

# Functional renormalization group study of anomalous magnetic moment in a low-energy effective theory

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The quark anomalous magnetic moments (AMMs) are investigated in a two-flavor low-energy effective theory within the functional renormalization group (FRG) approach under an external magnetic field. The Schwinger formalism is adopted for quark propagators, and Fierz-complete four-quark scatterings are self-consistently included through the renormalization group flows. We find that the quark AMMs are dynamically generated with the chiral symmetry breaking, and the magnitude of the AMM of the down quark is around four times larger than that of the up quark. The transverse AMMs and the longitudinal d-quark AMM monotonically decrease with the magnetic field strength, while the longitudinal u-quark AMM slightly increases with the magnetic field strength. At  $B = 0$ , the magnetic moments of proton and neutron are computed using the constituent quark model, which are close to the experimental values.

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## I. INTRODUCTION

Strong-interaction matter in an external magnetic field has attracted much attention in recent years. It can be created through noncentral heavy-ion collisions and can exist in magnetars and the early Universe [1–5]. Noncentral collisions of two high-speed nuclei moving in opposite directions can create strong magnetic fields of up to order  $\sim 10^{18}$  Gauss [1,2]. Magnetic fields on the surfaces of magnetars are estimated to be in the range of  $\sim 10^{10}$ – $10^{15}$  Gauss [6,7] and may reach up to  $\sim 10^{18}$  Gauss in their interiors [8]. Such strong magnetic fields influence the chiral and deconfinement phase transitions and give rise to various novel observable phenomena, such as the chiral magnetic effect [9,10], magnetic catalysis and inverse magnetic catalysis [11–18], the nonmonotonic behavior of the charged pion mass [19], and diamagnetism around the critical temperature [20]. Many investigations have been carried out from Lattice QCD simulations [18–27]; effective model studies, such as the Nambu–Jona-Lasinio (NJL) model [28–37] and the quark-meson model [38–41];

and theoretical approaches such as renormalization group [11,42,43] and Dyson–Schwinger equations [44]. For recent reviews, see Refs. [45,46].

The anomalous magnetic moment (AMM), which is already discussed in standard textbooks on QED [47,48], can also be dynamically generated for quarks through dynamical chiral symmetry breaking in QCD [49]. Recent studies have shown that the quark AMM influences the QCD phase diagram and mesonic properties in the presence of external magnetic fields [50–60], providing a possible explanation for the inverse magnetic catalysis puzzle. However, most of these studies treated the quark AMMs as free parameters, and some inappropriate parameter choices and regularization led to unphysical results. Therefore, a self-consistent calculation of the quark AMM is valuable. In Ref. [61], the AMM is calculated from the fermion-photon vertex using the Schwinger proper-time formalism in a low magnetic field. In Ref. [62], the authors employ the two-flavor NJL model in the mean-field approximation, calculate the photon-quark-antiquark vertex, and obtain the quark AMM under external magnetic fields. In Ref. [63], the authors calculate the quark AMM using perturbative QCD with the lowest-Landau level approximation and effective gluon mass.

In this work, we employ a two-flavor low-energy effective theory for calculating the quark mass and anomalous magnetic moments under a magnetic field within the functional renormalization group (FRG) approach. FRG is a functional realization of Wilson's renormalization group (RG) ideas [64,65]. It serves as a powerful tool for

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calculations in nonperturbative theories. The Schwinger formalism of quark propagators under a magnetic field is employed, and Fierz-complete four-quark scatterings are self-consistently included through the RG flows. The up- and down-quark AMMs in the directions parallel and perpendicular to the magnetic field are calculated, which could serve as inputs for other model studies.

This paper is organized as follows. In Sec. II, we introduce the two-flavor low-energy effective theory under a magnetic field within the FRG approach. The calculation of the anomalous magnetic moment is presented in Sec. III. In Sec. IV, we show the numerical results in our calculations, including the quark masses, four-quark couplings, and anomalous magnetic moments as functions of the strength of magnetic field. The summary and outlook are given in Sec. V.

## II. TWO-FLAVOR LOW-ENERGY EFFECTIVE THEORY

At high momentum and energy, QCD is described in terms of quarks and gluons as fundamental degrees of freedom. At low energies, gluonic degrees of freedom effectively decouple from the dynamics due to the mass gap, and their effects are encoded in the couplings of low-energy effective theories. In this work, we adopt a two-flavor low-energy effective theory with the kinetic term of the quarks and the four-quark scattering terms [66,67]. The effective action in Euclidean space reads

$$\Gamma_k[\bar{q}, q] = \int_x \bar{q}(\gamma_\mu D_\mu + m)q + \sum_\alpha \lambda_{4q}^{(\alpha)} T_{4q,ijklm}^{(\alpha)} \bar{q}_i q_j \bar{q}_l q_m \quad (1)$$

with a shorthand notation  $\int_x = \int d^4x$ . Here,  $D_\mu \equiv \partial_\mu - iQA_{\mu,0}$ , and  $Q$  denotes the quark electric charge. For the two-flavor case,  $Q = \text{diag}(2/3, -1/3)e$  with  $q = (u, d)^T$ .  $A_{\mu,0}$  denotes a background homogeneous magnetic field. Without loss of generality, we assume a magnetic field along the  $z$  direction and adopt the Landau gauge, i.e.,  $A_{\mu,0} = (0, 0, xB, 0)$ .

The second term of Eq. (1) denotes the four-quark interaction  $\Gamma_{4q}$  with couplings  $\lambda_{4q}^{(\alpha)}$  and tensor structures  $T_{4q,ijklm}^{(\alpha)}$ . In this work, we consider the isospin-symmetric Fierz-complete tensor structures and ignore the momentum dependence, which are listed in Appendix B and Ref. [68]:

$$\alpha \in \{\pi, \sigma, \eta, a, (V \pm A), (V - A)^{\text{adj}}, (S \pm P)_-^{\text{adj}}, (S + P)_+^{\text{adj}}\}. \quad (2)$$

$T_{4q,ijklm}^{(\alpha)}$  are antisymmetric under the commutation of pairs of indices  $i, j$  and  $l, m$ , i.e.,

$$T_{4q,ijklm}^{(\alpha)} = -T_{4q,lmij}^{(\alpha)}. \quad (3)$$

There is no doubt that the magnetic field breaks isospin symmetry, which leads to splitting into 20 tensor structures. These will be considered in future work.

The quark propagator in a homogeneous magnetic field is  $\hat{G} = \text{diag}[G(x, y; q_u), G(x, y; q_d)]$ , with the Schwinger formalism of quark propagators [69]

$$G(x, y; q_f) = \Phi(x, y) \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \tilde{G}(p; q_f). \quad (4)$$

Here,  $\Phi(x, y) = q_f B (x^1 + y^1)(x^2 - y^2)/2$  is the Schwinger phase factor, and  $\tilde{G}(p; q_f)$  is the translation-invariant part. In Euclidean space, the translation-invariant part  $\tilde{G}(p; q_f)$  reads

$$\begin{aligned} \tilde{G}(p; q_f) = & \int_0^\infty ds \exp \left[ -s \left( (1 + r_{f,s})^2 \left( p_\parallel^2 + p_\perp^2 \frac{\tanh(q_f B s)}{q_f B s} \right) + m^2 \right) \right] \\ & \cdot [(-ip_\parallel \gamma_\parallel (1 + r_{f,s}) + m)(1 + i\gamma_1 \gamma_2 \tanh(q_f B s)) \\ & - ip_\perp \gamma_\perp (1 + r_{f,s})(1 - \tanh^2(q_f B s))]. \end{aligned} \quad (5)$$

Here,  $r_{f,s}$  denotes the magnetic-dependent four-dimensional regulator of quarks. In this work, we use a simple exponential regulator,

$$r_{f,s} = \frac{e^{-x}}{x}, \quad x = \frac{p_\parallel^2 + p_\perp^2 \tanh(q_f B s)/(q_f B s)}{k^2}. \quad (6)$$

Here,  $k$  denotes the IR cutoff scale, i.e., the RG scale. The regulator  $r_{f,s}$  temporarily suppresses quantum fluctuations of momenta below the scale  $k$  and is eventually removed as  $k \rightarrow 0$ . This regulator is a common choice for momentum-dependent approximations. We also employ the Litim regulator and get qualitatively the same results. For more discussion about the regulator dependence, see Appendix A and Ref. [67].

The flow equations for the two- and four-point quark correlation functions are derived from the Wetterich equation [64], which are depicted in Fig. 1. The flow equation for the quark mass is obtained by projecting the flow of the two-point correlation function onto the scalar channel:

$$\partial_t m = \int \frac{dq^4}{(2\pi)^4} \frac{1}{4N_c N_f} \text{Tr}[\Gamma_{4q} \tilde{\partial}_t \hat{G}]. \quad (7)$$

Here, the RG time  $t \equiv \ln(k/\Lambda)$ , and  $\Lambda$  denotes the ultra-violet cutoff scale. The partial derivative with a tilde,  $\tilde{\partial}_t$ , denotes that it acts only on the RG scale through the regulators in propagators, i.e.,  $\tilde{\partial}_t \equiv \partial_t r_{f,s} \cdot \frac{\delta}{\delta r_{f,s}}$ . Traces are taken over color, flavor, and Dirac space. The detailed expression of the flow equation for the quark mass is



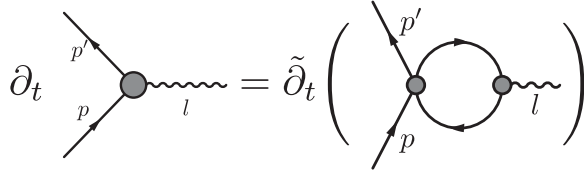


FIG. 2. Feynman diagram of the flow equation of the quark-photon vertex.

The Schwinger phase factor cancels out in this diagram. Furthermore, under a background magnetic field, the form factors with external photon momentum parallel (longitudinal) or perpendicular (transverse) to the magnetic field are split. As a consequence, even in the zero-momentum limit, the tensor structures corresponding to the parallel and

perpendicular components remain distinct, leading to different AMMs in the two directions. With the aid of the FormTracer package [70], we obtain the flow equations for the u-quark AMMs,

$$\begin{aligned} \partial_t \kappa_{u,\parallel/\perp} = & -\frac{1}{6q_u} \left[ 2(3\lambda_{4q}^a - 3\lambda_{4q}^\pi - 8\lambda_{4q}^{(S+P)\text{adj}} \right. \\ & + 8\lambda_{4q}^{(S+P)\text{adj}}) q_d \mathcal{L}_{\kappa,\parallel/\perp}(d) \\ & + (3\lambda_{4q}^a - 3\lambda_{4q}^\eta - 3\lambda_{4q}^\pi + 3\lambda_{4q}^\sigma \\ & \left. + 16\lambda_{4q}^{(S+P)\text{adj}}) q_u \mathcal{L}_{\kappa,\parallel/\perp}(u) \right]. \end{aligned} \quad (20)$$

Here, the loop functions are defined as

$$\begin{aligned} \mathcal{L}_{\kappa,\parallel}(f) = & \int \frac{d^4 q}{(2\pi)^4} \int_0^\infty ds \int_0^\infty dr \mathcal{E}(f, r) \mathcal{E}(f, s) m [\partial_t r_{f,r} (1 - 2(1 + r_{f,r})^2) (q_\parallel^2 r + q_\perp^2 \tanh(q_f B r) / (q_f B)) \\ & - 2\partial_t r_{f,s} (1 + r_{f,r}) (1 + r_{f,s}) (q_\parallel^2 s + q_\perp^2 \tanh(q_f B s) / (q_f B))] (-1 + \tanh(q_f B r) \tanh(q_f B s) / 3) \end{aligned} \quad (21)$$

and

$$\begin{aligned} \mathcal{L}_{\kappa,\perp}(f) = & \int \frac{d^4 q}{(2\pi)^4} \int_0^\infty ds \int_0^\infty dr \mathcal{E}(f, r) \mathcal{E}(f, s) m [\partial_t r_{f,r} (1 - 2(1 + r_{f,r})^2) (q_\parallel^2 r + q_\perp^2 \tanh(q_f B r) / (q_f B)) \\ & - 2\partial_t r_{f,s} (1 + r_{f,r}) (1 + r_{f,s}) (q_\parallel^2 s + q_\perp^2 \tanh(q_f B s) / (q_f B))] (-1 + \tanh^2(q_f B r)). \end{aligned} \quad (22)$$

For the expression of the d-quark, change  $u \leftrightarrow d$ , and  $\partial_t F_{2,f}(0) = 2m\partial_t \kappa_f$ . Obviously, in the limit of a vanishing external magnetic field, the expressions of the AMM in the two directions Eqs. (21) and (22) become identical. As the external magnetic field increases, the AMM in the two directions begins to differ.

Notably, if we ignore the nondominant channels, i.e.,  $\alpha \notin \{\pi, \sigma\}$ , and the splitting between the scalar and pseudoscalar couplings, the u-quark AMM depends only on the d-quark loop contribution, and vice versa. Based on this, we obtain  $\kappa_d / \kappa_u = (q_u^2 / q_d^2) \cdot (\mathcal{L}_\kappa(u) / \mathcal{L}_\kappa(d)) = 4\mathcal{L}_\kappa(u) / \mathcal{L}_\kappa(d)$ , which are consistent with Ref. [62].

The magnetic moments of quarks are connected to the form factors, which are given as [50,62]

$$\mu_f = q_f (1 + F_{2,f}(0)) \frac{m_N}{m} \mu_N, \quad f \in \{u, d\}, \quad (23)$$

where  $m_N = 938$  MeV is the nucleon mass and  $\mu_N = e / (2m_N)$  denotes the nuclear magneton. In the constituent quark model, the proton and neutron magnetic moments are given as

$$\mu_{\text{proton}} = \frac{1}{3} (4\mu_u - \mu_d), \quad (24)$$

$$\mu_{\text{neutron}} = \frac{1}{3} (4\mu_d - \mu_u). \quad (25)$$

#### IV. NUMERICAL RESULTS

The ultraviolet cutoff scale  $\Lambda$  of low-energy effective theories is generally chosen below the scale where gluonic degrees of freedom decouple and above the chiral symmetry breaking scale, typically in the range of 0.7 to 1 GeV. In this work, we start the evolution of the FRG flows at an ultraviolet cutoff scale  $\Lambda = 1$  GeV. At this scale, we only keep the scalar and pseudoscalar couplings nonvanishing, i.e.,  $\lambda_{4q,\Lambda}^{\alpha \notin \{\pi, \sigma\}} = 0$ . The initial conditions  $m_\Lambda = 13.5$  MeV,  $\lambda_{4q,\Lambda}^\pi = \lambda_{4q,\Lambda}^\sigma = 10.6\Lambda^{-2}$  are tuned to reproduce the quark mass  $m(T=0, B=0) = 350$  MeV and the pole mass of the pion  $m_\pi = 140$  MeV in the vacuum. Here, the pole mass of the pion is determined by analytic continuation from the Euclidean to Minkowski spacetime using Padé approximants; for more details, see Refs. [67,68,75].

In the left panel of Fig. 3, we present the quark masses as functions of RG scale  $k$  for several values of the strength of magnetic field. The quark masses, which reflect chiral symmetry breaking, increase with the magnetic field. The chiral symmetry-breaking scales also increase with the magnetic field strength. These features reflect the magnetic catalysis effect of our theory at zero temperature. In the right panel of Fig. 3, we show the quark masses as functions of the strength of magnetic field strength. The lattice QCD results are constructed from the quark chiral condensates in Ref. [19]. We also compare our full calculations, in which

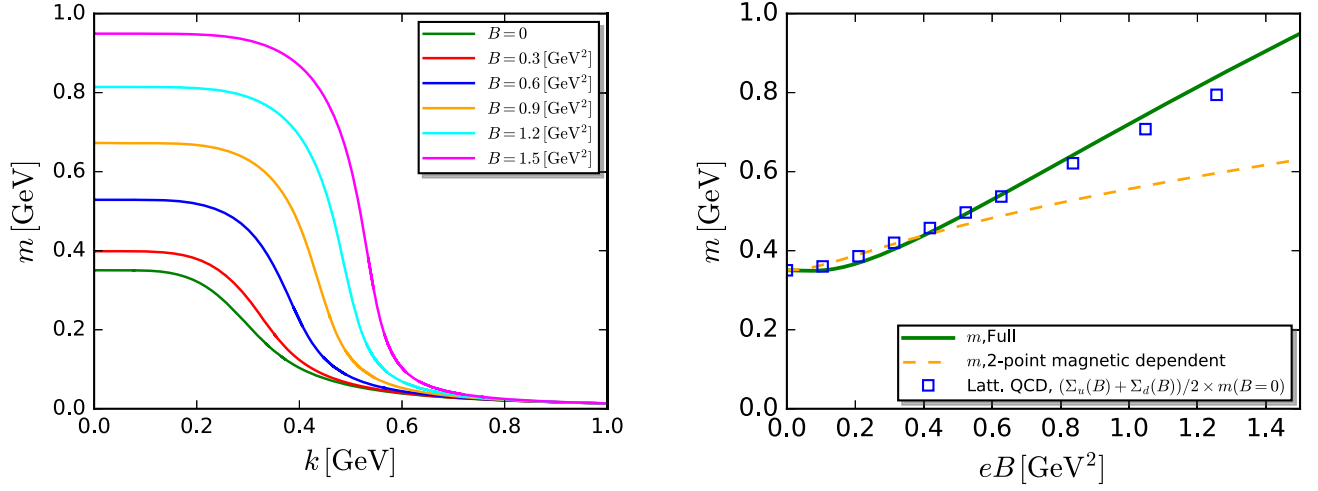


FIG. 3. Left panel: the quark masses as functions of RG scale  $k$  with several values of the strength of the magnetic field. Right panel: the quark masses as functions of the strength of magnetic field.

both two- and four-point quark correlation functions are computed using the magnetic-dependent quark propagators, with those from a simplified truncation scheme. In the simplified truncation scheme, the magnetic-dependent quark propagators are only used in the calculation of the flow equations of two-point correlation functions, while free quark propagators are employed in calculation of the flow equations of four-point quark correlation functions. Both truncation results exhibit the magnetic catalysis behavior. However, the full calculation shows a stronger dependence on the magnetic field strength and agrees better with the lattice QCD results. This indicates that an accurate treatment of the magnetic field dependence in the four-point quark correlation functions is important.

In Fig. 4, we plot the four-quark couplings as functions of the strength of the magnetic field. Obviously, the  $\pi$  and  $\sigma$  channels are dominant, and  $\lambda_{4q}^\pi > \lambda_{4q}^\sigma$  at  $B = 0$ , since pions

are pseudo-Goldstone bosons. As mentioned in Sec. II,  $\lambda_{4q}^\pi$  splits into  $\lambda_{4q}^{\pi^0}$  and  $\lambda_{4q}^{\pi^\pm}$  under magnetic fields. In this work, the splitting of  $\lambda_{4q}^\pi$  is not taken into account, and the presented results can be regarded as an averaged approximation of these couplings. The couplings of  $\pi$  and  $\sigma$  channels rapidly decrease with the strength of the magnetic field in the small magnetic field region and tend to become constant at large magnetic fields. In the right panel of Fig. 4, we show the nondominant four-quark couplings of the Fierz complete channels. We find that these nondominant couplings contribute approximately 6% to the quark masses and quark AMMs. The absolute values of these couplings also decrease with the strength of the magnetic field at  $eB > 0.2 \text{ GeV}^2$ .

In Fig. 5, we show the comparison of the dominant four-quark couplings as functions of the strength of the magnetic

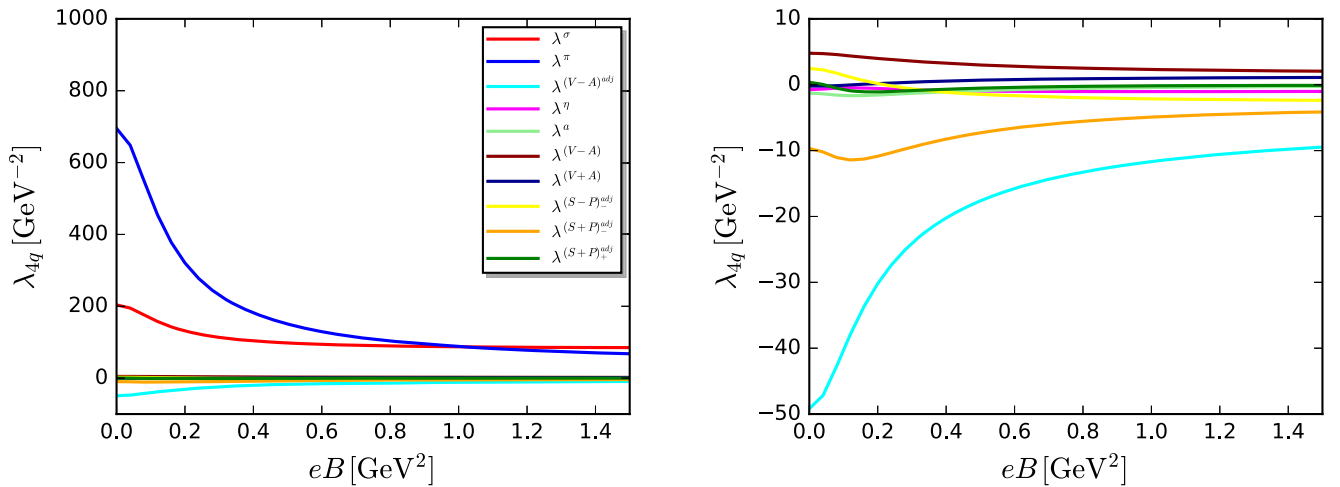


FIG. 4. Left panel: the four-quark couplings of Fierz complete channels as functions of the strength of the magnetic field. Right panel: nondominant four-quark couplings of the Fierz complete channels, which is the enlarged version of the left panel.

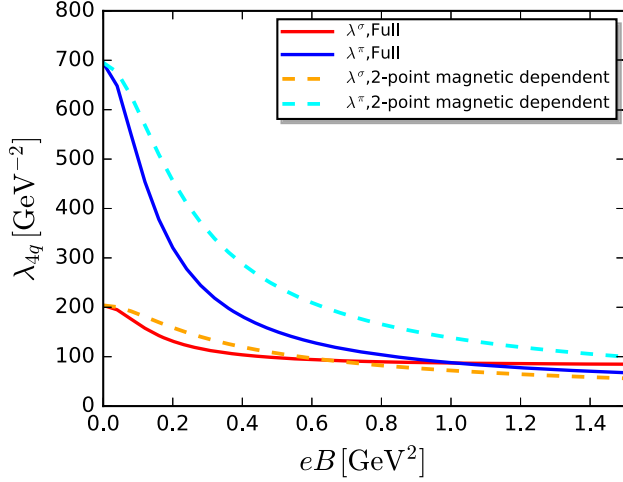


FIG. 5. Comparison of the dominant four-quark couplings as functions of the strength of the magnetic field between full calculation and two-point magnetic-dependent truncation.

field between the full calculation and the two-point magnetic-dependent truncation. The magnetic dependence of the four-quark couplings in the two-point magnetic-dependent truncation just comes from the increase of the quark mass. The four-quark interactions in the full calculation are more rapidly suppressed with increasing magnetic field than those in the two-point magnetic-dependent truncation. Notably, the four-quark interactions can also be effectively interpreted as an interaction generated by meson exchange. According to Refs. [19,38], while the  $\pi^0$  mass slightly decreases with the magnetic field, the masses of the  $\pi^\pm$  and  $\sigma$  mesons increase rapidly, leading to a suppression of the effective four-point interaction at large magnetic fields. This picture is consistent with our results. Following this perspective, the relation between the two-point magnetic-dependent truncation and the full calculation is analogous to the difference between a mean-field NJL approximation and a beyond-mean-field treatment including mesonic fluctuations. Our results are also in qualitative agreement with Ref. [36], where the inclusion of charged pion fluctuations leads to a stronger magnetic-field dependence of the quark mass.

The dimensionless form factors as functions of the magnetic field strength are plotted in Fig. 6, and the quark anomalous magnetic moments are plotted in Fig. 7. The magnitude of the AMM of the down quark is around four times larger than that of the up quark, as analyzed in Sec. III. The transverse form factors and quark AMMs monotonically decrease with the magnetic field strength, while the longitudinal u-quark AMM and form factor slightly increase with the magnetic field strength. To gain a deep understanding of these behaviors, we also plot the quark AMMs with the two-point magnetic-dependent truncation (corresponding to the orange dashed line in Fig. 3) in Fig. 8. We expand the loop function Eq. (21) in a Taylor series around  $B = 0$  and ignore

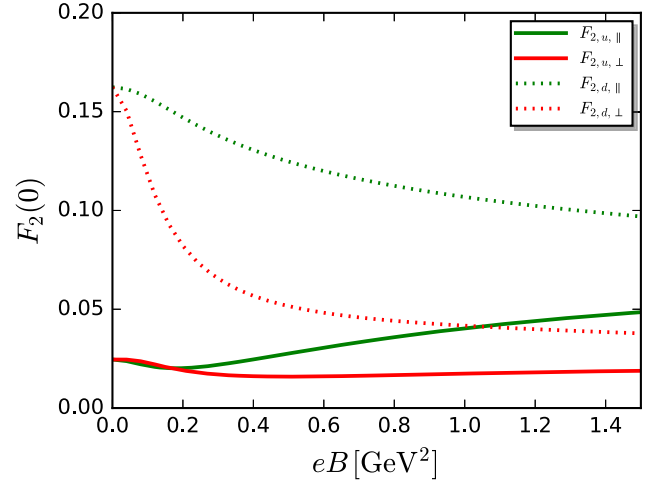


FIG. 6. The form factors as functions of the magnetic field strength. “Parallel” (longitudinal) or “perpendicular” (transverse) refers to the angle between the direction in which the photon’s external momentum vanishes and the external magnetic field.

the magnetic dependence of the regulators:

$$\mathcal{L}_{\kappa,\parallel}(f) = \int \frac{q^3 dq}{(2\pi)^2} \tilde{\partial}_t \left\{ -\frac{m(r_q + 1)}{2(m^2 + q^2(r_q + 1)^2)^2} + \frac{m(r_q + 1)(m^2 - 5q^2(r_q + 1)^2)}{6(m^2 + q^2(r_q + 1)^2)^5} (q_f B)^2 + \mathcal{O}(q_f B)^4 \right\}. \quad (26)$$

This expression can also be obtained from the weak-field expansion of the quark propagators [38,76]. The next-to-leading-order term of longitudinal AMM is predominantly negative, which enhances the AMM. For the transverse loop function Eq. (22),

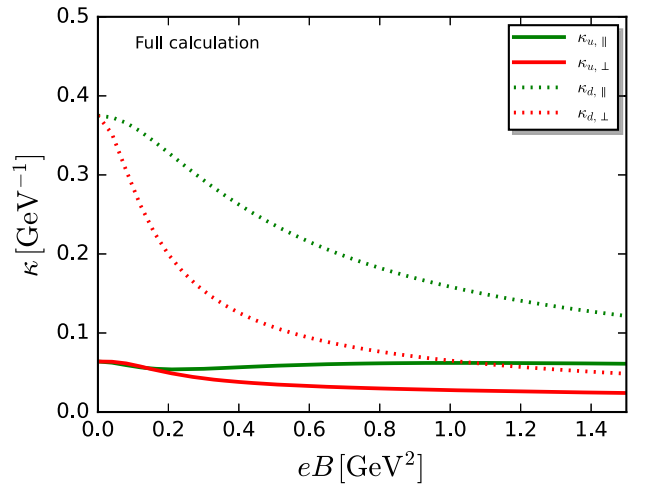


FIG. 7. The anomalous magnetic moments as functions of the magnetic field strength.

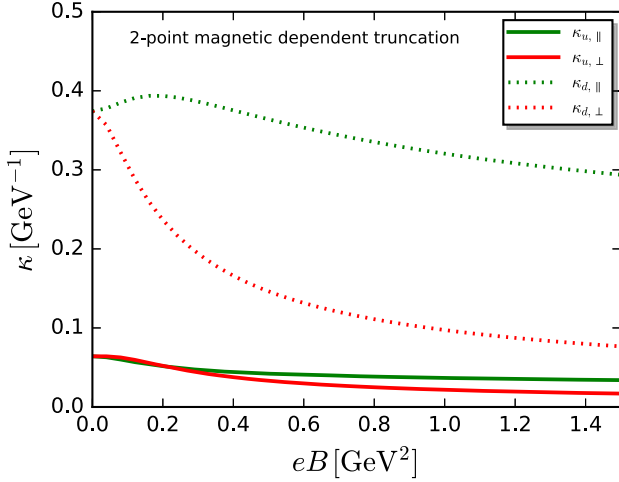


FIG. 8. The anomalous magnetic moments as functions of the magnetic field strength with two-point magnetic-dependent truncation.

$$\begin{aligned} \mathcal{L}_{\kappa,\perp}(f) = \int \frac{q^3 dq}{(2\pi)^2} \tilde{\partial}_t \left\{ -\frac{m(r_q + 1)}{2(m^2 + q^2(r_q + 1)^2)^2} \right. \\ \left. + \frac{m^3(r_q + 1)}{(m^2 + q^2(r_q + 1)^2)^5} (q_f B)^2 \right. \\ \left. + \mathcal{O}(q_f B)^4 \right\}. \end{aligned} \quad (27)$$

The sign of the next-to-leading-order term is opposite to that of the leading-order term, leading to a reduction in the transverse AMM. This analysis is consistent with the results in the small magnetic field region shown in Fig. 8. However, in the full calculation, both longitudinal and transverse AMMs are suppressed due to the rapidly decreasing four-quark couplings; see Figs. 4 and 5. From another perspective, this suppression can be understood as originating from mesonic fluctuations.

The transverse quark AMMs influence the dispersion relations of quarks under magnetic fields in other model studies, and a feedback calculation is left for future work. The transverse quark AMMs obtained in our calculation, which predominantly decrease with magnetic field strength, could avoid the unphysical “jumps” in the case of constant  $\kappa$  in Ref. [52].

In Fig. 9, we take  $\kappa_{d,||}$  as an example and plot it as a function for scale  $k$  with several values of magnetic field strength. The energy scale  $k$  at which the quark AMMs are generated, decreases with the magnetic field strength. By comparing with the quark mass results in Fig. 3, we find that the quark AMM generation energy scale is strongly correlated with the chiral symmetry breaking energy scale. This behavior is analogous to the picture proposed in Ref. [49], where anomalous (chromo)magnetic moments emerge as a consequence of dynamical chiral symmetry breaking through the nonperturbative dressing of quark

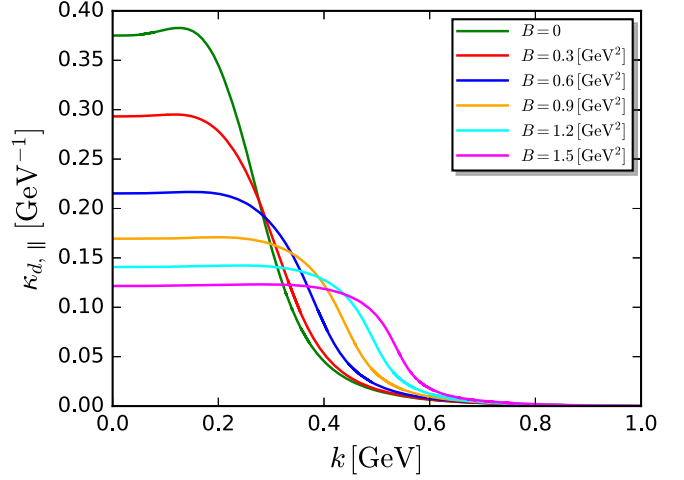


FIG. 9. The anomalous magnetic moment  $\kappa_{d,||}$  as functions of RG scale  $k$  for several values of the magnetic field strength.

interaction vertices. In our framework, this physics is effectively encoded in the rapid enhancement of the dominant channels of  $\lambda_{4q}^{(\alpha)}$ , i.e.,  $\lambda_{4q}^\pi$  and  $\lambda_{4q}^\sigma$ , at the chiral symmetry breaking scale [67], and the quark AMMs are proportional to the four-quark couplings  $\lambda_{4q}^{(\alpha)}$ ; see Eq. (17). As a result, the quark AMMs are generated with the chiral symmetry breaking.

In vacuum, i.e., at  $B = 0$ , we also compute the proton and neutron magnetic moments using the constituent quark model [see Eqs. (23)–(25)]. We obtain

$$\mu_{\text{proton}}/\mu_N = 2.787 \quad (28)$$

$$\mu_{\text{neutron}}/\mu_N = -1.995. \quad (29)$$

These results are close to the experimental results, i.e.,  $\mu_{\text{proton}}/\mu_N = 2.7928$  and  $\mu_{\text{neutron}}/\mu_N = -1.9130$  [77].

## V. CONCLUSIONS

We build a two-flavor Nambu-Jona-Lasinio-type effective theory with four-quark scatterings under external magnetic fields within the FRG approach, in which Fierz-complete four-quark scatterings are self-consistently included through the RG flows. The quark mass increases with the strength of the magnetic field, reflecting the magnetic catalysis property. The quark mass as functions of the strength of magnetic field is in good agreement with the lattice QCD results. The  $\pi$  and  $\sigma$  channels are dominant, and their couplings decrease rapidly with the strength of magnetic field.

Based on this low-energy effective theory, we calculate quark AMMs and form factors. By comparing the RG flows of the quark mass and AMMs, we find that the quark AMMs are dynamically generated with chiral symmetry breaking. The magnitude of the AMM of the down quark is

around four times larger than that of the up quark. The transverse quark AMMs monotonically decrease with magnetic field strength. The longitudinal d-quark AMM also monotonically decreases with the magnetic field strength due to the rapid decrease in the four-quark couplings, and the longitudinal u-quark AMM slightly increases with the magnetic field strength. At  $B = 0$ , the magnetic moments of proton and neutron are computed using the constituent quark model, which are close to the experimental values.

Our quark AMMs and four-quark couplings results could serve as inputs for calculations in other effective models. We will extend our calculations to finite temperatures and chemical potential after considering the momentum dependence of the correlation functions. The first-principles QCD calculation of quark AMMs based on Ref. [75] will be carried out in the upcoming works. In addition, a lattice QCD calculation of the quark AMMs could provide valuable guidance and a benchmark for further investigation.

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### DATA AVAILABILITY

The data that support the findings of this article are openly available [71]; embargo periods may apply.

### APPENDIX A: REGULATOR DEPENDENCE

In this appendix, we will discuss the choice of regulator and the dependence of the results on it. In principle, the choice of FRG regulators is arbitrary, as long as the regulator satisfies the infrared cutoff scale constraints. To this end, we also employed another regulator, which shape functions is a smooth version of the Litim regulator [78],

$$r(x) = \left( \frac{1}{\sqrt{x}} - 1 \right) \cdot \frac{1}{1 + e^{\frac{x-1}{a}}}, \quad (\text{A1})$$

where we set  $a = 0.1$ . Obviously, in the limit  $a \rightarrow 0$ , the second term will become the Heaviside step function  $\Theta(1 - x)$ . Here,  $x$  denotes the square of the dimensionless momentum. Notably, the translation-invariant part of the quark propagator in Euclidean space without a regulator reads

$$\begin{aligned} \tilde{G}(p; q_f) &= \int_0^\infty ds \exp \left[ -s \left( \left( p_\parallel^2 + p_\perp^2 \frac{\tanh(q_f Bs)}{q_f Bs} \right) + m^2 \right) \right] \\ &\cdot [(-ip_\parallel \gamma_\parallel + m)(1 + i\gamma_1 \gamma_2 \tanh(q_f Bs)) \\ &- ip_\perp \gamma_\perp (1 - \tanh^2(q_f Bs))]. \end{aligned} \quad (\text{A2})$$

In the limit  $B \rightarrow 0$ , the exponent term in the Schwinger representation integrates to  $p^2 + m^2$ . This motivates regarding  $p_\parallel^2 + p_\perp^2 \tanh(q_f Bs)/(q_f Bs)$  as the magnetic-field-modified momentum squared, and the  $x$  in the regulators is defined as

$$x = \frac{p_\parallel^2 + p_\perp^2 \tanh(q_f Bs)/(q_f Bs)}{k^2}. \quad (\text{A3})$$

This is analogous to the treatment in the Landau-level representation, where the discrete Landau levels are effectively regarded as transverse momentum modes [11,40,79]. Besides, we also consider B-independent regulators for comparison, for which  $x$  in Eqs. (6) and (A1) is

$$x = \frac{p^2}{k^2}. \quad (\text{A4})$$

For the smooth Litim regulator, we keep the ultraviolet cutoff scale  $\Lambda = 1$  GeV unchanged and tune the initial conditions  $m_\Lambda = 30$  MeV,  $\lambda_{4q,\Lambda}^\pi = \lambda_{4q,\Lambda}^\sigma = 27.1\Lambda^{-2}$ , and  $\lambda_{4q,\Lambda}^{\alpha \notin \{\pi,\sigma\}} = 0$  to reproduce the quark mass  $m = 350$  MeV and the pole mass of the pion  $m_\pi = 140$  MeV in the vacuum.

In Fig. 10, we compare the quark masses as functions of the strength of the magnetic field  $B$  with different regulators. In all cases, the quark mass increases with the magnetic field, reflecting the magnetic catalysis effect. The exponential and smooth Litim regulators with Eq. (A3) show good agreement with each other, especially in the region  $eB < 1$  GeV<sup>2</sup>; beyond this range, the slight differences may originate from boundary effects associated with the ultraviolet cutoff. The B-independent regulator results, however, differ from the others even at small magnetic field strengths. This may be because B-independent regulators break the spatial  $\mathcal{O}(4)$  symmetry, thereby introducing a dependence on the shape functions of the regulators.

In Fig. 11, we also present the anomalous magnetic moments as functions of the magnetic field strength with the smooth Litim regulator Eqs. (A1) and (A3). Compared with Fig. 7, the results of the Litim regulator are qualitatively and even quantitatively consistent with those of the exponential regulator.

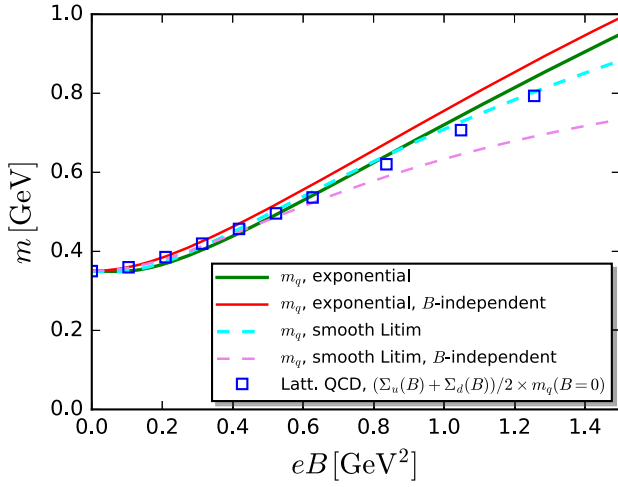


FIG. 10. Comparison of the quark masses as functions of the strength of the magnetic field for different choices of regulators.

## APPENDIX B: FIERZ-COMPLETE TENSOR STRUCTURES

In this appendix, we list the Fierz-complete tensor structures used in this work:

$$\mathcal{T}_{ijlm}^\sigma = (T^0)_{ij}(T^0)_{lm}, \quad (\text{B1})$$

$$\mathcal{T}_{ijlm}^\pi = -(\gamma_5 T^a)_{ij}(\gamma_5 T^a)_{lm}, \quad (\text{B2})$$

$$\mathcal{T}_{ijlm}^a = (T^a)_{ij}(T^a)_{lm}, \quad (\text{B3})$$

$$\mathcal{T}_{ijlm}^\eta = -(\gamma_5 T^0)_{ij}(\gamma_5 T^0)_{lm}, \quad (\text{B4})$$

$$\begin{aligned} \mathcal{T}_{ijlm}^{(V-A)} &= (\gamma_\mu T^0)_{ij}(\gamma_\mu T^0)_{lm} \\ &\quad - (i\gamma_\mu \gamma_5 T^0)_{ij}(i\gamma_\mu \gamma_5 T^0)_{lm}, \end{aligned} \quad (\text{B5})$$

$$\begin{aligned} \mathcal{T}_{ijlm}^{(V+A)} &= (\gamma_\mu T^0)_{ij}(\gamma_\mu T^0)_{lm} \\ &\quad + (i\gamma_\mu \gamma_5 T^0)_{ij}(i\gamma_\mu \gamma_5 T^0)_{lm}, \end{aligned} \quad (\text{B6})$$

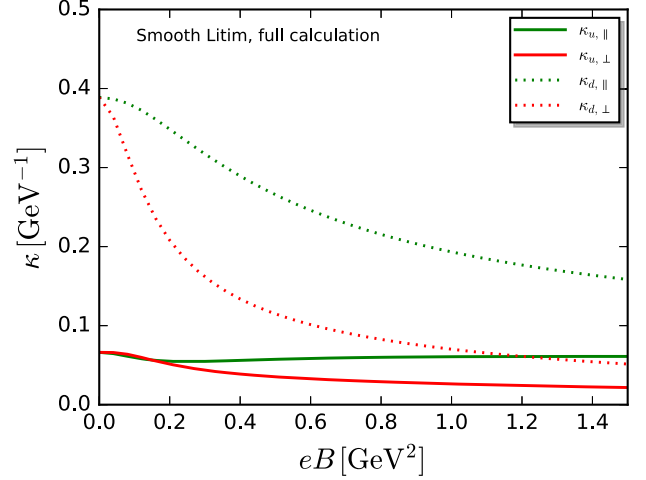


FIG. 11. The anomalous magnetic moments as functions of the magnetic field strength with a smooth version of Litim regulator Eq. (A1).

$$\begin{aligned} \mathcal{T}_{ijlm}^{(V-A)\text{adj}} &= (\gamma_\mu T^0 t^a)_{ij}(\gamma_\mu T^0 t^a)_{lm} \\ &\quad - (i\gamma_\mu \gamma_5 T^0 t^a)_{ij}(i\gamma_\mu \gamma_5 T^0 t^a)_{lm}, \end{aligned} \quad (\text{B7})$$

$$\begin{aligned} \mathcal{T}_{ijlm}^{(S+P)\text{adj}}_- &= (T^0 t^a)_{ij}(T^0 t^a)_{lm} + (\gamma_5 T^0 t^a)_{ij}(\gamma_5 T^0 t^a)_{lm} \\ &\quad - (T^a t^b)_{ij}(T^a t^b)_{lm} - (\gamma_5 T^a t^b)_{ij}(\gamma_5 T^a t^b)_{lm}, \end{aligned} \quad (\text{B8})$$

$$\begin{aligned} \mathcal{T}_{ijlm}^{(S-P)\text{adj}}_- &= (T^0 t^a)_{ij}(T^0 t^a)_{lm} - (\gamma_5 T^0 t^a)_{ij}(\gamma_5 T^0 t^a)_{lm} \\ &\quad - (T^a t^b)_{ij}(T^a t^b)_{lm} + (\gamma_5 T^a t^b)_{ij}(\gamma_5 T^a t^b)_{lm}, \end{aligned} \quad (\text{B9})$$

$$\begin{aligned} \mathcal{T}_{ijlm}^{(S+P)\text{adj}}_+ &= (T^0 t^a)_{ij}(T^0 t^a)_{lm} + (\gamma_5 T^0 t^a)_{ij}(\gamma_5 T^0 t^a)_{lm} \\ &\quad + (T^a t^b)_{ij}(T^a t^b)_{lm} + (\gamma_5 T^a t^b)_{ij}(\gamma_5 T^a t^b)_{lm}. \end{aligned} \quad (\text{B10})$$

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