

Goldstone bosons at nonzero temperature

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Spontaneous symmetry breaking in quantum field theories at nonzero temperature still holds fundamental open questions, in particular what happens to vacuum Goldstone bosons when the temperature is increased. By investigating a complex scalar field theory on the lattice we demonstrate that Goldstone bosons at nonzero temperature behave like screened massless particlelike excitations, so-called thermparticles, which continue to exist even in the symmetry-restored phase of the theory. We provide nonperturbative evidence for the functional form of the Goldstone mode's dissipative behavior, and determine its corresponding spectral properties. Since the persistence of thermal Goldstone modes within symmetry-restored phases is predicted to be a model-independent characteristic, this has fundamental consequences for systems in which continuous symmetries are restored at high temperatures.

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Introduction. The spontaneous breaking of continuous symmetries plays a fundamental role for many physical systems. In the relativistic regime, Goldstone's theorem [1,2] states that the spontaneous breaking of a continuous symmetry implies the existence of massless bosons. Although the theoretical and experimental consequences of this theorem are well-understood at zero temperature, relatively little is known from first principles about systems at nonzero temperature, and in particular those which undergo phase transitions at high temperatures [3–5]. Understanding these characteristics is important for describing numerous phenomena, including phase transitions in the early Universe, or the phase diagram of nuclear matter.

For relativistic systems at zero-temperature, Goldstone bosons leave distinct signatures on the correlation functions of the theory, in particular the appearance of $\delta(p^2)$ singularities in the Fourier transform of the two-point function $\langle [j_0(x), A(y)] \rangle$, where j_μ is the conserved current associated with the symmetry, and A is a local field whose transformation under the symmetry has a nonvanishing vacuum expectation value, $\langle \delta A \rangle \neq 0$ [6]. For nonzero temperatures $T = 1/\beta > 0$ the boost invariance of the system is lost, and it is no longer clear that stable massless Goldstone modes will exist. Nevertheless, the Fourier transform of the thermal commutator $\langle [j_0(x), A(y)] \rangle_\beta$ continues to contain a zero-energy singularity $\delta(\omega)$ in the limit

$\vec{p} \rightarrow 0$, a Goldstone quasi-particle [6]. In Ref. [7] the authors made a significant breakthrough by further demonstrating that Goldstone modes for $T > 0$ have the structure of distinct particlelike excitations. To reach this conclusion the authors considered finite-temperature systems which satisfy the fundamental nonperturbative QFT constraints proposed in Ref. [8], namely that one has local causal fields¹ which transform covariantly under spacetime translations and spatial rotations, and a thermal ground state which is invariant with respect to these transformations. Under these conditions one can demonstrate that the thermal expectation values of field commutators satisfy a nonperturbative spectral representation [8,12]. In particular, for a complex scalar field $\phi(x)$ at nonzero temperature the spectral function $\rho(\omega, \vec{p})$, defined as the Fourier transform of the thermal commutator $\langle [\phi(x), \phi^\dagger(0)] \rangle_\beta$, has the following general form:

$$\rho(\omega, \vec{p}) = \int_0^\infty ds \int \frac{d^3 \vec{u}}{(2\pi)^2} \epsilon(\omega) \times \delta(\omega^2 - (\vec{p} - \vec{u})^2 - s) \tilde{D}_\beta(\vec{u}, s), \quad (1)$$

where $\epsilon(\omega)$ is the sign function. This represents the $T > 0$ generalization of the well-known Källén-Lehmann representation that exists for QFTs at $T = 0$ [13,14]. An important implication of Eq. (1) is that the behavior of the spectral function is fixed by the thermal spectral density

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¹For a complex scalar theory causality means that the fields satisfy $[\phi(x), \phi^\dagger(y)] = 0$ for $(x - y)^2 < 0$, which guarantees that spacelike separated measurements commute with one another and are therefore causal [9–11].

$\tilde{D}_\beta(\vec{u}, s)$, and so its properties hold the key for determining the type of excitations that can exist, and how they are modified by changes in T . Since the Fourier transform of $\langle [j_0(x), A(y)] \rangle_\beta$ also satisfies an analogous representation [7], it follows from Goldstone's theorem that the position space thermal spectral density must satisfy the condition, $D_\beta(\vec{x}, s) \rightarrow \delta(s)$ for $T \rightarrow 0$, since then Eq. (1) implies the existence of a distinct massless component $\delta(p^2)$. In Ref. [7] it is explicitly demonstrated that the vacuum Goldstone singularity in $D_\beta(\vec{x}, s)$ persists for $T > 0$, even if the symmetry is restored at high temperatures. In particular, this means that the thermal spectral density contains a distinguished Goldstone contribution of the form,

$$D_\beta^G(\vec{x}, s) = D_\beta^G(\vec{x})\delta(s). \quad (2)$$

When $D_\beta^G(\vec{x})$ is nontrivial this causes the stable massless Goldstone peak in the spectral function at $p^2 = 0$ to become broadened, which describes the dissipative effects that the Goldstone boson experiences as it moves through the thermal medium. Since $D_\beta^G(\vec{x})$ also reduces the amplitude of the Goldstone propagator, it represents a thermal damping factor. The particlelike structure described by Eq. (2) is the massless realization of a general proposition $D_\beta(\vec{x}, s) = D_\beta(\vec{x})\delta(s - m^2)$, first put forward in Ref. [12] for how stable vacuum particle states with mass m should behave when $T > 0$. These were later referred to as *thermoparticles* in order to draw a distinction between other types of thermal excitations such as quasiparticles [15]. Evidence for the existence of massive thermoparticles has since been found in scalar theories [16], as well as more complex theories such as quantum chromodynamics (QCD) [17,18].

Signatures in euclidean correlation functions. If Goldstone bosons behave like massless thermoparticles when $T > 0$, these excitations will leave distinct signatures on the correlation functions of the theory. Understanding the impact this has on the behavior of Euclidean correlation functions is important since many of the nonperturbative techniques for studying thermal correlation functions, such as lattice calculations, are restricted to or optimized for calculations in imaginary time. In this work we will focus on QFTs involving complex scalar fields, where the real-time thermal two-point function is defined $\mathcal{W}(x_0, \vec{x}) = \langle \phi(x_0, \vec{x})\phi^\dagger(0) \rangle_\beta$. Due to the spectral representation in Eq. (1), and the condition of thermal equilibrium,² it

²The Kubo-Martin-Schwinger (KMS) condition [19]: $\mathcal{W}(x_0, \vec{x}) = \mathcal{W}(-x_0 - i\beta, -\vec{x})$ defines the notion of thermal equilibrium, and implies the following connection between the momentum-space two-point function and spectral function: $\tilde{\mathcal{W}}(\omega, \vec{p}) = \rho(\omega, \vec{p})(1 - e^{-\beta\omega})^{-1}$.

follows that the two-point function also possesses a spectral representation,

$$\mathcal{W}(x_0, \vec{x}) = \int_0^\infty ds \mathcal{W}_\beta^{(s)}(x_0, \vec{x}) D_\beta(\vec{x}, s), \quad (3)$$

where $\mathcal{W}_\beta^{(s)}(x_0, \vec{x})$ is the thermal two-point function for a free particle³ with mass $\sqrt{s}$. For the Euclidean two-point function $C(\tau, \vec{x}) = \langle \phi(\tau, \vec{x})\phi^\dagger(0) \rangle_\beta$, understanding the spatial variation of $C(\tau = 0, \vec{x})$ amounts to determining $\mathcal{W}(x_0 = 0, \vec{x})$. Equation (3) therefore implies that the Goldstone mode in Eq. (2) gives the following discrete contribution to the spatial Euclidean two-point function:

$$C^G(0, \vec{x}) = \frac{\coth(\frac{\pi|\vec{x}|}{\beta})}{4\pi\beta|\vec{x}|} D_\beta^G(\vec{x}). \quad (4)$$

Since $D_\beta^G(\vec{x}) \xrightarrow{T \rightarrow 0} \alpha_0$, with α_0 a constant, the Goldstone contribution to the two-point function reduces to that of a massless vacuum particle in this limit

$$C^G(0, \vec{x}) \xrightarrow{T \rightarrow 0} \frac{\alpha_0}{4\pi^2|\vec{x}|^2}. \quad (5)$$

Another Euclidean correlation function of particular relevance is the spatial screening correlator along the z -axis, which has the form,

$$\begin{aligned} C(z) &= \int dx dy d\tau C(\tau, \vec{x}) \\ &= \frac{1}{2} \int_0^\infty ds \int_{|z|}^\infty dR e^{-R\sqrt{s}} D_\beta(R, s), \end{aligned} \quad (6)$$

where the representation in the second line follows from Eq. (1) [17]. Here one defines $D_\beta(R, s) = D_\beta(|\vec{x}| = R, s)$, which makes use of the fact that the thermal spectral density only depends on $|\vec{x}|$ due to rotational invariance. In this case the Goldstone mode gives the contribution,

$$C^G(z) = \frac{1}{2} \int_{|z|}^\infty dR D_\beta^G(R). \quad (7)$$

Now that we understand the impact of the thermal Goldstone modes derived in Ref. [7] on Euclidean correlation functions, we will investigate the presence of these modes in lattice data.

Analysis of $U(1)$ scalar lattice field theory. The simplest QFT model for which a continuous symmetry is spontaneously broken in vacuum and restored at high

³This two-point function has the form $\mathcal{W}_\beta^{(s)}(x_0, \vec{x}) = \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot x} 2\pi\epsilon(\omega)\delta(\omega^2 - |\vec{p}|^2 - s)(1 - e^{-\beta\omega})^{-1}$.

temperatures is the U(1) complex scalar field theory. In the broken phase of the theory at $T = 0$, the scalar field has a nonvanishing vacuum expectation value $|v|^2 = \langle \phi \rangle \langle \phi^\dagger \rangle$, and the model contains a massless Goldstone mode and a resonancelike σ mode, which is unstable on account of its decay into Goldstone bosons. The model is expected to undergo a second-order phase transition at some critical temperature T_c , above which $|v|^2 = 0$ and the global U(1) symmetry is restored.⁴ For our analysis we work with the following lattice discretization of the action:

$$S = a^4 \sum_{x \in \Lambda_a} \left[\sum_{\mu} \left(\frac{1}{2} \Delta_{\mu}^f \phi^*(x) \Delta_{\mu}^f \phi(x) \right) + \frac{m_0^2}{2} \phi^*(x) \phi(x) + \frac{g_0}{4!} (\phi^*(x) \phi(x))^2 \right], \quad (8)$$

where Δ_{μ}^f is the lattice forward derivative, and $a > 0$ is the lattice spacing. In order to avoid the potential triviality of the model we keep the lattice spacing fixed throughout, so that it is sufficient to consider nonrenormalized correlation functions, while $T = (aN_{\tau})^{-1}$ can be varied in discrete steps only. Whether one is in the broken phase or not depends on the specific value of the bare parameters m_0 and g_0 . For fixed g_0 , it turns out that $(am_0)^2$ is necessarily negative in the broken phase, and hence am_0 must be a pure imaginary number. We further require a sufficiently fine and large lattice to ensure that the lattice temperature covers both the symmetry-broken and restored phases for large and small values of N_{τ} , respectively. Having scanned a range of bare parameter values, we found that $(am_0, g_0) = (0.297i, 0.85)$ was a good choice. In the vacuum broken symmetry phase the expectation value $|v|$ sets the physical scale, and separates long $|\vec{x}||v| > 1$ and short $|\vec{x}||v| < 1$ distances. As we shall see, our choice of parameters results in a short distance range $|\vec{x}| \lesssim 11a$, and a UV cutoff $\Lambda/|v| = \pi/(a|v|) \approx 36$, such that cutoff effects should be very small for lattice correlators beyond a few lattice spacings.

Determining the phase of any lattice model numerically is a nontrivial task. Spontaneous symmetry breaking cannot occur in a finite spatial volume V , and thus requires a $V \rightarrow \infty$ extrapolation of the lattice correlation functions. On a spatially symmetric space-time volume $L^3 \times L_{\tau}$, where $L_{\tau} = aN_{\tau}$ and $L = aN_s$, the finite spatial-volume Euclidean two-point function $C_L(\tau, \vec{x}) = \langle \phi(\tau, \vec{x}) \phi^\dagger(0) \rangle_L$ satisfies the condition

$$\lim_{L \rightarrow \infty} C_L(\tau, \vec{x}) \xrightarrow{|\vec{x}| \rightarrow \infty} |v|^2. \quad (9)$$

There are different approaches for extracting $|v|^2$ from lattice data [22,23], but a simple way is to consider the limit of correlators evaluated at their largest spatial extent,

⁴For discussions regarding the perturbative characteristics of this model see [20,21] and the references within.

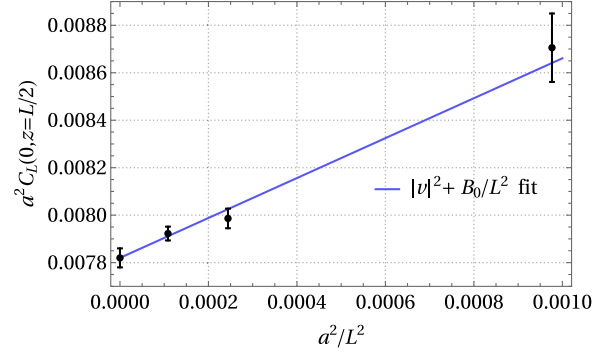


FIG. 1. Infinite-volume extrapolation of $a^2 C_L(0, L/2)$.

$$|v|^2 = \lim_{L \rightarrow \infty} C_L(0, |\vec{x}| = L/2). \quad (10)$$

An estimate of $|v|^2$ can then be obtained by extrapolating the $L \rightarrow \infty$ behavior using a range of correlators at sufficiently large values of N_s .

For $N_{\tau} = 32$ we expect the system to be in the low-temperature, vacuumlike broken phase, where the Goldstone mode dominates the behavior of $C_L(0, \vec{x})$, and the connected correlator should approach the massless particle form in Eq. (5) for sufficiently large volumes. To test this hypothesis, we performed fits of lattice correlator data for different volumes ($N_s = 32, 64, 96$) with the finite-volume functional form,

$$C_L(0, z) = c_0 + b_0 \left[\frac{1}{z^2} + \{z \rightarrow (L - z)\} \right], \quad (11)$$

where $C_L(0, z) = C_L(0, |\vec{x}| = z)$, and the final term accounts for the spatially periodic boundary conditions of our lattices. The fits were performed over a spatial range $[z_{\min}, L/2]$, where $z = L/2$ is the maximal lattice correlator extent, and $z_{\min} < L/2$ was varied in order to assess the fit stability. Keeping $z_{\min}/a > 5$ in order to avoid cutoff effects, we found that the continuum form Eq. (11) provided a very good description of the data across large ranges for each of the volumes considered. To estimate $|v|^2$ we therefore fit $C_L(0, z = L/2)$ to the functional form $|v|^2 + B_0/L^2$, as shown in Fig. 1, resulting in the nonzero infinite-volume extrapolation

$$a^2 |v|^2 = 0.00782(4). \quad (12)$$

The ratio $T/|v| = 1/(N_{\tau} a |v|)$ provides a qualitative measure for the temperature of the system, which in this case $T/|v| \approx 0.35$ confirms that it is in a cold vacuumlike state.⁵

⁵Strictly speaking, $|v|^2$ also requires an infinite N_{τ} extrapolation in order to represent the true vacuum value. However, the precise value is not important for our qualitative analysis, only that $|v|^2$ is significantly nonvanishing.

Overall, these results demonstrate that the $N_\tau = 32$ data is in the broken phase in the infinite-volume limit, and that the scalar correlation function is dominated by a massless vacuumlike Goldstone state, as is the case in the broken phase of the $O(4)$ model [24–28].

By contrast, for $N_\tau = 2$ we have $T/|v| \approx 5.65$, which indicates that the system is in the symmetry-restored phase. As discussed above, the Goldstone mode can still exist in this phase even though the symmetry is no longer broken, but it has the structure of a massless thermoparticle [7]. If this mode provides a dominant contribution, then $C(0, \vec{x})$ will take the form of Eq. (4). By investigating the properties of the spatial screening correlator $C(z)$, one can use Eq. (7) to determine the qualitative structure of the Goldstone damping factor $D_\beta^G(\vec{x})$. For this purpose, we performed fits to the lattice spatial screening correlator data with the finite-volume single-exponential ansatz

$$C_L(z) = d_L [e^{-m_L z} + \{z \rightarrow (L - z)\}]. \quad (13)$$

We found that Eq. (13) provides an excellent description of the data across the full range $[0, L/2]$ for each of the volumes considered ($N_s = 64, 96, 128$), with $\chi^2/\text{d.o.f.} \lesssim 1$ in all cases. The fit values obtained for m_L are plotted as dashed lines in Fig. 2, and have a relatively weak volume dependence. This strongly suggests that the theory is indeed in the symmetry-restored phase in the infinite-volume limit, and that due to Eq. (7) the Goldstone mode must have an exponential damping factor: $D_\beta^G(\vec{x}) = \alpha e^{-\gamma|\vec{x}|}$, and hence the spatial two-point function has the form

$$C^G(0, \vec{x}) = \frac{\coth(\frac{\pi|\vec{x}|}{\beta})}{4\pi\beta|\vec{x}|} \alpha e^{-\gamma|\vec{x}|}. \quad (14)$$

To test the consistency of this conclusion we fit the lattice spatial two-point function data using the finite-volume ansatz,

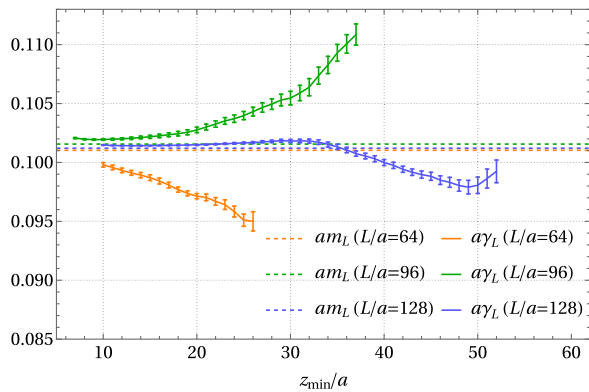


FIG. 2. Fitted values of γ_L as a function of $z_{\min}$ for different volumes. The horizontal dashed lines indicate the best-fit values of m_L extracted from the spatial screening correlator data.

$$C_L(0, z) = b_L \left[\frac{\coth(\frac{\pi z}{\beta})}{z} e^{-\gamma_L z} + \{z \rightarrow (L - z)\} \right]. \quad (15)$$

The fits were performed over a range $[z_{\min}, L/2]$, and the quality and stability of the fits were assessed by computing their sensitivity to $z_{\min}$. We found that Eq. (15) described the data increasingly better for larger volumes, as can be seen in Fig. 2 by the improved stabilization of the fit values for γ_L as a function of $z_{\min}$. In this case, the volume dependence of the fits was significantly more pronounced than for the spatial screening correlator. As in the broken phase, the $C_L(0, z)$ fits were restricted to $z_{\min}$ values larger than a few lattice spacings in order to avoid cutoff effects. Since $\coth(\pi z/\beta) z^{-1} \approx z^{-1}$ for almost all values of z , to further assess the robustness of Eq. (15) we also performed fits using a range of parametrizations of the form $B_L[z^{-n} e^{-\gamma_L z} + \{z \rightarrow (L - z)\}]$ with $n \neq 1$, and also without exponential factors. In all of these cases we found that the fit parameter values were highly sensitive to $z_{\min}$, and hence did not provide a good description of the data. If lattice cutoff effects are small, then a highly nontrivial test that the Goldstone mode has a massless thermoparticle structure is that the screening mass m_L and damping factor exponent γ_L must converge in the infinite-volume limit:

$$\lim_{L \rightarrow \infty} \gamma_L = \lim_{L \rightarrow \infty} m_L = \gamma. \quad (16)$$

In Fig. 2 we plot the values of γ_L obtained in the fit range $[z_{\min}, L/2]$ for which $\chi^2/\text{d.o.f.} \lesssim 1$, and where the fit errors are less than 1%. One can see that the m_L and γ_L values increasingly approach one another for larger volumes, which strongly indicates that the condition in Eq. (16) is satisfied.

Using the representation in Eq. (1) together with the damping factor $D_\beta^G(\vec{x})$, one can compute the corresponding spectral function of the Goldstone boson $\rho_G(\omega, \vec{p})$, which has the explicit form

$$\rho_G(\omega, \vec{p}) = \frac{4\alpha\omega\gamma}{(\omega^2 - |\vec{p}|^2 - \gamma^2)^2 + 4\omega^2\gamma^2}. \quad (17)$$

In Fig. 3 we plot ρ_G using the results obtained from the fits on the largest lattice volume $N_s = 128$. In contrast to the vacuumlike case at $N_\tau = 32$, where the Goldstone boson has the structure of a massless particle state with spectral function proportional to $\delta(p^2)$, at $N_\tau = 2$ the Goldstone mode experiences appreciable dissipative effects due to its interactions with the thermal medium, resulting in a spectral peak broadened around the vacuum singularity $p^2 = 0$. The Goldstone spectral function has similar properties to that of a vacuum resonance state, except that for ρ_G the width arises from purely collisional processes, as opposed to mixing effects brought about by the intrinsic instability of the state.

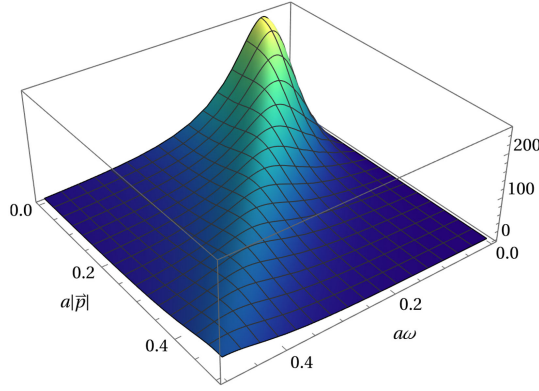


FIG. 3. Goldstone spectral function $\rho_G(a\omega, a|\vec{p}|)/a^2$ on the largest hot lattice, $N_s = 128$ at $N_\tau = 2$.

Physical implications. In this work we have focussed on the U(1) complex scalar theory, which confirms the fundamental prediction of Ref. [7] that the Goldstone mode can persist in the symmetry-restored phase, and has a massless thermoparticle structure. This prediction characterizes a model-independent property of Goldstone modes in QFTs at finite temperature, and therefore applies to all such systems in which a continuous symmetry is restored by thermal effects. Two particularly consequential implications are

- (i) In the nonrelativistic limit, any Goldstone mode which continues to exist in the low-temperature phase of a system should also leave distinct observable signatures above the corresponding critical temperature T_c . This applies, for example, to zero or low-density condensed matter systems, and could be used to determine the $T > T_c$ spectral properties of thermal Goldstone modes, such as spin waves in ferromagnets, or Landau phonons in superfluids [6].
- (ii) In Ref. [17] it was shown that pions in QCD persist above the pseudocritical temperature, and have the structure of *massive* thermoparticles. In the chiral limit this indicates that pions should behave like massless thermoparticle states above the critical temperature, which is precisely what one expects from Ref. [7], since pions are genuine Goldstone bosons in this limit.⁶ These observations suggest that the high-temperature phase transitions in both chiral

and physical QCD are not indicative of a change in the physical degrees of freedom associated with deconfinement, but may in fact be a reflection of the change in dissipative effects experienced by the (pseudo-)Goldstone bosons as the system increases in temperature. This is also supported by the Goldstone damping factor $D_\beta^G(\vec{x})$ in the U(1) scalar theory above T_c having the same qualitative form as that observed for pions in Ref. [17], even though the states in the scalar theory need not be composite.

Conclusions. We have explored the fundamental question of how spontaneously broken symmetries manifest themselves at nonzero temperature, and in particular what happens to the Goldstone bosons that exist in the vacuum theory as the temperature is increased. In Ref. [7] the authors established the nonperturbative signatures of thermal Goldstone bosons, and found that they need not cease to exist, even above the critical temperature T_c where the symmetry is restored. In this work, we investigated U(1) scalar field theory on the lattice, analysing how the scalar correlation functions behave in the broken and high-temperature symmetry-restored phases of the theory. We demonstrated that the Goldstone boson does indeed continue to exist above T_c , and has the properties of a screened massless particlelike excitation, a so-called *thermoparticle*. Since these characteristics are predicted to be model independent, they have implications for systems in which a continuous symmetry is restored at high temperatures. In particular, for zero or low-density nonrelativistic systems this indicates that Goldstone modes should leave distinct signatures for $T > T_c$, and in QCD it suggests that the thermal evolution of pions around the chiral crossover may not be due to a fundamental change in the degrees of freedom, but could instead be driven by changes in the dissipative effects experienced by these states.

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⁶For a recent discussion of this issue see Ref. [29].

Data availability. The data that support the findings of this article are openly available [30].

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