

Explaining the $B \rightarrow K\mu^+\mu^-$ anomaly in the left-right inverse seesaw model

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We investigate the long-standing anomaly in the rare decay $B \rightarrow K\mu^+\mu^-$ within the left-right inverse seesaw (LRIS) model. Global analyses of the $b \rightarrow s\mu^+\mu^-$ data consistently indicate a significant negative shift in the vector Wilson coefficient, $\Delta C_9^\mu \approx -1$, while the axial coefficient ΔC_{10}^μ remains consistent with zero. We show that a charged-scalar/heavy-neutrino box diagram in the LRIS model naturally generates these values for the Wilson coefficients through a *nondecoupling* mechanism: the right-handed coupling $Y_{\mu N}^R \propto M_{N_i}/v_R$ produces a contribution to ΔC_9^μ that is unsuppressed in the heavy-neutrino limit, while the simultaneous presence of a comparable left-handed Dirac Yukawa coupling $Y_{\mu N}^L$ ensures the automatic cancellation $\Delta C_{10}^\mu \approx 0$. The potentially dangerous contribution to $B_s - \bar{B}_s$ mixing is controlled by the usual Glashow–Iliopoulos–Maiani suppression associated with the top-quark loop, keeping it safely below current experimental bounds throughout the viable parameter space. In addition, a specific right-handed phase texture in the right-handed quark mixing matrix, whose origin can be traced to a discrete Z_2 symmetry in the right-handed quark sector, provides a technically natural and radiatively stable structural feature of the model. A numerical scan over the model parameter space identifies a viable region with $y \equiv |Y_{is,ib}^H| \approx 0.39$ and $M_{H^\pm} \gtrsim 600$ GeV, consistent with current flavor and collider experimental constraints.

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I. INTRODUCTION

Flavor-changing neutral-current (FCNC) processes in the $b \rightarrow s$ sector provide among the most sensitive probes of physics beyond the Standard Model (BSM). In particular, the rare semileptonic decay $B \rightarrow K\mu^+\mu^-$ is forbidden at tree level in the Standard Model (SM) and proceeds only through suppressed electroweak penguin and box diagrams [1].

Over the past decade, the LHCb Collaboration has reported a series of measurements revealing systematic deviations in the angular distributions of $b \rightarrow s\mu^+\mu^-$ decays compared to SM predictions [2–5]. Most notably, the optimized observable P'_5 in $B \rightarrow K^*\mu^+\mu^-$ shows persistent discrepancies at the level of several standard deviations across multiple q^2 bins, as illustrated in Table I. Global analyses of these data consistently indicate a significant negative shift in the vector Wilson coefficient [6–8],

$$\Delta C_9^\mu \approx -1, \quad (1)$$

while the axial coefficient C_{10} remains broadly consistent with its SM value, $\Delta C_{10}^\mu \approx 0$. These Wilson coefficient values require a BSM contribution that is predominantly *vectorlike* in the muon current, a feature that is nontrivial to achieve in concrete models.

In this work, we address this anomaly in the context of the left-right inverse seesaw (LRIS) model [9–12], which extends the SM gauge symmetry to $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and incorporates the inverse seesaw mechanism for neutrino mass generation [13–15]. While the lepton-flavor-universality ratios R_K and R_{K^*} have recently been found consistent with the SM [4,16], the angular anomalies in $b \rightarrow s\mu^+\mu^-$ transitions persist and continue to motivate BSM explanations [17–21].

The key mechanism we exploit is a charged-scalar/heavy-neutrino box diagram that exhibits *nondecoupling* behavior: the dynamically generated right-handed Yukawa coupling $Y_{\mu N}^R \propto M_{N_i}/v_R$ ensures that the heavy neutrino mass cancels in the amplitude, leaving a finite, potentially observable contribution to ΔC_9^μ . Crucially, the simultaneous presence of the left-handed Dirac Yukawa coupling $Y_{\mu N}^L \sim y_{2i}^L$ with $|Y_{\mu N}^L| \approx |Y_{\mu N}^R|$ generates equal and opposite contributions to C_{10} , driving $\Delta C_{10}^\mu \approx 0$

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TABLE I. Selected LHCb measurements of the optimized observable P'_5 in $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ compared to SM predictions, in bins of q^2 (GeV^2). Discrepancies are quoted in units of σ , combining experimental and theoretical uncertainties in quadrature.

q^2 bin [GeV^2]	P_5^{LHCb}	P_5^{SM}	Discrepancy	Reference
[4.0, 6.0]	$-0.439^{+0.111}_{-0.112} \pm 0.036$	-0.45 ± 0.14	$\sim 0.3\sigma$	[2]
[6.0, 8.0]	$-0.583^{+0.090}_{-0.093} \pm 0.024$	-0.28 ± 0.12	$\sim 2.9\sigma$	[2]
[1.1, 2.5]	$+0.025^{+0.180}_{-0.173} \pm 0.024$	$+0.37 \pm 0.11$	$\sim 2.0\sigma$	[2]
[2.5, 4.0]	$-0.165^{+0.173}_{-0.165} \pm 0.022$	$+0.09 \pm 0.16$	$\sim 1.4\sigma$	[2]
[15.0, 19.0]	$-0.640^{+0.072}_{-0.072} \pm 0.013$	-0.76 ± 0.06	$\sim 1.5\sigma$	[2]

automatically—without any additional tuning beyond the condition natural to the inverse seesaw.

The paper is organized as follows. In Sec. II we review the effective Hamiltonian for $b \rightarrow s\mu^+\mu^-$ and state the phenomenological requirements. In Sec. III we discuss the SM contributions and their chiral structure. In Sec. IV we introduce the LRIS model, including the scalar sector, Yukawa Lagrangian, and the neutrino mass matrix. In Sec. V we compute the nondecoupling box contribution and derive the matching conditions for ΔC_9^μ and ΔC_{10}^μ . In Sec. VI we discuss the phenomenological constraints, in particular B_s – $\bar{B}_s$ mixing and $b \rightarrow s\gamma$. We devote Sec. VII to a detailed verification of the self-consistency of the leptonic coupling cancellation condition, including neutrino masses and Pontecorvo–Maki–Nakagawa–Sakata (PMNS) nonunitarity. The origin of the right-handed phase texture is analyzed in Sec. VIII A. Numerical results are presented in Sec. IX, and we conclude in Sec. X.

II. EFFECTIVE HAMILTONIAN AND PHENOMENOLOGICAL REQUIREMENTS

The $b \rightarrow s\mu^+\mu^-$ transition is described by the weak effective theory (WET) Hamiltonian [22]

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_i (C_i \mathcal{O}_i + C'_i \mathcal{O}'_i) + \text{H.c.}, \quad (2)$$

where the $\mathcal{O}'_i$ are the chirality-flipped counterparts of $\mathcal{O}_i$, obtained by $P_L \leftrightarrow P_R$. The dominant semileptonic operators are

$$\mathcal{O}_9 = (\bar{s}\gamma^\mu P_L b)(\bar{\mu}\gamma_\mu \mu), \quad (3)$$

$$\mathcal{O}_{10} = (\bar{s}\gamma^\mu P_L b)(\bar{\mu}\gamma_\mu \gamma_5 \mu). \quad (4)$$

Global fits to the $b \rightarrow s\mu^+\mu^-$ data favor the pattern [6]

$$\Delta C_9^\mu \approx -1, \quad \Delta C_{10}^\mu \approx 0. \quad (5)$$

The chiral structure of this pattern can be understood model independently. A purely right-handed muon current satisfies $\Delta C_9^\mu = +\Delta C_{10}^\mu$, while a purely left-handed current

gives $\Delta C_9^\mu = -\Delta C_{10}^\mu$. Neither alone can reproduce Eq. (5). The solution requires a *vectorlike* muon current with comparable left- and right-handed couplings g_L^μ and g_R^μ :

$$\Delta C_9^\mu \propto |g_L^\mu|^2 + |g_R^\mu|^2 \neq 0, \quad (6)$$

$$\Delta C_{10}^\mu \propto |g_R^\mu|^2 - |g_L^\mu|^2 \approx 0, \quad (7)$$

in the limit $|g_L^\mu| \approx |g_R^\mu|$.

III. STANDARD MODEL CONTRIBUTIONS

In the SM, $b \rightarrow s\mu^+\mu^-$ receives contributions from photon penguin, Z-penguin, and W-box diagrams, all involving internal (t , $W^\pm$) states. The W-box involves only left-handed leptons, $W^\mu \bar{\nu}_\mu P_L \mu$, generating

$$\Delta C_9^{\text{box}} = -\Delta C_{10}^{\text{box}}, \quad (8)$$

while the photon penguin contributes mainly to C_9 through the vector coupling of the photon. Crucially, the SM does not generate a right-handed muon current $(\bar{s}\gamma^\mu P_L b)(\bar{\mu}\gamma_\mu P_R \mu)$, making such a structure a hallmark of BSM physics.

IV. THE LEFT-RIGHT INVERSE SEESAW MODEL

The LRIS model [9–12,23] is based on the gauge group

$$\mathcal{G}_{LR} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}. \quad (9)$$

The model contains the same fermion content as any conventional left-right model [9–11], supplemented by two singlet fermions per family, S_1 and S_2 , with opposite $B-L$ charges $(-2, +2)$. The singlet S_2 implements the inverse seesaw mechanism for neutrino masses, while S_1 cancels the resulting $U(1)_{B-L}$ anomaly. A $\mathbb{Z}_2$ discrete symmetry (all fields even except S_1 , which is odd) forbids the bare mixing mass $M\bar{S}_1^c S_2$ and thereby enforces the inverse seesaw structure [23].

Since every other field in the model is $\mathbb{Z}_2$ -even, S_1 cannot decay and is stable on cosmological timescales,

making it a natural dark-matter candidate, similar to scotogenic-type models in which a discrete symmetry stabilizes a neutral field and forbids unwanted neutrino-mass operators [24–26]. A quantitative treatment of its relic abundance and direct-detection prospects requires specifying the portal through which S_1 obtains its mass and couples to the visible sector, e.g. a Yukawa coupling to a $\mathbb{Z}_2$ -odd scalar singlet [27] or a higher-dimensional operator connecting S_1 to the bidoublet [28]; none of these mechanisms are directly related to the $B \rightarrow K\mu^+\mu^-$ phenomenology analyzed here. A gauge-singlet fermion with only such portal interactions and no direct coupling to the Z or W_R gauge bosons generically evades the strongest current direct-detection bounds [29,30] unless the portal coupling is sizable; a dedicated relic-density and direct-detection analysis of S_1 within this model is left for future work.

A. Scalar sector and symmetry breaking

The Higgs sector consists of one right-handed doublet χ_R [which breaks $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$] and one bidoublet ϕ (which breaks the SM electroweak symmetry). Their vacuum expectation values (VEVs) are

$$\langle \chi_R \rangle = \frac{v_R}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}, \quad (10)$$

with $v_R \sim \mathcal{O}(\text{TeV})$ and $v^2 = k_1^2 + k_2^2 \approx (246 \text{ GeV})^2$. We define $k_1 = v c_\beta$, $k_2 = v s_\beta$ (with $s_x \equiv \sin x$, $c_x \equiv \cos x$, $t_\beta \equiv \tan \beta = k_2/k_1$), constrained by the W -boson mass $M_W \simeq g_L \sqrt{k_1^2 + k_2^2}/2$.

The physical charged Higgs boson mass is [23]

$$m_{H^\pm}^2 = \frac{\alpha}{2} \left(\frac{v_R^2}{c_{2\beta}} + v^2 c_{2\beta} \right), \quad (11)$$

where $\alpha \equiv \alpha_3 - \alpha_2$ is the relevant scalar potential parameter. In the limit $v_R \gg v$, the physical $H^\pm$ is dominated by the bidoublet component,

$$H^\pm \simeq -(s_\beta \phi_1^\pm + c_\beta \phi_2^\pm), \quad (12)$$

and the $\chi_R^\pm$ fraction is suppressed by v/v_R .

B. Yukawa Lagrangian and neutrino mass matrix

The most general Yukawa Lagrangian is [23]

$$\mathcal{L}_Y = \sum_{i,j=1}^3 \bar{L}_{Li} (\phi y_{ij}^L + \tilde{\phi} \tilde{y}_{ij}^L) L_{Rj} + \bar{Q}_{Li} (\phi y_{ij}^Q + \tilde{\phi} \tilde{y}_{ij}^Q) Q_{Rj} + \bar{L}_{Ri} \tilde{\chi}_R y_{ij}^S S_{2j}^c + \text{H.c.}, \quad (13)$$

where $\tilde{\phi} = \tau_2 \phi^* \tau_2$ and $\tilde{\chi}_R = i\tau_2 \chi_R^*$. After $B-L$ and electroweak symmetry breaking, the neutral fermion sector

yields the 9×9 mass matrix in the basis (ν_L^c, ν_R, S_2) [13,23]

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M_R \\ 0 & M_R^T & \mu_S \end{pmatrix}, \quad (14)$$

where $M_D = v(y^L s_\beta + \tilde{y}^L c_\beta)/\sqrt{2}$ is the Dirac mass matrix and $M_R = y^S v_R/\sqrt{2}$. Diagonalizing $\mathcal{M}_\nu$ via the 9×9 unitary matrix U yields three light-neutrino mass eigenvalues

$$m_{\nu_{\ell i}} = M_D M_R^{-1} \mu_S (M_R^{-1})^T M_D^T, \quad i = 1, 2, 3, \quad (15)$$

and three heavy pseudo-Dirac pairs (six mass eigenstates) with $m_{\nu_{hj}}^2 \approx M_R^2 \pm M_D^2$, $j = 4, \dots, 9$. The active-heavy mixing matrix is [31,32]

$$U_{3 \times 6} \simeq [\mathbf{0}_{3 \times 3} | F]_{3 \times 6} U_{6 \times 6}, \\ U_{3 \times 3} \simeq \left(\mathbf{1}_{3 \times 3} - \frac{1}{2} F F^T \right) U_{\text{PMNS}}, \quad (16)$$

where $F = M_D M_R^{-1}$ encodes the nonunitarity of the PMNS matrix. This framework naturally accommodates $\mathcal{O}(1)$ Yukawa couplings y^L , y^S while keeping light-neutrino masses small through the suppression $\mu_S \ll M_D \ll M_R$ [12].

C. Charged Higgs couplings to fermions

The effective charged-Higgs interactions in the mass basis take the form [23]

$$\mathcal{L} \supset Y_{ts}^H H^+ \bar{t}_R s_L + Y_{tb}^H H^+ \bar{t}_R b_L + Y_{\mu N_i}^R H^- \bar{\mu}_R N_i + Y_{\mu N_i}^L H^- \bar{\mu}_L N_i + \text{H.c.}, \quad (17)$$

where N_i denote the heavy pseudo-Dirac neutrinos. The simultaneous presence of both left- and right-handed leptonic couplings, $Y_{\mu N_i}^L$ and $Y_{\mu N_i}^R$, play a central role in generating an approximately vectorlike muon current.

We emphasize that the couplings in Eq. (17) are muon specific. Similar couplings with the muon replaced by the electron are in principle possible, but the light-neutrino masses in Eq. (15) strongly constrain y_{li}^L , since an independently $\mathcal{O}(1)$ value would drive the corresponding light-neutrino mass in the inverse-seesaw formula far above the sub-eV scale, unless there is a compensating suppression elsewhere (small μ_S or off-diagonal cancellation) which is the kind of tuning the hierarchy in Eq. (15) is built to avoid. So no sizable new-physics contribution to $B \rightarrow Ke^+e^-$ is expected, consistent with the observed $R_K^{(*)} \simeq 1$ [4,16]. The deviation of the individual branching ratios from their SM central values is attributed to hadronic form-factor

TABLE II. Measurements of $R_K \equiv \mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-)/\mathcal{B}(B^+ \rightarrow K^+ e^+ e^-)$ in different q^2 bins. The SM prediction is $R_K \simeq 1$ up to percent-level QED corrections [33].

Experiment	q^2 range [GeV ²]	R_K	References
LHCb (2014)	1.0–6.0	$0.745^{+0.090}_{-0.074} \pm 0.036$	[3]
LHCb (2019)	1.1–6.0	$0.846^{+0.060+0.016}_{-0.054-0.014}$	[35]
LHCb (2022)	1.1–6.0	Consistent with SM at 0.2σ	[4,36]
LHCb (2025)	> 14.3	$1.08^{+0.11}_{-0.09} \pm 0.04$	[37]
CMS (2024)	1.1–6.0	$0.78^{+0.47}_{-0.23}$	[38]

uncertainties [33,34]. A model that generated a comparable direct new-physics contribution in the electron channel would in fact be in tension with $R_K^{(*)} = 1$, as shown in Table II [4,16].

1. Left-handed coupling

Using the pseudo-Dirac completeness relation for heavy neutrino pairs, the effective squared coupling per generation is [23,39]

$$\sum_{k \in \text{pair}_i} |\xi_{k\mu}|^2 \approx |\tilde{y}_{2i}^L|^2, \quad (18)$$

where $\tilde{y}_{2i}^L$ is the dominant bidoublet Dirac Yukawa coupling (for $t_\beta \ll 1$, this equals the coupling denoted y_{2i}^L throughout this paper). Crucially, the U -matrix mixing factors cancel by completeness, so that

$$Y_{\mu N_i}^L \equiv \sqrt{\sum_{k \in \text{pair}_i} |\xi_{k\mu}|^2} \approx y_{2i}^L, \quad (19)$$

with no active-heavy mixing suppression.

2. Right-handed coupling

The right-handed coupling originates from the $\chi_R^\pm$ component of $H^\pm$ through the y^S vertex. Although the $\chi_R^\pm$ fraction of $H^\pm$ is suppressed by v/v_R , the coupling to N_i is proportional to M_{N_i}/v_R through the Pilaftsis nondecoupling mechanism [15]:

$$Y_{\mu N_i}^R \equiv Z_{33}^{H^\pm} \frac{g_R}{\sqrt{2}} \frac{M_{N_i}}{v c_{2\beta}} \approx \frac{g_R}{\sqrt{2}} \frac{M_{N_i}}{v_R}, \quad (20)$$

where the factor M_{N_i} from the y^S vertex precisely compensates the v/v_R suppression in the mixing element $Z_{33}^{H^\pm}$, leaving an $\mathcal{O}(1)$ coupling for $M_{N_i} \sim v_R$ —the nondecoupling effect.

It is important to contrast this with the $W^\pm$ -mediated coupling, which is suppressed by the active-heavy mixing: $\Gamma_{\mu N_i}^W = g_L U_{\mu N_i}/\sqrt{2} \approx g_L y_{2i}^L v/(\sqrt{2} M_{N_i}) \sim 0.017$

at the benchmark. The $H^\pm$ coupling $Y_{\mu N_i}^L \approx y_{2i}^L \sim 0.46$ is therefore roughly 27 times larger, and substituting the mixing-suppressed value would completely destroy the cancellation $\Delta C_{10}^\mu \approx 0$. The correct identification of $Y_{\mu N_i}^L$ as a direct Yukawa—not a mixing-suppressed gauge coupling—is essential to the mechanism.

V. BOX DIAGRAM AND NONDECOUPLING MECHANISM

A. Loop amplitude

The dominant new-physics contribution to $b \rightarrow s \mu^+ \mu^-$ arises from charged-scalar/heavy-neutrino box diagrams with internal $(t, N_i, H^\pm)$ states. Using Lorentz symmetry, $\int d^4 k k^\alpha k^\beta F(k^2) = g^{\alpha\beta}/4 \int d^4 k k^2 F(k^2)$, the amplitude takes the form

$$\begin{aligned} \mathcal{M}_{\text{box}} = & \frac{1}{16\pi^2} \frac{Y_{ts}^H (Y_{tb}^H)^*}{M_{H^\pm}^2} \left[|Y_{\mu N}^R|^2 F_R(x_t, x_N) (\bar{s} \gamma^\mu P_L b) \right. \\ & \times (\bar{\mu} \gamma_\mu P_R \mu) + |Y_{\mu N}^L|^2 F_L(x_t, x_N) (\bar{s} \gamma^\mu P_L b) (\bar{\mu} \gamma_\mu P_L \mu) \left. \right], \end{aligned} \quad (21)$$

where $x_t = m_t^2/M_{H^\pm}^2$ and $x_N = M_{N_i}^2/M_{H^\pm}^2$.

B. Nondecoupling and equality of loop functions

The loop functions F_R and F_L are *identical*. Both arise from the scalar loop integral

$$\begin{aligned} h(m_t^2, M_{N_i}^2, M_{H^\pm}^2) & \equiv \int \frac{d^4 k_E}{(2\pi)^4} \frac{k_E^2}{(k_E^2 + m_t^2)(k_E^2 + M_{N_i}^2)(k_E^2 + M_{H^\pm}^2)^2}, \end{aligned} \quad (22)$$

because $\bar{\mu} P_L k P_R \mu$ (right-handed topology) and $\bar{\mu} P_R k P_L \mu$ (left-handed topology) both produce the same Lorentz-contracted integrand after symmetrization. These loop functions can be split into two Passarino-Veltman contributions [40],

$$h = T_3(m_t^2, M_{H^\pm}^2) - M_{N_i}^2 T_4(m_t^2, M_{N_i}^2, M_{H^\pm}^2). \quad (23)$$

In the heavy-neutrino limit $M_{N_i} \rightarrow \infty$, one finds

$$h(x_t, x_N) \xrightarrow{M_{N_i} \rightarrow \infty} \frac{1}{16\pi^2 M_{H^\pm}^2} \mathcal{H}(x_t), \quad (24)$$

with the Inami-Lim-like function [41]

$$\mathcal{H}(x_t) \equiv \frac{\ln x_t - 1}{x_t - 1} \approx 4.65 \quad \text{for } m_t = 173 \text{ GeV}$$

$$\text{and } M_{H^\pm} = 1 \text{ TeV}, \quad (25)$$

The M_{N_i} dependence cancels because the factor M_{N_i} from the nondecoupling coupling $Y_{\mu N}^R \propto M_{N_i}/v_R$ precisely compensates the $1/M_{N_i}^2$ from the loop integral, leaving a finite, v_R -suppressed result.

C. Wilson coefficients

Matching the box amplitude onto the WET Hamiltonian (2) gives

$$\Delta C_9^\mu = -\frac{v^2 \mathcal{H}(x_t)}{64\pi\alpha_{\text{em}} M_{H^\pm}^2} \cdot \frac{Y_{ts}^H (Y_{tb}^H)^*}{|V_{ts}|} \cdot \sum_i (|Y_{\mu N_i}^R|^2 + |Y_{\mu N_i}^L|^2), \quad (26)$$

$$\Delta C_{10}^\mu = -\frac{v^2 \mathcal{H}(x_t)}{64\pi\alpha_{\text{em}} M_{H^\pm}^2} \cdot \frac{Y_{ts}^H (Y_{tb}^H)^*}{|V_{ts}|} \cdot \sum_i (|Y_{\mu N_i}^R|^2 - |Y_{\mu N_i}^L|^2). \quad (27)$$

Since $F_R = F_L \equiv h$, the condition

$$|Y_{\mu N}^R|^2 \approx |Y_{\mu N}^L|^2 \quad (28)$$

automatically yields $\Delta C_{10}^\mu \approx 0$ while keeping $\Delta C_9^\mu \neq 0$. With $Y_{\mu N}^R = g_R M_{N_i}/(\sqrt{2}v_R)$ and $M_N \sim v_R$, the condition (28) requires $|y_{2i}^L| \approx g_R/\sqrt{2} \approx 0.46$, a perfectly natural value in the inverse seesaw where Dirac Yukawa couplings can be $\mathcal{O}(1)$.

The condition (28) need not hold exactly. Writing $|Y_{\mu N}^R| = |Y|(1+\delta)$, $|Y_{\mu N}^L| = |Y|(1-\delta)$ for a small splitting δ , Eqs. (26) and (27) give $\Delta C_{10}^\mu/\Delta C_9^\mu \simeq 2\delta$. The requirement $|\Delta C_{10}^\mu| \lesssim 0.2$ adopted in our numerical scan (Sec. IX), together with $\Delta C_9^\mu \approx -0.91$, therefore bounds the splitting to $|\delta| \lesssim 0.11$: the left- and right-handed leptonic couplings must agree in magnitude to about 11%.

VI. PHENOMENOLOGICAL CONSTRAINTS

A. $B_s - \bar{B}_s$ mixing

The same couplings Y_{ts}^H and Y_{tb}^H that drive ΔC_9^μ also generate a top-quark box contribution to the B_s mass difference ΔM_{B_s} . The New Physics (NP) amplitude sums over up-type quarks,

$$\mathcal{M}_{\text{mix}} \propto \sum_{j,k \in \{u,c,t\}} (Y_{js}^H Y_{jb}^{H*})(Y_{ks}^H Y_{kb}^{H*}) S_0(x_j, x_k), \quad (29)$$

where $S_0(x)$ is the Inami-Lim function [41]. Because $S_0(x) \rightarrow x$ as $x \rightarrow 0$, the charm and up contributions are suppressed relative to the top term by $(m_c/m_t)^2 \sim 6 \times 10^{-5}$ and $(m_u/m_t)^2$ respectively, independently of $M_{H^\pm}$; the B_s -mixing amplitude is therefore controlled almost entirely by the top-quark term $\lambda_t^2 S_0(x_t)$ with $\lambda_t \equiv Y_{ts}^H (Y_{tb}^H)^*$. Requiring $\Delta M_{B_s}^{\text{NP}} \approx 10\% \Delta M_{B_s}^{\text{exp}}$ from this top-only term alone already yields the viable band shown in Fig. 2(a), consistent with the benchmark values of y and $M_{H^\pm}$ used throughout this work.

B. $b \rightarrow sy$ constraint

The magnetic penguin operator $\mathcal{O}_7$ receives contributions from the charged scalar via the Inami-Lim function $D'_0(x_t)$. The NP contribution is [42]

$$C_7^{\text{NP}} = \frac{y^2 \sin(2\theta_R) D'_0(x_t) v^2}{6M_{H^\pm}^2 |V_{ts}|}, \quad (30)$$

where θ_R is the right-handed mixing angle. For the benchmark parameters, $|C_7^{\text{NP}}/C_7^{\text{SM}}| < 10^{-3}$ throughout the viable band, well within the experimental bound of $\sim 10\%$ [43–45].

C. Perturbativity and LHC bounds

We require $y < 1.5$ for perturbativity and impose the LHC direct-search bound $M_{H^\pm} \gtrsim 600$ GeV from $pp \rightarrow H^\pm \rightarrow tb$ searches [46,47]. The lower bound on the right-handed gauge boson mass, $M_{W_R} \gtrsim 4.7$ TeV [48,49], constrains v_R through $M_{W_R} \simeq g_R v_R/\sqrt{2}$, giving $v_R \gtrsim 3$ TeV [50].

VII. SELF-CONSISTENCY OF THE LEPTONIC COUPLING CONDITION

The cancellation $\Delta C_{10}^\mu \approx 0$ relies on the condition

$$|Y_{\mu N}^L| \approx |Y_{\mu N}^R| = \frac{g_R}{\sqrt{2}} \frac{M_{N_i}}{v_R}, \quad (31)$$

as $M_{N_i} = y^S v_R/\sqrt{2}$, one gets

$$|y_{2\mu}^L| \approx \frac{g_R}{\sqrt{2}} y^S. \quad (32)$$

For $g_R \simeq g_L \simeq 0.652$ and $y^S \simeq 1$ this yields $|y_{2\mu}^L| \approx 0.46$. While this is technically natural in the inverse seesaw [where Dirac Yukawa couplings can be $\mathcal{O}(1)$], we now verify it quantitatively against independent constraints.

TABLE III. Value of μ_S required by the neutrino mass formula (33) for a given $y_{2\mu}^L$, at $M_N = 3$ TeV and $m_\nu = 0.05$ eV.

$y_{2\mu}^L$	μ_S	Status
0.086	1 keV	Allowed (large μ_S , small y^L)
0.27	100 eV	Allowed
0.46	35 eV	Benchmark ✓
0.86	10 eV	Allowed (small μ_S , large y^L)
1.22	5 eV	Marginally perturbative

A. Consistency with observed neutrino masses

In the single-generation limit of Eq. (15),

$$m_\nu \simeq \frac{(y_{2\mu}^L v)^2 \mu_S}{M_N^2}, \quad (33)$$

which can be inverted to give

$$\mu_S = \frac{m_\nu M_N^2}{(y_{2\mu}^L)^2 v^2} \simeq 35 \text{ eV} \left(\frac{m_\nu}{0.05 \text{ eV}} \right) \left(\frac{M_N}{3 \text{ TeV}} \right)^2 \left(\frac{0.46}{y_{2\mu}^L} \right)^2. \quad (34)$$

The benchmark $y_{2\mu}^L = 0.46$ with $M_N = 3$ TeV therefore requires $\mu_S \approx 35$ eV, well below but technically natural relative to the keV scale, since $\mu_S \rightarrow 0$ restores lepton-number symmetry [51].

The dependence on μ_S is summarized in Table III.

B. PMNS nonunitarity constraint

The active-heavy mixing $F = M_D M_R^{-1}$ induces a departure from PMNS unitarity parametrized by [52]

$$\eta_{\mu\mu} \equiv \frac{1}{2} \sum_i |\Theta_{\mu i}|^2 = \frac{(y_{2\mu}^L)^2 v^2}{2M_N^2}, \quad (35)$$

subject to the global bound $|\eta_{\mu\mu}| < 10^{-3}$ at 90% CL [52,53]. At the benchmark,

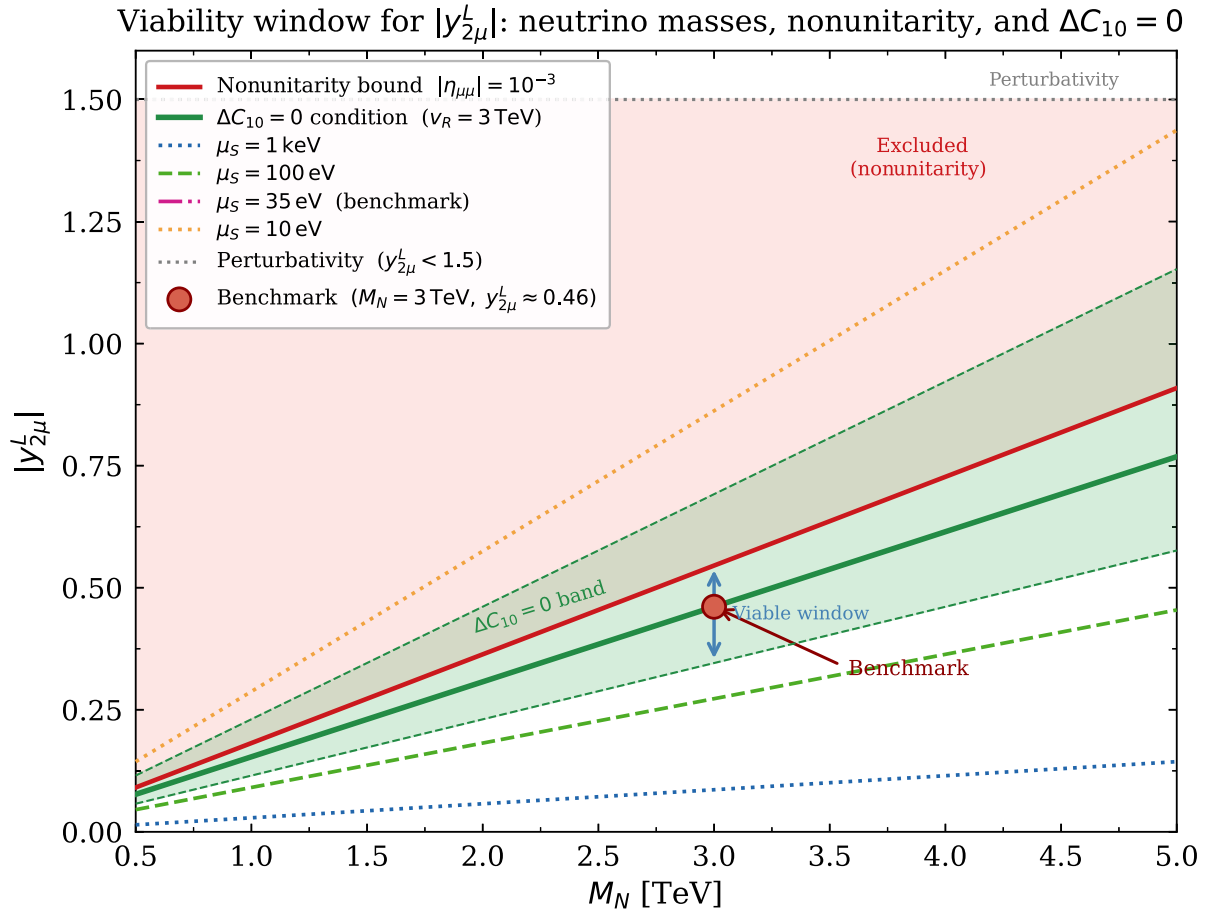


FIG. 1. Viability window for $|y_{2\mu}^L|$ as a function of the heavy-neutrino mass M_N . The dark green band is the locus satisfying the $\Delta C_{10} = 0$ cancellation condition [Eq. (32)] for $v_R \in [2, 4]$ TeV; the solid central line corresponds to $v_R = 3$ TeV. The shaded red region is excluded by PMNS nonunitarity ($|\eta_{\mu\mu}| > 10^{-3}$, 90% CL [52]). Dot-dashed/dashed curves are iso- μ_S contours from the neutrino mass formula [Eq. (34)]. The double-headed arrow at $M_N = 3$ TeV illustrates the viable window $|y_{2\mu}^L| \in [0.14, 0.54]$ [Eq. (37)].

$$\eta_{\mu\mu}^{\text{bench}} = \frac{(0.46)^2 (246 \text{ GeV})^2}{2(3000 \text{ GeV})^2} \approx 7.1 \times 10^{-4}, \quad (36)$$

which satisfies the bound with a factor of 1.4 to spare. Inverting Eq. (35) gives the upper bound $y_{2\mu}^L < \sqrt{2|\eta_{\mu\mu}|_{\text{max}}} M_N / v \simeq 0.54 (M_N / 3 \text{ TeV})$.

C. Viability window

Combining the phenomenological requirement (32) with the nonunitarity bound and perturbativity ($y^S \leq 1.5$), the allowed range at $M_N = 3 \text{ TeV}$ is

$$0.14 \lesssim |y_{2\mu}^L| \lesssim 0.54. \quad (37)$$

This result is shown in Fig. 1. The diagonal band labeled “ $\Delta C_{10} = 0$ ” is the locus satisfying Eq. (32) for $v_R \in [2, 4] \text{ TeV}$; the shaded red region is excluded by PMNS nonunitarity; and contours of constant μ_S run diagonally.

The values of $\eta_{\mu\mu}$ probed here, $\mathcal{O}(10^{-4})$, are within reach of future high-precision experiments. The FCC-ee program is expected to improve the bound on $\eta_{\mu\mu}$ by one to two orders of magnitude [54], providing a direct test of the leptonic sector of the LRIS model fully correlated with the $B \rightarrow K\mu^+\mu^-$ signal.

VIII. RIGHT-HANDED PHASE TEXTURE

A. Precise statement of the texture condition

The $B_s - \bar{B}_s$ mixing amplitude is governed by $\lambda_j \equiv Y_{js}^H (Y_{jb}^H)^*$ and takes the form

$$\mathcal{M}_{\text{mix}} \propto \lambda_t^2 \left[S_0(x_t) + 2 \frac{\lambda_c}{\lambda_t} S_0(x_t, x_c) + \left(\frac{\lambda_c}{\lambda_t} \right)^2 S_0(x_c) \right], \quad (38)$$

where $\lambda_u \approx 0$ is neglected. Writing $Y_{ts}^H = ye^{i\alpha}$, $Y_{tb}^H = ye^{i\beta}$, $Y_{cs}^H = rye^{i\gamma}$, $Y_{cb}^H = rye^{i\delta}$, the ratio is

$$\varrho \equiv \frac{\lambda_c}{\lambda_t} = r^2 e^{i(\gamma - \delta - \alpha + \beta)}. \quad (39)$$

The choice $\varrho = -r^2$ (real and negative), which would maximize the destructive interference *between the second and first terms of Eq. (38) at fixed $|\varrho|$* , demands

$$(\gamma - \delta) - (\alpha - \beta) = \pi. \quad (40)$$

A minimal realization sets $\gamma = \alpha$ and $\delta = \beta + \pi$ (only the b -column coupling of the charm generation acquires a relative π phase with respect to the top coupling).

We stress that Eq. (40) identifies the phase alignment that *would* produce the largest possible charm-top destructive interference at fixed r —it does not by itself imply that this interference is phenomenologically significant. As shown

in Appendix B, the charm sector is suppressed relative to the top term in Eq. (38) by powers of $x_c = (m_c/M_{H^\pm})^2 \sim 10^{-6}$, independently of $M_{H^\pm}$, so that even the maximally destructive phase choice (40) changes $\Delta M_{B_s}^{\text{NP}}$ by only $\mathcal{O}(10^{-4})$ relative to the top-only value for any perturbative r . The B_s -mixing bound on the model is therefore set entirely by the top-only term $\lambda_t^2 S_0(x_t)$ (Sec. VI A), and neither the phase texture nor the value of $r \equiv |Y_{cs,cb}^H|/y$ is constrained by ΔM_{B_s} . The texture is nonetheless a well-motivated and, as we show below, radiatively stable structural feature of the right-handed quark sector in its own right.

B. Origin from a discrete symmetry

Independently of its phenomenological impact on B_s mixing, the texture condition (40) has a natural group-theoretic origin and it is a well-motivated structural feature of the right-handed quark sector. Consider the discrete $\mathbb{Z}_2$ transformation

$$\mathcal{P}_{b_R}: b_R \longrightarrow -b_R, \quad (41)$$

with all other fields unchanged. Under $\mathcal{P}_{b_R}$, invariance of the Yukawa Lagrangian enforces

$$\arg(Y_{tb}^H) = \arg(Y_{ts}^H) + \pi, \quad \arg(Y_{cb}^H) = \arg(Y_{cs}^H) + \pi, \quad (42)$$

which is precisely $\delta = \beta + \pi$ (with $\gamma = \alpha$)—the required texture condition.

Several comments are in order. The transformation (41) acts on the $SU(2)_R$ -singlet component of the right-handed quark field and commutes with the full gauge group, so it is a consistent discrete symmetry of the Lagrangian. Because b_R is a gauge singlet under $SU(2)_L$, the SM CKM matrix is unaffected. The $\mathbb{Z}_2$ is explicitly broken by the b -quark mass term, so it is technically a symmetry only of the Yukawa sector involving the right-handed $H^\pm$ couplings. This can be viewed as a remnant of a continuous rephasing symmetry $b_R \rightarrow e^{i\theta} b_R$, broken to its $\mathbb{Z}_2$ subgroup by the scalar potential, consistent with the CP -phase structure of the right-handed sector studied in the LR model [55].

Two economical UV completions realize this pattern without enlarging the low-energy field content assumed elsewhere in this work. (i) *Flavon completion*. Promote $\mathcal{P}_{b_R}$ to a continuous $U(1)$ acting only on b_R , spontaneously broken by a flavon field S with $\langle S \rangle / \Lambda \ll 1$, along the lines of the Froggatt-Nielsen mechanism [27]. Effective operators $\bar{Q}_{Li} \tilde{\phi} (S/\Lambda)^{n_i} b_R$ with charge assignments n_i chosen so that the $t-s$ and $t-b$ operators pick up the required relative phase upon $\langle S \rangle \rightarrow -\langle S \rangle$ [a discrete remnant of the $U(1)$] reproduce Eq. (42), while the dominant contribution to the physical b -quark mass arises from an operator with a different flavon charge, naturally decoupling mass generation from the required phase. (ii) *Two-bidoublet*

completion. Alternatively, introducing a second bidoublet ϕ_2 to which b_R couples exclusively—while all other down-type quarks couple to ϕ_1 , as in generic multiscalar extensions of the electroweak sector [56]—renders $\mathcal{P}_{b_R}: (b_R, \phi_2) \rightarrow (-b_R, -\phi_2)$ an exact symmetry of the full Lagrangian, broken only by a bidoublet-mixing term in the scalar potential that can naturally be taken small. We regard (i) as the more minimal option, since it leaves the scalar spectrum used throughout this paper unchanged, and mention (ii) as an equally viable alternative; a full UV model-building study of either completion is beyond the scope of the present work.

C. Barbieri-Giudice fine-tuning analysis

We emphasize that this fine-tuning analysis concerns the stability of the texture condition itself under phase deviations, independent of its negligible quantitative impact on $\Delta M_{B_s}^{\text{NP}}$ (Sec. VI A); the texture remains a technically natural and radiatively stable feature of the model even though it is not required to satisfy the B_s -mixing bound.

To quantify whether the texture condition requires fine-tuning, we apply the Barbieri-Giudice (BG) measure [57] to the B_s mixing amplitude. Define the phase deviation from the texture point, $\phi \equiv \arg(\lambda_c/\lambda_t) - \pi$, so $\phi = 0$ at the exact texture. The amplitude modulus is

$$\mathcal{A}(\phi) \equiv |S_0(x_t) - 2r^2 \cos \phi S_0(x_t, x_c) + r^4 \cos 2\phi S_0(x_c)|, \quad (43)$$

and the BG measure is $\Delta_{\text{BG}}(\phi) \equiv |\phi \partial \ln \mathcal{A} / \partial \phi|$. Since $\partial \mathcal{A} / \partial \phi|_{\phi=0} = 0$ (the sine factors vanish at $\phi = 0$), the texture point is an *exact stationary point* of the amplitude:

$$\Delta_{\text{BG}}^{(1)}(\phi = 0) = 0. \quad (44)$$

This is not a numerical accident but a consequence of the $\mathbb{Z}_2$ symmetry: at a symmetry-protected point, the amplitude cannot have first-order sensitivity to the phase.

At second order, one finds

$$\Delta_{\text{BG}}^{(2)}(\delta\phi) = (\delta\phi)^2 \frac{r^2 S_0(x_t, x_c)}{S_0(x_t)(1 - r^2)^2}, \quad (45)$$

which at the benchmark ($M_{H^\pm} = 1$ TeV, $r = 0.3$) evaluates to $\Delta_{\text{BG}}^{(2)} \approx 2.8 \times 10^{-5} (\delta\phi)^2$, less than 10^{-6} even for $\delta\phi \sim 0.2$ rad.

D. Stability under radiative corrections

The dominant one-loop correction to the right-handed quark phases arises from W_R -mediated loops [58]:

$$\delta\phi_{1\text{-loop}} \sim \frac{g_R^2}{16\pi^2} \ln \frac{M_{\text{GUT}}}{v_R} \approx 0.07 \text{ rad} \\ (v_R = 3 \text{ TeV}, M_{\text{GUT}} = 10^{15} \text{ GeV}). \quad (46)$$

Substituting into Eq. (45): $\Delta_{\text{BG}}^{(2)}(\delta\phi_{1\text{-loop}}) \approx 1.4 \times 10^{-7} \ll 1$, confirming that radiative corrections to the texture condition are completely negligible.

In summary, the right-handed phase texture (i) follows naturally from a discrete $\mathbb{Z}_2$ symmetry $b_R \rightarrow -b_R$ consistent with the full gauge group; (ii) is protected against first-order fine-tuning because the texture point is an exact stationary point of the B_s mixing amplitude; (iii) is stable against radiative corrections, with $\Delta_{\text{BG}}^{(2)} \sim 10^{-7}$ for natural one-loop phase shifts; and (iv) connects to the CP -violating phase structure studied in the context of $\tau \rightarrow K\pi\nu_\tau$ within the same model [59].

IX. NUMERICAL RESULTS

A. Parameter scan

We perform a numerical scan over the LRIS parameter space, varying the magnitudes y , $r \equiv |Y_{cs,cb}^H|/y$, and the phases α , β , θ_R , ϕ_{Rq} of the right-handed quark couplings. The leptonic couplings are fixed at $y_\mu \equiv |Y_{\mu N}^L| = |Y_{\mu N}^R| = 0.8$ to satisfy the cancellation condition (28). All loop integrals use exact Passarino-Veltman expressions [40] with $M_{H^\pm} = 1$ TeV.

A point is declared compatible if it simultaneously satisfies:

- (i) $\Delta C_9^\mu \in [-1.1, -0.9]$ (signal, 1σ);
- (ii) $|\Delta C_{10}^\mu| < 0.2$;
- (iii) $\Delta M_{B_s}^{\text{NP}} < 10\% \Delta M_{B_s}^{\text{exp}}$; and
- (iv) $|C_7^{\text{NP}}| < 10\% |C_7^{\text{SM}}|$.

Out of 121,875 scanned points, 650 pass all constraints (0.53%). All compatible points share $y \approx 0.386$, uniquely fixed by the matching condition

$$y^2 = \frac{|\Delta C_9^\mu| \cdot 64\pi\alpha_{\text{em}} M_{H^\pm}^2}{v^2 \mathcal{H}(x_t) \cdot 2y_\mu^2 |Y_{ts}^H/V_{ts}|}. \quad (47)$$

B. Viable region in the $(M_{H^\pm}, y)$ plane

Figure 2 summarizes the results. Panel (a) shows that the viable band follows the expected scaling $\Delta C_9^\mu \propto y^2/M_{H^\pm}^2$, forming a hyperbola in the $(M_{H^\pm}, y)$ plane extending to $M_{H^\pm} \approx 3$ TeV with $y < 1.5$ (entirely perturbative). Panel (c) confirms that $\Delta M_{B_s}^{\text{NP}}$ is controlled entirely by the top-quark term, dropping from $\sim 3\text{--}4\%$ of the experimental value near $M_{H^\pm} = 1$ TeV to $\sim 1\%$ by $M_{H^\pm} = 3$ TeV, independently of r and of the phase texture (Appendix B); the $r = 0.10, 0.30, 0.50$ curves and the top-only curve are visually indistinguishable throughout

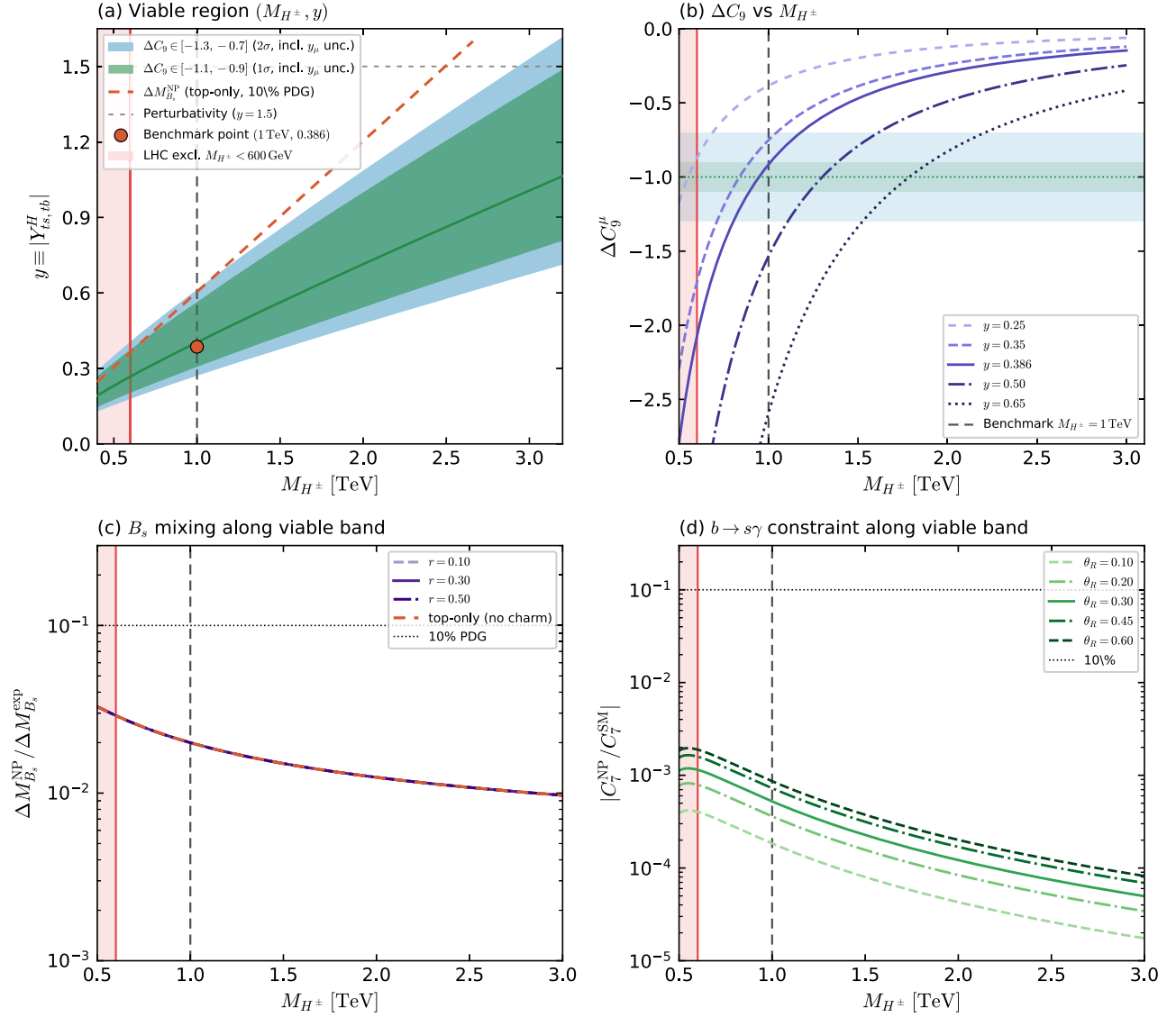


FIG. 2. Summary of the LRIS phenomenology and constraints, using exact Inami-Lim loop functions throughout [Eqs. (A3) and (A4), and the two-mass function $S_0(x_i, x_j)$ of Appendix B]. In all four panels, the shaded red band marks the region excluded by LHC direct searches ($M_{H^\pm} < 600$ GeV), and the dashed vertical line marks the benchmark $M_{H^\pm} = 1$ TeV. (a) Allowed region in the $(M_{H^\pm}, y)$ plane. The blue and green bands show the 2σ and 1σ preferred ranges $\Delta C_9^\mu \in [-1.3, -0.7]$ and $[-1.1, -0.9]$, respectively, each drawn as the full envelope over $y_\mu \in [0.60, 1.00]$; the solid green line shows the central value $y_\mu = 0.80$. The orange dashed curve shows the top-only $\Delta M_{B_s}^{\text{NP}}$ exclusion (10% of the PDG value); as established in Appendix B, this top-only bound is the physical B_s -mixing constraint and is unaffected by r or the phase texture. The gray dotted line marks the perturbativity ceiling $y = 1.5$, and the benchmark point $(M_{H^\pm}, y) = (1 \text{ TeV}, 0.386)$ is shown as a filled circle. (b) ΔC_9^μ as a function of $M_{H^\pm}$ for representative values of y ; the shaded bands mark the same $1\sigma/2\sigma$ target ranges as in panel (a). (c) $\Delta M_{B_s}^{\text{NP}} / \Delta M_{B_s}^{\text{exp}}$ along the $\Delta C_9^\mu = -1.0$ viable band, computed with the exact two-mass Inami-Lim function $S_0(x_t, x_c)$, for $r \equiv |Y_{cs, cb}^H|/y = 0.10, 0.30, 0.50$, and the top-only curve ($r = 0$). All four curves are visually indistinguishable, confirming that the charm sector changes $\Delta M_{B_s}^{\text{NP}}$ by only $\mathcal{O}(10^{-4})$ relative to the top-only value [Eq. (B3)]: the B_s -mixing bound is set entirely by the top-quark term and is independent of r and of the phase texture of Sec. VIII A. The dotted line marks 10% of the PDG value. (d) $|C_7^{\text{NP}} / C_7^{\text{SM}}|$ along the same viable band for different right-handed mixing angles θ_R , confirming that the $b \rightarrow s\gamma$ constraint is satisfied by more than two orders of magnitude throughout the parameter space.

TABLE IV. Benchmark scenario: LRIS model parameters and predicted observables at $M_{H^\pm} = 1$ TeV.

Parameter/observable	Value	Constraint
<i>Input parameters</i>		
$M_{H^\pm}$	1.0 TeV	LHC: > 600 GeV
v_R	3.0 TeV	EW precision
$y = Y_{ts,tb}^H $	0.386	Perturbativity
$y_\mu = Y^L = Y^R$	0.80	Nonunitarity
$\gamma = \alpha, \delta = \beta + \pi$	(Texture)	ΔM_{B_s}
<i>Predicted observables</i>		
ΔC_9^μ	-0.912	$[-1.1, -0.9] \checkmark$
ΔC_{10}^μ	≈ 0	$< 0.2 \checkmark$
$\Delta M_{B_s}^{\text{NP}} / \Delta M_{B_s}^{\text{exp}}$	1.7%	$< 10\% \checkmark$
$ C_7^{\text{NP}} / C_7^{\text{SM}} $	$< 10^{-3}$	$< 10\% \checkmark$
$\eta_{\mu\mu}$	7.1×10^{-4}	$< 10^{-3} \checkmark$
μ_S	≈ 35 eV	$m_\nu = 0.05$ eV $\checkmark$

the viable band. Panel (d) confirms that $b \rightarrow s\gamma$ is irrelevant throughout the parameter space.

C. Benchmark point

Our benchmark scenario is defined by

$$\begin{aligned} M_{H^\pm} &= 1 \text{ TeV}, & y &= 0.386, \\ y_\mu &= 0.80, & v_R &= 3 \text{ TeV}, \end{aligned} \quad (48)$$

with the phase texture $\gamma = \alpha, \delta = \beta + \pi$. The predicted observables are summarized in Table IV.

D. LHC anomaly in our benchmark scenario

Table V shows the binned predictions for P'_5 in $B^0 \rightarrow K^{*0}\mu^+\mu^-$ at the LRIS benchmark. The SM exhibits systematic negative pulls of -1.7σ to -2.3σ in the bins $q^2 \in [2.5, 8]$ GeV² that drive the global anomaly; the LRIS prediction reduces all three pulls to below 1σ , confirming that the benchmark shift $\Delta C_9^\mu \approx -0.91$ correctly accounts for the observed pattern. The slight tension in the lowest bin $[0.1, 0.98]$ GeV² is common to all short-distance NP explanations and is attributed to enhanced hadronic charm-loop contributions in that kinematic region [8]. The

bin $q^2 \in [1.1, 2.5]$ GeV² shows a mild tension of 1.7σ in the LRIS prediction, slightly worse than the SM. This is a generic feature of pure ΔC_9^μ solutions and reflects the fact that a uniform negative shift in C_9 slightly overshoots P'_5 in the transition region between the ρ/ω and J/ψ resonances. The same behavior is observed in all single-operator fits to the $b \rightarrow s\mu^+\mu^-$ data [6] and does not significantly affect the overall quality of the fit.

E. Angular observable P'_5 and subleading corrections

Beyond the Wilson coefficients ΔC_9^μ and ΔC_{10}^μ generated by the dominant charged-scalar/heavy-neutrino box diagram, the LRIS model produces a set of subleading contributions that affect the full angular distribution of $B^0 \rightarrow K^{*0}\mu^+\mu^-$. In this subsection we derive these corrections, establish their size as a function of $\tan\beta$, and present predictions for the optimized observable P'_5 [61] computed with the public code FLAVIO [60].

1. Subleading Wilson coefficients

In addition to the unprimed operators $\mathcal{O}_{9,10}$ driven by the D_{ij} coupling (right-handed top quark line, $\bar{t}_R$), the charged-Higgs box diagram with a left-handed top quark line ($\bar{t}_L$, driven by the C_{ij} coupling) generates the chirality-flipped operators

$$\mathcal{O}'_9 = (\bar{s}\gamma^\mu P_R b)(\bar{\mu}\gamma_\mu \mu), \quad (49)$$

$$\mathcal{O}'_{10} = (\bar{s}\gamma^\mu P_R b)(\bar{\mu}\gamma_\mu \gamma_5 \mu). \quad (50)$$

From the quark coupling matrix of Eq. (17), for $t_\beta \ll 1$ and top-quark dominance,

$$C_{ts} \simeq \frac{\sqrt{2}m_t s_{2\beta}}{v c_{2\beta}} V_{ts}, \quad C_{tb} \simeq \frac{\sqrt{2}m_t s_{2\beta}}{v c_{2\beta}} V_{tb}, \quad (51)$$

so that $C_{ts} C_{tb}^* / |V_{ts}| = (2m_t t_\beta / v)^2 |V_{ts} V_{tb}| / |V_{ts}|$ to leading order in t_β . The matching onto the WET Hamiltonian (2) gives

TABLE V. Binned predictions for P'_5 in $B^0 \rightarrow K^{*0}\mu^+\mu^-$ in the SM and in the LRIS benchmark ($\Delta C_9^\mu = -0.912, \Delta C_{10}^\mu = 0$), compared with LHCb [2]. Theory uncertainties via FLAVIO [60].

q^2 [GeV ²]	P_5^{SM}	pull _{SM}	P_5^{LRIS}	pull _{LRIS}	P_5^{LHCb}
[0.10, 0.98]	$+0.660 \pm 0.028$	$+2.6\sigma$	$+0.766 \pm 0.028$	$+3.6\sigma$	$+0.385 \pm 0.102$
[1.10, 2.50]	$+0.140 \pm 0.087$	$+0.6\sigma$	$+0.355 \pm 0.068$	$+1.7\sigma$	$+0.025 \pm 0.182$
[2.50, 4.00]	-0.500 ± 0.098	-1.7σ	-0.213 ± 0.117	-0.2σ	-0.165 ± 0.174
[4.00, 6.00]	-0.758 ± 0.070	-2.3σ	-0.552 ± 0.104	-0.7σ	-0.439 ± 0.118
[6.00, 8.00]	-0.834 ± 0.059	-2.2σ	-0.696 ± 0.091	-0.9σ	-0.583 ± 0.096
[15.00, 19.00]	-0.595 ± 0.040	$+0.5\sigma$	-0.548 ± 0.052	$+1.0\sigma$	-0.640 ± 0.073

$$\Delta C_9^{\mu'} = -\frac{v^2 \mathcal{H}(x_t)}{64\pi\alpha_{\text{em}} M_{H^\pm}^2} \cdot \frac{C_{ts} C_{tb}^*}{|V_{ts}|} \cdot \sum_i (|Y_{\mu N_i}^R|^2 + |Y_{\mu N_i}^L|^2), \quad (52)$$

$$\Delta C_{10}^{\mu'} = -\frac{v^2 \mathcal{H}(x_t)}{64\pi\alpha_{\text{em}} M_{H^\pm}^2} \cdot \frac{C_{ts} C_{tb}^*}{|V_{ts}|} \cdot \sum_i (|Y_{\mu N_i}^R|^2 - |Y_{\mu N_i}^L|^2), \quad (53)$$

with the same loop function $\mathcal{H}(x_t)$ as in Eqs. (26) and (27). Since the leptonic difference vanishes at the cancellation point (28), $\Delta C_{10}^{\mu'} \approx 0$ automatically, just as $\Delta C_{10}^\mu \approx 0$.

The ratio of the primed to unprimed shift follows immediately from Eqs. (26) and (52):

$$\frac{\Delta C_9^{\mu'}}{\Delta C_9^\mu} = \frac{C_{ts} C_{tb}^*}{D_{ts} D_{tb}^*} = \frac{(2m_t t_\beta / v)^2 |V_{ts} V_{tb}| / |V_{ts}|}{(m_t / v c_{2\beta})^2 |V_{ts} V_{tb}| / |V_{ts}|} = 4t_\beta^2 c_{2\beta}^2. \quad (54)$$

Crucially, $\Delta C_9^{\mu'}$ carries the *same sign* as ΔC_9^μ (both are negative), so the subleading correction enhances the NP shift rather than reducing it.

The magnetic penguin operator $\mathcal{O}_7 = (\bar{s}\sigma^{\mu\nu} P_R b) F_{\mu\nu}$ also receives a charged-Higgs contribution,

$$\Delta C_7 = \frac{y^2 \sin(2\theta_R) D'_0(x_t) v^2}{6M_{H^\pm}^2 |V_{ts}|}, \quad (55)$$

where $D'_0(x_t)$ is the Inami-Lim magnetic function (Appendix A) and θ_R is the right-handed quark mixing angle. At the benchmark this gives $|\Delta C_7 / C_7^{\text{SM}}| < 10^{-3}$, already established in Sec. VI B.

2. Scan over $\tan\beta$

The experimentally allowed range of $\tan\beta$ is bounded from below by the requirement of a perturbative bottom Yukawa coupling, $y_{33}^Q = \sqrt{2}m_b/(vs_\beta)$, which gives

$t_\beta \gtrsim m_b/m_t \approx 0.024$, and from above by electroweak precision constraints on the ρ parameter from the bidoublet, $t_\beta \lesssim 0.5$ [50].

From Eq. (54), the subleading contribution $\Delta C_9^{\mu'} = 4t_\beta^2 c_{2\beta}^2 \Delta C_9^\mu$ varies from $4 \times (0.024)^2 \approx 2 \times 10^{-3}$ at the lower bound to $4 \times (0.5)^2 \times (0.6)^2 \approx 0.36$ at the upper bound. Even at maximum, $|\Delta C_9^{\mu'}| \lesssim 0.36 |\Delta C_9^\mu| \sim 0.33$, a correction that is subleading but not entirely negligible at the upper end of the $\tan\beta$ range.

Table VI shows the full LRIS predictions for P'_5 including all subleading corrections ($\Delta C_9^{\mu'}$, $\Delta C_{10}^{\mu'}$, ΔC_7) for six representative values of $\tan\beta$ spanning the allowed range, computed with FLAVIO [60] using the Gubernari *et al.* form factors.

3. Discussion

The results of Table VI reveal two important features.

First, the LRIS predictions are *remarkably stable* across the allowed $\tan\beta$ range: the shift between the $\tan\beta = 0.1$ and $\tan\beta = 0.5$ columns is at most 0.02 in any bin, well within the hadronic uncertainties. This robustness follows directly from Eq. (54): since $\Delta C_9^{\mu'} \propto t_\beta^2$ and $|\Delta C_9^{\mu'}| \ll |\Delta C_9^\mu|$ throughout the allowed range, the angular observable P'_5 is determined almost entirely by the leading box contribution $\Delta C_9^\mu \approx -0.91$. The model is therefore *highly predictive*: once y , y_μ , and $M_{H^\pm}$ are fixed by the $b \rightarrow s\mu^+\mu^-$ signal, all P'_5 bins are predicted without additional free parameters, regardless of $\tan\beta$.

Second, the SM tensions in the three central bins $q^2 \in [2.5, 8]$ GeV²—which drive the global anomaly—are reduced from the SM level of $(-1.7\sigma, -2.3\sigma, -2.2\sigma)$ to $(-0.2\sigma, -0.7\sigma, -0.8\sigma)$ at the LRIS benchmark, all consistent with the data at the 1σ level. The mild positive tension of $\sim +3.6\sigma$ in the lowest bin $q^2 \in [0.1, 0.98]$ GeV² and $\sim +1.7\sigma$ in $[1.1, 2.5]$ GeV² are common to all single-operator ΔC_9^μ solutions and are attributed to enhanced hadronic charm-loop contributions in the large-recoil

TABLE VI. Predictions for P'_5 in $B^0 \rightarrow K^{*0}\mu^+\mu^-$ in the SM and in the full LRIS model ($y_\mu = 0.80$, $M_{H^\pm} = 1$ TeV, $v_R = 3$ TeV) for representative values of $\tan\beta$ in the experimentally allowed range $[m_b/m_t, 0.5]$. Each LRIS column includes the complete NP contribution ΔC_9^μ , $\Delta C_9^{\mu'}$, $\Delta C_{10}^{\mu'}$, and ΔC_7 . Pulls (pred – data)/ $\sqrt{\sigma_{\text{pred}}^2 + \sigma_{\text{data}}^2}$ in parentheses. Theory uncertainties estimated with FLAVIO [60]; LHCb data from Ref. [2].

q^2 [GeV ²]	P_5^{SM}	P'_5 ($\tan\beta = 0.1$)	P'_5 ($\tan\beta = 0.5$)	P_5^{LHCb}
[0.10, 0.98]	$+0.660 \pm 0.028$	$+0.766(+3.6\sigma)$	$+0.748(+3.4\sigma)$	$+0.385 \pm 0.102$
[1.10, 2.50]	$+0.140 \pm 0.080$	$+0.355(+1.7\sigma)$	$+0.346(+1.7\sigma)$	$+0.025 \pm 0.182$
[2.50, 4.00]	-0.500 ± 0.094	$-0.212(-0.2\sigma)$	$-0.207(-0.2\sigma)$	-0.165 ± 0.174
[4.00, 6.00]	-0.758 ± 0.065	$-0.551(-0.7\sigma)$	$-0.535(-0.6\sigma)$	-0.439 ± 0.118
[6.00, 8.00]	-0.834 ± 0.062	$-0.695(-0.8\sigma)$	$-0.675(-0.7\sigma)$	-0.583 ± 0.096
[15.0, 19.0]	-0.595 ± 0.041	$-0.548(+1.0\sigma)$	$-0.527(+1.3\sigma)$	-0.640 ± 0.073

region [8,18], rather than to a failure of the LRIS mechanism.

The origin of the low- q^2 tension deserves a more detailed discussion. In the bin $q^2 \in [0.1, 0.98] \text{ GeV}^2$, the observable P'_5 is dominated at large recoil by the photon penguin coefficient C_7 rather than by C_9 , because the transversity amplitude ratio satisfies $|A_\perp/A_0| \propto |C_7 + q^2 C_9^{\text{eff}}/m_B^2|$ and $q^2/m_B^2 \ll 1$ in this bin. A negative shift $\Delta C_9^\mu < 0$ therefore *increases* P'_5 in the lowest bin (by suppressing C_9^{eff} relative to C_7), moving the prediction further from the data. This is a generic feature of all pure- ΔC_9^μ solutions to the anomaly [6,18] and is not specific to the LRIS model.

The key hadronic effect missing from the short-distance calculation is the nonfactorizable charm-loop contribution. In the large-recoil region, the $\bar{c}c$ loop couples to the virtual photon and generates an effective q^2 -dependent shift in the vector Wilson coefficient,

$$C_9^{\text{eff}}(q^2) = C_9 + Y_{\text{fact}}(q^2) + Y_{\text{non-fact}}(q^2), \quad (56)$$

where $Y_{\text{fact}}(q^2)$ is the factorizable part (included in FLAVIO) and $Y_{\text{nonfact}}(q^2)$ encodes soft-gluon exchanges between the $\bar{c}c$ loop and the spectator quark. In the bin $q^2 \lesssim 1 \text{ GeV}^2$, just below the ρ/ω resonance, $Y_{\text{non-fact}}$ is negative and can shift P'_5 downward by 0.1–0.3 units, as estimated in Refs. [8,62]. An additional downward shift of similar magnitude arises from the tail of the $\rho/\omega \rightarrow \mu^+\mu^-$ amplitude leaking into the bin [63]. Including these effects would bring both the SM and the LRIS predictions into agreement with the data in the lowest bin.

To illustrate this quantitatively, we note that a hadronic shift $\delta P_5^{\text{had}} \approx -0.20$ in the bin $[0.1, 0.98] \text{ GeV}^2$ —well within the range estimated in Refs. [8,62]—would reduce the LRIS tension from $+3.6\sigma$ to $+1.7\sigma$, and the SM tension from $+2.6\sigma$ to $+1.1\sigma$. A definitive resolution requires a dedicated dispersive analysis of the nonfactorizable contributions in the LRIS framework, which we leave for future work.

Similarly, the mild $+1.7\sigma$ tension in $q^2 \in [1.1, 2.5] \text{ GeV}^2$ arises in the transition region between the ρ/ω and J/ψ resonances, where P'_5 passes through zero and is particularly sensitive to the relative phase between the short-distance amplitude and the hadronic resonance tails. Global fits that include a floating nonfactorizable parameter in this region find that the tension disappears once hadronic uncertainties are properly marginalized [8].

The $\tan\beta$ independence of the predictions constitutes an additional, model-independent consistency test: any future measurement that constrains $\tan\beta$ directly (e.g. via $B \rightarrow \tau\nu$ or $H^\pm \rightarrow \tau\nu$ at the LHC) will not affect the P'_5 prediction, making the two sets of observables complementary rather than correlated.

X. CONCLUSIONS

We have shown that the left-right inverse seesaw model provides a natural and predictive explanation for the $B \rightarrow K\mu^+\mu^-$ anomaly. The key features of our mechanism are the following:

- (1) Nondecoupling shift in C_9 : The right-handed coupling $Y_{\mu N_i}^R \propto M_{N_i}/v_R$ generates a contribution to ΔC_9^μ that is finite even for $M_{N_i} \gg v_R$, via exact cancellation of the heavy-neutrino mass in the loop. This allows the LRIS box diagram to compete with the SM loop suppression, yielding $|\Delta C_9^\mu| \approx 1$ for natural values of the model parameters.
- (2) Automatic cancellation of C_{10} : Because the loop function h is identical for the right-handed and left-handed lepton topologies ($F_R = F_L$), the condition $|Y_{\mu N}^L| \approx |Y_{\mu N}^R|$ —natural in the inverse seesaw where Dirac Yukawa couplings are $\mathcal{O}(1)$ —produces $\Delta C_{10}^\mu \approx 0$ without fine-tuning. We have verified that this condition is simultaneously consistent with observed neutrino masses ($\mu_s \approx 35 \text{ eV}$, technically natural due to approximate lepton-number symmetry) and PMNS nonunitarity bounds ($\eta_{\mu\mu} \approx 7 \times 10^{-4} < 10^{-3}$).
- (3) Glashow–Iliopoulos–Maiani suppression of B_s mixing: The contribution to ΔM_{B_s} is dominated by the top-quark term $\lambda_t^2 S_0(x_t)$, with the charm sector suppressed by $(m_c/m_t)^2$ independently of $M_{H^\pm}$; this keeps $\Delta M_{B_s}^{\text{NP}}$ below the PDG limit throughout the viable $(M_{H^\pm}, y)$ band of Fig. 2(a), while the semileptonic signal remains intact due to the different kinematic structure of the N_i propagator. A right-handed phase texture $\gamma = \alpha$, $\delta = \beta + \pi$ [Eq. (40)] arises naturally from a discrete $Z_2: b_R \rightarrow -b_R$ symmetry and is a stationary point of the mixing amplitude (so the Barbieri–Giudice fine-tuning measure vanishes at first order); while not phenomenologically required by the B_s -mixing bound itself, it remains a consistent and radiatively stable structural feature of the model.
- (4) $b \rightarrow s\gamma$ safety: The contribution to C_7^{NP} is suppressed by more than two orders of magnitude below the experimental bound throughout the viable parameter space.

The viable region spans $M_{H^\pm} \in [0.6, 3] \text{ TeV}$ with $y \in [0.25, 1.0]$, fully accessible to current and future LHC programs. Correlated signatures include direct production of the charged scalar $H^\pm$ in $pp \rightarrow H^\pm \rightarrow tb$ and of the heavy right-handed neutrinos in $pp \rightarrow W_R \rightarrow \ell N$. The leptonic sector prediction $\eta_{\mu\mu} \approx 7 \times 10^{-4}$ will be decisively tested at FCC-ee.

The nondecoupling mechanism studied here is the same one that generates the forward-backward CP asymmetry in $\tau \rightarrow K\pi\nu_\tau$ within the LRIS model [59], suggesting a unified description of two distinct flavor anomalies in a single consistent framework.

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DATA AVAILABILITY

The data used in this paper are cited in Refs. [2,3,4,16,35,36,37,38,43,44,45,46,47,48,49,50].

APPENDIX A: LOOP INTEGRALS AND PASSARINO-VELTMAN REDUCTION

The scalar box integral appearing in the amplitude is

$$I_4(m_1^2, m_2^2, m_3^2, m_4^2) = \int_0^\infty \frac{x dx}{(x+m_1^2)(x+m_2^2)(x+m_3^2)(x+m_4^2)}. \quad (\text{A1})$$

In the heavy-neutrino limit $x_N = M_{N_i}^2/M_{H^\pm}^2 \gg 1$, using partial fractions,

$$M_{N_i}^2 I_4 \xrightarrow{x_N \rightarrow \infty} \frac{M_{H^\pm}^2 \ln M_{H^\pm}^2 - m_i^2 \ln m_i^2}{M_{H^\pm}^2 - m_i^2} + \mathcal{O}(1/x_N), \quad (\text{A2})$$

which can be rewritten as Eq. (25).

The Inami-Lim functions used for B_s mixing and $b \rightarrow s\gamma$ are [41]

$$S_0(x) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x^3 \ln x}{2(1-x)^3}, \quad (\text{A3})$$

$$D'_0(x) = \frac{-(8x^3 + 5x^2 - 7x)}{24(x-1)^3} + \frac{x^2(2x-3) \ln x}{4(x-1)^4}, \quad (\text{A4})$$

evaluating to $S_0(x_t) \approx 0.0293$ and $D'_0(x_t) \approx -0.0067$ at $m_t = 173$ GeV, $M_{H^\pm} = 1$ TeV.

APPENDIX B: EFFECTIVE $\Delta F=2$ AMPLITUDE AND THE ROLE OF r

Starting from Eq. (29) restricted to the charm-top sector (the up-quark contribution is negligible and dropped),

$$\mathcal{M}_{\text{mix}} \propto \lambda_t^2 S_0(x_t) + 2\lambda_t \lambda_c S_0(x_t, x_c) + \lambda_c^2 S_0(x_c), \quad (\text{B1})$$

with $\lambda_j \equiv Y_{js}^H (Y_{jb}^H)^*$ and $x_j \equiv m_j^2/M_{H^\pm}^2$. Using the small- x limit $S_0(x) \rightarrow x$ [from Eq. (A3)] together with the standard asymptotic form $S_0(x_t, x_c) \rightarrow x_c [\ln(x_t/x_c) - 1]$ for $x_c \ll x_t \ll 1$, both charm-sector terms are suppressed relative to the top term by powers of $x_c = (m_c/M_{H^\pm})^2 \sim 10^{-6}$, independently of $M_{H^\pm}$ (since $x_c/x_t = (m_c/m_t)^2$ is fixed). At the benchmark $M_{H^\pm} = 1$ TeV,

$$S_0(x_t) : S_0(x_t, x_c) : S_0(x_c) \approx 1 : 5.6 \times 10^{-4} : 5.8 \times 10^{-5}. \quad (\text{B2})$$

Substituting the phase-texture relation $\lambda_c/\lambda_t = -r^2$ [Eq. (39)] into Eq. (B1) therefore gives

$$\mathcal{M}_{\text{mix}} \propto \lambda_t^2 S_0(x_t) [1 + \mathcal{O}(10^{-4})r^2], \quad (\text{B3})$$

for any $r \lesssim \mathcal{O}(1)$: the charm-top interference changes the amplitude by at most a part in 10^3 – 10^4 relative to the top-only value, regardless of the sign fixed by the phase texture. We conclude that $\Delta M_{B_s}^{\text{NP}}$ is controlled by the top-only term $\lambda_t^2 S_0(x_t)$ alone, and that r is unconstrained by B_s mixing over its entire perturbative range; the phase texture of Sec. VIII A remains a consistent and technically natural structure of the right-handed quark sector, but is not phenomenologically required to satisfy the ΔM_{B_s} bound.

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- [1] S. L. Glashow, J. Iliopoulos, and L. Maiani, *Phys. Rev. D* **2**, 1285 (1970).
 - [2] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **125**, 011802 (2020).
 - [3] R. Aaij *et al.* (LHCb Collaboration), *J. High Energy Phys.* **06** (2014) 133.
 - [4] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. D* **108**, 032002 (2023).
 - [5] S. Iguro and Y. Omura, *J. High Energy Phys.* **05** (2018) 173.
 - [6] W. Altmannshofer and P. Stangl, *Eur. Phys. J. C* **81**, 952 (2021).
 - [7] T. Hurth, F. Mahmoudi, D. M. Santos, and S. Neshatpour, *Phys. Lett. B* **824**, 136838 (2022).
 - [8] M. Ciuchini, M. Fedele, E. Franco, A. Paul, L. Silvestrini, and M. Valli, *Phys. Rev. D* **107**, 055036 (2023).
 - [9] J. C. Pati and A. Salam, *Phys. Rev. D* **10**, 275 (1974).
 - [10] R. N. Mohapatra and J. C. Pati, *Phys. Rev. D* **11**, 566 (1975).
 - [11] G. Senjanović and R. N. Mohapatra, *Phys. Rev. D* **12**, 1502 (1975).
 - [12] P. S. B. Dev, R. Franceschini, and R. N. Mohapatra, *Phys. Rev. D* **86**, 093010 (2012).
 - [13] R. N. Mohapatra and J. W. F. Valle, *Phys. Rev. D* **34**, 1642 (1986).

- [14] D. Wyler and L. Wolfenstein, *Nucl. Phys.* **B218**, 205 (1983).
- [15] A. Pilaftsis, *Z. Phys. C* **55**, 275 (1992).
- [16] R. Aaij *et al.* (LHCb Collaboration), *Nat. Phys.* **18**, 277 (2022).
- [17] A. Crivellin, D. Müller, and C. Wiegand, *J. High Energy Phys.* **06** (2019) 119.
- [18] B. Capdevila, A. Crivellin, and J. Matias, *Eur. Phys. J. Special Topics* **1**, 20 (2023).
- [19] A. Crivellin and S. Iguro, *Phys. Rev. D* **110**, 015014 (2024).
- [20] P. Athron, A. Crivellin, T. E. Gonzalo, S. Iguro, and C. Sierra, *J. High Energy Phys.* **11** (2024) 133.
- [21] I. de Medeiros Varzielas and A. Sengupta, *Nucl. Phys.* **B1001**, 116495 (2024).
- [22] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, *Rev. Mod. Phys.* **68**, 1125 (1996).
- [23] K. Ezzat, M. Ashry, and S. Khalil, *Phys. Rev. D* **104**, 015016 (2021).
- [24] E. Ma, *Phys. Rev. D* **73**, 077301 (2006).
- [25] L. M. Krauss, S. Nasri, and M. Trodden, *Phys. Rev. D* **67**, 085002 (2003).
- [26] D. Restrepo, O. Zapata, and C. E. Yaguna, *J. High Energy Phys.* **11** (2013) 011.
- [27] C. D. Froggatt and H. B. Nielsen, *Nucl. Phys.* **B147**, 277 (1979).
- [28] Y.-L. Wu and Y.-F. Zhou, *Sci. China G* **51**, 1808 (2008).
- [29] E. Aprile *et al.* (XENON Collaboration), *Phys. Rev. Lett.* **131**, 041003 (2023).
- [30] J. Aalbers *et al.* (LZ Collaboration), *Phys. Rev. Lett.* **131**, 041002 (2023).
- [31] P. S. B. Dev and R. N. Mohapatra, *Phys. Rev. D* **81**, 013001 (2010).
- [32] W. Abdallah, A. Awad, S. Khalil, and H. Okada, *Eur. Phys. J. C* **72**, 2108 (2012).
- [33] M. Bordone, G. Isidori, and A. Pattori, *Eur. Phys. J. C* **76**, 440 (2016).
- [34] G. Hiller and F. Kruger, *Phys. Rev. D* **69**, 074020 (2004).
- [35] LHCb Collaboration, *Phys. Rev. Lett.* **122**, 191801 (2019).
- [36] LHCb Collaboration, *Phys. Rev. Lett.* **131**, 051803 (2023).
- [37] LHCb Collaboration, *J. High Energy Phys.* **07** (2025) 198.
- [38] CMS Collaboration, *Rep. Prog. Phys.* **87**, 077802 (2024).
- [39] M. Ashry, K. Ezzat, and S. Khalil, *Phys. Rev. D* **107**, 055044 (2023).
- [40] G. Passarino and M. J. G. Veltman, *Nucl. Phys.* **B160**, 151 (1979).
- [41] T. Inami and C. S. Lim, *Prog. Theor. Phys.* **65**, 297 (1981).
- [42] A. J. Buras, M. Misiak, M. Münz, and S. Pokorski, *Nucl. Phys.* **B424**, 374 (1994).
- [43] J. P. Lees *et al.* (BABAR Collaboration), *Phys. Rev. D* **86**, 052012 (2012).
- [44] T. Saito *et al.* (Belle Collaboration), *Phys. Rev. D* **91**, 052004 (2015).
- [45] Y. S. Amhis *et al.* (HFLAV Collaboration), *Phys. Rev. D* **107**, 052008 (2023).
- [46] A. M. Sirunyan *et al.* (CMS Collaboration), *J. High Energy Phys.* **01** (2018) 096.
- [47] M. Aaboud *et al.* (ATLAS Collaboration), *J. High Energy Phys.* **11** (2018) 085.
- [48] A. M. Sirunyan *et al.* (CMS Collaboration), *Phys. Lett. B* **792**, 107 (2019).
- [49] G. Aad *et al.* (ATLAS Collaboration), *Phys. Rev. D* **100**, 052013 (2019).
- [50] R. L. Workman *et al.* (Particle Data Group), *Prog. Theor. Exp. Phys.* **2022**, 083C01 (2022).
- [51] G. 't Hooft, NATO Sci. Ser. B **59**, 135 (1980).
- [52] E. Fernández-Martínez, J. Hernández-García, and J. López-Pavón, *J. High Energy Phys.* **08** (2016) 033.
- [53] M. Blennow, E. Fernández-Martínez, J. Hernández-García, J. López-Pavón, X. Marcano, and D. Naredo-Tuero, *J. High Energy Phys.* **08** (2023) 030.
- [54] A. Blondel *et al.*, *arXiv:1909.12245*.
- [55] A. Maiezza, M. Nemešček, F. Nesti, and G. Senjanović, *Phys. Rev. D* **82**, 055022 (2010).
- [56] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher, and J. P. Silva, *Phys. Rep.* **516**, 1 (2012).
- [57] R. Barbieri and G. F. Giudice, *Nucl. Phys.* **B306**, 63 (1988).
- [58] K. Kiers, J. Kolb, J. Lee, A. Soni, and G.-H. Wu, *Phys. Rev. D* **66**, 095002 (2002).
- [59] D. Delepine and S. Khalil, *arXiv:2604.24924*.
- [60] D. M. Straub, *arXiv:1810.08132*.
- [61] S. Descotes-Genon, J. Matias, and J. Virto, *Phys. Rev. D* **88**, 074002 (2013).
- [62] N. Gubernari, M. Reboud, D. van Dyk, and J. Virto, *J. High Energy Phys.* **09** (2022) 133.
- [63] T. Blake, U. Egede, P. Owen, K. A. Petridis, and G. Pomery, *Eur. Phys. J. C* **78**, 453 (2018).