

Laboratory cosmology with free spinorial wave packets

Andre G. Campos^{✉,*}, Karen Z. Hatsagortsyan[✉], and Christoph H. Keitel[✉]

Max Planck Institute for Nuclear Physics, Heidelberg 69117, Germany



(Received 13 December 2024; accepted 4 August 2025; published 8 October 2025)

The electron quantum dynamics at crossing the black hole horizon is investigated. Exploiting the equivalence principle, we show a way to emulate the electron's dynamics via an appropriately chirped free electron wave packet, where the dynamical characteristics is encoded in the interference fringes of the wave packet. Apart from creating an analog of the gravitational effect, we deduce the electron's explicit wave function at the horizon of a black hole and provide quantitative predictions on the quantum properties of the electron crossing the horizon. These findings potentially pave the way for a new research avenue in laboratory cosmology based on the use of specially designed free electron wave packets.

DOI: [10.1103/jsb7-23q6](https://doi.org/10.1103/jsb7-23q6)

Introduction. Direct experimental investigation of black holes is elusive. To circumvent this limitation, the analog gravity method has emerged as a powerful tool for simulating gravitational phenomena using diverse physical systems with a focus on classical and semiclassical phenomena in black-hole physics. This approach proved instrumental in verifying the existence of Hawking (Unruh) radiation, a concept originally proposed by Zel'dovich over half a century ago [1,2]. Since then, analog gravity examples have been developed involving a wide range of physical media, including sound waves in fluid, microwave scattering, condensed matter systems, Bose-Einstein condensates, superfluid vortices, etc. [3–15] (see reviews in Refs. [16,17]), recently emulating black hole superradiance as well [18–21].

Analog gravity experiments provide *qualitative* counterparts of gravitational phenomena, such as an event horizon, entanglement of emitted particles, black hole back reaction, trying to address the problem of information loss due to black holes evaporation, etc. [22]. In contrast, we aim at a *quantitative* characterization of the quantum dynamics of electrons near a simulated black hole.

For describing the electron quantum dynamics in a gravitational field, we invoke the equivalence principle, according to which the gravitational field can be locally eliminated by an appropriate *local* Lorentz transformation. This correspondence between gravitational and noninertial local frames allows us to map electron behavior in a

gravitational field onto that of a free electron in the laboratory frame. As a result we access the complete quantum information of the electron dynamics in the vicinity of black holes by reconstructing the electron's wave function via a free spinorial wave packet. Moreover, the exact solutions derived in this paper, forming a complete basis, will facilitate the treatment of quantum electrodynamics (QED) processes accompanying the near-horizon dynamics. Successful implementation of this approach would open a new frontier in laboratory cosmology, enabling the study of particle quantum dynamics in gravitational fields by examining the behavior of specially engineered free electron wave packets and related observables in experiments.

To develop the approach mentioned in this paper, we begin by deriving the analytical solution of the Dirac equation for electron dynamics in the near-horizon region of a black hole as the benchmark for our simulation. To derive the mapping between the electron's wave function in the gravitation field and the free electron wave packet, we specially design a local accelerated frame (local Rindler frame) and find the free electron wave function in this frame. The latter solution in the form of spherical waves leads us directly to the design of the specially chirped free electron wave packet, which mimics the electron wave function at the black hole horizon and allows us to investigate the quantum features of the electron dynamics during crossing the horizon.

Quantum near horizon dynamics in a black hole. As the benchmark for our simulation with a free electron wave packet, we employ the analytical solution of the Dirac equation for the electron in the near-horizon region of a nonrotating black hole (see the derivation in the companion paper [23]), described by the standard Schwarzschild metric

$$ds^2 = e^A dt^2 - e^{-A} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

*Contact author: agontijo@mpi-hd.mpg.de

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where $A = \ln(1 - \frac{1}{r\kappa_*})$, with $r\kappa_* > 1$, $\kappa_* = \frac{1}{2M}$, M is the black hole mass. Relativistic units with $\hbar = c = G = 1$ (Planck constant $\hbar$, speed of light c , gravitational constant G) are applied throughout. In order to facilitate the application of the near-horizon approximation, we adapt to our purpose the known Kruskal-Szekeres coordinates to avoid singularities at the horizon via the following change of coordinates $T = \rho \sinh(\kappa_* \tau)$, $R = \rho \cosh(\kappa_* \tau)$, where the new radial coordinate ρ is related to the original radial coordinate r by the expression $\kappa_*^2 \rho^2 = (\kappa_* r - 1)e^{\kappa_* r}$. These equations have a convenient form without singularity at the horizon. In the near-horizon approximation $\kappa_* \rho \ll 1$, the metric reads

$$ds^2 = (\rho'^2 \kappa_*^2 d\tau^2 - d\rho'^2) - \kappa_*^{-2} (\sin^2 \theta d\phi^2 + d\theta^2). \quad (2)$$

where $\rho' = 2\rho/e^{1/2}$. With this metric, we find the analytical solution of the Dirac equation for the near-horizon of a black hole (see Ref. [23])

$$\psi = C_1 e^{-i\epsilon\tau} \begin{pmatrix} f(\rho) \\ 0 \\ ig(\rho) \cos(\theta) \\ ig(\rho) \sin(\theta) e^{i\phi} \end{pmatrix}. \quad (3)$$

Here $\kappa = -1$, corresponding to the spin-up electron with the orbital angular momentum $l = 0$, and

$$\begin{aligned} \frac{f(\rho)}{C_1 \sqrt{\kappa_* \rho}} &= K_{\frac{1}{2} + \frac{i\epsilon}{\kappa_*}} \left(\frac{2|\kappa| \kappa_* \rho}{\sqrt{e}} \right) - K_{\frac{1}{2} - \frac{i\epsilon}{\kappa_*}} \left(\frac{2|\kappa| \kappa_* \rho}{\sqrt{e}} \right), \\ \frac{ig(\rho)}{C_1 \sqrt{\kappa_* \rho}} &= K_{\frac{1}{2} - \frac{i\epsilon}{\kappa_*}} \left(\frac{2|\kappa| \kappa_* \rho}{\sqrt{e}} \right) + K_{\frac{1}{2} + \frac{i\epsilon}{\kappa_*}} \left(\frac{2|\kappa| \kappa_* \rho}{\sqrt{e}} \right), \end{aligned} \quad (4)$$

where C_1 is a normalization constant.

The probability density for the near-horizon spinor (3) is shown in Fig. 1. We choose a bound state with a high principal quantum number, indicating the number of fringes. The solution is stationary which does not include decay due to crossing the horizon. The decay of the state will be described by the imaginary part of energy (decay rate) of the exact eigenvalues, which generally needs the solution beyond the near-horizon approximation, see, e.g., the numerical solutions of Ref. [24]. However, we proceed in an alternative way. We will mimic the near-horizon gravitational solution by the free-electron chirped wave packet, and further generalize it to describe the decay of the gravitational quantum state because of the falling to the singularity at the center. This will offer an alternative way to study the electron quantum dynamics near a black hole. For as long as tidal forces within the extension of the wave packet are negligible, our solution remains valid.

Spherically symmetric Rindler coordinates. As an intermediate step toward mimicking a gravitational wave function (3) by a free electron wave packet, we use the

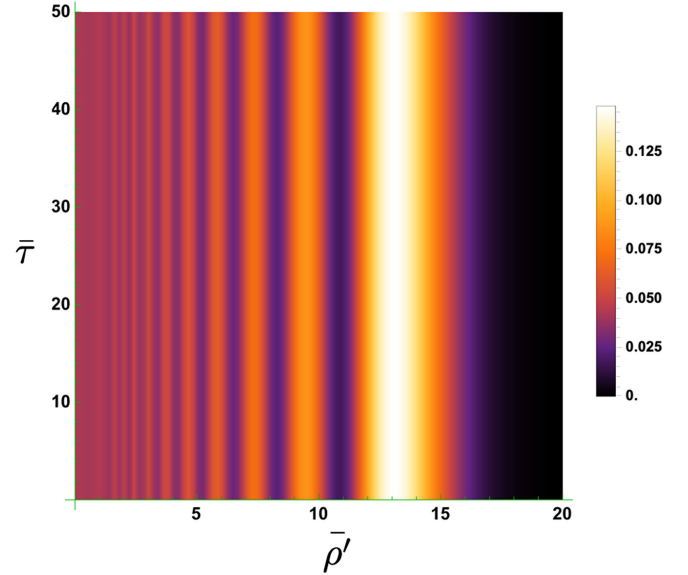


FIG. 1. Spacetime profile of the electron density as a function of $(\bar{\rho}', \bar{\tau})$, with $\bar{\rho}' = \rho'/\lambda$ and $\bar{\tau} = \tau/\lambda$, for the electron wave function in the near-horizon approximation with $\epsilon/\kappa_* = 15$ and $\lambda\kappa_* = 0.1$.

fact that the near-horizon Schwarzschild solution can be reproduced by the solution to the free Dirac equation in a local accelerated frame described by a flat metric in specially designed 3 + 1D Rindler coordinates. One sees the hint to design the latter from the fact that the metric Eq. (2) has the same form as the 1 + 1D Rindler metric [25] at $\theta = \phi = \text{const}$, i.e., considering only radial motion. While the solution to the Dirac equation in the 1D Rindler coordinates, i.e., for a free electron in a constant accelerating frame of reference is known [26], here we need the solution of the Dirac equation in 3D Rindler coordinates, which corresponds to the locally constant acceleration along the radial direction at each fixed angular coordinates (θ, ϕ) . These coordinates are related to the Minkowski coordinates by the transformation: $t = \xi \sinh(\kappa\tau)$, $x = \xi \cosh(\kappa\tau) \sin \theta \sin \phi$, $y = \xi \cosh(\kappa\tau) \sin \theta \cos \phi$, $z = \xi \cosh(\kappa\tau) \cos \theta$, which yields the metric

$$ds^2 = \xi^2 \kappa^2 d\tau^2 - d\xi^2 - \xi^2 \cosh^2(\kappa\tau) (\sin^2 \theta d\phi^2 + d\theta^2), \quad (5)$$

where $\kappa > 0$ is proportional to the norm of the acceleration four-vector of the electron. The metric (5) describes a flat spacetime which is the generalization of the Rindler metric to 3 + 1-dimensional spacetime, and it is similar to the near-horizon metric Eq. (2), besides the time dependent term $\cosh^2(\kappa\tau)$ in the angular part.

In the limit $\kappa\tau \gg 1$, which implies that $r \approx t$ [as $\kappa\tau = \tanh^{-1}(t/r)$], i.e., tending toward the light cone, the exact solution takes the simplified form in the lab frame

$$\psi \approx \frac{4(-1)^{1/4} c_1 e^{-i\zeta\kappa\tau} \text{sech}(\kappa\tau)}{\pi\xi} \begin{pmatrix} \psi_1 \\ 0 \\ \psi_3 \cos \theta \\ \psi_3 e^{i\phi} \sin \theta \end{pmatrix}, \quad (6)$$

where c_1 is a normalization constant and

$$\begin{aligned} \psi_1 &= -K_{i\zeta+\frac{1}{2}}(\xi/\lambda)(2\zeta-i) - K_{i\zeta-\frac{1}{2}}(\xi/\lambda)(2i\zeta+1), \\ \psi_3 &= K_{i\zeta-\frac{1}{2}}(\xi/\lambda)(-2i\zeta-1) + K_{i\zeta+\frac{1}{2}}(\xi/\lambda)(2\zeta-i). \end{aligned}$$

It is an eigenstate in the 3 + 1D Rindler metric with the eigenvalue $\varepsilon = \kappa\zeta$. The behavior of the wave function in the lab frame for $\kappa\tau \gg 1$ can be illustrated upon inspecting the electron probability density

$$\frac{\sqrt{-g}\psi^\dagger\psi}{\sin(\theta)} = \frac{64c_1^2\kappa\xi}{\pi^2} [(4\zeta^2+1)K_{i\zeta-\frac{1}{2}}(\xi/\lambda)K_{i\zeta+\frac{1}{2}}(\xi/\lambda)], \quad (7)$$

where g is the determinant of the metric tensor, and ζ the real valued variable separation constant. There is a remarkable equivalence between the Rindler spinor and the near-horizon spinor Eq. (3). This becomes obvious when we compare (7) with the electron probability density given by Eq. (3)

$$\frac{64}{\kappa_*} |C_1|^2 \sin(\theta) \rho' K_{-\frac{i\varepsilon}{\kappa_*}+\frac{1}{2}}(\kappa_*\rho') K_{\frac{i\varepsilon}{\kappa_*}+\frac{1}{2}}(\kappa_*\rho'). \quad (8)$$

There is proportionality to each other, which suggests the following identifications of the Rindler wave function parameters

$$\xi = \lambda\kappa_*\rho', \quad \zeta = \frac{\varepsilon}{\kappa_*}. \quad (9)$$

The probability density of the 3D Rindler solution is identical to the Schwarzschild one Fig. 1 for the same parameters. The density in the Rindler case demonstrates the fringes with the same qualitative behavior as the Schwarzschild one. In particular, the number of fringes is the same, and there is no decay of the probability.

Free electron wave packet as a counterpart of the Rindler solution. We have shown the mapping of the Rindler spinor to the Schwarzschild near-horizon solution. In the next step we will show that the Rindler spinor can be constructed by a suitable superposition of free electron spherical waves. To this end we note that the Rindler solution of Eq. (6) is expressed via the Bessel $K_{i\zeta\pm 1/2}$ functions, use the integral representation of the latter, and conclude that the free electron counterpart of the Rindler solution for the radial wave function can be represented as a wave packet, see Ref. [23]

$$\hat{\psi}_\alpha = \int \frac{d^3p}{(2\pi)^{3/2}} \sqrt{\rho_s} \psi_p e^{-\gamma^2 \gamma^1 [E_p t + ab(p)]}, \quad (10)$$

constructed from the free electron spherical Dirac waves ψ_p , with $\kappa = -1$. Here, $E_p = \sqrt{\mathbf{p}^2 + m^2}$ is the electron energy, and $b(p) = \tanh^{-1}(p/E_p)$ the rapidity, and the parameter α determines the chirping. The necessity of chirping follows from the desire to produce the imaginary index of the Bessel function in the wave function of Eq. (6). The expression similar to the stationary Rindler solution of Eq. (6) is obtained when choosing $\alpha = \varepsilon/\kappa_*$ and $\rho_s = \text{const}$.

We introduce heuristically a finite momentum width of the wave packet via an envelop function $\sqrt{\rho_s} = e^{-\beta E_p/m}$, with the parameter β . The nonvanishing parameter β , causes the stationary wave packet to decay, and as we will see below, it will describe the particle crossing the horizon and the decay of the stationary state in the gravitational setting. We confirm the choice of the parameter β with a quasi-classical analysis. The constructed free electron wave packet has the following spacetime form in the lab frame, (see Ref. [23])

$$\hat{\psi}_\alpha = \frac{ie^{\pi\alpha} \mathcal{N} \left(\frac{t'-r}{t'+r} \right)^{-\frac{1}{4}+\frac{i\alpha}{2}}}{\pi^{3/2} \lambda r \sqrt{r^2-t'^2}} \begin{pmatrix} \psi_1 \\ 0 \\ \frac{\psi_3 \cos \theta}{r} \\ \frac{\psi_3 e^{i\phi} \sin \theta}{r} \end{pmatrix}, \quad (11)$$

where

$$\begin{aligned} \psi_1 &= \frac{K_{i\alpha-\frac{1}{2}}(\bar{\xi}')}{\bar{\xi}'^2 \lambda^2} [2rr_- + \lambda(2\alpha+i)r_+] \\ &\quad + iK_{i\alpha+\frac{1}{2}}(\bar{\xi}') \left[\frac{2r}{\lambda} + (2\alpha-i)\frac{r_-}{r_+} \right], \end{aligned} \quad (12)$$

$$\begin{aligned} \psi_3 &= \frac{iK_{i\alpha-\frac{1}{2}}(\bar{\xi}')}{\bar{\xi}'^2 \lambda^2} [-2r^2r_- + \lambda r_+ [(2\alpha+i)r + 2ir_-]] \\ &\quad - K_{i\alpha+\frac{1}{2}}(\bar{\xi}') \left[\frac{2r^2}{\lambda} - \frac{r_- [(2\alpha-i)r - 2ir_+]}{r_+} \right] \end{aligned} \quad (13)$$

with $\bar{\xi}' = \sqrt{r^2 - t'^2}/\lambda$, $r_\pm \equiv r \pm t'$, and $t' = t - i\beta/m$.

For the simulation of the electron dynamics near the black hole, we constructed the free electron chirped wave packet, which in the lab frame is represented by the wave function of Eq. (11). The electron probability density in the Rindler frame is shown in Fig. 2, for the cases of $\beta = 0$ and for $\beta = 0.5$, respectively. In the case $\beta = 0$, the wave packet fringes are similar to that of the near-horizon gravitational counterpart of Fig. 1 at times larger $\bar{\tau} \sim 15$. However, at smaller times below the fringe breach, the fringes are compressed due to the time-dependent term in the metric in

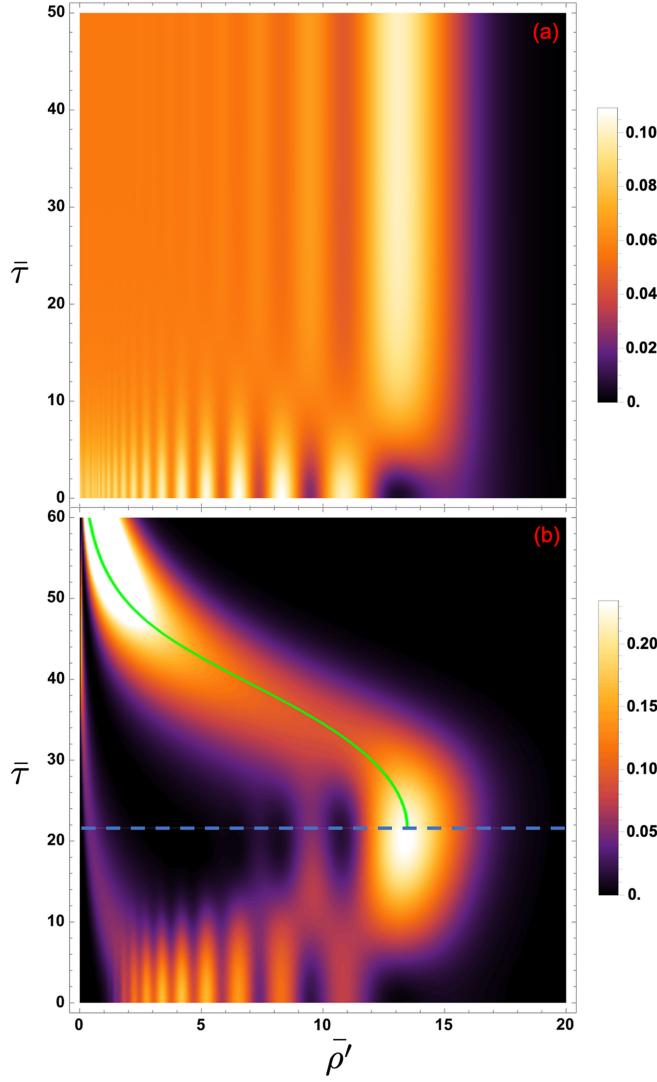


FIG. 2. Spacetime profile of the electron density in the Rindler frame as a function of $(\bar{\rho}', \bar{\tau})$, with $\bar{\rho}' = \rho'/\lambda$ and $\bar{\tau} = \tau/\lambda$ with $\alpha = 15$, $\lambda\kappa = 0.1$, and: (a) $\beta = 0$, (b) $\beta = 0.25$. The dashed horizontal line shows the point from which the classical particle starts. It corresponds to the time in which the wave-packet velocity is zero.

Eq. (5), but still maintain the same topological structure as in the gravitation case.

In the case of nonvanishing β the behavior of fringes is significantly modified. In the gravitational case, we have a stationary state without decay, which is due to the approximation applied for the analytical near-horizon solution. However, Fig. 2(b) shows an essentially time-dependent picture. There is a critical slow time dependence in Fig. 3, when fringes corresponding to the bound state dynamics gradually merge into one fringe, which later crosses the horizon, and plunges into the black hole abyss. This behavior of the wave function density emerges when we apply a finite momentum width for the free electron wave packet, described by the parameter β in Eq. (10).

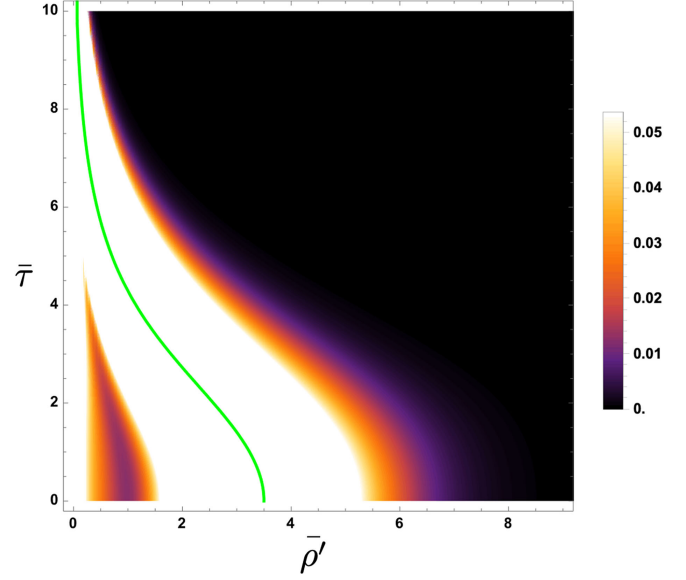


FIG. 3. Spacetime profile of the electron density in the Rindler frame as a function of $(\bar{\rho}', \bar{\tau})$, with $\bar{\rho}' = \rho'/\lambda$ and $\bar{\tau} = \tau/\lambda$, for the free electron modulated wave packet with $\alpha = 1$, $\beta = 4$, $\lambda\kappa = 0.5$. The classical trajectory is given by the full green curve for $\lambda\kappa_* = 0.5$, zero orbital angular momentum, initial radial velocity $\dot{r}(0) = 0$ and total energy $\epsilon = \alpha\kappa_*$.

Decay rate of the electron at crossing the horizon. For adapting the parameter β of the free electron wave packet to the electron physical dynamics of crossing the horizon, we recall the classical-to-quantum correspondence principle. As is known from the numerical solution of Ref. [24], the decay rate of the quasistationary bound state in a black hole corresponds to the classical falling time of the electron, which starts at the most probable point of the bound state. For the given energy of the state, which determines the chirping parameter α of the free electron wave packet, we identify the dominant fringe (i.e., the rightmost fringe in the electron density), and solve the classical geodesic equation for (τ, ρ) in the Rindler-Kruskal metric originating at the center of the aforementioned fringe and at the time $\bar{\tau}$ for which the wave packet velocity is zero. The choice of the parameter β is deduced by fitting the most probable classical trajectory to the major interference fringe of the electron wave packet, which simulates the dynamics of falling into the black hole. The full-green curve in Fig. 2(b) describes the classical geodesic adjusted according to the procedure described above and fitting well to the wave packet main fringe. We note in the figure three main regimes of the wave function. Up to $\bar{\tau} \approx 10$ we see fringes, a signature of quantum effects in the bound state, which begin to collapse (described by the parameter β) due to the black hole influence. During the time $10 \leq \bar{\tau} \leq 24$ the electron wave packet consolidates into a single fringe, which finally after $\bar{\tau} > 24$ plunges into the abyss of the black hole, focusing around the main trajectory, and disappears there.

We verified the procedure outlined above for the parameter β in [23] by comparing the expectation value of position ξ with the wave packet in electron's comoving frame with the one calculated with the exact gravitational wave function found by numerically solving the Dirac equation in the Schwarzschild metric.

Experimental concept. Let us briefly discuss the experimental conditions for the shaping of the free electron wave packet for the associated laboratory cosmology. The wave packet consists of a superposition of free electron spherical waves. Two parameters, α and β , define the wave packet: β determines the energy width of the wave packet, and α is the wave packet chirping parameter. For the considered example of the black hole $\alpha = 15$ and $\beta = 0.25$ are required. Thus, the energy width of the wave packet should be $\Delta E_p \sim m/\beta = 4m \sim 2$ MeV. The specially tailored momentum chirping of the wave packet is given by the phase $\varphi(p) = ab = \alpha \tanh^{-1}(p/E_p)$. This chirping induces a spatial shift of each momentum component in the laser field $\delta r(p) \sim D_2(p)\Delta p$, according to the group delay dispersion $|D_2(p)| = |\partial^2 \varphi(p)/\partial p^2| = p/E_p^3$, which yields $\delta r(p) \sim \alpha/\Delta E_p$. The created wave packet of the electron will be chirped if the particle with the corresponding momentum value is created with the corresponding spatial delay $\delta r(p)$. As the energy width is $\Delta E_p \sim 4m$, the total time delay of the energy components across the wave packet is $\Delta t \sim 0.25\alpha\hbar/c \sim 10^{-20}$ s [the Fourier transform limited wave packet duration is $1/\Delta E_p \sim 1/(4m) \sim 3 \times 10^{-22}$ s]. This kind of chirped free electron wave packet of spherical waves could be created, for instance, due to ionization of a high-Z ion with a chirped ultrashort

hard x-ray pulse of 10 zeptosecond duration with a broad spectrum spanning up to 2 MeV [27–30]. The free electron wave packet with desired properties will be created asymptotically via the ionization.

Conclusion. We have shown that with a specially constructed free electron wave packet, and a judicial choice of the wave packet chirping as well as the width in momentum space, one can simulate the electron quantum dynamics in a gravitational field, in particular, the electron dynamics near the horizon of a Schwarzschild black hole. The blueprint for the construction of the free electron wave packet is deduced by developing the intermediate wave function in the 3D Rindler space. In this way, we obtain the nonstationary wave function of the electron for the Schwarzschild problem, which contains much more information about quantum dynamics than just the decay rate. In particular, this wave function can provide information on the electron spin evolution when approaching and crossing the horizon, see Ref. [23].

With the same approach it would be interesting to study the electron dynamics near the horizon of the so-called *regular black holes*, where the singularity at the center is remedied via a quantum corrected black hole metric [31–34]. In such cases the near horizon dynamics will be modified, leading to different quantum features of the electron near the horizon in contrast to those discussed in this paper. The predictions of our model for the quantum features of the dynamics would then serve as a guide for identifying the true metric of the black hole.

Data availability. No data were created or analyzed in this study.

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