_{z,1} | \sigma | s_{z,1} \rangle \times \left(\frac{\mathbf{p}_1}{2m_1} - \frac{\mathbf{p}'_1}{2m'_1} \right) \right] \langle s_{z,5} \bar{s}_{z,4} | \sigma | 0 \rangle \\ &\quad - \langle s'_{z,1} | \sigma | s_{z,1} \rangle \left[\left(\frac{\mathbf{p}_5}{2m_5} + \frac{\mathbf{p}_4}{2m_4} \right) \langle s_{z,5} \bar{s}_{z,4} | I | 0 \rangle - i \langle s_{z,5} \bar{s}_{z,4} | \sigma | 0 \rangle \times \left(\frac{\mathbf{p}_4}{2m_4} - \frac{\mathbf{p}_5}{2m_5} \right) \right] \\ &\quad \left. + \langle s'_{z,1} | \sigma | s_{z,1} \rangle \left(\frac{\mathbf{p}'_1}{2m'_1} + \frac{\mathbf{p}_1}{2m_1} \right) \langle s_{z,5} \bar{s}_{z,4} | I | 0 \rangle \right\} \hat{\alpha}_1^{(-)} \hat{I}_P, \\ \mathcal{H}_{1,1 \rightarrow 3}^{\text{PV}} &= \frac{G_F}{\sqrt{2}} V_{\text{CKM}} V'_{\text{CKM}} \frac{\beta}{(2\pi)^3} \delta^3(\mathbf{p}_1 - \mathbf{p}'_1 - \mathbf{p}_4 - \mathbf{p}_5) \left(-\langle s'_{z,1} | I | s_{z,1} \rangle \langle s_{z,5} \bar{s}_{z,4} | I | 0 \rangle + \langle s'_{z,1} | \sigma | s_{z,1} \rangle \langle s_{z,5} \bar{s}_{z,4} | \sigma | 0 \rangle \right) \hat{\alpha}_1^{(-)} \hat{I}_P, \end{aligned} \quad (2.2)$$

in which the PC and PV Hamiltonians are distinguished by an odd or even number of γ_5 , respectively. I is the two-rank identity matrix and $\sigma = (\sigma^1, \sigma^2, \sigma^3)$ are the Pauli matrices. In addition, $|s_{z,i}\rangle$ is an abbreviation of $|s_i, s_{z,i}\rangle$ and represents the spin of the i th quark. The effective Hamiltonian $\mathcal{H}_{2,1 \rightarrow 3}^{\text{PC,PV}}$, which is associated with $c_2(\mu)$, can be obtained by making the replacements $(p'_1, m'_1) \leftrightarrow (p_5, m_5)$ and $s'_{z,1} \leftrightarrow s_{z,5}$ in Eq. (2.2).

Generally, the decay amplitudes of W -emission diagrams of the $\mathcal{B}_i(\mathbf{P}_i; J^i, J_z^i) \rightarrow \mathcal{B}_f(\mathbf{P}_f; J^f, J_z^f) + P(\mathbf{k})$ ($P \equiv$ pseudoscalar meson) process can be evaluated as

$$\begin{aligned} \mathcal{M}_T^{J_z^f, J_z^i} &= \left\langle \mathcal{B}_f(\mathbf{P}_f; J^f, J_z^f) P(\mathbf{k}) \left| \left[c_1(\mu) \mathcal{H}_{1,1 \rightarrow 3} + \frac{c_2(\mu)}{N_c} \mathcal{H}_{2,1 \rightarrow 3} \right] \right| \mathcal{B}_i(\mathbf{P}_i; J^i, J_z^i) \right\rangle, \\ \mathcal{M}_C^{J_z^f, J_z^i} &= \left\langle \mathcal{B}_f(\mathbf{P}_f; J^f, J_z^f) P(\mathbf{k}) \left| \left[\frac{c_1(\mu)}{N_c} \mathcal{H}_{1,1 \rightarrow 3} + c_2(\mu) \mathcal{H}_{2,1 \rightarrow 3} \right] \right| \mathcal{B}_i(\mathbf{P}_i; J^i, J_z^i) \right\rangle, \\ \mathcal{M}_{C'}^{J_z^f, J_z^i} &= \left\langle \mathcal{B}_f(\mathbf{P}_f; J^f, J_z^f) P(\mathbf{k}) \left| \left[-\frac{c_1(\mu)}{N_c} \mathcal{H}_{1,1 \rightarrow 3} + \frac{c_2(\mu)}{N_c} \mathcal{H}_{2,1 \rightarrow 3} \right] \right| \mathcal{B}_i(\mathbf{P}_i; J^i, J_z^i) \right\rangle, \end{aligned} \quad (2.3)$$

where $J^i = J^f = 1/2$ are the total spins of initial and final baryons and $N_c = 3$ is a color-suppressed factor. Besides, $\mathbf{P}_i = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3$ is the three-momentum of the initial double-charmed baryon, and $\mathbf{P}_f = \mathbf{p}'_1 + \mathbf{p}_2 + \mathbf{p}_3$ ($\mathbf{P}_f = \mathbf{p}_2 + \mathbf{p}_3 + \mathbf{p}_5$, $\mathbf{P}_f = \mathbf{p}'_1 + \mathbf{p}_2 + \mathbf{p}_5$) and $\mathbf{k} = \mathbf{p}_4 + \mathbf{p}_5$ ($\mathbf{k} = \mathbf{p}'_1 + \mathbf{p}_4$, $\mathbf{k} = \mathbf{p}_3 + \mathbf{p}_4$) are the three-momenta of the final charmed baryon and pseudoscalar meson, respectively, for $T(C, C')$ diagrams. In the rest frame of the initial state, we have $\mathbf{P}_i = 0$ and $\mathbf{P}_f = -\mathbf{k}$.

Taking the T diagram as an example, we can separate the decay amplitude as

$$\begin{aligned} \mathcal{M}_T^{J_z^f, J_z^i} &= \left\langle \mathcal{B}_f(\mathbf{P}_f; J^f, J_z^f) P(\mathbf{k}) \left| \left[c_1(\mu) \mathcal{H}_{1,1 \rightarrow 3} + \frac{c_2(\mu)}{N_c} \mathcal{H}_{2,1 \rightarrow 3} \right] \right| \mathcal{B}_i(\mathbf{P}_i; J^i, J_z^i) \right\rangle \\ &= \frac{G_F}{\sqrt{2}} V_{\text{CKM}} V'_{\text{CKM}} \frac{\beta}{(2\pi)^3} \delta^3(\mathbf{p}_1 - \mathbf{p}'_1 - \mathbf{p}_4 - \mathbf{p}_5) \sum \langle S^f, S_z^f; L^f, L_z^f | J^f, J_z^f \rangle \langle S^i, S_z^i; L^i, L_z^i | J^i, J_z^i \rangle \\ &\quad \times \left[c_1(\mu) \langle \phi_f \chi_f^{S^f, S_z^f}; \phi_P \chi_P | \hat{O}_{1,1 \rightarrow 3}^{\text{spin}} \hat{O}_{1,1 \rightarrow 3}^{\text{flavor}} | \phi_i \chi_i^{S^i, S_z^i} \rangle \langle \psi_P(\mathbf{k}) \psi_{L^f, L_z^f}(\mathbf{P}_f) | \hat{O}_{1,1 \rightarrow 3}^{\text{spatial}} | \psi_{L^i, L_z^i}(\mathbf{P}_i) \rangle \right. \\ &\quad \left. + \frac{c_2(\mu)}{N_c} \langle \phi_f \chi_f^{S^f, S_z^f}; \phi_P \chi_P | \hat{O}_{2,1 \rightarrow 3}^{\text{spin}} \hat{O}_{2,1 \rightarrow 3}^{\text{flavor}} | \phi_i \chi_i^{S^i, S_z^i} \rangle \langle \psi_P(\mathbf{k}) \psi_{L^f, L_z^f}(\mathbf{P}_f) | \hat{O}_{2,1 \rightarrow 3}^{\text{spatial}} | \psi_{L^i, L_z^i}(\mathbf{P}_i) \rangle \right], \end{aligned} \quad (2.4)$$

where $\langle S, S_z; L, L_z | J, J_z \rangle$ is the Clebsch-Gordan coefficient, while ϕ , χ , and ψ are the flavor, spin, and spatial wave functions of corresponding hadrons, respectively. The overlap of color wave functions is reflected in the factor $1/N_c$. Similarly, the concrete expressions for the decay amplitudes of C and C' diagrams can also be written down.

In Appendix A, we present the spin-flavor matrix elements $\langle \mathcal{B}_f \downarrow | \hat{\mathcal{O}} | \mathcal{B}_i \downarrow \rangle$ in Tables IX–XI, which contribute to the PC and PV amplitudes in the T , C , and C' diagrams, respectively. We emphasize that, particularly for the T diagram, the spin matrix elements associated with $\langle s_{z,5} \bar{s}_{z,4} | \sigma_{x,y,z} | 0 \rangle$ vanish [39]. Likewise, those associated with $\langle s'_{z,1} \bar{s}_{z,4} | \sigma_{x,y,z} | 0 \rangle$ vanish for the C diagram. This is a consequence of the pseudoscalar meson spin wave function. Similarly, the spin-flavor matrix elements $\langle \mathcal{B}_f \uparrow | \hat{\mathcal{O}} | \mathcal{B}_i \uparrow \rangle$ can also be obtained, and satisfy

$$\langle \mathcal{B}_f \uparrow | \hat{\mathcal{O}} | \mathcal{B}_i \uparrow \rangle = \begin{cases} -\langle \mathcal{B}_f \downarrow | \hat{\mathcal{O}} | \mathcal{B}_i \downarrow \rangle, & \text{for PC amplitude,} \\ \langle \mathcal{B}_f \downarrow | \hat{\mathcal{O}} | \mathcal{B}_i \downarrow \rangle, & \text{for PV amplitude.} \end{cases} \quad (2.5)$$

In addition, we also have $\langle \mathcal{B}_f \uparrow | \hat{\mathcal{O}} | \mathcal{B}_i \downarrow \rangle = \langle \mathcal{B}_f \downarrow | \hat{\mathcal{O}} | \mathcal{B}_i \uparrow \rangle = 0$.

After separating out the spin-flavor parts, the remaining spatial wave function overlaps can be expressed as

$$\begin{aligned} \mathcal{I}_{i,T}^{L_i, L_z^i; L_f^f, L_z^f} &= \langle \psi_P(\mathbf{k}) \psi_{L_f, L_z^f}(\mathbf{P}_f) | \hat{\mathcal{O}}_{i,1 \rightarrow 3}^{\text{spatial}}(\mathbf{p}_i) | \psi_{L_i, L_z^i}(\mathbf{P}_i) \rangle \\ &= \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}'_1 d\mathbf{p}_4 d\mathbf{p}_5 \delta^3(\mathbf{P}_f - \mathbf{p}'_1 - \mathbf{p}_2 - \mathbf{p}_3) \delta^3(\mathbf{P}_i - \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3) \delta^3(\mathbf{k} - \mathbf{p}_5 - \mathbf{p}_4) \\ &\quad \times \delta^3(\mathbf{p}_1 - \mathbf{p}_4 - \mathbf{p}_5 - \mathbf{p}'_1) \mathcal{O}_{i,1 \rightarrow 3}^{\text{spatial}}(\mathbf{p}_i) \psi_P^*(\mathbf{p}_5, \mathbf{p}_4) \psi_{L_f, L_z^f}^*(\mathbf{p}'_1, \mathbf{p}_3, \mathbf{p}_2) \psi_{L_i, L_z^i}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3), \end{aligned} \quad (2.6)$$

$$\begin{aligned} \mathcal{I}_{i,C}^{L_i, L_z^i; L_f^f, L_z^f} &= \langle \psi_P(\mathbf{k}) \psi_{L_f, L_z^f}(\mathbf{P}_f) | \hat{\mathcal{O}}_{i,1 \rightarrow 3}^{\text{spatial}}(\mathbf{p}_i) | \psi_{L_i, L_z^i}(\mathbf{P}_i) \rangle \\ &= \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}'_1 d\mathbf{p}_4 d\mathbf{p}_5 \delta^3(\mathbf{P}_f - \mathbf{p}_5 - \mathbf{p}_2 - \mathbf{p}_3) \delta^3(\mathbf{P}_i - \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3) \delta^3(\mathbf{k} - \mathbf{p}'_1 - \mathbf{p}_4) \\ &\quad \times \delta^3(\mathbf{p}_1 - \mathbf{p}_4 - \mathbf{p}_5 - \mathbf{p}'_1) \mathcal{O}_{i,1 \rightarrow 3}^{\text{spatial}}(\mathbf{p}_i) \psi_P^*(\mathbf{p}'_1, \mathbf{p}_4) \psi_{L_f, L_z^f}^*(\mathbf{p}_3, \mathbf{p}_5, \mathbf{p}_2) \psi_{L_i, L_z^i}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3), \end{aligned} \quad (2.7)$$

$$\begin{aligned} \mathcal{I}_{i,C'}^{L_i, L_z^i; L_f^f, L_z^f} &= \langle \psi_P(\mathbf{k}) \psi_{L_f, L_z^f}(\mathbf{P}_f) | \hat{\mathcal{O}}_{i,1 \rightarrow 3}^{\text{spatial}}(\mathbf{p}_i) | \psi_{L_i, L_z^i}(\mathbf{P}_i) \rangle \\ &= \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}'_1 d\mathbf{p}_4 d\mathbf{p}_5 \delta^3(\mathbf{P}_f - \mathbf{p}'_1 - \mathbf{p}_5 - \mathbf{p}_2) \delta^3(\mathbf{P}_i - \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3) \delta^3(\mathbf{k} - \mathbf{p}_3 - \mathbf{p}_4) \\ &\quad \times \delta^3(\mathbf{p}_1 - \mathbf{p}_4 - \mathbf{p}_5 - \mathbf{p}'_1) \mathcal{O}_{i,1 \rightarrow 3}^{\text{spatial}}(\mathbf{p}_i) \psi_P^*(\mathbf{p}_3, \mathbf{p}_4) \psi_{L_f, L_z^f}^*(\mathbf{p}'_1, \mathbf{p}_5, \mathbf{p}_2) \psi_{L_i, L_z^i}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3), \end{aligned} \quad (2.8)$$

where $\mathcal{O}_{i,1 \rightarrow 3}^{\text{spatial}}(\mathbf{p}_i)$ is the function of $\mathbf{p}_i/(2m_i)$ for the PC amplitude and $\mathcal{O}_{i,1 \rightarrow 3}^{\text{spatial}}(\mathbf{p}_i) \equiv 1$ is for the PV amplitude.

After synthesizing the above discussion, especially considering Eq. (2.5), we have the relations

$$\mathcal{M}_{\text{PC}}^{-1/2, -1/2} = -\mathcal{M}_{\text{PC}}^{1/2, 1/2}, \quad \mathcal{M}_{\text{PV}}^{-1/2, -1/2} = \mathcal{M}_{\text{PV}}^{1/2, 1/2}, \quad \mathcal{M}_{\text{PV,PC}}^{-1/2, 1/2} = \mathcal{M}_{\text{PV,PC}}^{1/2, -1/2} = 0, \quad (2.9)$$

for the T , C , and C' diagrams.

To describe the spatial wave function of a pseudoscalar meson, we employ the simple harmonic oscillator wave function [17,30,31]

$$\psi_P(\mathbf{k}_1, \mathbf{k}_2) = \frac{1}{\pi^{3/4} R_P^{3/2}} \exp \left[-\frac{(\mathbf{k}_1 - \mathbf{k}_2)^2}{8R_P^2} \right], \quad (2.10)$$

where R_P is a phenomenal parameter. In this work, we adopt $R_\pi = 0.28$ GeV, $R_K = 0.50$ GeV, $R_\eta = 0.40$ GeV, and $R_{\eta'} = 0.90$ GeV [31]. Besides, in order to describe the spatial wave function of a charmed baryon, we adopt the expression

$$\psi_{L,L_z}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \sum_{n_\rho, n_\lambda} \sum_{m_\rho, m_\lambda} \langle l_\rho, m_\rho; l_\lambda, m_\lambda | L, L_z \rangle \times C^{(n_\rho, n_\lambda)} \psi_{n_\rho, l_\rho, m_\rho}(\boldsymbol{\rho}) \psi_{n_\lambda, l_\lambda, m_\lambda}(\boldsymbol{\lambda}), \quad (2.11)$$

where the three-momenta of the ρ -mode and λ -mode are defined as

$$\boldsymbol{\rho} = \frac{m_1 \mathbf{k}_2 - m_2 \mathbf{k}_1}{m_1 + m_2}, \quad \boldsymbol{\lambda} = \frac{(m_1 + m_2) \mathbf{k}_3 - m_3 (\mathbf{k}_1 + \mathbf{k}_2)}{m_1 + m_2 + m_3}, \quad (2.12)$$

respectively. $C^{(n_\rho, n_\lambda)}$ is a set of coefficients and $\psi_{n,l,m}(\mathbf{p})$ is the Gaussian basis function [41]

$$\psi_{n,l,m}(\mathbf{p}) = N_{nl} k^l e^{-\nu_n p^2} Y_{lm}(\hat{\mathbf{p}}), \quad N_{nl} = \left(\frac{2^{l+2} (2\nu_n)^{l+3/2}}{\sqrt{\pi} (2l+1)!!} \right)^{1/2}, \quad (2.13)$$

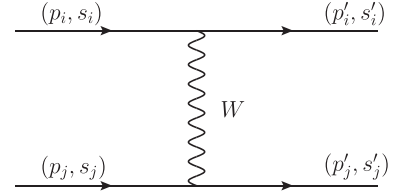


FIG. 3. Kinematic conventions of the W -exchange process.

where $\hat{\mathbf{p}} = \mathbf{p}/p$ is a unit vector and ν_n is the Gaussian size parameter. The details of wave functions of charmed baryons will be presented in Sec. II C.

B. Decay amplitudes of W -exchange diagrams

Next, we compute the decay amplitudes for the W -exchange diagrams. Adopting the conventions of Fig. 3, the effective Hamiltonian associated with $c_1(\mu)$ can be written as [17,30,31]

$$\begin{aligned} \mathcal{H}_{1,2 \rightarrow 2}^{\text{PC}} &= \frac{G_F}{\sqrt{2}} V_{\text{CKM}} V'_{\text{CKM}} \frac{1}{(2\pi)^3} \sum_{i \neq j} \hat{\alpha}_i^{(-)} \hat{\beta}_j^{(+)} \delta^3(\mathbf{p}'_i + \mathbf{p}'_j - \mathbf{p}_i - \mathbf{p}_j) (1 - \langle s'_{z,i} | \boldsymbol{\sigma}_i | s_{z,i} \rangle \langle s'_{z,j} | \boldsymbol{\sigma}_j | s_{z,j} \rangle), \\ \mathcal{H}_{1,2 \rightarrow 2}^{\text{PV}} &= \frac{G_F}{\sqrt{2}} V_{\text{CKM}} V'_{\text{CKM}} \frac{1}{(2\pi)^3} \sum_{i \neq j} \hat{\alpha}_i^{(-)} \hat{\beta}_j^{(+)} \delta^3(\mathbf{p}'_i + \mathbf{p}'_j - \mathbf{p}_i - \mathbf{p}_j) \\ &\times \left\{ -(\langle s'_{z,i} | \boldsymbol{\sigma}_i | s_{z,i} \rangle - \langle s'_{z,j} | \boldsymbol{\sigma}_j | s_{z,j} \rangle) \left[\left(\frac{\mathbf{p}_i}{2m_i} - \frac{\mathbf{p}_j}{2m_j} \right) + \left(\frac{\mathbf{p}'_i}{2m'_i} - \frac{\mathbf{p}'_j}{2m'_j} \right) \right] \right. \\ &\left. + i(\langle s'_{z,i} | \boldsymbol{\sigma}_i | s_{z,i} \rangle \times \langle s'_{z,j} | \boldsymbol{\sigma}_j | s_{z,j} \rangle) \left[\left(\frac{\mathbf{p}_i}{2m_i} - \frac{\mathbf{p}_j}{2m_j} \right) - \left(\frac{\mathbf{p}'_i}{2m'_i} - \frac{\mathbf{p}'_j}{2m'_j} \right) \right] \right\}, \end{aligned} \quad (2.14)$$

where, $\hat{\alpha}_i^{(-)}$ and $\hat{\beta}_j^{(+)}$ are flavor-changing operators acting on the i th and j th quarks, respectively. In particular, for the indicated transitions, we take $\hat{\alpha}_i^{(-)} c = s$ and $\hat{\beta}_j^{(+)} d = u$ for the $cd \rightarrow su$ transition, $\hat{\alpha}_i^{(-)} c = d$ and $\hat{\beta}_j^{(+)} d = u$ for the $cd \rightarrow du$ transition, $\hat{\alpha}_i^{(-)} c = s$ and $\hat{\beta}_j^{(+)} s = u$ for the $cs \rightarrow su$ transition, and $\hat{\alpha}_i^{(-)} c = d$ and $\hat{\beta}_j^{(+)} s = u$ for the $cs \rightarrow du$ transition. Furthermore, the effective Hamiltonian $\mathcal{H}_{2,2 \rightarrow 2}^{\text{PC,PV}}$ associated with $c_2(\mu)$ can be obtained from Eq. (2.14) by the replacements $(p'_i, m'_i) \leftrightarrow (p'_j, m'_j)$ and $s'_{z,i} \leftrightarrow s'_{z,j}$.

In this work, we adopt the pole model to estimate the decay amplitudes for W -exchange diagrams. The pole model has been widely and successfully applied to evaluate the nonfactorizable amplitudes of charmed-baryon nonleptonic decays [17–29,33,34]. In the pole model, the general expressions for the decay amplitudes can be written as [17,30,31,39]

$$i\mathcal{M}_{\mathcal{B}_m - \text{pole}}^{J_z^f, J_z^i} = \langle \mathcal{B}_f(\mathbf{P}_f; J_f^f, J_z^f) | \mathcal{H}_P | \mathcal{B}_m(\mathbf{P}_m; J_m^m, J_z^m) \rangle \frac{i}{\not{p}_m - M_m + i\Gamma_m/2} \langle \mathcal{B}_m(\mathbf{P}_m; J_m^m, J_z^m) | \mathcal{H}_{2 \rightarrow 2} | \mathcal{B}_i(\mathbf{P}_i; J_i^i, J_z^i) \rangle, \quad (2.15)$$

$$i\mathcal{M}_{\mathcal{B}_m - \text{pole}}^{J_z^f, J_z^i} = \langle \mathcal{B}_f(\mathbf{P}_f; J_f^f, J_z^f) | \mathcal{H}_{2 \rightarrow 2} | \mathcal{B}_m(\mathbf{P}_m; J_m^m, J_z^m) \rangle \frac{i}{\not{p}_m - M_m + i\Gamma_m/2} \langle \mathcal{B}_m(\mathbf{P}_m; J_m^m, J_z^m) | \mathcal{H}_P | \mathcal{B}_i(\mathbf{P}_i; J_i^i, J_z^i) \rangle, \quad (2.16)$$

for $E_{1,2}$ and E' diagrams, respectively, according to Fig. 4. The state $\mathcal{B}_m$ denotes an intermediate baryon with $J^P = 1/2^+$ or $1/2^-$. Particularly in this work, we take $\mathcal{B}_m = (\Xi_c^{(\prime)+}, \Lambda_c^+, \Sigma_c^+)$ for the $E_{1,2}$ diagrams and $\mathcal{B}_m = (\Xi_{cc}^+, \Omega_{cc}^+)$ for the E' diagrams. The p_m is the four-momentum, and M_m and Γ_m are the mass and total width, respectively, of the intermediate state $\mathcal{B}_m$. Besides, it is worthy to mention that we have $\mathbf{P}_m = \mathbf{P}_i$ and $(J_m^m, J_z^m) = (J_i^i, J_z^i)$ in Eq. (2.15), while we have

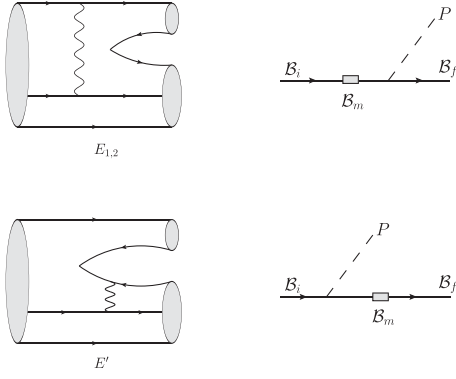


FIG. 4. Corresponding pole diagrams of the W -exchange processes.

$\mathbf{P}_m = \mathbf{P}_f$ and $(J^m, J_z^m) = (J^f, J_z^f)$ in Eq. (2.16). In the numerical evaluation, the approximation of the propagator

$$\frac{1}{\not{p} - m + i\Gamma/2} \approx \frac{2m}{p^2 - m^2 + i\Gamma m} \quad (2.17)$$

is employed. Moreover, considering the narrow widths of involved charmed baryons and large mass differences between double-charmed baryons and single-charmed baryons, we further neglect the $i\Gamma m$ term in the numerical calculation.

The Hamiltonian $\mathcal{H}_P$ is adopted as the quark-pseudoscalar meson coupling as

$$\mathcal{H}_P = \sum_j \int d\mathbf{x} \frac{1}{f_P} \bar{q}_j(\mathbf{x}) \gamma_\mu \gamma_5 q_j(\mathbf{x}) \partial^\mu \phi_P(\mathbf{x}), \quad (2.18)$$

where f_P is the pseudoscalar meson decay constant, and $q(\mathbf{x})$ and ϕ_P denote the quark and pseudoscalar meson

fields, respectively. In our calculation, we adopt the experimental values $f_\pi = 130$ MeV and $f_K = 156$ MeV [40], and the lattice QCD results $f_q = 139$ MeV and $f_s = 171$ MeV for the $\eta^{(\prime)}$ mesons [42]. In the nonrelativistic limit, the effective Hamiltonian can be expanded as [17,30,31]

$$\mathcal{H}_P = \frac{1}{\sqrt{(2\pi)^3 2\omega_P}} \sum_j \frac{1}{f_P} \left[\omega_P \left(\frac{\boldsymbol{\sigma} \cdot \mathbf{p}_f^j}{2m_f} + \frac{\boldsymbol{\sigma} \cdot \mathbf{p}_i^j}{2m_i} \right) - \boldsymbol{\sigma} \cdot \mathbf{k} \right] \times \hat{I}_P^j \delta^3(\mathbf{p}_f^j + \mathbf{k} - \mathbf{p}_i^j), \quad (2.19)$$

where ω_P and $\mathbf{k}$ are the energy and three-momentum of the pseudoscalar meson in the rest frame of the initial state, and $\mathbf{p}_i^j$ and $\mathbf{p}_f^j$ are the initial and final three-momenta of the j th quark, respectively. Besides, $\hat{I}_P^j$ represents the isospin operator acting on the j th quark, with the explicit form being [31]

$$\hat{I}_P = \begin{cases} b_d^\dagger b_u, & \text{for } \pi^+, \\ \frac{1}{\sqrt{2}} [b_u^\dagger b_u - b_d^\dagger b_d], & \text{for } \pi^0, \\ b_s^\dagger b_u, & \text{for } K^+, \\ b_{u(d)}^\dagger b_s, & \text{for } K^-(\bar{K}^0), \\ \frac{\cos \zeta}{\sqrt{2}} [b_u^\dagger b_u + b_d^\dagger b_d] - \sin \zeta [b_s^\dagger b_s], & \text{for } \eta, \\ \frac{\sin \zeta}{\sqrt{2}} [b_u^\dagger b_u + b_d^\dagger b_d] + \cos \zeta [b_s^\dagger b_s], & \text{for } \eta'. \end{cases} \quad (2.20)$$

Here, $\zeta = 39.3^\circ$ [43] is the mixing angle of η and η' , which is introduced by

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \zeta & -\sin \zeta \\ \sin \zeta & \cos \zeta \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}. \quad (2.21)$$

Taking the decay amplitude in Eq. (2.15) as an example, the concrete expressions of the baryon weak transition matrix elements for the PC and PV amplitudes can be obtained by performing the integrations

$$\begin{aligned} \langle \mathcal{B}_m(\mathbf{P}_i; J^i, J_z^i) | \mathcal{H}_{1(2),2 \rightarrow 2}^{\text{PC}} | \mathcal{B}_i(\mathbf{P}_i; J^i, J_z^i) \rangle &= (\pm 1) \times \frac{G_F}{\sqrt{2}} V_{\text{CKM}} V'_{\text{CKM}} \frac{6}{(2\pi)^3} \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}'_1 d\mathbf{p}'_2 d\mathbf{p}'_3 \delta^3(\mathbf{p}'_3 - \mathbf{p}_3) \\ &\times \sum_{S_z^i, S_z^m} \sum_{L_z^i, L_z^m} \langle S^m, S_z^m; L^m, L_z^m | J^i, J_z^i \rangle \langle S^i, S_z^i; L^i, L_z^i | J^i, J_z^i \rangle \\ &\times \langle \mathcal{B}_m(S^m, S_z^m) | \hat{\alpha}_1^{(-)} \hat{\beta}_2^{(+)} \hat{O}_{1(2),2 \rightarrow 2}^{\text{spin}} | \mathcal{B}_i(S^i, S_z^i) \rangle \delta^3(\mathbf{p}'_1 + \mathbf{p}'_2 - \mathbf{p}_1 - \mathbf{p}_2) \\ &\times \psi_{L^m, L_z^m}^*(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{p}'_3) \psi_{L^i, L_z^i}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \delta^3(\mathbf{P}_i - \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3) \delta^3(\mathbf{P}_i - \mathbf{p}'_1 - \mathbf{p}'_2 - \mathbf{p}'_3), \end{aligned} \quad (2.22)$$

$$\begin{aligned}
\langle \mathcal{B}_m(\mathbf{P}_i; J^i, J_z^i) | \mathcal{H}_{1(2),2 \rightarrow 2}^{\text{PV}} | \mathcal{B}_i(\mathbf{P}_i; J^i, J_z^i) \rangle &= (\pm 1) \times \frac{G_F}{\sqrt{2}} V_{\text{CKM}} V'_{\text{CKM}} \frac{6}{(2\pi)^3} \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}'_1 d\mathbf{p}'_2 d\mathbf{p}'_3 \delta^3(\mathbf{p}'_3 - \mathbf{p}_3) \\
&\times \sum_{S_z^i, S_z^m} \sum_{L_z^i, L_z^m} \langle S^m, S_z^m; L^m, L_z^m | J^i, J_z^i \rangle \langle S^i, S_z^i; L^i, L_z^i | J^i, J_z^i \rangle \\
&\times \langle \mathcal{B}_m(S^m, S_z^m) | \hat{\alpha}_1^{(-)} \hat{\beta}_2^{(+)} \hat{\mathcal{O}}_{1(2),2 \rightarrow 2}^{\text{spin}} | \mathcal{B}_i(S^i, S_z^i) \rangle \delta^3(\mathbf{p}'_1 + \mathbf{p}'_2 - \mathbf{p}_1 - \mathbf{p}_2) \\
&\times \psi_{L^m, L_z^m}^*(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{p}'_3) \mathcal{O}_{1(2),2 \rightarrow 2}^{\text{spatial}}(\mathbf{p}_i) \psi_{L^i, L_z^i}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \delta^3(\mathbf{P}_i - \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3) \\
&\times \delta^3(\mathbf{P}_i - \mathbf{p}'_1 - \mathbf{p}'_2 - \mathbf{p}'_3), \tag{2.23}
\end{aligned}$$

where the factor ± 1 is the color wave function overlap and 6 is the flavor symmetry factor. Besides, the concerned spin-flavor matrix elements for PC and PV amplitudes of W -exchange diagrams are summarized in Tables XII and XIII in Appendix B, respectively.

As for the transition matrix element $\langle \mathcal{B}_f(\mathbf{P}_f) | \mathcal{H}_P | \mathcal{B}_m(\mathbf{P}_i) \rangle$, we can evaluate it as

$$\begin{aligned}
\langle \mathcal{B}_f(\mathbf{P}_f; J^f, J_z^f) | \mathcal{H}_P | \mathcal{B}_m(\mathbf{P}_i; J^i, J_z^i) \rangle &= \frac{1}{\sqrt{(2\pi)^3 2\omega_P f_P}} \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}'_1 d\mathbf{p}'_2 d\mathbf{p}'_3 \delta^3(\mathbf{p}'_1 - \mathbf{p}_1) \delta^3(\mathbf{p}'_2 - \mathbf{p}_2) \\
&\times \sum_{S_z^f, S_z^m} \sum_{L_z^f, L_z^m} \langle S^f, S_z^f; L^f, L_z^f | J^f, J_z^f \rangle \langle S^m, S_z^m; L^m, L_z^m | J^i, J_z^i \rangle \\
&\times \langle \mathcal{B}_f(S^f, S_z^f) | \hat{I}_P(-\boldsymbol{\sigma} \cdot \mathbf{k}) | \mathcal{B}_m(S^m, S_z^m) \rangle \frac{\omega_P + 2(m_1 + m_2 + m_3)}{2(m_1 + m_2 + m_3)} \delta^3(\mathbf{p}'_3 - \mathbf{p}_3 + \mathbf{k}) \\
&\times \psi_{L^f, L_z^f}^*(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{p}'_3) \psi_{L^m, L_z^m}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \delta^3(\mathbf{P}_i - \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3) \delta^3(\mathbf{P}_f - \mathbf{p}'_1 - \mathbf{p}'_2 - \mathbf{p}'_3), \tag{2.24}
\end{aligned}$$

in which the spin-flavor matrix element can be divided as

$$\langle \mathcal{B}' | \hat{I}_P(-\boldsymbol{\sigma} \cdot \mathbf{k}) | \mathcal{B} \rangle = -k \times \langle \mathcal{B}' | \hat{I}_P \sigma_z | \mathcal{B} \rangle, \tag{2.25}$$

where $k \equiv |\mathbf{k}|$. We summarize the values of matrix elements $\langle \mathcal{B}' | \hat{I}_P \sigma_z | \mathcal{B} \rangle$ in Tables XIV and XV in Appendix B. Obviously, we have $\langle \mathcal{B}'_3 | \hat{I}_P \sigma_z | \mathcal{B}_3 \rangle = 0$ due to the heavy quark symmetry.

Analogously, the decay amplitude in Eq. (2.16) can also be obtained. Besides, based on the above discussion, we have the relations

$$\mathcal{M}_{\text{PC}}^{-1/2, -1/2} = -\mathcal{M}_{\text{PC}}^{1/2, 1/2}, \quad \mathcal{M}_{\text{PV}}^{-1/2, -1/2} = \mathcal{M}_{\text{PV}}^{1/2, 1/2}, \quad \mathcal{M}_{\text{PV,PC}}^{-1/2, 1/2} = \mathcal{M}_{\text{PV,PC}}^{1/2, -1/2} = 0 \tag{2.26}$$

for the W -exchange diagram.

C. Nonrelativistic potential and baryon wave functions

In this section, we illustrate how to obtain the wave functions of baryons. For this purpose, we employ a nonrelativistic Hamiltonian [44–46],

$$\mathcal{H} = \sum_{i=1,2,3} \left(\frac{p_i^2}{2m_i} + m_i \right) + \sum_{i < j} V_{ij}, \tag{2.27}$$

to describe a baryon system, where m_i and p_i are the mass and momentum of the i th quark, respectively. The nonrelativistic quark-quark interaction is given by $V_{ij} = V_{ij}^{\text{con}} + V_{ij}^{\text{hyp}} + V_{ij}^{\text{so(cm)}} + V_{ij}^{\text{so(tp)}}$, where the confinement potential V_{ij}^{con} , hyperfine potential V_{ij}^{hyp} , color-magnetic spin-orbit potential $V_{ij}^{\text{so(cm)}}$, and Thomas-precession spin-orbit potential $V_{ij}^{\text{so(tp)}}$ are given explicitly as [47–50]

TABLE II. Parameters used in the nonrelativistic potential model. Besides, the masses of constituent quarks are taken as $m_{u,d} = 370$ MeV, $m_s = 600$ MeV, and $m_c = 1880$ MeV [47–49].

Parameters	Values	Parameters	Values
α_s	0.560	b (GeV ²)	0.125
σ (GeV)	1.600	C (GeV)	−0.644

$$V_{ij}^{\text{conf}} = -\frac{2\alpha_s}{3r_{ij}} + \frac{b}{2}r_{ij} + \frac{C}{2}, \quad (2.28)$$

$$V_{ij}^{\text{hyp}} = \frac{2\alpha_s}{3m_i m_j} \left[\frac{8\pi}{3} \tilde{\delta}(r_{ij}) \mathbf{S}_i \cdot \mathbf{S}_j + \frac{1}{r_{ij}^3} \left(\frac{3\mathbf{s}_i \cdot \mathbf{r}_{ij} \mathbf{s}_j \cdot \mathbf{r}_{ij}}{r_{ij}^2} - \mathbf{S}_i \cdot \mathbf{S}_j \right) \right], \quad (2.29)$$

$$V_{ij}^{\text{so(cm)}} = \frac{2\alpha_s}{3r_{ij}^3} \left(\frac{\mathbf{r}_{ij} \times \mathbf{p}_i \cdot \mathbf{s}_i}{m_i^2} - \frac{\mathbf{r}_{ij} \times \mathbf{p}_j \cdot \mathbf{s}_j}{m_j^2} - \frac{\mathbf{r}_{ij} \times \mathbf{p}_j \cdot \mathbf{s}_i - \mathbf{r}_{ij} \times \mathbf{p}_i \cdot \mathbf{s}_j}{m_i m_j} \right), \quad (2.30)$$

$$V_{ij}^{\text{so(tp)}} = -\frac{1}{2r_{ij}} \frac{\partial V_{ij}^{\text{conf}}}{\partial r_{ij}} \left(\frac{\mathbf{r}_{ij} \times \mathbf{p}_i \cdot \mathbf{s}_i}{m_i^2} - \frac{\mathbf{r}_{ij} \times \mathbf{p}_j \cdot \mathbf{s}_j}{m_j^2} \right). \quad (2.31)$$

In the nonrelativistic quark potential, there are four phenomenological parameters:

- (1) the one-gluon exchange coupling constant α_s ,
- (2) the linear-confinement strength b ,
- (3) the constant C , and
- (4) the smearing parameter σ , which is introduced by

$$\tilde{\delta}(r_{ij}) = \frac{\sigma^3}{\pi^{3/2}} \exp(-\sigma^2 r_{ij}^2). \quad (2.32)$$

The phenomenological parameters of the nonrelativistic potential model are shown in Table II, which can be constrained by the well-established low-lying charmed baryon spectrum.

In the study of the baryon spectrum, the masses and wave functions can be obtained by the solving Schrödinger equation with the nonrelativistic Hamiltonian

$$\mathcal{H}|\Psi_{J,M_J}\rangle = E|\Psi_{J,M_J}\rangle, \quad (2.33)$$

in which the baryon wave function Ψ_{J,M_J} can be constructed using the $L-S$ coupling:

$$\Psi_{J,M_J} = \sum_{\alpha} C^{(\alpha)} \Psi_{J,M_J}^{(\alpha)},$$

$$\Psi_{J,M_J}^{(\alpha)} = \chi^{\text{color}} [[s_{q_1} s_{q_2}]_{S_{\rho}} s_{q_3}]_S \psi_{l_{\rho}, l_{\lambda}, L}^{\text{spatial}}(\mathbf{r}_{\rho}, \mathbf{r}_{\lambda})]_{J,M_J} \psi^{\text{flavor}}. \quad (2.34)$$

Here, $C^{(\alpha)}$ is a coefficient, and α represents all possible quantum numbers. Besides, the spatial wave function $\psi_{l_{\rho}, l_{\lambda}, L}^{\text{spatial}}$ consists of ρ -mode and λ -mode excitations as

$$\psi_{l_{\rho}, l_{\lambda}, L}^{\text{spatial}}(\mathbf{r}_{\rho}, \mathbf{r}_{\lambda}) = \sum_{m_{\rho}, m_{\lambda}} \langle l_{\rho}, m_{\rho}; l_{\lambda}, m_{\lambda} | L, m_{\rho} + m_{\lambda} \rangle \times \psi_{l_{\rho}, m_{\rho}}(\mathbf{r}_{\rho}) \psi_{l_{\lambda}, m_{\lambda}}(\mathbf{r}_{\lambda}), \quad (2.35)$$

with the corresponding Jacobi coordinates being defined as

$$\mathbf{r}_{\rho} = \mathbf{r}_2 - \mathbf{r}_1,$$

$$\mathbf{r}_{\lambda} = \mathbf{r}_3 - \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2}{m_1 + m_2}, \quad (2.36)$$

respectively. This is consistent with the picture in which a single-charmed baryon is treated as a bound state of a light-quark cluster and a charm quark, whereas a double-charmed baryon is regarded as a bound state of a (cc) cluster and a light quark, as shown in Fig. 5.

In the Gaussian expansion method, the infinitesimal-shift Gaussian basis [41,51]

$$\phi_{nlm}^G(\mathbf{r}) = \phi_{nl}^G(r) Y_{lm}(\hat{\mathbf{r}})$$

$$= \sqrt{\frac{2^{l+2} (2\nu_n)^{l+3/2}}{\sqrt{\pi} (2l+1)!!}} \lim_{\epsilon \rightarrow 0} \frac{1}{(\nu_n \epsilon)^l} \sum_{k=1}^{k_{\max}} C_{lm,k} e^{-\nu_n (\mathbf{r} - \epsilon \mathbf{D}_{lm,k})^2} \quad (2.37)$$

is adopted to expand the spatial wave functions $\psi_{l_{\rho}, m_{\rho}}$ and $\psi_{l_{\lambda}, m_{\lambda}}$, in which the Gaussian size parameter ν_n in our computations follows a geometric progression sequence [18,52,53],

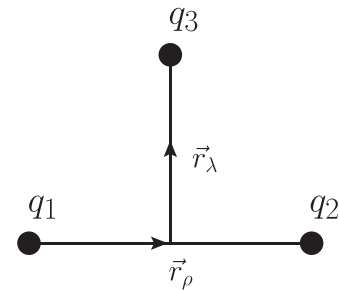


FIG. 5. Definition of Jacobi coordinates of the ρ -mode and λ -mode of the charmed baryon. We set $q_1 q_2 q_3 = (qqc)$ for single-charmed baryons and $q_1 q_2 q_3 = (ccq)$ for double-charmed baryons.

$$\begin{aligned} \nu_n &= 1/r_n^2, & r_n &= r_{\min} a^{n-1}, \\ a &= (r_{\max}/r_{\min})^{\frac{1}{n_{\max}-1}}, \end{aligned} \quad (2.38)$$

with $(r_{\rho_{\min}}, r_{\rho_{\max}}, n_{\rho_{\max}}) = (0.1 \text{ fm}, 3.0 \text{ fm}, 10)$ and $(r_{\lambda_{\min}}, r_{\lambda_{\max}}, n_{\lambda_{\max}}) = (0.1 \text{ fm}, 3.0 \text{ fm}, 10)$. The Gaussian basis in momentum space can be obtained with the replacements $r \rightarrow p$ and $\nu_n \rightarrow 1/(4\nu_n)$.

Finally, with the nonrelativistic potential and baryon wave function prepared, we further solve the Schrödinger equation using the Gaussian expansion method [41,51]:

$$(T^{\alpha',\alpha} + V^{\alpha',\alpha})C^{(\alpha)} = EN^{\alpha',\alpha}C^{(\alpha)}, \quad (2.39)$$

where the matrix elements can be evaluated as

$$\begin{aligned} T^{\alpha',\alpha} &= \left\langle \Psi_{J,M_J}^{(\alpha')} \left| \left[\sum_{i=1,2,3} \left(\frac{p_i^2}{2m_i} + m_i \right) \right] \right| \Psi_{J,M_J}^{(\alpha)} \right\rangle, \\ V^{\alpha',\alpha} &= \left\langle \Psi_{J,M_J}^{(\alpha')} \left| \sum_{i<j} V_{ij} \right| \Psi_{J,M_J}^{(\alpha)} \right\rangle, \\ N^{\alpha',\alpha} &= \left\langle \Psi_{J,M_J}^{(\alpha')} \left| \Psi_{J,M_J}^{(\alpha)} \right\rangle. \end{aligned} \quad (2.40)$$

III. NUMERICAL RESULTS

Before calculating the weak decay amplitudes, we first discuss the masses and spatial wave functions of the relevant low-lying charmed baryons. Adopting the non-relativistic potential in Eqs. (2.28)–(2.31) with the phenomenological parameters in Table II, we are able to get the masses of corresponding charmed baryons along with their spatial wave functions after solving the three-body

TABLE III. Comparison of theoretical and experimental values of masses of concerned charmed baryons.

States	Quantum numbers $(l_\rho, l_\lambda, L, S_\rho, S, J^P)$	PDG name	This work [MeV]	Expt. [40] [MeV]
$\Lambda_c(1S)$	$(0, 0, 0, 0, \frac{1}{2}, \frac{1}{2}^+)$	Λ_c^+	2280	2286
$\Lambda_c(2S)$	$(0, 0, 0, 0, \frac{1}{2}, \frac{1}{2}^+)$	$\Lambda_c(2765)^+$	2786	2765
$\Lambda_c(1^2P_\lambda)$	$(0, 1, 1, 0, \frac{1}{2}, \frac{1}{2}^-)$	$\Lambda_c(2595)^+$	2593	2595
$\Sigma_c(1S)$	$(0, 0, 0, 1, \frac{1}{2}, \frac{1}{2}^+)$	$\Sigma_c(2455)^{++,+,0}$	2470	2455
$\Sigma_c(2S)$	$(0, 0, 0, 1, \frac{1}{2}, \frac{1}{2}^+)$		2953	
$\Sigma_c(1^2P_\lambda)$	$(0, 1, 1, 1, \frac{1}{2}, \frac{1}{2}^-)$		2798	
$\Sigma_c(1^4P_\lambda)$	$(0, 1, 1, 1, \frac{3}{2}, \frac{1}{2}^-)$		2787	
$\Xi_c(1S)$	$(0, 0, 0, 0, \frac{1}{2}, \frac{1}{2}^+)$	$\Xi_c^{+,0}$	2481	2470
$\Xi_c(2S)$	$(0, 0, 0, 0, \frac{1}{2}, \frac{1}{2}^+)$	$\Xi_c(2970)^{+,0}$	2961	2970
$\Xi_c(1^2P_\lambda)$	$(0, 1, 1, 0, \frac{1}{2}, \frac{1}{2}^-)$	$\Xi_c(2790)^{+,0}$	2780	2790
$\Xi'_c(1S)$	$(0, 0, 0, 1, \frac{1}{2}, \frac{1}{2}^+)$	$\Xi'_c{}^{+,0}$	2589	2580
$\Xi'_c(2S)$	$(0, 0, 0, 1, \frac{1}{2}, \frac{1}{2}^+)$		3058	
$\Xi'_c(1^2P_\lambda)$	$(0, 1, 1, 1, \frac{1}{2}, \frac{1}{2}^-)$		2908	
$\Xi'_c(1^4P_\lambda)$	$(0, 1, 1, 1, \frac{3}{2}, \frac{1}{2}^-)$		2894	
$\Omega_c(1S)$	$(0, 0, 0, 1, \frac{1}{2}, \frac{1}{2}^+)$	Ω_c^0	2701	2695
$\Omega_c(2S)$	$(0, 0, 0, 1, \frac{1}{2}, \frac{1}{2}^+)$		3163	
$\Omega_c(1^2P_\lambda)$	$(0, 1, 1, 1, \frac{1}{2}, \frac{1}{2}^-)$		3018	
$\Omega_c(1^4P_\lambda)$	$(0, 1, 1, 1, \frac{3}{2}, \frac{1}{2}^-)$		3000	
$\Xi_{cc}(1S)$	$(0, 0, 0, 1, \frac{1}{2}, \frac{1}{2}^+)$	Ξ_{cc}^{++}	3649	3621
$\Xi_{cc}(2S)$	$(0, 0, 0, 1, \frac{1}{2}, \frac{1}{2}^+)$		4064	
$\Xi_{cc}(1^2P_\rho)$	$(1, 0, 1, 0, \frac{1}{2}, \frac{1}{2}^-)$		3965	
$\Xi_{cc}(1^2P_\lambda)$	$(0, 1, 1, 1, \frac{1}{2}, \frac{1}{2}^-)$		4064	
$\Xi_{cc}(1^4P_\lambda)$	$(0, 1, 1, 1, \frac{3}{2}, \frac{1}{2}^-)$		4011	
$\Omega_{cc}(1S)$	$(0, 0, 0, 1, \frac{1}{2}, \frac{1}{2}^+)$		3751	
$\Omega_{cc}(2S)$	$(0, 0, 0, 1, \frac{1}{2}, \frac{1}{2}^+)$		4185	
$\Omega_{cc}(1^2P_\rho)$	$(1, 0, 1, 0, \frac{1}{2}, \frac{1}{2}^-)$		4077	
$\Omega_{cc}(1^2P_\lambda)$	$(0, 1, 1, 1, \frac{1}{2}, \frac{1}{2}^-)$		4149	
$\Omega_{cc}(1^4P_\lambda)$	$(0, 1, 1, 1, \frac{3}{2}, \frac{1}{2}^-)$		4093	

TABLE IV. Decay amplitudes $\mathcal{M}_{\text{PC,PV}}^{-1/2,-1/2}$ (in the units of $10^{-4}G_F \text{ GeV}^{3/2}$) of CF decays.

$\mathcal{M}_{\text{PC}}^{-1/2,-1/2}$			$\mathcal{M}_{\text{PV}}^{-1/2,-1/2}$	
$\Omega_{cc}^+ \rightarrow \Omega_c^0 \pi^+$	T -diagram	−53.96	T -diagram	22.21
$\Omega_{cc}^+ \rightarrow \Xi_c^+ \bar{K}^0$	C -diagram	17.69	C -diagram	−35.61
	C' -diagram	−3.61	C' -diagram	12.14
	$\Xi_{cc}^+(1S)$ -pole	−54.75	$\Xi_{cc}^+(1^2P_\rho)$ -pole	×(s)
	$\Xi_{cc}^+(2S)$ -pole	−2.57	$\Xi_{cc}^+(1^2P_\lambda)$ -pole	−1.90
			$\Xi_{cc}^+(1^4P_\lambda)$ -pole	70.04
	Total	−43.24	Total	44.67
$\Omega_{cc}^+ \rightarrow \Xi_c'^+ \bar{K}^0$	C -diagram	−43.65	C -diagram	20.32
	C' -diagram	0	C' -diagram	0
	$\Xi_{cc}^+(1S)$ -pole	×(w)	$\Xi_{cc}^+(1^2P_\rho)$ -pole	×(s)
	$\Xi_{cc}^+(2S)$ -pole	×(w)	$\Xi_{cc}^+(1^2P_\lambda)$ -pole	0
			$\Xi_{cc}^+(1^4P_\lambda)$ -pole	×(w)
	Total	−43.65	Total	20.32

Schrödinger equation. The computed results of masses are summarized in Table III. Overall, our results for the masses of low-lying charmed baryons are comparable to the experimental values.

Having prepared the charmed-baryon wave functions, we are subsequently able to calculate the weak decay amplitudes numerically. The CKM matrix elements are adopted in the Wolfenstein parametrization as

$$\begin{aligned} V_{ud} &= 1 - \lambda^2/2, & V_{us} &= \lambda, \\ V_{cd} &= -\lambda, & V_{cs} &= 1 - \lambda^2/2, \end{aligned} \quad (3.1)$$

where $\lambda = 0.22501$ [40]. The decay amplitudes for CF, SCS, and DCS processes are presented in Tables IV, V, and VI, respectively. Here, we use the symbols ×(w) and ×(s) to mark that the corresponding decay amplitude vanishes due to the relevant weak or strong interaction vertex, respectively. According to the pole model, the nonfactorizable amplitudes arising from *W*-exchange diagrams are evaluated by considering intermediate low-lying poles, specifically, Λ_c^+ , Σ_c^+ , $\Xi_c^{(\prime)+}$, Ξ_{cc}^+ , and Ω_{cc}^+ in this work. In our calculation, for the PC amplitudes induced by double-charmed baryon poles, the contribution from the 2*S* pole is significantly suppressed compared with that of the 1*S* pole. However, for single-charmed baryon poles, the 2*S* pole contribution remains comparable to that of the 1*S* pole. For the PV amplitudes, the main contribution arises from double-charmed baryon poles, who have quantum numbers 1^4P_λ . In comparison, the contributions from either single-charmed or double-charmed baryon poles with quantum numbers 1^2P_λ are small.

In Table VII, we compare our results of factorizable and nonfactorizable amplitudes of PV and PC amplitudes of CF decays with other theoretical works [32–35,54], in which particularly the nonfactorizable contributions from

W-exchange diagrams were estimated using the covariant confined quark model [35], or the pole model, with relevant nonperturbative parameters being derived from the MIT bag model [33] or flavor symmetry [32,34]. In addition, the authors of Ref. [54] considered long-distance contributions arising from final-state-interaction (FSI) effects. In this table, the decay amplitudes *A* and *B* are defined by

$$\mathcal{M}[\mathcal{B}_i \rightarrow \mathcal{B}_f + P] = \bar{u}_f(A - B\gamma_5)u_i, \quad (3.2)$$

and they can be converted to $\mathcal{M}_{\text{PV,PC}}$ by

$$\begin{aligned} \mathcal{M}_{\text{PV}} &= \frac{1}{4\pi} \sqrt{\frac{m_f}{\pi E_f E_P}} \sqrt{\frac{E_f + m_f}{2m_f}} \chi_f^\dagger \chi_i A, \\ \mathcal{M}_{\text{PC}} &= \frac{1}{4\pi} \sqrt{\frac{m_f}{\pi E_f E_P}} \sqrt{\frac{E_f + m_f}{2m_f}} \chi_f^\dagger \frac{\boldsymbol{\sigma} \cdot \mathbf{P}_f}{E_f + m_f} \chi_i B. \end{aligned} \quad (3.3)$$

Clearly, the decay mode $\Omega_{cc}^+ \rightarrow \Omega_c^0 \pi^+$ receives a contribution only from the *T* diagram and is therefore purely factorizable. Our computed PV and PC amplitudes are comparable with those of Refs. [32–34], except that the PC amplitude is almost a factor of 2 larger than that of Ref. [34]. The decay mode $\Omega_{cc}^+ \rightarrow \Xi_c^+ \bar{K}^0$ receives substantial contributions from nonfactorizable amplitudes. For both the PV and PC amplitudes, the factorizable and nonfactorizable parts interfere destructively. This is consistent with the results of Refs. [33,35]. In contrast, the decay mode $\Omega_{cc}^+ \rightarrow \Xi_c'^+ \bar{K}^0$ has a small nonfactorizable contribution in the pole model. However, a theoretical study suggested that the FSI effects may play a significant role in the nonfactorizable amplitude of $\Omega_{cc}^+ \rightarrow \Xi_c'^+ \bar{K}^0$ decay [54]. More theoretical analyses are required to verify the nonfactorizable contributions.

Moving forward, we present a systematic investigation of the branching fractions and up-down decay asymmetry

TABLE V. Same as Table IV but for SCS decays.

$\mathcal{M}_{\text{PC}}^{-1/2,-1/2}$			$\mathcal{M}_{\text{PV}}^{-1/2,-1/2}$			$\mathcal{M}_{\text{PC}}^{-1/2,-1/2}$			$\mathcal{M}_{\text{PV}}^{-1/2,-1/2}$		
$\Omega_{cc}^+ \rightarrow \Xi_c^0 \pi^+$	T -diagram	2.78	T -diagram	-4.43	$\Omega_{cc}^+ \rightarrow \Xi_c^0 \pi^+$	T -diagram	-6.91	T -diagram	2.49		
	$\Xi_c^+(1S)$ -pole	$\times(s)$	$\Xi_c^+(1^2P_\lambda)$ -pole	$\times(s)$		$\Xi_c^+(1S)$ -pole	16.20	$\Xi_c^+(1^2P_\lambda)$ -pole	-0.14		
	$\Xi_c^+(2S)$ -pole	$\times(s)$	$\Xi_c^{\prime+}(1^2P_\lambda)$ -pole	0		$\Xi_c^+(2S)$ -pole	8.48	$\Xi_c^{\prime+}(1^2P_\lambda)$ -pole	0		
	$\Xi_c^{\prime+}(1S)$ -pole	$\times(w)$	$\Xi_c^{\prime+}(1^4P_\lambda)$ -pole	0		$\Xi_c^{\prime+}(1S)$ -pole	$\times(w)$	$\Xi_c^{\prime+}(1^4P_\lambda)$ -pole	0		
	$\Xi_c^{\prime+}(2S)$ -pole	$\times(w)$				$\Xi_c^{\prime+}(2S)$ -pole	$\times(w)$				
	Total	2.78	Total	-4.43		Total	17.77	Total	2.35		
$\Omega_{cc}^+ \rightarrow \Omega_c^0 K^+$	T -diagram	-24.56	T -diagram	13.54	$\Omega_{cc}^+ \rightarrow \Sigma_c^{++} K^-$	$\Xi_c^+(1S)$ -pole	17.15	$\Xi_c^+(1^2P_\lambda)$ -pole	-0.17		
	$\Xi_c^+(1S)$ -pole	-30.26	$\Xi_c^+(1^2P_\lambda)$ -pole	0.18		$\Xi_c^+(2S)$ -pole	8.27	$\Xi_c^{\prime+}(1^2P_\lambda)$ -pole	0		
	$\Xi_c^+(2S)$ -pole	-9.55	$\Xi_c^{\prime+}(1^2P_\lambda)$ -pole	0		$\Xi_c^{\prime+}(1S)$ -pole	$\times(w)$	$\Xi_c^{\prime+}(1^4P_\lambda)$ -pole	0		
	$\Xi_c^{\prime+}(1S)$ -pole	$\times(w)$	$\Xi_c^{\prime+}(1^4P_\lambda)$ -pole	0		$\Xi_c^{\prime+}(2S)$ -pole	$\times(w)$				
	$\Xi_c^{\prime+}(2S)$ -pole	$\times(w)$				Total	25.42	Total	-0.17		
	Total	-64.37	Total	13.72							
$\Omega_{cc}^+ \rightarrow \Xi_c^+ \eta$	C -diagram	-3.43	C -diagram	7.14	$\Omega_{cc}^+ \rightarrow \Xi_c^+ \eta$	C -diagram	8.45	C -diagram	-4.09		
	C' -diagram	0.29	C' -diagram	-1.72		C' -diagram	0	C' -diagram	0		
	$\Omega_{cc}^+(1S)$ -pole	10.43	$\Omega_{cc}^+(1^2P_\rho)$ -pole	$\times(s)$		$\Omega_{cc}^+(1S)$ -pole	$\times(w)$	$\Omega_{cc}^+(1^2P_\rho)$ -pole	$\times(s)$		
	$\Omega_{cc}^+(2S)$ -pole	0.62	$\Omega_{cc}^+(1^2P_\lambda)$ -pole	0.34		$\Omega_{cc}^+(2S)$ -pole	$\times(w)$	$\Omega_{cc}^+(1^2P_\lambda)$ -pole	0		
	$\Xi_c^+(1S)$ -pole	$\times(s)$	$\Omega_{cc}^+(1^4P_\lambda)$ -pole	-7.20		$\Xi_c^+(1S)$ -pole	18.09	$\Omega_{cc}^+(1^4P_\lambda)$ -pole	$\times(w)$		
	$\Xi_c^+(2S)$ -pole	$\times(s)$	$\Xi_c^+(1^2P_\lambda)$ -pole	$\times(s)$		$\Xi_c^+(2S)$ -pole	7.76	$\Xi_c^+(1^2P_\lambda)$ -pole	-0.15		
	$\Xi_c^{\prime+}(1S)$ -pole	$\times(w)$	$\Xi_c^{\prime+}(1^2P_\lambda)$ -pole	0		$\Xi_c^{\prime+}(1S)$ -pole	$\times(w)$	$\Xi_c^{\prime+}(1^2P_\lambda)$ -pole	0		
	$\Xi_c^{\prime+}(2S)$ -pole	$\times(w)$	$\Xi_c^{\prime+}(1^4P_\lambda)$ -pole	0		$\Xi_c^{\prime+}(2S)$ -pole	$\times(w)$	$\Xi_c^{\prime+}(1^4P_\lambda)$ -pole	0		
	Total	7.91	Total	-1.44		Total	34.3	Total	-4.24		
$\Omega_{cc}^+ \rightarrow \Xi_c^+ \eta'$	C -diagram	1.17	C -diagram	-9.04	$\Omega_{cc}^+ \rightarrow \Xi_c^+ \eta'$	C -diagram	-2.43	C -diagram	5.42		
	C' -diagram	-0.30	C' -diagram	2.19		C' -diagram	0	C' -diagram	0		
	$\Omega_{cc}^+(1S)$ -pole	-12.64	$\Omega_{cc}^+(1^2P_\rho)$ -pole	$\times(s)$		$\Omega_{cc}^+(1S)$ -pole	$\times(w)$	$\Omega_{cc}^+(1^2P_\rho)$ -pole	$\times(s)$		
	$\Omega_{cc}^+(2S)$ -pole	-0.42	$\Omega_{cc}^+(1^2P_\lambda)$ -pole	-0.34		$\Omega_{cc}^+(2S)$ -pole	$\times(w)$	$\Omega_{cc}^+(1^2P_\lambda)$ -pole	0		
	$\Xi_c^+(1S)$ -pole	$\times(s)$	$\Omega_{cc}^+(1^4P_\lambda)$ -pole	6.74		$\Xi_c^+(1S)$ -pole	-3.44	$\Omega_{cc}^+(1^4P_\lambda)$ -pole	$\times(w)$		
	$\Xi_c^+(2S)$ -pole	$\times(s)$	$\Xi_c^+(1^2P_\lambda)$ -pole	$\times(s)$		$\Xi_c^+(2S)$ -pole	-0.62	$\Xi_c^+(1^2P_\lambda)$ -pole	0.020		
	$\Xi_c^{\prime+}(1S)$ -pole	$\times(w)$	$\Xi_c^{\prime+}(1^2P_\lambda)$ -pole	0		$\Xi_c^{\prime+}(1S)$ -pole	$\times(w)$	$\Xi_c^{\prime+}(1^2P_\lambda)$ -pole	0		
	$\Xi_c^{\prime+}(2S)$ -pole	$\times(w)$	$\Xi_c^{\prime+}(1^4P_\lambda)$ -pole	0		$\Xi_c^{\prime+}(2S)$ -pole	$\times(w)$	$\Xi_c^{\prime+}(1^4P_\lambda)$ -pole	0		
	Total	-12.19	Total	-0.45		Total	-6.49	Total	5.44		
$\Omega_{cc}^+ \rightarrow \Xi_c^+ \pi^0$	C -diagram	1.37	C -diagram	-2.18	$\Omega_{cc}^+ \rightarrow \Xi_c^+ \pi^0$	C -diagram	-3.40	C -diagram	1.22		
	$\Xi_c^+(1S)$ -pole	$\times(s)$	$\Xi_c^+(1^2P_\lambda)$ -pole	$\times(s)$		$\Xi_c^+(1S)$ -pole	11.45	$\Xi_c^+(1^2P_\lambda)$ -pole	-0.10		
	$\Xi_c^+(2S)$ -pole	$\times(s)$	$\Xi_c^{\prime+}(1^2P_\lambda)$ -pole	0		$\Xi_c^+(2S)$ -pole	5.99	$\Xi_c^{\prime+}(1^2P_\lambda)$ -pole	0		
	$\Xi_c^{\prime+}(1S)$ -pole	$\times(w)$	$\Xi_c^{\prime+}(1^4P_\lambda)$ -pole	0		$\Xi_c^{\prime+}(1S)$ -pole	$\times(w)$	$\Xi_c^{\prime+}(1^4P_\lambda)$ -pole	0		
	$\Xi_c^{\prime+}(2S)$ -pole	$\times(w)$				$\Xi_c^{\prime+}(2S)$ -pole	$\times(w)$				
	Total	1.37	Total	-2.18		Total	14.04	Total	1.12		
$\Omega_{cc}^+ \rightarrow \Sigma_c^+ \bar{K}^0$	C' -diagram	0	C' -diagram	0	$\Omega_{cc}^+ \rightarrow \Lambda_c^+ \bar{K}^0$	C' -diagram	0.85	C' -diagram	-2.60		
	$\Xi_{cc}^+(1S)$ -pole	$\times(w)$	$\Xi_{cc}^+(1^2P_\rho)$ -pole	$\times(s)$		$\Xi_{cc}^+(1S)$ -pole	10.71	$\Xi_{cc}^+(1^2P_\rho)$ -pole	$\times(s)$		
	$\Xi_{cc}^+(2S)$ -pole	$\times(w)$	$\Xi_{cc}^+(1^2P_\lambda)$ -pole	0		$\Xi_{cc}^+(2S)$ -pole	0.57	$\Xi_{cc}^+(1^2P_\lambda)$ -pole	0.40		
	$\Xi_c^+(1S)$ -pole	12.13	$\Xi_{cc}^+(1^4P_\lambda)$ -pole	$\times(w)$		$\Xi_c^+(1S)$ -pole	$\times(s)$	$\Xi_{cc}^+(1^4P_\lambda)$ -pole	-15.66		
	$\Xi_c^+(2S)$ -pole	5.85	$\Xi_c^+(1^2P_\lambda)$ -pole	-0.12		$\Xi_c^+(2S)$ -pole	$\times(s)$	$\Xi_c^+(1^2P_\lambda)$ -pole	$\times(s)$		
	$\Xi_c^{\prime+}(1S)$ -pole	$\times(w)$	$\Xi_c^{\prime+}(1^2P_\lambda)$ -pole	0		$\Xi_c^{\prime+}(1S)$ -pole	$\times(w)$	$\Xi_c^{\prime+}(1^2P_\lambda)$ -pole	0		
	$\Xi_c^{\prime+}(2S)$ -pole	$\times(w)$	$\Xi_c^{\prime+}(1^4P_\lambda)$ -pole	0		$\Xi_c^{\prime+}(2S)$ -pole	$\times(w)$	$\Xi_c^{\prime+}(1^4P_\lambda)$ -pole	0		
	Total	17.98	Total	-0.12		Total	12.13	Total	-17.86		

TABLE VI. Same as Table IV but for DCS decays.

$\mathcal{M}_{\text{PC}}^{-1/2,-1/2}$				$\mathcal{M}_{\text{PV}}^{-1/2,-1/2}$				$\mathcal{M}_{\text{PC}}^{-1/2,-1/2}$				$\mathcal{M}_{\text{PV}}^{-1/2,-1/2}$			
$\Omega_{cc}^+ \rightarrow \Xi_c^0 K^+$	T -diagram	1.35		T -diagram	-2.73			$\Omega_{cc}^+ \rightarrow \Xi_c^{\prime 0} K^+$	T -diagram	-3.34		T -diagram	1.56		
	$\Lambda_c^+(1S)$ -pole	$\times(s)$		$\Lambda_c^+(1^2P_\lambda)$ -pole	$\times(s)$				$\Lambda_c^+(1S)$ -pole	-3.41		$\Lambda_c^+(1^2P_\lambda)$ -pole	0.028		
	$\Lambda_c^+(2S)$ -pole	$\times(s)$		$\Sigma_c^+(1^2P_\lambda)$ -pole	0				$\Lambda_c^+(2S)$ -pole	-1.64		$\Sigma_c^+(1^2P_\lambda)$ -pole	0		
	$\Sigma_c^+(1S)$ -pole	$\times(w)$		$\Sigma_c^+(1^4P_\lambda)$ -pole	0				$\Sigma_c^+(1S)$ -pole	$\times(w)$		$\Sigma_c^+(1^4P_\lambda)$ -pole	0		
	$\Sigma_c^+(2S)$ -pole	$\times(w)$							$\Sigma_c^+(2S)$ -pole	$\times(w)$					
Total		1.35		Total	-2.73			Total		-8.39		Total	1.59		
$\Omega_{cc}^+ \rightarrow \Xi_c^+ K^0$	C -diagram	-0.94		C -diagram	1.90			$\Omega_{cc}^+ \rightarrow \Xi_c^{\prime +} K^0$	C -diagram	2.33		C -diagram	-1.08		
	$\Lambda_c^+(1S)$ -pole	$\times(s)$		$\Lambda_c^+(1^2P_\lambda)$ -pole	$\times(s)$				$\Lambda_c^+(1S)$ -pole	3.41		$\Lambda_c^+(1^2P_\lambda)$ -pole	-0.028		
	$\Lambda_c^+(2S)$ -pole	$\times(s)$		$\Sigma_c^+(1^2P_\lambda)$ -pole	0				$\Lambda_c^+(2S)$ -pole	1.64		$\Sigma_c^+(1^2P_\lambda)$ -pole	0		
	$\Sigma_c^+(1S)$ -pole	$\times(w)$		$\Sigma_c^+(1^4P_\lambda)$ -pole	0				$\Sigma_c^+(1S)$ -pole	$\times(w)$		$\Sigma_c^+(1^4P_\lambda)$ -pole	0		
	$\Sigma_c^+(2S)$ -pole	$\times(w)$							$\Sigma_c^+(2S)$ -pole	$\times(w)$					
Total		-0.94		Total	1.90			Total		7.38		Total	-1.11		
$\Omega_{cc}^+ \rightarrow \Sigma_c^+ \pi^0$	$\Lambda_c^+(1S)$ -pole	4.40		$\Lambda_c^+(1^2P_\lambda)$ -pole	-0.044			$\Omega_{cc}^+ \rightarrow \Lambda_c^+ \pi^0$	$\Lambda_c^+(1S)$ -pole	$\times(s)$		$\Lambda_c^+(1^2P_\lambda)$ -pole	$\times(s)$		
	$\Lambda_c^+(2S)$ -pole	2.64		$\Sigma_c^+(1^2P_\lambda)$ -pole	$\times(s)$				$\Lambda_c^+(2S)$ -pole	$\times(s)$		$\Sigma_c^+(1^2P_\lambda)$ -pole	0		
	$\Sigma_c^+(1S)$ -pole	$\times(w)$		$\Sigma_c^+(1^4P_\lambda)$ -pole	0				$\Sigma_c^+(1S)$ -pole	$\times(w)$		$\Sigma_c^+(1^4P_\lambda)$ -pole	0		
	$\Sigma_c^+(2S)$ -pole	$\times(w)$							$\Sigma_c^+(2S)$ -pole	$\times(w)$					
Total		7.04		Total	-0.044			Total		0		Total	0		
$\Omega_{cc}^+ \rightarrow \Sigma_c^0 \pi^+$	$\Lambda_c^+(1S)$ -pole	4.40		$\Lambda_c^+(1^2P_\lambda)$ -pole	-0.044			$\Omega_{cc}^+ \rightarrow \Sigma_c^{++} \pi^-$	$\Lambda_c^+(1S)$ -pole	-4.40		$\Lambda_c^+(1^2P_\lambda)$ -pole	0.044		
	$\Lambda_c^+(2S)$ -pole	2.64		$\Sigma_c^+(1^2P_\lambda)$ -pole	0				$\Lambda_c^+(2S)$ -pole	-2.64		$\Sigma_c^+(1^2P_\lambda)$ -pole	0		
	$\Sigma_c^+(1S)$ -pole	$\times(w)$		$\Sigma_c^+(1^4P_\lambda)$ -pole	0				$\Sigma_c^+(1S)$ -pole	$\times(w)$		$\Sigma_c^+(1^4P_\lambda)$ -pole	0		
	$\Sigma_c^+(2S)$ -pole	$\times(w)$							$\Sigma_c^+(2S)$ -pole	$\times(w)$					
Total		7.04		Total	-0.044			Total		-7.04		Total	0.044		
$\Omega_{cc}^+ \rightarrow \Sigma_c^+ \eta$	C' -diagram	0		C' -diagram	0			$\Omega_{cc}^+ \rightarrow \Lambda_c^+ \eta$	C' -diagram	-0.068		C' -diagram	0.37		
	$\Omega_{cc}^+(1S)$ -pole	$\times(w)$		$\Omega_{cc}^+(1^2P_\rho)$ -pole	$\times(s)$				$\Omega_{cc}^+(1S)$ -pole	-2.16		$\Omega_{cc}^+(1^2P_\rho)$ -pole	$\times(s)$		
	$\Omega_{cc}^+(2S)$ -pole	$\times(w)$		$\Omega_{cc}^+(1^2P_\lambda)$ -pole	0				$\Omega_{cc}^+(2S)$ -pole	-0.16		$\Omega_{cc}^+(1^2P_\lambda)$ -pole	-0.077		
	$\Lambda_c^+(1S)$ -pole	$\times(s)$		$\Omega_{cc}^+(1^4P_\lambda)$ -pole	$\times(w)$				$\Lambda_c^+(1S)$ -pole	$\times(s)$		$\Omega_{cc}^+(1^4P_\lambda)$ -pole	1.66		
	$\Lambda_c^+(2S)$ -pole	$\times(s)$		$\Lambda_c^+(1^2P_\lambda)$ -pole	$\times(s)$				$\Lambda_c^+(2S)$ -pole	$\times(s)$		$\Lambda_c^+(1^2P_\lambda)$ -pole	$\times(s)$		
	$\Sigma_c^+(1S)$ -pole	$\times(w)$		$\Sigma_c^+(1^2P_\lambda)$ -pole	0				$\Sigma_c^+(1S)$ -pole	$\times(w)$		$\Sigma_c^+(1^2P_\lambda)$ -pole	$\times(s)$		
	$\Sigma_c^+(2S)$ -pole	$\times(w)$		$\Sigma_c^+(1^4P_\lambda)$ -pole	0				$\Sigma_c^+(2S)$ -pole	$\times(w)$		$\Sigma_c^+(1^4P_\lambda)$ -pole	0		
Total		0		Total	0			Total		-2.39		Total	1.95		
$\Omega_{cc}^+ \rightarrow \Sigma_c^+ \eta'$	C' -diagram	0		C' -diagram	0			$\Omega_{cc}^+ \rightarrow \Lambda_c^+ \eta'$	C' -diagram	0.082		C' -diagram	-0.49		
	$\Omega_{cc}^+(1S)$ -pole	$\times(w)$		$\Omega_{cc}^+(1^2P_\rho)$ -pole	$\times(s)$				$\Omega_{cc}^+(1S)$ -pole	2.86		$\Omega_{cc}^+(1^2P_\rho)$ -pole	$\times(s)$		
	$\Omega_{cc}^+(2S)$ -pole	$\times(w)$		$\Omega_{cc}^+(1^2P_\lambda)$ -pole	0				$\Omega_{cc}^+(2S)$ -pole	0.14		$\Omega_{cc}^+(1^2P_\lambda)$ -pole	0.093		
	$\Lambda_c^+(1S)$ -pole	$\times(s)$		$\Omega_{cc}^+(1^4P_\lambda)$ -pole	$\times(w)$				$\Lambda_c^+(1S)$ -pole	$\times(s)$		$\Omega_{cc}^+(1^4P_\lambda)$ -pole	-1.87		
	$\Lambda_c^+(2S)$ -pole	$\times(s)$		$\Lambda_c^+(1^2P_\lambda)$ -pole	$\times(s)$				$\Lambda_c^+(2S)$ -pole	$\times(s)$		$\Lambda_c^+(1^2P_\lambda)$ -pole	$\times(s)$		
	$\Sigma_c^+(1S)$ -pole	$\times(w)$		$\Sigma_c^+(1^2P_\lambda)$ -pole	0				$\Sigma_c^+(1S)$ -pole	$\times(w)$		$\Sigma_c^+(1^2P_\lambda)$ -pole	$\times(s)$		
	$\Sigma_c^+(2S)$ -pole	$\times(w)$		$\Sigma_c^+(1^4P_\lambda)$ -pole	0				$\Sigma_c^+(2S)$ -pole	$\times(w)$		$\Sigma_c^+(1^4P_\lambda)$ -pole	0		
Total		0		Total	0			Total		3.08		Total	-2.27		

parameters α for two-body nonleptonic decays of Ω_{cc}^+ . The decay width can be obtained using

$$\Gamma(\mathcal{B}_i \rightarrow \mathcal{B}_f P) = \frac{8\pi^2}{2J_i + 1} \frac{k E_f E_P}{m_i} \sum_{s_z^f, s_z^i} (|\mathcal{M}_{\text{PC}}^{s_z^f, s_z^i}|^2 + |\mathcal{M}_{\text{PV}}^{s_z^f, s_z^i}|^2), \quad (3.4)$$

where m_i is the mass of particle $\mathcal{B}_i$, E_f and E_P are the energies of $\mathcal{B}_f$ and P , respectively, and J is the total spin of

the initial state $\mathcal{B}_i$. At the same time, we also evaluate the asymmetry parameter α , defined by

$$\alpha = \frac{2\text{Re}[(\mathcal{M}_{\text{PV}}^{-1/2,-1/2})^* \mathcal{M}_{\text{PC}}^{-1/2,-1/2}]}{|\mathcal{M}_{\text{PC}}^{-1/2,-1/2}|^2 + |\mathcal{M}_{\text{PV}}^{-1/2,-1/2}|^2}. \quad (3.5)$$

We present our computed results together with those from other theoretical studies for a comprehensive comparison in Table VIII. In general, the CF modes exhibit larger

TABLE VII. Comparison of factorizable and nonfactorizable amplitudes (in units of $10^{-2}G_F \text{ GeV}^{-2}$), as well as the branching fractions and asymmetry parameters α of CF decays by various theoretical works. It should be mentioned that the lifetime of Ω_{cc}^+ is chosen as $\tau_{\Omega_{cc}^+} = 128 \text{ fs}$ in this table.

$\Omega_{cc}^+ \rightarrow \Omega_{cc}^0 \pi^+$ process	A^{fac}	A^{nf}	A^{tot}	B^{fac}	B^{nf}	B^{tot}	$\mathcal{B}$	α
This work	4.78	0	4.78	-71.52	0	-71.52	4.41%	-0.70
Ref. [33]	5.71	0	5.71	-67.48	0	-67.48	3.96%	-0.83
Ref. [32] ^a NRQM	6.25	0	6.25	-81.75	0	-81.75	6.02%	-0.77
HQET	7.32	0	7.32	-95.32	0	-95.32	8.19%	-0.77
Ref. [34]	-4.11	0	-4.11	32.96	0	32.96	1.33%	-0.96
Ref. [54]	$-7.39^{+0.07}_{-0.07}$	0	$-7.39^{+0.07}_{-0.07}$	$61.60^{+0.57}_{-0.57}$	0	$61.60^{+0.57}_{-0.57}$	$(6.81^{+0.13}_{-0.13})\%$	-0.96
$\Omega_{cc}^+ \rightarrow \Xi_c^+ \bar{K}^0$ process								
This work	-8.45	19.05	10.60	22.03	-75.88	-53.85	6.11%	-1.00
Ref. [35]	-4.02	12.17	8.15	6.20	-19.23	-13.02	1.98%	-0.54
Ref. [33]	2.62	-8.90	-6.28	-5.29	13.40	8.11	1.15%	-0.45
Ref. [32] ^a NRQM + FI	3.43	0	3.43	-8.13	-25.26	-33.39	1.47%	-0.84
HQET + FI	5.58	0	5.58	-11.55	-25.26	-36.81	2.26%	-0.97
NRQM + FD	3.43	0	3.43	-8.13	-53.00	-61.13	4.14%	-0.54
HQET + FD	5.58	0	5.58	-11.55	-53.00	-64.56	5.13%	-0.75
Ref. [54]	$0.12^{+0.00}_{-0.00}$	$(2.41^{+1.13}_{-0.98} - 4.31^{+1.79}_{-2.14}i)$	$(2.53^{+1.13}_{-0.98} - 4.31^{+1.79}_{-2.14}i)$	$-0.96^{+0.01}_{-0.01}$	$(3.36^{+1.24}_{-1.23} + 11.10^{+4.39}_{-5.21}i)$	$(2.40^{+1.24}_{-1.23} + 11.10^{+4.39}_{-5.21}i)$	$(1.33^{+1.56}_{-0.89})\%$	$-0.54^{+0.01}_{-0.03}$
$\Omega_{cc}^+ \rightarrow \Xi_c'^+ \bar{K}^0$ process								
This work	4.64	0	4.64	-59.24	0	-59.24	3.21%	-0.77
Ref. [35]	2.26	-0.11	2.14	-17.34	0.69	-16.64	0.31%	-0.97
Ref. [33]	-1.68	-0.04	-1.72	17.44	0.06	17.50	0.29%	-0.88
Ref. [32] ^a NRQM	-2.15	0	-2.15	26.9	0	26.9	0.67%	-0.78
HQET	-2.96	0	-2.96	37.6	0	37.6	1.30%	-0.77
Ref. [54]	$0.08^{+0.00}_{-0.00}$	$(0.99^{+0.53}_{-0.44} - 1.63^{+0.64}_{-0.73}i)$	$(1.10^{+0.53}_{-0.44} - 1.63^{+0.64}_{-0.73}i)$	$-0.68^{+0.01}_{-0.01}$	$(5.86^{+2.08}_{-2.05} + 8.35^{+3.00}_{-3.39}i)$	$(4.90^{+2.08}_{-2.05} + 8.35^{+3.00}_{-3.39}i)$	$(0.27^{+0.28}_{-0.16})\%$	$-0.42^{+0.02}_{-0.04}$

^a“NRQM” and “HQET” represent that the factorizable amplitudes are obtained using the nonrelativistic quark model and heavy quark effective theory, respectively, and “FI” and “FD” represent that the pole amplitudes are obtained without or with considering the flavor-dependent effects, respectively.

TABLE VIII. Branching fractions and up-down decay asymmetry parameters α of two-body nonleptonic decays of the double-charmed baryon Ω_{cc}^+ . It should be noted that the lifetime of Ω_{cc}^+ is adopted as 128 fs in this work, 300 fs in Ref. [32], and 206 fs in Ref. [54].

	This work		Ref. [32]		Ref. [54]	
	$\mathcal{B}$	α	$\mathcal{B}$	α	$\mathcal{B}$	α
CF decays						
$\Omega_{cc}^+ \rightarrow \Omega_{cc}^0 \pi^+$	4.41%	-0.70	(12.6, 17.2)%	(-0.794, -0.800)	$(6.81^{+0.13}_{-0.13})\%$	-0.96
$\Omega_{cc}^+ \rightarrow \Xi_c^+ \bar{K}^0$	6.11%	-1.00	(3.1, 4.9)%	(-0.855, -0.982)	$(1.33^{+1.56}_{-0.89})\%$	$-0.54^{+0.01}_{-0.03}$
$\Omega_{cc}^+ \rightarrow \Xi_c'^+ \bar{K}^0$	3.21%	-0.77	(1.4, 2.7)%	(-0.791, -0.789)	$(2.74^{+2.83}_{-1.66}) \times 10^{-3}$	$-0.42^{+0.02}_{-0.04}$
SCS decays						
$\Omega_{cc}^+ \rightarrow \Xi_c^0 \pi^+$	0.45×10^{-3}	-0.90	$(1.6, 3.7) \times 10^{-3}$	(0.640, 0.198)	$(2.99^{+0.34}_{-0.26}) \times 10^{-3}$	$-0.34^{+0.07}_{-0.07}$
$\Omega_{cc}^+ \rightarrow \Xi_c'^0 \pi^+$	4.80×10^{-3}	0.26	$(1.6, 3.9) \times 10^{-3}$	(-0.934, -0.871)	$(0.96^{+0.19}_{-0.10}) \times 10^{-3}$	$-0.74^{+0.37}_{-0.19}$
$\Omega_{cc}^+ \rightarrow \Omega_{cc}^0 K^+$	5.09%	-0.41	(1.2, 1.5)%	(-0.759, -0.780)	$(5.03^{+0.35}_{-0.29}) \times 10^{-3}$	$-0.98^{+0.00}_{-0.00}$

(Table continued)

TABLE VIII. (Continued)

	This work		Ref. [32]		Ref. [54]	
	$\mathcal{B}$	α	$\mathcal{B}$	α	$\mathcal{B}$	α
$\Omega_{cc}^+ \rightarrow \Sigma_c^{++} K^-$	1.04%	-0.013	8.0×10^{-4}	—	$(1.67_{-1.07}^{+1.91}) \times 10^{-4}$	$0.33_{-0.24}^{+0.18}$
$\Omega_{cc}^+ \rightarrow \Xi_c^+ \eta$	1.01×10^{-3}	-0.35	$(0.86, 1.7) \times 10^{-3}$	$(-0.997, -0.970)$		
$\Omega_{cc}^+ \rightarrow \Xi_c'^+ \eta$	1.62%	-0.24	$(2.0, 3.0) \times 10^{-3}$	$(-0.516, -0.578)$		
$\Omega_{cc}^+ \rightarrow \Xi_c^+ \eta'$	1.76×10^{-3}	0.074	$(8.4, 0.14) \times 10^{-3}$	$(0.995, 0.892)$		
$\Omega_{cc}^+ \rightarrow \Xi_c'^+ \eta'$	0.66×10^{-3}	-0.98	$(4.8, 7.8) \times 10^{-5}$	$(-0.856, -0.906)$		
$\Omega_{cc}^+ \rightarrow \Xi_c^+ \pi^0$	0.11×10^{-3}	-0.90	$(4.2, 5.3) \times 10^{-4}$	$(0.893, 0.992)$	$(2.65_{-1.75}^{+3.03}) \times 10^{-4}$	$0.28_{-0.07}^{+0.08}$
$\Omega_{cc}^+ \rightarrow \Xi_c'^+ \pi^0$	2.96×10^{-3}	0.16	$(0.43, 1.3) \times 10^{-4}$	$(-0.320, -0.972)$	$(3.01_{-2.05}^{+3.72}) \times 10^{-4}$	$-0.24_{-0.07}^{+0.07}$
$\Omega_{cc}^+ \rightarrow \Sigma_c^+ \bar{K}^0$	5.20×10^{-3}	-0.013	4.0×10^{-4}	—	$(2.09_{-1.39}^{+2.43}) \times 10^{-4}$	$-0.37_{-0.08}^{+0.04}$
$\Omega_{cc}^+ \rightarrow \Lambda_c^+ \bar{K}^0$	8.83×10^{-3}	-0.93	4.0×10^{-3}	—	$(3.93_{-2.57}^{+4.59}) \times 10^{-4}$	$0.34_{-0.20}^{+0.15}$
DCS decays						
$\Omega_{cc}^+ \rightarrow \Xi_c^0 K^+$	1.47×10^{-4}	-0.79	$(1.3, 3.3) \times 10^{-4}$	$(0.100, -0.162)$	$(0.17_{-0.01}^{+0.01}) \times 10^{-3}$	$-0.50_{-0.01}^{+0.01}$
$\Omega_{cc}^+ \rightarrow \Xi_c'^0 K^+$	1.01×10^{-3}	-0.37	$(3.3, 6.0) \times 10^{-4}$	$(-0.684, -0.711)$	$(0.13_{-0.00}^{+0.01}) \times 10^{-3}$	$-0.85_{-0.08}^{+0.10}$
$\Omega_{cc}^+ \rightarrow \Xi_c^+ K^0$	0.71×10^{-4}	-0.79	$(3.2, 6.1) \times 10^{-5}$	$(0.962, 0.634)$	$(2.10_{-1.27}^{+2.05}) \times 10^{-5}$	$-0.32_{-0.07}^{+0.08}$
$\Omega_{cc}^+ \rightarrow \Xi_c'^+ K^0$	7.72×10^{-4}	-0.29	$(0.85, 1.4) \times 10^{-4}$	$(-0.564, -0.615)$	$(1.32_{-0.89}^{+1.70}) \times 10^{-5}$	$0.25_{-0.17}^{+0.12}$
$\Omega_{cc}^+ \rightarrow \Sigma_c^+ \pi^0$	8.46×10^{-4}	-0.012	3.7×10^{-5}	—	$(0.88_{-0.59}^{+1.05}) \times 10^{-5}$	$0.93_{-0.09}^{+0.02}$
$\Omega_{cc}^+ \rightarrow \Lambda_c^+ \pi^0$	0	—			$(0.43_{-0.27}^{+0.46}) \times 10^{-5}$	$0.17_{-0.30}^{+0.26}$
$\Omega_{cc}^+ \rightarrow \Sigma_c^0 \pi^+$	8.46×10^{-4}	-0.012	3.7×10^{-5}	—	$(1.34_{-0.90}^{+1.55}) \times 10^{-5}$	$0.99_{-0.01}^{+0.01}$
$\Omega_{cc}^+ \rightarrow \Sigma_c^{++} \pi^-$	8.46×10^{-4}	-0.012	3.7×10^{-5}	—	$(0.69_{-0.45}^{+0.83}) \times 10^{-5}$	$0.49_{-0.24}^{+0.15}$
$\Omega_{cc}^+ \rightarrow \Sigma_c^+ \eta$	0	—				
$\Omega_{cc}^+ \rightarrow \Lambda_c^+ \eta$	1.78×10^{-4}	-0.98	1.2×10^{-4}	—		
$\Omega_{cc}^+ \rightarrow \Sigma_c^+ \eta'$	0	—				
$\Omega_{cc}^+ \rightarrow \Lambda_c^+ \eta'$	2.29×10^{-4}	-0.96	4.2×10^{-7}	—		

branching fractions, whereas the SCS and DCS modes are suppressed by CKM matrix elements. Our results indicate that the branching fractions of the CF decays $\Omega_{cc}^+ \rightarrow \Omega_c^0 \pi^+$ and $\Xi_c^{(\prime)+} \pi^+$ reach up to a few percent. Interestingly, several SCS channels— $\Omega_{cc}^+ \rightarrow \Omega_c^0 K^+$, $\Sigma_{cc}^{++} K^-$, $\Xi_c'^+ \eta$, and $\Lambda_c^+ \bar{K}^0$ —can also reach the few-percent level. Particularly for the $\Omega_{cc}^+ \rightarrow \Omega_c^0 K^+$ decay, owing to the large factorizable amplitude and constructive pole contributions, the branching fraction is comparable to that of the CF decays. Besides, the purely W -exchanged process $\Omega_{cc}^+ \rightarrow \Sigma_c^{++} K^-$ has large nonfactorizable contributions from $\Xi_c^+(nS)$ poles, which is different to the result of Ref. [32]. These decay channels can serve as the discovery channels for the double-charmed baryon Ω_{cc}^+ in future experiments, like LHCb and Belle II.

According to previous theoretical studies of meson [55–60] and baryon [14,54,61–68] nonleptonic decays, the FSI effects, as an important source of long-distance contributions, may be non-negligible. Particularly for our

concerned $\Omega_{cc} \rightarrow \mathcal{B}_c P$ processes, the FSI effects were systematically considered in Ref. [54]. In their theoretical framework, the factorizable amplitudes arising from T and C diagrams were computed using the factorizable method, while the nonfactorizable long-distance contributions were evaluated by considering the rescattering processes $\Omega_{cc} \rightarrow \mathcal{B}_c' P'(V) \rightarrow \mathcal{B}_c P$. For the channels that are contributed by the T diagram, the factorizable contributions are dominated, and the branching fractions of long-distance contributions are suppressed by $10^{-2} - 10^{-1}$ in magnitude, while for other channels lacking a T diagram, the long-distance contributions are dominated. In our computation, the decay amplitudes of $\Omega_{cc}^+ \rightarrow \Lambda_c^+ \pi^0$ and $\Sigma_c^+ \eta^{(\prime)}$ are vanishing in the framework of the pole model. However, after considering the long-distance contributions from FSI effects, they predicted that these channels will have considerable branching fractions with an order of magnitude of 10^{-5} [54]. Future experimental measurements of these

channels will help reveal the long-distance contributions in weak decay mechanisms of charmed baryons.

IV. CONCLUSION

The discovery of double-charmed baryons is a forefront research topic in heavy-flavor physics. As their ground states are prohibited from decaying via the strong and electromagnetic interactions while they can only decay via the weak interaction, a comprehensive theoretical investigation of their weak decay modes is essential for guiding future experimental searches. However, due to the non-perturbative nature of low-energy QCD and the complex dynamics in weak decays, particularly the nonfactorizable contributions, the theoretical study of nonleptonic decays remains challenging.

In this paper, we presented a theoretical study of the two-body nonleptonic decays $\Omega_{cc}^+ \rightarrow \mathcal{B}_c + P$, where $\mathcal{B}_c$ denotes a single-charmed baryon and P denotes a light-flavor pseudoscalar meson, such as π , K , or $\eta^{(\prime)}$. The relevant decay amplitudes were calculated within the framework of the nonrelativistic quark model, and in particular the nonfactorizable amplitudes contributed from W -exchange diagrams were evaluated under the pole model assumption. Moreover, in order to reduce the dependence of decay amplitudes on phenomenological parameters, especially those in the baryon wave functions, we adopted the charmed-baryon wave functions obtained from a non-relativistic quark potential model.

With the obtained decay amplitudes, we investigated the corresponding branching fractions. In our calculation, the

branching fractions of CF decays $\Omega_{cc}^+ \rightarrow \Omega_c^0 \pi^+$ and $\Xi_c^{(\prime)+} \pi^+$, together with several SCS decays $\Omega_{cc}^+ \rightarrow \Omega_c^0 K^+$, $\Sigma_{cc}^{++} K^-$, $\Xi_c^{'+} \eta$, and $\Lambda_c^+ \bar{K}^0$, can reach up to a few percent. Notably, the branching fraction of the SCS mode $\Omega_{cc}^+ \rightarrow \Omega_c^0 K^+$ is comparable to those of CF decays, owing to large constructive pole contributions. These decay channels can serve as the discovery channels for the double-charmed baryon Ω_{cc}^+ in future experiments, such as LHCb and Belle II.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [32,33,34,54].

APPENDIX A: SPIN-FLAVOR MATRIX ELEMENTS IN W -EMISSION PROCESSES

In this appendix, the spin-flavor matrix elements in W -emission processes are presented. Before calculating the concerned values, we give the spin-flavor wave functions of concerned charmed baryons. The spin-flavor wave functions of S -wave double-charmed baryons are chosen as [69]

$$|\mathcal{B}_{ccq}\rangle = -\frac{1}{\sqrt{3}}[ccq\chi_S + (13) + (23)], \quad (\text{A1})$$

and that of S -wave single-charmed baryons are chosen as

$$\begin{aligned} |\Omega_c^0\rangle &= \frac{1}{\sqrt{3}}[ssc\chi_S + (13) + (23)], & |\Lambda_c^+\rangle &= \frac{1}{\sqrt{6}}[(udc - duc)\chi_A + (13) + (23)], \\ |\Xi_c^+\rangle &= \frac{1}{\sqrt{6}}[(usc - suc)\chi_A + (13) + (23)], & |\Xi_c^{\prime+}\rangle &= \frac{1}{\sqrt{6}}[(usc + suc)\chi_S + (13) + (23)], \\ |\Xi_c^0\rangle &= \frac{1}{\sqrt{6}}[(dsc - sdc)\chi_A + (13) + (23)], & |\Xi_c^{\prime0}\rangle &= \frac{1}{\sqrt{6}}[(dsc + sdc)\chi_S + (13) + (23)], \\ |\Sigma_c^{++}\rangle &= \frac{1}{\sqrt{3}}[uuc\chi_S + (13) + (23)], & |\Sigma_c^+\rangle &= \frac{1}{\sqrt{6}}[(udc + duc)\chi_S + (13) + (23)], \\ |\Sigma_c^0\rangle &= \frac{1}{\sqrt{3}}[ddc\chi_S + (13) + (23)]. \end{aligned} \quad (\text{A2})$$

Here, we have

TABLE IX. Spin-flavor matrix elements of PC (top panel) and PV (bottom panel) amplitudes for T diagrams. For simplicity, the spin and flavor wave functions of pseudoscalar meson are omitted here.

Matrix elements	$\langle \Omega_c^0 \downarrow \hat{O} \Omega_{cc}^+ \downarrow \rangle$	$\langle \Xi_c^0 \downarrow \hat{O} \Omega_{cc}^+ \downarrow \rangle$	$\langle \Xi_c^{\prime 0} \downarrow \hat{O} \Omega_{cc}^+ \downarrow \rangle$
$\langle s'_{z,1} I s_{z,1} \rangle \langle s_{z,5} \bar{s}_{z,4} \sigma_z 0 \rangle$	0	0	0
$\langle s'_{z,1} \sigma_z s_{z,1} \rangle \langle s_{z,5} \bar{s}_{z,4} I 0 \rangle$	$\frac{5\sqrt{2}}{9}$	$-\frac{\sqrt{3}}{9}$	$\frac{5}{9}$
$(\langle s'_{z,1} \sigma s_{z,1} \rangle \times \langle s_{z,5} \bar{s}_{z,4} \sigma 0 \rangle)_z$	0	0	0
$\langle s_{z,5} I s_{z,1} \rangle \langle s'_{z,1} \bar{s}_{z,4} \sigma_z 0 \rangle$	$\frac{5\sqrt{2}}{18}$	$-\frac{\sqrt{3}}{18}$	$\frac{5}{18}$
$\langle s_{z,5} \sigma_z s_{z,1} \rangle \langle s'_{z,1} \bar{s}_{z,4} I 0 \rangle$	$\frac{5\sqrt{2}}{18}$	$-\frac{\sqrt{3}}{18}$	$\frac{5}{18}$
$(\langle s_{z,5} \sigma s_{z,1} \rangle \times \langle s'_{z,1} \bar{s}_{z,4} \sigma 0 \rangle)_z$	$-\frac{5\sqrt{2}i}{9}$	$\frac{\sqrt{3}i}{9}$	$-\frac{5i}{9}$
$\langle s'_{z,1} I s_{z,1} \rangle \langle s_{z,5} \bar{s}_{z,4} I 0 \rangle$	$-\frac{\sqrt{2}}{3}$	$\frac{\sqrt{3}}{3}$	$-\frac{1}{3}$
$\langle s'_{z,1} \sigma_x s_{z,1} \rangle \langle s_{z,5} \bar{s}_{z,4} \sigma_x 0 \rangle$	0	0	0
$\langle s'_{z,1} \sigma_y s_{z,1} \rangle \langle s_{z,5} \bar{s}_{z,4} \sigma_y 0 \rangle$	0	0	0
$\langle s'_{z,1} \sigma_z s_{z,1} \rangle \langle s_{z,5} \bar{s}_{z,4} \sigma_z 0 \rangle$	0	0	0
$\langle s_{z,5} I s_{z,1} \rangle \langle s'_{z,1} \bar{s}_{z,4} I 0 \rangle$	$-\frac{\sqrt{2}}{6}$	$\frac{\sqrt{3}}{6}$	$-\frac{1}{6}$
$\langle s_{z,5} \sigma_x s_{z,1} \rangle \langle s'_{z,1} \bar{s}_{z,4} \sigma_x 0 \rangle$	$-\frac{\sqrt{2}}{6}$	$\frac{\sqrt{3}}{6}$	$-\frac{1}{6}$
$\langle s_{z,5} \sigma_y s_{z,1} \rangle \langle s'_{z,1} \bar{s}_{z,4} \sigma_y 0 \rangle$	$-\frac{\sqrt{2}}{6}$	$\frac{\sqrt{3}}{6}$	$-\frac{1}{6}$
$\langle s_{z,5} \sigma_z s_{z,1} \rangle \langle s'_{z,1} \bar{s}_{z,4} \sigma_z 0 \rangle$	$-\frac{\sqrt{2}}{6}$	$\frac{\sqrt{3}}{6}$	$-\frac{1}{6}$

$$abc\chi_S^\downarrow = \frac{1}{\sqrt{6}}(-2a^\downarrow b^\downarrow c^\uparrow + a^\uparrow b^\downarrow c^\downarrow + a^\downarrow b^\uparrow c^\downarrow),$$

$$abc\chi_A^\downarrow = \frac{1}{\sqrt{2}}(a^\uparrow b^\uparrow c^\downarrow - a^\downarrow b^\uparrow c^\downarrow) \quad (A3)$$

for the baryons of $|S, S_z\rangle = |1/2, -1/2\rangle$, and

$$abc\chi_S^\uparrow = \frac{1}{\sqrt{6}}(2a^\uparrow b^\uparrow c^\downarrow - a^\uparrow b^\downarrow c^\uparrow - a^\downarrow b^\uparrow c^\uparrow),$$

$$abc\chi_A^\uparrow = \frac{1}{\sqrt{2}}(a^\uparrow b^\downarrow c^\uparrow - a^\downarrow b^\uparrow c^\uparrow) \quad (A4)$$

for that of $|S, S_z\rangle = |1/2, +1/2\rangle$. Besides, the spin wave function of a pseudoscalar meson is adopted as $\chi_P = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$. For calculating the spin matrix element, the particle-hole conjugation $\langle j, -m | \rightarrow (-1)^{j+m} | j, m \rangle$ is useful. This relation makes the antiquark spin transform as $\langle \uparrow | \rightarrow | \downarrow \rangle$ and $\langle \downarrow | \rightarrow -| \uparrow \rangle$.

The spin-flavor matrix elements $\langle \mathcal{B}_f \downarrow | \hat{O} | \mathcal{B}_i \downarrow \rangle$ associated with W -emission processes are presented in Tables IX–XI, which contribute to the PC and PV amplitudes in the T , C , and C' diagrams, respectively. The values of $\langle \mathcal{B}_f \uparrow | \hat{O} | \mathcal{B}_i \uparrow \rangle$, $\langle \mathcal{B}_f \uparrow | \hat{O} | \mathcal{B}_i \downarrow \rangle$, and $\langle \mathcal{B}_f \downarrow | \hat{O} | \mathcal{B}_i \uparrow \rangle$ can also be obtained.

TABLE X. Same as Table IX but for C diagrams.

Matrix elements	$\langle \Xi_c^+ \downarrow \hat{O} \Omega_{cc}^+ \downarrow \rangle$	$\langle \Xi_c^{\prime +} \downarrow \hat{O} \Omega_{cc}^+ \downarrow \rangle$
$\langle s'_{z,1} I s_{z,1} \rangle \langle s_{z,5} \bar{s}_{z,4} \sigma_z 0 \rangle$	$-\frac{\sqrt{3}}{18}$	$\frac{5}{18}$
$\langle s'_{z,1} \sigma_z s_{z,1} \rangle \langle s_{z,5} \bar{s}_{z,4} I 0 \rangle$	$-\frac{\sqrt{3}}{18}$	$\frac{5}{18}$
$(\langle s'_{z,1} \sigma s_{z,1} \rangle \times \langle s_{z,5} \bar{s}_{z,4} \sigma 0 \rangle)_z$	$\frac{\sqrt{3}i}{9}$	$-\frac{5i}{9}$
$\langle s_{z,5} I s_{z,1} \rangle \langle s'_{z,1} \bar{s}_{z,4} \sigma_z 0 \rangle$	0	0
$\langle s_{z,5} \sigma_z s_{z,1} \rangle \langle s'_{z,1} \bar{s}_{z,4} I 0 \rangle$	$-\frac{\sqrt{3}}{9}$	$\frac{5}{9}$
$(\langle s_{z,5} \sigma s_{z,1} \rangle \times \langle s'_{z,1} \bar{s}_{z,4} \sigma 0 \rangle)_z$	0	0
$\langle s'_{z,1} I s_{z,1} \rangle \langle s_{z,5} \bar{s}_{z,4} I 0 \rangle$	$\frac{\sqrt{3}}{6}$	$-\frac{1}{6}$
$\langle s'_{z,1} \sigma_x s_{z,1} \rangle \langle s_{z,5} \bar{s}_{z,4} \sigma_x 0 \rangle$	$\frac{\sqrt{3}}{6}$	$-\frac{1}{6}$
$\langle s'_{z,1} \sigma_y s_{z,1} \rangle \langle s_{z,5} \bar{s}_{z,4} \sigma_y 0 \rangle$	$\frac{\sqrt{3}}{6}$	$-\frac{1}{6}$
$\langle s'_{z,1} \sigma_z s_{z,1} \rangle \langle s_{z,5} \bar{s}_{z,4} \sigma_z 0 \rangle$	$\frac{\sqrt{3}}{6}$	$-\frac{1}{6}$
$\langle s_{z,5} I s_{z,1} \rangle \langle s'_{z,1} \bar{s}_{z,4} I 0 \rangle$	$\frac{\sqrt{3}}{3}$	$-\frac{1}{3}$
$\langle s_{z,5} \sigma_x s_{z,1} \rangle \langle s'_{z,1} \bar{s}_{z,4} \sigma_x 0 \rangle$	0	0
$\langle s_{z,5} \sigma_y s_{z,1} \rangle \langle s'_{z,1} \bar{s}_{z,4} \sigma_y 0 \rangle$	0	0
$\langle s_{z,5} \sigma_z s_{z,1} \rangle \langle s'_{z,1} \bar{s}_{z,4} \sigma_z 0 \rangle$	0	0

TABLE XI. Same as Table IX but for C' diagrams.

Matrix elements	$\langle \Xi_c^+ \downarrow \hat{O} \Omega_{cc}^+ \downarrow \rangle$	$\langle \Xi_c'^+ \downarrow \hat{O} \Omega_{cc}^+ \downarrow \rangle$	$\langle \Sigma_c^+ \downarrow \hat{O} \Omega_{cc}^+ \downarrow \rangle$	$\langle \Lambda_c^+ \downarrow \hat{O} \Omega_{cc}^+ \downarrow \rangle$
$\langle s'_{z,1} I s_{z,1} \rangle \langle s_{z,5} \bar{s}_{z,4} \sigma_z 0 \rangle$	$-\frac{\sqrt{3}}{36}$	$-\frac{1}{36}$	$-\frac{1}{36}$	$-\frac{\sqrt{3}}{36}$
$\langle s'_{z,1} \sigma_z s_{z,1} \rangle \langle s_{z,5} \bar{s}_{z,4} I 0 \rangle$	$\frac{\sqrt{3}}{36}$	$\frac{5}{36}$	$\frac{5}{36}$	$\frac{\sqrt{3}}{36}$
$\langle (s'_{z,1} \sigma s_{z,1} \rangle \times \langle s_{z,5} \bar{s}_{z,4} \sigma 0 \rangle)_z$	$\frac{\sqrt{3}i}{18}$	$-\frac{i}{6}$	$-\frac{i}{6}$	$\frac{\sqrt{3}i}{18}$
$\langle s_{z,5} I s_{z,1} \rangle \langle s'_{z,1} \bar{s}_{z,4} \sigma_z 0 \rangle$	$\frac{\sqrt{3}}{36}$	$-\frac{1}{36}$	$-\frac{1}{36}$	$\frac{\sqrt{3}}{36}$
$\langle s_{z,5} \sigma_z s_{z,1} \rangle \langle s'_{z,1} \bar{s}_{z,4} I 0 \rangle$	$-\frac{\sqrt{3}}{36}$	$\frac{5}{36}$	$\frac{5}{36}$	$-\frac{\sqrt{3}}{36}$
$\langle (s_{z,5} \sigma s_{z,1} \rangle \times \langle s'_{z,1} \bar{s}_{z,4} \sigma 0 \rangle)_z$	$-\frac{\sqrt{3}i}{18}$	$-\frac{i}{6}$	$-\frac{i}{6}$	$-\frac{\sqrt{3}i}{18}$
$\langle s'_{z,1} I s_{z,1} \rangle \langle s_{z,5} \bar{s}_{z,4} I 0 \rangle$	$-\frac{\sqrt{3}}{12}$	$-\frac{1}{12}$	$-\frac{1}{12}$	$-\frac{\sqrt{3}}{12}$
$\langle s'_{z,1} \sigma_x s_{z,1} \rangle \langle s_{z,5} \bar{s}_{z,4} \sigma_x 0 \rangle$	$\frac{\sqrt{3}}{12}$	$-\frac{1}{36}$	$-\frac{1}{36}$	$\frac{\sqrt{3}}{12}$
$\langle s'_{z,1} \sigma_y s_{z,1} \rangle \langle s_{z,5} \bar{s}_{z,4} \sigma_y 0 \rangle$	$\frac{\sqrt{3}}{12}$	$-\frac{1}{36}$	$-\frac{1}{36}$	$\frac{\sqrt{3}}{12}$
$\langle s'_{z,1} \sigma_z s_{z,1} \rangle \langle s_{z,5} \bar{s}_{z,4} \sigma_z 0 \rangle$	$\frac{\sqrt{3}}{12}$	$-\frac{1}{36}$	$-\frac{1}{36}$	$\frac{\sqrt{3}}{12}$
$\langle s_{z,5} I s_{z,1} \rangle \langle s'_{z,1} \bar{s}_{z,4} I 0 \rangle$	$\frac{\sqrt{3}}{12}$	$-\frac{1}{12}$	$-\frac{1}{12}$	$\frac{\sqrt{3}}{12}$
$\langle s_{z,5} \sigma_x s_{z,1} \rangle \langle s'_{z,1} \bar{s}_{z,4} \sigma_x 0 \rangle$	$-\frac{\sqrt{3}}{12}$	$-\frac{1}{36}$	$-\frac{1}{36}$	$-\frac{\sqrt{3}}{12}$
$\langle s_{z,5} \sigma_y s_{z,1} \rangle \langle s'_{z,1} \bar{s}_{z,4} \sigma_y 0 \rangle$	$-\frac{\sqrt{3}}{12}$	$-\frac{1}{36}$	$-\frac{1}{36}$	$-\frac{\sqrt{3}}{12}$
$\langle s_{z,5} \sigma_z s_{z,1} \rangle \langle s'_{z,1} \bar{s}_{z,4} \sigma_z 0 \rangle$	$-\frac{\sqrt{3}}{12}$	$-\frac{1}{36}$	$-\frac{1}{36}$	$-\frac{\sqrt{3}}{12}$

APPENDIX B: SPIN-FLAVOR MATRIX ELEMENTS IN W -EXCHANGE PROCESSES

In this appendix, the spin-flavor matrix elements in W -exchange processes are presented. For the evaluation of these spin-flavor matrix elements, we employ the following spin-flavor wave functions for excited baryons. For λ -mode excited charmed baryons with total spin $|S, S_z\rangle = |1/2, \pm 1/2\rangle$, the spin-flavor wave functions $|\mathcal{B}\rangle_{\lambda}$ are taken to be the same as those of the S -wave states. In addition, for λ -mode excited double-charmed baryons with total spin $|S, S_z\rangle = |3/2, \pm 1/2\rangle$, we adopt

$$|\mathcal{B}_{ccq}\rangle_{\lambda} = -\frac{1}{\sqrt{3}}[ccq\chi_s + (13) + (23)], \quad (\text{B1})$$

while for λ -mode excited single-charmed baryons we adopt

$$\begin{aligned}
|\Omega_c^0\rangle_{\lambda} &= \frac{1}{\sqrt{3}}[ssc\chi_s + (13) + (23)], & |\Sigma_c^{++}\rangle_{\lambda} &= \frac{1}{\sqrt{3}}[uuc\chi_s + (13) + (23)], \\
|\Sigma_c^+\rangle_{\lambda} &= \frac{1}{\sqrt{6}}[(udc + duc)\chi_s + (13) + (23)], & |\Sigma_c^0\rangle_{\lambda} &= \frac{1}{\sqrt{3}}[ddc\chi_s + (13) + (23)], \\
|\Xi_c^{'+}\rangle_{\lambda} &= \frac{1}{\sqrt{6}}[(usc + suc)\chi_s + (13) + (23)], & |\Xi_c'^0\rangle_{\lambda} &= \frac{1}{\sqrt{6}}[(dsc + sdc)\chi_s + (13) + (23)],
\end{aligned} \quad (\text{B2})$$

where

$$\begin{aligned}
abc\chi_s^{\downarrow} &= \frac{1}{\sqrt{3}}(a^{\downarrow}b^{\downarrow}c^{\uparrow} + a^{\uparrow}b^{\downarrow}c^{\downarrow} + a^{\downarrow}b^{\uparrow}c^{\downarrow}), \\
abc\chi_s^{\uparrow} &= \frac{1}{\sqrt{3}}(a^{\downarrow}b^{\uparrow}c^{\uparrow} + a^{\uparrow}b^{\uparrow}c^{\downarrow} + a^{\uparrow}b^{\downarrow}c^{\uparrow}).
\end{aligned} \quad (\text{B3})$$

Besides, the spin-flavor wave functions of ρ -mode excited double-charmed baryons with total spin $|S, S_z\rangle = |1/2, \pm 1/2\rangle$ can be written as

TABLE XII. Spin-flavor matrix elements for PC amplitudes of W -exchange diagrams with $\mathcal{B}_{cc}^+ = (\Xi_{cc}^+, \Omega_{cc}^+)$.

Matrix elements	$\langle \Xi_c^+ \downarrow \hat{O} \mathcal{B}_{cc}^+ \downarrow \rangle$	$\langle \Xi_c'^+ \downarrow \hat{O} \mathcal{B}_{cc}^+ \downarrow \rangle$	$\langle \Lambda_c^+ \downarrow \hat{O} \mathcal{B}_{cc}^+ \downarrow \rangle$	$\langle \Sigma_c^+ \downarrow \hat{O} \mathcal{B}_{cc}^+ \downarrow \rangle$
$\hat{\alpha}_1^{(-)} \hat{\beta}_2^{(+)} (1 - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$	$\sqrt{\frac{2}{3}}$	0	$\sqrt{\frac{2}{3}}$	0
$\hat{\alpha}_1^{(-)} \hat{\beta}_2^{(+)} (1 - \tilde{\boldsymbol{\sigma}}_1 \cdot \tilde{\boldsymbol{\sigma}}_2)$	$\sqrt{\frac{2}{3}}$	0	$\sqrt{\frac{2}{3}}$	0

$$|\mathcal{B}_{ccq}\rangle_{2p} = -\frac{1}{\sqrt{3}}[ccq\chi_A + (13) + (23)]. \quad (\text{B4})$$

With the above preparations, the spin-flavor matrix elements for PC and PV amplitudes of W -exchange

diagrams can be obtained. The concerned values are summarized in Tables XII and XIII for PC and PV amplitudes, respectively. Besides, the matrix elements $\langle \mathcal{B}' \downarrow | \hat{I}_P \sigma_z | \mathcal{B} \downarrow \rangle$ of strong interaction vertices are presented in Tables XIV and XV.

TABLE XIII. Spin-flavor matrix elements for PV amplitudes of W -exchange diagrams, in which $\mathcal{B}_{cc}^+ = (\Xi_{cc}^+, \Omega_{cc}^+)$, and the subscripts ρ and λ represent ρ -mode and λ -mode excitations, respectively.

Matrix elements	$\langle \Xi_c^+ \downarrow \hat{O} \mathcal{B}_{cc}^+ \downarrow \rangle_{2\lambda}$	$\langle \Xi_c^+ \downarrow \hat{O} \mathcal{B}_{cc}^+ \downarrow \rangle_{2\rho}$	$\langle \Xi_c^+ \downarrow \hat{O} \mathcal{B}_{cc}^+ \downarrow \rangle_{4\lambda}$	$\langle \Xi_c'^+ \downarrow \hat{O} \mathcal{B}_{cc}^+ \downarrow \rangle_{2\lambda}$	$\langle \Xi_c'^+ \downarrow \hat{O} \mathcal{B}_{cc}^+ \downarrow \rangle_{2\rho}$	$\langle \Xi_c'^+ \downarrow \hat{O} \mathcal{B}_{cc}^+ \downarrow \rangle_{4\lambda}$
$-\hat{\alpha}_1^{(-)} \hat{\beta}_2^{(+)} (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)_z$	$\frac{1}{3\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{2}{3\sqrt{3}}$	$\frac{1}{3\sqrt{2}}$	$\frac{1}{3\sqrt{6}}$	0
$\hat{\alpha}_1^{(-)} \hat{\beta}_2^{(+)} (\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2)_z$	$-\frac{i}{3\sqrt{6}}$	$\frac{i}{3\sqrt{2}}$	$\frac{2i}{3\sqrt{3}}$	$\frac{i}{3\sqrt{2}}$	$\frac{i}{3\sqrt{6}}$	0
$-\hat{\alpha}_1^{(-)} \hat{\beta}_2^{(+)} (\tilde{\boldsymbol{\sigma}}_1 - \tilde{\boldsymbol{\sigma}}_2)_z$	$\frac{1}{3\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{2}{3\sqrt{3}}$	$\frac{1}{3\sqrt{2}}$	$\frac{1}{3\sqrt{6}}$	0
$\hat{\alpha}_1^{(-)} \hat{\beta}_2^{(+)} (\tilde{\boldsymbol{\sigma}}_1 \times \tilde{\boldsymbol{\sigma}}_2)_z$	$-\frac{i}{3\sqrt{6}}$	$\frac{i}{3\sqrt{2}}$	$\frac{2i}{3\sqrt{3}}$	$\frac{i}{3\sqrt{2}}$	$\frac{i}{3\sqrt{6}}$	0
Matrix elements	$\langle \Lambda_c^+ \downarrow \hat{O} \mathcal{B}_{cc}^+ \downarrow \rangle_{2\lambda}$	$\langle \Lambda_c^+ \downarrow \hat{O} \mathcal{B}_{cc}^+ \downarrow \rangle_{2\rho}$	$\langle \Lambda_c^+ \downarrow \hat{O} \mathcal{B}_{cc}^+ \downarrow \rangle_{4\lambda}$	$\langle \Sigma_c^+ \downarrow \hat{O} \mathcal{B}_{cc}^+ \downarrow \rangle_{2\lambda}$	$\langle \Sigma_c^+ \downarrow \hat{O} \mathcal{B}_{cc}^+ \downarrow \rangle_{2\rho}$	$\langle \Sigma_c^+ \downarrow \hat{O} \mathcal{B}_{cc}^+ \downarrow \rangle_{4\lambda}$
$-\hat{\alpha}_1^{(-)} \hat{\beta}_2^{(+)} (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)_z$	$\frac{1}{3\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{2}{3\sqrt{3}}$	$\frac{1}{3\sqrt{2}}$	$\frac{1}{3\sqrt{6}}$	0
$\hat{\alpha}_1^{(-)} \hat{\beta}_2^{(+)} (\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2)_z$	$-\frac{i}{3\sqrt{6}}$	$\frac{i}{3\sqrt{2}}$	$\frac{2i}{3\sqrt{3}}$	$\frac{i}{3\sqrt{2}}$	$\frac{i}{3\sqrt{6}}$	0
$-\hat{\alpha}_1^{(-)} \hat{\beta}_2^{(+)} (\tilde{\boldsymbol{\sigma}}_1 - \tilde{\boldsymbol{\sigma}}_2)_z$	$\frac{1}{3\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{2}{3\sqrt{3}}$	$\frac{1}{3\sqrt{2}}$	$\frac{1}{3\sqrt{6}}$	0
$\hat{\alpha}_1^{(-)} \hat{\beta}_2^{(+)} (\tilde{\boldsymbol{\sigma}}_1 \times \tilde{\boldsymbol{\sigma}}_2)_z$	$-\frac{i}{3\sqrt{6}}$	$\frac{i}{3\sqrt{2}}$	$\frac{2i}{3\sqrt{3}}$	$\frac{i}{3\sqrt{2}}$	$\frac{i}{3\sqrt{6}}$	0
Matrix elements	$_{2\lambda} \langle \Xi_c^+ \downarrow \hat{O} \mathcal{B}_{cc}^+ \downarrow \rangle$	$_{2\lambda} \langle \Xi_c'^+ \downarrow \hat{O} \mathcal{B}_{cc}^+ \downarrow \rangle$	$_{4\lambda} \langle \Xi_c'^+ \downarrow \hat{O} \mathcal{B}_{cc}^+ \downarrow \rangle$	$_{2\lambda} \langle \Lambda_c^+ \downarrow \hat{O} \mathcal{B}_{cc}^+ \downarrow \rangle$	$_{2\lambda} \langle \Sigma_c^+ \downarrow \hat{O} \mathcal{B}_{cc}^+ \downarrow \rangle$	$_{4\lambda} \langle \Sigma_c^+ \downarrow \hat{O} \mathcal{B}_{cc}^+ \downarrow \rangle$
$-\hat{\alpha}_1^{(-)} \hat{\beta}_2^{(+)} (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)_z$	$\frac{1}{3\sqrt{6}}$	$\frac{1}{3\sqrt{2}}$	$\frac{1}{3}$	$\frac{1}{3\sqrt{6}}$	$\frac{1}{3\sqrt{2}}$	$\frac{1}{3}$
$\hat{\alpha}_1^{(-)} \hat{\beta}_2^{(+)} (\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2)_z$	$-\frac{i}{3\sqrt{6}}$	$\frac{i}{3\sqrt{2}}$	$\frac{i}{3}$	$-\frac{i}{3\sqrt{6}}$	$\frac{i}{3\sqrt{2}}$	$\frac{i}{3}$
$-\hat{\alpha}_1^{(-)} \hat{\beta}_2^{(+)} (\tilde{\boldsymbol{\sigma}}_1 - \tilde{\boldsymbol{\sigma}}_2)_z$	$\frac{1}{3\sqrt{6}}$	$\frac{1}{3\sqrt{2}}$	$\frac{1}{3}$	$\frac{1}{3\sqrt{6}}$	$\frac{1}{3\sqrt{2}}$	$\frac{1}{3}$
$\hat{\alpha}_1^{(-)} \hat{\beta}_2^{(+)} (\tilde{\boldsymbol{\sigma}}_1 \times \tilde{\boldsymbol{\sigma}}_2)_z$	$-\frac{i}{3\sqrt{6}}$	$\frac{i}{3\sqrt{2}}$	$\frac{i}{3}$	$-\frac{i}{3\sqrt{6}}$	$\frac{i}{3\sqrt{2}}$	$\frac{i}{3}$

TABLE XV. Same as Table XIV but for $\Omega_{cc}^+ \rightarrow \Xi_c^{(\prime)+} \eta^{(\prime)}$ decays, where the first value in brackets corresponds to the E_1 diagram, while the second corresponds to the E_2 diagram.

Matrix elements	Values	Matrix elements	Values	Matrix elements	Values
$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle$	$\{0, 0\}$	$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle_{2\lambda}$	$\{0, 0\}$	$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle_{4\lambda}$	$\{\frac{1}{\sqrt{3}} \cos \zeta, \sqrt{\frac{2}{3}} \sin \zeta\}$
$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle$	$\{\frac{1}{\sqrt{6}} \cos \zeta, \frac{1}{\sqrt{3}} \sin \zeta\}$	$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle_{2\lambda}$	$\{\frac{1}{\sqrt{6}} \cos \zeta, \frac{1}{\sqrt{3}} \sin \zeta\}$	$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle_{4\lambda}$	$\{\frac{1}{\sqrt{3}} \cos \zeta, \sqrt{\frac{2}{3}} \sin \zeta\}$
$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle$	$\{\frac{1}{\sqrt{6}} \cos \zeta, \frac{1}{\sqrt{3}} \sin \zeta\}$	$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle_{2\lambda}$	$\{\frac{1}{\sqrt{6}} \cos \zeta, \frac{1}{\sqrt{3}} \sin \zeta\}$	$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle_{4\lambda}$	$\{\frac{1}{\sqrt{3}} \cos \zeta, \sqrt{\frac{2}{3}} \sin \zeta\}$
$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle$	$\{-\frac{\sqrt{2}}{3} \cos \zeta, \frac{2}{3} \sin \zeta\}$	$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle_{2\lambda}$	$\{-\frac{\sqrt{2}}{3} \cos \zeta, \frac{2}{3} \sin \zeta\}$	$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle_{4\lambda}$	$\{\frac{1}{3} \cos \zeta, -\frac{\sqrt{2}}{3} \sin \zeta\}$
$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle$	$\{0, 0\}$	$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle_{2\lambda}$	$\{0, 0\}$	$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle_{4\lambda}$	$\{\frac{1}{\sqrt{3}} \sin \zeta, -\sqrt{\frac{2}{3}} \cos \zeta\}$
$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle$	$\{\frac{1}{\sqrt{6}} \sin \zeta, -\frac{1}{\sqrt{3}} \cos \zeta\}$	$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle_{2\lambda}$	$\{\frac{1}{\sqrt{6}} \sin \zeta, -\frac{1}{\sqrt{3}} \cos \zeta\}$	$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle_{4\lambda}$	$\{\frac{1}{\sqrt{3}} \sin \zeta, -\sqrt{\frac{2}{3}} \cos \zeta\}$
$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle$	$\{\frac{1}{\sqrt{6}} \sin \zeta, -\frac{1}{\sqrt{3}} \cos \zeta\}$	$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle_{2\lambda}$	$\{\frac{1}{\sqrt{6}} \sin \zeta, -\frac{1}{\sqrt{3}} \cos \zeta\}$	$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle_{4\lambda}$	$\{\frac{1}{\sqrt{3}} \sin \zeta, -\sqrt{\frac{2}{3}} \cos \zeta\}$
$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle$	$\{-\frac{\sqrt{2}}{3} \sin \zeta, -\frac{2}{3} \cos \zeta\}$	$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle_{2\lambda}$	$\{-\frac{\sqrt{2}}{3} \sin \zeta, -\frac{2}{3} \cos \zeta\}$	$\langle \Xi_c^+ \downarrow \hat{I}_\eta \sigma_z \Xi_c^+ \downarrow \rangle_{4\lambda}$	$\{\frac{1}{3} \sin \zeta, \frac{\sqrt{2}}{3} \cos \zeta\}$

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