

Where multipartite entanglement localizes: The junction law for genuine multientropy

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We uncover a “junction law” for genuine multipartite entanglement, suggesting that in gapped local systems multipartite entanglement is controlled and effectively localized near junctions where subsystem boundaries meet. Using the Rényi-2 genuine multientropy $\text{GM}_2^{(\alpha)}$ as a diagnostic of genuine α -partite entanglement, we establish this behavior in $(2+1)$ -dimensional gapped free-fermion lattices with correlation length ξ . For partitions with a single junction, $\text{GM}_2^{(\alpha)}$ exhibits a universal scaling crossover in L/ξ , growing for $L \lesssim \xi$ and saturating to a ξ -dependent constant for $L \gg \xi$, up to $\mathcal{O}(e^{-L/\xi})$ corrections. In sharp contrast, for partitions without a junction, $\text{GM}_2^{(\alpha)}$ is exponentially suppressed in L/ξ and drops below numerical resolution once $L \gg \xi$. We observe the same pattern for $\alpha = 3$ (tripartite) and $\alpha = 4$ (quadripartite) cases, and further corroborate this localization by translating the junction at fixed system size. We also provide a geometric explanation of the junction law in holography. Altogether, these results show that in this gapped free-fermion setting genuine multipartite entanglement is localized within a correlation length neighborhood of junctions.

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I. INTRODUCTION

Locality strongly constrains entanglement in quantum many-body systems. For gapped states, bipartite entanglement entropy obeys the famous area law [1–4], reflecting the fact that entanglement is generated near entangling surfaces. While genuine multipartite entanglement is expected to play a fundamental role in quantum many-body physics, quantum information theory, and holography, its spatial structure remains much less understood. Despite the profound role of locality in physics, extracting its implications for genuine multipartite entanglement has remained difficult, in part because systematic and computable diagnostics for multipartite entanglement are still limited. Consequently, determining whether an analogous organizing principle exists for *genuine multipartite entanglement* remains a fundamental open question [5–8].

In this work, we answer this question affirmatively by identifying the structure of a *junction law* for genuine multientropy [9,10]. Genuine multientropy $\text{GM}^{(\alpha)}$ is a *new* and sharp diagnostic of genuine α -partite correlations composed of multientropy [11–14]. It isolates *irreducible*

α -partite entanglement by subtracting all lower-partite contributions, and vanishes for states whose correlations can be decomposed entirely into contributions involving fewer than α subsystems. Using $\text{GM}^{(\alpha)}$, we show that, in the mass-gapped systems studied here, genuine multipartite entanglement is effectively localized near *junctions* where subsystem boundaries meet. We demonstrate this behavior explicitly in a solvable gapped free-fermion lattice model. A particularly nontrivial aspect is that our framework enables direct tests of higher-partite entanglement beyond the tripartite case. Although connected two-point correlations decay beyond the correlation length ξ in gapped phases, it is not *a priori* clear how *irreducible* multipartite correlations organize in space. The nontrivial question addressed here is not merely whether correlations are short-ranged, but whether the genuinely multipartite component that remains after subtracting all lower-partite contributions is controlled by junctions.

Concretely, we study the Rényi-2 genuine multientropy $\text{GM}_2^{(\alpha)}$ in $(2+1)$ -dimensional free-fermion lattice models. For tripartitions ($\alpha = 3$) and quadripartitions ($\alpha = 4$) containing a single junction, we find a universal crossover controlled by the ratio L/ξ , where L is the system size and ξ is the correlation length. For $L \lesssim \xi$, $\text{GM}_2^{(\alpha)}$ grows with L , while for $L \gg \xi$ it saturates to a constant determined by ξ and the local opening angles at the junction. Strikingly, when the partition geometry contains no junction, $\text{GM}_2^{(\alpha)}$ is

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exponentially suppressed and approaches zero in the limit $L \gg \xi$.

These results identify junctions as the spatial locus of genuine multipartite entanglement and suggest a multipartite analog of the area law structure of entanglement entropy.

Before we proceed, let us clarify why a junction is special. In a local state with finite correlation length ξ , connected correlations decay as $\langle O_X O_Y \rangle_c \sim e^{-\text{dist}(X,Y)/\xi}$, and well-separated regions approximately factorize up to $O(e^{-d/\xi})$ corrections. For a $\mathfrak{q}$ -partition with typical subsystem size L , any *irreducible* $\mathfrak{q}$ -partite correlation must couple all $\mathfrak{q}$ subsystems. If the boundaries do not meet, their minimal separation is $O(L)$, so such couplings are suppressed as $O(e^{-L/\xi})$, implying $\text{GM}_n^{(\mathfrak{q})} \rightarrow 0$ for $L \gg \xi$. A junction is, therefore, the natural geometry where all $\mathfrak{q}$ subsystems approach within $O(\xi)$, allowing an $O(1)$ contribution.

However, this reasoning should be understood as a locality-based intuition, not as a consequence of the ordinary bipartite area law. The decay of two-point functions, or of bipartite correlations, does not by itself determine how the irreducible component extracted by $\text{GM}^{(\mathfrak{q})}$ organizes spatially after all lower-partite contributions are subtracted. Our analysis provides a direct test of this expectation in a computable setting and shows that the surviving genuine multipartite contribution is localized at junctions.

II. SETUP

We use the Rényi- n genuine multientropy $\text{GM}_n^{(\mathfrak{q})}$ introduced in Refs. [9,10], which isolates irreducible $\mathfrak{q}$ -partite correlations by subtracting all lower-partite contributions. They are written in terms of multientropies as

$$\text{GM}_n^{(3)}(A:B:C) = S_n^{(3)}(A:B:C) - \frac{1}{2}(S_n^{(2)}(AB:C) + \dots), \quad (1)$$

$$\begin{aligned} \text{GM}_n^{(4)}(A:B:C:D) &= S_n^{(4)}(A:B:C:D) - \frac{1}{3}(S_n^{(3)}(AB:C:D) + \dots) \\ &\quad + \frac{1}{3}(S_n^{(2)}(ABC:D) + \dots) - aI_{3,n}(A:B:C:D), \end{aligned} \quad (2)$$

where $S_n^{(\mathfrak{q})}$ is the n th Rényi multientropy for the $\mathfrak{q}$ -partite system [11–14], $I_{3,n}$ is the n th Rényi triple mutual information, and $\dots$ represents symmetrization for subsystems. More explicit expressions for $\text{GM}_n^{(\mathfrak{q})}$, $S_n^{(\mathfrak{q})}$, and $I_{3,n}$ are given in Supplemental Material A [15]. Throughout this work we focus on the Rényi index $n = 2$.

We consider a $(2+1)$ -dimensional square lattice of spinless fermions with open boundary conditions, governed by a nearest-neighbor hopping Hamiltonian with a staggered mass term,

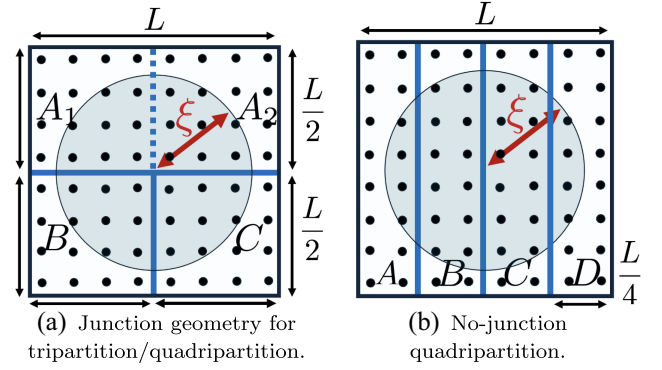


FIG. 1. Junction vs no-junction geometries. (a) Junction: subsystem boundaries meet at a single point—the shaded disk indicates the correlation length scale ξ . (b) No-junction: four-strip partition A, B, C, D with no point adjacent to all subsystems.

$$H = -\sum_{\langle ij \rangle} c_i^\dagger c_j + m \sum_i (-1)^{x_i+y_i} c_i^\dagger c_i, \quad (3)$$

where $\langle i, j \rangle$ denotes a pair of nearest-neighbor sites, and $x_i, y_i \in \{1, 2, \dots, L\}$ are the x and y coordinates of site i , respectively. This Hamiltonian is gapped for $m \neq 0$. We assume that $m \geq 0$. We study the many-body ground state in the half-filled sector [the Slater determinant obtained by filling all negative-energy levels, corresponding to the lowest $L^2/2$ and $(L^2 - 1)/2$ single-particle levels for even and odd L , respectively] and controlled low-lying Slater determinant excitations obtained by applying hole creation operators. All of these states remain Gaussian and hence amenable to the correlation matrix method.

As illustrated in Fig. 1(a), we partition the system into subsystems whose linear sizes scale with the system size L [16]. For the tripartite case ($\mathfrak{q} = 3$), the system is divided into three subsystems A, B , and C , where the region A consists of two disconnected components, $A \equiv A_1 \cup A_2$, separated by the other subsystems. For the quadrupartite case ($\mathfrak{q} = 4$), the system is divided into four subsystems, $A \equiv A_1, B, C$, and $D \equiv A_2$, each occupying one quadrant of the lattice.

In both cases, all subsystem boundaries intersect at a single point, which we refer to as a *junction* [17]. Our central focus is on such junction geometries. For comparison, we also study partitions in which subsystem boundaries do not meet at a common point [no-junction geometries shown in Fig. 1(b)].

We define the correlation length ξ from the exponential decay of equal-time single-particle correlators in the corresponding Gaussian state. To reduce lattice anisotropy effects and the sensitivity to the precise junction location, we extract ξ from a shell-averaged correlator with the source placed near the junction,

$$C(r) \equiv \frac{1}{\mathcal{N}_r} \sum_{\substack{j \in \mathcal{J} \\ i: ||i-j||-r \leq \Delta r/2}} |\langle c_i c_j^\dagger \rangle| \sim \frac{e^{-r/\xi}}{r^\alpha}, \quad (4)$$

where the sum averages over j in a small neighborhood $\mathcal{J}$ around the junction and over all sites i at distance r from j within a shell of thickness Δr , with $\mathcal{N}_r$ the number of pairs included, and α accounting for the power-law correction. We take the absolute value to suppress sign oscillations. We then fit $\log C(r)$ over an intermediate-distance window away from the boundaries (for details, see Supplemental Material B [15]). For the half-filled ground state, the extracted ξ is consistent with the continuum estimate $\xi \propto 1/|m|$ as $m \rightarrow 0$. For excited Slater determinant states we use the fitted ξ from (4).

The observable of interest is the Rényi-2 genuine multi-entropy $\text{GM}_2^{(\alpha)}$ for $\alpha = 3, 4$. To probe a wide range of correlation lengths ξ , we vary the mass parameter m and also consider controlled low-lying Slater determinant excitations, which remain Gaussian, but can effectively change the correlation length ξ .

Although Gaussianity allows us to use the correlation matrix method, evaluating $\alpha = 4$ (and higher- α) multi-entropies involves many multipartitions and becomes computationally heavy if implemented naively for a large number of spins. To streamline the computation, we use a new recursion relation that expresses a α -partite Rényi-2 multi-entropy in terms of a $(\alpha - 1)$ -partite one via canonical purification. In Supplemental Material [15], we summarize the recursion in Supplemental Material C and the resulting algorithm in Supplemental Material E, which generalizes the $\alpha = 3$ techniques of [18] to arbitrary α . Supplemental Material D provides supplementary definitions of the entanglement measures that enter the recursion, and Supplemental Material F describes the explicit construction of the Gaussian states (half-filled ground state and low-lying excitations) used in our numerics.

III. JUNCTION LAW

A. Partitions with a junction

We first focus on the tripartite case ($\alpha = 3$). Figure 2 summarizes our main results for partitions containing a single junction.

In the top panel of Fig. 2, we plot the genuine multi-entropy normalized by its large- L/ξ saturation value, $\text{GM}_2^{(3)}/\text{GM}_{2,\infty}^{(3)}$, as a function of L/ξ for the half-filled ground state at several mass parameters m . The data collapse onto a single curve and approach unity for $L/\xi \gg 1$, showing a universal saturation controlled by the correlation length ξ .

The bottom panel of Fig. 2 shows the same analysis for representative low-lying Slater determinant excitations. While the crossover regime ($L/\xi \lesssim 1$) displays nonuniversal deviations, the large- L/ξ saturation remains unchanged, indicating that short-distance details affect only the crossover.

We now turn to the quadripartite case ($\alpha = 4$), which provides a more stringent test of the junction law. To reach system sizes up to $L \sim 50$, we go beyond a direct

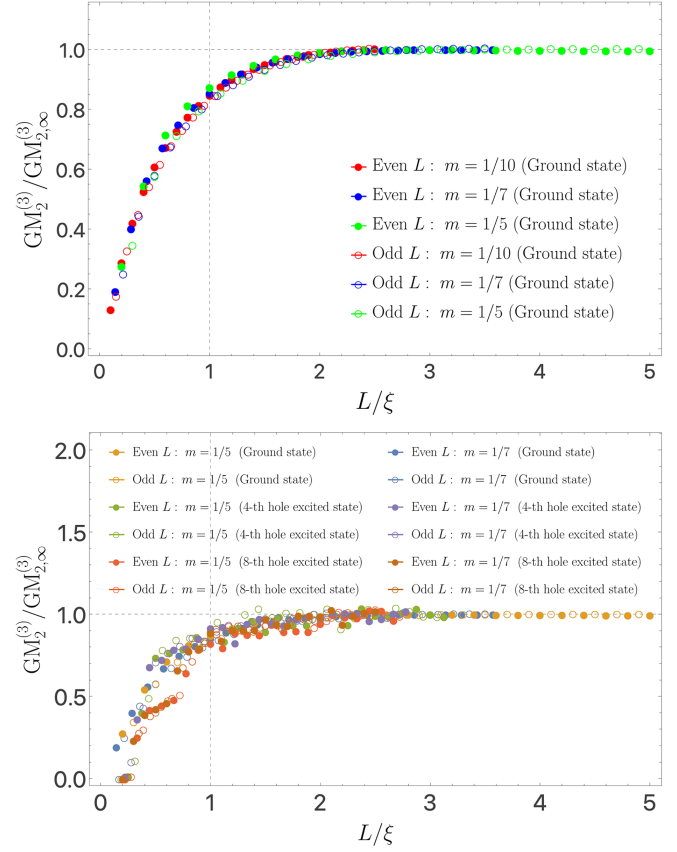


FIG. 2. Junction present: tripartite case ($\alpha = 3$). Scaling collapse of the normalized genuine multi-entropy $\text{GM}_2^{(3)}/\text{GM}_{2,\infty}^{(3)}$ versus L/ξ for single-junction partitions at several masses m . Top: half-filled ground states. Bottom: representative low-lying excited states. While excited states exhibit nonuniversal deviations in the crossover regime ($L/\xi \lesssim 1$), all data approach the same universal saturation for $L/\xi \gg 1$.

correlation matrix implementation and use our recursion algorithm based on canonical purification, which reduces the required $\alpha = 4$ quantities to tractable $\alpha = 3$ ones. Irreducible four-party correlations can be probed by two independent diagnostics, $\text{GM}_2^{(4)}|_{a=1/3}$ and $I_{3,n=2}$ [10,19]. In Fig. 3 we show that both $\text{GM}_2^{(4)}|_{a=1/3}$ and $I_{3,n=2}$ collapse as functions of L/ξ across multiple masses and saturate for $L/\xi \gg 1$. We note that $I_{3,n=2}$ can be negative in the crossover regime (and even at small L/ξ), but its sign does not affect the large- L/ξ saturation. The agreement of these independent measures provides a stringent, basis-independent confirmation that the junction law persists beyond $\alpha = 3$ and that genuine quadripartite entanglement is localized within an $\mathcal{O}(\xi)$ neighborhood of the junction.

B. Partitions without a junction

The contrast is shown in Fig. 4. For no-junction geometries, genuine four-partite correlations are exponentially suppressed in the separation scale: both independent

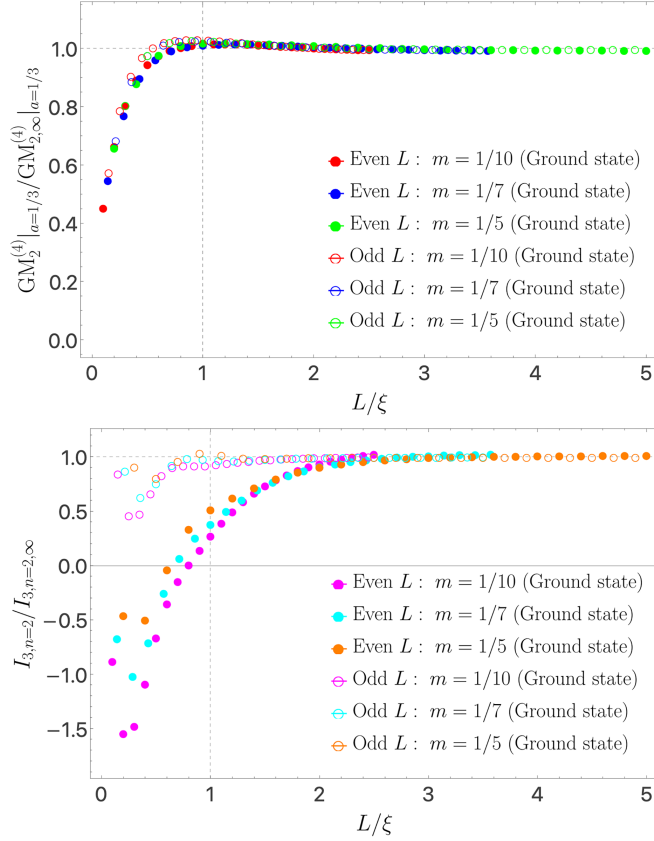


FIG. 3. Junction present: quadripartite case ($q = 4$). Scaling collapse of the two independent Rényi-2 four-partite diagnostics for single-junction quadripartitions in the ground state. Top: $GM_2^{(4)}|_{a=1/3}/GM_{2,\infty}^{(4)}|_{a=1/3}$. Bottom: $I_{3,n=2}/I_{3,n=2,\infty}$. In both panels, data for different m collapse versus L/ξ and saturate at large L/ξ . The bottom panel exhibits strong short- L/ξ nonuniversal deviations, including negative values.

diagnostics, $GM_2^{(4)}|_{a=1/3}$ and $I_{3,n=2}$, decrease rapidly with L/ξ and are consistent with

$$GM_2^{(4)}, \quad I_{3,n=2} \sim \mathcal{O}(e^{-cL/\xi}) \quad (L \gg \xi), \quad (5)$$

with $c = \mathcal{O}(1)$, a geometry-dependent constant, dropping below our numerical resolution once $L \gtrsim \xi$. The agreement of both diagnostics confirms that the suppression is geometric, dictated by the gapped locality. Note that for $L \lesssim \xi$, all subsystems coexist within an $\mathcal{O}(\xi)$ neighborhood, so the partition is effectively junctionlike on the scale ξ and the diagnostics need not be small.

C. Moving junction configuration

As an additional spatial test of the junction law, we shift the internal $B|C$ interface [the vertical cut that extends downward from the junction in Fig. 1(a)] while keeping L fixed. Figure 5 shows that when the junction is farther than ξ from the boundaries by more than ξ , $GM_2^{(3)}$ forms a constant plateau. As the junction approaches the boundary

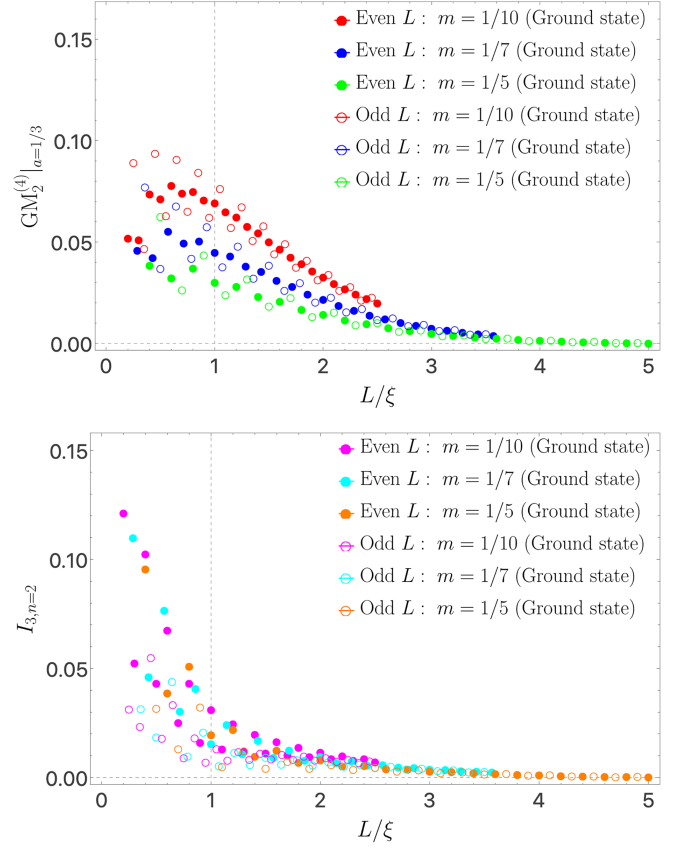


FIG. 4. No junction: genuine multipartite entanglement is exponentially suppressed. Rényi-2 quadripartite diagnostics $GM_2^{(4)}|_{a=1/3}$ and $I_{3,n=2}$ for the junction-free partition in Fig. 1(b). Both decay rapidly with L/ξ and are strongly suppressed for $L \gg \xi$.

within a distance $\sim \xi$, $GM_2^{(3)}$ is suppressed. This confirms that irreducible multipartite correlations are localized within an $\mathcal{O}(\xi)$ neighborhood of the junction.

IV. HOLOGRAPHIC PICTURE

Although our main results are purely nonholographic, a closely parallel statement holds in holography for genuine multientropy, assuming that holographic multientropy is computed by the minimal multiway-cut prescription [11]. Consider a gapped AdS/CFT setup with boundary correlation length ξ . In holographic models, a mass gap is geometrically realized by a bulk spacetime that smoothly caps off in the IR, $0 \leq z \lesssim \xi$, [20,21]. Accordingly, Ryu–Takayanagi (RT) surfaces cannot extend beyond the IR cap, and may end on it when the boundary region is sufficiently large [22–25]. As illustrated in Fig. 6, consider a partition of the boundary *without junctions*. If at least one boundary subsystem has linear size $\ell \lesssim \xi$, the bulk admits a nontrivial multiway cut with a “Mercedes-Benz” Y-junction, and the genuine multientropy is positive [9]. Note that for $\ell \lesssim \xi$, the partition behaves as an *effective* junction configuration on the scale ξ , where the relevant subsystems coexist within

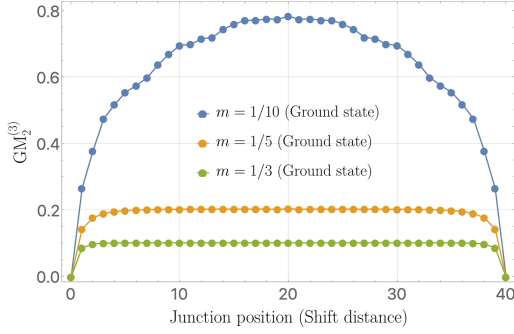


FIG. 5. Moving junction configuration. $GM_2^{(3)}$ of the half-filled ground state versus the junction position, i.e., the shift of the internal $B|C$ cut in Fig. 1(a), at fixed $L = 40$. Colors label m . A plateau appears when the junction is farther than $\sim \xi (\simeq 2/m)$ from the boundaries, and $GM_2^{(3)}$ is suppressed within $\sim \xi$ of a boundary.

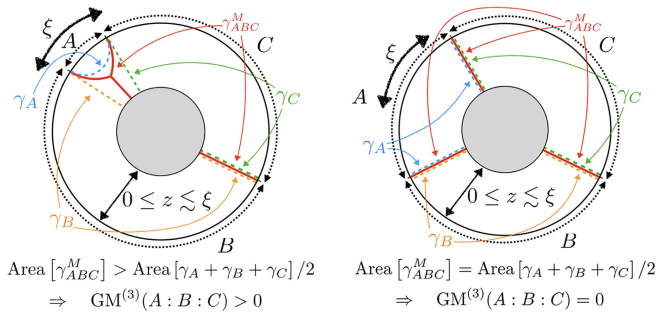


FIG. 6. Holographic setup with a mass gap. The gray shaded region is excluded, corresponding to the presence of the mass gap. γ_{ABC}^M denotes the multiway cut, and γ_α ($\alpha = A, B, C$) denotes the RT surface for a boundary subsystem α . Left: $|A| \lesssim \xi$. This is an *effective* junction configuration on the scale ξ , so a Y-junction forms in the bulk, resulting in $GM > 0$. Right: $|A| \gg \xi$ case. The RT surfaces match with the multiway cut, ending on the IR cap without forming a Y-junction, resulting in $GM \rightarrow 0$.

a neighborhood $\mathcal{O}(\xi)$. In contrast, if all subsystems satisfy $\ell \gg \xi$, the would-be Y-junction would lie behind the IR cap and cannot be realized as an extremal multiway cut, thus the minimal multiway cut becomes junction-free and coincides with the union of RT surfaces. In this case, the genuine multientropy vanishes. This provides a geometric explanation of the junction law.

V. DISCUSSION

Our results suggest a simple organizing principle for genuine multipartite entanglement in gapped local systems, in particular, short range-entangled gapped systems: irreducible q -partite correlations are supported only within an $\mathcal{O}(\xi)$ neighborhood of junctions where subsystem boundaries meet (or effectively coexist on the scale ξ), and are strongly suppressed otherwise. In particular, Fig. 5 strongly suggests the localization of genuine multipartite entanglement around the junction. This junction law suggests a

multipartite analog of the entanglement area law, identifying not only the scaling but also the spatial locus of higher-partite correlations. Importantly, while the correlation length ξ is conventionally defined through *two-point* or *bipartite* correlations, there is no general organizing principle establishing that the spatial scale of irreducible multipartite entanglement must also be controlled by ξ . Our results provide direct evidence for such a structure in the short range-entangled systems studied here.

The locality picture provides a natural intuition for the junction law, but it should not be regarded as a derivation from the ordinary bipartite area law or from two-point clustering alone. Since connected correlations decay as $e^{-\text{dist}/\xi}$ in a gapped phase, one may expect an irreducible q -partite contribution to be strongly suppressed unless all q subsystems approach within $\mathcal{O}(\xi)$ of a common region. This intuition is not tied to Gaussianity or free-fermion structure and, therefore, suggests that a similar junction-dominated structure may persist beyond the free-fermion models studied here, in interacting short range-entangled gapped phases.

Beyond gapped local phases, irreducible q -partite contributions may delocalize—e.g., in gapless systems (including CFTs), topologically ordered phases, or with long-range interactions. In such systems, additional nonlocal multipartite contributions associated with long-range entanglement or topology may coexist with, or dominate over, the local junction term. This contrast endows the junction law with diagnostic power: the presence or absence of junction-dominated genuine multientropy may itself distinguish short range-entangled gapped phases from long range-entangled ones, in the same spirit that the area law and topological entanglement entropy diagnose distinct phases. Clarifying the minimal conditions for a junction-dominated term is an important direction.

Furthermore, one should distinguish the junction law from the familiar corner contributions to bipartite entanglement entropy [26,27]. In bipartite entanglement entropy, corner terms arise as geometric corrections associated with singularities of the entangling surface. By construction, however, genuine multientropy isolates irreducible q -partite correlations after subtracting all lower-partite contributions, so it characterizes entanglement at a level distinct from these bipartite corner effects.

That said, it is natural to expect that genuine multientropy possesses its own characteristic dependence on junction geometry. In particular, changing the local corner/junction structure may modify the resulting irreducible multipartite contribution. In the present work, we focused only on the behavior of the junction law and restricted attention to a single junction geometry. Our framework, however, can be directly extended to study the detailed dependence of $GM^{(q)}$ on junction geometry while varying L . Clarifying this junction geometry dependence is an important future direction toward a deeper understanding of the junction law.

A key next step is to test the junction law beyond Gaussian/free-fermion settings: (i) interacting gapped lattices (tensor network or Monte Carlo-accessible), (ii) bosonic systems and alternative UV regularizations, and (iii) excited/finite energy-density states and higher dimensions, including holographic realizations.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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