

Systematic study of exotic 1^{-+} tetraquark spectroscopy

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The masses of exotic quantum-number 1^{-+} compact tetraquark states are calculated in a constituent quark model, where a Cornell-like potential is employed as the central potential, spin-spin and spin-orbit coupling derived from the Breit-Fermi interaction are treated as hyperfine corrections, and model parameters are taken from previous works. The ground state 1^{-+} P-wave tetraquarks are predicted at 1.9, 4.2, and 6.6 GeV for the light, charmoniumlike, and fully charm sectors, respectively. The decay width ratios of 1^{-+} tetraquark states are calculated for two-body strong decay channels within the rearrangement mechanism, including ωh_1 and ηf_1 for isospin $I = 0$ light tetraquarks, ρh_1 and πf_1 for isospin $I = 1$ light tetraquarks, $\pi/\eta + \chi_{c1}$ and $\rho/\omega + h_c$ for charmoniumlike tetraquarks, and $\eta_c \chi_{c1}$ and $J/\psi h_c$ for fully charm tetraquarks. The theoretical results are compared with the observed exotic 1^{-+} states, and promising search channels for 1^{-+} tetraquarks are discussed. The work suggests that $\eta_1(1855)$ is unlikely to be a compact tetraquark state.

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I. INTRODUCTION

Understanding the spectrum and internal structure of hadrons remains a central goal of quantum chromodynamics (QCD), and states beyond the conventional $q\bar{q}$ mesons and qqq baryons, such as multiquark configurations and hybrids, are now an active frontier of experiment and theory.

Charmoniumlike states with total angular momentum $J = 1$ have been widely searched over the last two decades and are promising P-wave tetraquark candidates. Several exotic states in the charmonium region, known as Y states with 1^{-+} quantum numbers that mix with conventional ψ mesons, could be P-wave tetraquark candidates: Y(4230) [1], Y(4360) [2], Y(4660) [3], the more recent Y(4484) [4], and Y(4544) [5].

In conventional mesons composed of a quark and an antiquark $q\bar{q}$, the quantum numbers J^{PC} follow simple rules: parity $P = (-1)^{L+1}$ and charge conjugation $C = (-1)^{L+S}$,

where L is orbital angular momentum and S is total spin [6]. These rules allow common states such as 1^{--} (the ρ meson), 1^{+-} (the h_1 meson), or 1^{++} (the f_1 meson) but not 1^{-+} . Thus, a 1^{-+} state, once observed, is naturally considered to have a nonconventional structure, such as a hybrid meson, hadronic molecule, or tetraquark.

The experimental status of exotic 1^{-+} states has advanced through complementary approaches. The existing data provide a foundation for evaluating their properties, which we briefly review below:

The vertex spectrometer (VES) experiment observed the exotic 1^{-+} $\pi_1(1400)$ state in $\pi^- p \rightarrow \pi \eta n$ decays at 100 GeV [7], with a mass of 1406 ± 20 MeV and a width of 180 ± 30 MeV. The analysis of the $\eta\pi$ channel showed a clear exotic signal inconsistent with conventional $q\bar{q}$ states.

The $\pi(1600)$ state was first observed by the Brookhaven E852 experiment [8], which identified an exotic 1^{-+} resonance in the $\rho\pi$ channel. The analysis found a mass of $1593 \pm 8^{+29}_{-47}$ MeV and a width of $168 \pm 20^{+150}_{-12}$ MeV. Later, E852 [9] observed two exotic 1^{-+} resonances in $\pi^- p \rightarrow f_1(1285)\pi^- p$ collisions at 18 GeV: the known $\pi_1(1600)$ with mass $M = 1709 \pm 24 \pm 41$ MeV and width $\Gamma = 403 \pm 80 \pm 115$ MeV, and a new $\pi_1(2015)$ state at $M = 2001 \pm 30 \pm 92$ MeV with width $\Gamma = 333 \pm 52 \pm 49$ MeV.

In the updated Particle Data Group (PDG) review, $\pi(1400)$ and $\pi(1600)$ are suggested to be a single state, as coupled-channel analyses favor one broad isovector resonance in the 1.4–1.6 GeV region [6]. Of the reported candidates, the $\pi_1(2015)$ has only been observed by the

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E852 experiment [9], and its existence remains inconclusive.

The BESIII Collaboration recently observed the isoscalar exotic state $\eta_1(1855)$ with 1^{-+} in $J/\psi \rightarrow \gamma\eta\eta'$ decays [10], measuring its mass as $(1855 \pm 9_{-1}^{+6})$ MeV, its width as $(188 \pm 18_{-8}^{+3})$ MeV with $> 19\sigma$ significance, and the product branching fraction $\mathcal{B}(J/\psi \rightarrow \gamma\eta_1(1855)) \times \mathcal{B}(\eta_1(1855) \rightarrow \eta\eta') = (2.70 \pm 0.41_{-0.35}^{+0.16}) \times 10^{-6}$. The narrow width of $\eta_1(1855)$ contrasts sharply with the broad $\pi_1(1600)$, suggesting different underlying dynamics between these exotic states.

In this work, we study the mass spectrum and decay properties of the 1^{-+} P-wave light, charmoniumlike, and fully charm compact tetraquark states within a constituent quark model framework.

The paper is organized as follows. In Sec. II, all possible configurations of the color, spin, flavor, and spatial degrees of freedom of tetraquark states are introduced, and a constituent quark model developed in our previous work [11–13] is briefly reviewed. In Sec. III, the 1^{-+} P-wave tetraquark mass spectra and relative decay ratios are evaluated within the constituent quark model. The theoretical results are compared with experimental data, and discussions of $\pi_1(2015)$ and $\eta_1(1855)$ are provided. A summary is given in Sec. IV.

II. THEORETICAL MODEL

A. Quark configurations of tetraquark

The construction of tetraquark states follows the principle that a tetraquark must be a color singlet, meaning the color wave function must correspond to a $[222]_1$ singlet within the $SU_c(3)$ group. Furthermore, the total wave function of any cluster of identical fermions must be antisymmetric.

In this work, we consider 1^{-+} light ($q_1 q_2 \bar{q}_3 \bar{q}_4$), charmoniumlike ($q_1 c_2 \bar{q}_3 \bar{c}_4$), and fully charm ($c_1 c_2 \bar{c}_3 \bar{c}_4$) tetraquark states, where q and c denote a light quark and a charm quark, and $\bar{q}$ and $\bar{c}$ denote a light antiquark and a charm antiquark, respectively.

We employ a diquark-antidiquark coupling scheme and classify each cluster by $SU(3)$ color Young tableaux. The permutation symmetry of the two-quark cluster (qq , qc , or cc) in tetraquark states is described by the Young tableaux $[2]_6$ and $[11]_3$ of the $SU_c(3)$ group, while the color configuration of the two-antiquark cluster ($\bar{q}\bar{q}$, $\bar{q}\bar{c}$, or $\bar{c}\bar{c}$) consists of a $[211]_3$ triplet and a $[22]_6$ antisextet.

Thus, a $[222]_1$ color singlet for tetraquark states requires the following color configurations: $[2]_6 \otimes [22]_6$ and $[11]_3 \otimes [211]_3$, which correspond to the color sextet-antisextet ($6_c \otimes \bar{6}_c$) and triplet-antitriplet ($\bar{3}_c \otimes 3_c$) configurations, respectively.

The 1^{-+} tetraquark states correspond to $L = 1$, $S = 1$ configurations according to the parity ($P = (-1)^L$) and charge-conjugation ($C = (-1)^{L+S}$) quantum numbers,

where L is the total orbital angular momentum and S is the total spin of the tetraquark. Thus, the 1^{-+} tetraquark requires an odd L , with the lowest possible value being $L = 1$. The complete basis is constructed by coupling harmonic-oscillator wave functions, which takes the general form

$$\psi_{NL} = \sum_{n_{\chi_i}, l_{\chi_i}} A(n_{\chi_1}, n_{\chi_2}, n_{\chi_3}, l_{\chi_1}, l_{\chi_2}, l_{\chi_3}) \times \psi_{n_{\chi_1} l_{\chi_1}}(\vec{\chi}_1) \otimes \psi_{n_{\chi_2} l_{\chi_2}}(\vec{\chi}_2) \otimes \psi_{n_{\chi_3} l_{\chi_3}}(\vec{\chi}_3) \quad (1)$$

where $\psi_{n_{\chi_i} l_{\chi_i}}$ are harmonic oscillator wave functions. The sum n_{χ_i}, l_{χ_i} runs over $n_{\chi_1}, n_{\chi_2}, n_{\chi_3}, l_{\chi_1}, l_{\chi_2}, l_{\chi_3}$. N and L are, respectively, the total principal quantum number and total angular momentum, where $N = (2n_{\chi_1} + l_{\chi_1}) + (2n_{\chi_2} + l_{\chi_2}) + (2n_{\chi_3} + l_{\chi_3})$, and $L = l_{\chi_1} + l_{\chi_2} + l_{\chi_3} = 1$ for the P wave tetraquark. The relative Jacobi coordinates $\vec{\chi}_1, \vec{\chi}_2$, and $\vec{\chi}_3$ are defined as

$$\begin{aligned} \vec{\chi}_1 &= \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2), \\ \vec{\chi}_2 &= \frac{1}{\sqrt{2}}(\vec{r}_3 - \vec{r}_4), \\ \vec{\chi}_3 &= \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} - \frac{m_3 \vec{r}_3 + m_4 \vec{r}_4}{m_3 + m_4}, \end{aligned} \quad (2)$$

where $\vec{r}_j$ and m_j are the coordinate and the mass of the j th quark, respectively. The reduced quark masses are defined as

$$\begin{aligned} u_1 &= \frac{2m_1 m_2}{m_1 + m_2}, \\ u_2 &= \frac{2m_3 m_4}{m_3 + m_4}, \\ u_3 &= \frac{(m_1 + m_2)(m_3 + m_4)}{m_1 + m_2 + m_3 + m_4}, \end{aligned} \quad (3)$$

The size of complete bases can be expanded by increasing the principal quantum number N . The complete bases with $N = 1$ are listed in Table I for reference. In this work, we employ complete bases of $N \leq 13$ to calculate the mass spectrum.

For $S = 1$ configurations, the possible spin combinations are $[\psi_{[s=1]}^{q_1 q_2} \otimes \psi_{[s=1]}^{\bar{q}_3 \bar{q}_4}]_{S=1}$, $\psi_{[s=1]}^{q_1 q_2} \otimes \psi_{[s=0]}^{\bar{q}_3 \bar{q}_4}$, and $\psi_{[s=0]}^{q_1 q_2} \otimes \psi_{[s=1]}^{\bar{q}_3 \bar{q}_4}$. As a $qq\bar{q}\bar{q}$ or $cc\bar{c}\bar{c}$ tetraquark must be a color singlet and simultaneously antisymmetric under any permutation of identical quarks, the qq -cluster spatial-spin-flavor configurations for $qq\bar{q}\bar{q}$ states are $\psi_{[11]}^{osf}$ for the color configuration $\psi_{[2]}^c$ and $\psi_{[2]}^{osf}$ for $\psi_{[11]}^c$. All possible configurations are as follows:

TABLE I. The complete bases of tetraquark with the lowest quantum number, $N = 2n + L = 1$ and $L = 1$.

Ψ_{NL}	Bases	
	$\{l_{\chi_1}, l_{\chi_2}, l_{\chi_3}\}$	$\psi_{n_{\chi_1} l_{\chi_1}}(\vec{\chi}_1) \psi_{n_{\chi_2} l_{\chi_2}}(\vec{\chi}_2) \psi_{n_{\chi_3} l_{\chi_3}}(\vec{\chi}_3)$
Ψ_{11}	$\{0, 0, 1\}$	$\psi_{0,0}(\vec{\chi}_1) \psi_{0,0}(\vec{\chi}_2) \psi_{0,1}(\vec{\chi}_3),$
	$\{0, 1, 0\}$	$\psi_{0,0}(\vec{\chi}_1) \psi_{0,1}(\vec{\chi}_2) \psi_{0,0}(\vec{\chi}_3),$
	$\{1, 0, 0\}$	$\psi_{0,1}(\vec{\chi}_1) \psi_{0,0}(\vec{\chi}_2) \psi_{0,0}(\vec{\chi}_3)$

TABLE II. $I = 0$ and $I = 1$ 1^{-+} Light tetraquark configurations in a complete harmonic oscillator (HO) basis. Internal orbitals $l_{\chi_1}, l_{\chi_2}, l_{\chi_3} \in \{0, 1\}$; $l_{\chi_1} + l_{\chi_2} + l_{\chi_3} = 1$. Pairwise Pauli is enforced for (12) and (34).

$\{l_{\chi_1}, l_{\chi_2}, l_{\chi_3}\}$	(qq)			$(\bar{q} \bar{q})$			Total I
	ψ_{qq}^c	s_{qq}	I_{qq}	$\psi_{\bar{q}\bar{q}}^c$	$s_{\bar{q}\bar{q}}$	$I_{\bar{q}\bar{q}}$	
$\{1, 0, 0\}$	$\bar{\mathbf{3}}_c$	1	0	$\mathbf{3}_c$	0	0	0
$\{0, 1, 0\}$	$\mathbf{6}_c$	1	0	$\bar{\mathbf{6}}_c$	0	0	0
$\{0, 0, 1\}$	$\mathbf{6}_c$	1	0	$\bar{\mathbf{6}}_c$	1	0	0
$\{0, 0, 1\}$	$\bar{\mathbf{3}}_c$	1	1	$\mathbf{3}_c$	0	0	1
$\{0, 1, 0\}$	$\bar{\mathbf{3}}_c$	1	1	$\mathbf{3}_c$	1	0	1
$\{1, 0, 0\}$	$\bar{\mathbf{3}}_c$	1	0	$\mathbf{3}_c$	1	1	1
$\{0, 0, 1\}$	$\mathbf{6}_c$	1	0	$\bar{\mathbf{6}}_c$	0	1	1
$\{0, 1, 0\}$	$\mathbf{6}_c$	1	0	$\bar{\mathbf{6}}_c$	1	1	1
$\{1, 0, 0\}$	$\mathbf{6}_c$	1	1	$\bar{\mathbf{6}}_c$	1	0	1

$$\begin{aligned}
& \psi_{[2]}^c \psi_{[2]}^o \psi_{[11]}^s \psi_{[2]}^f, & \psi_{[2]}^c \psi_{[2]}^o \psi_{[2]}^s \psi_{[11]}^f, \\
& \psi_{[2]}^c \psi_{[11]}^o \psi_{[11]}^s \psi_{[11]}^f, & \psi_{[2]}^c \psi_{[11]}^o \psi_{[2]}^s \psi_{[2]}^f
\end{aligned} \quad (4)$$

for color sextet $\psi_{[2]}^c$, and

$$\begin{aligned}
& \psi_{[11]}^c \psi_{[2]}^o \psi_{[11]}^s \psi_{[11]}^f, & \psi_{[11]}^c \psi_{[2]}^o \psi_{[2]}^s \psi_{[2]}^f, \\
& \psi_{[11]}^c \psi_{[11]}^o \psi_{[11]}^s \psi_{[2]}^f, & \psi_{[11]}^c \psi_{[11]}^o \psi_{[2]}^s \psi_{[11]}^f
\end{aligned} \quad (5)$$

for color triplet $\psi_{[11]}^c$, where, $\psi_{[2]}^o$ and $\psi_{[11]}^o$ correspond to $l_{\chi_1} = 0$ and $l_{\chi_1} = 1$, respectively, as defined by the basis in Table I. $\psi_{[2]}^s$ and $\psi_{[11]}^s$ correspond to spin $s_{qq} = 1$ and $s_{qq} = 0$, respectively. $\psi_{[2]}^f$ and $\psi_{[11]}^f$ correspond to isospin $I_{qq} = 1$ and $I_{qq} = 0$, respectively; analogous definitions apply to the antiquark cluster. Thus, the configurations of total isospin $I = 0$ and $I = 1$ light tetraquarks are listed in Table II.

For qq pair, color $\bar{\mathbf{3}}_c$ is antisymmetric (A), whereas $\mathbf{6}_c$ is symmetric (S). Analogous definitions for $\bar{q} \bar{q}$ pair, $\mathbf{3}_c$ and $\bar{\mathbf{6}}_c$ are A and S, respectively. Spin $s = 0$ is A and $s = 1$ is S. The internal spatial wave function is S for even l_{χ} and A for odd l_{χ} . In $SU_f(2)$ flavor symmetry, $I = 0$ and $I = 1$ correspond to flavor A and S configurations, respectively.

TABLE III. Possible 1^{-+} fully charm tetraquark configurations for 1^{-+} in a complete HO basis. Internal orbitals $l_{\chi_1}, l_{\chi_2}, l_{\chi_3} \in \{0, 1\}$; $l_{\chi_1} + l_{\chi_2} + l_{\chi_3} = 1$.

$\{l_{\chi_1}, l_{\chi_2}, l_{\chi_3}\}$	(cc)		$(\bar{c} \bar{c})$	
	ψ_{cc}^c	s_{cc}	$\psi_{\bar{c}\bar{c}}^c$	$s_{\bar{c}\bar{c}}$
$\{1, 0, 0\}$	$\mathbf{6}_c$	1	$\bar{\mathbf{6}}_c$	0
$\{0, 1, 0\}$	$\bar{\mathbf{3}}_c$	1	$\mathbf{3}_c$	0
$\{0, 0, 1\}$	$\bar{\mathbf{3}}_c$	1	$\mathbf{3}_c$	1

TABLE IV. Possible 1^{-+} charmoniumlike tetraquark configurations in a complete HO basis. Internal orbitals $l_{\chi_1}, l_{\chi_2}, l_{\chi_3} \in \{0, 1\}$; $l_{\chi_1} + l_{\chi_2} + l_{\chi_3} = 1$.

$\{l_{\chi_1}, l_{\chi_2}, l_{\chi_3}\}$	(qc)		$(\bar{q} \bar{c})$	
	ψ_{qc}^c	s_{qc}	$\psi_{\bar{q}\bar{c}}^c$	$s_{\bar{q}\bar{c}}$
$\{0, 0, 1\}$	$\bar{\mathbf{3}}_c$	1	$\mathbf{3}_c$	0
$\{0, 1, 0\}$	$\bar{\mathbf{3}}_c$	1	$\mathbf{3}_c$	0
$\{1, 0, 0\}$	$\bar{\mathbf{3}}_c$	1	$\mathbf{3}_c$	0
$\{0, 0, 1\}$	$\bar{\mathbf{3}}_c$	1	$\mathbf{3}_c$	1
$\{0, 1, 0\}$	$\bar{\mathbf{3}}_c$	1	$\mathbf{3}_c$	1
$\{1, 0, 0\}$	$\bar{\mathbf{3}}_c$	1	$\mathbf{3}_c$	1
$\{0, 0, 1\}$	$\mathbf{6}_c$	1	$\bar{\mathbf{6}}_c$	0
$\{0, 1, 0\}$	$\mathbf{6}_c$	1	$\bar{\mathbf{6}}_c$	0
$\{1, 0, 0\}$	$\mathbf{6}_c$	1	$\bar{\mathbf{6}}_c$	0
$\{0, 0, 1\}$	$\mathbf{6}_c$	1	$\bar{\mathbf{6}}_c$	1
$\{0, 1, 0\}$	$\mathbf{6}_c$	1	$\bar{\mathbf{6}}_c$	1
$\{1, 0, 0\}$	$\mathbf{6}_c$	1	$\bar{\mathbf{6}}_c$	1

For the configuration in which both $I_{qq} = 1$ and $I_{\bar{q}\bar{q}} = 1$, the total isospin I can take the values 0, 1, or 2, which may lead to ambiguous isospin eigenstates. This issue is not considered in this work.

For $cc\bar{c}\bar{c}$ tetraquarks, the cc cluster has a symmetric flavor component $\psi_{[2]}^f$, so the possible configurations are as follows:

$$\psi_{[2]}^c \psi_{[2]}^o \psi_{[11]}^s \psi_{[2]}^f, \quad \psi_{[2]}^c \psi_{[11]}^o \psi_{[2]}^s \psi_{[2]}^f \quad (6)$$

for color sextet $\psi_{[2]}^c$,

$$\psi_{[11]}^c \psi_{[2]}^o \psi_{[2]}^s \psi_{[2]}^f, \quad \psi_{[11]}^c \psi_{[11]}^o \psi_{[11]}^s \psi_{[2]}^f \quad (7)$$

for color triplet $\psi_{[11]}^c$, and similarly for charm antiquark cluster. The possible configurations of fully charm tetraquarks are listed in Table III.

For $qc\bar{q}\bar{c}$ tetraquarks, there is no Pauli-antisymmetry constraint tying flavor to color, spin, or space, so no pair contains two identical quarks or two identical antiquarks. The color-spin-orbit (CSO) sector can therefore mix freely,

independent of flavor, and charmoniumlike tetraquark states are isospin-blind. In this work, we use the notation $\psi_{\bar{3}\otimes 3}^c \psi_{(1\otimes 1)}^{s=1}$, $\psi_{\bar{3}\otimes 3}^c \psi_{(1\otimes 0)}^{s=1}$, $\psi_{\bar{6}\otimes \bar{6}}^c \psi_{(1\otimes 1)}^{s=1}$, $\psi_{\bar{6}\otimes \bar{6}}^c \psi_{(1\otimes 0)}^{s=1}$ to denote these configurations. The possible configurations of charmoniumlike tetraquark are listed in Table IV.

B. Hamiltonian

We employ a widely used nonrelativistic Hamiltonian [13–15] to study meson and tetraquark systems, which takes the form

$$\begin{aligned} H &= H_0 + H_{so}, \\ H_0 &= \sum_{k=1}^N \left(\frac{1}{2} M_k^{\text{ave}} + \frac{p_k^2}{2m_k} \right) \\ &\quad + \sum_{i<j}^N \left(-\frac{3}{16} \vec{\lambda}_i \cdot \vec{\lambda}_j \right) (V_0(r_{ij}) + V_{ss}(r_{ij})), \\ H_{so} &= \sum_{i<j}^N \left(-\frac{3}{16} \vec{\lambda}_i \cdot \vec{\lambda}_j \right) (V_{so}(r_{ij})), \end{aligned} \quad (8)$$

where M_k^{ave} denotes the spin-averaged mass as $\frac{1}{4}M_{PS} + \frac{3}{4}M_V$. m_k are the constituent quark masses. $\vec{\lambda}_i$ are the quark color operator in SU(3). The central potential $V_0(r_{ij})$, the spin-spin interaction $V_{ss}(r_{ij})$ and spin-orbit interaction $V_{so}(r_{ij})$ take the form

$$\begin{aligned} V_0 &= A_{ij} r_{ij} - \frac{B_{ij}}{r_{ij}}, \\ V_{ss} &= \frac{1}{6m_i m_j} \Delta V_V(r) \vec{\sigma}_i \cdot \vec{\sigma}_j, \\ V_{so} &= \frac{1}{r_{ij}} \frac{dV_V}{dr_{ij}} \frac{1}{4} \left[\left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4}{m_i m_j} \right) \vec{L}_{ij} \cdot \vec{S}_{ij} \right. \\ &\quad \left. + \left(\frac{1}{m_i^2} - \frac{1}{m_j^2} \right) \vec{L}_{ij} \cdot (\vec{s}_i - \vec{s}_j) \right] \\ &\quad - \frac{1}{r_{ij}} \frac{dV_S}{dr_{ij}} \left(\frac{\vec{L}_{ij} \cdot \vec{s}_i}{2m_i^2} + \frac{\vec{L}_{ij} \cdot \vec{s}_j}{2m_j^2} \right), \end{aligned} \quad (9)$$

where $\vec{\sigma}_i$ are the quark spin operator in SU(2). Note that we have employed $V_V(r) = -B \text{Erf}[\sigma r]/r$ and $V_S(r) = Ar$, taken from Ref. [16]. m_i and m_j are constituent quark masses of i th and j th quark. $\vec{s}_i$ represents the spin operator for i th quark. $\vec{S}_{ij} = \vec{s}_i + \vec{s}_j$ is the spin operator for (ij) th quark pair. $\vec{L}_{ij}$ is relative orbital operator, taking the form, $\vec{L}_{ij} = \vec{r}_{ij} \times \vec{p}_{ij}$, where $\vec{p}_{ij} = \frac{m_i \vec{p}_i - m_j \vec{p}_j}{m_i + m_j}$.

In line with the previous works [11–13], A_{ij} , B_{ij} , and σ_{ij} are taken to be mass-dependent coupling parameters, taking the form

TABLE V. Imported model coupling parameters.

a	b	B_0	σ_0	m_u	m_c
67413 MeV ²	35 MeV	31.7 MeV ^{1/2}	0.7	420 MeV	1270 MeV

$$A_{ij} = a + b m_{ij}, \quad B_{ij} = B_0 \sqrt{\frac{1}{m_{ij}}}, \quad \sigma_{ij} = \sigma_0 m_{ij}. \quad (10)$$

with m_{ij} being the reduced mass of i th and j th quarks, defined as $m_{ij} = \frac{2m_i m_j}{m_i + m_j}$. a , b , B_0 , and σ_0 are constants.

The central potential V_0 and the spin-spin interaction V_{ss} are treated as leading effects, while the spin-orbit interaction V_{so} is regarded as a perturbation that shifts the spectrum. The Schrödinger equation is solved for mesons and tetraquarks with the Hamiltonian H_0 , yielding the eigenvalue E_0 and eigenstates ψ_0 . The mass spectrum is then obtained by adding the diagonalized H_{so} in the basis of these eigenstates.

The coupling constants a , b , B_0 , and σ_0 , as well as the constituent quark masses m_u and m_c , are taken from previous work [11,13]. These parameters were fixed by reproducing the S- and P-wave mass spectra of light, charmed, bottom, charmonium, and bottomonium mesons within the Hamiltonian in Eq. (8). The adopted parameters and constituent quark masses are listed in Table V, and the corresponding fits can be found in Ref. [13].

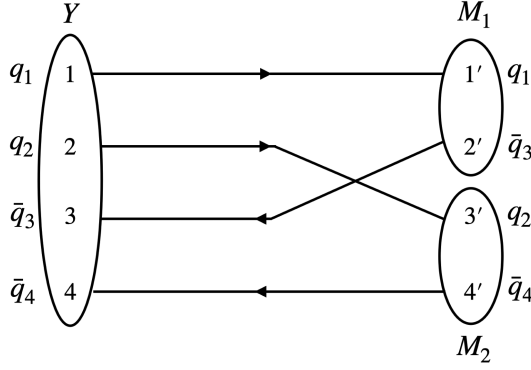
C. Two-body strong decay

We study the two-body strong decay properties of 1^{++} Y tetraquark states within the rearrangement mechanism, as shown in Fig. 1, which is believed to be the dominant mechanism for tetraquarks decaying into two mesons. The transition matrix of the color-spin-flavor takes the forms,

$$\begin{aligned} T_{csf} &= \langle \psi_f^{csf} | \psi_i^{csf} \rangle \\ &= \langle \psi_{M_1 M_2}^c | \psi_Y^c \rangle \langle \psi_{M_1 M_2}^s | \psi_Y^s \rangle \langle \psi_{M_1 M_2}^f | \psi_Y^f \rangle \end{aligned} \quad (11)$$

where ψ_Y^c , ψ_Y^s , and ψ_Y^f are color, spin, and flavor wave functions of the initial tetraquark states taken from the mass spectrum calculation. $\psi_{M_1 M_2}^c$, $\psi_{M_1 M_2}^s$ and $\psi_{M_1 M_2}^f$ are the color, spin, and flavor wave functions of the final states of possible two-body strong decay channels: ωh_1 and ηf_1 for isospin $I = 0$ light tetraquarks; ρh_1 and πf_1 for isospin $I = 1$ light tetraquarks; $\pi/\eta + \chi_{c1}$ and $\rho/\omega + h_c$ for charmoniumlike tetraquarks; and $\eta_c \chi_{c1}$ and $J/\psi h_c$ for fully charm tetraquarks.

To partially account for kinematic effects beyond the color-spin-flavor overlap, we further weight each channel by the corresponding two-body phase space evaluated at the tetraquark mass, and estimate the relative partial widths as

FIG. 1. Quark rearranged diagram for 1^{--} tetraquark.

$$\Gamma_\alpha \propto \Phi_\alpha(M_Y) |T_{csf}^{(\alpha)}|^2, \\ \Phi_\alpha(M_Y) = k_\alpha \frac{E_1(k_\alpha) E_2(k_\alpha)}{M_Y}, \quad (12)$$

where α labels a given decay mode $Y \rightarrow M_1 M_2$, M_Y is the mass of the initial tetraquark, and $E_{1,2}(k) = \sqrt{m_{1,2}^2 + k^2}$ are the on-shell energies of the final mesons with masses $m_{1,2}$. The breakup momentum k_α in the Y rest frame is

$$k_\alpha = \frac{\lambda^{1/2}(M_Y^2, m_1^2, m_2^2)}{2M_Y}, \quad (13)$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$.

In the work, the ratios between Γ_α of various decay channels are approximated as the decay branching ratios (relative decay ratios), supposing that the contributions of the spatial part of different decay channels are not significantly different.

III. RESULTS AND DISCUSSION

The color operator $\vec{\lambda}_i \cdot \vec{\lambda}_j$ in the Hamiltonian in Eq. (8), along with the mass dependent coefficient in the Cornell-like potential, mixes different color configurations, leading to nonzero cross terms for tetraquarks.

Similarly, the color-spin operator $\vec{\lambda}_i \cdot \vec{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j$, together with the mass dependent coefficient in the spin-spin interaction,

$$\vec{\lambda}_i \cdot \vec{\lambda}_j V_{ss} = \frac{2B_{ij}\sigma_{ij}^3}{3m_i m_j \sqrt{\pi}} e^{-\sigma_{ij}^2 r_{ij}^2} \vec{\lambda}_i \cdot \vec{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j, \quad (14)$$

mixes different color-spin configurations.

For reference, the expectation values of $O_{ij} = \vec{\lambda}_i \cdot \vec{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j$ for $S = 1$ states are listed in Table VI, with all components given in the order $(O_{12}, O_{13}, O_{23}, O_{14}, O_{24}, O_{34})$. The mass dependent factor in Eq. (14) induces nonzero cross terms between different color-spin configurations.

In conclusion, the $I = 0$ light tetraquarks are linear combinations of $|\psi_{3\bar{3}\bar{3}}^c \psi_{(1\bar{0})}^{S=1}\rangle$, $|\psi_{6\bar{6}\bar{6}}^c \psi_{(1\bar{0})}^{S=1}\rangle$, $|\psi_{6\bar{6}\bar{6}}^c \psi_{(1\bar{0})}^{S=1}\rangle$, and $I = 1$ light tetraquarks are linear combinations of $|\psi_{6\bar{6}\bar{6}}^c \psi_{(1\bar{0})}^{S=1}\rangle$, $|\psi_{3\bar{3}\bar{3}}^c \psi_{(1\bar{0})}^{S=1}\rangle$, $|\psi_{6\bar{6}\bar{6}}^c \psi_{(1\bar{0})}^{S=1}\rangle$, $|\psi_{3\bar{3}\bar{3}}^c \psi_{(1\bar{0})}^{S=1}\rangle$. For fully charm tetraquark, $|\psi_{6\bar{6}\bar{6}}^c \psi_{(1\bar{0})}^{S=1}\rangle$, $|\psi_{3\bar{3}\bar{3}}^c \psi_{(1\bar{0})}^{S=1}\rangle$, $|\psi_{6\bar{6}\bar{6}}^c \psi_{(1\bar{0})}^{S=1}\rangle$ configurations are considered due to the quark configurations we have mentioned in Sec. II A. The charmoniumlike tetraquarks are linear combinations of $|\psi_{6\bar{6}\bar{6}}^c \psi_{(1\bar{0})}^{S=1}\rangle$, $|\psi_{3\bar{3}\bar{3}}^c \psi_{(1\bar{0})}^{S=1}\rangle$, $|\psi_{6\bar{6}\bar{6}}^c \psi_{(1\bar{0})}^{S=1}\rangle$, $|\psi_{3\bar{3}\bar{3}}^c \psi_{(1\bar{0})}^{S=1}\rangle$.

We tested convergence with increasing bases and adopt $N_{\max} = 13$ (corresponding to $n_{\max} = 6$ for $L = 1$), which yields stable masses in all three tetraquark configurations. The masses obtained with this setting are taken as the final results in this section.

A. Results of light tetraquark

The mass results for the mixed light tetraquark states are given in Table VII, where En ($n = 1, 2, 3, \dots$) represents the n th eigenstate of the Hamiltonian in Eq. (8) with color-spin configuration mixing. The relative decay ratios of the E1 to E5 with various mixed configurations are also listed in Table VII, which are normalized to the E3 ωh_1 channel = 1 for $I = 0$ states and E3 ρh_1 channel = 1 for $I = 1$ states. THR represents the threshold of two meson final states $M_1 + M_2$, with the units of MeV. The symbol “...” in table indicates that the predicted mass lies below the corresponding two-body threshold; therefore this decay channel is kinematically forbidden in our framework.

For an S-wave decay of an exotic 1^{--} state, the only two-meson combinations allowed by the quantum numbers are $(0^{--} \oplus 1^{++})$ and $(1^{--} \oplus 1^{+-})$. All other $S + P$ (S-wave + P-wave meson) combinations either fail to satisfy $C = +$ or

TABLE VI. Expectation values of hyperfine interaction of $S = 1$ $qc\bar{q}\bar{c}\langle\psi_c^k \psi_s^{h1} | \vec{\lambda}_i \cdot \vec{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j | \psi_c^k \psi_s^{h2}\rangle$.

$\vec{\lambda}_i \cdot \vec{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j$	$ \psi_{3\bar{3}\bar{3}}^c \psi_{(1\bar{0})}^{S=1}\rangle$	$ \psi_{3\bar{3}\bar{3}}^c \psi_{(1\bar{0})}^{S=1}\rangle$	$ \psi_{6\bar{6}\bar{6}}^c \psi_{(1\bar{0})}^{S=1}\rangle$	$ \psi_{6\bar{6}\bar{6}}^c \psi_{(1\bar{0})}^{S=1}\rangle$
$ \psi_{3\bar{3}\bar{3}}^c \psi_{(1\bar{0})}^{S=1}\rangle$	$(-\frac{8}{3}, 0, 0, 0, 0, 8)$	$(0, -\frac{4\sqrt{2}}{3}, -\frac{4\sqrt{2}}{3}, \frac{4\sqrt{2}}{3}, \frac{4\sqrt{2}}{3}, 0)$	$(0, 0, 0, 0, 0, 0)$	$(0, -4, 4, -4, 4, 0)$
$ \psi_{3\bar{3}\bar{3}}^c \psi_{(1\bar{0})}^{S=1}\rangle$	$(0, -\frac{4\sqrt{2}}{3}, -\frac{4\sqrt{2}}{3}, \frac{4\sqrt{2}}{3}, \frac{4\sqrt{2}}{3}, 0)$	$(-\frac{8}{3}, \frac{4}{3}, \frac{4}{3}, \frac{4}{3}, \frac{4}{3}, -\frac{8}{3})$	$(0, -4, 4, -4, 4, 0)$	$(0, 2\sqrt{2}, -2\sqrt{2}, -2\sqrt{2}, 2\sqrt{2}, 0)$
$ \psi_{6\bar{6}\bar{6}}^c \psi_{(1\bar{0})}^{S=1}\rangle$	$(0, 0, 0, 0, 0, 0)$	$(0, -4, 4, -4, 4, 0)$	$(-\frac{4}{3}, 0, 0, 0, 0, -4)$	$(0, -\frac{10\sqrt{2}}{3}, -\frac{10\sqrt{2}}{3}, \frac{10\sqrt{2}}{3}, \frac{10\sqrt{2}}{3}, 0)$
$ \psi_{6\bar{6}\bar{6}}^c \psi_{(1\bar{0})}^{S=1}\rangle$	$(0, -4, 4, -4, 4, 0)$	$(0, 2\sqrt{2}, -2\sqrt{2}, -2\sqrt{2}, 2\sqrt{2}, 0)$	$(0, -\frac{10\sqrt{2}}{3}, -\frac{10\sqrt{2}}{3}, \frac{10\sqrt{2}}{3}, \frac{10\sqrt{2}}{3}, 0)$	$(\frac{4}{3}, \frac{10}{3}, \frac{10}{3}, \frac{10}{3}, \frac{10}{3}, \frac{4}{3})$

TABLE VII. Masses of mixed $qq\bar{q}\bar{q}$ tetraquark states. The mass results are given in units of MeV. THR denotes the threshold of the two-meson final state $M_1 + M_2$, also in units of MeV. The decay ratios are normalized to the E3 ωh_1 channel ($= 1$) for $I = 0$ states and to the E3 ρh_1 channel ($= 1$) for $I = 1$ states.

$qq\bar{q}\bar{q}$	THR	E1	E2	E3	E4	E5
$M_{\text{cal}} I = 0$		1961	2231	2242	2320	2516
$\omega h_1(1170)$	1952	0.01	0.01	1	32	26
$\eta f_1(1285)$	1835	0.03	0.01	0.1	7	4
$M_{\text{cal}} I = 1$		1884	1924	2112	2247	2270
$\rho h_1(1170)$	1940	...	...	1	1.6	2.0
$\pi f_1(1285)$	1420	38	42	2.0	1.2	0.2

cannot produce $J = 1$ when the relative orbital angular momentum between the two mesons is $L_{\text{rel}} = 0$.

It is worth mentioning that, in principle, the $I = 0$ 1^{-+} tetraquark can decay through the $\eta\eta'$ channel when the relative orbital angular momentum is $L_{\text{rel}} = 1$, which allows $J = 1$ and negative parity $P = -$. For a two-meson final state composed of two different self-conjugate mesons $M_1 \neq M_2$ (each with a well-defined intrinsic $C_{M_1}, C_{M_2} = \pm 1$), the total two-body C-parity is simply $C_{\text{tot}} = C_{M_1} C_{M_2}$, because there is no interchange symmetry between the two distinct mesons. Since η and η' are distinct, no exchange symmetry factor applies. Thus, for the $\eta\eta'$ channel, the charge conjugation can take $C_{\text{tot}} = C_{M_1} C_{M_2} = +$ [17,18].

However, in the work, the spin-color-flavor factor of the transition amplitude is zero in the two-body strong decay to $\eta\eta'$ from $I = 0$ 1^{-+} light tetraquarks.

B. $\pi_1(2015)$

E852 observed two 1^{-+} states while studying $\pi^- p$ collisions in the decay channel $f_1(1285)\pi$ [9] around 1.6 and 2.0 GeV respectively. The higher-mass peak is as $M = 2001 \pm 30 \pm 92$ MeV and $\Gamma = 333 \pm 52 \pm 49$ MeV, commonly denoted $\pi_1(2015)$ [also known as $\pi_1(2000)$]. Later, the $\pi_1(2015)$ state is observed again by E852 in the decay channel $b_1(1235)\pi$ [19], with mass and decay width $M = 2014 \pm 20 \pm 16$ MeV and $\Gamma = 230 \pm 32 \pm 73$ MeV which are consistent with that observed in the $f_1(1285)\pi$ final state.

VES performed a similar partial-wave analysis, and the $\pi_1(1600)$ is observed in both $f_1(1285)\pi$ and $b_1(1235)\pi$ decay channels [20]. Their intensity of the 1^{-+} partial wave in both channels above 1.9 GeV is considered to be consistent with that observed by E852 [21]. However, VES did not comment on this and did not claim the existence of the $\pi_1(2015)$.

COMPASS recently presented preliminary results of partial-wave decomposition of the diffractive reaction $\pi^- p \rightarrow \pi^- \pi^+ \pi^- \eta p$ [22], and no clear evidence for an excited state as previously reported by E852 [9] was found,

and they highlighted the model dependence of such interpretations.

A recent light hybrid nonet calculation in a semirelativistic quark potential model organizes the light 1^{-+} $\pi_1(1600)$ states into a hybrid nonet with 1669 MeV correspond to the hybrid configurations $(u\bar{u} - d\bar{d})g/\sqrt{2}$. The following analyses accommodate $\pi_1(2015)$ to be first radial excitation of $\pi_1(1600)$ near 2.1 GeV [23]. In this view, $\pi_1(2015)$ may have sizable hybrid content but requires confirmation in $f_1\pi$ and $b_1\pi$ with modern amplitude methods.

Flux-tube models [24] and lattice-QCD [25] place the lightest 1^{-+} near 1.7–1.9 GeV and expect dominant $S + P$ decays, consistent with the observed $f_1\pi$ and $b_1\pi$ modes. The slightly higher mass of $\pi_1(2015)$ can be explained by hybrid-tetraquark mixing or by identifying it as an excited partner.

A QCD sum rule study of the light 1^{-+} hybrid mesons gives a conservative mass range of 1.7–2.6 GeV, which naturally accommodates the $\pi_1(2015)$ as a hybrid but disfavors the $\pi_1(1600)$ as a pure hybrid, indicating possible additional nonhybrid components [26]. A related light-cone QCD sum rule analysis finds a large $\pi_1 \rightarrow b_1\pi$ width increasing across the 1.6–2.0 GeV region, but a very small $\rho\pi$ width, consistent with the expectation of dominant $S + P$ decays. This suggests that a predominantly hybrid $\pi_1(2015)$ should couple strongly to $b_1\pi$, while the $\rho\pi$ channel remains suppressed [27].

The observed $S + P$ decay pattern matches expectations for hybrids, yet is also natural for a compact tetraquark. A QCD sum rule analysis of exotic tetraquark states, using diquark-antidiquark currents with $I^G J^{PC} = 1^{-+} 1^{-+}$ and $0^{+} 1^{-+}$ [28,29] can accommodate $\pi_1(2015)$, with the $qs\bar{q}\bar{s}$ configuration around 2.0 GeV, which supports the assignment of $\pi_1(2015)$ to a compact tetraquark that naturally prefers $S + P$ decays such as $b_1\pi$ and $f_1\pi$.

In this work, the $I = 1$ light tetraquark E2 state with a mass of around 2 GeV is comparable to the $\pi_1(2015)$ within the experimental uncertainties. The $\pi_1(2015)$ may be assigned to the E2 tetraquark state based on its large decay ratio to πf_1 . We suggest searching for $I = 1$ tetraquark states in $S + P$ channels around 1.9–2.0 GeV, and for $I = 0$ tetraquark states in $S + P$ channels around 2.3 GeV and 2.5 GeV, motivated by the large decay ratios predicted for several tetraquark states.

C. $\eta_1(1855)$

In 2022, the BESIII Collaboration reported the discovery of a new isoscalar exotic state, $\eta_1(1855)$ with quantum numbers 1^{-+} , observed in the decay process $J/\psi \rightarrow \gamma\eta\eta'$ [10]. The measured mass is $(1855 \pm 9_{-1}^{+6})$ MeV, and the width is $(188 \pm 18_{-8}^{+3})$ MeV, with a statistical significance greater than 19σ .

BESIII also estimated the product branching fraction as $\mathcal{B}(J/\psi \rightarrow \gamma \eta_1(1855)) \times \mathcal{B}(\eta_1(1855) \rightarrow \eta \eta') = (2.70 \pm 0.41_{-0.35}^{+0.16}) \times 10^{-6}$ [10]. This state is considered a candidate for a hybrid meson, in agreement with lattice QCD predictions [30].

In the hybrid meson framework, the observation of $\eta_1(1855)$, together with the isovector $\pi_1(1600)$, provides valuable information for understanding the 1^{-+} hybrid nonet [31–38]. A semirelativistic quark potential model was used to estimate the masses of light hybrid mesons with 1^{-+} [37]. This study suggests that $\pi_1(1600)$ and $\eta_1(1855)$ might correspond to the hybrid configurations $(u\bar{u} - d\bar{d})g/\sqrt{2}$ and $s\bar{s}g$, respectively.

However, another recent study modeled hybrid mesons as three-body systems (quark-antiquark-gluon), treating the gluon as a real constituent [39]. Their results indicate that both $\pi_1(1600)$ and $\eta_1(1855)$ cannot be interpreted as 1^{-+} hybrids at the same time, due to inconsistencies in their decay widths. In particular, they found that the decay width of $\eta_1(1855) \rightarrow \eta \eta'$ is nearly zero at leading order, which challenges its hybrid interpretation.

Alternatively, in the hadronic molecule picture, $\eta_1(1855)$ could be a bound state of $K\bar{K}_1(1400)$, as its mass is about 40 MeV below this threshold [40,41]. Ref. [40] used the one-boson exchange model to explore the binding and decay properties of such a molecule and found results supporting this interpretation. Similarly, Ref. [41] studied its radiative and strong decays using the effective Lagrangian approach and compared them with experimental data.

On the other hand, a study using the chiral quark model did not support the $K\bar{K}_1$ molecular interpretation of $\eta_1(1855)$ [42]. In another analysis using QCD sum rules [43], the exotic $K\bar{K}_1(1270)$ and $K\bar{K}_1(1400)$ molecular states with 1^{-+} were investigated, but the spectral functions turned out negative in physical regions, making the results unreliable.

A tetraquark molecular picture was also considered. In Ref. [44], configurations like $[1_c]s\bar{s} \otimes [1_c]q\bar{q}$ and $[1_c]s\bar{q} \otimes [1_c]s\bar{q}$ were studied using QCD sum rules. The mass calculations suggest that $\eta_1(1855)$ could match the $[1_c]s\bar{s} \otimes [1_c]q\bar{q}$ configuration.

In this work, the theoretical relative decay ratios for the decay of the 1^{-+} light tetraquark state into $\eta \eta'$ are found to be nearly zero. Consequently, the $\eta_1(1855)$ state observed in the $\eta \eta'$ channel [10] cannot be accommodated within the present tetraquark picture when both the mass spectrum and decay properties are taken into account.

The $\eta_1(1855)$ may instead correspond to other configurations, such as a hadronic molecular state or a hybrid meson, as discussed above. In particular, if the $\eta_1(1855)$ is interpreted as a $K_1\bar{K}$ hadronic molecule and decays into $\eta \eta'$, the corresponding decay width is expected to be sizable, as discussed in detail in Ref. [45]. While some models support such exotic interpretations, others

challenge these assignments, highlighting the need for further theoretical and experimental studies.

D. Results of charmoniumlike 1^{-+} tetraquark

Lattice QCD predicts that the lightest 1^{-+} charmonium hybrid (η_{c1}) lies around 4.2–4.4 GeV and decays mainly to open-charm $S + P$ channels such as $D_1\bar{D}$, $D\bar{D}$, and $D\bar{D}^*$. Its hidden-charm branching fractions are small, which makes it hard to observe in clean but suppressed radiative modes [46].

At present, no charmoniumlike state with 1^{-+} has been confirmed. Extensive studies of the XYZ sector by BESIII, LHCb, Belle, and Belle II have found many states with nonexotic, non- 1^{-+} quantum numbers (e.g., 1^{++} , 1^{+-} , 0^{++} , 2^{++}), while searches for 1^{-+} have so far found nothing. A dedicated BESIII search, $e^+e^- \rightarrow \gamma \eta^{(\prime)} \eta_c$ at $\sqrt{s} = 4.258 - 4.681$ GeV, saw no significant signal and set 90% CL upper limits on the cross sections [47].

Promising search modes that act as quantum-number filters for $C = +$, odd parity, and $J = 1$ are: (i) radiative production at BESIII/Belle II, $e^+e^- \rightarrow \gamma Y(1^{-+})$ with $Y \rightarrow S + P$ meson pairs ($\eta + \chi_{cJ}$, $\rho + h_c$), which provide clean angular observables for amplitude fits [48,49]; and (ii) B -meson decays at LHCb/Belle II, $B \rightarrow KY(1^{-+})$ with $Y \rightarrow S + P$ meson pairs, enabling direct tests of the exotic 1^{-+} hypothesis.

Higher-luminosity scans and better electromagnetic calorimeter (EMC) calibrations at BESIII (below 4.66 GeV) and Belle II (10.58 GeV), plus targeted angular-moment analyses for 1^{-+} are especially promising [50]. Notably, BESIII's first direct production of a $C = +$ state, $e^+e^- \rightarrow \chi_{c1}$, shows that e^+e^- experiments can access $C = +$ states [51]. The work confirms key tools, photon reconstruction and interference control required for a 1^{-+} search.

The mass spectrum and S-wave relative decay ratios of the charmoniumlike tetraquarks are listed in Table VIII, and the ratios are normalized to the $E3 \eta \chi_{c1}$ channel = 1. Unlike the cases for light tetraquarks and fully charm tetraquarks, where the flavor configurations impose constraints on color-spin-orbit ones, the color-spin-orbit configurations for charmoniumlike tetraquark are independent of flavors. Therefore, the present work is isospin-blind for charmoniumlike tetraquark states. As a result, the masses of the $I = 0$ and $I = 1$ states are the same in this work, and both the $\pi/\eta + \chi_{c1}$ and $\rho/\omega + h_c$ decay channels are included. The relative decay ratios differ due to phase-space factors.

The lowest mass is around 4.2 GeV with a relatively large ratio to $\eta \chi_{c1}$ channel. In the work, the theoretical results of relative decay ratios for 1^{-+} charmoniumlike tetraquark state decaying to $\eta \eta_c$ are very small. The work suggests searching for tetraquark states in $\eta \chi_{c1}$ channels in the mass range of 4.2–4.3 GeV due to the large decay ratio of two tetraquark states.

TABLE VIII. Masses of mixed charmoniumlike tetraquark states. The theoretical mass results M_{cal} are given in units of MeV. THR denotes the threshold of the two-meson final state $M_1 + M_2$, also in units of MeV. The decay ratios are normalized to the E3 $\eta\chi_{c1}$ channel ($= 1$).

$q\bar{c}\bar{q}\bar{c}$	THR	E1	E2	E3	E4	E5	E6
$M_{\text{cal}} I = 0$		4245	4287	4289	4307	4421	4441
$\eta\chi_{c1}$	4057	13	23	1	4.5	0.1	0.6
ωh_c	4307	...	...	...	0.03	0.3	0.6
$M_{\text{cal}} I = 1$		4245	4287	4289	4307	4421	4441
$\pi\chi_{c1}$	3650	27	41	1.8	8	0.1	0.8
ρh_c	4295	...	...	...	0.2	0.3	0.7

E. Results of fully charm 1^{-+} tetraquark

To date there is no confirmed observation of a 1^{-+} fully charm state. The established structures in the di- J/ψ spectrum, most prominently the narrow $X(6900)$ and companion peaks near 6.6 and 7.1 GeV observed by LHCb [52], CMS [53], and ATLAS [54], are studied in the previous work [11], and tentatively assigned as S-wave tetraquark states. A recent CMS angular analysis determines the parity and charge-conjugation of the fully charm peaks to be 2^{++} and disfavors $J = 1$ assignments, thus excluding 1^{-+} for those structures [55]. Some theoretical studies typically locate low-lying fully charm tetraquarks with 0^{++} , 2^{++} (and sometimes 1^{+-}), not 1^{-+} [56,57].

The most sensitive venues are CMS/ATLAS and LHCb, which provide large prompt-charm samples and clean muon triggers for dicharmonium studies. However, the main channel $J/\psi J/\psi$ favors even- J and $C = +$ waves; $J = 1$ in S-wave is strongly suppressed near threshold and disfavored by the Bose symmetry of two identical vectors, so sensitivity to 1^{-+} is low. Inclusive production also dilutes signals with large nonresonant dicharmonium backgrounds, and limited data in $S + P$ modes hinder full amplitude analyses. A better strategy is to move beyond identical-vector final states to $S + P$ charmonium pairs with definite $C = +$ and richer angular structure (e.g., $\eta_c\chi_{cJ}$, $J/\psi h_c$).

On the detector side, LHC Run 3 upgrades, higher-rate dimuon triggers, better mass resolution, and dedicated dicharmonium selections, should increase sensitivity [53,58]. At the $\bar{p}p$ threshold, PANDA will enable formation scans with excellent resolution and access to exclusive $S + P$ final states, which could be decisive for exotic J^{PC} tests [59]. Belle II can also contribute via double-charmonium production at 10.58 GeV, with full-amplitude analyses in $J/\psi h_c$ and $\eta_c\chi_{cJ}$, though cross sections are lower than at the LHC.

The mass spectra and S-wave relative decay ratio are listed in Tables IX for the fully charm tetraquark, and the ratios are normalized to the E3 $J/\psi h_c$ channel $= 1$. The lowest mass is around 6.6 GeV with a relatively large ratio

TABLE IX. Masses of $1^{-+} cc\bar{c}\bar{c}$ tetraquark states. The mass results are given in units of MeV. THR denotes the threshold of the two-meson final state $M_1 + M_2$, also in units of MeV. The decay ratios are normalized to the E3 $J/\psi h_c$ channel ($= 1$).

$cc\bar{c}\bar{c}$	THR	E1	E2	E3	E4	E5	E6
M_{cal}		6595	6762	6881	6908	7063	7079
$J/\psi h_c$	6622	...	0.1	1	36	47	0.2
$\eta_c\chi_{c1}$	6496	2	0.1	1.5	51	60	0.3

to $\eta_c\chi_{c1}$ channel. The work suggests searching for tetraquark states in $\eta_c\chi_{c1}$ channels around 6.6 GeV, and in $J/\psi h_c$ and $\eta_c\chi_{c1}$ channels in the mass range of 6.9–7.1 GeV, due to the large decay ratio of several tetraquark states.

IV. SUMMARY

While the hybrid interpretation has been extensively explored in the literature, the tetraquark scenario remains particularly compelling due to the rich internal dynamics inherent to four-quark systems.

The masses and decay patterns of 1^{-+} light, charmoniumlike, and fully charm tetraquark states are studied using a constituent quark model that incorporates a Cornell-like potential and the Breit-Fermi interaction. The theoretical results for tetraquarks are compared with those of 1^{-+} exotic states.

The lowest 1^{-+} light tetraquark state is around 1.9 GeV, which is significantly above the observed $\pi_1(1400)$ and $\pi_1(1600)$ states.

The observed states $\eta_1(1855)$ and $\pi_1(2015)$ are located within the light tetraquark range. 1^{-+} Y tetraquark states decaying to $\eta\eta'$ is strongly suppressed in our CSF overlap within the rearrangement mechanism. Thus, the $\eta_1(1855)$ state observed from $\eta\eta'$ by BESIII cannot be accommodated in the current tetraquark picture.

The $\pi_1(2015)$ is assigned as $I = 1$ E2 1^{-+} light tetraquark state according to both mass agreement and the large decay ratio. More experimental data and theoretical studies are essential for making unambiguous assignments.

For the 1^{-+} charmoniumlike tetraquark state, the lowest mass is predicted near 4.2 GeV and could decay into $\eta\chi_{c1}$ channel.

The lowest 1^{-+} fully charm tetraquark state is around 6.6 GeV and may be searched in $\eta_c\chi_{c1}$ channel.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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