

Light fields in various patches of de Sitter spacetime

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We start with the consideration of the loop effects for light fields with nonzero mass in the expanding Poincaré patch of de Sitter spacetime. We derive the Dyson-Schwinger equation, which sums up the leading infrared (growing with time) loop corrections in certain limit for small initial perturbations above the Bunch-Davies state. The solution of this equation shows the destiny of the initial state at the future infinity. Then we discuss the case of the contracting Poincaré patch and global de Sitter spacetime and briefly the case of different initial conditions in the expanding Poincaré patch.

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I. INTRODUCTION

The problem of loop corrections in different expanding spacetimes has gained significant attention over the past two decades [1–13]. First, quantum fluctuations in the very early Universe are believed to have determined its present large-scale structure [14–20]. Second, resolving the cosmological constant problem may be connected to the large infrared (IR) contributions from loop corrections, as these corrections might lead to a secular screening of the cosmological constant [21–26]. Third, understanding the backreaction of quantum matter on strong gravitational fields is incomplete without accounting for loop corrections (see, e.g., for the reviews [26,27]). Overall, it is of great interest to reduce our lack of understanding about the role of quantum effects in interacting quantum field theory in various rapidly expanding backgrounds. These questions remain unresolved even in the case of de Sitter (dS) spacetime, which possesses the highest degree of symmetry. The challenges arise due to the vacuum instability of dS [25], the singular behavior of the light fields [2], and the breaking of global dS isometries at loop level [25,26].

There are several types of IR loop effects in dS space. Secular contributions are often studied in the context of quantum effects in inflationary cosmology [28–32]. In this context, massless nonconformal fields such as gravitons

and massless minimally coupled scalars are usually considered. For such fields, secularly growing contributions take place even in local tadpole-type diagrams [32], while the IR and UV divergences in dS space can be taken into account self-consistently using renormalization of the coupling constants [33].

Separately, the secular growth of loop corrections can take place even in Minkowski space if the interactions of fields are switched on and off adiabatically. In [34] it is argued that such secular growth is an artifact of the perturbative approximation in quantum field theory and can be neglected for equilibrium states. The fact that there are no secular effects for stationary states is quite obvious and known for long time. Meanwhile, secular effects in Minkowski space do appear for nonstationary initial distributions (even for spatially inhomogeneous cases and for massive fields), which is a context of textbooks since the 1960s. The subject is related to the thermalization process. See, e.g., [35] for a respectively recent review, which contains earlier necessary references. However, an interesting observation of [34] is that for spatially inhomogeneous interactions secular effects can be neglected. In this paper, we consider expanding and contracting Poincaré patches of dS space. In the expanding patch, every state sooner or later becomes homogeneous, while in the contracting patch inhomogeneities make our observations even stronger.

In this paper, we study IR effects which are a bit different from the standard ones considered in dS space for massless nonconformal fields. We are interested in the IR secular effects which are somewhat similar to those mentioned in the previous paragraph and are present even for massive fields. The presence of the secular effects of interest for us is due to nonstationarity of the situation that we consider. The effects that we consider do *not* appear from the tadpole-type diagrams. They follow from the loop

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(e.g., sunset) diagrams, which do not lead to local terms in the IR limit. In relation to these observations, it is important to note that the issue of thermalization (returning back to the equilibrium) in different regions of dS space is generally a controversial and open question. We continue investigating these questions, following the approach of [10,12,13,23,26]. We mainly focus on the case of the massive real scalar fields from the so-called complementary series ($m < \frac{D-1}{2}$ in units of the Hubble constant) in expanding Poincaré patch (EPP), except for some discussion in Sec. VI and the Appendixes. We discuss the differences in the situations in the EPP, contracting Poincaré patch (CPP) and in global dS. In order to set up the notations we first introduce the standard procedure of the quantization in dS and the Schwinger-Keldysh diagrammatic technique, which we will use to calculate the quantities we are interested in.

A. Setup of the quantization in de Sitter spacetime

We consider the massive real scalar self-interacting theory:

$$S[\phi] = \int d^D x \sqrt{|g|} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{6} \phi^3 \right], \quad (1)$$

in the EPP of dS spacetime:

$$ds^2 = \frac{1}{\eta^2} (d\eta^2 - d\mathbf{x}^2), \quad (2)$$

where we take the Hubble parameter to be unit $H = 1$, and $\eta = e^{-t}$ is the conformal time in the EPP. It is ranging from $+\infty$ at the past infinity, $t \rightarrow -\infty$, to 0 at the future one, $t \rightarrow +\infty$; $D = d + 1$ is the spacetime dimension. The cubic potential is taken to simplify equations below. The phenomena that we observe below have nothing to do with the instability of the cubic potential and are present also in quartic theory (see, e.g., [26,36]).

The free quantum scalar field in EPP can be expanded as

$$\hat{\phi}(\eta, \mathbf{x}) = \int \frac{d^{D-1} \mathbf{p}}{(2\pi)^{D-1}} [\hat{a}_{\mathbf{p}} f_{\mathbf{p}}(\eta) e^{i\mathbf{p}\mathbf{x}} + \hat{a}_{\mathbf{p}}^\dagger f_{\mathbf{p}}^*(\eta) e^{-i\mathbf{p}\mathbf{x}}],$$

$$[\hat{a}_{\mathbf{p}}, \hat{a}_{\mathbf{q}}^\dagger] = (2\pi)^{D-1} \delta(\mathbf{p} - \mathbf{q}),$$

$$\text{where } f_{\mathbf{p}}(\eta) = \eta^{\frac{D-1}{2}} h_\nu(p\eta), \quad \mathbf{p} \equiv |\mathbf{p}|. \quad (3)$$

The field operator obeys the canonical commutation relations with its conjugate momentum. To achieve the latter the modes $f_{\mathbf{p}}(\eta) e^{i\mathbf{p}\mathbf{x}}$ should obey the Klein-Gordon equation in the inflationary metric (2). Then $h_\nu(p\eta)$ can, e.g., be expressed in terms of the Hankel function of the first kind as follows:

$$h_\nu(p\eta) = \frac{\sqrt{\pi}}{2} H_\nu^{(1)}(p\eta), \quad \nu = \sqrt{\frac{(D-1)^2}{4} - m^2}. \quad (4)$$

These are the so-called Bunch-Davies (BD) modes [37]. Other options are discussed at the end of the paper.

For the future use we show here the asymptotic form of these functions:

$$\begin{aligned} h_\nu(p\eta) &\simeq \frac{1}{\sqrt{2p\eta}} e^{ip\eta} e^{-i\frac{\pi}{2}(\nu+\frac{1}{2})}, \quad \text{as } p\eta \gg |\nu|; \\ h_\nu(p\eta) &\simeq iA_- \frac{1}{(p\eta)^\nu} + iA_+ (p\eta)^\nu + iB(p\eta)^{-\nu+2} + \dots, \\ \text{as } p\eta \ll |\nu|; \quad \text{where } A_- &= -\frac{\Gamma(\nu)}{2\sqrt{\pi}} 2^\nu, \\ A_+ &= \frac{\sqrt{\pi} e^{-i\pi\nu}}{2^{\nu+1} \Gamma(\nu+1) \sin(\pi\nu)}, \quad B = \frac{2^\nu \Gamma(\nu)}{8\sqrt{\pi} 2-\nu}. \end{aligned} \quad (5)$$

Note that the term $iB(p\eta)^{-\nu+2}$ and higher terms in the expansion on the right-hand side in the second line are needed for the calculations below with small enough masses (such that ν is real and $\nu > 1$). Moreover, to simplify calculations below we assume that $\nu < \frac{D-1}{6}$ and $D \leq 4$. These restrictions will be explained below.

In dS spacetime one distinguishes massive fields, $m > (D-1)/2$, corresponding to the pure imaginary ν . They are from the so-called principal series of the representations of the dS isometry group. In dS quantum field theory there are also light fields, $m < (D-1)/2$, corresponding to real ν . They are from the so-called complementary series. In this paper we will be mostly considering light fields, giving only a brief comment on the principal series in the Appendixes.

B. Schwinger-Keldysh formalism and the goal of the paper

Below we perform calculations in the Schwinger-Keldysh diagrammatic technique after the Keldysh rotation (see, e.g., [38–43] for the review):

$$\phi_{cl} = \frac{\phi_+ + \phi_-}{2}, \quad \phi_q = \phi_+ - \phi_-. \quad (6)$$

Then the interaction term in the theory under consideration takes the form

$$\begin{aligned} S_{\text{int}}[\phi_{cl}, \phi_q] &= S_{\text{int}}[\phi_+] - S_{\text{int}}[\phi_-] \\ &= \int d^D x \sqrt{|g|} \left[\frac{\lambda}{24} \phi_q^3 + \frac{\lambda}{2} \phi_{cl}^2 \phi_q \right]. \end{aligned} \quad (7)$$

Below in this technique we perform calculations with the BD state taken as the initial one at the lightlike initial Cauchy surface—the boundary of the EPP. The state is

specified by the condition $\hat{a}_p|\text{BD}\rangle = 0$ for the above defined modes and annihilation operators.

In the technique in question the scalar field is characterized by the following three propagators:

$$\begin{aligned} iG_0^K(\eta_1, \mathbf{x}_1 | \eta_2, \mathbf{x}_2) &= \overbrace{\phi_{cl}(\eta_1, \mathbf{x}_1) \phi_{cl}(\eta_2, \mathbf{x}_2)} \\ &= \frac{1}{2} \langle \text{BD} | \{ \hat{\phi}(\eta_1, \mathbf{x}_1), \hat{\phi}(\eta_2, \mathbf{x}_2) \} | \text{BD} \rangle, \\ iG_0^A(\eta_1, \mathbf{x}_1 | \eta_2, \mathbf{x}_2) &= \overbrace{\phi_{cl}(\eta_1, \mathbf{x}_1) \phi_q(\eta_2, \mathbf{x}_2)} \\ &= \theta(\eta_1 - \eta_2) \rho(\eta_1, \mathbf{x}_1 | \eta_2, \mathbf{x}_2), \\ iG_0^R(\eta_1, \mathbf{x}_1 | \eta_2, \mathbf{x}_2) &= \overbrace{\phi_q(\eta_1, \mathbf{x}_1) \phi_{cl}(\eta_2, \mathbf{x}_2)} \\ &= -\theta(\eta_2 - \eta_1) \rho(\eta_1, \mathbf{x}_1 | \eta_2, \mathbf{x}_2), \end{aligned}$$

$$\text{where } \rho(\eta_1, \mathbf{x}_1 | \eta_2, \mathbf{x}_2) = [\hat{\phi}(\eta_1, \mathbf{x}_1), \hat{\phi}(\eta_2, \mathbf{x}_2)]. \quad (8)$$

Here the subscript zero stands for the tree-level value of these two-point functions. The BD state is spatially homogeneous. Hence, it is convenient to perform the spatial Fourier transformation of the propagators. After the transformation we obtain that

$$\begin{aligned} iG_0^K(\mathbf{p} | \eta_1, \eta_2) &= \text{Re}\{f_p(\eta_1) f_p^*(\eta_2)\}, \\ \rho(\mathbf{p} | \eta_1, \eta_2) &= -2 \text{Im}\{f_p(\eta_1) f_p^*(\eta_2)\}. \end{aligned} \quad (9)$$

The retarded and advanced propagators, $G_0^{R,A}$, are sensitive to the spectrum of the theory, but are not sensitive to the state of it (at the tree level). Meanwhile, the Keldysh propagator, G_0^K , is sensitive to the state of the theory.

Our goal is to trace the destiny of the state of the theory at the future infinity. That is the reason why we are interested in the behavior of the loop-corrected Keldysh propagator in the limit of large external times $\eta = \sqrt{\eta_1 \eta_2} \rightarrow 0$, while $\eta_1/\eta_2 = \text{const}$, in the units of the cosmological constant. In the momentum space (for each mode separately) this limit corresponds to $p\eta \ll |\nu|$. In this limit the loop corrected Keldysh propagator acquires the form [26]

$$\begin{aligned} G^K(\mathbf{p} | \eta_1, \eta_2) &\simeq \left(\frac{1}{2} + n_p(\eta) \right) f_p(\eta_1) f_p^*(\eta_2) \\ &+ \kappa_p(\eta) f_p(\eta_1) f_p(\eta_2) + \text{c.c.}, \end{aligned} \quad (10)$$

where in general $n_p(\eta)$ and $\kappa_p(\eta)$ [perhaps multiplied by the mode functions as in (10) at the coincident points] are the level populations and the anomalous quantum averages, correspondingly.

In the future infinity limit under consideration, $p\eta \rightarrow 0$, for the principal series ($m > \frac{D-1}{2}$) in the EPP the leading contributions to such products of modes as $f_p(\eta_1) f_p^*(\eta_2)$ and $f_p(\eta_1) f_p(\eta_2)$ are different functions of η 's, as follows from (5). Then, in such a case one can separate IR loop

contributions to $n_p(\eta)$ and $\kappa_p(\eta)$ from each other. However, for the complementary series in the EPP the leading contributions in the IR limit are the same, $f_p(\eta_1) f_p^*(\eta_2) \simeq -f_p(\eta_1) f_p(\eta_2)$. As a result, one obtains that

$$G^K(\mathbf{p} | \eta_1, \eta_2) \simeq \frac{A_-^2 \eta^{D-1}}{(p\eta)^{2\nu}} N_p(\eta), \quad (11)$$

where $N_p(\eta) = 1 + 2n_p(\eta) - 2\text{Re}\{\kappa_p(\eta)\}$ and we have used the asymptotic form of the modes (5). Moreover, in this case the leading first loop contributions to $n_p(\eta)$ and $\kappa_p(\eta)$ cancel each other within $N_p(\eta)$, as is shown in [12]. We will show that below again.

As demonstrated in several works [10–13, 26, 36], in the future infinity limit, $p\eta \ll |\nu|$, of the EPP the Keldysh propagator at one-loop order (in ϕ^3 theory) receives secularly growing contributions of the form $\sim \lambda^2 \log(p\eta/|\nu|)$. They become comparable in magnitude to the tree-level contribution (8) for a long enough time of evolution of the system. This is true both for the heavy fields from the principal series and for the light ones from the complementary series (see, however, the discussion in the Appendixes below). At the same time, one-loop corrections to the retarded and advanced propagators are not growing as $p\eta \rightarrow 0$ and are just suppressed by λ^2 . These large IR secular contributions drastically correct the physical effects seen at the tree level, making it necessary to resum at least the leading multiloop corrections of the form $\sim (\lambda^2 \log(p\eta/|\nu|))^n$ [assuming that λ^2 is small, while $\log(p\eta/|\nu|)$ is large].

In this paper, we build upon and refine previous works on loop calculations for light fields in dS in the limit that was described above. Specifically, in Secs. II–IV we compute the leading secularly growing contributions at one-loop level and demonstrate that multiloop “bubble inside bubble” diagrams, as well as corrections to multi-point correlation functions, are suppressed. Based on these observations in Sec. V we write down the Dyson-Schwinger equation, which resums the leading corrections from all loops in the limit under consideration in the EPP. These observations have already been made in [13], but no correct resummation of the leading contributions was provided.

In Sec. VI we perform similar calculations for the light fields in the CPP. The latter are having such a form as $\lambda^2 \log(\frac{\eta}{\eta_0})$. Here η_0 is the position of the initial Cauchy surface in the CPP.

The case of CPP is discussed here for three reasons. First, the consideration of CPP is quite an important question even from the observational point of view and not only from the academic perspective. In fact, the same type of secular divergence as we just mentioned is present in the global dS spacetime. That can be understood, e.g., from the observation that the global dS is the union of EPP and CPP.

At the same time we do not really know what was the initial state of our Universe at the start of the inflation. We do not know either the initial Fock space state or the geometry of the initial Cauchy surface. The state definitely was not the BD one at the lightlike initial surface of the EPP. Otherwise even galaxies will not exist because the situation would be absolutely symmetric. The natural question is, once the start of the inflation was from a nonsymmetric state, what was achieved the first—the equilibration or the strong backreaction on the background dS geometry [27]?

Second, in CPP we encounter a long-memory effect in dS [11] (see also [26,44]), manifesting itself as the secular divergence. To cut this divergence one needs to introduce into the loop corrections the dependence on the position of the initial Cauchy surface, η_0 , thus, breaking the symmetry of the tree-level theory under the dS isometry group. This phenomenon was previously treated in works [26,44] for the principal series in CPP and global dS within the kinetic approximation. There it was shown that whether the symmetry under the dS isometry is restored in the exact correlation functions or not depends on the initial conditions.

The third reason to consider the situation in CPP is as follows. Unlike the case of the resummation in the EPP, the corresponding resumming equations in CPP are nonlinear. The reason for that is as follows: due to the secular divergence different types of diagrams give the leading corrections. This fact is making the resummation for light fields in CPP and global dS more challenging, particularly

because the kinetic approximation is not applicable in this context. We will address this issue in future work.

Additionally, in Appendix A, we revisit the resummation of leading logarithms in EPP for the principal series. We argue that for heavy fields, loop corrections exhibit an additional suppression of the form $e^{-\pi\nu}$, on top of the λ^2 , thus, behaving as $\sim e^{-\pi\nu} \lambda^2 \log(p\eta/|\nu|)$. Finally, we show that for sufficiently large masses, $\nu \gtrsim \frac{2}{\pi} \log(\frac{1}{\lambda})$, the level populations and the anomalous quantum averages attain small values at late times, resulting in only a slight perturbation to the initial BD state.

II. ONE-LOOP CORRECTIONS TO THE PROPAGATORS

In this section we single out the leading contribution from the one-loop correction to the Keldysh propagator in the limit $p\sqrt{\eta_1\eta_2} \equiv p\eta \rightarrow 0$ for the BD initial state. Having the tree-level vertices from (7) and the propagators from (8) we can write all the relevant one-loop diagrams, which are shown in Fig. 1. Keeping in mind that we are interested in secular effects in the limit, when $(t_1 + t_2)/2 \gg |t_1 - t_2|$, where $\eta_{1,2} = e^{-t_{1,2}}$, in the arguments of the corrected propagator [13], we can neglect the difference between the external time coordinates. Then the leading one-loop correction in the momentum space is contained in the following expression:

$$\begin{aligned} \Delta_{1\text{-loop}} G^K(\mathbf{p}|\eta, \eta) = & \lambda^2 \int \frac{d^{D-1}\mathbf{k}}{(2\pi)^{D-1}} \frac{d\eta_3}{\eta_3^D} \frac{d\eta_4}{\eta_4^D} \left[-\frac{1}{2} \theta(\eta_3 - \eta) \theta(\eta_4 - \eta) \right. \\ & \times \rho(\mathbf{p}|\eta, \eta_3) \left(G_0^{K^2}(\mathbf{k} - \mathbf{p}|\eta_3, \eta_4) - \frac{1}{4} \rho^2(\mathbf{k} - \mathbf{p}|\eta_3, \eta_4) \right) \rho(\mathbf{p}|\eta_4, \eta) \\ & + \theta(\eta_4 - \eta_3) \theta(\eta_3 - \eta) \rho(\mathbf{p}|\eta, \eta_3) \rho(\mathbf{k} - \mathbf{p}|\eta_3, \eta_4) G_0^K(\mathbf{k} - \mathbf{p}|\eta_3, \eta_4) G_0^K(\mathbf{p}|\eta_4, \eta) \\ & \left. + \theta(\eta_3 - \eta_4) \theta(\eta_4 - \eta) G_0^K(\mathbf{p}|\eta, \eta_3) \rho(\mathbf{k} - \mathbf{p}|\eta_3, \eta_4) G_0^K(\mathbf{k} - \mathbf{p}|\eta_3, \eta_4) \rho(\mathbf{p}|\eta_4, \eta) \right]. \end{aligned} \quad (12)$$

First, the analysis of the expression (12) shows that the dominant contribution in the limit under consideration comes from the region of the momentum integrals, where the external momentum, $\mathbf{p}$, can be neglected in

comparison with the internal one, $\mathbf{k}$ (see, e.g., [11] and Appendix B). Furthermore, to single out the leading correction one can assume that the internal conformal times, η_3, η_4 , are such that the IR asymptotic

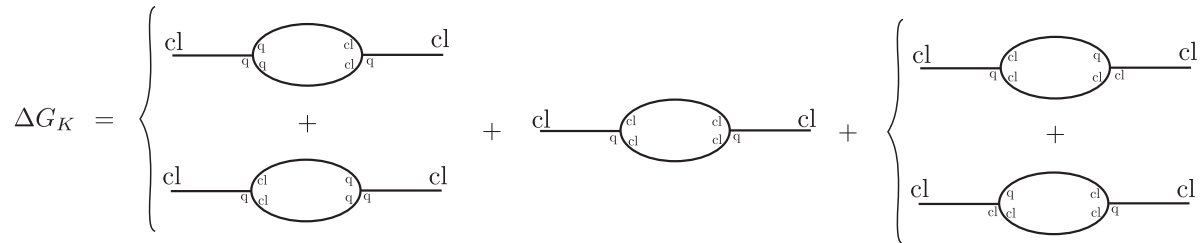


FIG. 1. Bubble diagram.

form (5) for the modes can be used in the external propagators [11].

Second, the terms in the last two lines of (12) are actually equal after the exchange of the integration variables

$\eta_3 \leftrightarrow \eta_4$. Let us denote the sum of these two terms as $\Delta_1 G^K(\mathbf{p}|\eta, \eta)$ and use the expressions for the propagators via the modes to write the expression in the limit discussed above as

$$\Delta_1 G^K(\mathbf{p}|\eta, \eta) \simeq -\frac{4A_-^2}{(p\eta)^{2\nu}} \eta^{D-1} \lambda^2 \int_1^\infty \frac{d\tau_4}{\tau_4} \int_{\tau_4}^\infty \frac{d\tau_3}{\tau_3} (\tau_3 \tau_4)^{\frac{D-1}{2}} \text{Im}\{h_\nu(p\eta\tau_3)\} \text{Re}\{h_\nu(p\eta\tau_4)\} \\ \times \int \frac{d^{D-1}\mathbf{q}}{(2\pi)^{D-1}} \text{Im}\{h_\nu(q\tau_3)h_\nu(|\mathbf{q}-\mathbf{p}\eta|\tau_3)h_\nu^*(q\tau_4)h_\nu^*(|\mathbf{q}-\mathbf{p}\eta|\tau_4)\}, \quad (13)$$

where we have made the following change of the integration variables: $\tau_3 = \frac{\eta_3}{\eta}$, $\tau_4 = \frac{\eta_4}{\eta}$, $\mathbf{q} = \mathbf{k}\eta$. It can be seen that under the integral over $\mathbf{q}$ we can take the limit $p\eta \rightarrow 0$ right away assuming the condition that $\nu < \frac{D-1}{4}$ for the IR convergence of these expressions with $\mathbf{p}$ taken to be zero.¹ This is in accordance with the similar limit in [11,26] and that is the region of integration over $\mathbf{q}$ from which the leading secular contribution is coming from, when $p\eta \ll |\nu|$. Let us now make the further change of variables $u = \sqrt{\tau_3\tau_4}$, $v = \sqrt{\frac{\tau_3}{\tau_4}}$, $\ell = \mathbf{q}u$ to get

$$\Delta_1 G^K(\mathbf{p}|\eta, \eta) \simeq -\frac{8A_-^2}{(p\eta)^{2\nu}} \eta^{D-1} \lambda^2 \int_1^\infty \frac{dv}{v} \int_v^\infty \frac{du}{u} u^{D-1} \cdot \text{Im}\{h_\nu(p\eta uv)\} \text{Re}\left\{h_\nu\left(p\eta \frac{u}{v}\right)\right\} \\ \times \int \frac{d^{D-1}\ell}{(2\pi)^{D-1}} \text{Im}\left\{h_\nu^2(\ell v)h_\nu^{*2}\left(\frac{\ell}{v}\right)\right\}. \quad (14)$$

Analyzing the asymptotics (5), one can see that there is a large contribution in the expression (14). It is large in the limit $p\eta \ll |\nu|$ and comes from the region where the Hankel functions are expanded at small arguments $p\eta uv < |\nu|$, $p\eta \frac{u}{v} < |\nu|$, while the other contributions are suppressed in the limit $p\eta \rightarrow 0$. Hence, we use the IR asymptotic form (5) for $h_\nu(p\eta uv)$ and $h_\nu(p\eta \frac{u}{v})$, to obtain that

$$\Delta_1 G^K(\mathbf{p}|\eta, \eta) \simeq \frac{8A_-^3 \text{Im}\{A_+\}}{(p\eta)^{2\nu}} \eta^{D-1} \int_1^{\frac{\sqrt{|\nu|}}{\sqrt{p\eta}}} \frac{dv}{v^{2\nu+1}} \int_v^{\frac{|\nu|}{p\eta v}} \frac{du}{u} \int \frac{d^{D-1}\ell}{(2\pi)^{D-1}} \text{Im}\left\{h_\nu^2(\ell v)h_\nu^{*2}\left(\frac{\ell}{v}\right)\right\} \\ \simeq \frac{A_-^2}{(p\eta)^{2\nu}} \eta^{D-1} \left[A \cdot \lambda^2 \log\left(\frac{p\eta}{|\nu|}\right) + B \cdot \lambda^2\right], \quad (15)$$

where

$$A \equiv -8A_- \text{Im}\{A_+\} \int_1^\infty \frac{dv}{v^{2\nu+1}} \text{Im}\{F(v)\}, \\ F(v) \equiv \int \frac{d^{D-1}\ell}{(2\pi)^{D-1}} \text{Im}\left\{h_\nu^2(\ell v)h_\nu^{*2}\left(\frac{\ell}{v}\right)\right\}, \quad (16)$$

and B is some irrelevant constant for further discussion. To obtain the second line of (15) we calculated the integral over u and set the upper limit of the integration over v to infinity due to the rapid convergence of these integrals on much shorter intervals. One should keep in mind that we mainly restrict ourselves to the dimensions $D \leq 4$.

¹Actually, an accurate calculation shows that nothing much different happens when $\nu > \frac{D-1}{4}$, but the coefficients in the resulting expressions are slightly changed.

We see that for sufficiently large times (or low conformal times in EPP) the logarithm in (15) becomes large and the correction can dominate even over the tree-level Keldysh propagator, while the other contributions are suppressed as $\mathcal{O}(\lambda^2)$ in this limit. In the same way it can be shown that other terms in (12) give no additional large or growing with time contributions (see also Appendix B). Hence, in the one-loop order the dominant correction to the Keldysh propagator (12) is given by

$$\Delta_{1\text{-loop}} G^K(\mathbf{p}|\eta, \eta) \simeq G_0^K(\mathbf{p}|\eta, \eta) \cdot A \lambda^2 \log\left(\frac{p\eta}{|\nu|}\right), \quad \text{as } p\eta \rightarrow 0. \quad (17)$$

This is the secular correction that we have been talking about in the Introduction. It should be stressed at the same time that retarded and advanced propagators do not receive growing with time contributions in the limit $\sqrt{\eta_1\eta_2} \rightarrow 0$,

$\eta_1/\eta_2 = \text{const}$ [26]. Thus, only the Keldysh propagator contains the secular contribution, which, hence, cannot be, e.g., absorbed into the self-energy and/or mode frequency shift.

III. MULTILoop CONTRIBUTIONS TO THE TWO-POINT FUNCTIONS

At late times, when $\lambda^2 \log(p\eta) \sim 1$, the perturbation expansion breaks down, and we need to sum up at least the leading secular corrections from all loops. In order to write the system of appropriate Dyson-Schwinger equations, which does the summation of the leading corrections, we have to estimate which type of diagrams should be taken into account. Obviously, there are ladder (bead or chaplet type) diagrams which will give powers of the form $[\lambda^2 \log(p\eta)]^n$, $n \geq 2$.

In this section we show that the “bubble inside bubble” diagrams, such as those shown in Fig. 2, contribute subleading corrections. In view of the fact that one-loop corrections to the retarded and advanced propagators do not contain any secular growth, one simple way to see this fact is to substitute the leading one-loop correction (17) into the internal Keldysh lines of Fig. 1. Estimating the correction in the internal line of the diagrams as

$$\begin{aligned} \Delta G^K(\mathbf{q}|\eta_3, \eta_4) &\sim \lambda^2 \log(q\sqrt{\eta_3\eta_4}) G^K(\mathbf{q}|\eta_3, \eta_4) \\ &= \lambda^2 \log(\ell) G^K(\mathbf{q}|\eta_3, \eta_4), \end{aligned} \quad (18)$$

we get

$$\begin{aligned} \Delta^{(2)} G^K(\mathbf{p}|\eta, \eta) &\sim \frac{\lambda^4}{(p\eta)^{2\nu}} \int_1^{\frac{1}{\sqrt{p\eta}}} \frac{dv}{v^{2\nu+1}} \int_v^{\frac{1}{p\eta v}} \frac{du}{u} \\ &\quad \times \int d^{D-1} \ell \log(\ell) h_v^2(\ell v) h_v^{*2}\left(\frac{\ell}{v}\right) \\ &\sim \frac{\lambda^4 \eta^{D-1}}{(p\eta)^{2\nu}} \log(p\eta) \\ &\sim \lambda^4 \log(p\eta) G_0^K(\mathbf{p}|\eta, \eta). \end{aligned} \quad (19)$$

The resulting expression (19) is, thus, suppressed by the additional power of λ^2 , which is not accompanied by an additional power of the logarithm. This very important

remark was not taken into account in [12,26]. Then it was corrected in [13], but without the revision of the Dyson-Schwinger equation and its solutions. In this work we fill in this loophole in the research. The point is that if the diagrams shown in Fig. 2 are included in the leading approximation, then the resulting integral equation on the corrected Keldysh propagator is not linear and has redundant singular solutions [12]. Nevertheless, in [26] it was shown that nonlinear equations resumming diagrams appear in CPP and global dS. That was shown for the case of the principal series, and we will demonstrate the same for the light fields from the complementary series below.

IV. CORRECTIONS TO MULTIPOINT CORRELATION FUNCTIONS

The system of Dyson-Schwinger equations is imposed on propagators and multipoint correlation functions. Hence, to single out the leading corrections we should check for potentially dangerous (growing with time) loop contributions to higher-point correlators as well. At the one-loop level the correction to the three-point correlation function consists of diagrams shown in Fig. 3.

The loop corrections from Fig. 3 have the following form modulo the overall coefficient:

$$\begin{aligned} \Delta_{1\text{-loop}} \langle \phi_1 \phi_2 \phi_3 \rangle &\sim \delta^{(D-1)}(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3) \cdot \lambda^3 \cdot \int_{\eta_1}^{\infty} \frac{d\eta_4}{\eta_4^D} \int_{\eta_2}^{\infty} \frac{d\eta_5}{\eta_5^D} \int_{\eta_3}^{\infty} \frac{d\eta_6}{\eta_6^D} \\ &\quad \times \rho(\mathbf{p}_1|\eta_1, \eta_4) \rho(\mathbf{p}_2|\eta_5, \eta_2) \rho(\mathbf{p}_3|\eta_6, \eta_3) \\ &\quad \times \int \frac{d^{D-1} \mathbf{k}}{(2\pi)^{D-1}} G_0^K(\mathbf{k} - \mathbf{p}_2|\eta_4, \eta_5) G_0^K(\mathbf{k}|\eta_5, \eta_6) \\ &\quad \times G_0^K(\mathbf{k} + \mathbf{p}_3|\eta_6, \eta_4). \end{aligned} \quad (20)$$

There are potentially large IR contributions from the loop momentum integral in (20) in the case $\nu > \frac{D-1}{6}$ in view of the asymptotic behavior (5). To make an estimation of this correction let us take the integral in one region where all propagators of this diagram can be expanded at small arguments: $k < \min\{\frac{1}{\eta_5}, \frac{1}{\eta_6}, p_2, p_3\}$, $\eta_4 < \frac{1}{\max\{p_1, p_2, p_3\}}$, $\eta_5 < \frac{1}{p_2}$, $\eta_6 < \frac{1}{p_3}$:

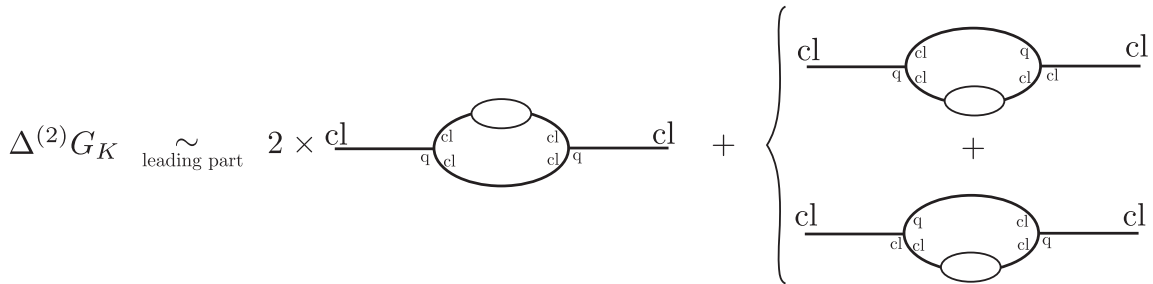


FIG. 2. Bubble inside bubble.

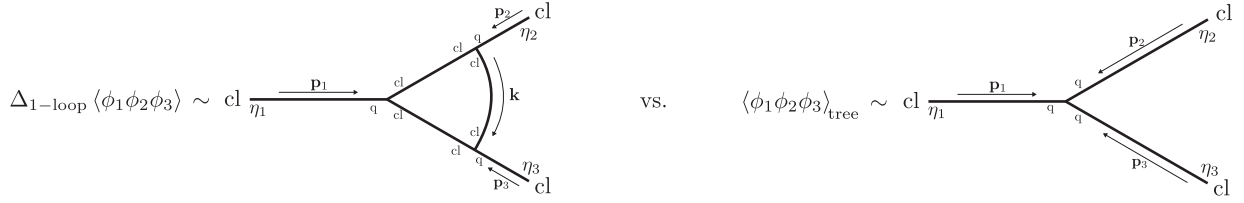


FIG. 3. One-loop correction to the three-point function to be compared with the tree-level value in the limit $p_{1,2,3}\eta_{1,2,3} \rightarrow 0$. The subscripts of the fields in the correlators symbolically mean the corresponding time coordinate and the diagrams are written in the momentum representation.

$$\Delta_{1\text{-loop}} \langle \phi_1 \phi_2 \phi_3 \rangle \sim A_-^0 \text{Im}^3 \{A_+\} \delta^{(D-1)}(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3) \cdot \lambda^3 \cdot \frac{(\eta_1 \eta_2 \eta_3)^{D-1-\nu}}{(p\eta_3)^\nu (p\eta_2)^\nu (p\eta_1)^\nu} \times \frac{p_1^\nu \min^{D-1-2\nu}\{p_2, p_3\}}{p_2^\nu p_1^\nu} \frac{1}{(\eta_1 \max\{p_1, p_2, p_3\})^{\frac{D-1}{2}-\nu}} \frac{1}{(p_2 \eta_2)^{\frac{D-1}{2}-\nu}} \frac{1}{(p_3 \eta_3)^{\frac{D-1}{2}-\nu}}. \quad (21)$$

At the same time the typical contribution to the three-point function at tree level (the second picture in Fig. 3) has the form

$$\langle \phi_1 \phi_2 \phi_3 \rangle_{\text{tree}} \sim \delta^{(D-1)}(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3) \cdot \lambda \cdot \int \frac{d\eta_4}{\eta_4^D} G^A(\mathbf{p}_1 | \eta_1, \eta_4) G^R(\mathbf{p}_2 | \eta_4, \eta_3) G^R(\mathbf{p}_3 | \eta_4, \eta_3) \sim \delta^{(D-1)}(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3) \cdot \lambda \cdot \frac{(\eta_1 \eta_2 \eta_3)^{\frac{D-1}{2}}}{(p_1 \eta_1)^\nu (p_2 \eta_2)^\nu (p_3 \eta_3)^\nu} \frac{(p_1 p_2 p_3)^\nu}{(\max\{p_1, p_2, p_3\})^{\frac{D-1}{2}+3\nu}}. \quad (22)$$

Now let us introduce dimensionless variables $x_{1,2,3}$ and $\tau_{1,2,3}$ through $p_{1,2,3} = p x_{1,2,3}$, $\eta_{1,2,3} = \eta \tau_{1,2,3}$. As in the case of the Keldysh propagator we are interested in the secular effects at late times. Hence, we choose the external parameters to be of the order $x_{1,2,3} \sim 1$ and $\tau_{1,2,3} \sim 1$ and take the limit $p\eta \rightarrow 0$. Then it is evident that the one-loop correction is just suppressed in this limit:

$$\frac{\Delta_{1\text{-loop}} \langle \phi_1 \phi_2 \phi_3 \rangle}{\langle \phi_1 \phi_2 \phi_3 \rangle_{\text{tree}}} \sim \lambda^2 \cdot \frac{\eta^{3(D-1-\nu)}}{(p\eta)^{3\nu}} \frac{p^{D-1-3\nu}}{(p\eta)^{\frac{3D-1}{2}-3\nu}} / \frac{\eta^{\frac{3D-1}{2}}}{(p\eta)^{3\nu}} \frac{p^{3\nu}}{p^{\frac{D-1}{2}+3\nu}} \sim \lambda^2. \quad (23)$$

More accurate estimates of the other corrections to the three-point functions also show no additional growing with time contributions in the limit under consideration. It happens because in the multipoint correlation functions the “volume” factors $\eta^{\frac{D-1}{2}}$ in the modes (3) and propagators (9) suppress the peculiar IR behavior of the light fields (5) when integrated over the internal conformal times of the diagrams.

Thus, we do not need to write the Dyson-Schwinger equations for the multipoint correlation functions when

we sum up the leading secularly growing corrections of the form (15), and can keep these correlation functions of the tree-level form in the equations for the propagators.

However, let us stress that there is a strong IR correction at the one-loop order for the vertex function, which is the amputated diagram of the form shown in Fig. 3, i.e., with removed external propagators in the expression (20). In fact, in this case the corresponding estimates give

$$\Delta_{1\text{-loop}} \langle \phi_1 \phi_2 \phi_3 \rangle^{\text{Amp.}} \sim \delta^{(D-1)}(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3) \cdot \lambda^3 \cdot (\eta_1 \eta_2 \eta_3)^{D-1-2\nu} \frac{\max\{p_2, p_3\}}{p_2^{2\nu} p_3^{2\nu}}, \quad \langle \phi_1 \phi_2 \phi_3 \rangle_{\text{tree}}^{\text{Amp.}} \sim \delta^{(D-1)}(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3) \cdot \lambda \cdot \delta(\eta_1 - \eta_2) \delta(\eta_2 - \eta_3) (\eta_1 \eta_2)^D. \quad (24)$$

Hence, in the limit $p\eta \rightarrow 0$ the ratio of these two contributions is as follows:

$$\frac{\Delta_{1\text{-loop}} \langle \phi_1 \phi_2 \phi_3 \rangle^{\text{Amp.}}}{\langle \phi_1 \phi_2 \phi_3 \rangle_{\text{tree}}^{\text{Amp.}}} \sim \lambda^2 (p\eta)^{D-1-6\nu}, \quad (25)$$

which is singular if $\nu > \frac{D-1}{6}$. In [12] it is pointed out that such a behavior of the vertex functions is connected to the appearance of the discrete set of states from the complementary series in the Källén-Lehmann decomposition for small enough masses of the scalar field. Nevertheless, as we discussed above, these observations do not complicate the

procedure of the resummation of the leading contributions of the form of powers of (17).

V. RESUMMATION

The observations that have been made in the previous sections allow us to state that at the leading approximation in the EPP in the limit $p\sqrt{\eta_1\eta_2} \rightarrow 0$, $\eta_1/\eta_2 = \text{const}$ we should sum up the bead or chaplet type diagrams

constructed from the one depicted in Fig. 1. Such a summation has to be done only for the Keldysh propagator, while retarded and advanced propagators, and vertices can be taken of the tree-level form. (We care only about the leading secular IR contributions and assume that all the constants in the theory have their physical values, i.e., are UV renormalized.)

The corresponding Dyson-Schwinger equation is

$$\begin{aligned} G^K(\mathbf{p}|\eta, \eta) &\simeq G_0^K(\mathbf{p}|\eta, \eta) + \lambda^2 \int \frac{d^{D-1}\mathbf{k}}{(2\pi)^{D-1}} \frac{d\eta_3}{\eta_3^D} \frac{d\eta_4}{\eta_4^D} \left[-\frac{1}{2} \theta(\eta_3 - \eta) \theta(\eta_4 - \eta) \right. \\ &\quad \times \rho(\mathbf{p}|\eta, \eta_3) \left(G_0^{K^2}(\mathbf{k} - \mathbf{p}|\eta_3, \eta_4) - \frac{1}{4} \rho^2(\mathbf{k} - \mathbf{p}|\eta_3, \eta_4) \right) \rho(\mathbf{p}|\eta_4, \eta) \\ &\quad + \theta(\eta_4 - \eta_3) \theta(\eta_3 - \eta) \rho(\mathbf{p}|\eta, \eta_3) \rho(\mathbf{k} - \mathbf{p}|\eta_3, \eta_4) G_0^K(\mathbf{k} - \mathbf{p}|\eta_3, \eta_4) G^K(\mathbf{p}|\eta_4, \eta) \\ &\quad \left. + \theta(\eta_3 - \eta_4) \theta(\eta_4 - \eta) G_0^K(\mathbf{p}|\eta, \eta_3) \rho(\mathbf{k} - \mathbf{p}|\eta_3, \eta_4) G_0^K(\mathbf{k} - \mathbf{p}|\eta_3, \eta_4) \rho(\mathbf{p}|\eta_4, \eta) \right]. \end{aligned} \quad (26)$$

On the rhs of this equation the exact form has only the Keldysh propagator and only in one place.

In order to solve this equation, we will use the following ansatz for the “exact” Keldysh propagator:

$$G^K(\mathbf{p}|\eta_1, \eta_2) = A_-^2 \eta^{D-1} \frac{N(p\eta)}{(p\eta)^{2\nu}}, \quad \eta = \sqrt{\eta_1\eta_2}, \quad (27)$$

which solves this equation under certain conditions in the limit that we consider, as we will see now. In general, we are interested in solving the Eq. (26) for different initial conditions for the Keldysh propagator, which carries the information about the quantum state of the scalar field at the exit surface² $p\eta \sim |\nu|$.

In this paper we use the simplest choice. Namely, we assume some initial value for $N(p\eta)$ in (27), $N_0 \equiv N(P_0) = 1 + 2n(P_0) - 2\text{Re}\{\chi(P_0)\}$, which is imposed for some initial physical momentum $P_0 = (p\eta)_0$. Such a choice preserves at least the following part of the dS isometry³: $p \rightarrow \frac{1}{\alpha} p$, $\eta \rightarrow \alpha \eta$ for some constant α . Therefore, the resulting equation for the evolution of the quantum state takes the form

²One should keep in mind that loop corrections respect the dS isometry in the EPP for the BD state [45–48]. But it is more interesting to trace the destiny of perturbations (perhaps spatially homogeneous) of the BD state that violate the dS isometry at the initial moment.

³It must be stressed here that this is a rather specific choice of the initial conditions from the physical perspective (see, e.g., [49] for the related discussion). There are many more different ways to set up initial conditions, and they normally break the dS isometry. This point is discussed in a bit more detail in the concluding section of this paper.

$$\begin{aligned} N(p\eta) &\simeq N_0 + 4A_- \text{Im}\{A_+\} N_0 \lambda^2 \int_1^{\frac{1}{\sqrt{p\eta}}} \frac{dv}{v^{2\nu+1}} \int_v^{\frac{1}{p\eta}} \frac{du}{u} \\ &\quad \times [N(p\eta\sqrt{uv}) + N_0] \text{Im}\{F(v)\} + \mathcal{O}(\lambda^2), \end{aligned} \quad (28)$$

where $A_{\pm}$ are defined in (5) and $F(v)$ is defined in (16). Let us stress again that the equation is valid in the approximations discussed in Sec. II.

The Eq. (28) in principle may have different solutions for different initial conditions. An obvious ansatz for the solution can be taken of the form $N(p\eta) = C(p\eta)^\alpha$. If $\alpha > 0$, then $N(p\eta\sqrt{uv}) \ll N_0$ in Eq. (28), as $p\eta \rightarrow 0$, and, hence, the ansatz is not appropriate, because it does not solve the equation under consideration.

In contrast, if we assume that $\alpha < 0$, then $N(p\eta\sqrt{uv}) \gg N_0$ and we obtain from (28) the equation defining α :

$$\alpha \simeq -4\lambda^2 N_0 A_- \text{Im}\{A_+\} \int_1^\infty \frac{dv}{v^{2\nu+1}} \text{Im}\{F(v)\} v^{\frac{\alpha}{2}}, \quad (29)$$

where we set the upper limit of the integral over v to infinity due to its rapid convergence on a much smaller interval. In the leading order the solution for α is very close to zero:

$$\alpha \simeq -4\lambda^2 N_0 A_- \text{Im}\{A_+\} \int_1^\infty \frac{dv}{v^{2\nu+1}} \text{Im}\{F(v)\} + \mathcal{O}(\lambda^4), \quad (30)$$

providing that the initial value N_0 , which can have both signs, should be such that $\alpha < 0$. In this case the exact Keldysh propagator blows up in comparison with its tree-level value. In fact, the rate is

$$\frac{G^K(\mathbf{p}|\eta, \eta)}{G_0^K(\mathbf{p}|\eta, \eta)} \sim (p\eta)^{-|\alpha|}.$$

Nevertheless this power-law growth of the Keldysh propagator does not overcome the volume factor, η^{D-1} , in (3), (9) or (27) and generally can be considered as an IR renormalization of the mass, making the situation somewhat similar to the one considered in [50–52] in large- N models. (The difference in details of our answer with respect to the one in those papers can be attributed to the fact that we consider a different limit.) We will return to the Dyson-Schwinger equation (26) to consider more generic initial states and types of self-interactions in our future works.

VI. A COMMENT ON THE SITUATION IN THE CONTRACTING POINCARÉ PATCH

In order to illustrate the situation where the dS isometry is inevitably broken at the loop order (even if at the tree level it was respected) we examine here similar calculations in CPP for the complementary series.

In CPP we consider the same initial BD state and the corresponding modes (4) as in EPP, i.e., we also consider a spatially homogeneous state, which of course in CPP, unlike the case of EPP, is unstable with respect to spatially inhomogeneous perturbations. However, the effects that we consider below have nothing to do with this instability and are present, e.g., in the upper part of the global dS spacetime.

The metric in CPP being expressed in conformal time is the same as in EPP, but the key difference is now the relation between conformal and cosmological time, $\eta = e^t$, and the arguments of theta functions in (8) must be swapped between 1 and 2. Hence, the conformal time, η , is ranging from $\eta = 0$ at past infinity, $t = -\infty$, to $\eta = +\infty$ in the future infinity, $t = +\infty$.

This choice of the mode functions and the state in CPP corresponds to the propagators that respect the proper Hadamard behavior and the dS isometry at the tree level. But this is not the only option for the initial state. In fact, it may be appropriate to consider in modes in CPP, which are proportional to the Bessel function $h_\nu \sim Y_\nu(p\eta)$ (see, e.g., [26,36]) rather than Hankel ones. This Bessel function is some linear combination of the Hankel functions $H_\nu^{(1)}$ and $H_\nu^{(2)}$. (In the case of the principal series these modes behave as single oscillating exponents of the cosmological time at past infinity.) Then the corresponding (to the Fock space ground state) propagators also respect the dS isometry, but they do not have the proper Hadamard behavior. Furthermore, one should keep in mind that for other so-called alpha states, different from the BD one, which respect the dS isometry at tree level, this symmetry is broken in the loops [48] even in EPP. In a certain sense the latter fact is related to the non-Hadamard behavior of the corresponding propagators.

In the case of CPP we encounter the secular divergence [11,13] rather than the secular growth. In fact, one can straightforwardly see from (12) that in EPP the $\eta_{3,4}$

integrals are bounded from below by the external time η due to the causality of the Schwinger-Keldysh technique. Then the resulting expression for the correction in EPP (17) is singular in the limit $\eta \rightarrow 0$. Now, in the CPP the external conformal time, $\eta = \sqrt{\eta_1 \eta_2}$, is the upper limit of the $\eta_{3,4}$ integrals rather than the lower one. Then the analogs of integrals in (12) for CPP are divergent, because the lower bound in them is zero. To cut the divergence we must introduce the lower bound of $\eta_{3,4}$ integration by imposing some initial conformal time η_0 . This is the position of an initial Cauchy surface that cannot be taken to zero (or $t_0 = \log \eta_0$ to past infinity) because of the singularity. This introduction of the initial Cauchy surface breaks the dS isometry at one-loop level.

Then the reasonable question arises whether this symmetry is restored after the resummation of the loop corrections or not, i.e., can one remove the cut η_0 from the loop-corrected answer or not? In the loop calculations in CPP for the light fields there are two situations that should be considered separately. The first situation appears for the modes with momentum $p\sqrt{\eta_1 \eta_2} = p\eta > |\nu|$. Then in the external legs we cannot use for the Hankel functions their approximate form for small arguments (5). Hence, one can distinguish between the quantum level populations n_p and the anomalous quantum average κ_p inside the whole correction to the Keldysh propagator as in (10). To estimate the leading corrections to n_p and κ_p , as $p\eta_0 \rightarrow 0$, one can use for the Hankel functions inside $\eta_{3,4}$ integrals their asymptotic form for small arguments (5) when $\eta_{3,4} \leq \frac{|\nu|}{p}$. Then the gained contributions to these quantities in the one-loop order are as follows:

$$\begin{aligned} n_p(\eta) \simeq & \frac{\lambda^2}{(p\eta_0)^{2\nu}} \cdot \frac{A_-^2}{\nu} \int_1^\infty \frac{dv}{v^{2\nu+1}} \text{Re}\{F(v)\} \\ & - 2\lambda^2 \log\left(\frac{p\eta_0}{|\nu|}\right) \cdot A_- \int_1^\infty \frac{dv}{v} \\ & \times \text{Re}\left\{A_+ v^{2\nu} + A_+^* \frac{1}{v^{2\nu}} F(v)\right\} \\ & + \mathcal{O}(\lambda^2), \end{aligned} \quad (31)$$

$$\begin{aligned} \kappa_p(\eta) \simeq & \frac{\lambda^2}{(p\eta_0)^{2\nu}} \cdot \frac{A_-^2}{\nu} \int_1^\infty \frac{dv}{v^{2\nu+1}} F(v) \\ & - 2\lambda^2 \log\left(\frac{p\eta_0}{|\nu|}\right) \cdot A_- A_+^* \int_1^\infty \frac{dv}{v} \left[v^{2\nu} + \frac{1}{v^{2\nu}}\right] F(v) \\ & + \mathcal{O}(\lambda^2). \end{aligned} \quad (32)$$

Here we see not only the logarithmic, but also the powerlike divergence in the first terms. Such contributions to n_p and κ_p also appear in EPP, but in that case they exactly cancel each other in the correction to the Keldysh propagator (10). We see that in CPP for the modes with $p\eta > |\nu|$ this cancellation does not happen.

The second case that should be considered in CPP is for the modes with the momentum $p\eta < |\nu|$. In this case one cannot separate the corrections to n_p and κ_p as in (10) and should consider the correction to the whole Keldysh propagator as in (11). Then the one-loop correction is expressed as

$$\begin{aligned} \Delta_{1\text{-loop}} G^K(p|\eta, \eta) &= -\frac{A_-^2}{(p\eta)^{2\nu}} \eta^{D-1} \cdot \lambda^2 \log\left(\frac{\eta}{\eta_0}\right) \cdot 8A_- \text{Im}\{A_+\} \\ &\times \int_1^\infty \frac{dv}{v} v^{2\nu} \text{Im}\{F(v)\} + \mathcal{O}(\lambda^2), \end{aligned} \quad (33)$$

and the leading nonlogarithmic corrections cancel each other, as was the case in EPP.

Let us summarize and compare our observations both in CPP and in EPP. In the case of EPP the integrals in (12) are convergent, but have large contributions for such external momenta, p , that $p\eta \ll |\nu|$. For other momenta the corrections are just suppressed by the coupling constant, λ^2 .

At the same time, in CPP independently of the value of momentum p such integrals as in (12) are divergent at the lower bound of integration over internal times $\eta_{3,4}$, because the upper and lower limits of these integrals are exchanged in CPP, as compared to the case of EPP. One has to introduce the cutoff, η_0 , at the lower bound of integration independently of the value of p . This is the position of the initial Cauchy surface. Furthermore, for such p that $p\eta_0 < |\nu|$ there are large corrections to the Keldysh propagator. At the same time, for such p that $p\eta_0 > |\nu|$ there are no large corrections. What is peculiar now, for such p that $p\eta_0 < |\nu|$, but $p\eta > |\nu|$ ($\eta > \eta_0$) we have large powerlike (in $p\eta_0$) rather than logarithmic corrections, which are separately present in $n_p(\eta)$ and $\kappa_p(\eta)$. At the same time for such p that $p\eta < |\nu|$ we have only large logarithmic corrections, while powerlike corrections cancel each other in the whole expression for the corrected Keldysh propagator.

Furthermore, it is straightforward to see that for the case⁴ $p\eta < |\nu|$ in CPP the diagrams of the type in Fig. 2 will behave as $\lambda^4 \log^2(\frac{\eta}{\eta_0})$, as was shown in [13]. Therefore, in contrast with respect to the case of EPP, we must take into account the “loops inside loops” in the leading order in powers of the parameter $\lambda^2 \log(\frac{\eta}{\eta_0})$. Hence, in the corresponding Dyson-Schwinger equation, which sums leading loop corrections, we should use the exact Keldysh propagator also inside internal lines. This will lead to a nonlinear integral equation for the exact Keldysh propagator in CPP. As shown for principal series in [26,44] the nonlinear integral equations can lead to both singular solutions with the “explosive” behavior of the stress-energy tensor for a

finite proper time and smooth solutions, depending on the initial conditions. The complication with the complementary series is inapplicability of the kinetic approximation, which works when the function $N(p\eta)$ [or $n_p(\eta)$ and $\kappa_p(\eta)$] can be treated as a slow function in comparison with the modes. We will return to the situation in CPP and in global dS for the fields from the complementary series in future work.

VII. CONCLUSION

In this paper we revised the secular effects in dS spacetime, focusing on the complementary series. We derived the Dyson-Schwinger equation, which sums up the leading secular contributions in the case of light fields in EPP. We corrected the observations of [12] according to the observations that have been made in [13]. Namely, we excluded the diagrams of the type shown in Fig. 2, as these diagrams are actually suppressed as compared to the bead or chaplet diagrams, and provide subleading contributions.

Additionally, we have shown that the higher-point correlation functions should not be considered in the resulting system of Dyson-Schwinger equations, even for sufficiently small masses $\nu > \frac{D-1}{6}$, and can be taken to have its tree-level form. We then found the solution to this Dyson-Schwinger equation in one particular situation, when the initial values of quantum level populations and anomalous quantum averages are set at the initial physical momentum $P_0 = (p\eta)_0$. First, with this choice we preserve at least a part of the dS isometry, consisting of the rescalings $p \rightarrow \frac{1}{\alpha} p, \eta \rightarrow \alpha \eta$ (see also [1,2] for the related discussion). Second, it allows for a straightforward solution of the Dyson-Schwinger equation with a clear physical meaning.

It is interesting to explore different forms of initial conditions in EPP. One option is to fix some initial time η_0 , which can be considered as the time after which the self-interaction of the fields was adiabatically turned on, then define the initial values of $n_p(\eta_0)$ and may be even $\kappa_p(\eta_0)$. However, taking η_0 to past infinity in this case is not possible, as it would lead to a divergent physical energy density. The second option is to use the momentum-dependent Bogolyubov coefficients when constructing modes (3):

$$h_\nu(p\eta) \propto \alpha_p H_\nu^{(1)}(p\eta) + \beta_p H_\nu^{(1)}(p\eta). \quad (34)$$

In this case we must suppose that the coefficient β_p approaches zero for large momenta rapidly enough to obtain propagators (for the Fock space ground state) with the proper Hadamard behavior. These options for the initial conditions violate the dS isometry at the tree level. But their consideration still may provide further insights into the solutions of the Dyson-Schwinger equations for large IR

⁴The case $p\eta > |\nu|$ should be considered separately.

contributions, depending on the initial state of the system at the beginning of its evolution.

In addition, we have considered loop corrections in CPP in Sec. VI. While in the case of EPP the integrals in (12) are convergent, but have large contributions for such external momenta, p , that $p\eta \ll |\nu|$, in CPP we encounter quite a different situation. In the latter case independently of the value of momentum p such integrals as in (12) are divergent at the lower bound of integration over internal times $\eta_{3,4}$, because the upper and lower limits of these integrals are exchanged in CPP, as compared to the case of EPP. One has to introduce the cutoff, η_0 , at the lower bound of integration independently of the value of p . This is the position of the initial Cauchy surface. Furthermore, for such p that $p\eta_0 < |\nu|$ there are large corrections to the Keldysh propagator. At the same time, for such p that $p\eta_0 > |\nu|$ there are no large corrections. What is peculiar now, for such p that $p\eta_0 < |\nu|$, but $p\eta > |\nu|$ ($\eta > \eta_0$) we have large powerlike (in $p\eta_0$) rather than logarithmic corrections, which are separately present in $n_p(\eta)$ and $\kappa_p(\eta)$. At the same time for such p that $p\eta < |\nu|$ we have only large logarithmic corrections, while powerlike corrections cancel each other in the complete expression for the corrected Keldysh propagator. We will derive the Dyson-Schwinger equation, which sums these corrections up, in our future work and also consider the situation in global dS.

Finally, in Appendix A we demonstrated that for the principal series, unlike the complementary series, there is a static solution for the exact $n_p(\eta)$ and $\kappa_p(\eta)$ when the scalar field has a sufficiently large mass.

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: A BRIEF COMMENT ON THE PRINCIPAL SERIES IN THE EXPANDING POINCARÉ PATCH

Secular effects in EPP for the principal series were investigated in [11,26]. In this appendix we just reconsider the derivation of the Dyson-Schwinger equation, making

sure that the “bubble inside bubble” diagrams from Fig. 2 are not included as the corrections of subleading order.

In the field quantization (3) for principal series ($m > (D-1)/2$) the modes are as follows:

$$f_p(\eta) = \eta^{\frac{D-1}{2}} h_{iv}(p\eta), \quad h_{iv}(p\eta) = H_{iv}^{(1)}(p\eta) \frac{\sqrt{\pi}}{2} e^{-\frac{\pi}{2}\nu},$$

$$\nu = \sqrt{m^2 - \frac{D-1}{2}}. \quad (\text{A1})$$

In this case the behavior of the Hankel function at small arguments is oscillatory rather than powerlike:

$$h_{iv}(p\eta) \simeq A_+(p\eta)^{i\nu} + A_-(p\eta)^{-i\nu},$$

$$A_+ = \frac{2^{-i\nu} e^{-\frac{\pi}{2}\nu} \sqrt{\pi} (1 + \coth(\pi\nu))}{2\Gamma(1+i\nu)},$$

$$A_- = -\frac{i2^{i\nu}}{2\sqrt{\pi}} e^{-\frac{\pi}{2}\nu} \Gamma(i\nu). \quad (\text{A2})$$

Then at one-loop order the corrections to the level populations and to the anomalous quantum averages are [11,26]

$$n_p(\eta) \simeq -\Gamma \cdot e^{-2\pi\nu} \lambda^2 \log(p\eta),$$

$$\text{Re}\{\kappa_p(\eta)\} \simeq \frac{\Gamma}{2} \cdot e^{-\pi\nu} \lambda^2 \log(p\eta),$$

$$\text{Im}\{\kappa_p(\eta)\} \simeq C \cdot e^{-\pi\nu} \lambda^2 \log(p\eta),$$

$$\Gamma = \frac{S_{D-2}}{(2\pi)^{D-1}} \frac{1}{2\nu} \left| \int_0^\infty dx x^{\frac{D-3}{2}-i\nu} h_{iv}^2(x) \right|^2,$$

$$C = \frac{2}{\nu} \int_1^\infty \frac{dv}{v} \cos(\nu \log(v)) \int \frac{d^{D-1}\ell}{(2\pi)^{D-1}}$$

$$\times \text{Im} \left\{ h_{iv}^2(\ell n) h_{iv}^{*2}\left(\frac{\ell}{v}\right) \right\}, \quad (\text{A3})$$

where S_{D-2} is the volume of the unit $(D-2)$ -dimensional sphere. Please note here the presence of the extra suppressing factor $e^{-\pi\nu}$, which is present on top of the coupling constant λ^2 . Furthermore, every higher loop brings extra power of this factor. Then for large masses there is perhaps no even necessity to sum up the loop corrections, because it demands too many e-foldings to make the loop corrections comparable to the tree-level contributions to the correlation functions.

Nevertheless let us consider the Dyson-Schwinger equation, which resums the leading corrections, because for sufficiently small ν the factor $e^{-\pi\nu}$ is essentially absent. Then in the kinetic approximation we can write the Dyson-Schwinger equation for the leading logarithms (A3) [26]:

$$\begin{aligned}
G^K(\mathbf{p}|\eta, \eta) &= G_0^K(\mathbf{p}|\eta, \eta) + \lambda^2 \int \frac{d^{D-1}\mathbf{k}}{(2\pi)^{D-1}} \frac{d\eta_3}{\eta_3^D} \frac{d\eta_4}{\eta_4^D} \left[-\frac{1}{2} \theta(\eta_3 - \eta) \theta(\eta_4 - \eta) \right. \\
&\quad \times \rho(\mathbf{p}|\eta, \eta_3) \left(G_0^{K^2}(\mathbf{k} - \mathbf{p}|\eta_3, \eta_4) - \frac{1}{4} \rho^2(\mathbf{k} - \mathbf{p}|\eta_3, \eta_4) \right) \rho(\mathbf{p}|\eta_4, \eta) \\
&\quad + \theta(\eta_4 - \eta_3) \theta(\eta_3 - \eta) \rho(\mathbf{p}|\eta, \eta_3) \rho(\mathbf{k} - \mathbf{p}|\eta_3, \eta_4) G_0^K(\mathbf{k} - \mathbf{p}|\eta_3, \eta_4) G^K(\mathbf{p}|\eta_4, \eta) \\
&\quad \left. + \theta(\eta_3 - \eta_4) \theta(\eta_4 - \eta) G^K(\mathbf{p}|\eta, \eta_3) \rho(\mathbf{k} - \mathbf{p}|\eta_3, \eta_4) G_0^K(\mathbf{k} - \mathbf{p}|\eta_3, \eta_4) \rho(\mathbf{p}|\eta_4, \eta) \right]. \quad (\text{A4})
\end{aligned}$$

In order to find an approximate solution to this equation we use the ansatz (10) for the exact Keldysh propagator and for simplicity choose the BD vacuum as the initial state. Then we have the following equations for $n_p(\eta) \equiv n(p\eta)$ and $\kappa_p(\eta) \equiv \kappa(p\eta)$:

$$\begin{aligned}
n(p\eta) &\simeq -\Gamma \cdot e^{-2\pi\nu} \lambda^2 \log(p\eta) \\
&\quad + \lambda^2 \int_{\eta}^{\infty} \frac{d\eta_4}{\eta_4} \int_{\eta_4}^{\infty} \frac{d\eta_3}{\eta_3} (\eta_3 \eta_4)^{\frac{D-1}{2}} \text{Im}\{h_{i\nu}^*(p\eta_3) h_{i\nu}(p\eta_4) n(p\sqrt{\eta\eta_3}) + h_{i\nu}(p\eta_3) h_{i\nu}(p\eta_4) \kappa(p\sqrt{\eta\eta_3})\} \\
&\quad \cdot \int \frac{d^{D-1}\mathbf{k}}{(2\pi)^{D-1}} \text{Im}\{h_{i\nu}(k\eta_3) h_{i\nu}(|\mathbf{k} - \mathbf{p}|\eta_3) h_{i\nu}^*(k\eta_4) h_{i\nu}^*(|\mathbf{k} - \mathbf{p}|\eta_4)\} + \mathcal{O}(\lambda^2), \quad (\text{A5})
\end{aligned}$$

$$\begin{aligned}
\kappa(p\eta) &\simeq \left[\frac{\Gamma}{2} + iC \right] \cdot e^{-\pi\nu} \lambda^2 \log(p\eta) \\
&\quad + i\lambda^2 \int_{\eta}^{\infty} \frac{d\eta_4}{\eta_4} \int_{\eta_4}^{\infty} \frac{d\eta_3}{\eta_3} (\eta_3 \eta_4)^{\frac{D-1}{2}} [h_{i\nu}^*(p\eta_3) h_{i\nu}^*(p\eta_4) n(p\sqrt{\eta\eta_3}) + h_{i\nu}(p\eta_3) h_{i\nu}^*(p\eta_4) \kappa(p\sqrt{\eta\eta_3})] \\
&\quad \cdot \int \frac{d^{D-1}\mathbf{k}}{(2\pi)^{D-1}} \text{Im}\{h_{i\nu}(k\eta_3) h_{i\nu}(|\mathbf{k} - \mathbf{p}|\eta_3) h_{i\nu}^*(k\eta_4) h_{i\nu}^*(|\mathbf{k} - \mathbf{p}|\eta_4)\} + \mathcal{O}(\lambda^2). \quad (\text{A6})
\end{aligned}$$

The Eqs. (A5) and (A6), in contrast with the case of the complementary series discussed in Sec. V, have static solution $n(p\eta) \simeq n_f$, $\kappa(p\eta) \simeq \kappa_f$ with

$$\begin{aligned}
n_f &\simeq \frac{\Gamma^2 + 3C^2}{\Gamma^2 + C^2} \cdot e^{-2\pi\nu}, \\
\kappa_f &\simeq 2 \frac{\Gamma^2 + 2C^2 - i\Gamma C}{\Gamma^2 + C^2} \cdot e^{-\pi\nu}. \quad (\text{A7})
\end{aligned}$$

This solution is meaningful if $n_f \ll 1$ and $\kappa_f \ll 1$, which is certainly true under the condition $\nu \gtrsim \frac{2}{\pi} \log(\frac{1}{\lambda})$. Therefore, in the case of sufficiently large masses of a scalar field with

BD vacuum as the initial state, the system only slightly restructures the initial state and passes into some energy density on top of the out vacuum as a result of a long evolution in the expanding background.

APPENDIX B: DETAILS OF THE LOOP CALCULATION

In this appendix we show why the approximation of taking the external momentum to zero in the loop integral (13) is justified. Let us consider the region of integration, in which $v \in [1, \sqrt{\frac{|\nu|}{p\eta}}]$, $u \in [v, \frac{|\nu|}{p\eta v}]$, where for the “external” mode functions one can use their asymptotic from (5):

$$\begin{aligned}
\Delta G^K(\mathbf{p}|\eta, \eta) &\simeq \frac{8A_+^3 \text{Im}\{A_+\}}{(p\eta)^{2\nu}} \eta^{D-1} \lambda^2 \int_1^{\sqrt{\frac{|\nu|}{p\eta}}} \frac{dv}{v^{2\nu+1}} \int_v^{\frac{|\nu|}{p\eta v}} \frac{du}{u} \\
&\quad \times \int \frac{d^{D-1}\boldsymbol{\ell}}{(2\pi)^{D-1}} \text{Im}\left\{ h_{i\nu}(\ell v) h_{i\nu}(|\boldsymbol{\ell} - \mathbf{p}\eta u|v) h_{i\nu}^*\left(\frac{\ell}{v}\right) h_{i\nu}^*\left(\frac{|\boldsymbol{\ell} - \mathbf{p}\eta u|}{v}\right) \right\} \\
&\equiv \frac{8A_+^3 \text{Im}\{A_+\}}{(p\eta)^{2\nu}} \eta^{D-1} \lambda^2 \int_1^{\sqrt{\frac{|\nu|}{p\eta}}} \frac{dv}{v^{2\nu+1}} \int_v^{\frac{|\nu|}{p\eta v}} \frac{du}{u} \times \mathcal{F}(v, p\eta u). \quad (\text{B1})
\end{aligned}$$

Here we have changed the variables to u, v, ℓ as is explained in the main text, but have not yet taken the limit $p \rightarrow 0$ in the internal loop integral denoted by $\mathcal{F}(v, p\eta u)$. The integral in (B1) is convergent in both UV and IR limits. We can estimate it approximately by taking the leading terms in the expansion over small parameters. Namely, in the regions $\ell < p\eta u$ and $\ell > p\eta u$ we can assume that $|\ell - p\eta u| \simeq p\eta u$ and $|\ell - p\eta u| \simeq \ell$ correspondingly, tracing the most singular part of the integral, while the corrections to these leading terms will be suppressed.

Let us stress at this point that there is no problem with the fact that when $\ell \sim p\eta u$ the relations $|\ell - p\eta u| \simeq \ell$ and

$|\ell - p\eta u| \simeq \ell$ are violated. In fact, one can further subdivide the regions of integration and estimate the integral when ℓ is in the vicinity of the vector $p\eta u$ to find that these contributions do not change the leading result in the limit $p\eta \rightarrow 0$. The idea is that due to IR convergence of this loop integral and the suppression of the corresponding correction to the propagator from this integration region in comparison with the leading secularly growing term, we can safely take the limit $p\eta \rightarrow 0$ in these expressions. Thus, in the first region $\ell < p\eta u$ we can approximate:

$$\begin{aligned} \mathcal{F}_I(v, p\eta u) &\simeq \frac{\Omega_{D-2}}{(2\pi)^{D-1}} \int_0^{p\eta u} d\ell \ell^{D-2} \operatorname{Im} \left\{ h_\nu(\ell v) h_\nu^* \left(\frac{\ell}{v} \right) h_\nu(p\eta u v) h_\nu^* \left(\frac{p\eta u}{v} \right) \right\} \\ &\simeq \frac{\Omega_{D-2}}{(2\pi)^{D-1}} A_-^3 \operatorname{Im}\{A_+\} \cdot v^{2\nu} \cdot \int_0^{p\eta u} d\ell \ell^{D-2} \left[\frac{1}{\ell^{2\nu}} + \frac{1}{(p\eta u)^{2\nu}} \right] = C_I \cdot v^{2\nu} \cdot (p\eta u)^{D-1-2\nu}, \end{aligned} \quad (\text{B2})$$

where C_I is some numerical constant. In the last line of (B2) we used the appropriate asymptotics (5) and neglected $\frac{1}{v^{2\nu}}$ in comparison with $v^{2\nu}$ as $v > 1$. We can now further take the integrals over u and v and find the corresponding correction to the Keldysh propagator:

$$\begin{aligned} \Delta_I G^K(p|\eta, \eta) &\simeq \frac{8A_-^3 \operatorname{Im}\{A_+\}}{(p\eta)^{2\nu}} \eta^{D-1} \lambda^2 \int_1^{\sqrt{\frac{v}{p\eta}}} \frac{dv}{v^{2\nu+1}} \int_v^{\frac{v}{p\eta}} \frac{du}{u} \times C_I \cdot v^{2\nu} \cdot (p\eta u)^{D-1-2\nu} \\ &= \text{const} \cdot \frac{\lambda}{(p\eta)^{2\nu}} \eta^{D-1} \cdot (p\eta)^{\frac{D-1-2\nu}{2}} = \text{const} \cdot G_0^K(p|\eta, \eta) \cdot \lambda^2 \cdot (p\eta)^{\frac{D-1-2\nu}{2}}. \end{aligned} \quad (\text{B3})$$

It can be seen now that the correction from this region goes to zero in the limit $p\eta \rightarrow 0$ as compared to the secularly growing term (17), because $D - 1 - 2\nu > 0$ for the fields from the complementary series.

In a similar way, we can make the estimation of the contribution to the integral coming from the region $p\eta u < \ell < \frac{v}{v}$:

$$\begin{aligned} \mathcal{F}_{II}(v, p\eta u) &\simeq \frac{\Omega_{D-2}}{(2\pi)^{D-1}} \int_{p\eta u}^{\frac{v}{v}} d\ell \ell^{D-2} \operatorname{Im} \left\{ h_\nu^2(\ell v) h_\nu^{*2} \left(\frac{\ell}{v} \right) \right\} \\ &\simeq C_{II} \cdot v^{2\nu} \cdot \left[\left(\frac{v}{v} \right)^{D-1-2\nu} - (p\eta u)^{D-1-2\nu} \right]. \end{aligned} \quad (\text{B4})$$

We see that in the second term of (B4) the dependence on the parameter $p\eta$ is the same as in the Eq. (B2), which leads to the suppressed contribution of the form (B3). In the remaining integration region $\ell > \frac{v}{v}$ the parameter $p\eta$ does not appear in the leading approximation of the integrals. Moreover, due to the IR convergence of the integrals above, one can neglect the contribution (B3) and set $p\eta u \rightarrow 0$ in (B4) with the same precision. Effectively, this leads to the expressions (14)–(16), where we omitted the external

momentum p in the loop momentum integral from the beginning and put $\mathcal{F}(v, 0) \equiv F(v)$. The similar analysis of different integration regions shows that actually the first term in the second line (B4) gives the dominant contribution to the loop correction and determines the coefficient A in (16):

$$\begin{aligned} A &\simeq -8A_- \operatorname{Im}\{A_+\} \int_1^\infty \frac{dv}{v^{2\nu+1}} \cdot C_{II} \cdot \frac{v^{D-1-2\nu}}{v^{D-1-4\nu}} \\ &= -8A_- \operatorname{Im}\{A_+\} \frac{v^{D-1-2\nu}}{D-1-2\nu} C_{II}. \end{aligned} \quad (\text{B5})$$

We would like to stress here that we consider the massive $m \neq 0$ fields from the complementary series. Hence, all the loop momentum integrals are IR convergent. In the pure massless $m = 0$ case there are other dangerous IR effects in de Sitter spacetime, which are beyond the consideration in our paper here [13].

Finally, let us explain why the first two lines of the Eq. (12) do not contribute to the secular growth. Consider the product of “external legs” of this diagram:

$$\begin{aligned}\rho(\mathbf{p}|\eta, \eta_3)\rho(\mathbf{p}|\eta_4, \eta) &= 4 \cdot \text{Im} \{h_\nu(p\eta)h_\nu^*(p\eta\tau_3)\} \text{Im} \{h_\nu(p\eta\tau_4)h_\nu^*(p\eta)\} \\ &\simeq -\frac{4A_-^2}{(p\eta)^{2\nu}} \text{Re}\{h_\nu^*(p\eta\tau_3)\} \text{Re}\{h_\nu(p\eta\tau_4)\} + \dots,\end{aligned}\quad (\text{B6})$$

where the ellipsis stand for the terms suppressed by the factor $(p\eta)^{2\nu}$ in the limit $p\eta \rightarrow 0$. In the leading approximation the mode functions h_ν at small arguments are pure imaginary (5); therefore we must take the subleading term for both external mode functions in this region. As the result, repeating the steps between Eqs. (13) and (16) we do not obtain any secular logarithms:

$$\begin{aligned}\Delta_2 G^K(\mathbf{p}|\eta, \eta) &\simeq \frac{4A_-^2}{(p\eta)^{2\nu}} \eta^{D-1} \frac{\lambda^2}{2} \int_1^\infty \frac{d\tau_4}{\tau_4} \int_1^\infty \frac{d\tau_3}{\tau_3} (\tau_3\tau_4)^{\frac{D-1}{2}} \text{Re}\{h_\nu(p\eta\tau_3)\} \text{Re}\{h_\nu(p\eta\tau_4)\} \\ &\quad \times \int \frac{d^{D-1}\mathbf{q}}{(2\pi)^{D-1}} \text{Re}\{h_\nu(q\tau_3)h_\nu(|\mathbf{q}-\mathbf{p}\eta|\tau_3)h_\nu^*(q\tau_4)h_\nu^*(|\mathbf{q}-\mathbf{p}\eta|\tau_4)\} \\ &\simeq \lambda^2 \cdot \frac{C}{(p\eta)^{2\nu}} \eta^{D-1}, \quad C = \frac{8A_-^2 \text{Im}^2\{A_+\}}{2\nu} \int_1^\infty \frac{dv}{v^{2\nu+1}} \int \frac{d^{D-1}\ell}{(2\pi)^{D-1}} \text{Re}\left\{h_\nu^2(\ell v)h_\nu^{*2}\left(\frac{\ell}{v}\right)\right\}.\end{aligned}\quad (\text{B7})$$

The expression in the last line is well defined under the condition $\nu < \frac{D-1}{4}$, which we impose in the article for convenience. However, even in the case when $\nu > \frac{D-1}{4}$ still there will be no IR divergence or secular growth from this term, as we already pointed out in the footnote after the

Eq. (13): one just need to keep nonzero $p\eta$ in the loop momentum integral, then estimate all the integrals in the first and second lines of (B7) and get the correct coefficient in the last line of this equation.

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