

Structure of divergences in the higher-derivative supersymmetric 6D gauge theory

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Using the harmonic superspace approach, we perform a comprehensive study of the structure of divergences in the higher-derivative 6D, $\mathcal{N} = (1, 0)$ supersymmetric Yang-Mills theory coupled to the hypermultiplet in the adjoint representation. The effective action is constructed in the framework of the superfield background field method with the help of the $\mathcal{N} = (1, 0)$ supersymmetric higher-derivative regularization scheme which preserves all symmetries of the theory. The one-loop divergences are calculated in a manifestly gauge invariant and 6D, $\mathcal{N} = (1, 0)$ supersymmetric form, hopefully admitting a generalization to higher loops. The β function in the one-loop approximation is found and analyzed. In particular, it is shown that the one-loop β function for an arbitrary regulator function is specified by integrals of double total derivatives in momentum space, like it happens in 4D, $\mathcal{N} = 1$ superfield gauge theories. This points to the potential possibility to derive the *all-loop* Novikov-Shifman-Vainshtein-Zakharov-like exact β function in the considered theory.

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I. INTRODUCTION

The higher-derivative field theories attract an increasing interest. In spite of the well-known problems with ghosts, they possess a number of remarkable properties in the quantum domain, which suggests treating them as useful effective models. There exists a huge literature on the quantum structure of higher-derivative theories, mainly in the context of quantum gravity (see, e.g., a review [1] and references therein, as well as recent papers [2,3]).

It is well known that the supersymmetric theories in four dimensions have a better ultraviolet behavior compared to their nonsupersymmetric counterparts due to the non-renormalization theorems (see, e.g., [4–6]). This entails

remarkable consequences for the structure of supersymmetric quantum field theories. For instance, because of the absence of divergent quantum corrections to the superpotential [7], it becomes possible to relate the renormalization of masses and Yukawa couplings to the renormalization of the chiral matter superfields. As a result, the mass anomalous dimension and the Yukawa β -function can be expressed in terms of the anomalous dimension of the matter superfields. It turns out that the gauge β -function can also be related to the anomalous dimension of the chiral matter superfields by the exact Novikov-Shifman-Vainshtein-Zakharov (NSVZ) equation [8–11]. In the particular case of pure 4D, $\mathcal{N} = 1$ supersymmetric Yang-Mills (SYM) theory, the NSVZ approach yields the all-order exact expression for the β function in the form of a geometric series. It is natural to expect that supersymmetry in higher-derivative theories is capable to further soften the ultraviolet behavior as compared to the conventional bosonic higher-derivative theories.

Supersymmetric theories in higher dimensions are under an intensive study at present (see, e.g., [12–19]), mainly due to their numerous relations to string and brane stuff. The conventional higher-dimensional Yang-Mills theories are not renormalizable by power counting because of the

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dimensionful coupling constant.¹ However, it is possible to construct UV complete supersymmetric theories by introducing higher derivatives in the Lagrangian. Depending on the degree of derivatives, this trick allows one to gain the higher-derivative renormalizable or even superrenormalizable theories. For instance, $6D$, $\mathcal{N} = (1, 0)$ SYM theory with higher derivatives constructed in [21] is renormalizable by power counting. The one-loop divergences for this theory have been considered² in [21,24] by making use of a nonmanifestly supersymmetric component formulation and the regularization by dimensional reduction [25]. The manifestly $\mathcal{N} = (1, 0)$ supersymmetric form of the action can be achieved in the framework of harmonic superspace. Originally, the latter has been developed for the $4D$ case; see [26–31]. A generalization to the case of $6D$ supersymmetric theories was given in [32–36]. The relevant techniques allow one to explicitly calculate supersymmetric quantum corrections, provided that the regularization preserves supersymmetry at each order of the loop expansion. For the higher-derivative $\mathcal{N} = (1, 0)$ theory formulated in the harmonic superspace, the one-loop divergences were found in [37,38], applying the regularization by dimensional reduction.³ Although this regularization is efficient enough and is very often used in the concrete calculations (see, e.g., [47,48], and references therein), it can violate supersymmetry in higher loops. It seems useful to reconsider the previously derived results on the structure of the effective action in the framework of a regularization which would guarantee unbroken supersymmetry. At present, the only regularization of supersymmetric gauge theories which meets such a demand is the one through higher covariant derivatives (see, e.g., [49] and references therein) originally proposed by Slavnov [50–52]. For $4D$ supersymmetric theories, this regularization can be formulated both in the $\mathcal{N} = 1$ superfield language [53,54] and using the harmonic superspace [55].

When applied to $4D$ supersymmetric gauge theories, the higher covariant derivative regularization makes it possible to express the β function through integrals of double total derivatives in the momentum space [56–58]. This property leads to the appearance of the NSVZ expression for the renormalization-group functions (RGFs) defined in terms of the bare couplings or, equally, for the standard RGFs in the higher-derivatives and minimal-subtractions-of-logarithms (HD + MSL) scheme [59,60]. It can be derived by summing the singular contributions (which appear when the double total derivatives act on the inverse-squared momenta) [60,61]

and taking into account the nonrenormalization of the triple gauge ghost vertices [62]. All these general statements have been confirmed by various multiloop calculations; see, e.g., [63–70]. In particular, the factorization into integrals of double derivatives can be observed already in the one-loop approximation [71]. Note that for an arbitrary form of the higher-derivative regulator this result is nontrivial and actually reflects some deep features of quantum $4D$ supersymmetric theories. This is the basic reason why we believe that the possibility of similar factorization for higher-dimensional supersymmetric theories could shed more light on their underlying quantum structure.

In this paper, we consider $6D$, $\mathcal{N} = (1, 0)$ supersymmetric theory with higher derivatives proposed in [21], with the action modified by adding the matter hypermultiplet in the adjoint representation of gauge group. The $6D$, $\mathcal{N} = (1, 0)$ supersymmetric theories are, in general, anomalous [72]. However, in the presence of a hypermultiplet, the theory under consideration is anomaly-free [73]. We will regularize it by higher covariant derivatives with the help of the manifestly $\mathcal{N} = (1, 0)$ supersymmetric technique based on the $6D$ version of harmonic superspace [32–36]. Using the regularization constructed, we calculate the one-loop divergences and demonstrate that the β function is determined by integrals of the double total derivatives for an arbitrary choice of the regulator function. The resemblance of the resulting expression to the analogous expression in the $4D$ case suggests that some other features of $4D$ supersymmetric theories could also be valid in the $6D$ case. In particular, we guess that the β function of the model under consideration is given by integrals of the double total derivatives to any loop and so can be brought in the NSVZ-like form.

The paper is organized as follows. In Sec. II, the basic features of the theory under consideration are overviewed. In particular, in Sec. II A, we briefly recall some information about the harmonic superspace. This technique is used in Sec. II B for constructing the manifestly $\mathcal{N} = (1, 0)$ supersymmetric and gauge-invariant action. The theory is regularized by higher covariant derivatives in Sec. II C. The quantization procedure is described in Sec. II D. Section III is devoted to the calculation of the one-loop divergent quantum corrections. In particular, we demonstrate that the one-loop β function can be written as an integral of the double total derivatives in the momentum space. Possible generalizations of this result and its consequences for the higher-order quantum corrections are discussed in Sec. IV. The results of the paper are briefly summarized in the Conclusion, while some technical details of the calculations are expounded in Appendixes A and B.

II. HIGHER-DERIVATIVE $6D$, $\mathcal{N} = (1, 0)$ SUPERSYMMETRIC YANG-MILLS THEORY

A. Harmonic superspace

The manifestly $\mathcal{N} = (1, 0)$ supersymmetric description of $6D$ theories is achieved in harmonic superspace

¹Nevertheless, such theories can inherit some properties of the underlying superstring theory, which may simplify their renormalization structure (see, e.g., a recent paper [20] and references therein).

²See also [22,23].

³For the conventional $6D$ $\mathcal{N} = (1, 0)$ and $\mathcal{N} = (1, 1)$ supersymmetric theories, various calculations of quantum corrections were performed in [20,39–46].

(see, e.g., [74]). It is parametrized by the ordinary space-time coordinates x^M (with $M = 0, \dots, 5$), the anticommuting left-handed spinors θ_i^a (with $a = 1, \dots, 4$ and $i = 1, 2$), and the harmonic variables $u^{\pm i}$, such that

$$u_i^- = (u^{+i})^*, \quad u^{+i}u_i^- = 1, \quad u_i^\pm = \varepsilon_{ij}u^{\pm j}. \quad (2.1)$$

The harmonic superspace contains a subspace which is closed under supersymmetry transformations and involves half the original Grassmann coordinates. This (analytic) subspace is parametrized by the coordinate set

$$x_A^M = x^M + \frac{i}{2}\theta^- \gamma^M \theta^+, \quad \theta^{+a} = u_i^+ \theta^{ai}, \quad u_i^\pm, \quad (2.2)$$

where the $6D$ γ matrices are denoted by γ^M .

Using the harmonic variables, one can construct the harmonic derivatives

$$D^{++} = u^{+i} \frac{\partial}{\partial u^{-i}}, \quad D^{--} = u^{-i} \frac{\partial}{\partial u^{+i}}, \quad D^0 = u^{+i} \frac{\partial}{\partial u^{+i}} - u^{-i} \frac{\partial}{\partial u^{-i}} \quad (2.3)$$

and the harmonic spinor covariant derivatives

$$D_a^+ = u_i^+ D_a^i, \quad D_a^- = u_i^- D_a^i. \quad (2.4)$$

By definition, an analytic superfield ϕ satisfies the Grassmann analyticity constraint $D_a^+ \phi = 0$.

For constructing the $6D$, $\mathcal{N} = (1, 0)$ supersymmetric actions, we use the superspace integration measures. In the notation adopted in this paper, they are defined as

$$\int d^{14}z \equiv \int d^6x d^8\theta, \quad \int d\zeta^{(-4)} \equiv \int d^6x d^4\theta^+ du, \quad (2.5)$$

where $z \equiv (x^M, \theta_i^a)$. The integration measures are related as

$$\int d\zeta^{(-4)} (D^+)^4 = \int d^{14}z du, \quad (2.6)$$

where

$$(D^+)^4 \equiv -\frac{1}{24} \varepsilon^{abcd} D_a^+ D_b^+ D_c^+ D_d^+. \quad (2.7)$$

B. The higher-derivative theory

We consider $6D$, $\mathcal{N} = (1, 0)$ higher-derivative SYM theory proposed in [21], to which we add the hypermultiplet in the adjoint representation. This theory is described by the action

$$S = \pm \frac{1}{2e_0^2} \text{tr} \int d\zeta^{(-4)} (F^{++})^2 - \frac{2}{e_0^2} \text{tr} \int d\zeta^{(-4)} \widetilde{q}^+ \nabla^{++} q^+. \quad (2.8)$$

Note that the sign of the usual Yang-Mills action is chosen so that the energy is positive. However, this requirement cannot, in general, be satisfied for theories with higher derivatives. That is why, following [37,38], in the gauge part of the action (2.8) we admit both signs. Below, the upper and lower signs will always correspond to the plus and minus signs in Eq. (2.8), respectively.

According to reasonings of [73], the theory (2.8) is anomaly-free due to the presence of the hypermultiplet in the adjoint representation. Moreover, we will see that the theory with hypermultiplet avoids one-loop quadratic divergences, which is very important for constructing the higher covariant derivative regularization in $6D$ space-time.⁴

In Eq. (2.8), e_0 is the dimensionless bare coupling constant (hereafter, all bare couplings are marked by the subscript 0). The gauge superfield $V^{++} = e_0 V^{++A} t^A$ and the hypermultiplet superfield $q^+ = e_0 q^{+A} t^A$ satisfy the analyticity conditions

$$D_a^+ V^{++} = 0, \quad D_a^+ q^+ = 0. \quad (2.9)$$

The generators of the gauge group t^A are taken in the fundamental representation for which the normalization condition and the commutation relations can be written as

$$\text{tr}(t^A t^B) = \frac{1}{2} \delta^{AB}, \quad [t^A, t^B] = i f^{ABC} t^C, \quad (2.10)$$

where f^{ABC} are real and totally antisymmetric structure constants.

The generalized conjugation is denoted by tilde, the gauge superfield being real with respect to it, $\widetilde{V^{++}} = V^{++}$. The conjugated hypermultiplet superfield $\widetilde{q^+}$ is also analytic.

In the harmonic approach, the covariant gauge field strength is the analytic superfield F^{++} defined as

$$F^{++} \equiv (D^+)^4 V^{--}, \quad (2.11)$$

where the (nonanalytic) superfield

⁴The hypermultiplet term in (2.8) breaks $\mathcal{N} = 2$ superconformal symmetry respected by the first term. As argued in [35], there exist no superconformally invariant $6D$ hypermultiplet actions. On the other hand, nothing forbids adding some higher-derivative hypermultiplet terms to (2.8) (e.g., like those discussed in [35]). We limit our consideration to (2.8) for minimality reasons.

$$V^{--}(z, u) \equiv \sum_{n=1}^{\infty} (-i)^{n+1} \int du_1 \dots du_n \times \frac{V^{++}(z, u_1) V^{++}(z, u_2) \dots V^{++}(z, u_n)}{(u^+ u_1^+)(u_1^+ u_2^+) \dots (u_n^+ u^+)} \quad (2.12)$$

satisfies the zero curvature constraint

$$D^{++}V^{--} - D^{--}V^{++} + i[V^{++}, V^{--}] = 0. \quad (2.13)$$

The harmonic covariant derivatives of hypermultiplet are written as

$$\nabla^{\pm\pm} q^+ = D^{\pm\pm} q^+ + i[V^{\pm\pm}, q^+]. \quad (2.14)$$

The theory (2.8) is invariant under the gauge transformations

$$V^{++} \rightarrow e^{i\lambda} V^{++} e^{-i\lambda} - i e^{i\lambda} D^{++} e^{-i\lambda}, \quad q^+ \rightarrow e^{i\lambda} q^+ e^{-i\lambda}, \quad (2.15)$$

where the parameter $\lambda = e_0 \lambda^A t^A$ is an analytic superfield, such that $\widetilde{\lambda}^A = \lambda^A$. Under these transformations, other quantities defined above behave as

$$V^{--} \rightarrow e^{i\lambda} V^{--} e^{-i\lambda} - i e^{i\lambda} D^{--} e^{-i\lambda}, \quad F^{++} \rightarrow e^{i\lambda} F^{++} e^{-i\lambda}, \\ \nabla^{\pm\pm} q^+ \rightarrow e^{i\lambda} \nabla^{\pm\pm} q^+ e^{-i\lambda}, \quad \widetilde{q}^+ \rightarrow e^{i\lambda} \widetilde{q}^+ e^{-i\lambda}. \quad (2.16)$$

C. Regularization by higher-order covariant derivatives

The higher covariant-derivative regularization is introduced by inserting a term with higher derivatives into the action (2.8). For constructing such a term, it is necessary to take into account the identity which is satisfied for an arbitrary analytic superfield ϕ :

$$\frac{1}{2} (D^+)^4 (D^{--})^2 \phi = \partial^2 \phi. \quad (2.17)$$

Obviously, the needed higher-derivative term should be gauge invariant. So, in order to construct it, we should use the analogous operator containing covariant derivatives:

$$\square \equiv \frac{1}{2} (D^+)^4 (\nabla^{--})^2. \quad (2.18)$$

Then the sum of the original action and the higher-derivative term can be written in the form

$$S_{\text{reg}} = \pm \frac{1}{2e_0^2} \text{tr} \int d\zeta^{(-4)} F^{++} R \left(\frac{\square}{\Lambda^2} \right) F^{++} - \frac{2}{e_0^2} \text{tr} \int d\zeta^{(-4)} \widetilde{q}^+ \nabla^{++} q^+, \quad (2.19)$$

where Λ is a dimensionful parameter which plays the role of an ultraviolet cutoff. The regulator function $R(x)$ should increase at $x \rightarrow \infty$ and satisfy the condition $R(0) = 1$ [this condition is required in order to recover the original theory (2.8) in the limit $\Lambda \rightarrow \infty$]. It is easy to verify that the action (2.19) is invariant under the gauge transformations (2.15) and (2.16).

However, according to [52], after adding the higher-derivative term to the classical action, divergences still survive in the one-loop approximation. To regularize these residual divergences, one is led to insert the Pauli-Villars fields with a mass parameter in the generating functional. The details of this procedure for the theory we deal with will be discussed in the next section.

D. Quantization

For quantizing gauge theories, it is convenient to use the background field method [75,76] (see also [77–80]), because it allows constructing the manifestly gauge-invariant effective action. The first step of background field method is the background-quantum splitting. In harmonic superspace, such a splitting is carried out as follows [30,81]:

$$V^{++} = \mathbf{V}^{++} + v^{++}, \quad (2.20)$$

where $\mathbf{V}^{++}$ and v^{++} are the background and quantum gauge superfields, respectively. In this case, the original gauge invariance (2.15) amounts to two different symmetries. The quantum gauge symmetry

$$\mathbf{V}^{++} \rightarrow \mathbf{V}^{++}, \\ v^{++} \rightarrow e^{i\lambda} (v^{++} + \mathbf{V}^{++}) e^{-i\lambda} - \mathbf{V}^{++} - i e^{i\lambda} D^{++} e^{-i\lambda}, \\ q^+ \rightarrow e^{i\lambda} q^+ \quad (2.21)$$

is broken by the gauge-fixing procedure down to the Becchi-Rouet-Stora-Tyutin invariance. The second, background transformations

$$\mathbf{V}^{++} \rightarrow e^{i\lambda} \mathbf{V}^{++} e^{-i\lambda} - i e^{i\lambda} D^{++} e^{-i\lambda}, \\ v^{++} \rightarrow e^{i\lambda} v^{++} e^{-i\lambda}, \quad q^+ \rightarrow e^{i\lambda} q^+ \quad (2.22)$$

remain the manifest symmetry of the effective action. Certainly, this occurs only providing we choose the gauge-fixing term which preserves this symmetry. For instance, it is convenient to fix a gauge by adding the term

$$S_{\text{gf}} = \mp \frac{1}{2e_0^2 \xi_0} \text{tr} \int d^4 z du_1 du_2 e^{ib_1} e^{-ib_2} (\nabla_2^{++} v_2^{++}) e^{ib_2} e^{-ib_1} \\ \times \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3} R \left(\frac{\square_1}{\Lambda^2} \right) \square_1 \nabla_1^{++} v_1^{++}, \quad (2.23)$$

where ξ_0 is the bare gauge-fixing parameter and the subscripts denote different sets of harmonic variables, e.g.,

$v_1^{++} \equiv v^{++}(z, u_1)$, etc. The background covariant derivatives

$$\nabla^{\pm\pm} v^{++} \equiv D^{\pm\pm} v^{++} + i[V^{\pm\pm}, v^{++}] \quad (2.24)$$

are constructed in terms of the background gauge superfield $V^{\pm\pm}$ [unlike the derivative (2.14), which includes the total gauge superfield $V^{\pm\pm}$]. The operator $\square$ in Eq. (2.23) is defined exactly as the operator $\square$ in Eq. (2.18), but the derivatives ∇^{--} should be replaced by the background-group covariant derivatives ∇^{--} . The superfield b present in the gauge-fixing term (2.23) is the so-called bridge; it is related to the background superfields $V^{\pm\pm}$ by the equation

$$V^{++} = -ie^{ib} D^{++} e^{-ib}, \quad V^{--} = -ie^{ib} D^{--} e^{-ib}. \quad (2.25)$$

The gauge-fixing term (2.23) is invariant under the transformations (2.22) which act on the bridge as $e^{ib} \rightarrow e^{i\lambda} e^{ib} e^{-i\tau}$, where $\tau = \tau(z)$ is independent of the harmonic variables.

The action of Faddeev-Popov ghosts corresponding to the gauge-fixing term (2.23) has the standard form

$$S_{\text{FP}} = \frac{2}{e_0^2} \text{tr} \int d\zeta^{(-4)} \nabla^{++} b (\nabla^{++} c + i[v^{++}, c]), \quad (2.26)$$

where b and c are anticommuting analytic superfields in the adjoint representation. When using the background field method for supersymmetric theories, it is also necessary to insert the Nielsen-Kallosh determinant. For the case at hand, it can be written as

$$\Delta_{\text{NK}} = \text{Det}^{1/2} \left[\square^2 R \left(\frac{\square}{\Lambda^2} \right) \right] \int D\varphi \exp(iS_{\text{NK}}), \quad (2.27)$$

where φ is a commuting analytic superfield in the adjoint representation:

$$S_{\text{NK}} = -\frac{1}{e_0^2} \text{tr} \int d\zeta^{(-4)} (\nabla^{++} \varphi)^2 \quad (2.28)$$

and the function $R(x)$ appears due to its presence in the expression (2.23). The corresponding factor can also be represented in the form of the functional integral

$$\begin{aligned} & \text{Det} \left[\square^2 R \left(\frac{\square}{\Lambda^2} \right) \right] \\ &= \int D\chi^{(+4)} D\sigma \exp \left\{ \frac{2i}{e_0^2} \text{tr} \int d\zeta^{(-4)} \chi^{(+4)} \square^2 R \left(\frac{\square}{\Lambda^2} \right) \sigma \right\}, \end{aligned} \quad (2.29)$$

where $\chi^{(+4)}$ and σ are anticommuting analytic superfields in the adjoint representation.

According to Ref. [52], in the generating functional of a theory regularized by higher covariant derivatives, it is also necessary to insert the Pauli-Villars determinant in order to cancel the one-loop divergences which survive after adding the higher-derivative term to the classical action. Construction of the Pauli-Villars determinant in the case under consideration is realized as follows. We introduce the commuting analytic superfield $P^{++} = e_0 P^{++A} t^A$. To find its action, one considers the terms quadratic in v^{++} in the sum of the regularized and gauge-fixing actions. Then, one replaces v^{++} by P^{++} and adds the masslike regularization parameter:

$$\begin{aligned} S_{\text{PV}}|_{v^{++}=0} &\equiv \int d\zeta_1^{(-4)} d\zeta_2^{(-4)} P_1^{++A} \frac{\delta^2(S_{\text{reg}} + S_{\text{gf}})}{\delta v_1^{++A} \delta v_2^{++B}} P_2^{++B} \\ &\pm \frac{M^4}{2} \int d\zeta^{(-4)} (P^{++A})^2, \end{aligned} \quad (2.30)$$

where the regularized action S_{reg} was defined by Eq. (2.19). The resulting action for the superfield P^{++} is almost the same as the part of the usual action quadratic in v^{++} but contains the mass term. Its sign is concordant with the one in Eq. (2.8) and is chosen in such a way that the part of the action which does not contain (both background and quantum) gauge superfields takes the form

$$\pm \frac{1}{2} \int d\zeta^{(-4)} P^{++A} \left[\partial^4 R \left(\frac{\partial^2}{\Lambda^2} \right) + M^4 \right] P^{++A}. \quad (2.31)$$

Therefore, all P^{++} vertices will be exactly the same as the v^{++} vertices, but the propagators will be different. As we will see later, the expression (2.30) is sufficient for one-loop calculations.

Note that in Eq. (2.30) we present only the result for vanishing quantum gauge superfield v^{++} . The dependence on v^{++} can be restored from the requirement of canceling one-loop divergences of the two-point Green function of the *quantum* gauge superfield. This Green function is not calculated in this paper, because it is not necessary for obtaining the one-loop β function. However, it can get essential in higher orders. The parameter M (with the dimension of mass) in the action (2.30) should be proportional to the parameter Λ in the regularized action (2.19):

$$M = a\Lambda, \quad (2.32)$$

where a is an arbitrary constant. In particular, it is possible to choose $a = 1$. However, an arbitrariness in the choice of both the constant a and the regulator function $R(x)$ implies that the higher covariant derivative regularization is not uniquely defined. The dependence of the renormalization-group functions on the regularization parameters is very similar to the scheme dependence. For the gauge β function, it becomes essential starting from the three-loop

approximation. The explicit dependence of the three-loop β function(s) on the parameter a and the regulator functions for $4D, \mathcal{N} = 1$ supersymmetric models can be found in [82].

Now it becomes possible to construct the Pauli-Villars determinant

$$\text{Det}^{-1}(PV, M) = \int DP^{++} \exp(iS_{PV}). \quad (2.33)$$

As we will see, in order to cancel the residual one-loop divergences, one should insert the determinant defined in (2.33) into the generating functional, which then takes the form

$$\begin{aligned} Z[\text{sources}] &= \int Dv^{++} D\widetilde{q}^+ Dq^+ DbDcD\varphi \text{Det}^{1/2} \left[\square^2 R \left(\frac{\square}{\Lambda^2} \right) \right] \\ &\times \text{Det}(PV, M) \exp(iS_{\text{reg}} + iS_{\text{gf}} + iS_{\text{FP}} + iS_{\text{NK}} + iS_{\text{sources}}). \end{aligned} \quad (2.34)$$

It is important that the expression for the generating functional contains the Pauli-Villars determinant to the degree one, while Eq. (2.33) defines the inverse of this determinant. This implies that any closed loop of the Pauli-Villars superfield contributes an additional minus sign exactly as a loop of fermions. Nevertheless, the superfield

P^{++} is commuting, and the minus sign comes from powers of the Pauli-Villars determinant in Eqs. (2.33) and (2.34).

The generating functional (2.34) also depends on the sources which are included into the term

$$\begin{aligned} S_{\text{sources}} &= \int d\zeta^{(-4)} (v^{++A} J^{(+2)A} + j^{(+3)A} q^{+A} \\ &+ \widetilde{j^{(+3)A}} \widetilde{q^{+A}} + \dots), \end{aligned} \quad (2.35)$$

where dots stand for possible sources for ghost superfields. The effective action is constructed according to the standard procedure as the Legendre transform of the generating functional $W = -i \ln Z$ for the connected Green functions. It will depend on the background gauge superfield as a parameter and will be manifestly gauge invariant. The manifest $\mathcal{N} = (1, 0)$ supersymmetry is also guaranteed due to the use of the harmonic superspace.

III. ONE-LOOP DIVERGENCES

A. The general structure of divergent contributions

Let us analyze possible divergent contributions to the effective action. The simplest way to perform such an analysis is to resort to the dimensional considerations. The dimensions of various quantities encountered in the harmonic superspace approach are

$$\begin{aligned} [d^{14}z] &= m^{-2}, & [d\zeta^{(-4)}] &= m^{-4}, & [D_a^+] &= m^{1/2}, & [(D^+)^4] &= m^2, \\ [q^+] &= [\widetilde{q}^+] = m^2, & [V^{\pm\pm}] &= 1, & [F^{++}] &= m^2, & [e_0] &= 1. \end{aligned} \quad (3.1)$$

Any divergent contribution to the effective action should be invariant under the background gauge transformations, be presentable as an integral over $d^{14}z$, and possess the dimension m^α with $\alpha \leq 0$. From the (background) gauge superfield, we can construct two such expressions, namely,

$$I_1 \equiv \text{tr} \int d\zeta^{(-4)} (F^{++})^2 = \text{tr} \int d^{14}z du V^{--} F^{++}, \quad (3.2)$$

$$I_2 \equiv \sum_{n=2}^{\infty} \frac{(-i)^n}{n} \text{tr} \int d^{14}z du_1 du_2 \dots du_n \frac{V_1^{++} V_2^{++} \dots V_n^{++}}{(u_1^+ u_2^+)(u_2^+ u_3^+) \dots (u_n^+ u_1^+)}. \quad (3.3)$$

The first one has the same structure as the gauge part of the action (2.8), while the second expression (up to an insignificant dimensionful constant) coincides with the action of standard $\mathcal{N} = (1, 0)$ SYM theory in the harmonic superspace formulation [34]. It is easy to see that the dimensions of the invariants (3.2) and (3.3) are

$$[I_1] = 1, \quad [I_2] = m^{-2}. \quad (3.4)$$

Therefore, the momentum integrals which determine the coefficients in these expressions are, respectively, logarithmically and quadratically divergent. This remains true at any loop, because the coupling constant e_0 is dimensionless.

The structure of harmonic superspace allows us to make immediately some statements about the expected counterterms. Indeed, it is impossible to construct the invariants from a hypermultiplet and a background gauge superfield that simultaneously satisfy all the requirements formulated above [after Eq. (3.1)]. For instance, the expression

$$I_3 \equiv \text{tr} \int d\zeta^{(-4)} \widetilde{q}^+ \nabla^{++} q^+ \quad (3.5)$$

cannot be written as an integral over $d^{14}z$ of the local expression, and the expression

$$I_4 \equiv \text{tr} \int d\zeta^{(-4)} \widetilde{q}^+ [F^{++}, q^+] = \text{tr} \int d^{14}z du \widetilde{q}^+ [V^{--}, q^+] \quad (3.6)$$

has the dimension m^2 . This implies that these structures cannot appear in the *divergent* part of the effective action.⁵ Similar arguments also apply to the ghost contributions. This is why in the theory under consideration the hypermultiplet and ghosts are not renormalized at any loop. More precisely, let us define the renormalization constants Z_q , Z_c , and Z_α (for the hypermultiplet, the Faddeev-Popov ghosts, and the coupling constant $\alpha = e^2/4\pi$, respectively) by the equations

$$\begin{aligned} q &= e_0 q^A t^A, & b &= e_0 b^A t^A, & c &= e_0 c^A t^A, \\ q_R &= e(q_R)^A t^A, & b_R &= e(b_R)^A t^A, & c_R &= e(c_R)^A t^A, \\ q &\equiv \sqrt{Z_q Z_\alpha^{-1}} q_R, & bc &\equiv Z_c Z_\alpha^{-1} b_R c_R, & \frac{1}{\alpha_0} &= \frac{Z_\alpha}{\alpha}, \end{aligned} \quad (3.7)$$

where the renormalized superfields are marked by the subscript R and $\alpha_0 = e_0^2/4\pi$ is the bare coupling constant. Then, the nonrenormalization of the hypermultiplet and ghosts implies that $Z_q = 1$ and $Z_c = 1$.

It is necessary to check that the contributions proportional to I_2 cancel each other and to explore the charge renormalization induced by the divergences proportional to I_1 . This charge renormalization is encoded in the β function

$$\tilde{\beta}(\alpha) \equiv \left. \frac{d\alpha}{d \ln \mu} \right|_{\alpha=\text{const}}. \quad (3.8)$$

Usually, it is defined in terms of the renormalized coupling constant α . However, when analyzing quantum corrections in supersymmetric theories, it is more expedient to use another definition of the β function, namely, in terms of the bare coupling constant α_0 ,

$$\beta(\alpha_0) \equiv \left. \frac{d\alpha_0}{d \ln \Lambda} \right|_{\alpha=\text{const}}. \quad (3.9)$$

The functions (3.8) and (3.9) are, in general, different [59] but, modulo the formal change of the argument, coincide with each other in the so-called HD + MSL scheme. In this scheme, a theory is regularized by the higher-order covariant derivatives and the divergences are removed by minimal subtractions of logarithms. This prescription means that only powers of $\ln \Lambda/\mu$ (where μ is a

⁵Nevertheless, the second expression can appear in the one-loop divergences for the ordinary $6D$, $\mathcal{N} = (1, 0)$ SYM theory, thanks to the dimensionful coupling constant (see [83,84] for details).

renormalization point) are included into the renormalization constants.

The β function can be calculated, starting from the two-point Green function of the background gauge superfield V^{++} . The corresponding contribution to the effective action can be presented in the form

$$\Gamma_{V^{++}}^{(2)} = \pm \frac{1}{8\pi} \text{tr} \int \frac{d^6 p}{(2\pi)^6} d^4 \theta^+ du F_{\text{linear}}^{++}(-p, \theta, u) \times F_{\text{linear}}^{++}(p, \theta, u) d^{-1}(\alpha_0, -p^2/\Lambda^2), \quad (3.10)$$

where

$$F_{\text{linear}}^{++}(z, u_1) = \int \frac{du_2}{(u_1^+ u_2^+)^2} (D_1^+)^4 V^{++}(z, u_2) \quad (3.11)$$

is that part of F^{++} which is linear in the background gauge superfield.

According to the definition of the renormalized coupling α , the function d^{-1} , being expressed through the renormalized coupling constant, cannot depend on the cutoff Λ in the limit $\Lambda \rightarrow \infty$. Therefore, the β function in the HD + MSL scheme can be evaluated according to the prescription

$$\begin{aligned} \tilde{\beta}(\alpha \rightarrow \alpha_0)|_{\text{HD+MSL}} \\ = \beta(\alpha_0) = \alpha_0^2 \lim_{\Lambda \rightarrow \infty} \frac{d}{d \ln \Lambda} (d^{-1} - \alpha_0^{-1})|_{\alpha=\text{const}}. \end{aligned} \quad (3.12)$$

In $4D$, $\mathcal{N} = 1$ supersymmetric theories regularized by higher covariant derivatives, the analogous expression is expressed as integrals of double total derivatives at any loop [58]. In what follows, we will see that for the theory (2.8) this is also true (at least, in the one-loop approximation).

B. One-loop divergences produced by the hypermultiplet and ghosts

To find the coefficients of the invariants I_1 and I_2 in the divergent part of the one-loop effective action, we should calculate the two-point Green function of the background gauge superfield in this approximation. It is contributed by the harmonic superdiagrams presented in Fig. 1.

In these diagrams, the external bold wavy lines correspond to the background gauge superfield V^{++} . Thin wavy lines denote propagators of the quantum gauge superfield v^{++} and the Pauli-Villars superfield P^{++} . The propagators of the Faddeev-Popov and Nielsen-Kallosh ghosts are denoted by dashed and dotted lines, respectively. The solid lines correspond to the propagators of the hypermultiplet. From the form of the hypermultiplet action, it is easy to conclude that there are no vertices with two hypermultiplets and two external gauge lines. It should also be noticed that the diagrams with a loop of the Nielsen-Kallosh ghosts consist of two different contributions. One of them

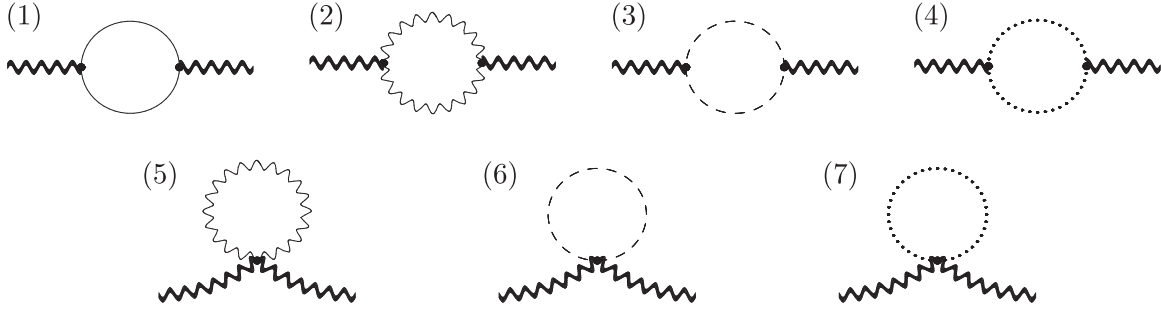


FIG. 1. One-loop harmonic supergraphs contributing to the two-point Green function of the background gauge superfield.

corresponds to the superfield φ , and another is produced by the superfields $\chi^{(+4)}$ and σ ; see Eqs. (2.27)–(2.29).

The hypermultiplet and ghost superdiagrams [except for the contributions of the superfields $\chi^{(+4)}$ and σ] are exactly the same as in the conventional $6D$, $\mathcal{N} = (1, 0)$ SYM theory. According to [84], their contribution to the part of the effective action producing the two-point Green function of the background gauge superfield (in the Minkowski space, before Wick rotation) can be written as

$$\begin{aligned} \Delta_{\text{ghost+hyper}} \Gamma_{V^{++}}^{(2)} &= -iC_2 \text{tr} \int \frac{d^6 p}{(2\pi)^6} d^8 \theta \frac{du_1 du_2}{(u_1^+ u_2^+)^2} V^{++}(p, \theta, u_1) V^{++}(-p, \theta, u_2) \\ &\times \int \frac{d^6 k}{(2\pi)^6} \frac{1}{k^2 (k+p)^2} (s_1 + s_3 + s_4 + s_6 + s_7), \end{aligned} \quad (3.13)$$

where the quadratic Casimir C_2 is defined by the equation

$$f^{ACD} f^{BCD} = C_2 \delta^{AB} \quad (3.14)$$

and each numerical constant s_n corresponds to the diagram (n) in Fig. 1. The values of the constants s_n based on the calculations made in [84] are as follows:

$$s_1 = 1, \quad s_3 = -2, \quad s_4 = 1, \quad s_6 = s_7 = 0. \quad (3.15)$$

The contributions of the supergraphs (2) and (5) have a different structure and will be considered separately.

Note that the action for the superfields $\chi^{(+4)}$ and σ [see Eq. (2.29)] now contains the regulator function $R(x)$ and differs from the one considered in [84]. However, the contribution of both diagrams with a loop of these superfields still vanishes for exactly the same reasons as in the conventional $6D$, $\mathcal{N} = (1, 0)$ SYM theory.

Inserting the values (3.15) into the expression (3.13), we see that the sum of the ghost and hypermultiplet contributions vanishes:

$$\Delta_{\text{ghost+hyper}} \Gamma_{V^{++}}^{(2)} = (1) + (3) + (4) + (6) + (7) = 0. \quad (3.16)$$

C. One-loop divergences produced by the quantum gauge superfield and the Pauli-Villars superfield

In the previous section, we demonstrated that the hypermultiplet and ghost contributions to the two-point Green function of the background gauge superfield cancel each other. Therefore, for obtaining the one-loop divergences, we need to evaluate only the superdiagrams (2) and (5) in Fig. 1. These diagrams contain a loop of either the quantum gauge superfield v^{++} or the Pauli-Villars superfield P^{++} . We will calculate them, using the analog of the Feynman gauge with $\xi_0 = 1$. Details of the calculation are presented in Appendix B. The result can be written in the form

$$\begin{aligned} \Delta_{\text{gauge}} \Gamma_{V^{++}}^{(2)} &= (2) + (5) \\ &= \text{tr} \int \frac{d^6 p}{(2\pi)^6} d^4 \theta^+ du F_{\text{linear}}^{++}(-p, \theta, u) F_{\text{linear}}^{++}(p, \theta, u) (I_v(-p^2/\Lambda^2) - I_P(-p^2/\Lambda^2)), \end{aligned} \quad (3.17)$$

where F_{linear}^{++A} defined by Eq. (3.11) is the part of F^{++A} linear in the background gauge superfield V^{++} . The integrals over the loop momenta I_v and I_P are generated by the supergraphs containing loops of the quantum gauge superfield v^{++} and the Pauli-Villars superfield P^{++} , respectively. Note that (as we discussed at the end of Sec. II D) a loop of the Pauli-Villars superfield produces an additional minus sign, so that it is reasonable to put a minus sign in front of I_P .

It is convenient to present the result of the calculation in Appendix B in the form

$$\begin{aligned} \frac{d}{d \ln \Lambda} I_v|_{\Lambda \rightarrow \infty} &= C_2 \frac{d}{d \ln \Lambda} \int \frac{d^6 K}{(2\pi)^6} \left(-\frac{4}{K^6} - \frac{R'_K}{K^4 R_K} - \frac{(R'_K)^2}{K^2 R_K^2} + \frac{R''_K}{K^2 R_K} \right) \\ &= \frac{C_2}{4} \frac{d}{d \ln \Lambda} \int \frac{d^6 K}{(2\pi)^6} \frac{\partial^2}{\partial K_\mu \partial K^\mu} \left[\frac{1}{K^4} \ln(K^4 R_K) \right], \end{aligned} \quad (3.18)$$

$$\begin{aligned} \frac{d}{d \ln \Lambda} I_P|_{\Lambda \rightarrow \infty} &= C_2 \frac{d}{d \ln \Lambda} \int \frac{d^6 K}{(2\pi)^6} \left\{ -\frac{1}{(K^4 R_K + M^4)^2} (4K^2 R_K^2 \right. \\ &\quad \left. + 4K^4 R_K R'_K + K^6 (R'_K)^2) + \frac{1}{K^4 R_K + M^4} (K^2 R''_K + 3R'_K) \right\} \\ &= \frac{C_2}{4} \frac{d}{d \ln \Lambda} \int \frac{d^6 K}{(2\pi)^6} \frac{\partial^2}{\partial K_\mu \partial K^\mu} \left[\frac{1}{K^4} \ln(K^4 R_K + M^4) \right], \end{aligned} \quad (3.19)$$

where $R_K \equiv R(K^2/\Lambda^2)$, the prime denotes the derivative with respect to K^2 , and the loop integrals are written in the Euclidean space, with the Wick rotation performed. The Euclidean momenta in our notation are denoted by capital letters in order to distinguish them from the Minkowski momenta. The derivatives with respect to $\ln \Lambda$ are needed to obtain the well-defined expressions. These derivatives should be calculated before fulfilling the integration.

It is important that the double total derivatives with respect to the loop momentum acting on $1/K^4$ produce singularities [see Eq. (4.2) below], so that the corresponding integrals should be accurately defined. Namely, by definition, we exclude a small neighborhood of the singular points ($K^\mu = 0$ in the case under consideration) from the

integration region. That is why the δ functions that appear after calculating the double total derivatives in Eqs. (3.18) and (3.19) do not contribute to the integrals. However, in this case when using the divergence theorem it is necessary to take into account the surface integrals over small spheres surrounding the singular points. It is possible to demonstrate that these surface integrals are equal to the sum of singular contributions (obtained by acting of the double total derivatives on $1/K^4$) taken with the opposite sign; see [58] for details.

Taking into account that the function d^{-1} defined by Eq. (3.10) is equal to α_0^{-1} at the tree level, in the one-loop approximation we have

$$\begin{aligned} \frac{d}{d \ln \Lambda} (d^{-1} - \alpha_0^{-1})|_{\Lambda \rightarrow \infty} &= \pm 8\pi \frac{d}{d \ln \Lambda} (I_v - I_P)|_{\Lambda \rightarrow \infty} + O(\alpha_0) \\ &= \mp 2\pi C_2 \frac{d}{d \ln \Lambda} \int \frac{d^6 K}{(2\pi)^6} \frac{\partial^2}{\partial K_\mu \partial K^\mu} \left[\frac{1}{K^4} \ln \left(1 + \frac{M^4}{K^4 R_K} \right) \right] + O(\alpha_0). \end{aligned} \quad (3.20)$$

This expression is related to the β function defined in terms of the bare coupling constant. It will be calculated in the next section.

D. The one-loop β function

From Eqs. (3.12) and (3.20), it follows that, in the one-loop approximation, the β function defined in terms of the bare coupling constant for the theory (2.8) is given by the integral of double total derivatives in the momentum space:

$$\beta(\alpha_0) = \mp 2\pi \alpha_0^2 C_2 \frac{d}{d \ln \Lambda} \int \frac{d^6 K}{(2\pi)^6} \frac{\partial^2}{\partial K_\mu \partial K^\mu} \left[\frac{1}{K^4} \ln \left(1 + \frac{M^4}{K^4 R_K} \right) \right] + O(\alpha_0^3). \quad (3.21)$$

This expression is very similar to its 4D analog derived in [71] and can be calculated by the same method. To this end, we first perform the differentiation with respect to $\ln \Lambda$, taking into account that the parameter M is proportional to Λ [see Eq. (2.32)]:

$$\beta(\alpha_0) = \mp 8\pi \alpha_0^2 C_2 \int \frac{d^6 K}{(2\pi)^6} \frac{\partial^2}{\partial K_\mu \partial K^\mu} \left[\frac{M^4}{K^4 (K^4 R_K + M^4)} + \frac{M^4 R'_K}{2K^2 R_K (K^4 R_K + M^4)} \right] + O(\alpha_0^3). \quad (3.22)$$

It is assumed that the integration domain in this integral does not contain a small vicinity of the singular point $K_\mu = 0$. In other words, we cut a small ball of the radius $\varepsilon \rightarrow 0$ from the integration region. Using the 6D analog of the divergence theorem, we reduce this integral to the integral over the sphere S_∞^5 with the infinitely large radius and the sphere S_ε^5 with an infinitely small radius ε :

$$\beta(\alpha_0) = \mp \frac{\alpha_0^2 C_2}{8\pi^5} \int_{S_\infty^5 + S_\varepsilon^5} dS_\mu^{(K)} \frac{\partial}{\partial K^\mu} \left[\frac{M^4}{K^4(K^4 R_K + M^4)} + \frac{M^4 R'_K}{2K^2 R_K(K^4 R_K + M^4)} \right] + O(\alpha_0^3). \quad (3.23)$$

The integral over the infinitely large sphere S_∞^5 vanishes due to the presence of the (rapidly growing at $K^2 \rightarrow \infty$) function R_K in the denominator. In the remaining integral over S_ε^5 , the only contribution that survives in the limit $\varepsilon \rightarrow 0$ is obtained if the derivative acts on $1/K^4$ in the denominator of the first term:

$$\beta(\alpha_0) = \pm \frac{\alpha_0^2 C_2}{2\pi^5} \int_{S_\varepsilon^5} dS_\mu^{(K)} \frac{K_\mu M^4}{K^6(K^4 R_K + M^4)} + O(\alpha_0^3). \quad (3.24)$$

Taking into account that the normal to the sphere S_ε^5 is directed inward and that the volume (understood as an analog of the surface area) of the sphere S^5 with the unit radius is equal to

$$\Omega_5 = \frac{2\pi^3}{\Gamma(3)} = \pi^3, \quad (3.25)$$

we obtain the final expression for the one-loop β function of the theory (2.8) in the form

$$\beta(\alpha_0) = \mp \frac{\alpha_0^2 C_2}{2\pi^2} + O(\alpha_0^3) \quad (3.26)$$

(the standard one-loop β function defined in terms of the renormalized coupling constant is obtained through a formal replacement $\alpha_0 \rightarrow \alpha$). This expression agrees with the results of the calculations made in [21,24,37,38], when one takes into account the contribution of the hypermultiplet in the adjoint representation.

IV. HIGHER ORDERS

The calculation of one-loop divergent quantum corrections for the theory (2.8) demonstrated, first, the absence of quadratic divergences in this approximation and, second, the factorization of the integrals specifying the one-loop β function into integrals of double total derivatives in the momentum space. This leads us to suggest that similar features are preserved in higher orders. If this guess is true, the absence of quadratic divergences in all orders would imply the renormalizability of the theory.

For 4D, $\mathcal{N} = 1$ supersymmetric theories, the factorization of integrals defining the β function into integrals of

double total derivatives in the momentum space led to the exact NSVZ expression for the β function. This exact expression was obtained by summing singular contributions which appear when double total derivatives act on the inverse-squared momenta⁶:

$$\frac{\partial^2}{\partial K_\mu \partial K^\mu} \left(\frac{1}{K^2} \right) = -4\pi^2 \delta^4(K). \quad (4.1)$$

Taking Eq. (3.25) into account, we find that the 6D analog of this equation is

$$\frac{\partial^2}{\partial K_\mu \partial K^\mu} \left(\frac{1}{K^4} \right) = -4\pi^3 \delta^6(K). \quad (4.2)$$

Exactly as the inverse-squared momenta come from various propagators in the 4D case, for the 6D theory (2.8) momenta to the inverse degree -4 arise from the gauge superfield propagators. Therefore, this theory in a certain sense is quite similar to the pure 4D, $\mathcal{N} = 1$ SYM theory, for which the NSVZ expression yields the exact β function in the form of the geometric series. This is why we are tempted to assert that the β function for the theory (2.8) can also be written in a similar way.

To construct the corresponding would-be exact expression, we recall how the NSVZ β function is obtained in the 4D case. If we consider a vacuum supergraph, then the sum of all superdiagrams obtained by attaching two external lines of the background gauge superfield is given by an integral of double total derivatives. These double total derivatives effectively cut internal lines in the original vacuum supergraph and produce the anomalous dimensions of various quantum superfields. Their sum coincides with the NSVZ equation written in the specific form proposed in [62]. Let us assume that this qualitative picture also takes place in the case under consideration.

The cut of the one-loop supergraph is schematically presented in Fig. 2 on the left. It is reasonable to suggest that in the higher orders in loops the double total derivatives cut the effective propagator, as is shown in Fig. 2 on the right. The effective propagator is obtained from the two-point Green function of the quantum gauge superfield.

⁶As we mentioned above, they should be taken with the opposite sign.

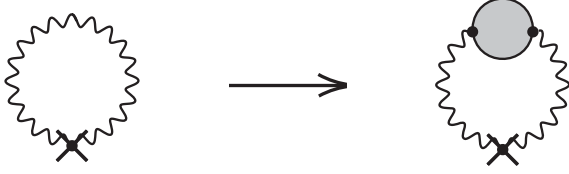


FIG. 2. The double total derivatives cut the gauge propagator in the one-loop approximation and the effective gauge propagator in higher orders. These cuts are denoted by the crosses.

Because of the Slavnov-Taylor identity [85,86], the quantum corrections to this Green function are written in the form

$$\Gamma_{v^{++}}^{(2)} - S_{\text{gf}}^{(2)} = \pm \frac{1}{8\pi} \text{tr} \int \frac{d^6 k}{(2\pi)^6} d^4 \theta^+ du f^{++}(-k, \theta, u) \times f^{++}(k, \theta, u) d_q^{-1}(\alpha_0, -k^2/\Lambda^2), \quad (4.3)$$

where

$$f^{++}(z, u_1) = \int \frac{du_2}{(u_1^+ u_2^+)^2} (D_1^+)^4 v^{++}(z, u_2) \quad (4.4)$$

is a linear part of gauge superfield strength for the quantum gauge superfield. In the tree approximation, the function d_q^{-1} (of the Euclidean momentum K_μ for finite values of Λ) is equal to $\alpha_0^{-1} R(K^2/\Lambda^2)$. Therefore, this function can be written in the form

$$d_q^{-1}(\alpha_0, K^2/\Lambda^2) = \alpha_0^{-1} R(K^2/\Lambda^2) - \alpha_0^{-1} \Pi(\alpha_0, K^2/\Lambda^2) \equiv \alpha_0^{-1} (R_K - \Pi_K), \quad (4.5)$$

where Π is the polarization operator of the quantum gauge superfield.

For 4D supersymmetric gauge theories, the renormalization of the quantum gauge superfield is related to the renormalization of the coupling constant and the Faddeev-Popov ghosts by the relation $Z_v Z_c Z_\alpha^{-1/2} = 1$, which follows from the nonrenormalization theorem for the triple gauge ghost vertices proved in [62]. As we demonstrated in Sec. III A, for 6D theory (2.8) the ghosts are not renormalized, so that $Z_c = 1$. Therefore, it is natural to assume that $Z_v Z_\alpha^{-1/2} = 1$ in this case, so that the quantum gauge superfields v^{++} is not renormalized:

$$v^{++} = e_0 v^{++A} t^A = e(v_R^{++})^A t^A = v_R^{++} \equiv Z_v Z_\alpha^{-1/2} v^{++}. \quad (4.6)$$

If this holds, from Eq. (4.3) we can see that the derivative of the function d_q (expressed in terms of the renormalized coupling constant) with respect to $\ln \Lambda$ vanishes in the limit $\Lambda \rightarrow \infty$. Hence, from Eq. (4.5) we obtain

$$\left. \frac{d \ln(R_K - \Pi_K)}{d \ln \Lambda} \right|_{\Lambda \rightarrow \infty} = \left. \frac{d \ln(R_K - \Pi_K)}{d \ln \Lambda} \right|_{K \rightarrow 0} = \frac{\beta(\alpha_0)}{\alpha_0}. \quad (4.7)$$

Looking at Fig. 2 and Eq. (4.5), we are tempted to suggest that the β function in higher orders can be obtained by replacing the function R_K in the one-loop result (3.21) by $R_K - \Pi_K$:

$$\begin{aligned} \frac{\beta(\alpha_0)}{\alpha_0^2} &= \pm 2\pi C_2 \int \frac{d^6 K}{(2\pi)^6} \frac{d}{d \ln \Lambda} \frac{\partial^2}{\partial K_\mu \partial K^\mu} \\ &\times \left[\frac{1}{K^4} \ln(K^4(R_K - \Pi_K)) \right. \\ &\left. - \frac{1}{K^4} \ln(K^4 R_K + M^4) - (PV) \right], \end{aligned} \quad (4.8)$$

where we explicitly wrote down the one-loop contribution of the Pauli-Villars superfield and denoted their contributions in higher orders by the symbol (PV) . In particular, if the replacement $R_K \rightarrow R_K - \Pi_K$ is valid for the supergraphs containing Pauli-Villars loops, then

$$\begin{aligned} &\frac{1}{K^4} \ln(K^4(R_K - \Pi_K) + M^4) \\ &= \frac{1}{K^4} \ln(K^4 R_K + M^4) + \frac{1}{K^4} \ln\left(1 - \frac{K^4 \Pi_K}{K^4 R_K + M^4}\right) \\ &= \frac{1}{K^4} \ln(K^4 R_K + M^4) - \frac{\Pi_K}{K^4 R_K + M^4} + \dots \\ &\equiv \frac{1}{K^4} \ln(K^4 R_K + M^4) + (PV). \end{aligned} \quad (4.9)$$

However, for understanding the structure of the Pauli-Villars contributions, it is better to rely on the results of the explicit 4D calculations made in higher orders (see, e.g., [49]). These calculations demonstrate that the Pauli-Villars superfields are essential only in the one-loop approximation, while in higher orders they either cancel one-loop subdivergences or give vanishing contributions, because the double total derivatives act on the expressions that do not involve singularities. Presumably, exactly the same situation should take place in the case under consideration. Then it is possible to omit the term (PV) and to keep only the one-loop term containing M . After that, we calculate one derivative with respect to K_μ and use the divergence theorem for the other one. As before, the integral over the infinitely large sphere S_∞^5 vanishes due to the presence of the regulator function. Therefore, the result can be written in the form

$$\begin{aligned} \frac{\beta(\alpha_0)}{\alpha_0^2} &= \pm \frac{C_2}{16\pi^5} \int_{S_\infty^5} dS_\mu^{(K)} \frac{d}{d \ln \Lambda} \left[\frac{K^\mu}{K^6} - \frac{2K^\mu}{K^6} \ln(K^4(R_K - \Pi_K)) \right. \\ &\left. + \frac{2K^\mu}{K^6} \ln(K^4 R_K + M^4) + O\left(\frac{1}{K^3}\right) \right], \end{aligned} \quad (4.10)$$

where $O(1/K^3)$ denote terms which grow as $1/K^3$ or slower in the limit $K \rightarrow 0$. Evidently, they do not contribute to the integral over the infinitely small sphere S_e^5 . Next, we calculate the derivative with respect to $\ln \Lambda$ and, after that, evaluate the integral, taking into account Eq. (3.25) and the inward direction of the normal. The result can be written as

$$\begin{aligned} \frac{\beta(\alpha_0)}{\alpha_0^2} &= \mp \frac{C_2}{2\pi^2} \pm \frac{C_2}{8\pi^2} \frac{d}{d \ln \Lambda} \ln(R_K - \Pi_K)|_{K \rightarrow 0} \\ &= \mp \frac{C_2}{2\pi^2} \pm \frac{C_2}{8\pi^2} \frac{\beta(\alpha_0)}{\alpha_0}, \end{aligned} \quad (4.11)$$

where we took into account Eq. (4.7). Solving Eq. (4.11) for $\beta(\alpha_0)$, we obtain the NSVZ-like expression for the β function defined in terms of the bare coupling constant:

$$\beta(\alpha_0) = \mp \frac{\alpha_0^2 C_2}{2\pi^2 (1 \mp \alpha_0 C_2 / 8\pi^2)}. \quad (4.12)$$

The signs in this equation are concordant with the ones in Eq. (2.8). Namely, the upper signs correspond to the upper sign in Eq. (2.8), and the lower signs correspond to the lower sign in Eq. (2.8). Their origin can be explained as follows. Changing the sign in the gauge part of the action (2.8), we actually change the sign in front of α_0 and, therefore, the sign of $\beta(\alpha_0)$.

Although the reasonings used in deriving this equation look quite rigorous, the result can, nevertheless, be considered merely as a guess about the form of the β function. Actually, the structure of the one-loop quantum corrections for the theory (2.8) is very similar to that in the 4D supersymmetric gauge theories. In particular, the factorization of loop integrals reducing the β function to integrals of double total derivatives may lead to recursive relations between the renormalization-group functions, exactly as it happens in the 4D case (see, e.g., [60]). To justify the rather lax reasoning used for deriving Eq. (4.12), we could refer to the fact that the same speculations for the 4D, $\mathcal{N} = 1$ SYM theory exactly reproduced the NSVZ β function. The two-loop verification of Eq. (4.12) and a possible all-loop derivation of this expression seem to be interesting problems for the future study.

If the expression (4.12) is correct and (for the β function defined in terms of the renormalized coupling constant) is valid in the HD + MSL scheme as in the 4D case, then it is possible to integrate the renormalization-group equation

$$\frac{d\alpha}{d \ln \mu} = \tilde{\beta}(\alpha) = \mp \frac{\alpha^2 C_2}{2\pi^2 (1 \mp \alpha C_2 / 8\pi^2)}. \quad (4.13)$$

It is convenient to present the result in the form of the renormalization-group invariant (analogous to those considered in [8,87,88])

$$\left(\frac{\alpha}{\mu^4}\right)^{C_2} \exp\left(\pm \frac{8\pi^2}{\alpha}\right) = \text{RGI}. \quad (4.14)$$

By definition, it does not depend on the renormalization point μ in all orders.

V. CONCLUSIONS

In this paper, we have developed an approach to studying the structure of quantum corrections for the 6D theory (2.8) regularized by higher covariant derivatives in the harmonic superspace. The one-loop divergences have been calculated and analyzed in detail. The obtained results on the structure of divergences are in agreement with the results of the papers [37,38], where the regularization by dimensional reduction has been used, and demonstrate some new interesting features of this structure which are displayed due to the use of the higher-derivative regularization. Certainly, when comparing the results, one should take into account the hypermultiplet contribution present in the theory under consideration (2.8).

The action of the theory under consideration already contains the fourth derivatives of the gauge superfield and, therefore, despite that we work in six dimensions, the degree of divergence does not grow unboundedly with a number of loops. Although the effective action could include the gauge-invariant term (3.3) as the potential source of the quadratically divergent contributions, at least in the one-loop approximation no quadratic divergences appear due to the presence of the hypermultiplet in the adjoint representation. Namely, the quadratic divergences produced by the Faddeev-Popov and Nielsen-Kallosh ghosts are canceled by the quadratic divergences produced by the hypermultiplet; see Eq. (3.13). We stress that the absence of the residual one-loop quadratic divergences is very important for constructing the higher-derivative covariant regularization in the case under consideration. The presence of the hypermultiplet also leads to the cancelation of anomalies [73]. The harmonic superspace formulation is very convenient, because in this case $\mathcal{N} = (1, 0)$ supersymmetry is manifest at all steps of calculating quantum corrections. The choice of the higher covariant derivative regularization is motivated by the fact that in the 4D case it makes manifest some interesting features of quantum corrections. For instance, the β function (defined in terms of the bare couplings) in all loops proves to be given by integrals of double total derivatives and to satisfy the NSVZ relation. This is why it would be interesting to understand if these features are also valid in the 6D higher derivative theory (2.8). Because of the arbitrariness in the choice of the higher-derivative regulator function, it becomes possible to make a nontrivial check of the factorization of the integrals which specify the β function in terms of integrals of the double total derivatives even in the one-loop approximation.

The calculations carried out in this paper demonstrated that the one-loop β function of the theory under

consideration is really determined by the integrals of the double total derivatives in momentum space for an arbitrary choice of the higher-derivative regulator function. These integrals are very similar to those one-loop integrals which appear in $4D$ supersymmetric gauge theories. Therefore, it is natural to assert that (exactly as in the $4D$ case) such a structure of the loop integrals also persists in higher orders of the perturbation theory. The double total derivatives allow calculating an integral over one of the loop momenta, thereby reducing the number of loop integrations by one. In $4D$ theories, this gives rise to the all-loop exact β function, and one can suppose that an analog of this β function also exists for the $6D$ higher-derivative theory (2.8).

In this paper, we tried to guess the form of the corresponding expression based on some nonstrict indirect arguments. Obviously, such a reasoning cannot be regarded as a proof. Nevertheless, it made possible to derive some trial expression which can be verified (or rejected) in future study. Definitely, it would be interesting to check it by an explicit two-loop calculation and/or even to prove it in all loops.

DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: PROPAGATORS AND VERTICES FOR THE THEORY REGULARIZED BY HIGHER COVARIANT DERIVATIVES

To calculate the diagrams presented in Fig. 1, we need the expressions for the quantum gauge superfield propagator and vertices with two lines of the quantum gauge superfields and one or two lines of the background gauge superfield. In order to get them, we start with the action (2.19) and calculate its second variation, assuming that $\delta V^{++} = v^{++}$. Then the terms which determine the propagator and vertices we are interested in are obtained from the expression

$$\frac{1}{2} \delta^2 S_{\text{reg}}|_{V^{++} \rightarrow V^{++}, q^+ \rightarrow 0} \quad (\text{A1})$$

(the hypermultiplet and ghost contributions to the one-loop effective action have already been presented in Sec. III B, and so we do not discuss here the corresponding Feynman rules).

The first variation of the gauge part of the action (2.19) is written as

$$\delta S_{\text{reg}} = \pm \frac{1}{2e_0^2} \text{tr} \int d^{14}z du \left\{ 2\delta V^{--} R\left(\frac{\square}{\Lambda^2}\right) F^{++} + V^{--} \delta R\left(\frac{\square}{\Lambda^2}\right) F^{++} \right\}. \quad (\text{A2})$$

Using the identity

$$\delta V^{--} = \frac{1}{2} (\nabla^{--})^2 \delta V^{++} - \frac{1}{2} \nabla^{++} \nabla^{--} \delta V^{--}, \quad \nabla^{++} F^{++} = 0, \quad (\text{A3})$$

this expression can be rewritten in the equivalent form

$$\delta S_{\text{reg}} = \pm \frac{1}{2e_0^2} \text{tr} \int d^{14}z du \left\{ 2V^{--} \square R\left(\frac{\square}{\Lambda^2}\right) \delta V^{++} + \nabla^{--} \delta V^{--} \left[\nabla^{++}, R\left(\frac{\square}{\Lambda^2}\right) \right] F^{++} + V^{--} \delta R\left(\frac{\square}{\Lambda^2}\right) F^{++} \right\}. \quad (\text{A4})$$

After that, we can calculate the second variation:

$$\begin{aligned} \delta^2 S_{\text{reg}} = & \pm \frac{1}{2e_0^2} \text{tr} \int d^{14}z du \left\{ 2\delta V^{--} \square R\left(\frac{\square}{\Lambda^2}\right) \delta V^{++} + 2V^{--} \delta \left[\square R\left(\frac{\square}{\Lambda^2}\right) \right] \delta V^{++} \right. \\ & + \delta(\nabla^{--}) \delta V^{--} \left[\nabla^{++}, R\left(\frac{\square}{\Lambda^2}\right) \right] F^{++} + \nabla^{--} \delta^2 V^{--} \left[\nabla^{++}, R\left(\frac{\square}{\Lambda^2}\right) \right] F^{++} + \nabla^{--} \delta V^{--} \\ & \times \delta \left[\nabla^{++}, R\left(\frac{\square}{\Lambda^2}\right) \right] F^{++} + \nabla^{--} \delta V^{--} \left[\nabla^{++}, R\left(\frac{\square}{\Lambda^2}\right) \right] \delta F^{++} + 2\delta V^{--} \delta R\left(\frac{\square}{\Lambda^2}\right) F^{++} + V^{--} \delta^2 R\left(\frac{\square}{\Lambda^2}\right) F^{++} \left. \right\}. \quad (\text{A5}) \end{aligned}$$

Replacing $V^{++} \rightarrow V^{++}$, we see that the first term in this expression does not contain F^{++} , while the remaining ones are proportional either to the first or to the second powers of F^{++} because of the identities

$$[\nabla^{++}, \square] \phi^{(+2)} = i[F^{++}, \phi^{(+2)}] \quad \text{and} \quad (D^+)^4 V^{--} = F^{++}, \quad (\text{A6})$$

where $\phi^{(+2)}$ is an analytic superfield in the adjoint representation.

Because any perturbative contribution to the effective action can be presented as an integral over $d^{14}z$, the terms quadratic in F^{++} correspond to the convergent loop integrals and do not affect the divergent part of the effective action; see Sec. III A for the detailed

discussion. Therefore, in what follows, they can be omitted.

Next, we should calculate the expression (A1) and add the gauge-fixing action (2.23). Integrating by parts with respect to the harmonic derivative ∇_2^{++} and using the identities

$$e^{-ib}\nabla^{++}e^{ib} = D^{++}, \quad D_1^{++} \frac{(u_1^- u_2^+)}{(u_1^+ u_2^+)^3} = \frac{1}{(u_1^+ u_2^+)^2} - \frac{1}{2} (D_1^{--})^2 \delta^{(2,-2)}(u_1, u_2), \quad (\text{A7})$$

it can be rewritten in the form

$$S_{\text{gf}} = \mp \frac{1}{2e_0^2 \xi_0} \text{tr} \int d^{14}z du_1 du_2 e^{ib_1} e^{-ib_2} v_2^{++} e^{ib_2} e^{-ib_1} \left\{ \left(\frac{1}{(u_1^+ u_2^+)^2} - \frac{1}{2} (D_1^{--})^2 \delta^{(2,-2)}(u_1, u_2) \right) \right. \\ \left. \times \square_1 R \left(\frac{\square_1}{\Lambda^2} \right) v_1^{++} + \frac{(u_1^- u_2^+)}{(u_1^+ u_2^+)^3} \left[\nabla_1^{++}, \square_1 R \left(\frac{\square_1}{\Lambda^2} \right) \right] v_1^{++} \right\}. \quad (\text{A8})$$

Taking into account that

$$\delta V_1^{--} = \int \frac{du_2}{(u_1^+ u_2^+)^2} e^{ib_1} e^{-ib_2} \delta V_2^{++} e^{ib_2} e^{-ib_1} = \int \frac{du_2}{(u_1^+ u_2^+)^2} e^{ib_1} e^{-ib_2} v_2^{++} e^{ib_2} e^{-ib_1}, \quad (\text{A9})$$

we see that the sum of such terms in Eqs. (A5) and (A8) which do not contain F^{++} in the Feynman gauge $\xi_0 = 1$ can be written in the form

$$\frac{1}{2} \delta^2 S_{\text{reg}}|_{V^{++} \rightarrow V^{++}, q^+ \rightarrow 0} + S_{\text{gf}}|_{\xi_0=1} = \pm \frac{1}{2e_0^2} \text{tr} \int d\zeta^{(-4)} v^{++} \square^2 R \left(\frac{\square}{\Lambda^2} \right) v^{++} + \dots, \quad (\text{A10})$$

where dots stand for terms which include F^{++} . The term that does not depend on the background gauge superfield determines the propagator of the quantum gauge superfield:

$$\frac{\delta^2 \ln Z_0}{\delta J_1^{(+2)A} \delta J_2^{(+2)B}} = \mp \frac{2i}{\partial^4 R} (D_1^+)^4 \delta^{14}(z_1 - z_2) \delta^{(-2,2)}(u_1, u_2) \delta^{AB}. \quad (\text{A11})$$

However, the expression (A10) also contributes to the vertices. These vertices are essential when calculating the one-loop divergences. In particular, after some calculations it is possible to obtain that the vertex containing two legs of the quantum gauge superfield and one leg of the background gauge superfield can be presented in the form

$$\text{Ver}_1 = \pm \frac{e_0}{8} f^{ABC} \int \frac{d^6 p}{(2\pi)^6} \frac{d^6 k}{(2\pi)^6} d^8 \theta du \frac{(k+p)^4 R_{k+p} - k^4 R_k}{(k+p)^2 - k^2} v^{++A}(-k-p, \theta, u) \\ \times [2V^{--B}(p, \theta, u) D^{--} v^{++C}(k, \theta, u) + D^{--} V^{--B}(p, \theta, u) v^{++C}(k, \theta, u)], \quad (\text{A12})$$

where $R_k \equiv R(-k^2/\Lambda^2)$. Strictly speaking, this expression also contains a term quadratic in V^{++} ; see Eq. (2.12). However, it is easy to see that, because of the total antisymmetry of the structure constants, these quadratic terms do not contribute to the tadpole diagram (5) in Fig. 1. In what follows, we will always assume that only the part linear in V^{++} is taken from V^{--} in the expression (A12).

Similarly, it is possible to evaluate that contribution to the vertex quadratic in the background gauge superfield which comes from the expression (A10). In this case, the calculations are more complicated, and the result is given by the rather bulky expression

$$\begin{aligned}
\text{Ver}_2 = & \mp \frac{e_0^2}{16} f^{ABC} f^{CDE} \int \frac{d^6 p}{(2\pi)^6} \frac{d^6 q}{(2\pi)^6} \frac{d^6 k}{(2\pi)^6} d^8 \theta du v^{++A} (-k - q - p, \theta, u) \\
& \times \left\{ \frac{2((k+q+p)^4 R_{k+q+p} - q^4 R_q)}{(k+q+p)^2 - q^2} V^{--B}(k, \theta, u) V^{--D}(p, \theta, u) v^{++E}(q, \theta, u) \right. \\
& - \left[\frac{(q+p)^4 R_{q+p}}{((q+p)^2 - (k+q+p)^2)((q+p)^2 - q^2)} + \frac{q^4 R_q}{(q^2 - (k+q+p)^2)(q^2 - (q+p)^2)} \right. \\
& + \left. \frac{(k+q+p)^4 R_{k+q+p}}{((k+q+p)^2 - (q+p)^2)((k+q+p)^2 - q^2)} \right] (2V^{--B}(k, \theta, u) D^{--} \\
& + D^{--} V^{--B}(k, \theta, u)) (D^+)^4 (2V^{--D}(p, \theta, u) D^{--} + D^{--} V^{--D}(p, \theta, u)) v^{++E}(q, \theta, u) \left. \right\}. \quad (\text{A13})
\end{aligned}$$

Strictly speaking, this expression is not quadratic in V^{++} , because each V^{--} is an infinite series in V^{++} . For constructing the vertex with two external lines of V^{++} , we need to take only linear parts from each V^{--} . It should be also noted that the terms quadratic in the background gauge superfield which are present in Eq. (A12) were not included into Eq. (A13).

We will also need the terms proportional to the first power of F^{++} . Some of them come from the expression (A5), namely, from the terms

$$\begin{aligned}
\frac{1}{2} \delta^2 S_{\text{reg}} \rightarrow & \pm \frac{1}{4e_0^2} \text{tr} \int d^{14} z du \left\{ 2V^{--} \delta \left[\square R \left(\frac{\square}{\Lambda^2} \right) \right] \delta V^{++} + 2\delta V^{--} \delta R \left(\frac{\square}{\Lambda^2} \right) F^{++} \right. \\
& + \nabla^{--} \delta V^{--} \delta \left[\nabla^{++}, R \left(\frac{\square}{\Lambda^2} \right) \right] F^{++} + \nabla^{--} \delta V^{--} \left[\nabla^{++}, R \left(\frac{\square}{\Lambda^2} \right) \right] \delta F^{++} \left. \right\} \quad (\text{A14})
\end{aligned}$$

after the replacement $\delta V^{++} \rightarrow v^{++}$ and $V^{++} \rightarrow \mathbf{V}^{++}$.

The terms proportional to F^{++} also come from the gauge-fixing action (2.23). In the Feynman gauge, the contribution of the considered structure reads

$$S_{\text{gf}}|_{\xi_0=1} \rightarrow \mp \frac{1}{2e_0^2} \text{tr} \int d^{14} z du_1 du_2 e^{ib_1} e^{-ib_2} v_2^{++} e^{ib_2} e^{-ib_1} \frac{(u_1^- u_2^+)}{(u_1^+ u_2^+)^3} \left[\nabla_1^{++}, \square_1 R \left(\frac{\square_1}{\Lambda^2} \right) \right] v_1^{++}. \quad (\text{A15})$$

Starting from the expressions (A14) and (A15), after some calculations we obtain the vertex with two legs of the quantum gauge superfield and one leg of the background gauge superfield corresponding to F^{++} . The result can be written in the form

$$\begin{aligned}
\text{Ver}_3 = & \pm \frac{e_0}{8} f^{ABC} \int \frac{d^6 p}{(2\pi)^6} \frac{d^6 k}{(2\pi)^6} d^8 \theta du_1 du_2 \left\{ \frac{2(u_1^- u_2^+)}{(u_1^+ u_2^+)^3} v^{++A} (-p - k, \theta, u_2) \mathbf{F}^{++B}(p, \theta, u_1) \right. \\
& \times v^{++C}(k, \theta, u_1) \frac{(k+p)^2 R_{k+p} - k^2 R_k}{(k+p)^2 - k^2} - \frac{1}{(u_1^+ u_2^+)^2} v^{++A} (-p - k, \theta, u_2) [D_1^{--} \mathbf{F}^{++B}(p, \theta, u_1) \\
& \times v^{++C}(k, \theta, u_1) - \mathbf{F}^{++B}(p, \theta, u_1) D_1^{--} v^{++C}(k, \theta, u_1)] \frac{k^2 R_k - p^2 R_p}{k^2 - p^2} + \int \frac{du_3}{(u_1^+ u_2^+)^2 (u_1^+ u_3^+)^2} \\
& \times \frac{R_{k+p} - R_p}{(k+p)^2 - p^2} \left[\left(D_1^{--} - \frac{2(u_1^- u_2^+)}{(u_1^+ u_2^+)} \right) (D_1^+)^4 v^{++A} (-p - k, \theta, u_2) \mathbf{F}^{++B}(p, \theta, u_1) \right. \\
& - (D_1^+)^4 v^{++A} (-p - k, \theta, u_2) D_1^{--} \mathbf{F}^{++B}(p, \theta, u_1) \left. \right] v^{++C}(k, \theta, u_3) + 2 \int \frac{du_3 (u_1^- u_2^+)}{(u_1^+ u_2^+)^3 (u_1^+ u_3^+)^2} \\
& \times \left[\frac{R_{k+p} - R_p}{(k+p)^2 - p^2} - \frac{R_{k+p} - R_k}{(k+p)^2 - k^2} \right] v^{++A} (-p - k, \theta, u_2) \mathbf{F}^{++B}(p, \theta, u_1) (D_1^+)^4 v^{++C}(k, \theta, u_3) \left. \right\}. \quad (\text{A16})
\end{aligned}$$

The contribution proportional to F^{++} in the vertex with two legs of the quantum gauge superfield and two legs of the background gauge superfield is rather bulky and complicated. That is why we do not present here the

general expression for it. Below, we write down only those terms which nontrivially contribute to the divergent part of the one-loop effective action. In particular, in the expression (A19) below, we do not include the following parts:

- (1) The terms that contain less than four spinor derivatives distributed among the quantum gauge superfields. They do not contribute to the supergraph (5) in Fig. 1 because of the identity

$$(D^+)^{\alpha 8}(\theta_1 - \theta_2)|_{\theta_1=\theta_2} = 0 \quad \text{for any } \alpha \leq 7 \quad (\text{A17})$$

(four spinor derivatives come from the gauge propagator).

- (2) The terms containing more than four spinor derivatives acting on the external legs, which produce loop integrals convergent in the ultraviolet region. Note that F^{++} already contains four spinor derivatives acting on the background gauge superfield.
- (3) The terms in which the number of the harmonic derivatives D^{--} is so small that the corresponding contributions to the tadpole graph vanish due to the identity $(u^+ u^+) = 0$.
- (4) The terms for which the corresponding contributions to the tadpole graph vanish due to the identities $f^{AAB} = 0$ or $(b_1 - b_2)|_{u_1=u_2} = 0$.

Moreover, as we will see in the next section, for calculating the contribution of the logarithmically divergent part of the supergraph (5) in Fig. 1 to the β function, we need to consider a limit of the vanishing external momentum. In this case, the background gauge superfields have zero momenta, and the momenta of the quantum gauge superfields differ only in sign. Therefore, for calculating the logarithmically divergent part of the supergraph (5), it is sufficient to take the vertex in the limit of the vanishing external momenta. Namely, we write the vertex under consideration in the form

$$\text{Ver}_4 = \int \frac{d^6 p}{(2\pi)^6} \frac{d^6 q}{(2\pi)^6} \frac{d^6 k}{(2\pi)^6} \text{Ver}_4(p, q, k), \quad (\text{A18})$$

where p and q denote the momenta of the background gauge superfield. The momentum of one quantum gauge superfield is denoted by k , and the second quantum gauge superfield has the momentum $-q - k - p$. Then (after some transformations), the vertex under consideration can be cast in the form

$$\begin{aligned} \text{Ver}_4(p=0, q=0, k) = & \pm \frac{e_0^2}{4} f^{ABC} f^{CDE} \int d^8 \theta du_1 du_2 \frac{(u_1^- u_2^+)}{(u_1^+ u_2^+)^3} (D_1^+)^4 v^{++A}(-k, \theta, u_2) \\ & \times F_{\text{linear}}^{++B}(0, \theta, u_1) V_{\text{linear}}^{--D}(0, \theta, u_1) ((k^2 R_k)'' + R_k') D_1^{--} v^{++E}(k, \theta, u_1) \\ & + \text{the terms that do not contribute to the supergraph (5),} \end{aligned} \quad (\text{A19})$$

where primes denote derivatives with respect to k^2 . The terms collected in Eq. (A19) are sufficient to calculate the supergraph (5) in Fig. 1. We briefly describe this calculation in the next section.

APPENDIX B: DETAILS OF THE HARMONIC SUPERGRAPH CALCULATION

With the help of the vertices constructed in the previous section, we can calculate the divergent contributions to the one-loop effective action produced by the superdiagrams containing a loop of the quantum gauge superfield. These superdiagrams appear in Fig. 1 with numbers (2) and (5). It is expedient to split them into parts containing vertices proportional to various powers of F^{++} . In Sec. III A, we demonstrated that the terms containing two (or more)

F^{++} correspond to finite contributions (because the result of the calculation should always be given by an integral over $d^{14}z du$). Therefore, the diagrams containing these terms can be omitted. The remaining supergraphs are presented in Fig. 3. They either do not contain F^{++} at all or are linear in it. The former superdiagrams are quadratically divergent, while the latter ones diverge logarithmically.

In the quadratically divergent graphs (2a) and (5a), the vertices are generated by the expression (A10). They are given by Eqs. (A12) and (A13). Note that, calculating these diagrams, it is impossible to set the external momentum p_μ equal to 0, because the subleading terms proportional to p^2 are logarithmically divergent. However, in fact, the contributions (2a) and (5a) vanish. Namely, let us first consider the superdiagram (2a). It is given by the expression

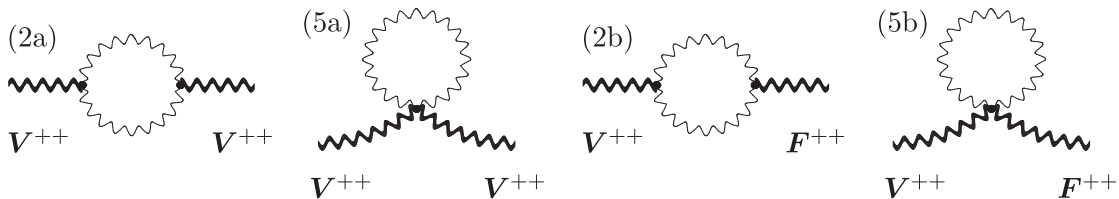


FIG. 3. Various contributions to the two-point Green function of the background gauge superfield from the harmonic supergraphs containing a loop of the quantum gauge superfield.

$$\begin{aligned}
& -\frac{iC_2}{4} \text{tr} \int \frac{d^6 p}{(2\pi)^6} \frac{d^6 k}{(2\pi)^6} d^8 \theta_1 d^8 \theta_2 \frac{du_1 du_2 du_3 du_4}{(u_1^+ u_3^+)^2 (u_2^+ u_4^+)^2} V^{++}(-p, \theta_1, u_3) V^{++}(p, \theta_2, u_4) \\
& \times \frac{((k+p)^4 R_{k+p} - k^4 R_k)^2}{k^4 R_k (k+p)^4 R_{k+p} ((k+p)^2 - k^2)^2} \left[\left(D_1^{--} - \frac{(u_1^- u_3^+)}{(u_1^+ u_3^+)} \right) \left(D_2^{--} - \frac{(u_2^- u_4^+)}{(u_2^+ u_4^+)} \right) (D_1^+)^4 \right. \\
& \times \delta^8(\theta_1 - \theta_2) \delta^{(-2,2)}(u_1, u_2) \cdot (D_2^+)^4 \delta^8(\theta_1 - \theta_2) \delta^{(-2,2)}(u_2, u_1) - \left(D_1^{--} - \frac{(u_1^- u_3^+)}{(u_1^+ u_3^+)} \right) \\
& \times (D_1^+)^4 \delta^8(\theta_1 - \theta_2) \delta^{(-2,2)}(u_1, u_2) \cdot \left(D_2^{--} - \frac{(u_2^- u_4^+)}{(u_2^+ u_4^+)} \right) (D_2^+)^4 \delta^8(\theta_1 - \theta_2) \delta^{(-2,2)}(u_2, u_1) \left. \right]. \quad (\text{B1})
\end{aligned}$$

First, we note that the minimal number of spinor covariant derivatives placed between two $\delta^8(\theta_1 - \theta_2)$ is equal to 8. Therefore, after integrating by parts with respect to $(D_2^+)^4$, all derivatives will act on $(D_1^+)^4 \delta^8(\theta_1 - \theta_2)$. After that, we use the identity

$$\delta^8(\theta_1 - \theta_2) (D_1^+)^4 (D_2^+)^4 \delta^8(\theta_1 - \theta_2) = (u_1^+ u_2^+)^4 \delta^8(\theta_1 - \theta_2). \quad (\text{B2})$$

Next, we note that no more than two harmonic derivatives D^{--} can act on this structure. Therefore, the result for the supergraph vanishes because

$$(u_1^+ u_2^+)|_{u_1=u_2} = 0. \quad (\text{B3})$$

The second quadratically divergent supergraph (5a) vanishes for the same reason. Indeed, various terms in the vertex (A13) contain no more than two harmonic derivatives. Therefore, they cannot produce a nonvanishing result when acting on the expression

$$(u_1^+ u_2^+)^4, \quad (\text{B4})$$

which comes from the gauge propagator, after setting $u_2 = u_1$ (for this purpose one would need at least four derivatives D^{--}). Thus, the graph (5a) also gives the zero contribution.

The contributions of the supergraphs (2a) and (5a) with a loop of the Pauli-Villars superfield vanish for the same reasons. The expressions for them are obtained by replacing the propagators $(k^4 R_k)^{-1}$ by $(k^4 R_k + M^4)^{-1}$ and changing the overall sign of a supergraph.

Taking into account that the supergraphs (2a) and (5a) do not contribute to the one-loop effective action, while the ghosts and hypermultiplet contributions cancel each other, we conclude that there are no quadratic one-loop divergences in the considered theory.

The supergraphs (2b) and (5b) are logarithmically divergent. It is these supergraphs that determine the one-loop β function of the theory (2.8). We calculate the β function according to Eq. (3.12), taking into account that the limit $\Lambda \rightarrow \infty$ is equivalent to the limit of the vanishing external momentum $p \rightarrow 0$. In what follows, for each supergraph we will present the corresponding contribution to the expression

$$\begin{aligned}
\frac{d}{d \ln \Lambda} \Delta \Gamma_{V^{++}, \infty}^{(2)} &= \pm \frac{1}{8\pi} \text{tr} \int \frac{d^6 p}{(2\pi)^6} d^8 \theta du V_{\text{linear}}^{--} F_{\text{linear}}^{++} \frac{d}{d \ln \Lambda} (d^{-1}(\alpha_0, -p^2/\Lambda^2) - \alpha_0^{-1})|_{\Lambda \rightarrow \infty} \\
&= \pm \frac{1}{8\pi} \frac{d}{d \ln \Lambda} (d^{-1}(\alpha_0, -p^2/\Lambda^2) - \alpha_0^{-1})|_{p \rightarrow 0} \text{tr} \int \frac{d^6 p}{(2\pi)^6} d^8 \theta du V_{\text{linear}}^{--}(-p, \theta, u) F_{\text{linear}}^{++}(p, \theta, u) \\
&= \pm \frac{1}{8\pi} \cdot \frac{\beta(\alpha_0)}{\alpha_0^2} \text{tr} \int d\zeta^{(-4)} (F_{\text{linear}}^{++})^2, \quad (\text{B5})
\end{aligned}$$

where $\Delta \Gamma$ is a sum of quantum corrections to the effective action. The subscript V^{++} and the superscript (2) mean that we consider only the terms corresponding to the two-point Green function of the background gauge superfield, and the subscript ∞ denotes their divergent part. The derivative with respect to $\ln \Lambda$ should be taken at a fixed value of the *renormalized* coupling constant α .

Let us start with the supergraph (2b) in Fig. 3. To avoid too large expressions, we describe in detail only the contribution produced by the first term in the expression (A16) for the right vertex in the case when the loop corresponds to the quantum gauge superfield v^{++} .⁷ The Feynman rules give for it the expression

⁷The contribution of the supergraph with the Pauli-Villars superfield will be added in what follows.

$$\begin{aligned}
& -\frac{ie_0^2 C_2}{8} \frac{d}{d \ln \Lambda} \int \frac{d^6 p}{(2\pi)^6} \frac{d^6 k}{(2\pi)^6} \int d^8 \theta_1 d^8 \theta_2 \int du_1 du_2 du_3 \frac{(u_2^- u_3^+)}{(u_2^+ u_3^+)^3} \frac{1}{k^4 R_k (k+p)^4 R_{k+p}} \\
& \times \frac{(k+p)^4 R_{k+p} - k^4 R_k}{(k+p)^2 - k^2} \cdot \frac{(k+p)^2 R_{k+p} - k^2 R_k}{(k+p)^2 - k^2} \mathbf{F}_{\text{linear}}^{++A}(p, \theta_2, u_2) \{ (2V_{\text{linear}}^{--A}(-p, \theta_1, u_1) D_1^{--} \\
& + D_1^{--} V_{\text{linear}}^{--A}(-p, \theta_1, u_1)) ((D_1^+)^4 \delta^{(-2,2)}(u_1, u_2)) (D_3^+)^4 \delta^8(\theta_1 - \theta_2) \delta^{(2,-2)}(u_1, u_3) \\
& - (2V_{\text{linear}}^{--A}(-p, \theta_1, u_1) D_1^{--} + D_1^{--} V_{\text{linear}}^{--A}(-p, \theta_1, u_1)) ((D_1^+)^4 \delta^8(\theta_1 - \theta_2) \delta^{(-2,2)}(u_1, u_3)) \\
& \times (D_2^+)^4 \delta^8(\theta_1 - \theta_2) \delta^{(2,-2)}(u_1, u_2) \}. \tag{B6}
\end{aligned}$$

Here, the propagators give $(k^4 R_k)^{-1}$ and $((k+p)^4 R_{k+p})^{-1}$, where $R_k \equiv R(-k^2/\Lambda^2)$, and the other functions of the momenta are produced by the vertices.

The products of the Grassmannian δ functions do not vanish only if there are at least eight spinor derivatives D^+ acting on them. Then, using the identity (B2) and doing the integral over θ_2 , the expression (B6) can be cast in the form

$$\begin{aligned}
& -\frac{ie_0^2 C_2}{8} \frac{d}{d \ln \Lambda} \int \frac{d^6 p}{(2\pi)^6} \frac{d^6 k}{(2\pi)^6} \int d^8 \theta \int du_1 du_2 du_3 \frac{(u_2^- u_3^+)}{(u_2^+ u_3^+)^3} \frac{1}{k^4 R_k (k+p)^4 R_{k+p}} \\
& \times \frac{(k+p)^4 R_{k+p} - k^4 R_k}{(k+p)^2 - k^2} \cdot \frac{(k+p)^2 R_{k+p} - k^2 R_k}{(k+p)^2 - k^2} \mathbf{F}_{\text{linear}}^{++A}(p, \theta, u_2) \{ (2V_{\text{linear}}^{--A}(-p, \theta, u_1) D_1^{--} \\
& + D_1^{--} V_{\text{linear}}^{--A}(-p, \theta, u_1)) ((u_1^+ u_3^+)^4 \delta^{(-2,2)}(u_1, u_2)) \delta^{(2,-2)}(u_1, u_3) - (2V_{\text{linear}}^{--A}(-p, \theta, u_1) D_1^{--} \\
& + D_1^{--} V_{\text{linear}}^{--A}(-p, \theta, u_1)) ((u_1^+ u_2^+)^4 \delta^{(-2,2)}(u_1, u_3)) \delta^{(2,-2)}(u_1, u_2) \}. \tag{B7}
\end{aligned}$$

Applying Eq. (B3) and using the relation

$$(u_1^+ u_3^+)^4 \delta^{(-2,2)}(u_1, u_2) = (u_2^+ u_3^+)^4 \delta^{(2,-2)}(u_1, u_2), \tag{B8}$$

this contribution can be written as

$$\begin{aligned}
& -\frac{ie_0^2 C_2}{4} \frac{d}{d \ln \Lambda} \int \frac{d^6 p}{(2\pi)^6} \frac{d^6 k}{(2\pi)^6} \int d^8 \theta \int du_1 du_2 du_3 \frac{(u_2^- u_3^+)(u_2^+ u_3^+)}{k^4 R_k (k+p)^4 R_{k+p}} \\
& \times \frac{(k+p)^4 R_{k+p} - k^4 R_k}{(k+p)^2 - k^2} \cdot \frac{(k+p)^2 R_{k+p} - k^2 R_k}{(k+p)^2 - k^2} \mathbf{F}_{\text{linear}}^{++A}(p, \theta, u_2) V_{\text{linear}}^{--A}(-p, \theta, u_1) \\
& \times \{ D_1^{--} \delta^{(2,-2)}(u_1, u_2) \delta^{(2,-2)}(u_1, u_3) - D_1^{--} \delta^{(2,-2)}(u_1, u_3) \delta^{(2,-2)}(u_1, u_2) \}. \tag{B9}
\end{aligned}$$

Next, we calculate the integral over u_1^+ with the help of the harmonic δ functions:

$$\begin{aligned}
& -\frac{ie_0^2 C_2}{4} \frac{d}{d \ln \Lambda} \int \frac{d^6 p}{(2\pi)^6} \frac{d^6 k}{(2\pi)^6} \int d^8 \theta \int du_2 du_3 \frac{(u_2^- u_3^+)(u_2^+ u_3^+)}{k^4 R_k (k+p)^4 R_{k+p}} \\
& \times \frac{(k+p)^4 R_{k+p} - k^4 R_k}{(k+p)^2 - k^2} \cdot \frac{(k+p)^2 R_{k+p} - k^2 R_k}{(k+p)^2 - k^2} \mathbf{F}_{\text{linear}}^{++A}(p, \theta, u_2) \{ V_{\text{linear}}^{--A}(-p, \theta, u_3) \\
& \times D_3^{--} \delta^{(2,-2)}(u_3, u_2) - V_{\text{linear}}^{--A}(-p, \theta, u_2) D_2^{--} \delta^{(2,-2)}(u_2, u_3) \}. \tag{B10}
\end{aligned}$$

After integrating the harmonic derivatives by parts and keeping in mind that

$$(u^+ u^-) = -(u^- u^+) = 1, \quad (u^+ u^+) = 0, \quad (u^- u^-) = 0, \tag{B11}$$

this expression can be transformed to the form

$$\begin{aligned}
& -\frac{ie_0^2 C_2}{2} \frac{d}{d \ln \Lambda} \int \frac{d^6 p}{(2\pi)^6} \int d^8 \theta du V_{\text{linear}}^{--A}(-p, \theta, u) \mathbf{F}_{\text{linear}}^{++A}(p, \theta, u) \int \frac{d^6 k}{(2\pi)^6} \frac{1}{k^4 R_k (k+p)^4 R_{k+p}} \\
& \times \frac{(k+p)^4 R_{k+p} - k^4 R_k}{(k+p)^2 - k^2} \cdot \frac{(k+p)^2 R_{k+p} - k^2 R_k}{(k+p)^2 - k^2}. \tag{B12}
\end{aligned}$$

The integral over the loop momentum k^μ in the limit $\Lambda \rightarrow \infty$ is logarithmically divergent and depends on p^2/Λ^2 . After differentiating this integral with respect to $\ln \Lambda$ in the limit $\Lambda \rightarrow \infty$ (or, equivalently, $p^\mu \rightarrow 0$), one obtains a finite constant. Therefore, the integral over k^μ should be calculated in the limit of the vanishing external momentum (and obviously after performing Wick rotation). The corresponding Euclidean momentum will be denoted by K^μ . Thus, the contribution to the expression (B5) considered is given by

$$-\frac{e_0^2 C_2}{2} \int d\zeta^{(-4)} (F_{\text{linear}}^{++A})^2 \frac{d}{d \ln \Lambda} \int \frac{d^6 K}{(2\pi)^6} \frac{(K^4 R_K)' (K^2 R_K)'}{K^8 R_K^2}, \quad (\text{B13})$$

where primes mark the derivatives with respect to K^2 .

It is also necessary to add the corresponding contribution coming from the supergraph with a loop of the Pauli-Villars superfield. By construction, the vertices containing the Pauli-Villars superfield are the same as the ones quadratic in the quantum gauge superfield v^{++} , while the propagator is different due to the presence of the mass term. Moreover, each loop of the Pauli-Villars superfield contributes a minus sign; see Sec. II D. Therefore, the Pauli-Villars contribution can be obtained through replacing the momentum part of the propagator as

$$\frac{1}{K^4 R_K} \rightarrow \frac{1}{K^4 R_K + M^4}, \quad (\text{B14})$$

as well as changing the overall sign. Then the considered contribution [corresponding to the first term in Eq. (A16)] takes the form

$$-C_2 \text{tr} \int d\zeta^{(-4)} (F_{\text{linear}}^{++})^2 \frac{d}{d \ln \Lambda} \int \frac{d^6 K}{(2\pi)^6} \left\{ \frac{(K^4 R_K)' (K^2 R_K)'}{K^8 R_K^2} - \frac{(K^4 R_K)' (K^2 R_K)'}{(K^4 R_K + M^4)^2} \right\}. \quad (\text{B15})$$

The contributions of other terms in the expression (A16) are obtained similarly, but for calculating some of them it is also necessary to use the identity

$$\delta^8(\theta_1 - \theta_2) (D_1^+)^4 (D_2^+)^4 (D_3^+)^4 \delta^{14}(z_1 - z_2) = (u_1^+ u_2^+)^2 (u_1^+ u_3^+)^2 (u_2^+ u_3^+)^2 \partial^2 \delta^{14}(z_1 - z_2). \quad (\text{B16})$$

The sum of all contributions is given by the expression

$$\Delta_{(2b)} = C_2 \text{tr} \int d\zeta^{(-4)} (F_{\text{linear}}^{++})^2 \frac{d}{d \ln \Lambda} \int \frac{d^6 K}{(2\pi)^6} \left\{ -\frac{(K^4 R_K)' (K^2 R_K)'}{K^8 R_K^2} - \frac{(K^4 R_K)'}{K^8 R_K} + \frac{(K^4 R_K)' (K^2 R_K)'}{(K^4 R_K + M^4)^2} + \frac{(K^4 R_K)' R_K}{(K^4 R_K + M^4)^2} \right\}. \quad (\text{B17})$$

Next, it is necessary to calculate the supergraph (5b) in Fig. 3. The corresponding vertex is given by Eq. (A19). Once again, for simplicity, we will assume that a loop corresponds to the quantum gauge superfield v^{++} . In order to avoid the appearance of ill-defined expressions, we present the vertex in the form

$$\begin{aligned} & \pm \frac{e_0^2}{4} f^{ABC} f^{CDE} \lim_{u_3 \rightarrow u_1} \int d^8 \theta du_1 du_2 \frac{(u_1^- u_2^+)}{(u_1^+ u_2^+)^3} (D_1^+)^4 v^{++A}(-k, \theta, u_2) F_{\text{linear}}^{++B}(0, \theta, u_1) \\ & \times V_{\text{linear}}^{--D}(0, \theta, u_1) ((k^2 R_k)'' + R_k') D_3^{--} v^{++E}(k, \theta, u_3) + \dots, \end{aligned} \quad (\text{B18})$$

where dots denote the terms which give zero contributions to the tadpole graph. Then the corresponding part of the expression (B5) obtained with the help of the Feynman rules is written as

$$\begin{aligned} & -\frac{ie_0^2 C_2}{2} \lim_{u_3 \rightarrow u_1} \int \frac{d^6 p}{(2\pi)^6} \int d^8 \theta du_1 du_2 V_{\text{linear}}^{--A}(-p, \theta, u_1) F_{\text{linear}}^{++A}(p, \theta, u_1) \frac{(u_1^- u_2^+)}{(u_1^+ u_2^+)^3} (D_1^+)^4 \\ & \times (D_2^+)^4 \delta^8(\theta_1 - \theta_2)|_{\theta_1=\theta_2=\theta} D_3^{--} \delta^{(-2,2)}(u_2, u_3) \frac{d}{d \ln \Lambda} \int \frac{d^6 k}{(2\pi)^6} \frac{(k^2 R_k)'' + R_k'}{k^4 R_k}. \end{aligned} \quad (\text{B19})$$

Note that here we took into account that, when calculating the derivative of the integral over the loop momentum k^μ with respect to $\ln \Lambda$ in the limit $\Lambda \rightarrow \infty$, the external momentum p^μ in the integral under consideration *should* be set equal to 0.

Taking into account that

$$(D_1^+)^4(D_2^+)^4\delta^8(\theta_1 - \theta_2)|_{\theta_1=\theta_2=\theta} = (u_1^+u_2^+)^4, \quad (\text{B20})$$

the expression (B6) can be rewritten as

$$\begin{aligned} & -\frac{ie_0^2 C_2}{2} \int \frac{d^6 p}{(2\pi)^6} \int d^8 \theta du_1 du_2 V_{\text{linear}}^{--A}(-p, \theta, u_1) F_{\text{linear}}^{++A}(p, \theta, u_1) \\ & \times (u_1^- u_2^+)(u_1^+ u_2^+) D_1^{--} \delta^{(-2,2)}(u_2, u_1) \frac{d}{d \ln \Lambda} \int \frac{d^6 k}{(2\pi)^6} \frac{(k^2 R_k)'' + R'_k}{k^4 R_k}. \end{aligned} \quad (\text{B21})$$

Integrating by parts with respect to the derivative D_1^{--} , making use of the identities (B11), and calculating the harmonic integral over u_2 with the help of the harmonic δ function, we cast the result in the form

$$\frac{ie_0^2 C_2}{2} \int \frac{d^6 p}{(2\pi)^6} \int d^8 \theta du V_{\text{linear}}^{--A}(-p, \theta, u) F_{\text{linear}}^{++A}(p, \theta, u) \frac{d}{d \ln \Lambda} \int \frac{d^6 k}{(2\pi)^6} \frac{(k^2 R_k)'' + R'_k}{k^4 R_k}. \quad (\text{B22})$$

After Wick rotation and adding the Pauli-Villars contribution [which is obtained by the replacement (B14) and changing the overall sign], we obtain the contribution of the supergraph (5b) in Fig. 3 to the expression (B5):

$$\Delta_{(5b)} = C_2 \text{tr} \int d\zeta^{(-4)} (F_{\text{linear}}^{++})^2 \frac{d}{d \ln \Lambda} \int \frac{d^6 K}{(2\pi)^6} \left\{ \frac{(K^2 R_K)'' + R'_K}{K^4 R_K} - \frac{(K^2 R_K)'' + R'_K}{K^4 R_K + M^4} \right\}. \quad (\text{B23})$$

Summing up the expressions (B17) and (B23), we obtain the contribution to the expression (B5) coming from the one-loop superdiagrams (2b) and (5b) in Fig. 3:

$$\begin{aligned} \frac{d}{d \ln \Lambda} \Delta \Gamma_{v^{++}, \infty}^{(2)} &= C_2 \text{tr} \int d\zeta^{(-4)} (F_{\text{linear}}^{++})^2 \frac{d}{d \ln \Lambda} \int \frac{d^6 K}{(2\pi)^6} \left\{ -\frac{4}{K^6} - \frac{R'_K}{K^4 R_K} - \frac{(R'_K)^2}{K^2 R_K^2} \right. \\ & \left. + \frac{R''_K}{K^2 R_K} + \frac{4K^2 R_K^2 + 4K^4 R_K R'_K + K^6 (R'_K)^2}{(K^4 R_K + M^4)^2} - \frac{K^2 R''_K + 3R'_K}{K^4 R_K + M^4} \right\} + O(\alpha_0). \end{aligned} \quad (\text{B24})$$

It is convenient to split this expression into two parts. One of them does not contain the mass M and comes from the supergraphs with a loop of the quantum gauge superfield v^{++} . The other part contains terms with massive propagators and comes from the superdiagrams involving a loop of the Pauli-Villars superfield P^{++} . Like in the case of $4D$, $\mathcal{N} = 1$ supersymmetric theories regularized by higher covariant derivatives [71], it is possible to show that each of these two parts can be written as an integral of the double total derivative:

$$\frac{d}{d \ln \Lambda} \Delta \Gamma_{v^{++}, \infty}^{(2)} = \frac{C_2}{4} \text{tr} \int d\zeta^{(-4)} (F_{\text{linear}}^{++})^2 \frac{d}{d \ln \Lambda} \int \frac{d^6 K}{(2\pi)^6} \frac{\partial^2}{\partial K_\mu \partial K^\mu} \left\{ \frac{1}{K^4} \ln(K^4 R_K) - \frac{1}{K^4} \ln(K^4 R_K + M^4) \right\} + O(\alpha_0). \quad (\text{B25})$$

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