

Constraints on axionlike particles from VERITAS observations of a flaring radio galaxy in the Perseus cluster

C. B. Adams^{1,*}, A. Archer², P. Bangale³, J. T. Bartkoske⁴, W. Benbow⁵, Y. Chen⁶, J. L. Christiansen⁷, A. J. Chromey⁵, A. Duerr⁴, M. Errando⁸, M. Escobar Godoy⁹, J. Escudero Pedrosa¹⁰, S. Feldman⁶, Q. Feng^{4,†}, S. Filbert⁴, L. Fortson¹¹, A. Furniss⁹, W. Hanlon⁵, O. Hervet⁹, C. E. Hinrichs¹², J. Holder³, Z. Hughes⁸, T. B. Humensky¹³, M. Iskakova⁸, W. Jin⁶, P. Kaaret¹⁴, M. Kertzman², M. Kherlakian¹⁵, D. Kieda⁴, T. K. Kleiner¹⁶, N. Korzoun³, F. Krennrich¹⁷, S. Kumar¹⁸, S. Kundu¹⁹, M. J. Lang²⁰, M. Lundy²¹, P. Moriarty²⁰, R. Mukherjee²², W. Ning⁶, R. A. Ong⁶, A. Pandey⁴, M. Pohl²³, E. Pueschel¹⁵, J. Quinn²⁴, P. L. Rabinowitz⁸, K. Ragan²¹, P. T. Reynolds²⁵, D. Ribeiro¹¹, E. Roache⁵, C. Rulten^{11,26}, I. Sadeh¹⁶, L. Saha⁵, M. Santander¹⁹, G. H. Sembroski²⁷, R. Shang²², D. Tak²⁸, A. K. Talluri¹¹, J. V. Tucci²⁹, J. Valverde³⁰, V. V. Vassiliev⁶, D. A. Williams⁹, S. L. Wong²¹, J. Woo³¹, T. Yoshikoshi³²

(VERITAS Collaboration) and M. Meyer³³

¹*Physics Department, Columbia University, New York, New York 10027, USA*

²*Department of Physics and Astronomy, DePauw University, Greencastle, Indiana 46135-0037, USA*

³*Department of Physics and Astronomy and the Bartol Research Institute, University of Delaware, Newark, Delaware 19716, USA*

⁴*Department of Physics and Astronomy, University of Utah, Salt Lake City, Utah 84112, USA*

⁵*Center for Astrophysics | Harvard & Smithsonian, Cambridge, Massachusetts 02138, USA*

⁶*Department of Physics and Astronomy, University of California, Los Angeles, California 90095, USA*

⁷*Physics Department, California Polytechnic State University, San Luis Obispo, California 94307, USA*

⁸*Department of Physics, Washington University, St. Louis, Missouri 63130, USA*

⁹*Santa Cruz Institute for Particle Physics and Department of Physics, University of California, Santa Cruz, California 95064, USA*

¹⁰*Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, Massachusetts 02138, USA*

¹¹*School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA*

¹²*Center for Astrophysics | Harvard & Smithsonian, Cambridge, Massachusetts 02138, USA and Department of Physics and Astronomy, Dartmouth College, 6127 Wilder Laboratory, Hanover, New Hampshire 03755, USA*

¹³*Department of Physics, University of Maryland, College Park, Maryland, USA and NASA GSFC, Greenbelt, Maryland 20771, USA*

¹⁴*Department of Physics and Astronomy, University of Iowa, Van Allen Hall, Iowa City, Iowa 52242, USA*

¹⁵*Fakultät für Physik & Astronomie, Ruhr-Universität Bochum, D-44780 Bochum, Germany*

¹⁶*DESY, Platanenallee 6, 15738 Zeuthen, Germany*

¹⁷*Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011, USA*

¹⁸*Department of Physics, University of Maryland, College Park, Maryland, USA*

¹⁹*Department of Physics and Astronomy, University of Alabama, Tuscaloosa, Alabama 35487, USA*

²⁰*School of Natural Sciences, University of Galway, University Road, Galway, H91 TK33, Ireland*

²¹*Physics Department, McGill University, Montreal, Quebec H3A 2T8, Canada*

²²*Department of Physics and Astronomy, Barnard College, Columbia University, New York 10027, USA*

²³*Institute of Physics and Astronomy, University of Potsdam, 14476 Potsdam-Golm, Germany and DESY, Platanenallee 6, 15738 Zeuthen, Germany*

²⁴*School of Physics, University College Dublin, Belfield, Dublin 4, Ireland*

²⁵*Department of Physical Sciences, Munster Technological University, Bishopstown, Cork T12 P928, Ireland*

²⁶*Centre for Advanced Instrumentation, Department of Physics, University of Durham, South Road, Durham DH1 3LE, United Kingdom*

²⁷*Department of Physics and Astronomy, Purdue University, West Lafayette, Indiana 47907, USA*

²⁸*SNU Astronomy Research Center, Seoul National University, Seoul 08826, Republic of Korea*

²⁹*Department of Physics, Indiana University Indianapolis, Indianapolis, Indiana 46202, USA*

³⁰*Department of Physics, University of Maryland, Baltimore County, Baltimore Maryland 21250, USA and NASA GSFC, Greenbelt, Maryland 20771, USA*

³¹*Columbia Astrophysics Laboratory, Columbia University, New York, New York 10027, USA*

³²*Institute for Cosmic Ray Research, University of Tokyo, 5-1-5, Kashiwa-no-ha,
Kashiwa, Chiba 277-8582, Japan*

³³*CP3-Origins, University of Southern Denmark, Campusvej 55, 5230 Odense M, Denmark*



(Received 1 August 2025; accepted 20 October 2025; published 24 November 2025)

Axion-like particles (ALPs) are hypothetical particles that emerge in numerous theoretical extensions to the Standard Model. Their coupling to electromagnetic field implies that ALPs would mix with photons in the presence of external magnetic fields. As ALP phenomenology is governed by the mass and strength of its coupling, there is a subset of this parameter space in which this mixing would be expected to leave an imprint on the spectra of TeV γ -ray sources. In 2017, the VERITAS γ -ray observatory recorded the second day of a dramatic flare of the radio galaxy NGC 1275, embedded at the center of the Perseus galaxy cluster. This serendipitous locale provides a spatially-extended magnetic field of strength $\mathcal{O}(10 \mu\text{G})$ through which escaping photons traverse, making it an excellent target to study ALPs. We analyze the VERITAS data of NGC 1275's 2017 flare with the gammapy analysis package. Extensive fitting and modeling are performed to ultimately conduct a likelihood analysis used to search for any evidence of a preference for ALPs and to explore the confidence with which constraints can be set. We adopt the CL_s method for this study for its conservative approach to setting limits in regimes where the search has limited sensitivity. No evidence for the existence of ALPs is found, and no combination of mass and coupling strength can be excluded at or above 95% confidence level. We provide a map showing the strength of our exclusions in the mass and coupling parameter space. The strongest exclusions are found in the mass range $2 \times 10^{-7} \text{ eV} \lesssim m_a \lesssim 4 \times 10^{-7} \text{ eV}$ and at the coupling strength of $g_{a\gamma} \gtrsim 3 \times 10^{-11} \text{ GeV}^{-1}$ up to 80% confidence level, which are consistent with previous studies. We find the CL_s to be a trustworthy approach, and advocate for its continued usage in future studies. We note that many of the limitations contributing to the limited sensitivity seen by VERITAS in this study will be improved with next-generation γ -ray instruments, such as the Cherenkov Telescope Array Observatory.

DOI: [10.1103/jydc-npy6](https://doi.org/10.1103/jydc-npy6)

I. INTRODUCTION

TeV γ -ray observations can be used to probe fundamental physics beyond the Standard Model (SM) of particle physics (see, e.g., [1]), including axions and axion-like particles (ALPs) (see, e.g., [2,3] and references therein). Axions are hypothetical pseudo Nambu-Goldstone (pNG) bosons postulated to solve the “strong charge-parity (CP) problem” in particle physics and, fortuitously, are excellent candidates for dark matter (DM) (see, e.g., [4]). Axions would couple with photon pairs, with a coupling strength proportional to their mass, which is unconstrained in theoretical models. ALPs, on the other hand, are predicted in many extensions of the SM, particularly in numerous theories of string compactifications, with a looser constraint: their coupling strength to photons $g_{a\gamma}$ is independent of mass m_a (e.g., [5–21]). ALPs do not necessarily solve the strong CP problem, but bear many phenomenological similarities to axions, making them

also viable DM candidates as weakly interacting sub-eV particles (WISPs).

As will be seen in Sec. II, the coupling between photon pairs and ALPs means that ALPs can interconvert with photons in the presence of an external magnetic field. This would lead to ALP-photon mixing, a process which is mathematically closely analogous to neutrino oscillations (e.g., [22]). This mixing could distort the photon spectra from astrophysical sources [23]. As an example, photons could be converted into ALPs in a magnetic field at or near a source, or in some intervening field encountered on the trajectory from the source to the observer, such as the intergalactic magnetic field (IGMF). The ALP-photon beam could then undergo further oscillations, including from ALP back to photon, in the Galactic magnetic field. This ALP-photon mixing may lead to irregularities in the observed spectrum of a source (see, e.g., [24] for a comprehensive review).

One such irregularity would be an increased probability that high-energy γ rays survive the pair absorption process with the extragalactic background light (EBL). This would effectively increase the transparency of the Universe to γ rays, leading to unusually hard γ -ray spectra and potentially the detection of γ -ray sources out to greater distances. Another spectral irregularity would manifest as oscillatory features in the observed spectrum—a result of the energy dependence of

*Contact author: ca2762@columbia.edu

†Contact author: qi.feng@utah.edu

the ALP-photon mixing. The ALP-induced spectral irregularity becomes more prominent when the photons travel through a stronger magnetic field or across a larger magnetic field domain. Therefore, stronger experimental constraints on the properties of ALPs can be obtained from observations of γ -ray sources that are either embedded in a relatively strong magnetic field, or distant enough that their emission would extensively traverse through the IGMF.

In addition to photons, axions would also couple with gluons and, in some models (e.g., the DFSZ model [25,26]), with fermions, such as electrons. In hadronic models (e.g., the KSVZ model [27,28]), the coupling between axions and electrons is negligible. ALPs, however, may generally couple to electrons. In this work, we assume that the coupling between ALPs and both gluons and electrons is subdominant.

Several experiments have directly searched for ALPs using various methods (see, e.g., [19] for a review). Purely laboratory experiments, the so-called “Light Shining through a Wall” (LSW) experiments such as Any Light Particle Search (ALPS) I [29], search for photons that appear to pass through a wall by oscillating into intermediate WISPs and have established upper limits on the ALP-photon coupling strength at the level of $g_{a\gamma} < \mathcal{O}(10^{-8}) \text{ GeV}^{-1}$ at masses $m_a \lesssim 1 \text{ meV}$. The ALPS II experiment [30] is expected to improve the limits to $g_{a\gamma} \lesssim 2 \times 10^{-11} \text{ GeV}^{-1}$ in a similar mass range. Helioscopes and haloscopes seek to convert ALPs from the Sun and the Galactic DM halo, respectively, into detectable photons to constrain the coupling strength between ALPs and photons. The CERN Axion Solar Telescope (CAST), a helioscope, has looked for solar axions converting into x rays in a laboratory magnetic field and established an upper limit on the axion-photon¹ coupling strength of $g_{a\gamma} \lesssim 6.6 \times 10^{-11} \text{ GeV}^{-1}$ at mass $m_a \lesssim 0.02 \text{ eV}$ [31]. The Axion Dark Matter Experiment (ADMX), a haloscope, has searched for dark matter axions converting into microwave photons in a large resonant cavity within a strong superconducting magnet and established upper limits on the axion-photon coupling strength of $g_{a\gamma} < \mathcal{O}(10^{-13}) \text{ GeV}^{-1}$ within a narrow mass range of $2.66 \lesssim m_a \lesssim 2.80 \text{ } \mu\text{eV}$ [32]. To constrain the coupling between ALPs and gluons and nuclear spin, the Cosmic Axion Spin Precession Experiment (CASPER) uses nuclear magnetic resonance to detect an oscillating torque on nuclear spins [33]. Constraints on ALPs have also been derived from observations of other astrophysical sources, such as giant stars [34], neutron stars [35], including magnetars [36], the supernova SN 1987A (e.g., [37,38]), globular clusters [39], spinning black holes [40], and active galactic nuclei (AGN), as well as from various cosmological observations [e.g., [41]].

¹By virtue of the shared phenomenology between axions and ALPs, all the constraints discussed in this section may be generalized to ALPs.

Constraints on ALPs using spectral irregularities from AGN, including blazars and radio galaxies, have been derived from x-ray and γ -ray observations. These constraints improved the upper limit on the ALP-photon coupling strength $g_{a\gamma}$ by more than an order of magnitude compared to the limits from CAST, covering an ALP mass range ($\lesssim \mu\text{eV}$) lower than what ADMX is sensitive to (e.g., [42]). Blazars and radio galaxies (RGs) exhibit nonthermal radiation powered by the central supermassive black hole, featuring smooth x-ray and γ -ray spectra that can often be described by a power law or log parabola model (e.g., [43,44]). Blazars and RGs that reside in the center of galaxy clusters are particularly interesting for ALP studies, as the intracluster medium (ICM) is permeated by large-scale turbulent magnetic fields where ALP-photon conversion may take place efficiently. These fields are particularly strong close to the center in cool-core clusters [45], where the ICM is so dense that the cooling time, primarily from free-free Bremsstrahlung [46], is short compared to the Hubble time [47,48], resulting in a rapidly cooled ICM [49,50].

The TeV radio galaxy NGC 1275, located at the center of the cool-core Perseus cluster at a distance of $\sim 75 \text{ Mpc}$ ($z = 0.01756$), has been extensively studied to search for evidence for ALPs. The Perseus cluster is the brightest galaxy cluster in the x-ray band, with a strong intracluster magnetic field (ICMF) of $\mathcal{O}(10 \text{ } \mu\text{G})$, as shown by radio rotation measures (RMs) [51]. In comparison, the IGMF is much weaker, constrained by several methods to be below $\mathcal{O}(1 \text{ nG})$ with various coherence lengths assumed [52–56]. The Chandra X-ray Observatory and the *Fermi*-Large Area Telescope (LAT) [57] have searched for spectral irregularities in NGC 1275’s x-ray and GeV γ -ray emission, respectively, to set limits on the ALP-photon coupling strength ($g_{a\gamma} \lesssim 6 \times 10^{-13} \text{ GeV}^{-1}$ at masses $m_a \lesssim 1 \times 10^{-12} \text{ eV}$ from X-ray, and $g_{a\gamma} \lesssim 5 \times 10^{-12} \text{ GeV}^{-1}$ in the mass range of $0.5 \text{ neV} \lesssim m_a \lesssim 5 \text{ neV}$ from γ -ray) [58–60].

At TeV γ -ray energies, the exceptional sensitivity on short timescales from ground-based imaging atmospheric Cherenkov telescopes (IACTs) offers an opportunity to search for spectral irregularities in fast and intense flares from blazars and radio galaxies. An exceptionally bright very-high energy (VHE) flare of NGC 1275 was recorded by the Major Atmospheric Gamma Imaging Cherenkov Telescopes (MAGIC) [61] beginning on the night spanning December 31st, 2016 to January 1st, 2017 (MJD 57753–57754) [62]. The brightest outburst, recorded during the first night of the flare, had a flux ~ 1.5 times the flux of the Crab Nebula above 100 GeV, falling on subsequent nights [63], during which it was further detected by the Very Energetic Radiation Imaging Telescope Array System (VERITAS) [64]. Observations of this flare present an excellent opportunity to search for ALPs through γ -ray spectral irregularity, as (1) the source is embedded in a strong ICMF and (2) the bright flare allows for a well-characterized TeV γ -ray

spectrum due to the superior signal-to-noise ratio. Motivated by this, MAGIC used their flaring and postflare data from NGC 1275, alongside long-term quiescent monitoring data of the source, to exclude at 99% confidence ALP coupling constants $g_{a\gamma} \gtrsim 3 \times 10^{-12} \text{ GeV}^{-1}$ in the range $4 \times 10^{-8} \text{ eV} \lesssim m_a \lesssim 9 \times 10^{-8} \text{ eV}$ [65].

During VERITAS observations of NGC 1275 spanning January 2nd(MJD 57755) to January 3rd(MJD 57756) of 2017, an average flux state at 50% of the Crab flux at TeV energies was recorded. During the flare, NGC 1275 was detected by VERITAS over 2.2 hours (MJD 57755) with three-telescope operations and 1.3 hours (MJD 57756) with four-telescope operations, noting a marginally higher measured flux on the first night observed. The two nights recorded respective statistical significances of $\sim 31\sigma$ and $\sim 22\sigma$. These observations overlap with a Bayesian-block-identified steady state in *Fermi*-LAT spanning roughly January 1st to January 6th of 2017 [66]. However, as the VHE spectra recorded by MAGIC appear to evolve significantly over this period [63,65], we opt to exclude the Bayesian-block *Fermi*-LAT data from this study. This disconnect between the evolution of the flare in GeV and TeV bands may be explained by stronger cooling at VHEs. In addition, to minimize evolution in the VHE band, we opt to include only VERITAS data from January 2nd in this analysis due to the higher VERITAS observed flux on this date.

In this work, we use these VERITAS observations to set limits on the ALP-photon coupling strength and the mass of ALPs. In Sec. II we discuss the theory of ALP-photon interactions. Then, in Sec. III, we describe the VERITAS observatory that recorded the data and the techniques used to analyze them. In Sec. IV, we delineate the statistical methods employed to compare the ALP and no-ALP hypotheses. In Sec. V, we report our findings, and compare our methodology and results to those reported previously by MAGIC [65].

II. ALP-PHOTON INTERACTIONS

The theory of ALP-photon interactions has been covered extensively in previous publications (see, e.g., [67–76], among many others). An abbreviated summary is provided here.

The ALP contribution to the SM Lagrangian² is given by [67,68,76]

$$\mathcal{L}_{\text{ALP}} = \underbrace{\frac{1}{2} \partial_\mu a \partial^\mu a - \frac{1}{2} m_a^2 a^2}_{\mathcal{L}_{aa}} + \underbrace{\frac{1}{4} g_{a\gamma} F_{\mu\nu} \tilde{F}^{\mu\nu} a}_{\mathcal{L}_{a\gamma}}, \quad (1)$$

$F_{\mu\nu}$ is the electromagnetic (EM) field tensor, and $\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$ is its dual. The contribution $\mathcal{L}_{aa}$ contains a

²Using natural Lorentz-Heaviside units such that $\hbar = c = k_B = 1$ throughout, and $\alpha = e^2/4\pi \simeq 1/137$.

kinetic and mass term to describe ALPs as a noninteracting, free scalar field a with mass m_a [68,77]. The rightmost term describes the ALP coupling to electromagnetism through the interaction,

$$\mathcal{L}_{a\gamma} = \frac{1}{4} g_{a\gamma} F_{\mu\nu} \tilde{F}^{\mu\nu} a = g_{a\gamma} (\mathbf{E} \cdot \mathbf{B}) a, \quad (2)$$

where $g_{a\gamma}$ is the coupling strength of the ALP field to photons. As a consequence, any photon in the presence of a magnetic field with some component oriented transversely to the photon propagation direction will mix with ALPs [67].

Because of the coupling described in Eq. (2), an ALP-photon beam will propagate as a mixing of three quantum states: two corresponding to photon polarization, and the third corresponding to the ALP [20]. For an initially polarized photon beam of energy E propagating through a single homogeneous magnetic field B (with component B_T that is transverse to photon propagation direction, and parallel to the polarization state), the equations of motion for an ALP-photon beam can be solved (see, e.g., [67,69,78]), and result in ALP-photon oscillations with amplitude:

$$P_{\gamma \rightarrow a}(E, z) = \left(\frac{g_{a\gamma} B_T}{\Delta_{\text{osc}}(E)} \right)^2 \sin^2 \left(\frac{\Delta_{\text{osc}}(E)(z - z_0)}{2} \right), \quad (3)$$

for a beam that has traveled from z_0 to z . The oscillation wave number is given by [79]

$$\begin{aligned} \Delta_{\text{osc}}(E) &= \left[\left\{ \frac{m_a^2 - \omega_{\text{pl}}^2}{2E} + E \left(\frac{7}{2} b + \chi + i \frac{\Gamma_{\gamma\gamma}}{2E} \right) \right\}^2 + (g_{a\gamma} B_T)^2 \right]^{1/2}, \\ & \quad (4) \end{aligned}$$

where ω_{pl} is the plasma frequency $\omega_{\text{pl}} \sim 0.037 \sqrt{n_e} \text{ neV}$, with the electron density n_e in units of cm^{-3} . χ and $\Gamma_{\gamma\gamma}$ give the terms for the dispersion and absorption for the surrounding photon fields, respectively, and b is the vacuum quantum electrodynamics (QED) term which describes dispersion of the magnetic field:

$$b = \frac{\alpha}{45\pi} \left(\frac{B_T}{B_{\text{cr}}} \right)^2, \quad (5)$$

where α is the fine structure constant, and B_{cr} is the critical magnetic field strength $B_{\text{cr}} = m_e^2/|e| \approx 4.41 \times 10^{13} \text{ G}$ with m_e the electron mass and e the fundamental electric charge.

Assuming that the absorption is small, it is possible to define a critical energy [69,73,79]

$$E_{\text{cr}}^{\text{low}} = \frac{|m_a^2 - \omega_{\text{pl}}^2|}{2g_{a\gamma} B_T}, \quad (6)$$

far above which the conversion probability is maximal and independent of energy (the *strong-mixing regime*), and below which the conversion probability is oscillatory, and increasing, with energy. The strong-mixing regime is bounded from above by

$$E_{\text{cr}}^{\text{high}} = \frac{g_{\text{ay}} B_T}{\frac{7}{2}b + \chi}. \quad (7)$$

The energy-dependent oscillatory behavior near $E_{\text{cr}}^{\text{low}}$ is of particular interest for this study, as these oscillations have the potential to imprint features in the spectra of astrophysical γ -ray emitters.

A. Modeling the ALP effect

The above is a simple and illustrative example of the effects one can expect from ALP-photon mixing; however, for the purposes of this paper it is necessary to consider the more complex scenario of an initially unpolarized ALP-photon beam traversing nonhomogeneous magnetic fields. This can be properly accounted for by splitting magnetic regions into slices of homogeneous field and by using the formalism of the density matrix [60,69,70,80,81]. The assumption of an initially unpolarized beam is necessary because, although the beam will likely have some degree of polarization, it is not known *a priori*, nor do current-generation γ -ray telescopes have the ability to measure it [82]. Further, it is expected to be small— $\mathcal{O}(\text{a few}\%)$ [83].

We model the propagation of the ALP-photon beam as it traverses three distinct environments. Since NGC 1275 is located at the center of the Perseus Galaxy cluster, the first region the beam will encounter will be the ICMF (see Sec. II A 1 for details). Provided the relatively small domain traversed by the emission (considering that NGC 1275 is a radio galaxy observed at large viewing angles with respect to the jet axis [2,84]) and the relative dominance of the cluster contribution to mixing over the jet component [74], we are able to neglect ALP-photon mixing in the AGN jet. We next include EBL absorption for propagation through intergalactic space, using the model of Domínguez *et al.* [85]. As noted in Sec. I, current limits place the maximum strength of the IGMF at $\sim 10^{-9}$ G. Given the additional constraint $g_{\text{ay}} < 6.6 \times 10^{-11} \text{ GeV}^{-1}$ [31], we find that $E_{\text{cr}}^{\text{low}} \lesssim 3.5 \text{ TeV}$ only for $m_a \lesssim 3 \times 10^{-9} \text{ eV}$. This is below the m_a range considered for this analysis, so we only consider the regime of m_a where the $E_{\text{cr}}^{\text{low}} \gtrsim 3.5 \text{ TeV}$, which is roughly the upper bound of our instrument's sensitivity in this analysis. Together with the relatively small redshift of NGC 1275, we do not expect strong irregularities in the spectrum to be induced by mixing in the IGMF, and therefore neglect its contribution as well. Finally, we consider beam propagation and mixing in the Milky Way's magnetic field, following the model of Jansson and Farrar [86].

In order to compute the probability to observe at Earth a γ ray (of either polarization) from an initially unpolarized

beam at the source, known as the photon survival probability $P_{\gamma\gamma}$, we use the gammaALPs³ software package [87]. The code numerically solves the equations of motion for the ALP-photon system using the transfer-matrix formalism and incorporates all relevant mixing terms in the mixing matrix, including dispersion from QED effects and the cosmic microwave background (CMB).

1. Perseus cluster modeling

Following from Refs. [60,70], we expect the ICMF of the Perseus cluster to be a turbulent field, with a strength that traces the electron density $n_e(r)$ of the ICM, $B(r) = B_0[n_e(r)/n_e(r=0)]^\eta$ [88]. The turbulent component is modelled as a divergence-free homogeneous isotropic field with Gaussian turbulence with a mean of zero and variance σ_B . We assume that the power spectrum of the turbulence follows a power law $M(k) \propto k^q$ in wave numbers k between the maximum and minimum turbulence scales $k_L = 2\pi/\Lambda_{\text{max}}$ and $k_H = 2\pi/\Lambda_{\text{min}}$, and 0 otherwise [70,76]. We use an analytical approximation for $n_e(r)$ derived from x-ray observations with *XMM-Newton* [Eq. (4) in Churazov *et al.* [89]] within 500 kpc of the cluster's center (r_{max}), and conservatively assume zero magnetic field beyond.⁴ RMs determined from Very Long Baseline Array (VLBA) observations of the innermost region (tens of pc) around NGC 1275 suggest a central magnetic field for the cluster of 25 μG [51]. Considering this, alongside an independently derived lower limit on $B_0 \gtrsim 2\text{--}13 \mu\text{G}$ for $0.3 \leq \eta \leq 0.7$ from the MAGIC observatory [90], we make the assumptions $\sigma_B = 10 \mu\text{G}$ and $\eta = 0.5$, which are consistent with measurements of similar cool-core clusters Hydra A [91] and A2199 [92]. In the absence of detailed RMs outside of the central-most region of the Perseus cluster, we assume values from A2199, which has a comparable number of member galaxies. A summary of the fiducial parameters used to model the magnetic field structure of the Perseus cluster can be found in Table I of Ajello *et al.* [60].

III. DATA ANALYSIS

Data for this study were recorded by VERITAS, a ground-based γ -ray observatory comprising an array of four IACTs located in southern Arizona at $31^\circ 40' 30''\text{N}$ $110^\circ 57' 07''\text{W}$, 1.3 km above sea level (a.s.l.) and capable of detecting γ rays from 85 GeV to $> 30 \text{ TeV}$ [93]. VERITAS, similar to other current ground-based γ -ray observatories, offers moderate energy resolution ($\sim 15\%$ at 1 TeV) limited by the air shower fluctuations associated with the air Cherenkov technique (e.g., [94]).

For this study, we used VERITAS observations of NGC 1275 in a flaring state on January 2nd, 2017 with an

³<https://github.com/me-manu/gammaALPs>.

⁴For ALP parameters and photon energies relevant to IACTs, a typical ALP-photon oscillation length for the Perseus cluster is $\mathcal{O}(10 \text{ kpc})$ [20].

exposure of 2.2 hr after data quality selection. Spectral analysis for these data was performed using the *gammapy* software package [95]. VERITAS data were preprocessed using the EventDisplay VERITAS analysis pipeline [96] with a reflected region background model [97] and were provided to *gammapy* at the event level, a data level 3 (DL3) product [see [98]]. From the DL3 data, a full region-based on-off spectral analysis was performed.

Two different intrinsic models for the data were examined for the null hypothesis case, an exponential cutoff power law (ECPL) and a log parabola (LP), neither of which was significantly preferred over the other. In an ongoing study using the same VERITAS data that overlaps with the integrated *Fermi*-LAT data for January 2nd [66], a Bayesian information criterion (BIC) was utilized to find that a LP fit was preferred to an exponentially cutoff power law. Therefore, we have chosen the LP as the base intrinsic model for our analysis:

$$\Phi_{\text{int}}(E) = N_0 \left(\frac{E}{E_0} \right)^{-\alpha - \beta \log(E/E_0)}, \quad (8)$$

where $\Phi_{\text{int}}(E)$ is the intrinsic differential photon flux at photon energy E , N_0 the spectral normalization, E_0 the reference energy, α the spectral index, and β the curvature. The null hypothesis model is then given by

$$\Phi_0(E, z) = e^{-\tau(E, z)} \Phi_{\text{int}}(E), \quad (9)$$

where $\tau(E, z)$ is the optical depth due to the EBL, from Domínguez *et al.* [85]. For the alternative hypothesis, the photon survival probability $P_{\gamma\gamma}(E, z; m_a, g_{a\gamma}, \mathbf{B})$, which includes both EBL attenuation and the ALP effect (dictated by $m_a, g_{a\gamma}$, and the magnetic field environment traversed by the beam $\mathbf{B}$), will be used instead, yielding

$$\Phi_{\text{ALP}}(E, z; m_a, g_{a\gamma}, \mathbf{B}) = P_{\gamma\gamma}(E, z; m_a, g_{a\gamma}, \mathbf{B}) \Phi_{\text{int}}(E). \quad (10)$$

$P_{\gamma\gamma}(E, z; m_a, g_{a\gamma}, \mathbf{B})$ is calculated with the *gammaALPs* [87] software package.

The models given by Eqs. (9) and (10) can be used to compute their respective likelihood given the observed data. The likelihood, across all energy bins i , can be written as

$$\mathcal{L}(\mu(m_a, g_{a\gamma}, \mathbf{B}, \boldsymbol{\theta}), b|\mathbf{D}) = \prod_i \mathcal{L}_i(\mu_i(m_a, g_{a\gamma}, \mathbf{B}, \boldsymbol{\theta}), b_i|\mathbf{D}_i), \quad (11)$$

where μ_i is the expected number of counts in the i th energy bin, computed by forward folding the detector response into the model given by Eqs. (9) or (10). This number depends on $m_a, g_{a\gamma}$, the realization of the magnetic field $\mathbf{B}$, and the spectral energy distribution (SED) nuisance parameters which define Eq. (8): $\boldsymbol{\theta} = (N_0, \alpha, \beta)$. The parameter b_i is the number of background counts expected in the source

region, and $\mathbf{D}_i$ the counts measured by the instrument in its source and background estimation regions, also for the i th energy bin.

As suggested above, there are two cases under consideration: the no-ALP case (the null hypothesis), corresponding to the case where ALPs do not couple to photons ($g_{a\gamma} = 0$), and the ALP case (the alternative hypothesis). Following from Eq. (11), the likelihood of the alternative hypothesis can be written as

$$\mathcal{L}_{\text{ALP}}(\mu(m_a, g_{a\gamma}, \mathbf{B}, \boldsymbol{\theta}), b|\mathbf{D}), \quad (12)$$

and the likelihood of the null hypothesis can be written simply as

$$\mathcal{L}_0(\mu(\boldsymbol{\theta}), b|\mathbf{D}). \quad (13)$$

For the fixed parameters ($m_a, g_{a\gamma}$), the nuisance parameters can be “profiled out” using the profile likelihood method of Rolke *et al.* [99]. This profile approach is employed by the WStat fit statistic used in *gammapy* [100]. Profiling is done for the expected background counts b_i by replacing them with the value $\hat{b}_i$ that analytically maximizes the likelihood, where $\hat{b}_i$ is effectively a background estimator. This technique is known to underestimate the predicted background counts, particularly in the higher-energy regime for IACTs, where the number of background counts will likely drop to zero [101]. This has been addressed by adaptively rebinning the data, ensuring that each energy bin contains > 0 background counts. Although this does not eliminate bias in the predicted counts, the relative scale of the bias is significantly reduced and, when tested on fits of simulated null-hypothesis data, was shown to adequately recover the injected model. The SED nuisance parameters $\boldsymbol{\theta}$ are profiled for a fixed ($m_a, g_{a\gamma}$) by performing an iMinuit [102,103] minimization⁵ of *gammapy*’s WStat fit statistic with the expected background counts profiled, yielding $\hat{\boldsymbol{\theta}}$.

To treat the magnetic field structure of Perseus statistically, for each ($m_a, g_{a\gamma}$) we perform fits over $N_B = 100$ random realizations of the turbulent magnetic field $\mathbf{B}_j$, $j = 1, \dots, N_B$, where each individual fit yields, following the profiling described above, $\mathcal{L}_{\text{ALP}}(\mu(m_a, g_{a\gamma}, \mathbf{B}_j, \hat{\boldsymbol{\theta}}), \hat{b}|\mathbf{D})$. However, it is highly unlikely that the magnetic field that is realized in nature is included in the simulated magnetic-field realizations. Therefore, instead of profiling over the magnetic-field realizations, we sort them by their $\mathcal{L}_{\text{ALP}}$, choosing the realization j associated with the 95% most likely fit, following from Refs. [2,60,79]. For $N_B = 100$ realizations, this corresponds to $j = 95$, or the quantile

⁵The WStat fit statistic implemented in *gammapy* follows $-2 \ln \mathcal{L}$.

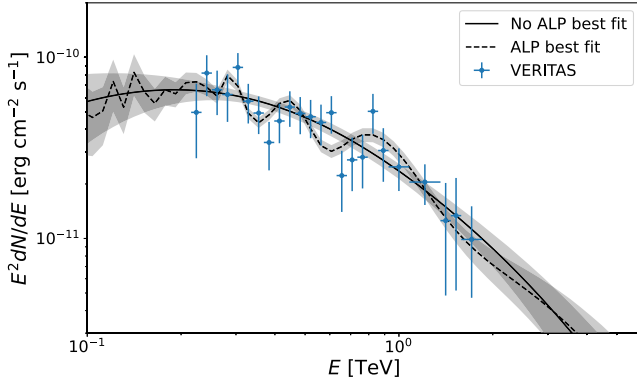


FIG. 1. SED for the January 2nd, 2017 observations of the NGC 1275 flare. The best-fit model of the joint likelihood without ALPs (with an ALP of $m_a = 1.61 \times 10^{-8}$ eV and $g_{a\gamma} = 3.16 \times 10^{-11}$ GeV $^{-1}$) is shown as a solid (dashed) black line for one realization of the ICMF.

$Q_B = 0.95$.⁶ The selected magnetic field realization can be denoted $\mathbf{B}_{95}$, taking particular care to note that $\mathbf{B}_{95}$ may be different for different ALP parameters, such that $\mathbf{B}_{95} \equiv \mathbf{B}_{95}(m_a, g_{a\gamma})$. The maximum likelihood for a fixed $(m_a, g_{a\gamma})$ will then be given by $\mathcal{L}_{\text{ALP}}(\mu(m_a, g_{a\gamma}, \mathbf{B}_{95}, \hat{\theta}), \hat{b}|\mathbf{D})$.

Since gammapy forward folds the model spectrum with the detector response to yield the predicted counts from the source, bin-by-bin correlations are self-consistently accounted for. As a result, it is possible to bin the VERITAS data below the level of the instrument's energy resolution—small enough to resolve ALP-photon spectral features—albeit, of course, with diminishing returns [60]. We have chosen 30 bins per decade ($\sim 15\%$ – 25% of the VERITAS energy resolution [93]) for this analysis, as a compromise between resolution and computational overhead. The SED for the flaring data is shown in Fig. 1, where it is fit by the null hypothesis (no-ALP) case modeled by Eq. (9). The overall best-fit model for the alternative hypothesis [Eq. (10)] is also shown.

IV. STATISTICAL METHODS

The goal of this section is to describe the process towards yielding statistics which can

- (i) Describe the likelihood of the existence of ALPs.
- (ii) Exclude regions of ALP parameter space.

In order to yield both of these results, it will be necessary to scan $(m_a, g_{a\gamma})$ space in the regime where we expect VERITAS to provide the greatest sensitivity. Following from Eq. (6), a reasonable choice is to cover $m_a \in [3, 700]$ neV and $g_{a\gamma} \in [0.1, 10] \times 10^{-11}$ GeV $^{-1}$. This range will allow $E_{\text{crit}}^{\text{low}}$ to fall in the energy range where VERITAS is most sensitive [93]. Note that this parameter space is consistent and complementary with previous studies done

with *Fermi* -LAT on this source [60,104]. We choose to scan a 14×9 logarithmically spaced grid in $(m_a, g_{a\gamma})$ space to perform this study, with the goal of balancing computational overhead with resolution in the results.

A. ALP likelihood

The first question we can address is whether our results indicate evidence for the existence of ALPs. We will seek to compare the likelihood of the most likely ALP case, i.e., the set of ALP parameters $(\hat{m}_a, \hat{g}_{a\gamma})$ which best represent the observed data, with the maximum likelihood of the no-ALP model (the null hypothesis).

This information is encapsulated in the test statistic (TS):

$$\text{TS} = -2 \ln \left(\frac{\mathcal{L}_0(\mu(\bar{\theta}), \bar{b}|\mathbf{D})}{\mathcal{L}_{\text{ALP}}(\mu(\hat{m}_a, \hat{g}_{a\gamma}, \hat{\mathbf{B}}_{95}, \hat{\theta}), \hat{b}|\mathbf{D})} \right), \quad (14)$$

a special case⁷ of the profile log-likelihood ratio test [99], which specifically measures the data's incompatibility with the null hypothesis as compared to the test hypothesis [2]. $\mathcal{L}_0(\mu(\bar{\theta}), \bar{b}|\mathbf{D})$ is the maximum likelihood for the no-ALP model, with profiled expected background $\bar{b}$ and parameters of best-fit $\bar{\theta}$, and $\mathcal{L}_{\text{ALP}}(\mu(\hat{m}_a, \hat{g}_{a\gamma}, \hat{\mathbf{B}}_{95}, \hat{\theta}), \hat{b}|\mathbf{D})$ is the maximum likelihood over the entire ALP grid (indicated by the additional hats). If we find the likelihood of the ALP case to be higher than that of the no-ALP case, then this would be a potential indicator of the existence of the ALP case.

However, it is not a trivial exercise to quantify the significance of this TS. Due to the nonlinearity in the relationship between the spectral oscillations and ALP parameters, as well as the degeneracy between $g_{a\gamma}$ and magnetic field strength in Eqs. (3) and (4), one can not simply convert TS value into a significance following the usual Wilks's theorem [105]. As such, the null distribution is *a priori* unknown, and must be derived from Monte Carlo (MC) simulations.

We perform MC simulations of the data assuming the no-ALP case, and examine the TS distribution to establish the confidence levels at which one could reject the no-ALP hypothesis. Simulations were generated starting from the best-fit null hypothesis model to the data. To do so, we convolve the null hypothesis model best fit to the observed data with the instrument response functions to make predictions for the number of counts. Then Poisson random numbers are generated, based on the expected number of events, to yield simulated counts. In a similar vein to the discussion in Sec. III, the lack of observed background counts in the highest-energy bins will bias the high-energy bins of simulations produced using the observed data. To account for this, we model the background counts above

⁶Note that this is the 0.05 quantile of $-2 \ln \mathcal{L}_{\text{ALP}}$.

⁷This is Eq. (15) for the case in which $(m_a, g_{a\gamma}) = \mathbf{0}$.

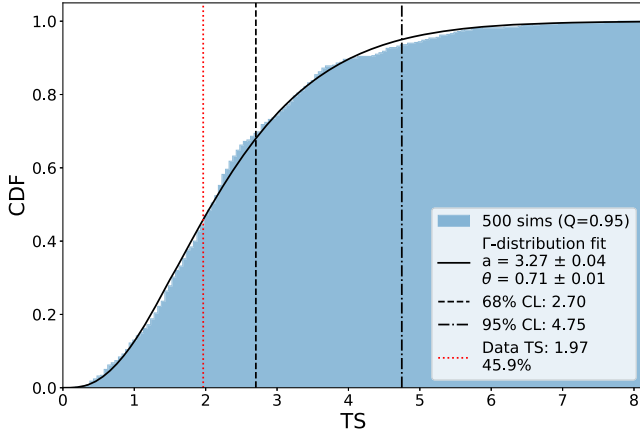


FIG. 2. Cumulative distribution function (CDF) for the TS values calculated from 500 simulations produced assuming the no-ALP case. The CDF is fitted with a modified Γ distribution. Black vertical lines show the 68% and 95% TS thresholds, and the red dotted line shows the TS value of the data, with its associated confidence level shown in the legend.

$E_{\text{thresh,bkg}} = 1$ TeV as a power law with index 3, chosen to closely match the background data at lower energies with higher statistics. The power law is normalized such that, integrated above $E_{\text{thresh,bkg}}$, it yields the total number of measured background counts above $E_{\text{thresh,bkg}}$ [106].⁸

A total of 500 simulations are generated, each of which is reanalyzed following the exact procedure described before, yielding a TS distribution from the fits to the simulations, shown in Fig. 2. Following from Abdalla *et al.* [2], we fit the TS distribution with a modified Γ distribution, from which we can extract the threshold values at 95% confidence. From the best fit modified Γ distribution, a $TS > 4.75$ is required to reject the no-ALP hypothesis with 95% confidence.

B. Exclusion regions of ALP parameter space

Regardless of whether any indication of ALPs is present, it is possible to exclude regions of the $(m_a, g_{a\gamma})$ parameter space which are inconsistent with the observations. This has historically been done with the generalized profile log-likelihood ratio test [99], see, e.g., Ref. [2]:

$$\lambda(m_a, g_{a\gamma}) = -2 \ln \left(\frac{\mathcal{L}_{\text{ALP}}(\mu(m_a, g_{a\gamma}), \mathbf{B}_{95}, \hat{\theta}), \hat{b}|\mathbf{D})}{\mathcal{L}_{\text{ALP}}(\mu(\hat{m}_a, \hat{g}_{a\gamma}), \hat{\mathbf{B}}_{95}, \hat{\theta}), \hat{b}|\mathbf{D})} \right), \quad (15)$$

which compares the best fit at each $(m_a, g_{a\gamma})$ point in the grid with the overall best fit among the test points. Larger values of λ will indicate that the best fit at the point under consideration is less representative of the data than the best fit overall.

⁸Note that this procedure is only applied to the data which is used to produce the simulations, not the simulations themselves.

A key component of the likelihood ratio test is that it is dependent on an unrestricted maximization (over the entire parameter space) [107]. In the presence of any systematic uncertainties (for example, in energy resolution, instrument response functions, modeling) that are unaccounted for in the maximum likelihood estimation, and especially in the generation of simulations, this unrestricted maximization may encounter (marginal) false positives, also known as Type I errors. Since the exclusion regions will be tied, by the denominator of Eq. (15), to the strongest signal found in the parameter space, such anomalous results would consequently result in artificially inflated exclusion confidence levels, which may extend exclusion regions to large parts of the unrestricted parameter space, including in regimes where the search ostensibly has no sensitivity [108]. In the case of a true (marginal) positive signal, this is the desired behavior; however, as the truth case is unknown, the possibility for this inflationary effect motivates a more conservative approach.

The so-called CL_s method is a well-established tool for searches in high-energy experiments, which normalizes the test statistic used for exclusion with respect to the null hypothesis, rather than the supremum of the space as in Eq. (15), and has an additional renormalization which reduces the risk of constraining regimes where a search has no sensitivity [17, 108, 109]. In light of these advantages, we opt to utilize the CL_s method instead. Recent articles pursuing this type of study have also adopted the CL_s method for the same reasons [110, 111].

To do so, we can generalize the test statistic in Eq. (14) to

$$Q(m_a, g_{a\gamma}) = -2 \ln \left(\frac{\mathcal{L}_0(\mu(\bar{\theta}), \bar{b}|\mathbf{D})}{\mathcal{L}_{\text{ALP}}(\mu(m_a, g_{a\gamma}), \mathbf{B}_{95}, \hat{\theta}), \hat{b}|\mathbf{D})} \right), \quad (16)$$

where again the numerator is the maximum likelihood under the null hypothesis, with profiled expected background $\bar{b}$ and parameters of best-fit $\bar{\theta}$. The denominator now, however, is no longer profiled over the unrestricted parameter space, and is rather the maximum likelihood under the alternative hypothesis for a given fixed $(m_a, g_{a\gamma})$. We can thus systematically scan the parameter space, finding a $Q_{\text{obs}}(m_a, g_{a\gamma})$ for the observed data at each test point, which can then be used to assess the strength of exclusion at that point.

For some example points $(m_a, g_{a\gamma})$, we seek to understand $Q_{\text{obs}}(m_a, g_{a\gamma})$ in the context of the underlying distribution of $Q(m_a, g_{a\gamma})$ in the cases where the null hypothesis is true and where the alternative hypothesis is true, respectively. From Sec. IV A, 500 simulations representing the null hypothesis truth case have already been produced. Each of these simulations was analyzed as described in Sec. III, the results from which were passed through Eq. (16), yielding a distribution $\{Q(m_a, g_{a\gamma})\}_0$ for each $(m_a, g_{a\gamma})$ pair tested that can again be fitted with a modified Γ distribution.

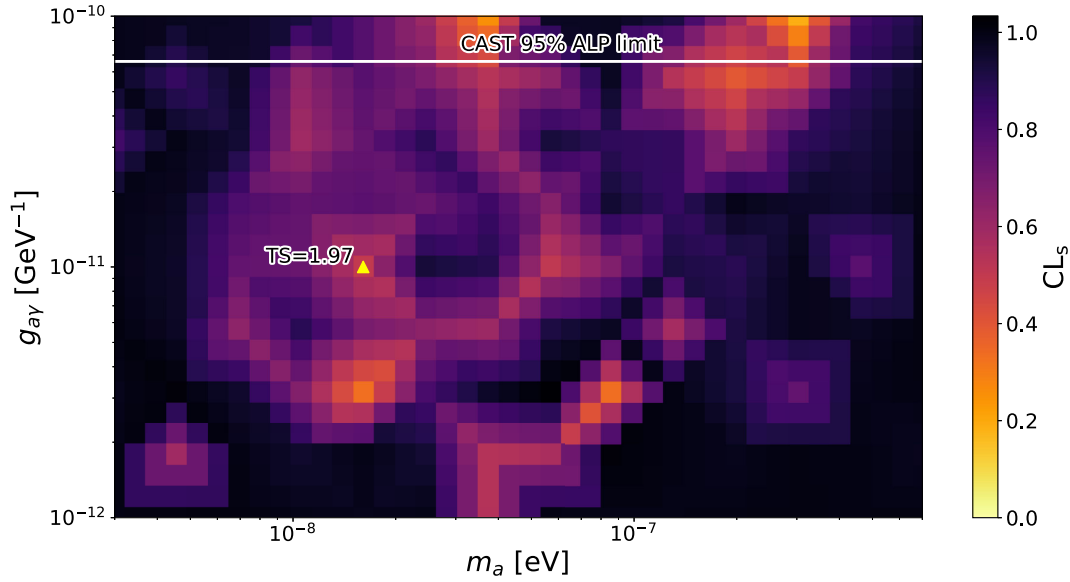


FIG. 3. Map of ALP parameter CL_s values. Points are excluded with confidence $1 - CL_s$. The point of highest VERITAS TS is indicated by a yellow triangle and annotated with its value. As a reference, 95% exclusions from CAST are above the white line. For visualization purposes, the sparse 14×9 grid has been upsampled by a factor of 3 and smoothed with linear interpolation.

Additionally, we now also simulate the injected ALP signal at each point in the parameter space to represent the alternative hypothesis truth case, with 500 simulations produced at each point. Following the same analysis, we find a distribution $\{Q(m_a, g_{a\gamma})\}_{ALP}$. Two example distributions of $\{Q(m_a, g_{a\gamma})\}_0$ and $\{Q(m_a, g_{a\gamma})\}_{ALP}$ along with the corresponding $Q_{obs}(m_a, g_{a\gamma})$ are shown in Appendix A.

The CL_s value is colloquially defined as $CL_s = p_\mu / (1 - p_b)$ [17], where μ and b refer to the signal and background-only models respectively. Applied in this context, with the alternative hypothesis H_{ALP} and null hypothesis H_0 , we define the CL_s value as

$$CL_s(m_a, g_{a\gamma}) = \frac{P_{ALP}}{1 - p_0} = \frac{P(\text{reject } H_{ALP} | H_{ALP} = \text{TRUE})}{P(\text{accept } H_0 | H_0 = \text{TRUE})}. \quad (17)$$

Thus, the CL_s value can be interpreted as a p value that has been normalized to the acceptance probability for H_0 . As a result, this means that the less separation that exists between the distributions $\{Q(m_a, g_{a\gamma})\}_{ALP}$ and $\{Q(m_a, g_{a\gamma})\}_0$ (i.e., the less sensitive we are in our search), the more severely penalized our CL_s value will be. This is seen in that p_{ALP} is the strict lower bound of the CL_s value. When the CL_s value is smaller than some number α , say for example $\alpha = 0.05$, the test point is considered incompatible with the observed data at the $1 - \alpha$ confidence level; in our example, this would correspond to an exclusion at 95% confidence. Both the numerator and denominator of Eq. (17) are computed simply as the cumulative distribution functions (CDFs) of the $\{Q(m_a, g_{a\gamma})\}_{ALP}$ and $\{Q(m_a, g_{a\gamma})\}_0$ distributions respectively, evaluated at $Q_{obs}(m_a, g_{a\gamma})$. Denoting the CDF

from the simulation with an injected ALP signal at $(m_a, g_{a\gamma})$ with $\mathcal{F}_{m_a, g_{a\gamma}}(Q(m_a, g_{a\gamma}))$ and without an ALP signal with $\mathcal{F}_0(Q(m_a, g_{a\gamma}))$, the CL_s therefore becomes

$$CL_s(m_a, g_{a\gamma}) = \frac{\mathcal{F}_{m_a, g_{a\gamma}}(Q_{obs}(m_a, g_{a\gamma}))}{\mathcal{F}_0(Q_{obs}(m_a, g_{a\gamma}))}. \quad (18)$$

V. RESULTS AND DISCUSSION

We find in our results no evidence for the existence of ALPs. Subsequently, we exclude portions of the ALP mass and coupling parameter space at up to the 80% confidence level.

The test statistic for the observed data was measured as $TS = 1.97$, found at the test point $(m_a = 1.61 \times 10^{-8} \text{ eV}, g_{a\gamma} = 3.16 \times 10^{-11} \text{ GeV}^{-1})$. This result is shown as the red dotted line which has been included in Fig. 2, which indicates a data TS which lies just below the 50% quantile of the modified Γ -distribution fit to the distribution. This confidence level can be converted to the one-tailed Gaussian equivalent standard deviation σ through the inverse of the complementary error function [17]: $\sigma = -\sqrt{2} \text{erfc}^{-1}(2 \cdot CL)$, corresponding to a rejection significance of the null hypothesis of -0.1σ .⁹ This indicates no evidence for a rejection of the null hypothesis in favor of the alternative hypothesis.

⁹The equivalent significance of the two-tailed Gaussian, which has been presented in other related works, is given by $\sigma = \sqrt{2} \text{erf}^{-1}(CL)$, where erf^{-1} is the inverse of the error function. It yields a two-tailed equivalent Gaussian significance of 0.61σ , which remains consistent with the conclusions drawn from the one-tailed Gaussian significance.

From the CL_s values, we can still examine the ability of our search to constrain the region of ALP parameter space tested. This is done as described in Sec. IV B, and is shown in Fig. 3. We note that Fig. 3 has been upsampled by a factor of 3 and smoothed with linear interpolation for visualization purposes. As can be seen, the region of the tested space excluded with the highest confidence is where $g_{a\gamma} \gtrsim 3 \times 10^{-11} \text{ GeV}^{-1}$ in the range $2 \times 10^{-7} \text{ eV} \lesssim m_a \lesssim 4 \times 10^{-7} \text{ eV}$, peaking at $\sim 80\%$ confidence exclusion. As our constraints do not anywhere reach the 95% confidence level, our results with the CL_s method are not considered constraining. Nonetheless, our results show no inconsistencies with exclusion regions found in previous studies (e.g., [65,79,112]).

As a cross-check, we also conducted the analysis using the profile likelihood method described at the start of Sec. IV B. This method yielded an exclusion map characterized by the same overall trend, with constraints not exceeding the 95% confidence level anywhere in the parameter space. However, all of the strongest constraints were found to be tighter in the profile likelihood case, emphasizing the less conservative nature of this approach (see Appendix B). Marked differences between the two methods do appear near the most likely ALP test point in the parameter space (corresponding to $\text{TS} = 1.97$), highlighting one of the key differences between the two approaches—the regime nearest to the most likely ALP point will always be minimally constrained in the profile likelihood approach, whereas the constraints set by the CL_s method will be insensitive to this feature.

Our results are considerably less constraining than those shown in the analysis of the same source, during the same flare, with MAGIC in Abe *et al.* [65]. We note as a potential interpretation for this discrepancy the differences in datasets, instruments, and methodology. MAGIC observed the target’s flaring state a day before VERITAS, recording a flux $\sim 3\times$ higher than that measured by VERITAS the following day, yielding a more robust signal-to-noise ratio. Additionally, postflare and quiescent state data were included in that work. We also note the presence of two points in the space examined by the MAGIC analysis, ($m_a = 1.00 \times 10^{-7} \text{ eV}, g_{a\gamma} = 2.71 \times 10^{-10} \text{ GeV}^{-1}$) and ($m_a = 2.15 \times 10^{-8} \text{ eV}, g_{a\gamma} = 3.81 \times 10^{-12} \text{ GeV}^{-1}$), which reject the null hypothesis at $\gtrsim 95\%$ confidence in favor of the alternative hypothesis. Both points lie in regimes that have been previously excluded at or above 95% confidence by other searches [31,113]. As the profile likelihood was utilized in this case, it is likely that the presence of these marginally significant points contributed to the depth and extent of their exclusion regions, as discussed in Sec. IV B.

In considering future applications for this analysis, the advent of the Cherenkov Telescope Array Observatory (CTAO) offers improved prospects for constraining the ALP parameter space. Notably, CTAO’s exceptional energy

resolution means an increased sensitivity to the predicted small-scale spectral irregularities induced by ALP oscillations in the presence of external magnetic fields. Further, the two-array design of CTAO will continue to provide the nearly full-sky VHE coverage offered by existing IACT arrays, meaning uninterrupted access to a plethora of potential sources residing in galaxy clusters that could be probed. A preliminary look at the CTAO capabilities was investigated on the same source, NGC 1275, in Abdalla *et al.* [2], which underscores the importance of using flaring data, as well as the value of improved energy resolution in next-generation instruments. The study also highlights the impact of systematic uncertainties in model assumptions and detector response. For the model assumptions, of particular focus were magnetic field strength, the index of magnetic field turbulence, the minimum turbulence scale, and the power law index η relating electron density with magnetic field strength. It shows that limits would be expected to weaken as any one of these parameters decreases (with the exception of η , which would yield weaker limits as it increases). When systematic uncertainties in the instrument responses were included, limits were also found to degrade. These findings apply to our study as well, and are therefore an important caveat to consider when appraising our results.

VI. CONCLUSION

Although they may not solve the strong CP problem like the axion, ALPs are still a compelling target for experimental searches for a number of reasons. Principal among these is that ALPs are theoretically well-motivated, emerging naturally from a number of string theories and other extensions to the SM. The large parameter space implied by the relaxation of the $m_a/g_{a\gamma}$ relationship in going from axions to ALPs allows for a diverse set of experimental approaches that may be employed to provide ALP constraints. In particular, x- and γ -ray observations of astrophysical objects, especially those embedded in strong magnetic fields over large spatial scales, have proven to be a useful tool in constraining the region of ALP parameter space for masses $m_a \lesssim 100 \text{ neV}$ (e.g., [20,58–60,65,75,79,114,115]). VERITAS, with its VHE observations of dramatic AGN flares, is well positioned to contribute in this regime in accordance with Eq. (6).

Motivated by these prospects, we have used the VERITAS spectrum of the radio galaxy NGC 1275 during a major flare to search for the imprint of ALPs. In particular, we exploit the turbulent magnetic field of the Perseus cluster, in which NGC 1275 is embedded, as the main mixing region for the ALP-photon beam. The large spatial scale of the cluster, along with its considerable $\mathcal{O}(10 \text{ } \mu\text{G})$ magnetic field strength, makes it a particularly promising target for this technique. In considering some of the pitfalls associated with the commonly used profile likelihood method, we adopted the CL_s method. This is a more conservative approach to determining the confidence of exclusion regions, and our

work is the first time it has been used to set constraints with the Perseus cluster. With this approach, our analysis yielded conservative constraints, peaking at an 80% confidence exclusion at $(m_a = 3.03 \times 10^{-7} \text{ eV}, g_{a\gamma} = 1.00 \times 10^{-10} \text{ GeV}^{-1})$, and did not contradict earlier measurements. In future studies of this type, we advocate for the continued adoption of the CL_s method considering the limitations of the profile likelihood approach.

The next-generation CTAO, with its improved energy resolution and extensive energy sensitivity, is expected to further the capabilities of IACTs to contribute meaningful constraints in these types of analyses, bringing this technique into a new era of γ -ray astronomy and astrophysics.

ACKNOWLEDGMENTS

This research is supported by grants from the U.S. Department of Energy Office of Science, the U.S. National Science Foundation and the Smithsonian Institution, by NSERC in Canada, and by the Helmholtz Association in Germany. This research used resources provided by the Open Science Grid, which is supported by the National Science Foundation and the U.S. Department of Energy's Office of Science, and resources of the National Energy Research Scientific Computing Center (NERSC), a U.S. Department of Energy Office of Science User Facility operated under Contract No. DE-AC02-05CH11231. We acknowledge the excellent work of the technical support staff at the Fred Lawrence Whipple Observatory and at the collaborating institutions in the construction and operation of the instrument. C. A. was supported by NSF Grant No. PHY-2110497. Q. F. acknowledges the support from NSF Grant No. PHY-2411860. M. M. acknowledges the support from the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy—EXC 2121 “Quantum Universe”—390833306 and from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation program Grant agreement No. 948689 (AxionDM).

DATA AVAILABILITY

The data that support the findings of this article are openly available [116].

APPENDIX A: TEST STATISTIC DISTRIBUTIONS FROM THE CL_s METHOD

To illustrate the performance of the CL_s method, Fig. 4 shows example distributions of the test statistic (Q) under both the null and ALP hypotheses at two test points ($m_a = 2 \times 10^{-7} \text{ eV}, g_{a\gamma} = 1 \times 10^{-11} \text{ GeV}^{-1}$) and ($m_a = 3 \times 10^{-7} \text{ eV}, g_{a\gamma} = 1 \times 10^{-12} \text{ GeV}^{-1}$). At each test point, including the test points not shown in Fig. 4, a test statistic distribution for the null hypothesis ($\{Q\}_0$) is calculated with the 500 null

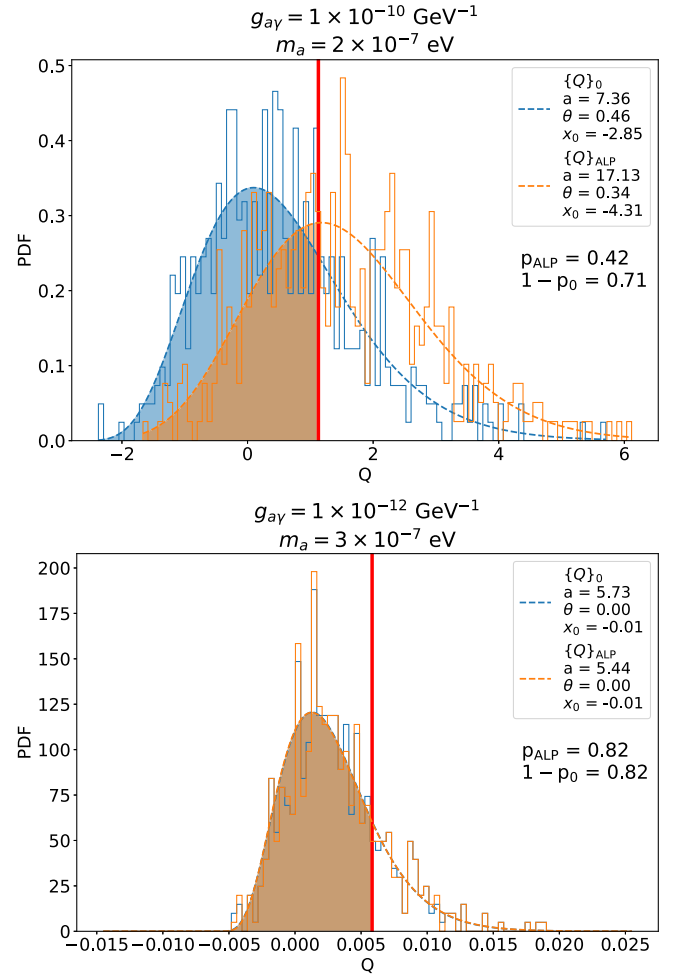


FIG. 4. The distribution of the test statistic (Q) values calculated using Eq. (16) at two test points ($m_a = 2 \times 10^{-7} \text{ eV}, g_{a\gamma} = 1 \times 10^{-10} \text{ GeV}^{-1}$) and ($m_a = 3 \times 10^{-7} \text{ eV}, g_{a\gamma} = 1 \times 10^{-12} \text{ GeV}^{-1}$), respectively. The blue distributions are for the null hypothesis, and the orange distributions are for the alternative ALP hypothesis. The red vertical line is the test statistic Q calculated from the data. a , θ , and x_0 are the shape, scale, and location parameters resulting from a fit to a gamma function.

simulations as described in Sec. IV A (shown in blue in Fig. 4). Similarly, a test statistic distribution for the alternative hypothesis ($\{Q\}_{\text{ALP}}$) is calculated using the 500 simulations with the injected ALP signal at each test point as described in Sec. IV B (shown in orange in Fig. 4).

When an instrument lacks sensitivity to an ALP signature in a certain region of the mass and coupling strength parameter space (e.g., Fig. 4 bottom panel; ($m_a = 3 \times 10^{-7} \text{ eV}, g_{a\gamma} = 1 \times 10^{-12} \text{ GeV}^{-1}$)), the null and alternative distributions are similar, leading to a CL_s close to 1 [see Eq. (17)]. A CL_s value of nearly 1 indicates low confidence in excluding the ALP hypothesis with the given parameters. Hence, the CL_s method does not exclude parameter space where the instrument has low sensitivity to the signal being searched for.

At the other test point [Fig. 4 top panel; ($m_a = 2 \times 10^{-7}$ eV, $g_{a\gamma} = 1 \times 10^{-11}$ GeV $^{-1}$)], the null and alternative distributions are clearly different, translating into a relatively low CL_s value [$p_{\text{ALP}}/(1 - p_0) \approx 0.6$] and some confidence to exclude the specific mass and coupling strength values.

APPENDIX B: EXCLUSION REGIONS USING THE PROFILE LIKELIHOOD METHOD

Although the CL_s method is chosen as the analysis method of this work, we include in this appendix the exclusion regions obtained using the profile likelihood method described in Sec. IV B. We include in this appendix the exclusion regions obtained using the profile likelihood method described in Sec. IV B, as shown in Fig. 5.

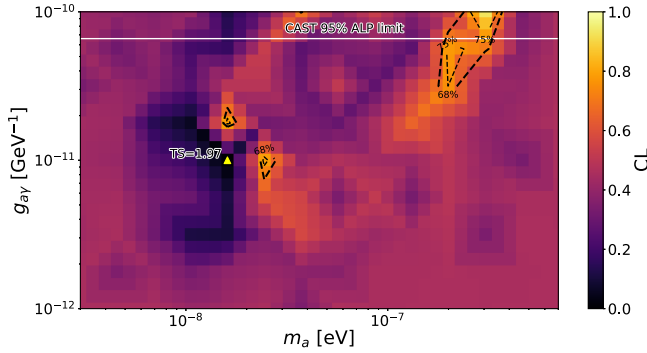


FIG. 5. The map of exclusion confidence levels derived from the λ values calculated using Eq. (15), covering the same ALP parameter space as Fig. 3. The dashed black contours indicate the regions excluded at 68% and 75%. The point of highest VERITAS TS is indicated by a yellow triangle and annotated with its value.

The confidence level of excluding a given combination of the ALP mass and coupling strength is derived from the profile likelihood λ , as defined in Eq. (15).

The region excluded at 68% and 75% confidence levels using the profile likelihood method is consistent with similar previous studies. The exclusion region also matches the expectation for where VERITAS would be most sensitive to ALP/photon oscillation effects. However, the confidence of the exclusion is much higher compared to the CL_s method, since the exclusion confidence is derived relative to the overall best fit ($\hat{m}_a, \hat{g}_{a\gamma}$) among the test points.

APPENDIX C: MAP OF ALP PARAMETER CL_s VALUES WITHOUT UPSAMPLING

Figure 3 is shown with an upsampling by a factor of 3 with linear interpolation for visualization purposes. For completeness, we show the raw map of ALP parameter CL_s values without any smoothing in Fig. 6.

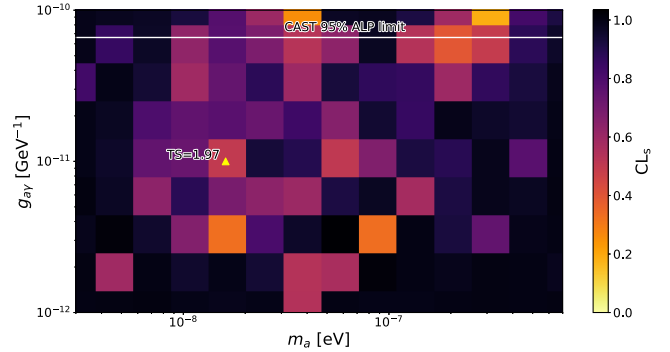


FIG. 6. Map of ALP parameter CL_s values, similar to Fig. 3, but without any upsampling/smoothing.

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