

Double parton interactions initiated by direct photons in γp and γA collisions revisited

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We consider the double parton scattering (DPS) initiated by direct photons in γp and γA collisions. We extend the previously known results for photoproduction to the case of electroproduction with nonzero photon virtuality Q^2 , and then consider the DPS for both photo and electroproduction for HERA and EIC (electron ion collider) kinematics. The number of DPS events at the EIC is shown to be of the same order as in HERA and is even larger, with increase in luminosity compensating the decrease of the center-of-mass energy.

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I. INTRODUCTION

The study of double parton scattering (DPS) and multi-parton interactions (MPI) started in early 1980s [1,2]. In recent years there was a considerable progress in the subject, both theoretically [3–14] (see also the recent review [15]) and experimentally. The recent applications however dealt mostly with pp and pA collisions. On the other hand, the MPI, including DPS in deep-inelastic scattering attracted relatively little attention, except some research on MPI in HERA, where it was claimed that the inclusion of MPI improves the agreement between the experimental and theoretical description of HERA measurements, although no concrete evidence for MPI in HERA was presented experimentally [16,17]. More recently, MPI in DIS were studied in [18] and in [19], where the contribution of MPI in the underlying event in the future electron-ion collider (EIC) was considered. The MPI and in particular DPS are important both from the point of view of searching for new physics as irreducible backgrounds, as well as for the study of the perturbative and nonperturbative correlations of partons in the nucleon and in the nuclei.

One of the new ideas suggested recently to explain the observed cross sections of DPS in pp and pA scattering was the so-called $1 \otimes 2$ mechanism. While in conventional DPS two independent partons from one nucleon interact with two independent partons from the second nucleon, in $1 \otimes 2$ mechanism [see Fig. 1(a)] perturbative gluon from one of the nucleons involved in the collision splits into two

and interacts with two independent partons of the second nucleon. The analysis in [9,10] shows that the contribution of this mechanism in pp and pA collisions has the same order of magnitude as the contribution of the conventional mechanism due to independent partons [see Fig. 1(b)]. However, in pp and pA collisions, it is not clear how to isolate this mechanism and get the conclusive confirmation of its existence.

The photon nucleus and photon nucleon collisions give a unique possibility to separate such mechanism. Namely, we can consider the so-called direct photons. The wave function of such photons can be viewed as corresponding to a perturbatively created quark-antiquark pair where the quarks can be either light or heavy, like charmed quark. In distinction to deep-inelastic scattering we can consider the processes where each of the created quarks interacts with a parton from the target nucleon, creating a quark and gluonic jets, where the gluon comes from the nucleon side. Such processes can be especially easily identified if one of the

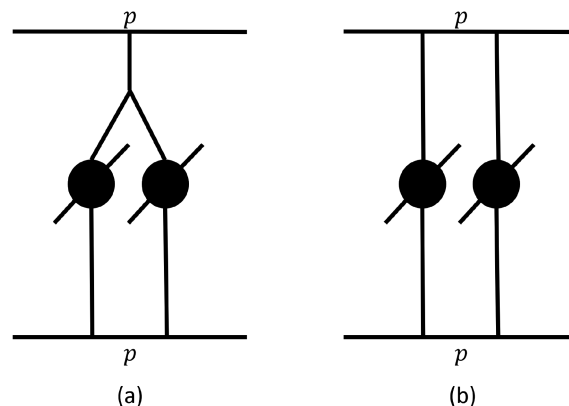


FIG. 1. The $1 \otimes 2$ (a) and $2 \otimes 2$ (b) mechanisms for DPS in pp collisions.

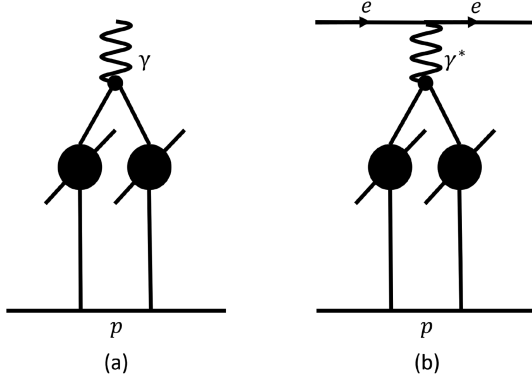


FIG. 2. The DPS processes with the direct photons for γp deep-inelastic collisions: (a) photoproduction and (b) electroduction.

quarks is a charmed quark, and we can neglect the charm-anticharm content of the nucleon. Such processes are the direct analogs of the $1 \otimes 2$ mechanism, see Fig. 2. We expect such processes to be dominant in DPS at large $x_\gamma \equiv x_1 + x_2 \sim 1$, where x_1, x_2 are the Bjorken fractions of quark jets. In this paper we consider the case when both $x_1, x_2 \geq 0.2$.

Such processes for the case of the photoproduction [see Fig. 2(a)] were first considered in [20], where the DPS associated with direct photons were considered for HERA and LHeC kinematics and for ultraperipheral collisions at the LHC. In this paper we extend the results of [20] to the general case of electroduction, i.e., we consider the case of virtual photons $Q^2 \neq 0$ [Fig. 2(b)]. In the limit $Q^2 \rightarrow 0$ our results correspond to photoproduction and we correct several arithmetic mistakes made in numerical simulations in [20]. We consider the DPS due to direct photons both in the case of HERA and the EIC. Such processes may be an important part of the underlying event (UE) in these colliders [19]. We see that although the energies of HERA are four times larger than the maximal energy at the EIC, this is compensated by much larger luminosity. We shall consider the maximum energy of $W^2 = 22500 = 150^2 \text{ GeV}^2$, where we shall assume luminosity as much as 100 times larger than in HERA, and the energies $W^2 = 10000 \text{ GeV}^2$ where we assume luminosity 1000 times larger than in HERA. For photoproduction with charmed jets we obtain for $p_t = 3 \text{ GeV}$ approximately 4×10^4 events per year for total luminosity $\sim 1.7 \times 10^{31} \text{ cm}^{-2} \text{ s}^{-1}$ for HERA, and of order 10^6 events per year at the EIC. The increase of the luminosity compensates for the decrease in the center-of-mass energy.

Our results may permit us to study both the short-range correlations in a nucleon and nuclei and study the structure of underlying event at the EIC [19]. We shall consider two important final-state cases. First is that each hard event has two light quark and two gluon jets, and the second case is two gluonic jets and two charmed quark states. Here, certainly, it is much more easy to observe the charm-gluon dijets.

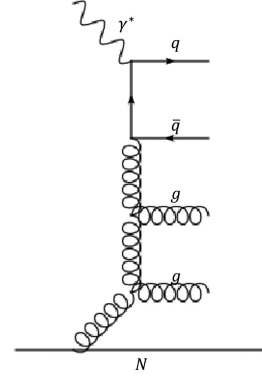


FIG. 3. The leading-twist final state with two gluon and two charmed jets.

The experience of HERA shows that it is possible to identify charmed jets with the scale $p_T > 3.5 \text{ GeV}$. Consequently, we shall show the results for DPS starting from $p_T > 3 \text{ GeV}$ [16,17,21]. We expect the same cutoff for the charmed jets at the EIC.

For light quark final states the cutoffs at HERA were two time larger (see minireview in [19]), i.e., of order 6 GeV . We expect for the EIC there will be the same scale. Nevertheless, it will be interesting to start from 3 GeV since this is the scale (2–3 GeV) of MPI transverse cutoffs in Pythia, and, on the other hand, the ability to distinguish jets is constantly improving although the relevant techniques are mostly developed for the LHC processes. Thus, our results can be also used as analytical counterpart of Monte Carlo (MC) simulations [19].

Since we find a large number of DPS events at HERA and the LHC, the question is whether we can actually observe them, and in particular to distinguish from leading twist $2 \rightarrow 4$ contributions depicted in Fig. 3.

As it was already discussed for the case of pp collisions, the DPS are dominant in the case when the disbalances of the dijets are rather small, in distinction to the hedgehog picture when four jets are produced as a result of a hard collision of two partons that produce a four-jet event. Thus, we will see in DPS events two pairs of nearly back-to-back jets. This kinematical preference is in stark contrast with “hedgehoglike” configurations of four jets stemming from a single collision [7].

Note that there is an additional effect that can help to observe DPS at already small p_T . It is the expected smaller level of activity (pedestal) in direct photon-nucleon interactions than in pp and even in generic photon-nucleon collisions. This is due to the selection by the process of the $c\bar{c}$ pair of small size in the collision.

The numerical results demand the MC calculations. Such calculations were done using Pythia and Madgraph for pp collisions, their authors showed that it is possible to introduce observables that are dominated by the DPS in the back-to-back kinematics. Note, however, that these simulations were carried through for $p_T > 10 \text{ GeV}$. When we go to small p_T

of order 3 GeV there are two factors influencing the result. On the one hand the DPS contribution, being higher twist than $2 \rightarrow 4$ contribution, is suppressed by μ^2/Q^2 , so the smaller the transverse scale the role of DPS is higher. On the other hand the MC simulations are based on the difference between hedgehog and back-to-back kinematics, and for small transverse momenta the disbalances may be of the same order as the transverse momenta of the jets themselves. Thus, further MC simulations must be carried to distinguish DPS from parasitical $2 \rightarrow 4$ leading-twist contributions depicted in Fig. 3.

The interesting question is in what kinematic region the direct gluon dominates over the resolved photon contribution. The preliminary calculations based on $2 \otimes 2$ analysis show that in the kinematics region considered in this paper $x_\gamma \equiv x_1 + x_2 \geq 0.4$ the direct photon contribution dominates even if we include the $1 \otimes 2$ contribution for resolved photon. We expect that the resolved contribution starts to dominate at $x_\gamma \leq 0.2$, when the resolved photon Parton Distribution Function (PDF) [22] rapidly increase; however, a detailed calculation in this kinematic region is needed [23]. Moreover, due to the small phase space available for HERA and especially at the EIC, the effect of the rapidly increasing PDFs at $x_i (i = 1-4)$ for resolved photon will be suppressed by phase-space constraints in hard matrix elements, decreasing the resolved photon contributions. We expect these constraints to be relaxed at LHeC.

One way to distinguish direct and resolved photons will be the large rapidity gap between the quark-jet pair (large x_1, x_2) and gluon jets (small x_3, x_4). Indeed the $q\bar{q}$ pair is created in the fragmentation region, while the gluon jets are created in the target region. This property will also help to identify the DPS event.

The paper is organized in the following way. In Sec. II we calculate the partonic model contributions to the DPS cross section for transverse and virtual photons (some technical details of the calculations are given in Appendix A). In particular we show how to use gauge invariance to calculate the contribution due to longitudinal photons. In Sec. III we calculate the total number of DPS events in different kinematics, both for photo and electroproduction, and include the pQCD evolution. In Sec. IV we reconsider numerical results for photoproduction in HERA and compare them with the results for the EIC. We also consider the numerical results for electroproduction for the EIC and HERA kinematics. Our results are summarized in Sec. V. In Appendix A we give the details of calculation of DPS cross sections in the Breit system. In Appendix B we study how the gauge invariance is realized.

II. BASIC FORMULAS FOR MPI IN THE DIRECT PHOTON-PROTON SCATTERING FOR GIVEN Q^2

A. DPS kinematics

The DPS kinematics is similar to the kinematics of DIS process [24],

$$\frac{d\sigma}{dQ^2 dy} = \frac{\pi e^2}{4W} \frac{L_{\alpha\beta} D^{\alpha\nu} D^{\beta\mu} W_{\mu\nu}}{(2\pi)^3}, \quad (1)$$

where we integrated over the azimuthal angle ϕ_e of the scattered electron. Here $W^2 = 2kp$ is the center-of-mass energy squared of the proton and electron, while $s = 2qp$ is the center-of-mass energy squared of the photon and proton, $s = yW^2$, $q = k' - k$ emit the virtual photon, and p is the momentum of the proton. Here q is the momentum of the virtual photon, $q^2 = -Q^2$, k', k are the final and initial momenta of the electrons.

The propagator of the virtual photon $D_{\mu\nu}$ is conveniently represented as,

$$D_{\mu\nu}(q) = -\frac{\left(g_{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)}{q^2} = -\left(e_L^\mu e_L^\nu - \sum_{\lambda=1,2} e_T^{\lambda\mu} e_T^{\lambda\nu}\right) / q^2, \quad (2)$$

where

$$e_L = (q + 2xp)/Q, \quad Q \equiv \sqrt{Q^2} \quad (3)$$

is the polarization vector of the longitudinal gluon and $e_T^\lambda, \lambda = 1, 2$ are the basic polarization vectors of the transverse photon, The leptonic tensor $L_{\mu\nu}$ is given by

$$L_{\mu\nu} = 4(k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu} Q^2/2), \quad (4)$$

the tensor $W^{\mu\nu}$ is the hadronic tensor which we shall start to calculate in the next subsection, and $x = Q^2/(2pq)$. This kinematics is depicted in Fig. 4.

B. Parton model

In this subsection we consider the process of production of two dijets in the parton model. The corresponding kinematics is depicted in Fig. 5, and is analogous to the

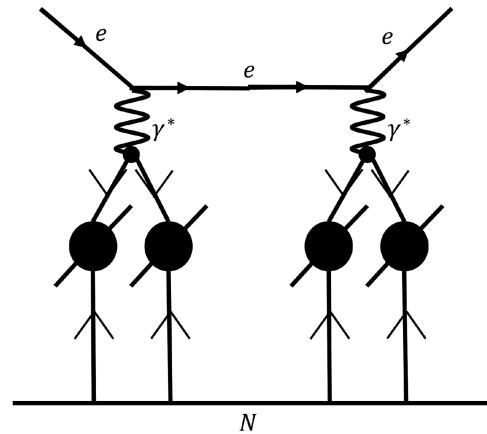


FIG. 4. The DPS kinematics in DIS.

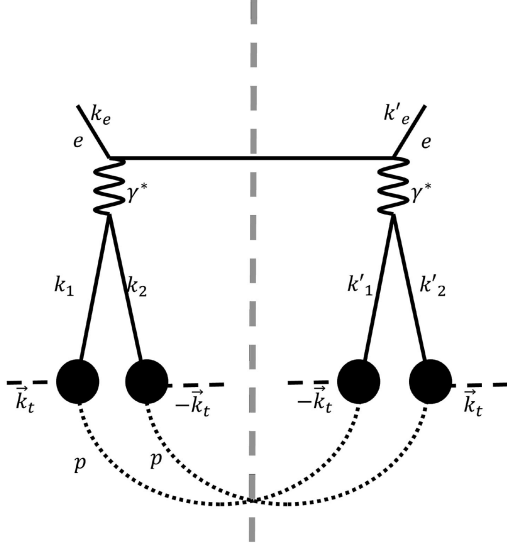


FIG. 5. Parton model electroproduction with DPS.

$1 \otimes 2$ transition in pp collisions as it was mentioned in the Introduction. Let us parametrize the momenta of quarks and gluons using Sudakov variables (k_1, k_2 are the momenta of virtual quarks and antiquark of the $q\bar{q}$ pair and k_3, k_4 are the gluon momenta). Let us analyze the lowest-order amplitude shown in Fig. 3 for the double-hard collision which involves photon splitting.

We decompose parton momenta k_i in terms of the Sudakov variables using the lightlike vectors q' and p along the incident photon and proton momenta,

$$\begin{aligned} k_1 &= x_1 q' + \beta p + k_t, & k_3 &\simeq (x_3 - \beta) p, \\ k_2 &= x_2 q' - (\beta + x) p - k_t, & k_4 &\simeq (x_4 + \beta - x) p, \\ \vec{k}_t &= \vec{\delta}_{12} = -\vec{\delta}_{34} \ (\delta' \equiv 0); & k_0 &\simeq (x_1 + x_2) q. \end{aligned}$$

Here $q' = q + xp, q'^2 = 0$, and we neglect the nucleon mass $p^2 = 0$. Note that $x_1 + x_2 = 1, q = k_1 + k_2$, and $x = Q^2/(2pq)$. For the case of massive, i.e., charmed quark, we shall neglect the charm quark masses except while dealing with infrared singularities and phase space constraints. The light cone fractions $x_i, (i = 1, \dots, 4)$, are determined by the jet kinematics (invariant masses and rapidities of the jet pairs). The momenta $\vec{\delta}_{12}, \vec{\delta}_{34}, \delta' \equiv \delta_{12} + \delta_{34}$ are dijet transverse imbalances in the parton model their sum is zero due to momentum conservation. The fraction β that measures the *difference* of the longitudinal momenta of the two partons coming from the hadron, is arbitrary and is integrated over. The fixed values of the parton momentum fractions $x_3 - \beta$ and $x_4 + \beta$ correspond to the plane wave description of the scattering process in which the longitudinal distance between the two scatterings is arbitrary. This description does not correspond to the physical picture of the process we are discussing, where two partons originate from the same bound state. In order to

ensure that partons 3 and 4 originate from the *same hadron* of a finite size, we have to introduce integration over β in the amplitude, in the region $\beta = \mathcal{O}(1)$, as was explained in detail in [7].

The Feynman amplitude contains the product of two virtual propagators. The virtualities k_1^2 and k_2^2 in the denominators of the propagators can be written in terms of the Sudakov variables as

$$k_1^2 = x_1 \beta s - k_t^2, \quad k_2^2 = -x_2 (\beta + x) s - k_t^2, \quad (5)$$

where $k_t^2 \equiv (\vec{k}_t)^2 > 0$ is the square of the two-dimensional transverse momentum vector.

The singular contribution we are looking for originates from the region $\beta \ll 1$. Hence, the precise form of the longitudinal smearing does not play role and the integral over β yields the amplitude A

$$\begin{aligned} A &\sim \int \frac{d\beta}{(x_1 \beta s - k_t^2 + i\epsilon)(-x_2 (\beta + x) s - k_t^2 + i\epsilon)} \\ &= \frac{1}{s(k_t^2 + x_1 x_2 Q^2)}. \end{aligned} \quad (6)$$

The numerator of the full amplitude is proportional to the *first power of the transverse momentum* k_t . As a result, the squared amplitude (and thus the differential cross section) acquires the necessary factor $1/k_t^2$ that enhances the back-to-back jet production.

The integration over k_t gives a single log contribution to the cross section $\alpha_{\text{em}} \log(Q_1^2/\mu^2)$ for light quarks and $\alpha_{\text{em}} \log(Q_1^2/m_c^2)$, where $Q_1 \sim Q_2$ is the characteristic transverse scale of the hard processes. The scale μ is the scale beyond which the nonperturbative dynamics becomes dominant and is of order of $m_\rho^2 \sim 4m_{\text{const}}^2$, where m_ρ is the ρ -meson mass, $m_{\text{const}} \sim 200 - 300$ MeV is the constituent quark mass for the light quark.

Note, that strictly speaking the answer due to diagram 3 is proportional to $\delta(\vec{k}_{1t} + \vec{k}_{2t})/(k_{1t}^2 + x_1 x_2 Q^2)$, where k_{1t} and k_{2t} are the transverse momenta of quark and antiquark ($\vec{k}_{1t} = -\vec{k}_{2t} \equiv \vec{k}_t$ in the parton model).

The parton model answer is only single collinearly enhanced while we are looking for the double collinear enhanced contributions [7]. It is well-known that these contributions originate from the gluon dressing of the parton model vertex, with the δ function becoming a new pole.

Assuming factorization theorem in order to calculate the vertex function it is enough to calculate the diagram 3. The direct calculation gives the answer for the t channel amplitude (see Appendix A for details),

$$W^{\mu\nu} \sim \int \frac{d^2 k_t}{(2\pi)^2} \frac{J^\mu J^\nu + \kappa_{\mu\nu}}{(k_t^2 + x_1 x_2 Q^2)^2}, \quad (7)$$

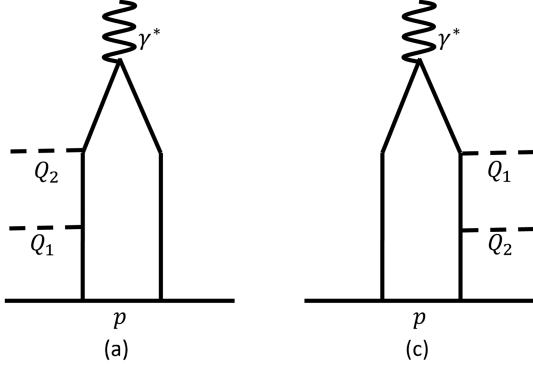


FIG. 6. Gauge invariance and parton model, Q_1 and Q_2 are the dijet momenta. The additional diagrams (a) and (c) needed for gauge invariance in parton model.

where

$$\begin{aligned} J^\mu &= 2x_1x_2q'^\mu - 2\vec{k}_{1t}^2 p^\mu + (x_2 - x_1)k_{1t}^\mu = J_L^\mu + J_T^\mu, \\ \kappa_\mu &= \frac{2i}{s} \epsilon_{\mu abc} k_{1t}^a q^b p^c, \quad J_T^\mu = (x_2 - x_1)k_{1t}^\mu, \\ J_L^\mu &\sim 2x_1x_2q'^\mu - (k_t^2/s)p^\mu. \end{aligned} \quad (8)$$

Note that $J_T \kappa = 0$, i.e., these two transverse currents are actually orthogonal to each other.

This is however not the end of the story. It is easy to see that the longitudinal part of the current J_L^μ [see Eq. (8)] is not conserved; $qJ_L^\mu \neq 0$, where J_L^μ is the contribution to the longitudinal current of the diagram Fig. 3, i.e., the expression in Eq. (8). As shown in Appendix B, in order to include the gauge invariance one has to include the contributions of diagrams 6(a) and 6(c); the full current is a sum $J_L^a + J_L^b + J_L^c$, where a and c correspond to contributions of the two latter diagrams.

We however do not need the explicit form of the full longitudinal vector current. Indeed it enters the amplitude in the form $J_L e_L$, where e_L is the polarization vector of the longitudinal photon that interacts with the hadron. Recall that from the condition $q e_L = 0$ we have $e_L = (q' + xp)/\sqrt{Q^2} = (q + 2xp)/\sqrt{Q^2}$. So the contribution to the amplitude has two parts proportional to qJ_L and pJ_L , respectively. For the full longitudinal current we have $qJ_L = 0$ due to the gauge invariance (the gauge invariance is explicitly checked in Appendix B). On the other hand it is easy to see that pJ_L is due only to the contribution of the diagram Fig. 5; for Figs. 6(a) and 6(c) this contribution is zero, since they are proportional to $\gamma^\mu u(p)$ or $\bar{u}(p)\gamma^\mu$ and these are zero since $\hat{p}u(p) = 0$, due to equations of motion. Then $e_L J_L = 2xpJ_L^b/Q = 2x_1x_2Q$, where $Q = \sqrt{Q^2}$.

Then remaining parts of the calculation is easily done and give

$$\begin{aligned} \frac{d\sigma}{dQ^2 dy} &= Q_q^2 \frac{\alpha_e^2}{2} \int \frac{d^2 k_t}{(2\pi)^2} \frac{y}{k_t^2 + x_1 x_2 Q^2} \\ &\times \left(\vec{k}_t^2 (x_1^2 + x_2^2) \left(\frac{(1 + (1-y)^2)}{y^2 Q^2} - 2m_e^4/Q^4 \right) \right. \\ &\left. + 8x_1^2 x_2^2 (1-y)/y^2 \right), \end{aligned} \quad (9)$$

where $W^2 = 2kp = ys$ is the squared center-of-mass energy of the e - p collider, and in parton model $x_1 + x_2 = 1$, and Q_q is the quark charge in electron units, i.e., $Q_u = Q_c = 2/3$, $Q_d = Q_s = -1/3$.

III. FULL DPS CROSS SECTION AND THE EVENT NUMBER

A. Hard matrix elements

The cross sections of hard processes $d\sigma/dt$ are the usual dijet cross sections calculated with $s \rightarrow yW^2$, where s is the invariant energy of the γp system.

We have

$$d\sigma/dp_{1t}^2 = \frac{M^2}{(x_1 x_3 \sqrt{x_1 x_3}) 16\pi s^{3/2} \sqrt{x_1 x_3 s - 4p_{1t}^2}}, \quad (10)$$

where p_{1t} is the dijet transverse scale, and x_1, x_3 are jet momentum fractions. The matrix element M^2 of the quark-gluon scattering is given by [25]

$$\begin{aligned} M^2 &= (4\pi\alpha_s(p_{1t}^2))^2 \left(-\frac{4}{9} \left(\frac{\hat{u}}{\hat{s}} + \frac{\hat{s}}{\hat{u}} \right) + \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2} \right) \\ &= 4\pi\alpha_s(p_{1t}^2)^2 \left(-(4/9) \left(1 + z + \frac{1}{1+z} \right) + 2(1+z^2) \right), \end{aligned} \quad (11)$$

where

$$\begin{aligned} \hat{s} &= x_1 x_3 s, \quad \hat{t} = -\hat{s}(1-z)/2, \quad \hat{u} = -\hat{s}(1+z)/2, \\ z &= \cos \theta = \tanh(y_1 - y_3)/2 = \sqrt{1 - 4p_{1t}^2/(x_1 x_3 s)}. \end{aligned} \quad (12)$$

The angle θ is the scattering angle in the center-of-mass frame of the dijet. The region of integration is given by

$$x_1 x_3 s - 4(p_{1t}^2 + m_e^2) \geq 0, \quad x_1 > 0.2. \quad (13)$$

The integration over the second dijet event goes in the same way, with $x_1 \rightarrow x_2, x_3 \rightarrow x_4$.

B. Inclusion of a target

The target can be either a proton or a nucleus with atomic weight A (the corresponding factors were calculated in [20]). Here we just recall the results.

1. The γp case

In order to estimate whether it is feasible to observe the MPI events discussed in the previous section, we have to calculate the double differential cross section and then to convolute it with the photon flux.

For the case of the proton target we have

$$\begin{aligned} \frac{d\sigma}{dx_1 x_2 dx_3 dx_4 dp_{1t}^2 dp_{2t}^2} \\ = D(x_1, x_2, p_{1t}^2, p_{2t}^2) G(p_{1t}^2, x_3) G(p_{2t}^2, x_4) \frac{d\sigma}{dt_1} \frac{d\sigma}{dt_2} \\ \times \int \frac{d^2\Delta}{(2\pi)^2} U(\Delta). \end{aligned} \quad (14)$$

Here we carried the integration over the momenta Δ conjugated to the distance between partons, obtaining the last multipliers in the equations above. This integral measures the parton wave function at zero transverse separation between the partons and hence it is sensitive to short-range parton-parton correlations. For the γp case, the factor $U(x_1, x_2, \Delta)$ in the approximation when two gluons are not correlated, is equal to a product of two gluon form factors of the proton,

$$U(\Delta, x_3, x_4) = F_{2g}(\Delta, x_3) F_{2g}(\Delta, x_4). \quad (15)$$

For the numerical estimates we use the following approximation for ${}_2GPD$ of the nucleon:

$$\begin{aligned} {}_2D(x_3, x_4, p_{1t}^2, p_{2t}^2, \Delta) = G(x_3, p_{1t}) G(x_4, p_{2t}) \\ \times F_{2g}(\Delta, x_3) F_{2g}(\Delta, x_4), \end{aligned} \quad (16)$$

where the two-gluon form factor,

$$F_{2g}(\Delta) = \frac{1}{(1 + \Delta^2/m_g^2)^2}, \quad (17)$$

and the parameter

$$m_g^2 = 8/\delta, \quad (18)$$

where

$$\delta = \max(0.28 \text{ fm}^2, 0.31 \text{ fm}^2 + 0.014 \text{ fm}^2 \log(0.1/x)), \quad (19)$$

was determined from the analysis of the exclusive J/Ψ diffractive photoproduction [26]. The functions G are the gluon PDFs of the proton, that we parametrize using [27]. Then

$$\int \frac{d^2\Delta}{(2\pi)^2} U(\Delta) = \frac{1}{4\pi} \frac{m_g^2(x_3) m_g^2(x_4) (m_g^4(x_3) - m_g^4(x_4) + 2m_g^2(x_3) m_g^2(x_4) \log(m_g^2(x_4)/m_g^2(x_3)))}{(m_g^2(x_3) - m_g^2(x_4))^3}. \quad (20)$$

In the limit $x_3 \sim x_4$ we recover

$$\int \frac{d^2\Delta}{(2\pi)^2} U(\Delta) = \frac{m_g^2}{(12\pi)}. \quad (21)$$

2. The γA case

The general expressions for a nuclear target is

$$\frac{d\sigma}{dx_1 x_2 dx_3 dx_4 dp_{1t}^2 dp_{2t}^2} = D(x_1, x_2, p_{1t}^2, p_{2t}^2) G(p_{1t}^2, x_3) G(p_{2t}^2, x_4) \frac{d\sigma}{dt_1} \frac{d\sigma}{dt_2} \int d^2\Delta F'_A(\Delta, -\Delta), \quad (22)$$

where

$$F'_A(\Delta, -\Delta) = F_A(\Delta, -\Delta) + AU(\Delta). \quad (23)$$

Here $F_A(\Delta, -\Delta)$ is the nucleus body form factor, and the form factor U was defined in Eq. (15). The first term in Eq. (23) corresponds to the processes when two gluons originate from the different nucleons in the nucleus while the second term in Eq. (23) corresponds to the case when they originate from the same nucleon. The first term is expected to dominate for heavy nuclei as it scales as $A^{4/3}$ [13,14].

For the nuclear target we have

$$\begin{aligned} F_A(\Delta, -\Delta) = F^2(\Delta), \\ F(\Delta) = \int d^2b \exp(i\vec{\Delta} \cdot \vec{b}) T(b), \end{aligned} \quad (24)$$

where

$$T(b) = \int dz \rho_A(b, z) dz \quad (25)$$

is the nucleus profile function, b is the impact parameter. The nuclear form factor integral is expressed through the

profile function as

$$G(A) = \int \frac{d^2\Delta}{(2\pi)^2} F(\Delta, -\Delta) = \int T^2(b) d^2b$$

$$= \pi \int T^2(b) db^2, \quad (26)$$

where $T(b)$ is calculated using the conventional mean field nuclear density [28–30],

$$\rho_A(b, z) = \frac{C(A)}{A} \frac{1}{1 + \exp(\sqrt{b^2 + z^2} - 5.5 \cdot A^{1/3})/(2.8)}. \quad (27)$$

The factor $C(A)$ is a normalization constant,

$$\int d^2b dz \rho_A(b, z) = A. \quad (28)$$

Here the distance scales are given in GeV^{-1} . Note that the nuclear form factor is independent of x_3, x_4 .

Then the only difference from the expression for γp collisions for photon nucleus scattering is the substitution,

$$U(x_3, x_4) \rightarrow AU(x_3, x_4) + G(A). \quad (29)$$

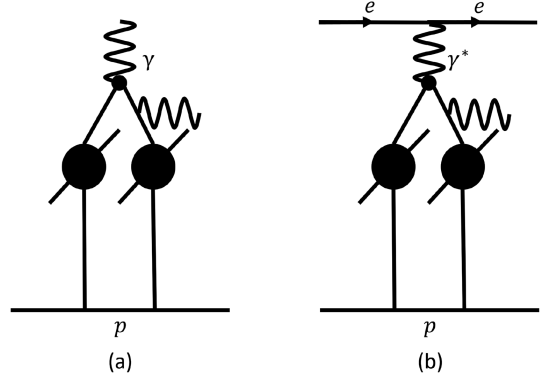


FIG. 7. DPS with direct photons in pQCD: (a) photoproduction and (b) electroproduction.

C. Account of gluon radiation and full expression

We can now take into account the gluon radiation in the leading logarithmic approximation (LLA) depicted in Fig. 7.

We include only the logarithmically enhanced Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP)-type diagrams both for the longitudinal and transverse gluons. The DGLAP evolution is easily accounted in $p_A = 0$ light cone gauge, see [9,10] for details. We obtain the expression for the total number of events for a unit of time,

$$N = L \int dk_t^2 dz dQ^2 dy dx_1 dx_2 dx_3 dx_4 dp_{1t}^2 dp_{2t}^2 \frac{d\sigma}{dQ^2 dy dx_1 dx_2 dx_3 dx_4 dp_{1t}^2 dp_{2t}^2}, \quad (30)$$

where L is the corresponding luminosity. We have the following conditions determining the integration phase space in p_{1t}, p_{2t} :

$$p_{1t}^2 < x_1 x_3 y W^2 / 4, \quad p_{2t}^2 < x_2 x_4 y W / 4, \quad (31)$$

$$x_1 < z < 1 - x_2, \quad (32)$$

$$\frac{Q^2}{W^2} < y < (Q^2 / (2m_e^2)) \left(-1 + \sqrt{1 + 4m_e^2 / Q^2} \right), \quad (33)$$

coming from $Q^2 / (yW^2) < 1$, $Q^2 > m_e^2 y^2 / (1 - y)$ [24]. These kinematic constraints together determine the integration region in the phase space. For the differential cross section we then have

$$\begin{aligned} \frac{d\sigma}{dQ^2 dy dx_1 dx_2 dx_3 dx_4 dp_{1t}^2 dp_{2t}^2} &= \frac{\alpha_e^2 N_c}{2} \int_{x_1}^{1-x_2} dz \int \frac{d^2 k_t}{(2\pi)^2} \frac{y}{(k_t^2 + x_1 x_2 Q^2)^2} (\vec{k}_t^2 (z^2 + (1-z)^2) \left(\frac{(1 + (1-y)^2)}{y^2 Q^2} - 2m_e^4 / Q^4 \right) \\ &\quad + 8z^2 (1-z)^2 (1-y) / y^2) G_{qA}(x_1/z, k_t^2, p_{1t}^2) G_{qB}(x_2/(1-z), k_t^2, p_{2t}^2) \frac{1}{z(1-z)} \\ &\quad \times \frac{M^2}{(x_1 x_3 \sqrt{x_1 x_3}) 16\pi (yW^2)^{3/2} \sqrt{x_1 x_3 y W^2 - 4p_{1t}^2}} \\ &\quad \times \frac{M^2}{(x_2 x_4 \sqrt{x_2 x_4}) 16\pi (yW^2)^{3/2} \sqrt{x_1 x_3 y W^2 - 4p_{2t}^2}} U(x_3, x_4) f(x_3, p_{1t}^2) f_g(x_4, p_{2t}^2), \end{aligned} \quad (34)$$

where the hard matrix element M^2 is given by Eq. (11), functions f are conventional gluon PDFs [27], U is a nonperturbative factor proportional determined in the previous subsection after integration in Δ^2 and proportional to m_g^2 for the proton target, $\bar{q}q$ is the quark-antiquark pair created in a photon splitting, and A, B are the resulting jet flavors.

The functions G_{qA}, G_{qB} are the fundamental solutions of the DGLAP equations that correspond to evolution from the scale k_t^2 to the scale p_{1t}^2 and p_{2t}^2 , respectively. If we neglect the evolution (i.e., gluon radiation from the quark model) these fundamental solutions become delta functions $\delta(1 - x_1/z), \delta(1 - x_2/(1 - z))$, and we obtain from Eq. (34) the parton model expression.

IV. NUMERICS

A. Photoproduction revisited

Let us first consider the photoproduction. We depict the photoproduction due to direct photons in Fig. 8; left for the case of the light quark jet, and right for the case of the charmed quark jet. The two quark jets come from the direct photon side and the two gluon jets come from the nucleon side.

We consider the 3 cases, with different values of W^2 : HERA with $W^2 = 90000 \text{ GeV}^2$, and electron-ion collider. We consider two regimes for EIC: $W^2 = 10000 \text{ GeV}^2$ and maximum, $W^2 = 22500 \text{ GeV}^2$. In the figures we calculate the number of the DPS events, that occur for $x_1 > 0.2$, $x_2 > 0.2$ in the logarithmic scale. For the case of light quark jet we sum the contributions of u, d and s quarks giving a multiplier 3/2, relative to the charm quark jets. We assume the luminosities $L = 1.7 \times 10^{31} \text{ sm}^{-2} \text{ s}^{-1}$ for HERA, $L = 10^{34} \text{ sm}^{-2} \text{ s}^{-1}$ for EIC and $W^2 = 10000 \text{ GeV}^2$, $L = 10^{33} \text{ sm}^{-2} \text{ s}^{-1}$ for EIC and $W^2 = 22500 \text{ GeV}^2$.

We see that due to higher luminosities the biggest number of DPS events occurs at EIC at $W^2 = 10000 \text{ GeV}^2$, and this is even a larger number than at HERA, i.e., increase in luminosity overcomes the decrease of a number of events due

to the decrease of the center-of-mass energy W^2 and the phase space available for hard processes.

For $p_t = 3 \text{ GeV}$ for example we obtain 2×10^6 events per year for light quark jets, and $\sim 10^6$ for charmed jets (i.e., final state with two charmed and two gluonic jets), while for HERA we have $3.6 \times 10^5, 1.2 \times 10^5$ per year, respectively.

We also consider in Fig. 9 the change of the number of events if we increase x_1, x_2 , and x_γ . We see that the decrease is rather slow with an increase of x_γ , similar to the results for such ratio in [20].

In our numerical calculations we used mathematica to integrate over k_t, z, y and then used Simpson's rule to integrate over x_i and p_t . We integrate over k_t^2 starting from $Q_0^2 \sim m_p^2 \sim 4m_{\text{const}}^2 \sim 0.6 \text{ GeV}^2$, where m_{const} is the constituent mass of the light quark. The states with small k_t splitting must be considered as nonperturbative and related to resolved photon contributions.

Note that when $x_1 + x_2$ become close to 1, the phase space for gluon radiation starts to be extremely small; $\sim Q_1^2(1 - x_1 - x_2)$, i.e., the transverse momenta of radiated gluons become small and the coupling constant large, i.e., the process becomes nonperturbative, and we come close to Landau pole. The situation seems to be close to what we encounter in the $x \rightarrow 1$ limit in PDF/Drell-Yan processes, where in analogous way the phase space available for radiating gluons becomes $Q^2(1 - x)$, and the transverse momenta of the radiated gluons are $k_t^2 \leq Q^2(1 - x)$. It was shown in [31,32] that the resummation of such gluons effectively corresponds to the change in the running coupling constant from $\alpha_s(Q^2)$ to $\alpha_s(Q^2(1 - x))$, meaning that, when $Q^2(1 - x) \sim \Lambda_{\text{QCD}}^2$, we hit the Landau pole and the perturbative QCD method due to DGLAP becomes unapplicable. It is well-known that the way to get rid of the Landau pole is to carry out the so-called resummation procedure, which essentially means adding higher-twist contributions to get rid of the Landau pole [33]. Our case is more complicated since we need to carry the resummation for a 3-point function. Consequently, in order not to hit the

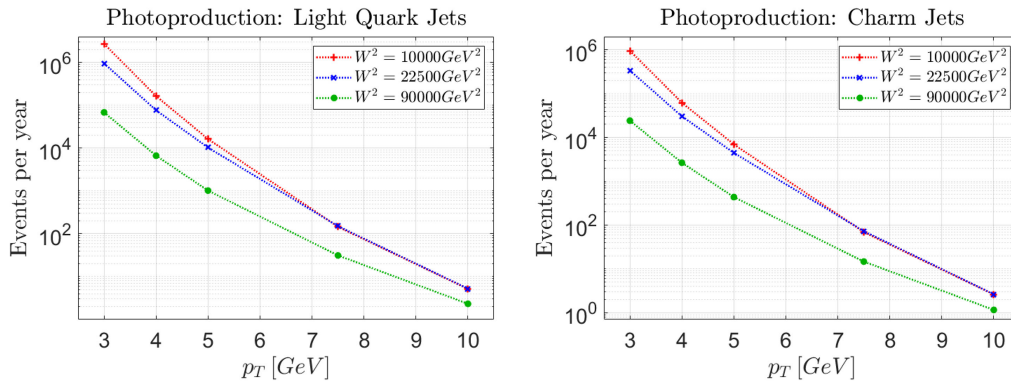


FIG. 8. The DPS processes with direct photons for γp deep inelastic collisions; (left) two light quark and two gluon jets, (right) two charmed jets and two gluon jets as final states. Here and in the figures below p_T is the scale of the leading jet.

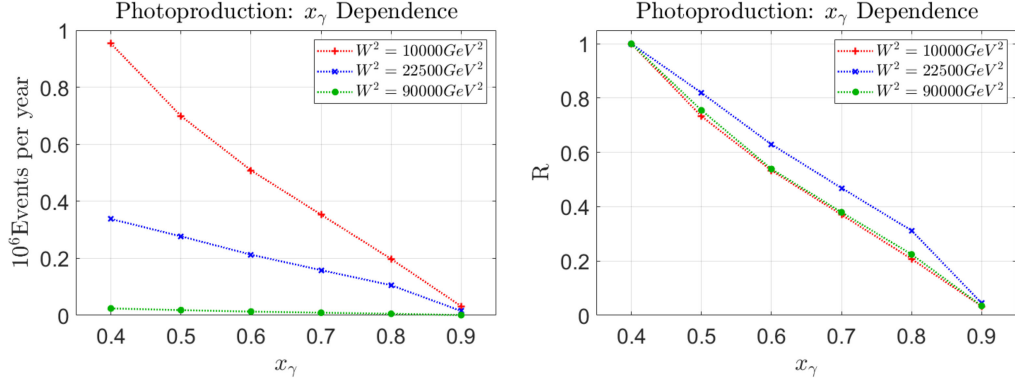


FIG. 9. The DPS processes with direct photons for γp deep inelastic collisions, charmed jets dependence of the events number on $x_\gamma = x_1 + x_2$. $R = \frac{N(x_\gamma)}{N(0.4)}$, where N is a number of events for x_γ calculated for $x_1 = x_2 = x_\gamma/2$. (Left) Absolute number of events per year, (Right) The ratio R defined above. (For this figure we considered $x_1 = x_2 = x_\gamma/2$ and the case of two charmed jets and two gluon jets as final states, however we expect that similar dependence on x_γ will continue for other final states and for electroproduction.

Landau pole, while calculating the number of events we imposed the kinematic constraint $Q^2(1 - x_1 - x_2) \geq \Lambda_{\text{QCD}}^2$.

B. Electroproduction

For the case of electroproduction we consider both the charm and light quark jets. We take the transverse scales of the jets for the examples and we made the numerical estimates to be 3 and 5 GeV.

In Figs. 10–13 we depict the DPS event numbers for HERA and EIC, with the same parameters as in the previous subsection, as a function of the photon virtuality Q . We consider separately the contribution of the longitudinal and the transverse photon. We see, however, that in all considered examples the contribution of the longitudinal photons is rather small, only 5–10% at most, relative to the transverse photon contribution. Consequently in these examples total number of the DPS is approximately equal

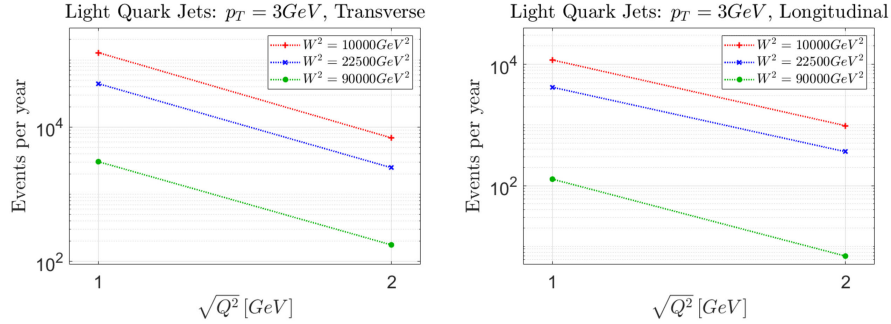


FIG. 10. The DPS processes with direct photons for γp deep inelastic collisions as function of Q for fixed p_T ; light quark jets with $p_T = 3$ GeV, electroproduction. (Left) Transverse photon contribution. (Right) Longitudinal photons.

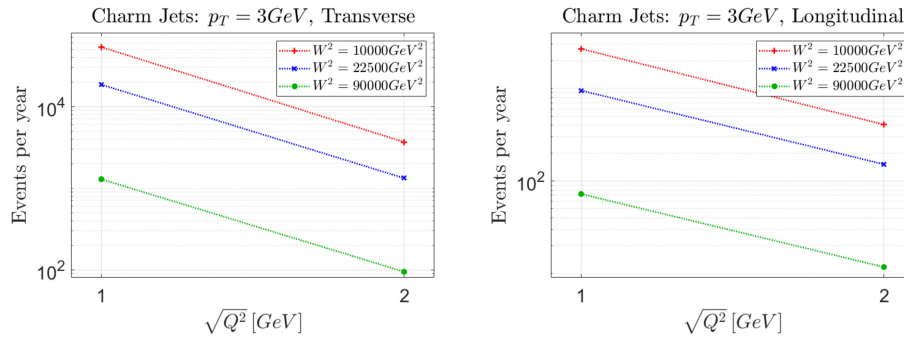


FIG. 11. The DPS processes with direct photons for γp deep inelastic collisions as function of Q for fixed p_T ; charm quark jets with $p_T = 3$ GeV, electroproduction. (Left) Transverse photon contribution. (Right) Longitudinal photons.

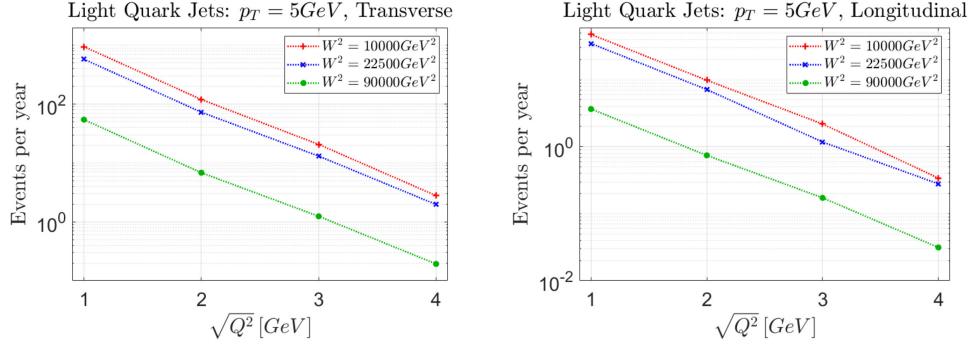


FIG. 12. The DPS processes with direct photons for γp deep inelastic collisions as function of Q for fixed p_T ; light quark jets with $p_T = 5$ GeV, electroproduction. (Left) Transverse photon contribution (Right) Longitudinal photons.

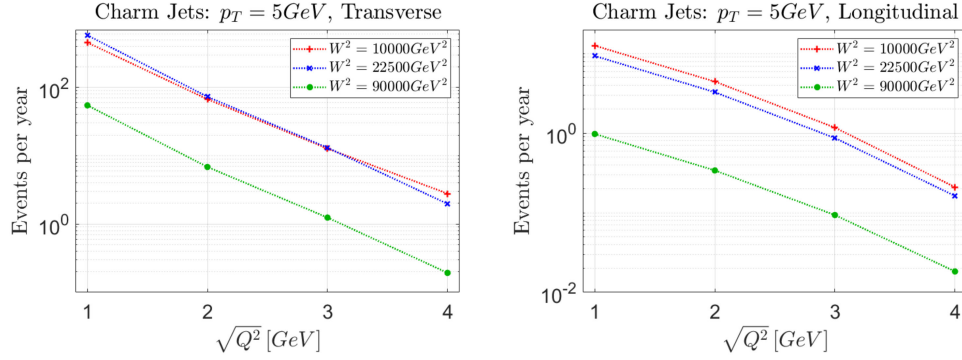


FIG. 13. The DPS processes with direct photons for γp deep inelastic collisions as function of Q for fixed p_T ; charm quark jets with $p_T = 5$ GeV, electroproduction. (Left) Transverse photon contribution. (Right) Longitudinal photons.

to the transverse photon contribution. Note that in this case we integrate over $k_t^2 \geq Q^2$, where Q is the momentum of the virtual photon, and effectively the invariant mass of the initial quark-antiquark pair. We also see that the number of DPS in electroproduction is significantly smaller than the number of DPS events in photoproduction.

C. SPS versus DPS

It is also interesting to estimate what is the percent of the DPS events relative to the number of corresponding SPS events, i.e., when we have one dijet=charm jet plus gluon

jet in the final state. We depict these results in Figs. 14–16. For longitudinal photons the form of graphs is similar but the ratios are by the order of magnitude smaller. We see also that the ratios decrease rather rapidly with the increase of p_T and increase of Q^2 for electroproduction.

D. Photon-nucleus reactions

The factor $G(A)$ for $A = 208$ is equal to $G(A) = 11.5$. On the other hand for $A = 208$ $AU(x_3, x_4)$ varies from 4.5 to 5.7 for different x_3, x_4 relevant for the integration region

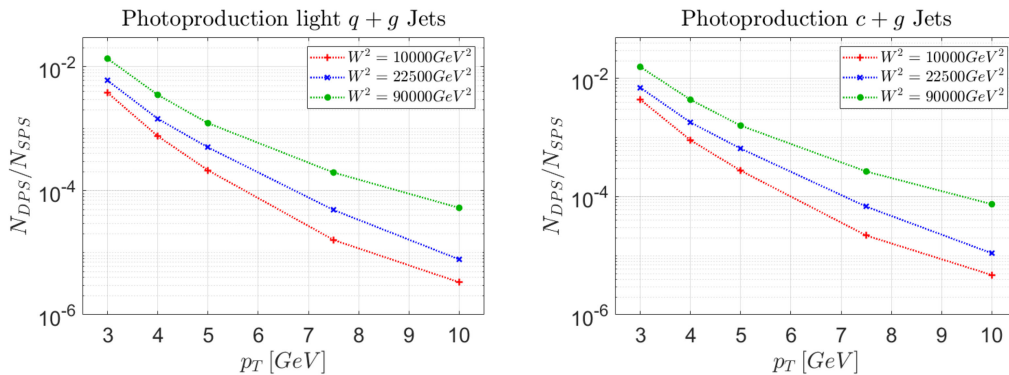


FIG. 14. The ratio of event numbers for DPS and SPS light quark + gluon dijets photoproduction (left) and charm quark + gluon photoproduction (right).

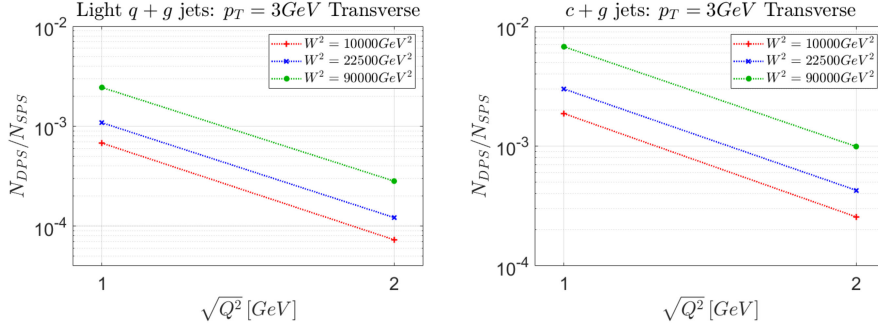


FIG. 15. The ratio of event numbers for DPS and SPS light quark + gluon dijets photoproduction (left) and charm quark + gluon electroproduction (right), $p_T = 3$ GeV.

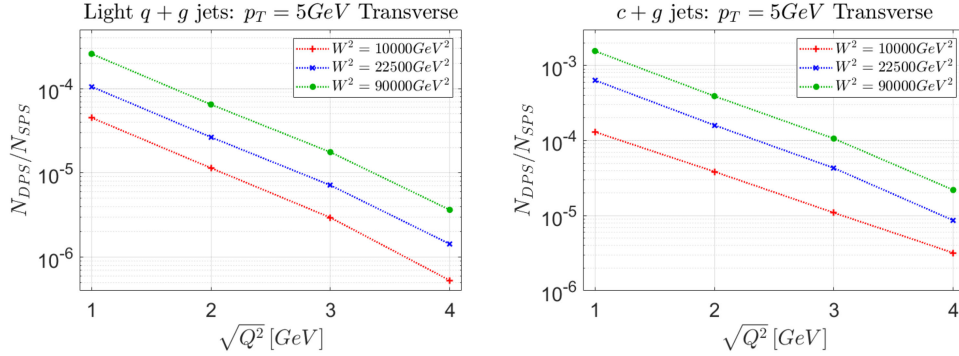


FIG. 16. The ratio of event numbers for DPS and SPS light quark + gluon dijets photoproduction (left) and charm quark + gluon electroproduction (right), $p_T = 5$ GeV.

for HERA, while for EIC the relevant region is even smaller—from 5.3 to 5.7 for $W^2 = 10000$ GeV², and 5.1 to 5.7 for $W^2 = 22500$ GeV²—thus the corresponding multipliers for $A = 208$ (Pb) is for EIC 16.9–17.3 and 16.7–17.3. In other words, with good accuracy, a number of the events in photon-nucleus collisions at the EIC and HERA is three times larger than for photon-proton DPS events for the whole region of x_1, x_2 considered here (and taken for the same luminosity).

V. CONCLUSION

We found that a rather large number of DPS events were due to direct events both in HERA and EIC. In EIC the number of events per year is of order 10^6 for $p_T = 3$ – 3.5 GeV jets, due to rather high luminosity. For HERA we have 4×10^4 , since although the energy is higher for HERA, the luminosity is significantly smaller. The corresponding ratio for the final state in SPS for the one dijet (quark + gluon) was found to be of order 10^{-2} . We also found the number of DPS events in electroproduction (see Figs. 10–13) as a function of Q^2 for different energies for both HERA and EIC. The use of the heavy nucleus as a target increases these numbers by the order of 3.

The study of these DPS-like events will permit us to study the specific properties of $1 \otimes 2$ mechanism and the

short-range transverse correlations in the nucleon. The DPS events in the limit $x_1 + x_2 \rightarrow 1$ deserve further study since they may be controlled by very soft gluons, and one needs to develop the resummation procedure along the lines of [33] to take them into account.

We shall also make some comments on next-to-leading order (NLO) calculations, although this problem goes beyond the scope of the current paper that deals with LLA results. The NLO corrections are most important as the corrections to hard processes. These corrections, as is widely known, lead to rather large K factors of order 1.5–2. Thus, the number of DPS events may be larger by a factor 2–4 relative to numerical estimates in the previous section. Nevertheless, we must note that most of these estimate were obtained using asymmetric jet cuts, i.e., rather large jet disbalances, i.e., the estimates for dijets at LHC with $p_T > 20$ GeV need the cut asymmetry of order 10 GeV [34] (or a complicated resummation procedure). Thus, further work must be carried out for studying NLO corrections. Nevertheless we expect that they only increase the number of DPS events and DPS/SPS ratios calculated above.

ACKNOWLEDGMENTS

We thank M. Strikman and Yu. Dokshitzer for very useful discussions, and M. Strikman for reading this article. This work was supported by BSF Grant No. 2033344.

DATA AVAILABILITY

The data that support the findings of this article are openly available [27].

**APPENDIX A: BREIT SYSTEM
CALCULATION FOR DPS**

In this appendix we show the details of the calculation of the electroproduction cross section for DPS. We shall carry the calculation in Breit system [35],

$$q = Q(0, 0, 0, 1), \quad p = E_p(1, 0, 0, -1), \quad (\text{A1})$$

where E_p is the energy of the proton, $s = 2E_p Q$.

In this system we can parametrize the momenta of initial and scattered electrons as

$$\begin{aligned} k &= \frac{1}{2}Q(\cosh(\psi), \sinh(\psi) \cos(\phi_e), \sinh(\psi) \sin(\phi_e), 1), \\ k' &= \frac{1}{2}Q(\cosh(\psi), \sinh(\psi) \cos(\phi_e), \sinh(\psi) \sin(\phi_e), -1). \end{aligned} \quad (\text{A2})$$

If $y = qp_p/kp_p$, then

$$\cosh(\psi) = (2 - y)/y, \quad \sinh(\psi) = 2\sqrt{1 - y}/y. \quad (\text{A3})$$

The leptonic tensor is

$$L_{\alpha\beta} \sim 4(k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu} Q^2/2). \quad (\text{A4})$$

The emitted photon polarization vectors are

$$\begin{aligned} e_T^1 &= (0, \cos(\phi_\gamma), \sin(\phi_\gamma), 0), \\ e_T^2 &= (0, -\sin(\phi_\gamma), \cos(\phi_\gamma), 0), \\ e_L &= (1, 0, 0, 0), \end{aligned} \quad (\text{A5})$$

where we choose linearly polarized transverse photons as a basis. It is easy to calculate

$$\begin{aligned} L^{\alpha\beta} e_{L\alpha} e_{L\beta} &= 2Q^2(1 - y)/y^2, \\ L^{\alpha\beta} e_{T1\alpha} e_{T1\beta} &= \frac{Q^2}{2}(1 + 2(ke_t^1)^2) = \frac{Q^2}{2}(1 + \sinh(\psi)^2 \cos(\phi_e - \phi_\gamma)^2), \\ L^{\alpha\beta} e_{T2\alpha} e_{T2\beta} &= \frac{Q^2}{2}(1 + \sinh(\psi)^2 \sin(\phi_e - \phi_\gamma)^2), \\ L^{\alpha\beta} e_{T1\alpha} e_{T2\beta} &= \frac{Q^2}{2} \sinh(\psi)^2 \sin(2(\phi_e - \phi_\gamma))/2, \\ L^{\alpha\beta} e_{L\alpha} e_{T1\beta} &= -\frac{Q^2}{2y} \sinh(\psi) \cos(\phi_e - \phi_\gamma), \\ L^{\alpha\beta} e_{L\alpha} e_{T2\beta} &= -\frac{Q^2}{2y} \sinh(\psi) \sin(\phi_e - \phi_\gamma). \end{aligned} \quad (\text{A6})$$

Next we consider the corresponding contractions with the hadronic tensor,

$$\begin{aligned} J e_L &= J_L e_L = 2x_1 x_2 Q, \\ J e_T^1 &= J_T e_T^1 + i k e_T^1 = -((x_1 - x_2) \cos(\phi_q - \phi_\gamma) - i \sin(\phi_q - \phi_\gamma)), \\ J e_T^2 &= J_T e_T^2 + i k e_T^2 = -((x_1 - x_2) \sin(\phi_q - \phi_\gamma) + i \cos(\phi_q - \phi_\gamma)), \end{aligned} \quad (\text{A7})$$

where in the Breit system,

$$\kappa_t^i = \epsilon^{ij} k_{1tj}, \quad \vec{k}_{1t} = (\cos(\phi_q), \sin(\phi_q)), \quad \vec{\kappa} = (-\sin(\phi_q), \cos(\phi_q)). \quad (\text{A8})$$

Then it is easy to see that the longitudinal-transverse cross terms are zero. For the part of the cross section due to longitudinal photons we have

$$\sigma_{LL} \sim (L_{\alpha\beta} e_L^\alpha e_L^\beta) (J e_L)^2 = 8Q^4 x_1^2 x_2^2 (1 - y)/y^2. \quad (\text{A9})$$

For the transverse part we have

$$\sigma_{TT} \sim \sum_{\lambda=1,2} L_{\alpha\beta} e_T^{\alpha\lambda} e_T^{\beta\lambda} (J e_T^\lambda)(J e_T^\lambda) + 2 L_{\alpha\beta} e_T^{\alpha 1} e_T^{\beta 2} (J e_T^1)(J e_T^2). \quad (\text{A10})$$

This can be rewritten as

$$\begin{aligned} \sigma_{TT} \sim & Q^2 (\sinh(\psi)^2 \cos(\phi_e - \phi_\gamma)^2 + 1) (x_1^2 + x_2^2) \cos(\phi_q - \phi_\gamma)^2 \\ & + Q^2 (\sinh(\psi)^2 \sin(\phi_e - \phi_\gamma)^2 + 1) (x_1^2 + x_2^2) \sin(\phi_q - \phi_\gamma)^2 \\ & - Q^2 (x_1 x_2) (\sinh(\psi)^2 \sin(2(\phi_q - \phi_\gamma)) \sin(2(\phi_e - \phi_\gamma))). \end{aligned} \quad (\text{A11})$$

We now average over the photon polarization angle ϕ_γ ,

$$\begin{aligned} \int d\phi_\gamma \cos(\phi_q - \phi_\gamma)^2 &= (1/2) * 2\pi, \\ \int d\phi_\gamma \cos(\phi_q - \phi_\gamma)^2 \cos(\phi_e - \phi_\gamma)^2 &= 2\pi * (1/4) (1 + \cos(2(\phi_q - \phi_e))/2), \\ \int d\phi_\gamma \sin(\phi_q - \phi_\gamma)^2 \sin(\phi_e - \phi_\gamma)^2 &= 2\pi * (1/4) (1 + \cos(2(\phi_q - \phi_e))/2), \\ \int d\phi_\gamma \sin(2(\phi_q - \phi_\gamma)) \sin(2(\phi_e - \phi_\gamma)) &= -\pi \cos(2(\phi_q - \phi_e)). \end{aligned} \quad (\text{A12})$$

This means that

$$\sigma_{TT} \sim 2Q^2 (x_1^2 + x_2^2) (1 + \sinh(\psi)^2/2) * (1 + \cos(2(\phi_q - \phi_e))/2) + Q^2 x_1 x_2 \sinh(\psi)^2 \cos(2(\phi_q - \phi_e))/2. \quad (\text{A13})$$

We see that the interaction with longitudinal gluons is isotropic while the transverse gluons have angular correlations. Averaging over angular correlations we get

$$\sigma_{TT} \sim Q^2 \vec{k}_{1t}^2 (x_1^2 + x_2^2) (1 + \sinh(\psi)^2/2) = Q^2 \vec{k}_{1t}^2 (x_1^2 + x_2^2) (1 + (1-y)^2)/y^2, \quad (\text{A14})$$

where ψ is determined by Eq. (A3). Thus, including the term with the mass of the electron, we get

$$\frac{d\sigma}{dQ^2 dy} = \frac{\alpha_e^2 N_c}{2} \int \frac{d^2 k_t}{(2\pi)^2} \frac{y}{(k_t^2 + x_1 x_2 Q^2)^2} \left(\vec{k}_t^2 (x_1^2 + x_2^2) \left(\frac{(1 + (1-y)^2)}{y^2 Q^2} - 2m_e^2/Q^4 \right) + 8x_1^2 x_2^2 (1-y)/y^2 \right), \quad (\text{A15})$$

where we have the same normalization as in [20] for the total cross section and $\alpha_e = 1/137$ is an electromagnetic constant. There are two kinematic boundaries. First, since

$$x = Q^2/s = Q^2/(yW^2) < 1, \quad (\text{A16})$$

then

$$y > Q^2/W^2. \quad (\text{A17})$$

The second boundary comes from

$$Q^2 > \frac{m_e^2 y^2}{1-y}. \quad (\text{A18})$$

Solving this inequality we obtain,

$$y \leq \frac{Q^2}{2m_e^2} \left(-1 + \sqrt{1 + 4m_e^2/Q^2} \right) \sim 1, \quad \text{if } Q^2 \gg m_e^2. \quad (\text{A19})$$

So for fixed Q^2 we have the limits

$$\frac{Q^2}{2m_e^2} \left(-1 + \sqrt{1 + 4m_e^2/Q^2} \right) > y > Q^2/W^2. \quad (\text{A20})$$

The latter equations means that

$$y \leq 1Q^2 \gg m_e^2; \quad y \leq Q/m_e, \quad Q^2 \ll m_e^2. \quad (\text{A21})$$

In addition there is a constraint due to hard processes (see Sec. III C).

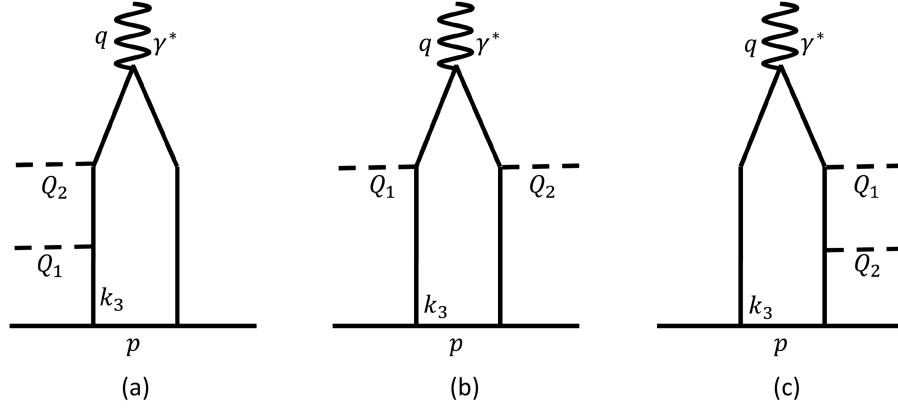


FIG. 17. The three amplitudes that contribute to the process.

APPENDIX B: GAUGE INVARIANCE

It is interesting to note that the longitudinal part of the current calculated in the previous section is not conserved. In order to achieve gauge invariance we need to consider not only the diagram 17(b), but all three diagrams in Fig. 17. It is easy to see that the diagrams 17(a) and 17(c) do not contribute to the scattering amplitudes, but account for the gauge invariance. We look at a proton with momentum p interacting with a photon with momentum q such that $q^2 = -Q^2 < 0$ via two quarks with momentum k_3 and k_4 such that $k_3 \parallel k_4 \parallel p$, resulting in two dijets with momenta Q_1 and Q_2 . In Fig. 17 we present the three amplitudes that contribute to the process and that need to be accounted for to satisfy the Ward-Takahashi identity.

We introduce the center-of-energy scale $s \equiv 2qp$ and the vector $q' \equiv q + xp$ where $x = Q^2/s$, we are going to assume $p^2 = 0$, and from that we can see that $(q')^2 = 0$. We can parametrize the outgoing momenta as $Q_1^\mu = x_1(q')^\mu + x_3 p^\mu + k_t^\mu$, $Q_2^\mu = x_2(q')^\mu + x_4 p^\mu - k_t^\mu$, where $x_1 + x_2 = 1$ and k_t is perpendicular to both q' and p , and $k_3 = (x_3 - \beta)p$, $k_4 = (x_4 + \beta + x)p$. With this, we can write out the amplitudes as

$$\begin{aligned}
 (a) & \propto \bar{u}(p) \frac{-x_1 \hat{q}' - \beta \hat{p} - \hat{k}_t - \hat{q}' - (x_4 + \beta) \hat{p}}{x_1 \beta s - \vec{k}_t^2 + i\epsilon} \frac{\hat{q}' - (x_4 + \beta) \hat{p}}{(x_4 + \beta) s + i\epsilon} \gamma^\mu u(p), \\
 (b) & \propto \bar{u}(p) \frac{-x_1 \hat{q}' - \beta \hat{p} - \hat{k}_t}{x_1 \beta s - \vec{k}_t^2 + i\epsilon} \gamma^\mu \frac{x_2 \hat{q}' - (\beta + x) \hat{p} - \hat{k}_t}{-x_2(\beta + x)s - \vec{k}_t^2 + i\epsilon} u(p), \\
 (c) & \propto \bar{u}(p) \gamma^\mu \frac{\hat{q}' + (x_3 - \beta - x) \hat{p}}{(x_3 - \beta - x)s + i\epsilon} \frac{x_2 \hat{q}' - (\beta + x) \hat{p} - \hat{k}_t}{-x_2(\beta + x)s - \vec{k}_t^2 + i\epsilon} \\
 & \quad \times u(p). \tag{B1}
 \end{aligned}$$

Taking all three amplitudes together we can see what we get if we take a product of them with different vectors. The useful vector basis that we can work with is the set q' , p , k_t , and κ , where κ is a vector perpendicular to all the other three and satisfies $\kappa^2 = k_t^2 = -\vec{k}_t^2$. We now define the sum of all three diagrams as a new “vector”,

$$\begin{aligned}
 V^\mu(\beta) &= \bar{u}(p) \left[\frac{-x_1 \hat{q}' - \hat{k}_t}{x_1 \beta s - \vec{k}_t^2 + i\epsilon} \frac{-\hat{q}' - (x_4 + \beta) \hat{p}}{(x_4 + \beta) s + i\epsilon} \gamma^\mu \right. \\
 & \quad + \frac{-x_1 \hat{q}' - \hat{k}_t}{x_1 \beta s - \vec{k}_t^2 + i\epsilon} \gamma^\mu \frac{x_2 \hat{q}' - \hat{k}_t}{-x_2(\beta + x)s - \vec{k}_t^2 + i\epsilon} \\
 & \quad \left. + \gamma^\mu \frac{\hat{q}' + (x_3 - \beta - x) \hat{p}}{(x_3 - \beta - x)s + i\epsilon} \frac{x_2 \hat{q}' - \hat{k}_t}{-x_2(\beta + x)s - \vec{k}_t^2 + i\epsilon} \right] \\
 & \quad \times u(p), \tag{B2}
 \end{aligned}$$

where the terms of the form $\bar{u}(p) \hat{p}$ and $\hat{p} u(p)$ are taken to be zero. Using the basis defined above we obtain,

$$\begin{aligned}
 q'V(\beta) &= \frac{-x_1 x_2 x s^2}{(x_1 \beta s - \vec{k}_t^2 + i\epsilon)(-x_2(\beta + x)s - \vec{k}_t^2 + i\epsilon)}, \\
 pV(\beta) &= \frac{-x_1 x_2 s^2}{(x_1 \beta s - \vec{k}_t^2 + i\epsilon)(-x_2(\beta + x)s - \vec{k}_t^2 + i\epsilon)}, \\
 k_t V(\beta) &= \left[\frac{1}{(x_1 \beta s - \vec{k}_t^2 + i\epsilon)((x_4 + \beta)s + i\epsilon)} \right. \\
 & \quad + \frac{(x_2 - x_1)}{(x_1 \beta s - \vec{k}_t^2 + i\epsilon)(-x_2(\beta + x)s - \vec{k}_t^2 + i\epsilon)} \\
 & \quad \left. - \frac{1}{((x_3 - \beta - x)s + i\epsilon)(-x_2(\beta + x)s - \vec{k}_t^2 + i\epsilon)} \right] \vec{k}_t^2 s, \\
 \kappa V(\beta) &= \left[\frac{1}{(x_1 \beta s - \vec{k}_t^2 + i\epsilon)((x_4 + \beta)s + i\epsilon)} \right. \\
 & \quad + \frac{1}{(x_1 \beta s - \vec{k}_t^2 + i\epsilon)(-x_2(\beta + x)s - \vec{k}_t^2 + i\epsilon)} \\
 & \quad \left. + \frac{1}{((x_3 - \beta - x)s + i\epsilon)(-x_2(\beta + x)s - \vec{k}_t^2 + i\epsilon)} \right] \\
 & \quad \times 2i\lambda \epsilon^{abcd} p_a(q')_b \kappa_c(k_t)_d, \tag{B3}
 \end{aligned}$$

where λ is the helicity of the quark.

Let us make several observations. First, if we look at the products with q'^μ and p^μ we can see that $q'V(\beta) = (q' - xp)V(\beta) = 0$, showing that the Ward-Takahashi identity is satisfied. What happens is that if we multiply the diagrams and c by q'^μ , the denominators proportional to $1/(x_4 + \beta)$ and $1/(x_3 - \beta - x)$ cancel out with nominators of the diagrams. We are left then with the some of two terms

one proportional to $1/(x_1\beta s - \vec{k}_t^2)$ and $1/(-x_2(\beta + x)s - \vec{k}_t^2)$ due to the diagrams 17(a) and 17(c), respectively. The sum of this terms gives the first line in Eq. (B3), the two multipliers give the convergent contribution,

$$\sim \frac{1}{(x_1\beta s - \vec{k}_t^2)(-x_2(\beta + x)s - \vec{k}_t^2)},$$

that is canceled by the contribution of diagram b.

Note that although the diagrams 17(a) and 17(c) seem to be divergent in β and cannot be calculated using residues due to β in the nominators. Their sum, convoluted with q , has no dependence on β in the nominator and is convergent and can be calculated using the residue analysis.

Second, in the transverse products, meaning $k_t V(\beta)$ and $\kappa V(\beta)$, we find that each amplitude has basic poles in β , but for amplitude (b) they are always on different sides of the real line, while for amplitudes (a) and (c) both poles are on the same side meaning they do not contribute to the amplitudes of the physical processes. Note that β dependence in the nominators is also canceled out, permitting the application of the residue technique.

We can also look at three diagrams in Fig. 17 from a slightly different point of view. Consider the sum of diagrams 17(a) and 17(c). We see that the sum of these two diagrams is the sum of two terms.

First, these are convergent terms that behave as $1/\beta^2$ at large β . These terms have denominators

$$(a) = \frac{1}{(x_4 + \beta + i\epsilon)(x_1\beta s - \vec{k}_t^2 + i\epsilon)} \quad (B4)$$

and

$$(c) = \frac{1}{((x_3 - \beta - x) + i\epsilon)(x_2(\beta + x)s + \vec{k}_t^2 + i\epsilon)}, \quad (B5)$$

and are zero since, as it was mentioned above, the residues in β lie in the same half-plane of complex β .

Second, there are two terms that do not depend on x_3, x_4 , but the corresponding integrals in β are divergent. It is easy to see that these terms are longitudinal and proportional to p^μ ,

$$2p^\mu \left(\frac{(x_1\hat{q}' + \hat{k}_t)}{(x_1\beta s - k_t^2 + i\epsilon)} + \frac{x_2\hat{q}' - \hat{k}_t}{-x_2(\beta + x)s - k_t^2 + i\epsilon} \right). \quad (B6)$$

Although each term behaves as $1/\beta$, in the sum the dependence of the nominator on β is canceled out and we have for this sum the same denominator as for diagram 17(b).

We can now sum the contributions of three diagrams together. We see that the longitudinal part of the sum is just

$$\sim \frac{2x_1x_2(q^\mu + 2xp^\mu)}{(x_1\beta s - k_t^2 + i\epsilon)(x_2(\beta + x)s + k_t^2 - i\epsilon)}. \quad (B7)$$

Then, after taking residues, we see that the longitudinal part of the vertex current is

$$J_L^\mu = 2x_1x_2(q^\mu + 2xp^\mu), \quad (B8)$$

which is proportional to e_L^μ . In the same way we obtain, for the transverse part of the current the result in Sec. II B in the main text, it is contributed only by the diagram of 17(b).

We conclude that we can carry all calculations using the diagram 17(b) but using the effective gauge-invariant vertex Eqs. (B7) and (B8).

Finally, in the LLA approximation we must attach two ladders to the parton model diagrams. It is easy to see, using for example the simple ladder building rules [36], that the DGLAP gluons factor out, and the resulting diagram has the spinor structure of the parton model times the numerical factor due to DGLAP gluons, as in the pp collisions [7]. In other words, the answer corresponds to the product of the vertex factor [given by Eq. (B7) and the corresponding expression for the transverse part of the current] and the DGLAP factors. Consequently the gauge invariance of the resulting expression remains intact.

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