

EFT origin of secluded dark matter

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The present study aims to unveil a scenario with a nonminimal secluded dark sector (DS) in an effective field theory (EFT) framework. To explore this, we have examined a suitable extension of the type-X two Higgs doublet model (2HDM) as a potential origin for the secluded DS. The DS comprises a dark matter (DM) candidate and a mediator particle “ a ” and possesses some nonminimal characteristics. It becomes nonthermally populated through diverse dim-6 four-Fermi operators, effectively generated by integrating out the heavier Higgs particles. The analysis further focuses on the consequences of the collision processes $DM + a \leftrightarrow a + a$ and $DM + DM \leftrightarrow a + a$ occurring within the DS. We have investigated the significance of employing an EFT approach in tracking the temperature evolution of the DS. Within the present framework, the observed relic abundance of the DM can be realized through both dark freeze-out and freeze-in mechanisms. Further, we have delineated the permissible ranges of the relevant parameters, viz., the DM mass ($m_\chi \gtrsim 20$ GeV), the portal coupling ($C_\tau \lesssim 10^{-14}$ GeV⁻²), and the DS coupling ($\lambda \lesssim 10^{-6}$ GeV⁻²), by taking into account the perturbativity of the involved couplings while reproducing the observed DM relic and complying with the bounds from successful big bang nucleosynthesis (BBN) and γ -ray searches.

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I. INTRODUCTION

The existence of dark matter (DM) has now been firmly established through various astrophysical and cosmological observations [1–5]. However, the particle nature of the DM and its interactions with the Standard Model (SM) fields are still unknown. The absence of a viable candidate for the DM in the SM spectrum has led to several decades of intense exploration of potential physics scenarios beyond the SM (BSM) possessing a DM particle of varying nature. This has further prompted us to carry out detailed studies of plausible cosmological mechanisms that could generate the experimentally observed relic abundance of the DM.

In this context, one of the prominently debated topics aiming to elucidate the particle nature of the DM is the

concept of weakly interacting massive particles (WIMPs) [6–9]. Under the WIMP paradigm, the DM particles are expected to be in thermal equilibrium with the SM particles in the early Universe and would decouple from the thermal bath later as they “freeze out.” The fact that such a weakly interacting DM particle, when having a mass in the electroweak scale, would possess a relic abundance matching the experimental measurements is commonly known as the “WIMP miracle” [10]. Despite being very appealing, such WIMPs have evaded detection so far. The absence of any convincing signature of the DM in direct and indirect searches severely restricts the viability of a WIMP DM [11–15]. The null results of these searches might also hint at the possibility of new DM production mechanisms in thermal and nonthermal scenarios. The most standard way of DM freeze-out in a conventional WIMP setup, following its decoupling from the thermal bath, typically involves an annihilation process like $DMDM \rightarrow aa$, where a is any lighter particle present in the thermal bath.

Another mechanism for the freeze-out of thermal DM has been proposed in [16], where the relic abundance is determined solely by the coannihilation of the DM with such a lighter species a via the process $DMa \rightarrow aa$. Some other coannihilation processes have been discussed in detail in Refs. [17–19].

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However, it is possible that, in the early Universe, the DM could never reach a thermal equilibrium if it had very feeble interactions with particles in the thermal bath. Under such circumstances, the so-called freeze-in mechanism [20–22] could come into play, which might still give rise to the observed DM relic abundance. Alternatively, it is also possible that a particle DM is an SM singlet residing in a dark sector (DS) which remains thermally decoupled (secluded) from the visible sector (VS). However, a strong conversion process inside the DS allows the DM to reach an internal thermal equilibrium [23–31]. In that case, evolutions of the DS temperature (T_D) and the VS temperature (T) turn out to be significantly different [23,24,27–29,32]. The ratio $\frac{T_D}{T}$ crucially depends on how the DS particles were originally produced. In the case of a minimal secluded DS scenario, the DM is depleted through annihilation with itself via DM number-changing self-interaction processes [31,33]. In a scenario with a nonminimal secluded sector, the DM annihilates into lighter DS states via $DMDM \rightarrow aa$ scattering process, where a (the mediator particle) is the lighter DS particle, possessing feeble couplings to the VS particles¹ [24,35,36].

In this work, our primary aim is to develop an EFT description of nonminimal secluded dark matter and subsequently examine its phenomenological ramifications.² In particular, we investigate how the DS gets populated in the first place in a generic EFT-motivated framework where both $DMDM \leftrightarrow aa$ and $DMa \leftrightarrow aa$ conversion processes could take place in a thermally decoupled or a secluded DS. Here, we have assumed that the DS is comprised of a single dark matter candidate (χ) and a lighter mediator particle (ξ ; can be identified with a above), which are fermionic in nature.

The population of the DS is originally induced and subsequently influenced by the VS through a series of dim-6 four-Fermi vertex operators. These operators are formulated within the framework of an EFT derived from a UV-complete model. The UV-complete model we have considered for our purpose is an extended version of the type-X two Higgs doublet model (2HDM). The choice of a 2HDM scenario facilitates the incorporation of Yukawa-like portal interactions. On the other hand, our preference for only lepton-specific portal couplings makes us choose the type-X variant. The charged Higgs bosons of the type-X 2HDM would trigger lepton flavor universality violation (LFUV) at the tree level in the decays of the tau lepton into an electron and a muon and hence draws a lower bound on its mass [42]. Such an extended type-X 2HDM with lepton-specific portal interactions is further motivated by different astrophysical and cosmological observations [43,44].

¹The process $DMa \rightarrow aa$ where a' is the lighter DS species, has been discussed in Ref. [34].

²The EFT description of thermal WIMPs can be found in [37–41].

The particular choice of an extended 2HDM framework is solely to ensure generation of the required effective operators in the theory for DM masses (m_{DM}) ranging from $\mathcal{O}(10)$ GeV to $\mathcal{O}(10)$ TeV. Such a scenario typically requires mass scales much heavier than m_{DM} , for example, additional, heavy Higgs states, which are to be integrated out to generate the effective scale Λ . In passing, we note that similar effective operators in the DS may also arise in more general or beyond-2HDM UV frameworks, provided that sufficiently heavy Higgs fields are present in the theory.

Additionally, we have also incorporated a global $U(1)_{\mu-\tau}$ symmetry that gives rise to the portal couplings between ξ and the μ , τ flavored leptons and also allows for both the conversion processes (as mentioned earlier) to become active within the DS. The $U(1)_{L_\mu-L_\tau}$ gauge theory stands out as one of the anomaly-free gauge extensions, as discussed in Refs. [45,46], and also in Ref. [47] in the context of neutrino mass generation. The global $U(1)_{\mu-\tau}$ symmetry is also well motivated as it offers an explanation to the quasidegenerate neutrino mass spectrum. At the leading order, this symmetry predicts the maximal atmospheric mixing (i.e., $\theta_{23} = 45^\circ$) and a vanishing reactor neutrino mixing angle, i.e., $\theta_{13} = 0$, with the solar angle (θ_{12}) remaining unconstrained [48]. To reproduce neutrino oscillation data [49], i.e., to obtain the observed nonzero values of θ_{12} and θ_{13} , the symmetry must be broken softly. It has been demonstrated in Refs. [50–57] that, depending on different soft-breaking patterns, one can reproduce the observed neutrino mass mixing. However, in the present work, we do not focus on the observed pattern of neutrino mass mixing, as this would require inducing additional degrees of freedom beyond the minimal setup considered here. Detailed discussions on neutrino mass and mixing parameters in this kind of a scenario can be found elsewhere [58,59]. These works reveal that the additional right-handed neutrinos of such scenarios have to be rather heavy ($M_R \sim 10^{14-15}$ GeV), which is far above the effective scale Λ considered in the present work. Hence, in a simple scenario that we propose in this work, we expect that a mechanism that generates the observed patterns in the neutrino mass mixing would not affect our analysis, which focuses on the DM sector.

Further, we assume the BSM Higgs bosons to be heavy enough such that they could be integrated out below the effective scale Λ . This results in not only the relevant operators that induce the DS population in the first place but also those that would subsequently control the internal dynamics of the DS. To track the phases through which the DS passes as the Universe evolves, we further calculate the DS temperature T_D by considering the most relevant production modes of ξ , followed by the production of χ from ξ . As the Universe cools down ($T_D < m_\chi$), χ annihilates to ξ via the processes $\bar{\xi}\chi \rightarrow \bar{\xi}^c\xi$ and $\bar{\chi}\chi \rightarrow \bar{\xi}\xi$,

thus undergoing an eventual freeze-out, assuming the DM to be stable.

In the scenario we propose in this work, the stability of the DM particle χ is primarily aided by the usual consideration of a mass hierarchy $m_\xi < m_\chi < 3m_\xi$ [16,34], which forbids its tree-level decay to ξ 's, i.e., $\chi \rightarrow \xi\xi^c$. It should, however, be noted that this arrangement may not suffice as the DM could then still undergo a loop-level decay, viz., $\chi \rightarrow \xi\bar{l}$, which might threaten its stability unless the involved interactions are feeble enough. As long as one assumes a universal coupling parameter governing the interactions among the DS particles, its feebleness would immediately favor the freeze-in mechanism (over freeze-out) to set in (via the processes $\xi^c\xi \rightarrow \bar{\xi}\chi$ and $\bar{\xi}\xi \rightarrow \chi\bar{\chi}$) that would eventually generate the observed relic abundance of a (stable) DM.

For a quantitative understanding, we study two representative situations. In the first case, both $\bar{\xi}\chi \leftrightarrow \xi^c\xi$ and $\chi\chi \leftrightarrow \bar{\xi}\xi$ processes are active within the DS, while in the second case, only the latter process is present. To estimate the DM relic abundance, we have constructed the set of coupled Boltzmann equations that govern the number density of the DS particles and the total energy density of the DS. We then solve them simultaneously in both cases mentioned above.

Furthermore, ensuring the smallness of the portal coupling that facilitates the initial nonthermal production or population of the DS is crucial for keeping the DS secluded. The structure of the proposed model allows for the decay of ξ into SM leptons at a later stage, via the same portal coupling, which might jeopardize the successful predictions of the big bang nucleosynthesis (BBN). Both of these considerations impose strong constraints on the VS-DS portal coupling. We duly consider these issues while looking for the viable region of the model parameter space that yields the correct DM relic abundance. Further, the annihilation processes of the DM into a pair of mediator particles may give rise to a γ -ray spectrum which could be probed by the Fermi-LAT [60–62] telescope.

The paper is organized as follows. In Sec. II, we briefly describe the model under consideration and write down the relevant effective operators. Sections III and IV are devoted to a detailed analysis of the Boltzmann equations and discussions of our numerical results by complying with the theoretical requirements that the VS and the DS have to be out of mutual thermal equilibrium and that the involved couplings should remain perturbative. We also discuss constraints from the BBN and shed light on the prospects of indirect detection of the DM in γ -ray searches. We further discuss that the constraint from LFUV on the mass of charged Higgs bosons would set a maximum value of the effective scale of our theory. The summary and conclusions are presented in Sec. V.

II. THE MODEL

In this study, we have investigated the type-X 2HDM model [63,64] (that constitutes the VS) by augmenting it

TABLE I. The $U(1)_{\mu-\tau}$ charge assignments for the BSM gauge singlet fields.

Fields	$U(1)_{\mu-\tau}$
ξ	1
S	-2
χ	3

with a DS comprised of an SM gauge singlet scalar (S) and two Dirac fermions: one acting as the mediator particle (ξ) and the other (χ) proposed as the candidate for dark matter. We have introduced a global $U(1)_{\mu-\tau}$ symmetry, assigning nontrivial charges to S , ξ , and χ , as detailed in Table I. The interactions that are relevant to this work are all contained in the Yukawa sector of the Lagrangian and are given as follows:

$$\mathcal{L} \supset y_{1i}\bar{L}_i\Phi_1 e_{Ri} + y_{2i}\bar{Q}_i\Phi_2 d_{Ri} + y_{3i}\bar{Q}_i\Phi_2^c u_{Ri} + y_4\bar{L}_\mu\Phi_1^c \xi_R + y_5\bar{L}_\tau\Phi_1^c \xi_R + y_6 S \xi^c \xi + y_7 S \bar{\xi} \chi + \text{H.c.}, \quad (1)$$

where Φ_1 and Φ_2 represent two distinct $SU(2)_L$ Higgs doublets. Notably, Φ_1 exclusively couples to SM leptons, while Φ_2 interacts exclusively with SM quarks. This coupling arrangement is specific to type-X 2HDM and usually achieved by enforcing a Z_2 symmetry [42], wherein Φ_2 , along with the right-handed up-type quarks u_{Ri} and right-handed down-type quarks d_{Ri} , exhibit odd parity under this symmetry. Once the electroweak symmetry breaks, the scenario gives rise to three Goldstone bosons that appear as the longitudinal modes of the massive $W^\pm$ and Z bosons. Additionally, the spectrum contains five massive scalar degrees of freedom: two CP -even states (h, H), one CP -odd state (A), and a pair of charged Higgs states ($H^\pm$). Here, both Φ_1 and Φ_2 acquire nonzero VEVs v_1 and v_2 , respectively, with $v_{\text{SM}} = \sqrt{v_1^2 + v_2^2} = 246$ GeV. The scalar doublets Φ_1 and Φ_2 , in terms of the physical scalar states, can be written as

$$\Phi_1 = \begin{pmatrix} -H^+ \sin \beta \\ \sqrt{\frac{1}{2}}(v_1 + H \cos \alpha - h \sin \alpha - iA \sin \beta) \end{pmatrix}, \quad (2)$$

$$\Phi_2 = \begin{pmatrix} H^+ \cos \beta \\ \sqrt{\frac{1}{2}}(v_2 + H \sin \alpha + h \cos \alpha + iA \cos \beta) \end{pmatrix}, \quad (3)$$

where $\tan \beta = \frac{v_2}{v_1}$ and α parametrize the mixing between the gauge and mass eigenstates of the CP -even Higgs bosons. In the alignment limit, i.e., $\beta - \alpha = \frac{\pi}{2}$, h can be identified with the observed SM-like Higgs boson having a mass $\simeq 125$ GeV [63,65].

The Yukawa couplings y_{1i} , y_{2i} , and y_{3i} are given by

$$y_{1i} = \frac{gm_{e_i}}{\sqrt{2}m_W \cos\beta}, \quad y_{2i} = \frac{gm_{u_i}}{\sqrt{2}m_W \sin\beta}, \quad y_{3i} = \frac{gm_{d_i}}{\sqrt{2}m_W \sin\beta}, \quad (4)$$

where $e_i = \{e, \mu, \tau\}$, $u_i = \{u, c, t\}$, and $d_i = \{d, s, b\}$ include different generations of leptons, up-type quarks, and down-type quarks, respectively, and m_{e_i} , m_{u_i} , and m_{d_i} are the corresponding masses. The mass of the W gauge boson is given by m_W , while g represents the $SU(2)_L$ gauge coupling. It may be noted that the most conservative upper limit on $\tan\beta$ comes from considering $y_{1\tau} < 4\pi$ and is given by

$$\tan\beta \lesssim 1255. \quad (5)$$

As for the interactions involving the DS particles, the existence of a global $U(1)_{\mu-\tau}$ symmetry dictates that only the fourth and fifth terms in Eq. (1) are permissible as portal couplings linking the VS and the DS. For the same reason, the last two terms in Eq. (1) are the only allowed ones that govern the interactions of DS particles among themselves. In subsequent sections, we will delve into the impact of these terms on the production mechanisms of the DS particles, the relic abundance of DM, and its detection prospects.

We impose a mass hierarchy of $m_{S,H,A,H^\pm} \gg m_{\chi,\xi}$, which enables us to adopt an effective field theory framework with a cutoff scale Λ . The coupling of ξ with the SM Higgs boson (h) is suppressed by $\tan\beta$ and can be safely ignored for our purpose. To ensure a reasonably accurate approach to EFT at a high temperature, i.e., when the DS particles are relativistic to start with, we set $\Lambda = 100m_\chi$ [66]. Below this cutoff, on integrating out the heavier Higgs fields, several dim-6 four-Fermi operators emerge that contribute significantly to the production of the DS particles in the early Universe. The specific dim-6 operators are listed in Table II. The Yukawa couplings y_{1i} are fixed by the masses of the leptons for a given $\tan\beta$, whereas y_4 and y_5 are the free parameters of the theory. Here, y_{2i} , y_{3i} are also fixed by SM quark masses, i being the quark generation index. Note that y_{2i} , y_{3i} do not contribute to the DS phenomenology since the quarks do not interact with the DS particles. It is further important to note that the requirement of non-thermal production of the DS forces y_4 and y_5 to be very small as compared to y_{1i} , y_{2i} , which in turn makes C_3 , C_4 , and C_5 significantly suppressed. We also note that, due to the known mass (Yukawa coupling) hierarchy of the SM leptons, the primary contribution to the nonthermal production of the DS particles would predominantly come from terms involving the coefficient $C_{1\tau}$, $C_{2\tau}$. Since both $C_{1\tau}$ and $C_{2\tau}$ have similar roles to play in our analysis, for simplicity, we have assumed $C_{1\tau} = C_{2\tau}$. We further define

TABLE II. Effective four-Fermi operators obtained by integrating out the heavier Higgs bosons. The coefficients can be identified in terms of Yukawa interactions as $C_{1\tau} = y_{1\tau}y_4$, $C_{2\tau} = y_{1\tau}y_5$, $C_3 = y_4y_5$, $C_4 = y_4^2$ and $C_5 = y_5^2$, $\tilde{\lambda}_1 = y_6y_7$, and $\tilde{\lambda}_2 = y_7^2$.

By integrating out $H^\pm$
$\frac{C_{1\tau}}{m_{H^\pm}^2} \sin^2 \beta \bar{\nu}_\tau \tau_R \bar{\mu}_L \xi_R, \frac{C_{2\tau}}{m_{H^\pm}^2} \sin^2 \beta \bar{\nu}_\tau \tau_R \bar{\tau}_L \xi_R^c, \frac{C_3}{m_{H^\pm}^2} \sin^2 \beta \bar{\mu}_L \xi_R \bar{\xi}_R^c \mu_L$
$\frac{C_4}{m_{H^\pm}^2} \sin^2 \beta \bar{\mu}_L \xi_R \bar{\xi}_R \mu_L, \frac{C_5}{m_{H^\pm}^2} \sin^2 \beta \bar{\tau}_L \xi_R \bar{\xi}_R^c \tau_L$
By integrating out H and A
$\frac{C_{1\tau}}{m_H^2} \cos^2 \alpha \bar{\tau} \tau \bar{\nu}_\mu \xi_R, \frac{C_{2\tau}}{m_H^2} \cos^2 \alpha \bar{\tau} \tau \bar{\nu}_\tau \xi_R^c, C_3 \left(\frac{\cos^2 \alpha}{m_H^2} - \frac{\sin^2 \alpha}{m_A^2} \right) \bar{\nu}_\mu \xi_R \bar{\nu}_\tau \xi_R^c,$
$C_4 \left(\frac{\cos^2 \alpha}{m_H^2} + \frac{\sin^2 \alpha}{m_A^2} \right) \bar{\nu}_\mu \xi_R \bar{\nu}_\mu \xi_R, C_5 \left(\frac{\cos^2 \alpha}{m_H^2} + \frac{\sin^2 \alpha}{m_A^2} \right) \bar{\nu}_\tau \xi_R \bar{\nu}_\tau \xi_R^c,$
$\frac{C_{1\tau}}{m_A^2} \sin^2 \beta \bar{\tau} \gamma_5 \tau \bar{\nu}_\mu \xi_R, \frac{C_{2\tau}}{m_A^2} \sin^2 \beta \bar{\tau} \gamma_5 \tau \bar{\nu}_\tau \xi_R^c, C_3 \left(\frac{\cos^2 \alpha}{m_H^2} - \frac{\sin^2 \alpha}{m_A^2} \right) \bar{\xi}_R \nu_\mu \bar{\nu}_\tau \xi_R^c,$
$C_4 \left(\frac{\cos^2 \alpha}{m_H^2} - \frac{\sin^2 \alpha}{m_A^2} \right) \bar{\nu}_\mu \xi_R \bar{\xi}_R \nu_\mu, C_5 \left(\frac{\cos^2 \alpha}{m_H^2} - \frac{\sin^2 \alpha}{m_A^2} \right) \bar{\nu}_\tau \xi_R \bar{\xi}_R^c \nu_\tau$
By integrating out S
$\frac{\tilde{\lambda}_1}{m_S^2} \bar{\xi} \chi \bar{\xi} \xi^c, \frac{\tilde{\lambda}_2}{m_S^2} \bar{\chi} \chi \bar{\xi} \xi$

$$C_\tau = \frac{C_{1\tau} \sin^2 \beta}{\Lambda^2}. \quad (6)$$

Similarly, after integrating out S from the last two terms of Eq. (1), we obtain two four-Fermi operators which are responsible for the conversion processes inside the DS³ and hence for the time evolution of the number densities of the DS particles. These are

$$\frac{\tilde{\lambda}_1}{m_S^2} \bar{\xi} \chi \bar{\xi} \xi^c \quad (\text{for the process } \bar{\xi} \chi \leftrightarrow \bar{\xi}^c \xi), \quad (7a)$$

$$\frac{\tilde{\lambda}_2}{m_S^2} \bar{\chi} \chi \bar{\xi} \xi \quad (\text{for the process } \bar{\chi} \chi \leftrightarrow \bar{\xi} \xi), \quad (7b)$$

with $\tilde{\lambda}_1 = y_6y_7$ and $\tilde{\lambda}_2 = y_7^2$, where $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$ are the coupling parameters exclusive to the DS (see Table II). A diagram elucidating the complete dynamics of the DS in the present setup is presented in Fig. 1.

Now, to understand the effect of the above-mentioned conversion processes on the DM production mechanism, we study the following two cases based on the relative magnitudes of $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$.

Case I: This is a minimal scenario where, for simplicity, we assume $\tilde{\lambda}_1 = \tilde{\lambda}_2 = \tilde{\lambda}$, and we define a universal DS coupling parameter as $\lambda = \frac{\tilde{\lambda}}{\Lambda}$. For this case, we note that the impact of the process $\bar{\xi} \chi \rightarrow \bar{\xi}^c \xi$ on DM freeze-out is

³To develop the effective theory, we have integrated out various mass scales at different stages; however, for convenience, in this work we have assumed the presence of a uniform cutoff scale considering the limit $\Lambda \simeq m_h, m_{H^\pm}, m_S$.

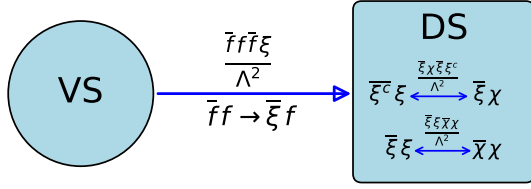


FIG. 1. Dynamics of DS phenomenology in the current setup. Both the initial population of the DS and the conversion processes within the DS are driven by dimension-6 four-Fermi operators, as indicated. Here, f represents the SM leptons.

significantly greater than that of the process $\bar{\chi}\chi \rightarrow \bar{\xi}\xi$. We discuss this further in Sec. III B. To ensure the stability of the DM, we first impose the kinematic condition $m_{\xi} < m_{\chi} < 3m_{\xi}$, which prevents the tree-level decay (3-body, $\chi \rightarrow \xi\xi\bar{\xi}^c$, driven by $\tilde{\lambda}_1$) of the DM. However, if $\tilde{\lambda}$ is critically large (and hence $\tilde{\lambda}_1$), the DM could still undergo the decay $\chi \rightarrow \xi\bar{l}l$ ($l \equiv \tau, \mu, \nu_{\mu}, \nu_{\tau}$) at one loop, which is also driven by $\tilde{\lambda}_1$. Opting for a smaller $\tilde{\lambda}$, in turn, would clearly impede the freeze-out process. As we shall see later, in this case, freeze-in is the only viable mechanism that can explain the observed DM abundance.

Case II: One possible avenue to explore the viability of the DM freeze-out for obtaining the correct relic abundance is thus suppressing the problematic loop-mediated decay of the DM by making $\tilde{\lambda}_1$ sufficiently small ($\tilde{\lambda}_1 \ll \tilde{\lambda}_2$). An extreme situation of this case is when $\tilde{\lambda}_1 = 0$. In this case, the process $\bar{\chi}\chi \rightarrow \bar{\xi}\xi$ driven by Eq. (7b) would have to be the dominant one contributing to the DM freeze-out. As in case I, there is still the possibility of attaining the observed relic abundance via the freeze-in mechanism.

In the subsequent sections, we discuss the dynamics of these cases in their analytical details with the help of Boltzmann equations, while the corresponding numerical estimates are presented by adopting some benchmark

points. The free parameters that control the DM phenomenology of the broad scenario are as follows:

$$\{C_{\tau}, \lambda_1, \lambda_2, m_{\xi}, m_{\chi}\}, \quad (8)$$

where $\lambda_1 = \frac{\tilde{\lambda}_1}{\Lambda^2}$ and $\lambda_2 = \frac{\tilde{\lambda}_2}{\Lambda^2}$. An issue that is important for our subsequent analyses is the perturbativity of the effective interaction strengths C_{τ} and $\lambda_{1,2}$, which sets upper bounds on their values. In this work, to meet this requirement, we demand

$$\lambda_{1,2}, C_{\tau} < \frac{16\pi^2}{(100m_{\chi})^2}, \quad (9)$$

which is consistent with (in fact, more stringent than) what has been considered in the literature [66].

III. STUDYING THE BOLTZMANN EQUATIONS

In this section, we discuss in detail the Boltzmann equations that govern the time evolution of the number densities of the DS particles. We also present an analytical understanding of these equations with reference to the two DS processes indicated in Eqs. (7a) and (7b).

A. Formulation of the Boltzmann equations

As stated earlier, the DS is comprised of the fields ξ and χ and decoupled from the VS bath. First, ξ is produced nonthermally from the VS mainly via processes of the kind SM + SM $\rightarrow \xi$ + SM. As discussed in Sec. II, the effective operators associated with the τ leptons play the dominant role in populating the DS. The contributing processes can be listed in two broad categories as follows:

- (i) Those proceeding via operators obtained by integrating out heavy $H^{\pm}$:

$$\begin{aligned} \bar{\tau}_R \nu_{\tau} &\rightarrow \bar{\mu}_L \xi_R, & \bar{\tau}_R \mu_L &\rightarrow \bar{\nu}_{\tau} \xi_R, & \mu_L \nu_{\tau} &\rightarrow \tau_R \xi_R, & \bar{\mu}_L \bar{\nu}_{\tau} &\rightarrow \bar{\tau}_R \bar{\xi}_R, & (+\text{H.c.}), \\ \bar{\tau}_R \nu_{\tau} &\rightarrow \bar{\tau}_L \xi_R^c, & \bar{\tau}_R \tau_L &\rightarrow \bar{\nu}_{\tau} \xi_R^c, & \tau_L \nu_{\tau} &\rightarrow \tau_R \xi_R^c, & \bar{\tau}_L \bar{\nu}_{\tau} &\rightarrow \bar{\tau}_R \bar{\xi}_R^c, & (+\text{H.c.}). \end{aligned} \quad (10)$$

- (ii) Those proceeding via operators obtained by integrating out heavy H, A :

$$\begin{aligned} \bar{\tau}\tau &\rightarrow \bar{\nu}_{\mu} \xi_R, & \bar{\tau}\nu_{\mu} &\rightarrow \bar{\tau} \xi_R, & \tau\nu_{\mu} &\rightarrow \tau \xi_R, & \bar{\tau}\bar{\nu}_{\mu} &\rightarrow \bar{\tau} \bar{\xi}_R, & (+\text{H.c.}), \\ \bar{\tau}\tau &\rightarrow \bar{\nu}_{\tau} \xi_R^c, & \bar{\tau}\nu_{\tau} &\rightarrow \bar{\tau} \xi_R^c, & \tau\nu_{\tau} &\rightarrow \tau \xi_R^c, & \bar{\tau}\bar{\nu}_{\tau} &\rightarrow \bar{\tau} \bar{\xi}_R^c, & (+\text{H.c.}). \end{aligned} \quad (11)$$

For an optimally strong mutual interaction between ξ and χ , with both having sufficient number densities, these two states could reach a local (within DS) kinetic equilibrium with unique temperature $T_D \neq T$ [67,68], where T corresponds to the temperature of the VS. As discussed in Sec. II, the proximity in masses of these two states makes it imperative to track their number densities (n_{χ} and n_{ξ}) simultaneously. In addition, since the dark sector particles have a temperature different from that of the VS ones, we should also consider the evolution of the dark sector energy density ($\rho_D = \rho_{\chi} + \rho_{\xi}$) as a function of T . The relevant Boltzmann equations with the collision term (CE) appearing on the rhs are as follows:

$$\frac{dn_{\chi_{\text{tot}}}}{dt} + 3\mathcal{H}n_{\chi_{\text{tot}}} = \frac{1}{2}\langle\sigma v\rangle_{\bar{\xi}\xi\rightarrow\bar{\chi}\chi}^{T_D}\left[n_{\xi_{\text{tot}}}^2 - n_{\chi_{\text{tot}}}n_{\xi_{\text{tot}}}^{\text{eq}}\frac{n_{\xi_{\text{tot}}}^{\text{eq}}(T_D)}{n_{\chi_{\text{tot}}}^{\text{eq}}(T_D)}\right] + \frac{1}{2}\langle\sigma v\rangle_{\bar{\xi}\xi\rightarrow\bar{\chi}\chi}^{T_D}\left[n_{\xi_{\text{tot}}}^2 - n_{\chi_{\text{tot}}}^2\frac{n_{\xi_{\text{tot}}}^{\text{eq}}(T_D)^2}{n_{\chi_{\text{tot}}}^{\text{eq}}(T_D)^2}\right], \quad (12)$$

$$\frac{dn_{\xi_{\text{tot}}}}{dt} + \frac{dn_{\chi_{\text{tot}}}}{dt} + 3\mathcal{H}(n_{\xi_{\text{tot}}} + n_{\chi_{\text{tot}}}) = \frac{1}{2}[\langle\Gamma_{\xi\rightarrow\text{SM}}\rangle^T n_{\xi_{\text{tot}}}^{\text{eq}}(T) - \langle\Gamma_{\xi\rightarrow\text{SM}}\rangle^{T_D} n_{\xi_{\text{tot}}}^{\text{eq}}(T)] + 2\Upsilon_{\text{SM,SM}\rightarrow\text{SM},\xi}(T), \quad (13)$$

$$\frac{d\rho_{D_{\text{tot}}}}{dt} + 3\mathcal{H}(\rho_{D_{\text{tot}}} + p_{D_{\text{tot}}}) = \frac{1}{2}\Gamma_{\xi\rightarrow\text{SM}}m_{\xi}(n_{\xi_{\text{tot}}}^{\text{eq}}(T) - n_{\xi_{\text{tot}}}) + 2\Upsilon_{\text{SM,SM}\rightarrow\text{SM},\xi}(T), \quad (14)$$

where $n_{i_{\text{tot}}} = n_{\bar{i}} + n_i$ is the total number density of the i th particle with $n_i^{\text{eq}}(T) = \frac{g_i}{2\pi^2} m_i^2 T K_2(\frac{m_i}{T})$, and $\rho_{D_{\text{tot}}} = \rho_{\bar{\chi}} + \rho_{\chi} + \rho_{\bar{\xi}} + \rho_{\xi}$ and $p_{D_{\text{tot}}} = p_{\bar{\chi}} + p_{\chi} + p_{\bar{\xi}} + p_{\xi}$ are the total energy density and pressure of the DS, respectively. In a relativistic regime of the DS particles, the equation of state is characterized by $p = \frac{1}{3}\rho$, while during matter domination, it simplifies to $p = 0$. For simplicity, we have assumed the Maxwell-Boltzmann statistics for both χ and ξ and express ρ_i and p_i (where $i = \{\chi, \xi\}$) in the following form:

$$\rho_i = n_i \left(3T_D + m_i \frac{K_1(\frac{m_i}{T_D})}{K_2(\frac{m_i}{T_D})} \right), \quad (15)$$

$$p_i = n_i T_D, \quad (16)$$

where m_i is the mass of the i th species and K_1 and K_2 are the modified Bessel functions of the second kind. While deriving the Boltzmann equations, we have considered a

matter-antimatter symmetry within the DS. This symmetry explains the presence of the $\frac{1}{2}$ and 2 factors on the rhs of Eqs. (12)–(14). Once the evolution of ρ_D as a function of T is known, it is possible to extract the evolution of T_D with T .

In Eqs. (12) and (13), $\langle\sigma v\rangle_{\bar{\xi}\xi\rightarrow\bar{\chi}\chi}^{T_D}$ and $\langle\sigma v\rangle_{\bar{\xi}\xi\rightarrow\bar{\chi}\chi}^{T_D}$ represent the thermally averaged cross sections for the processes $\bar{\xi}\xi \rightarrow \bar{\chi}\chi$ and $\bar{\xi}\xi \rightarrow \bar{\chi}\chi$, respectively, at temperature T_D . In Eq. (13), $\langle\Gamma_{\xi\rightarrow\text{SM}}\rangle^T = \Gamma_{\xi\rightarrow\text{SM}} \frac{K_1(\frac{m_{\xi}}{T})}{K_2(\frac{m_{\xi}}{T})}$ refers to the thermal average of the three-body decay width of ξ into the VS particles at temperature T , where $\Gamma_{\xi\rightarrow\text{SM}}$ is the total decay width of ξ . The terms $\gamma_{\text{SM,SM}\rightarrow\text{SM},\xi}(T)$ in Eq. (13) and $\Upsilon_{\text{SM,SM}\rightarrow\text{SM},\xi}$ in Eq. (14) correspond to the collision terms contributing to the production component (arising from the VS) feeding the number density and the energy density of the DS particles, respectively. The expressions for these terms are as follows:

$$\gamma_{ab\rightarrow c\xi}(T) = \frac{8\pi^2 g_a g_b T}{(2\pi)^6} \int_{s_{\min}}^{\infty} ds \lambda(s, m_c, m_{\xi})^2 s^{3/2} \sigma_{c\xi\rightarrow ab}(s) K_1(\sqrt{s}/T), \quad (17)$$

$$\Upsilon_{ab\rightarrow c\xi}(T) = \frac{4\pi^2 g_c g_d T}{(2\pi)^6} \int_{s_{\min}}^{\infty} ds \lambda(s, m_c, m_{\xi})^2 s^2 (1 - R(s, m_c, m_{\xi})) \sigma_{c\xi\rightarrow ab}(s) K_2(\sqrt{s}/T). \quad (18)$$

Here, $\lambda(s, m_a, m_b) = \frac{1}{2s} [(s - (m_a + m_b)^2)(s - (m_a - m_b)^2)]^{1/2}$, and $s_{\min} = \min[(m_a + m_b)^2, (m_c + m_d)^2]$, and $R(s, m_c, m_d) = \frac{m_d^2 - m_c^2}{s}$. Similarly, the two terms with positive coefficients in the rhs of Eq. (14) serve as the source terms for the DS energy density with origins in inverse decay and scattering of SM particles, respectively, at the early Universe. Due to the smallness of T_D , we have safely ignored the backward reaction processes ($\xi + \text{SM} \rightarrow \text{SM} + \text{SM}$) in the Boltzmann equation. The terms with a negative coefficient in the respective rhs of Eqs. (13) and (14) induce the fall of $n_{\xi_{\text{tot}}}$ and $\rho_{D_{\text{tot}}}$ due to the late time decay of ξ . The inclusion of the decay term for ξ in Eq. (13) also ensures that ξ is not sufficiently abundant in the present-day Universe, thus establishing the present setup as a single component DM framework.

B. Analytic understanding of the Boltzmann equations

In this subsection, we aim to analytically comprehend the Boltzmann equations presented in Eqs. (12)–(14). We study the time evolution of the DS temperature (T_D) and the roles played by the two key conversion processes [as indicated in Eqs. (7a) and (7b)] that determine the relic abundance of the DM.

First, let us concentrate on Eq. (14), which governs the evolution of the energy density of the DS. We assume that χ and ξ are in kinetic equilibrium after the initial nonthermal production of ξ from the VS. If the coupling within the DS is not enough, χ and ξ will not be in chemical equilibrium. Such a situation can open up the possibility of producing DM via a freeze-in mechanism from ξ 's produced initially from the VS. However, if the interactions within the DS are

strong enough, there would be a chemical equilibrium between χ 's and ξ 's. Driven by the existence of various processes within the DS, once a chemical equilibrium gets established, the principle of detailed balance would dictate $\mu_\chi = \mu_\xi = \mu_D$, where μ_χ and μ_ξ are the effective chemical potentials of χ 's and ξ 's, respectively. The expression for μ_D is

$$\mu_D = T_D \log \left(\frac{n_i}{n_i^{\text{eq}}} \right). \quad (19)$$

The effect of the nonzero chemical potential on the DM freeze-out has been discussed in Refs. [16,34,69,70]. Consequently, in such a scenario, the relativistic DS particles will have an effective equilibrium number density ($n^{\text{eq,eff}}$), which for the DM χ is given by

$$\begin{aligned} n_\chi^{\text{eq,eff}}(T) &= \frac{n_\xi(T)}{n_\xi^{\text{eq}}(T_D)} n_\chi^{\text{eq}}(T_D) \\ &= n_\chi^{\text{eq}}(T_D) e^{\frac{\mu_D}{T_D}}. \end{aligned} \quad (20)$$

To obtain an analytical estimate of T_D , we assume that μ_D is significantly smaller than the energies of the respective particles and hence can be neglected. Therefore, when the DS particles are relativistic in nature, we can write $\rho_D = \frac{\pi^2}{30} g_{D\rho} T_D^4$, where $g_{D\rho} = g_\chi + g_\xi$, with g_χ and g_ξ being the internal degrees of freedom for χ and ξ , respectively.⁴ It turns out that the impact of the decay term on the rhs of Eq. (14) on the production of ξ at early times is subdominant compared to that of the scattering term and hence can be safely ignored. We thus write $\sigma_{\xi\text{SM} \rightarrow \text{SM},\text{SM}} \simeq \frac{C_\tau^2}{16\pi} s$ in the limit $\sqrt{s} \gg m_{\chi,\xi}$, where C_τ is the effective coupling of ξ with the VS particles. Then, the collision term (involving the $2 \rightarrow 2$ process) appearing in the rhs of Eq. (14) can be estimated as

$$\begin{aligned} \Upsilon_{\text{SM},\text{SM} \rightarrow \text{SM}\xi}^{\text{appx}}(T) &= \frac{4\pi^2 g_a g_b T}{(2\pi)^6} \int_{s_{\min}}^{\infty} ds \lambda(s, m_a, m_b)^2 s^2 \\ &\quad \times \frac{C_\tau^2}{16\pi} s K_2(\sqrt{s}/T) \\ &= C_\tau^2 \frac{24}{\pi^5} T^9. \end{aligned} \quad (21)$$

Next, we derive an approximate analytical expression for T_D (a similar approach has been adopted in Ref. [27]) given by

$$\zeta(T) = \left[\int_{T_i=\Lambda}^T dT' \frac{30 \Upsilon_{\text{SM},\text{SM} \rightarrow \text{SM}\xi}^{\text{appx}}(T')}{\pi^2 g_{*D} \mathcal{H}(T') T'^5} \right]^{1/4}, \quad (22)$$

⁴For finite values of μ_D , ρ_D can be expressed as $\rho_D = \frac{\pi^2}{30} g_{D\rho} T_D^4 e^{\frac{\mu_D}{T_D}}$.

where $\zeta = \frac{T_D}{T}$ and T_i corresponds to the cutoff scale Λ of the EFT. If we consider the Universe to be radiation dominated at the time of production of the DS particles, then by using Eqs. (21) and (22), we find

$$T_D = \sqrt{C_\tau} T \left(\frac{80 M_P \Lambda^3}{14.94 \pi^7 g_{D*} \sqrt{g_\rho}} \right)^{1/4} \propto \sqrt{C_\tau} T \Lambda^{3/4} (M_P)^{1/4}. \quad (23)$$

Thus, in the early Universe, when the DS particles are relativistic, T_D turns out to be proportional to T . Such a correlation has also been reported in Ref. [26]. On the other hand, if the DS is induced by some renormalizable couplings, T_D does not redshift as radiation in the early Universe [27,32,36] even when all the DS particles are relativistic.

Note that μ_D may not always be negligible (as assumed till now). If that is the case, i.e., for $\mu_D \neq 0$, one also has to consider the evolution of $\frac{\mu_D}{T_D}$ with temperature (T). Under the circumstances, to obtain an accurate description of the DS dynamics, one needs to solve the coupled set of Boltzmann equations. Nevertheless, the relationship between T_D and T , as mentioned earlier [see Eq. (23)], remains valid, which can be gleaned from the numerical results presented in the next section (Sec. IV).

We now move on to the set of Boltzmann equations in Eqs. (12) and (13). Given that $m_\chi \approx m_\xi$, these two equations are to be solved simultaneously. For simplicity, let us assume $\mu_D = 0$ and thus consider the analytical expressions for T_D in Eq. (23). Here, both the terms on the rhs of Eq. (12) would contribute to the production of DM (χ). Once the DS reaches thermal equilibrium with a temperature T_D , the DS particles attain the effective equilibrium number density [16,34]. As T_D drops below m_χ , DM (χ) starts annihilating to ξ via $\bar{\xi}\chi \rightarrow \bar{\xi}^c \xi$ and $\bar{\chi}\chi \rightarrow \bar{\xi}\xi$. Consequently, the number density of ξ (n_ξ) starts increasing, thus prompting an eventual “freeze-in” for these particles. On the other hand, the number density of DM (χ) decreases while approaching a freeze-out. It is important to note that, due to nonrenormalizable couplings, $\langle \sigma_{\chi\bar{\chi} \rightarrow \xi\bar{\xi}} \rangle, \langle \sigma_{\bar{\xi}\chi \rightarrow \bar{\xi}^c \xi} \rangle \propto C_\tau^2 m_\chi^2$. Notably, the contributions from both the annihilation modes to the freeze-out process, at the time of DM freeze-out (characterized by the freeze-out temperature $T_{D_{\text{fo}}}$), can be understood by comparing the reaction rates of these processes, i.e.,

$$\frac{\Gamma_{\bar{\xi}\chi \rightarrow \bar{\xi}^c \xi}}{\Gamma_{\bar{\chi}\chi \rightarrow \bar{\xi}\xi}} = \frac{n_\xi(T_{D_{\text{fo}}}) \langle \sigma_{\bar{\xi}\chi \rightarrow \bar{\xi}^c \xi} \rangle}{n_\chi(T_{D_{\text{fo}}}) \langle \sigma_{\bar{\chi}\chi \rightarrow \bar{\xi}\xi} \rangle}. \quad (24)$$

Given that the effective operators that control these processes have similar strengths, we may consider $\frac{\langle \sigma_{\chi\bar{\chi} \rightarrow \xi\bar{\xi}} \rangle}{\langle \sigma_{\chi\bar{\xi}^c \rightarrow \chi\xi} \rangle} \simeq 1$. Then, by using the condition for the so-called sudden

freeze-out, we can arrive at $n_\chi(T_{D_{\text{fo}}}) = n_\chi^{\text{eq}}(T_{D_{\text{fo}}})$. On the other hand, a freeze-in production of ξ (during the DM freeze-out) implies $n_\xi(T_{D_{\text{fo}}}) > n_{\xi_{\text{eq}}}(T_{D_{\text{fo}}})$. Equation (24) then leads to the following relation:

$$\frac{\Gamma_{\bar{\xi}\chi \rightarrow \bar{\xi}^c \xi}}{\Gamma_{\bar{\chi}\chi \rightarrow \bar{\xi} \xi}} > \frac{n_\xi^{\text{eq}}(T_{D_{\text{fo}}})}{n_\chi^{\text{eq}}(T_{D_{\text{fo}}})},$$

$$> \left(\frac{m_\xi}{m_\chi}\right)^{3/2} \exp\left(\frac{m_\chi - m_\xi}{m_\chi} x_{D_{\text{fo}}}\right), \quad (25)$$

where $x_{D_{\text{fo}}} = \frac{m_\chi}{T_{D_{\text{fo}}}} \sim 20$. This implies that, in our scenario, the reaction rate of $\bar{\xi}\chi \rightarrow \bar{\xi}^c \xi$ would be much larger than that of $\bar{\chi}\chi \rightarrow \bar{\xi} \xi$ at the time of DM freeze-out.

As mentioned in Sec. II, in case I, both the DS processes are active, out of which the process $\bar{\xi}\chi \rightarrow \bar{\xi}^c \xi$ plays the dominant role in determining the DM relic abundance via the freeze-out mechanism. However, later we check that the freeze-out process for case I is ruled out by the stability of the DM. On the other hand, for feeble DS couplings, the correct DM relic abundance comes from the freeze-in production of the DM χ from ξ . In this situation, both the processes $\bar{\xi}^c \xi \rightarrow \bar{\xi}\chi$ and $\bar{\xi}\xi \rightarrow \bar{\chi}\chi$ become relevant. In case II, since only one DS process [as indicated in Eq. (7b)] is active, the DM relic abundance in both freeze-out (large DS coupling) and freeze-in (small DS coupling) scenarios is governed by this process.

IV. RESULTS

In this section, we discuss various constraints on the parameter space of the broad scenario under consideration, which come from the theory and relevant experiments. The former set includes the one originating from the requirement of the VS and the DS to be out of thermal equilibrium, and the others concern the perturbativity of the (effective) coupling parameters λ_i and C_τ . The latter set stems from the demand for achieving a successful BBN, from compliance with the photon flux observed by an experiment like the Fermi-LAT in its bid to find indirect signatures of the DM and also from the requirement of satisfying the LFUV constraint. We carry out a thorough study of the scenario by numerically solving the Boltzmann equations presented in Sec. III A and estimating the DM relic abundance for both cases introduced in Sec. II.

A. Regions (out) of thermal equilibrium and of a perturbative C_τ

We identify the region of parameter space where the VS and the DS are out of thermal equilibrium by first delineating the region where these sectors are indeed in thermal equilibrium. We follow the conventional approach that involves the computation of the collective reaction rates responsible for the production of the particles belonging to

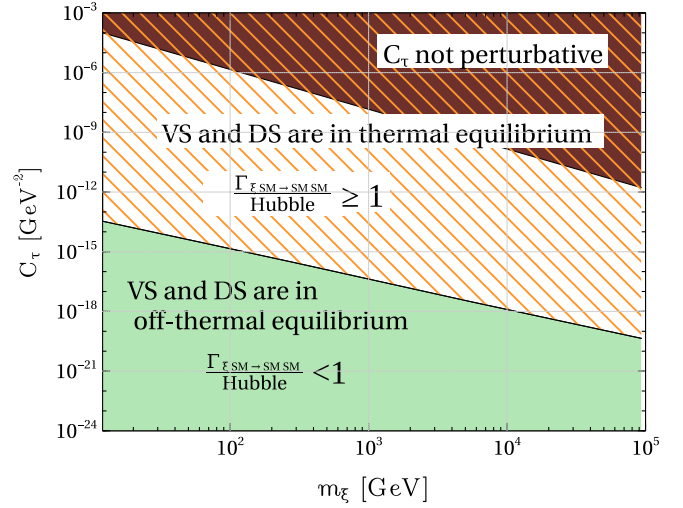


FIG. 2. Situation with a possible thermal equilibrium between the VS and the DS presented in the $m_\xi - C_\tau$ plane with $T = \Lambda (= 100m_\chi)$. Also shown is the region where C_τ ceases to be perturbative. See text for details.

the VS from the DS ones, juxtaposing them with the Hubble parameter ($\mathcal{H}$). The quantity that determines whether a thermal equilibrium is established or not between the two sectors is $\frac{\Gamma_{\xi \text{SM} \rightarrow \text{SM SM}}}{\mathcal{H}}$, where $\frac{\Gamma_{\xi \text{SM} \rightarrow \text{SM SM}}}{\mathcal{H}} > (<) 1$ implies the equilibrium is (not) realized.

In Fig. 2, we contrast the regions in the $m_\xi - C_\tau$ plane over which the VS and the DS could eventually move into thermal equilibrium as opposed to where those remain out of equilibrium, the latter being the target region of the present study. The choice of the indicated plane is prompted by the fact that m_ξ and C_τ are the two parameters that primarily control the ratio $\frac{\Gamma_{\xi \text{SM} \rightarrow \text{SM SM}}}{\mathcal{H}}$, the very quantity that decides, in the first place, the feasibility of thermal equilibrium. In addition, m_ξ restricts the DM mass m_χ such that the latter not only aids its stability but could also account for its observed value of relic abundance. As for C_τ , it plays a role in accounting for the observed relic abundance and ensuring a successful BBN. Furthermore, the figure is drawn for $T = \Lambda (= 100m_\chi)$. The choice of the cutoff scale Λ restricts the value of C_τ from above [see Eq. (9)], beyond which the theory violates the perturbativity of C_τ . The corresponding region is shown by the brown, wedge-shaped area in the top part of the figure.

The hashed strip in orange illustrates the region where the VS and the DS coexist in thermal equilibrium. Conversely, the coveted region over which the two sectors would be out of thermal equilibrium is shown in light green. Notably, as m_ξ increases, the reaction rates are enhanced; we thus look for smaller C_τ 's for larger values of m_χ so that the two sectors still remain out of thermal equilibrium.

At this point, note that the interactions between the VS and the DS are controlled by the dim-6 effective operators, as discussed in Sec. II. The processes that describe the production of the VS states from the DS ones are just the reverse ones of those mentioned in Eqs. (10) and (11). The expression for the relevant collision term at very high temperatures can be obtained in the same way as $\gamma_{ab \rightarrow c\xi}$ in Sec. III B and is found to vary as $\gamma_{ab \rightarrow c\xi} \propto C_\tau^2 T^8$. Consequently, the reaction rates for the processes $\xi\text{SM} \rightarrow \text{SM SM}$ are given by $\Gamma_{\xi\text{SM} \rightarrow \text{SM SM}} \propto C_\tau^2 T^5$. Thus, the quantity $\frac{\Gamma_{\xi\text{SM} \rightarrow \text{SM SM}}}{\mathcal{H}}$ drops as T decreases. This signifies that a DS of a secluded kind at a higher temperature would remain so at a lower temperature.

Thus, a departure from a thermal equilibrium between the VS and the DS may be allowed at a higher value of $T(> 100m_\chi)$, and hence such a possibility remains germane to the present work. However, this necessitates a smaller C_τ to throw the VS and the DS systems out of mutual equilibrium at that high temperature. As we will see later (see Fig. 7), such a smaller value of C_τ tends to attract constraints from BBN. Thus, these two requirements, in conjunction, restrict the value of C_τ from both above and below in our scenario with a secluded DS. It is also found that the requirement of the VS and the DS to be off equilibrium sets an upper bound on C_τ more stringently than the requirement of perturbativity does.

B. Constraints from the BBN

In the standard picture of BBN, around a temperature $\lesssim \mathcal{O}(1)$ MeV, the energy of the cosmic microwave background (CMB) photons becomes lower than the binding energies of the light elements, e.g., deuterium (D), ^3He , ^4He , and ^7Li ; hence, these elements or isotopes become synthesized [71]. There are strict observational constraints on the primordial abundances of these light elements (see Sec. 2 of Ref. [72] and references therein). If a long-lived particle is produced in the early Universe and it decays subsequently into SM particles at a late time, the BBN predictions might get altered. Thus, given that the relevant observables have now been precisely measured, such an effect can be used to constrain the model parameters.

In our framework, the particle ξ is unstable and can be long-lived depending on the smallness of the coupling coefficient C_τ . Once the DS freezes out or freezes in, ξ finally decays to the SM leptons. If such decays occur during the BBN, the newly produced charged leptons could emit energetic photons (an electromagnetic shower) which, in turn, could interact with the cosmic microwave background radiation (CMBR) and trigger the photodissociation process [72–75], thus altering the abundance of light elements. On the other hand, high-energy neutrinos that might appear from such decays could scatter off the

background leptons and produce charged leptons and charged pions. The former could, once again, induce electromagnetic showers that might destroy light elements that are already synthesized, while the latter impacts the $n - p$ ratio through nucleon and pion interactions [76]. Hence, to evade any significant impact on the outcome of BBN, it is apparently safe to make ξ decay early enough, $\tau_\xi < 1$ second, which translates to $T > 3$ MeV. The constraint on the relevant model parameters (m_ξ and C_τ) from BBN will be presented later in Sec. IV D.

C. DM relic abundance

The study of the DM relic abundance is entirely based on the analysis of the Boltzmann equations. Thus, in this section, we first thoroughly explore the implications of these coupled set of equations [presented in Eqs. (12)–(14)] for the DS processes by solving them numerically. To this aim, we track the evolution of the effective chemical potential μ_D (discussed in Sec. III) and the number densities ($n_{\chi, \xi}$) of χ and ξ as functions of $x = \frac{m_\chi}{T}$. We do this by choosing some benchmark points for the two cases (cases I and II) that are introduced in Sec. II. Throughout, as mentioned in Sec. II, we set the cutoff scale of the effective theory to be $\Lambda = 100m_\chi$.

1. Case I ($\lambda_1 = \lambda_2 = \lambda$)

In this case, we consider the Yukawa couplings related to the DS to be of equal strength. Consequently, we have $\frac{\lambda_1}{\Lambda^2} = \frac{\lambda_2}{\Lambda^2} = \lambda$ whose value is to be suitably chosen. To have a quantitative understanding of the DS dynamics, we consider a benchmark point (BP 1) consisting of three remaining free parameters as given by

$$\text{BP 1: } \{C_\tau, m_\xi, m_\chi\} \\ \equiv \{1.5 \times 10^{-16} \text{ GeV}^{-2}, 10^2 \text{ GeV}, 1.1 \times 10^2 \text{ GeV}\}. \quad (26)$$

The choice of such a benchmark point is guided primarily by the stability of the DM and compatibility with the BBN constraint, which, as we have discussed earlier, are in some tension.

In Fig. 3, we discuss the temperature evolutions of the number densities for χ and ξ (n_χ and n_ξ , respectively) for BP 1 in terms of more conventional quantities $\Omega_i/\Omega_{\text{obs}}$. Here, the abundance $\Omega_{i=\chi, \xi} = \frac{m_i s_0 Y_i}{\rho_{\text{crit}}} = 2.755 \times 10^8 \left(\frac{m_i}{1 \text{ GeV}}\right) Y_i(T)$, with $Y_i(T) = \frac{n_i(T)}{s(T)}$, where $s(T)$ is the entropy density of the Universe while s_0 indicates its present value. On the other hand, Ω_{obs} is the observed DM relic abundance. Such evolutions for χ and ξ are illustrated with solid and dashed lines, respectively, and the value of λ for which the DM relic abundance can be achieved within its experimentally reported range is also indicated. We stick to these line-type conventions throughout the rest of the

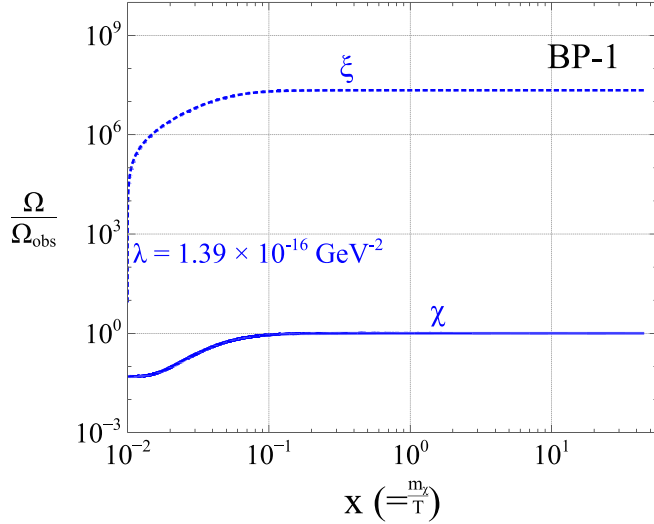


FIG. 3. Variations of $\Omega_i/\Omega_{\text{obs}}$ with respect to $x = m_\chi/T$ for the BP-1 scenario under case I. The value of λ for which the observed DM relic abundance is reproduced is indicated. Here, the solid (dashed) lines represent evolutions of χ (ξ). See text for details.

paper to represent the evolution of number densities of χ and ξ (in terms of $\Omega_i/\Omega_{\text{obs}}$) with x .

We consider a value of λ ($= 1.39 \times 10^{-16} \text{ GeV}^{-2}$) in Fig. 3, where the corresponding variations for $\Omega_i/\Omega_{\text{obs}}|_{i=\chi,\xi}$ are indicated in blue colors. For such a small value of λ , the DS is unable to reach an internal thermal equilibrium, and both processes $\bar{\xi}^c \xi \rightarrow \bar{\xi} \chi$ and $\bar{\xi} \xi \rightarrow \bar{\chi} \chi$ contribute in attaining the final (observed) relic abundance of the DM, via the freeze-in mechanism. Although, in our analysis, we respect the kinematic condition $m_\xi < m_\chi < 3m_\xi$ that prohibits the primary (tree-level) 3-body decay of the DM ($\chi \rightarrow \xi \bar{\xi} \xi^c$), the loop-mediated decay of the DM, $\chi \rightarrow \xi \bar{l} l$, could pose a serious threat to the stability of the DM. However, for the above choice of parameters, the DM is found to be stable with its lifetime ($\tau_\chi \simeq 3.3 \times 10^{29}$ sec) far exceeding the age of the Universe ($\simeq 10^{17}$ sec) and also exceeding its lower bound coming from γ -ray searches [$\mathcal{O}(10^{17})$ sec, see Refs. [77–79]].

We have further checked that for a much larger value of λ (in this case, $\lambda = 1.89 \times 10^{-8} \text{ GeV}^{-2}$), a chemical equilibrium between χ and ξ can set in, and as the Universe evolves, i.e., the temperature drops, χ starts annihilating to ξ via the DS processes $\bar{\xi} \chi \rightarrow \bar{\xi}^c \xi$ and $\bar{\chi} \chi \rightarrow \bar{\xi} \xi$. Hence, the number density of χ starts decreasing, and consequently, the DM seems to freeze out around the observed relic abundance. However, at a later time, the DM will decay into ξ via $\chi \rightarrow \xi \bar{l} l$ at loop level (as the tree-level decay is forbidden by the aforesaid kinematic condition). The lifetime of the DM in this case turns out to be $\tau_\chi \simeq 1.94 \times 10^{13}$ sec, which makes the DM decay too rapidly. Thus, the only viable mechanism that could give rise to the observed DM relic abundance in case I is the

freeze-in type. The freeze-out mechanism is found to have serious tension with the stability of the DM.

It should be noted that the small value of λ that is required to provide a stable DM while possibly ensuring a correct DM relic abundance falls short of the present-day sensitivity of not only the direct but also the indirect (DM) detection experiments. The issue is discussed in further detail in Sec. IV C 3.

2. Case II ($\lambda_1 \ll \lambda_2$)

In this section, we first investigate if the freeze-out mechanism could result in the observed DM relic abundance while keeping the DM stable. We note that the DM decay width ($\Gamma_{\chi \rightarrow \xi \bar{l} l}$) varies as λ_1^2 . Hence, a possible way to find a stable DM is by considering a suitably smaller value of λ_1 (as mentioned in Sec. II) such that $\lambda_1 \ll \lambda_2$. Consequently, the DS dynamics is now dominated by λ_2 , which is the Wilson coefficient for the four-Fermi operator $\bar{\chi} \chi \bar{\xi} \xi$. We further look for a viable freeze-in mechanism consistent with all relevant constraints.

With such a hierarchy between λ_1 and λ_2 , to carry out a quantitative study of the DS dynamics, we start with BP 1 which is characterized by relatively small $m_{\xi\chi} \sim 100 \text{ GeV}$. As we shall soon show, any lower value of m_ξ (and hence m_χ) quickly attracts constraints from the BBN (see Fig. 7) in the secluded DS scenario. This prompts us to explore a much heavier DM, which inevitably requires a smaller λ_2 due to the perturbativity constraint that we discuss by referring to Fig. 7. The corresponding benchmark point that we choose is given by

$$\text{BP 2: } \{C_\tau, m_\xi, m_\chi\} \equiv \{10^{-21} \text{ GeV}^{-2}, 10 \text{ TeV}, 11 \text{ TeV}\}, \quad (27)$$

which still maintains the same ratio between m_χ and m_ξ as in the case of BP 1. BP 2 is characterized by fairly large values of m_χ and m_ξ and hence will be consistent only with rather small values of λ_2 . As discussed earlier, we solve the coupled set of Boltzmann equations presented in Eqs. (12)–(14). For the sake of simplicity, we set $\lambda_1 = 0$ and work with finite values of λ_2 . We then look into the evolutions of $\frac{\mu_D}{T_D}$ and T_D and the number densities of χ and ξ , i.e., n_χ and n_ξ .

As pointed out in Sec. III, we work with the most general situation having $\mu_D \neq 0$. In the left plot of Fig. 4, we present the variation of $\frac{\mu_D}{T_D}$ with x for the BP-1 scenario. The corresponding value of λ_2 that is used for this purpose is also indicated. The value is chosen such that it reproduces the DM relic abundance within the range reported by experiments. It can be seen that, as the temperature (T) of the Universe drops (when the DM freezes out; see the right plot), the ratio $\frac{\mu_D}{T_D}$ starts increasing. This is what is expected from Eq. (19).

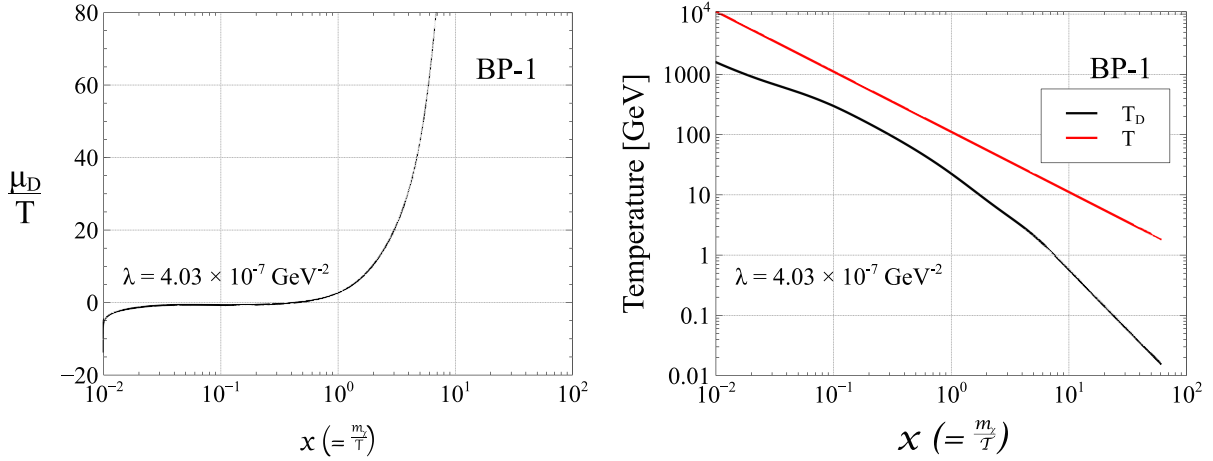


FIG. 4. Variations of $\frac{\mu_D}{T}$ (left) and T_D and T (right; in black and red, respectively) as functions of $x(=m_\chi/T)$ for the BP-1 scenario under case II. The appropriate value of λ_2 that results in the correct relic abundance of DM is also indicated. See text for details.

In the right plot of Fig. 4, we now choose to show the evolutions of T_D (in black) and T (in red) as functions of x . It is seen that, while T drops with increasing x (as defined) for a given m_χ , T_D initially increases sharply because of a transfer of energy from the VS bath to the DS as dictated by the set of Boltzmann equations presented in Sec. III. As the Universe cools down (i.e., T decreases), T_D drops as well. For $T_D < m_\xi$ (i.e., $x > 1$), when both DS particles χ and ξ become nonrelativistic, T_D starts redshifting like matter.

One can have a similar set of plots for the BP-2 scenario that would aid in understanding a possible freeze-out of DM in that scenario. However, we do not present those here as we have checked that the value of λ_2 ($3.27 \times 10^{-9} \text{ GeV}^{-2}$) that is required for yielding the right DM relic (within 1σ) would be ruled out by the perturbativity constraint, thus making the freeze-out mode no more viable for achieving the correct DM relic.

We now turn to the discussion of evolutions of the number densities n_χ and n_ξ in terms of the ratio $\Omega_i/\Omega_{\text{obs}}|_{i=\chi,\xi}$ (as introduced in the previous subsection) and check if they could result in the observed DM relic following the freeze-out (red lines) and/or freeze-in (blue lines) path(s). For freeze-out to take place, one needs an optimally large λ_2 such that there exists a chemical equilibrium between the DS states χ and ξ . Subsequently, when the DS temperature T_D becomes comparable to m_χ , χ starts annihilating to ξ via the process $\bar{\chi}\chi \rightarrow \bar{\xi}\xi$. Finally, when the reaction rate of this particular process drops below $\mathcal{H}$, the DM freezes out. Note that the other annihilation mode $\bar{\xi}\chi \rightarrow \bar{\xi}^c\xi$ is now closed as we have set $\lambda_1 = 0$.

In contrast, a much smaller value of the λ_2 suffices for the freeze-in mechanism to work. For such a small λ_2 , the DS is unable to reach a chemical equilibrium because of an insufficient interaction rate inside the DS. Here, the process

driven by λ_1 (i.e., $\bar{\xi}^c\xi \rightarrow \bar{\xi}\chi$) would be missing, and the process $\bar{\xi}\xi \rightarrow \bar{\chi}\chi$, driven by λ_2 , would only contribute in achieving the right DM relic abundance via freeze-in. In Fig. 5, we present the evolutions of the number densities ($\Omega_i/\Omega_{\text{obs}}|_{i=\chi,\xi}$) with $x = \frac{m_\chi}{T}$ for the BP-1 (left plot) and BP-2 (right plot) scenarios.

As can be seen from the left plot of Fig. 5, out of the two chosen values of λ_2 , the larger (smaller) one leads to, as expected, a freeze-out (freeze-in) of the DM (χ) indicated by the solid red (blue) lines. It also shows that as the DM freezes out, ξ attains a constant number density via freeze-in. It may, however, be noted that ξ 's would subsequently decay to SM leptons well before the BBN. The indicated values of λ_2 , though primarily chosen to reproduce the right relic abundance via the respective mechanisms, are also found to be consistent with various relevant theoretical and experimental constraints as elucidated later in this section by referring to Fig. 7.

It is further noted that for freeze-in to work in case II, we require a slightly larger value of λ_2 when compared to what is needed in the BP-1 scenario under case I. This is expected given that the contribution to the production of the DM from $\bar{\xi}^c\xi \rightarrow \bar{\xi}\chi$ is negligible (since $\lambda_1 = 0$) and hence $\bar{\xi}\xi \rightarrow \bar{\chi}\chi$ would be the only contributing process. Thus, a slightly larger value of λ_2 is required to compensate for the small deficit in the DM number density (which has previously been generated via the process $\bar{\xi}^c\xi \rightarrow \bar{\xi}\chi$ in case I) such that the DM could achieve the observed relic abundance.

The plot on the right in Fig. 5 simply tells us that, while remaining consistent with all relevant constraints, the only way in which the DM could attain the right relic abundance in BP 2 under case II is via freeze-in. The reasons behind this were pointed out earlier in this subsection.

In Table III, we present a comparative summary of the possibilities in the two cases we have discussed by referring

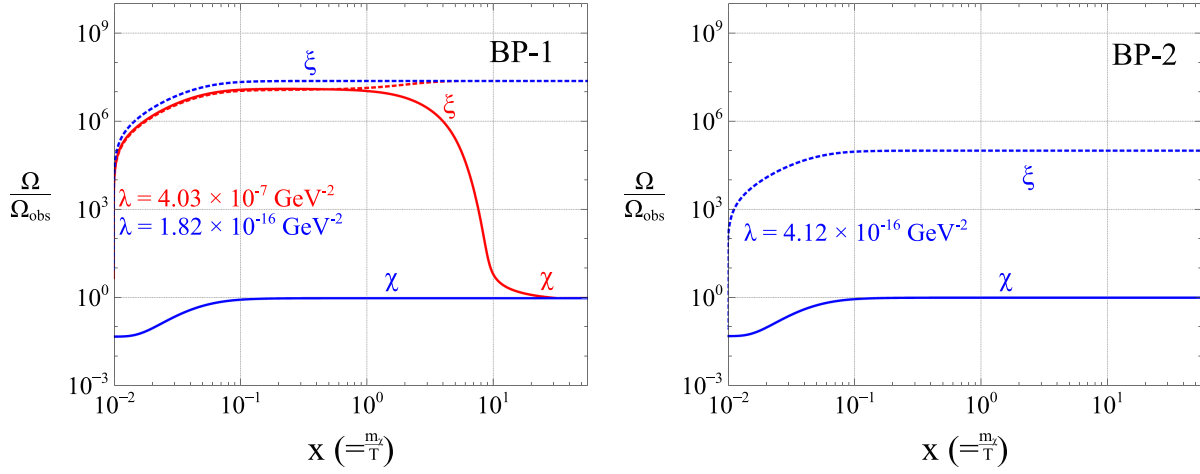


FIG. 5. Evolution of $\frac{\Omega}{\Omega_{\text{obs}}}$ as functions of $x(=m_\chi/T)$ for χ (solid) and ξ (dashed) for the BP-1 (left) and BP-2 (right) scenarios under case II. Results for both freeze-out (red) and freeze-in (blue) scenarios are shown. The value of λ_2 that results in the observed relic abundance of the DM for the freeze-out (freeze-in) scenario is indicated in red (blue).

to the chosen benchmark points and indicating the dominant processes responsible for DM (relic) production that contribute to the mechanism of freeze-out and/or freeze-in.

Before we end this discussion, we carry out an investigation on the effect of the ratio m_χ/m_ξ on such a study. We first consider the same C_τ and m_ξ as in BP 1 (for which both freeze-out and freeze-in are viable) under case II but consider $m_\chi = 2 \times m_\xi$ in place of $m_\chi = 1.1 \times m_\xi$, which we have used so far. We find that, for the freeze-out (freeze-in) mechanism to work, one needs to consider a smaller value of λ_2 to avoid an underabundant (overabundant) DM relic. Note that a smaller value of λ_2 is only constrained by the requirement of generating the observed DM relic while its larger values attract bounds from the demand on its perturbativity and the indirect DM-detection experiments searching for diffused γ rays. Hence, the smaller the value of m_χ/m_ξ , the more conservative the scenario becomes from the viewpoint of perturbativity and bounds coming from an indirect DM-detection experiment like the Fermi-LAT. Hence, we choose $m_\chi/m_\xi = 1.1$. Furthermore, the variation of the required value of $\lambda(\lambda_1 = \lambda_2)$ with an increasing DM mass is found to be very similar to that in case I.

3. Indirect detection of the DM in γ -ray searches

In the scenario presented in this work, prompt photons (γ) can be generated from one-step cascades of a pair of ξ 's produced in the mutual annihilation of the DM particles. Schematically, this is given by $\bar{\chi}\chi \rightarrow \bar{\xi}\xi$, followed by the decays $\xi_R \rightarrow \bar{\tau}_R \nu_\tau \mu_L$, $\xi_R^c \rightarrow \bar{\tau}_R \nu_\tau \tau_L$ which are governed by the portal coupling C_τ . The prompt photons have their origins in the QED and electroweak bremsstrahlung off the charged leptons. The differential photon flux originating from such a cascade is given by [80–82]

$$\frac{d\Phi}{dE_\gamma d\Omega} = \frac{\langle \sigma v \rangle_{\bar{\chi}\chi \rightarrow \bar{\xi}\xi}}{16\pi m_\chi^2} \rho_\odot^2 R_\odot \bar{J}_{\text{ann}} \sum_l \text{Br}(\xi \rightarrow l + \nu) \frac{dN_\gamma^l}{dE_\gamma}, \quad (28)$$

where $\langle \sigma v \rangle_{\bar{\chi}\chi \rightarrow \bar{\xi}\xi}$, $\text{Br}(\xi \rightarrow l + \nu)$, and m_χ are the model-dependent quantities. Here, we have adopted the Navarro-Frenk-White (NFW) density profile for the DM [83,84], which sets the DM mass density $\rho_\odot = 0.3 \text{ GeV} \cdot \text{cm}^{-3}$ at the solar location while $R_\odot$ is the distance vector between the Galactic center and the Sun. The term $\bar{J}_{\text{ann}}$ is completely determined by the distribution of the DM mass density $\rho(r)$

TABLE III. Possibilities of freeze-out and/or freeze-in as a means of yielding the observed DM relic abundance and the processes contributing to them in cases I and II for the chosen benchmark points.

Case	Benchmark	Mechanism	λ (λ_2) in case I (II) [GeV^{-2}]	Processes	Possibility
Case I	BP 1	Freeze-out	—	$\bar{\xi}\chi \rightarrow \bar{\xi}^c \xi$	$\times$
		Freeze-in	1.39×10^{-16}	$\bar{\xi}^c \xi \rightarrow \bar{\xi}\chi, \bar{\xi}\xi \rightarrow \bar{\chi}\chi$	$\checkmark$
Case II	BP 1	Freeze-out	4.03×10^{-7}	$\bar{\chi}\chi \rightarrow \bar{\xi}\xi$	$\checkmark$
		Freeze-in	1.82×10^{-16}	$\bar{\xi}\xi \rightarrow \bar{\chi}\chi$	$\checkmark$
	BP 2	Freeze-out	—	$\bar{\chi}\chi \rightarrow \bar{\xi}\xi$	$\times$
		Freeze-in	4.12×10^{-16}	$\bar{\xi}\xi \rightarrow \bar{\chi}\chi$	$\checkmark$

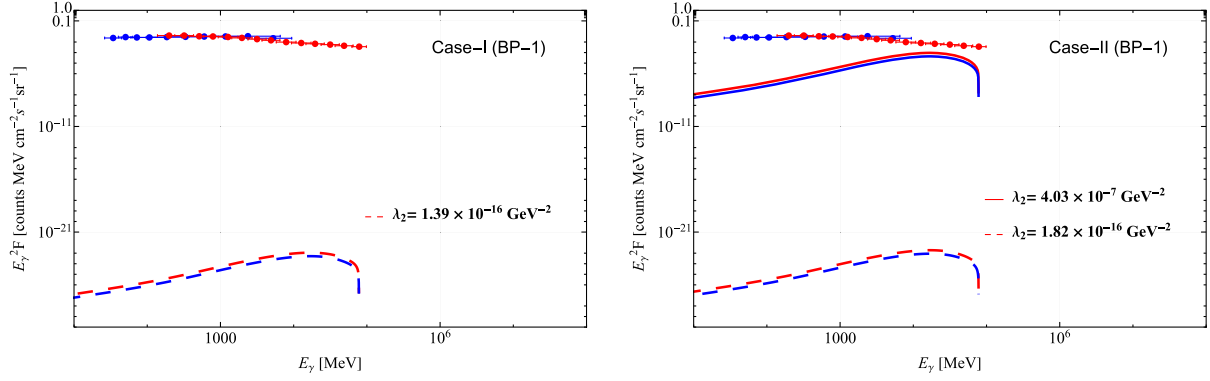


FIG. 6. Variations of the differential photon flux $E_\gamma^2 F$ (where $F = \frac{d\Phi}{dE_\gamma}$) as a function of E_γ for case I (left) and case II (right), in scenarios with freeze-out (solid line) and freeze-in (dashed line). In the top part of each plot, we show the data points from the Fermi-LAT [61,62] (in red) and the EGRET [86,87] (in blue) experiments. The corresponding values of λ are as indicated in Figs. 3 and 5.

and is defined as

$$\bar{J}_{\text{ann}} = \frac{1}{\Delta\Omega} \int d\Omega \int \frac{d\ell}{R_\odot} \left(\frac{\rho[r(\ell, \psi)]}{\rho_\odot} \right)^2, \quad (29)$$

where r is the distance of the source under survey from the Galactic center, ℓ is its distance from the observer along the line of sight, ψ is the angle between the line of sight and $R_\odot$, and $\text{Br}(\xi \rightarrow l + \nu)$ is the relevant decay branching fraction of ξ to charged leptons. The quantity $\frac{dN_\gamma^l}{dE_\gamma}$, i.e., the photon energy spectrum, is obtained by using the publicly available package PPC4DMID [85].

We now study the relevant photon spectra numerically and compare them with the latest experimental results (limits) from various experiments (like Fermi-LAT [61,62] and EGRET [86,87], in particular) that carry out indirect searches for the DM. In Fig. 6, we present the energy spectra of the photon flux, projected (at the bottom) for the experiments Fermi-LAT (red) and EGRET (blue), and compare them with the corresponding observations (at the top). The left (right) plot refers to the BP-1 scenario under case I (case II) where the indicated values of λ (λ_2) ensure a stable DM with the observed relic abundance (as discussed in Sec. IV C). In both cases, the smaller value of λ (λ_2) (previously shown in blue in Figs. 3 and 5) corresponds to the freeze-in scenario (shown by dashed lines), and the photon flux remains far below the level of current sensitivity of both Fermi-LAT and EGRET over the relevant range of E_γ .

Note that the left plot of Fig. 6 tells us that the prospects of detecting the DM at the above-mentioned experiments, via the γ -ray spectra, in case I is abysmal. The situation is no different in case II for the freeze-in scenario, as can be gleaned from the right plot of Fig. 6. However, the projected photon fluxes at these experiments for a larger value of λ_2 (shown by solid lines) that correspond to the freeze-out scenario in BP 1 under case II (presented in red in the left plot of Fig. 5) come much closer to the current

observations (sensitivities). This is because a larger value of λ_2 enhances the key annihilation rate of the DM (χ) to ξ , the photons contributing to the flux being subsequently radiated off the leptons to which these ξ 's decay. No doubt, the situation is much better compared to the freeze-in case. However, one needs to closely check if an increase in sensitivity by 1 to 2 orders of magnitude is possible in the future so that the flux projected here is experimentally meaningful.

D. Viable region of the parameter space

In this section, we delineate the viable region of the parameter space of the broad scenario under consideration by including the relevant theoretical and experimental constraints in our analysis. The theoretical constraints include the ones coming from the requirements of the VS and the DS to be out of mutual thermal equilibrium and the perturbativity of the interaction strengths in the form of λ_2 's, while the experimental ones refer to those required to obtain a successful BBN and not to exceed the photon flux as observed by the Fermi-LAT experiment in its bid to detect the DM indirectly. The necessary discussions on these constraints can be found in Secs. IV A, IV B, and IV C 3. In addition, we require compliance with the observed relic abundance of the DM.

For the purpose at hand, we choose to work under case II. This is because, for case I, freeze-in is the only possible relic production mechanism which already requires a rather small λ ($= \lambda_1 = \lambda_2$) and hence is far less sensitive to the current γ -ray searches, as can be seen from Fig. 6. Thus, the free parameters we use in this study are m_χ , m_ξ , C_τ , and λ_2 .

To understand the interplay of these parameters on the DM relic abundance and also on the production mechanisms of the DM, we thus scan over the suitable region of the parameter space under case II by retaining $m_\chi/m_\xi = 1.1$. Such a region is chosen such that it complies with all the constraints mentioned above. In particular, the need for the VS and the DS to remain out of thermal

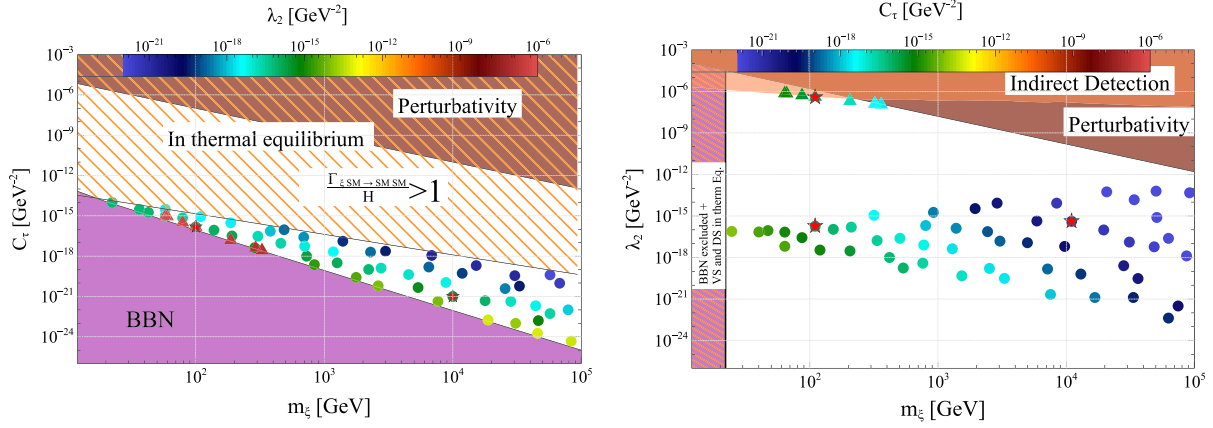


FIG. 7. Regions in the $m_\xi - C_\tau$ plane (left) and $m_\chi - \lambda_2$ plane (right) constrained by requiring a successful BBN (in purple) and perturbativity (in brown) of C_τ (left) and λ_2 (right). The colored points scattered over the white regions represent parameter values which, in addition, comply with the observed DM relic abundance, the colors being indicative of the values of λ_2 (left) and C_τ (right), as shown in the respective palettes. The freeze-out (filled triangles) and freeze-in (filled circles) processes via which the observed DM abundance is achieved are also presented. The asterisks (in pink) stand for the two benchmark points that we use for the numerical study. In the left plot, the region over which the VS and DS are in thermal equilibrium is delineated by orange hashed lines. In the right plot, the horizontal band in light brown is excluded by the Fermi-LAT [61,62] data.

equilibrium (as discussed previously in Sec. IV A) pushes one to small values of C_τ . On the other hand, smaller C_τ 's are constrained by requiring a successful BBN, as discussed in Sec. IV B.

In Fig. 7, we first delineate (in colors) the regions in the relevant parameter planes that are constrained by various theoretical and experimental considerations as discussed earlier. The leftover regions (in white) are then populated by points that satisfy the observed relic abundance (within 1σ of its observed central value). In either case, the color palette reflects the value of the remaining parameter (λ_2 or C_τ) that varies simultaneously. The particular mechanism that leads to the relic abundance is indicated by a solid triangle (freeze-out) and a solid circle (freeze-in), respectively. The specific scenarios we considered under case II in the previous section are marked by solid asterisks.

For the plot on the left in Fig. 7, we choose the plane $m_\xi - C_\tau$, while the variation of λ_2 is presented with the help of the color palette. The choice of the variable m_ξ in the horizontal axis is motivated by the fact that the appearance (or absence, which is what we try to explore and ensure in our study) of a thermal equilibrium between the VS and the DS is directly connected to m_ξ , rather than m_χ . The phenomenon further depends on C_τ .

The same is true for the case of BBN. As discussed in Sec. IV A, with increasing m_ξ , C_τ must decrease if the VS and the DS have to remain out of thermal equilibrium. In Fig 7, we highlight the region (purple-shaded) where the smallness of C_τ makes ξ decay at a later time, i.e., at a temperature below 3 MeV, thus spoiling the prospect of a successful BBN. As can be gleaned from the left plot of Fig. 7 (tip of the white wedge), the two aforesaid constraints, in conjunction, put a simultaneous lower bound on

m_ξ by excluding $m_\xi < 20$ GeV and an upper bound on C_τ by excluding $C_\tau > 10^{-14}$ GeV $^{-2}$ over the requirement of the VS and the DS to be thermally off equilibrium.

This plot further shows that the wedge-shaped white region in the $m_\xi - C_\tau$ plane that survives various constraints is moderately thin with smaller m_ξ (amounting to $m_\chi \lesssim 20$ GeV) being practically ruled out. We further note that a freeze-out of the DM could only happen in a restricted region with a DM that is not too heavy ($\lesssim 400$ GeV). For larger DM masses, freeze-in is found to be the only possibility. This is because the low values of λ_2 that we obtain as the upper bound coming from its perturbativity are drastically lowered as the DM mass increases (see Sec. II). This dependence is shown rather neatly in the right plot of Fig. 7. It is evident from the colors of the scattered points and the gradient of the color palette that, for a particular m_ξ , as C_τ increases the required value of λ_2 that satisfies the observed DM abundance decreases. This is because, for a particular m_ξ , a larger C_τ would imply a large number of ξ ; hence, the production rate of the DM via the freeze-in mechanism is also enhanced. Hence, to satisfy the observed relic abundance, we require a smaller λ_2 . In the right plot of Fig. 7, we present the same points that are presented in the left plot but in the $m_\chi - \lambda_2$ plane with C_τ now being presented via the color palette. The chosen plane suits the simultaneous presentation of the relevant constraints, including the one from the indirect detection of the γ ray by the Fermi-LAT experiment along with that for the DM relic abundance (and the mechanisms via which this is achieved) in a single frame. Here, we present how the γ -ray flux observed by the Fermi-LAT experiment [61,62] (the one from the EGRET experiment being less severe in the region we are interested in) restricts

the $m_\chi - \lambda_2$ plane. For this purpose, we perform a small scan over the parameters $\{\lambda_2, m_\chi\}$ while setting $m_\xi = \frac{m_\chi}{1.1}$. For some given sets of values for $\{m_\chi, \lambda\}$,⁵ we then estimate the differential photon flux of Eq. (28) in E_γ and ensure that the value of the flux thus obtained does not exceed its upper bound as set by Fermi-LAT [61,62]. This then rules out a region in the $m_\chi - \lambda_2$ plane which is delineated in the figure in light brown region.

The region that is ruled out by demanding perturbativity of λ_2 is delineated by dark brown. Also shown is a vertical band (orange hashes over purple region) at small values of m_χ , which arises from the simultaneous imposition of the requirement of a successful BBN and the need for the VS and the DS to be off equilibrium as discussed in the context of the left plot. In fact, this is a map in the current plane of a robust observation made from the plot on the left where $m_\chi \lesssim 20$ GeV is ruled out in such a numerical setup.

As may be seen, the points are now clearly grouped into two regions: for the few points having larger values of λ_2 and $m_\chi \lesssim 400$ GeV, the observed DM relic abundance is achieved, as expected, via freeze-out (solid triangles). On the other hand, for lower values of λ_2 , the freeze-in (solid circles) mechanism yields the correct DM relic abundance over the entire range of m_χ presented in the plot.

Note that the freeze-out route seems to be somewhat fine-tuned as the corresponding points appear at the edge of viability when seen from the angle of perturbativity of λ_2 and the constraints imposed on it from an indirect DM detection experiment like the Fermi-LAT. The fine-tuned situation arises since a somewhat lower value of λ_2 (with other parameters kept fixed) would lead to an overabundant DM via freeze-out. The only possibility to obtain the right DM relic is then by opting for a smaller C_τ , which would suppress the production of ξ at the early Universe leading to a smaller effective equilibrium number density ($n_\chi^{\text{eq,eff}}$) of the DM [see Eq. (20)]. Consequently, a reduced annihilation rate of the DM resulting from a lower value of λ_2 could be sufficient to optimally annihilate the lesser number of the DM given by the suppressed value of $n_\chi^{\text{eq,eff}}$, thus retrieving the observed value of DM relic abundance. However, it is clear from the left plot of Fig. 7 that such a smaller value of C_τ is ruled out by requiring a successful BBN—hence, the absence of any viable point at a lower value of λ_2 which results in such a fine-tuned situation.

As for the larger set of points with lower values of λ_2 for which observed DM relic abundance is achieved via freeze-in, the following patterns are essential.

- (i) For any given value of m_χ , as λ_2 increases, C_τ decreases. This is because with increasing λ_2 , the annihilation rate of ξ increases (via $\bar{\xi}\xi \rightarrow \bar{\chi}\chi$), and consequently, we end up with an overabundant DM. Hence, to obtain the DM relic abundance within the experimentally observed range, a lowering of C_τ is required so that the number density of ξ initially produced from the VS [via the generic process $\text{SM SM} \rightarrow \text{SM } \xi$, see Eqs. (10) and (11)] is reduced. However, lowering of C_τ is soon confronted by the requirement of a successful BBN. This is reflected by the absence of any viable point beyond a certain value of λ_2 depending on the value of m_χ . Conversely, λ_2 cannot indefinitely decrease as that would make the DM underabundant via freeze-in. To compensate for this, one could instead increase C_τ . However, this would soon restore the thermal equilibrium between the VS and the DS (see Sec. IV A), thus spoiling the very basic requirement for such a secluded scenario to work.
- (ii) For any given λ_2 , as m_χ (m_ξ) increases, C_τ is found to decrease, as can be gleaned from the color palette. This is because, with increasing m_χ , the reaction rate of the process $\bar{\xi}\xi \rightarrow \bar{\chi}\chi$ that controls the freeze-in mechanism is enhanced (as it is driven by a four-fermion operator which makes $\langle\sigma v\rangle$ vary like m_χ^2), thus resulting in an overabundant DM. A smaller C_τ could lower the abundance to its observed value by lowering n_ξ . The plot, however, displays a clear trend that a further increase in m_χ would soon attract the perturbative bound on λ_2 for its larger values.

The discussions above reveal that, in the case under consideration, the freeze-out mechanism that results in the right amount of DM relic requires a relatively large λ_2 ($\mathcal{O}(10^{-7} \text{ GeV}^{-2})$). Its viability, however, is somewhat strained due to looming constraints from the theory (perturbativity of λ_2) and the Fermi-LAT experiment. The discussions also shed light on how intimately the observed DM abundance achieved via the freeze-in mechanism is connected to the production of ξ from the SM bath at the early Universe and its annihilation rate.

E. Constraints from lepton-flavor universality violation

Apart from the astrophysical and cosmological constraints on the model parameter space, we have also examined the limits arising from LFUV. In the SM, the decays $\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau$ and $\tau \rightarrow e \bar{\nu}_e \nu_\tau$ proceed through a virtual $W^\pm$ boson. In our framework, integrating out the heavy charged Higgs $H^\pm$ induces additional dimension-6 operators that contribute to these processes (as shown in Fig. 8). Since the resulting decay amplitudes depend on the Yukawa interactions of the μ , τ , and e , they modify the LFUV observable (g_μ/g_e), which is constrained by the

⁵The differential photon flux is proportional to $\text{Br}(\xi \rightarrow l + \nu)$ as shown in Eq. (28). The quantity is insensitive to C_τ as it is canceled out in the definition of the branching fraction.

τ -decay rate and its leptonic branching fractions. In lepton-specific 2HDM, at tree level, (g_μ/g_e) can be expressed as [42]

$$\frac{g_\mu^2}{g_e^2} = \frac{1 + m_\mu^2 m_\tau^2 \tan^4 \beta / (4M_{H^\pm}^4) - (2m_\mu^2 \tan^2 \beta / M_{H^\pm}^2) g(\frac{m_\mu^2}{m_\tau^2}) / f(\frac{m_\mu^2}{m_\tau^2})}{1 + m_e^2 m_\tau^2 \tan^4 \beta / (4M_{H^\pm}^4) - (2m_e^2 \tan^2 \beta / M_{H^\pm}^2) g(\frac{m_e^2}{m_\tau^2}) / f(\frac{m_e^2}{m_\tau^2})}, \quad (30)$$

where $f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$ and $g(x) = 1 + 9x - 9x^2 - x^3 + 6x(1+x) \ln x$. The constraint on $(g_\mu/g_e) = 1.0018 \pm 0.0014$ is given by the HFLAV group [88]. As discussed in Ref. [42], these considerations make the following windows of $m_{H^\pm}$ viable:

$$(0.60 \tan \beta) \text{ GeV} \leq m_{H^\pm} \leq (0.66 \tan \beta) \text{ GeV}, \quad m_{H^\pm} \geq (3.15 \tan \beta) \text{ GeV}. \quad (31)$$

As discussed already, in our scenario, the relevant region of parameter space should have a large $m_{H^\pm} (> 100 m_\chi)$. Since we are working in an EFT scenario, this puts a lower limit on the effective scale Λ . Therefore, by using the definition of C_τ [in Eq. (6)], Eq. (31) can be expressed in terms of our model parameters as

$$C_\tau \leq \frac{g_2 m_\tau y_4 \cos \beta}{\sqrt{2} m_W (3.15)^2},$$

where g_2 is the $SU(2)_L$ gauge coupling of the SM. For the maximum value of $\tan \beta$ [as given in Eq. (5)],

$$\frac{C_\tau}{y_4} \leq 8.2 \times 10^{-7} \text{ GeV}^{-2}. \quad (32)$$

From Fig. 7, the maximum contribution to the ratio $\frac{C_\tau}{y_4}$ comes from the point $\{m_\xi, C_\tau\} = \{15.4 \text{ GeV}, 10^{-14} \text{ GeV}^{-2}\}$. Using the definition of C_τ , we recalculate the ratio $\frac{C_\tau}{y_4} \simeq 3.9 \times 10^{-9} \text{ GeV}^{-2}$ (for maximum $\tan \beta$). Thus, we can see that the ratio lies well below the conservative LFUV bound, ensuring that the model safely satisfies the experimental constraint on $\frac{g_\mu}{g_e}$ as mentioned above.

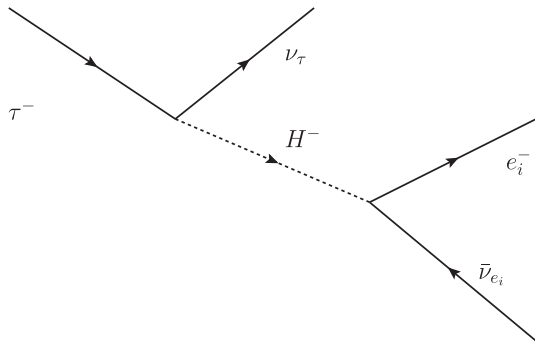


FIG. 8. Feynman diagram responsible for LFUV contribution from BSM, where $i = \mu, \tau$.

V. SUMMARY AND CONCLUSIONS

In this work, we present a theoretical description and the phenomenology of secluded dark matter having its origin in a setup motivated by an effective field theory. We have explored the type-X 2HDM with a global $U(1)_{\mu-\tau}$ symmetry extended by two fermionic fields χ (the DM) and ξ (the mediator) and a scalar field S , all belonging to a secluded dark sector. The choice of such a scenario is motivated by the possibilities that it allows for an initial thermal production of the dark sector particle ξ from the visible (SM) sector and the mutual conversion processes involving ξ and χ within the dark sector working in tandem to yield the observed DM relic abundance. The former set of processes is predominantly controlled by various dim-6 four-Fermi (contact interaction) operators that we have constructed by integrating out the heavier Higgs bosons ($H^\pm$, H , and A), with their strengths being given by the portal coupling parameter C_τ . The latter ones are constructed by integrating out the heavy dark-sector scalar excitation (S) whose strengths are parametrized by the DS couplings $\lambda_{1,2}$. Further, on the cosmology front, the scenario is able to ensure a successful BBN front; on the astrophysics front, it could also be sensitive to γ rays that are searched for as indirect probes to the DM in an experiment like the Fermi-LAT.

We have identified the region in the $m_\xi - C_\tau$ plane where the VS and the DS are not in thermal equilibrium, which is characterized by a relatively small C_τ . However, the requirement of a successful BBN in the early Universe places a lower bound on C_τ , for any given value of m_ξ (and hence m_χ , the mass of the DM particle). On the other hand, the stability of the DM (χ) requires a small enough DS coupling λ_1 . Furthermore, we have ensured the perturbativity of both C_τ and λ_2 .

With all these considerations in place, we have carried out a comprehensive numerical analysis of the scenario by solving the appropriate set of coupled Boltzmann equations after incorporating all relevant production and annihilation processes of χ and ξ . Depending on the coupling strengths

of the DS (λ_1, λ_2), we have considered two cases: case I ($\lambda_1 = \lambda_2 = \lambda$) and case II [$\lambda_1 (= 0) \ll \lambda_2$]. The salient observations of these two cases that can be made from the present analysis are as follows.

- (i) Requiring the VS and the DS to be out of mutual thermal equilibrium and simultaneously securing a successful BBN forbids the DM mass m_χ below around 20 GeV. This results in an absolute upper bound on C_τ ($\lesssim 10^{-14}$ GeV $^{-2}$) in the scenario under consideration. The physics behind both these requirements depends only on the portal coupling (C_τ) and the mass of the mediator (m_ξ), and is independent of the choice of $\lambda_{1,2}$. These requirements and the dependencies on the portal coupling and mediator mass can be considered as some general features in any scenario that can prompt nonthermal production of DM.
- (ii) In case I, the possibility of a large value of λ ($= \lambda_1 = \lambda_2$) which helps attain the right DM relic via freeze-out is ruled out by the stability of the DM, χ , decaying at a later time to ξ , via the loop-mediated process $\chi \rightarrow \xi \bar{l}$, even as its tree-level decay is forbidden due to the choice $m_\xi < m_\chi < 3m_\xi$. However, the correct relic abundance of the DM can still be achieved by exploiting the freeze-in mechanism, with a smaller λ , thus without jeopardizing the stability of the DM. This interplay of the DS couplings λ_1 and λ_2 in the production mechanism of DM is independent of the choice of the origin of DS production.
- (iii) The setup proposed in case II could explain the observed DM relic abundance achieved either via freeze-out or through freeze-in within the DS, depending on the strength of the interaction, λ_2 between χ and ξ . Here, the stability of the DM (χ) is ensured by setting $\lambda_1 = 0$, which prohibits the possible loop-level decay of χ , $\chi \rightarrow \xi \bar{l}$.
- (iv) In case II, one requires $\lambda_2 = \mathcal{O}(10^{-7}$ GeV $^{-2}$) for the freeze-out scenario to work. However, the adverse interplay of the model parameters, while complying simultaneously with different constraints (including indirect detection), leads to a rather fine-tuned region having smaller $m_{\xi,\chi}$ and larger λ_2 . For a large DM mass, $m_\chi \gtrsim 400$ GeV, the scenario is constrained by the perturbativity of λ_2 .
- (v) On the other hand, in case II, the freeze-in mechanism works with rather small values of λ_2 , as expected, but now over a large range of m_χ . The strength of the coupling parameter λ_2 (required to produce the observed DM relic abundance) has been found to be inversely proportional to the portal coupling C_τ .

- (vi) Note that the requirement of a larger DS coupling (λ_2) for the freeze-out scenario to work under case II would result in an enhanced γ -ray flux arising from DM annihilation. Such an enhanced γ -ray flux could easily reach the current limit of sensitivity of the Fermi-LAT experiment; hence, the scenario attracts a bound from the same flux.

Therefore, case II could be valid over a larger volume of parameter space that conforms with the constraints coming from the requirement of having off-equilibrium VS and DS, as well as from the BBN, the observed relic abundance of the DM, and those from the observed γ -ray flux in an experiment like the Fermi-LAT.

- (vii) The constraint on LFUV from the HFLAV experiment translates into a lower bound on the effective scale of the theory and hence results in an upper bound on the ratio $\frac{C_\tau}{y_4}$. The latter turns out to be far above our conservative limit, thus ensuring its viability.

To conclude, we would like to emphasize that we focus on a minimalistic setup in which the secluded dark matter can have an EFT-based origin in a type-X 2HDM scenario. Nonetheless, we have clarified that our approach would work in a more general setup as well. In that case, qualitatively, the phenomenological aspects remain the same, albeit there could be some major quantitative implications depending on the origin of the DS. As for a future γ -ray search experiment, the Cherenkov Telescope Array [89] has the potential to constrain the parameter space further. This can, in turn, rule out some larger values of λ_2 , thus restricting the possibility of attaining the observed DM relic in such a scenario via the freeze-out mechanism. As far as the BBN is concerned, some improvements in our analysis are still possible by performing a more precise computation of the abundances of the primordial elements in the presence of late decays of an otherwise long-lived ξ . We leave such studies for future works.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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