

Precise theoretical prediction on branching fractions and polarizations of $D \rightarrow VV$ decays

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We present a precise and systematic analysis of $D \rightarrow VV$ decays within the factorization-assisted topological-amplitude approach, where D denotes the set $\{D^0, D^+, D_s^+\}$ and V represents the vector mesons ρ, K^*, ω , and ϕ . Given the limited current experimental data, the factorization-assisted topological-amplitude approach serves as a available phenomenological framework for predicting charmed meson decays to both vector mesons. In this framework, incorporating flavor SU(3) symmetry-breaking effects, we can express nonfactorizable contributions of different modes as a minimal set of universal parameters globally fitted to experimental data. Utilizing 36 experimental data points for $D \rightarrow VV$ decays, we precisely extract ten nonfactorizable parameters associated with the C and E topological diagrams with $\chi^2/\text{d.o.f.} = 8.43$. We find that a large strong phase in the longitude E amplitude causes strong destructive interference with the C longitudinal component, yielding $f_{\parallel} > f_L$, contrary to the naive factorization predictions. Additionally, for modes processing exclusively by the E diagram, the amplitude hierarchy $|S| < |D|$ leads to a D -wave branching fraction larger than that of the S wave. This explains recent observations that contradict S -wave dominance predictions. The predicted branching fractions and polarizations for 28 decay modes are consistent with existing experimental data. Unobserved modes, especially those with branching fractions of the order of 10^{-3} – 10^{-2} , the D -wave-dominated modes, and modes exhibiting $f_{\parallel} > f_L$, await measurement by BESIII, STCF, Belle II, and LHCb.

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I. INTRODUCTION

Since 2003, *BABAR* [1] and *Belle* [2] have measured large transverse polarization fractions ($\sim 50\%$) in $B \rightarrow \phi K^*$, respectively, contradicting the fraction expectation of dominant longitude polarization fractions, $f_L = 1 - \mathcal{O}(m_V^2/m_B^2)$. Similar anomalies were observed in other penguin-dominated strangeness-changing decays, such as $B^+ \rightarrow \omega K^{*+}$, $B^+ \rightarrow \rho^0 K^{*+}$, and $B_s \rightarrow \phi \phi$, sparking extensive theoretical interest. Although transverse polarization fractions in the charm sector are not as heavily suppressed as in B decays, they still significantly deviate from naive factorization estimates. For instance, Mark III Collaboration reported a transverse branching fraction of $\mathcal{B}(D^0 \rightarrow \bar{K}^{*0} \rho^0)_T = (1.6 \pm 0.6)\%$, comparable to the total branching fraction of $\mathcal{B}(D^0 \rightarrow \bar{K}^{*0} \rho^0)_{\text{tot}} = (1.59 \pm 0.35)\%$

[3], and partial-wave analysis yields S -, P -, and D -wave components of $(3.1 \pm 0.6)\%$, $< 3 \times 10^{-3}$, and $(2.1 \pm 0.6)\%$, respectively. This hierarchy contradicts the naive prediction $|S|^2 > |P|^2 > |D|^2$. Subsequently, BESIII found D -wave dominance in $D^0 \rightarrow K^{*-} \rho^+, \bar{K}^{*0} \rho^0, \rho^+ \rho^-$ and P -wave dominance in $D_s^+ \rightarrow \bar{K}^{*0} \rho^+, K^{*-} \rho^0$ [4–6]. Additionally, the D -wave dominance in $D^0 \rightarrow \rho^0 \rho^0$ was reported by FOCUS [7], with partial-wave analyses further done in Ref. [8].

To understand these “polarization anomalies” in $D \rightarrow VV$ decays within QCD remains challenging due to the intermediate charm mass scale m_c . Unlike B decays, where heavy quark expansion ($1/m_b$) enables successful applications of QCD factorization, perturbative QCD, and soft-collinear effective theory, the mass m_c is not sufficiently large to allow for such an expansion. Early studies of $D \rightarrow VV$ decays relied on naive factorization estimates [9–12], pole-dominance model [13], heavy quark effective Lagrangian combined with chiral perturbative theory [14], or model-independent symmetry-based approaches, including flavor SU(3) symmetry [9] and broken flavor SU(3) symmetry models [15]. These studies typically focus on only a few specific modes. Recent works have extended

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this to systematic analyses of all Cabibbo-favored and singly Cabibbo-suppressed $D^0 \rightarrow VV$ decays using quark model with final state interactions (FSIs) [16], and all $D^0 \rightarrow VV$ decays have been studied in naive factorization [17]. While abundance data for $D \rightarrow PP$ and $D \rightarrow PV$ decays have enabled systematic phenomenological studies via topological diagram approach [17,18], the factorization-assisted topological-amplitude (FAT) approach [19,20], and quark model [21], $D \rightarrow VV$ decays are experimentally more challenging. Consequently, the resulting scarcity and limited precision of data have prevented their study within the topological diagram approach. Leveraging current available data, the FAT approach offers a viable framework to predict all $D_{(s)} \rightarrow VV$ mode, encompassing Cabibbo-favored, singly Cabibbo-suppressed, and doubly Cabibbo-suppressed decays.

The FAT approach builds upon the conventional topological diagram approach [22,23]. In the topological diagram approach, decay amplitudes are categorized into distinct topological diagrams according to electroweak interactions. As the weak interaction is factorized automatically from strong interaction due to their distant scales, the QCD effects including perturbative and nonperturbative contributions (including FSIs) are effectively encapsulated within these topological amplitudes. Consequently, when these amplitudes (excluding the common factor $G_F/\sqrt{2}V_{CKM}$) are extracted directly from experimental data, they inherently contain all QCD effects. However, the conventional topological diagram approach relies on flavor SU(3) symmetry to reduce the number of free parameters so as to improve fit quality. It is well established that the SU(3)-breaking effects can reach 20%–30% in B decays [24,25] and are expected to be even larger in D decays. This significant symmetry breaking limits the prediction power of the conventional diagrammatic approach. The FAT approach addresses this limitation by incorporating SU(3)-breaking effects assisted by factorization. Especially, we factor out the form factors and decay constants from the topological diagram amplitudes. As a result, the remaining nonfactorizable contributions become universal across all decay modes and can be characterized by a minimal set of free parameters, determined through a global fit to all available experimental data.

The FAT framework was originally developed for charm meson decays [19,20,26] and later extended to B -meson decays by one of us (S.-H. Z.) and collaborators [24,25,27] (see [28] for a review). It has been applied to $D^0 - \bar{D}^0$ mixing [29], $K_S^0 - K_L^0$ asymmetries [30], CP violation in charm decays [31], and the extraction of the Cabibbo-Kobayashi-Maskawa (CKM) phase γ from charmless two-body B decays [32]. Recent extensions include quasi-two-body B decays [33–36] and D decays [37,38]. Notably, the

FAT approach has successfully explained the “polarization anomalies” in $B \rightarrow VV$ decays [27] by predicting reduced branching fractions and longitudinal polarization fractions for color-suppressed decays and attributing large transverse polarization fractions observed in the penguin-dominant modes to a single transverse amplitude. In this work, we apply the FAT approach to systematically analyze $D \rightarrow VV$ decays. We aim to resolve the polarization anomalies in the charm sector and provide robust predictions as references for upcoming experimental measurements.

The remainder of this paper is organized as follows. In Sec. II, we introduce the theoretical framework. Numerical results and detailed discussions are presented in Sec. III. Finally, our conclusions are summarized in Sec. IV.

II. FACTORIZATION AMPLITUDES FOR TOPOLOGICAL DIAGRAMS

In this section, we first introduce the decay amplitudes for $D \rightarrow VV$ decays in various bases, along with their expressions within naive factorization. We then proceed to analyze the three polarization amplitudes under the FAT framework.

A. Decay amplitude decomposition and naive factorization

For a D meson with four-momentum p_D decaying into two vector mesons $V_1(m_1, p_1, \eta^*)$ and $V_2(m_2, p_2, \epsilon^*)$ with polarization vectors η^* and ϵ^* , the decay amplitude, based on Lorentz decomposition, reads

$$\mathcal{A}_{D \rightarrow V_1 V_2} = i\eta^{*\mu}\epsilon^{*\nu} \left(g_{\mu\nu} S_1 - \frac{p_{D\mu} p_{D\nu}}{m_D^2} S_2 + i\epsilon_{\mu\nu\rho\sigma} \frac{p_1^\rho p_2^\sigma}{p_1 \cdot p_2} S_3 \right). \quad (1)$$

The amplitudes with definite helicity ($h = 0, +, -$) are expressed as

$$\begin{aligned} \mathcal{A}^0 &= \mathcal{A}[D \rightarrow V_1(p_1, \eta_0^*) V_2(p_2, \epsilon_0^*)] = i \frac{m_D^2}{2m_1 m_2} \left(S_1 - \frac{S_2}{2} \right), \\ \mathcal{A}^\pm &= \mathcal{A}[D \rightarrow V_1(p_1, \eta_\pm^*) V_2(p_2, \epsilon_\pm^*)] = i(S_1 \mp S_3). \end{aligned} \quad (2)$$

Within the naive factorization framework, the helicity amplitudes can be formally expressed as

$$\mathcal{A}^h = \frac{G_F}{\sqrt{2}} V_{CKM} \langle V_1^h | (\bar{c}q)_{V-A} | D \rangle \langle V_2^h | (\bar{q}q')_{V-A} | 0 \rangle. \quad (3)$$

By neglecting m_i^2 contributions, the three helicity amplitudes are explicitly given by

$$\begin{aligned}
\mathcal{A}^0 &= i \frac{G_F}{\sqrt{2}} V_{\text{CKM}} f_2 m_D^2 A_0^{DV_1}(m_2^2), \\
\mathcal{A}^+ &= i \frac{G_F}{\sqrt{2}} V_{\text{CKM}} f_2 m_2 \{-(m_D + m_1) A_1^{DV_1}(m_2^2) + (m_D - m_1) V^{DV_1}(m_2^2)\}, \\
\mathcal{A}^- &= i \frac{G_F}{\sqrt{2}} V_{\text{CKM}} f_2 m_2 \{-(m_D + m_1) A_1^{DV_1}(m_2^2) - (m_D - m_1) V^{DV_1}(m_2^2)\},
\end{aligned} \tag{4}$$

where $A_0^{DV_1}(m_2^2)$, $A_1^{DV_1}(m_2^2)$, and $V^{DV_1}(m_2^2)$ denote the transition form factors, as defined in Refs. [10,39], and f_2 is the decay constant of the emitted vector meson. Given the m_2/m_D suppression of $\mathcal{A}^\pm$ relative to the $\mathcal{A}^0$ and the A_1 and V cancellation, the amplitudes follow the hierarchy

$$\mathcal{A}^0 : \mathcal{A}^- : \mathcal{A}^+ = 1 : \frac{\Lambda_{\text{QCD}}}{m_c} : \left(\frac{\Lambda_{\text{QCD}}}{m_c} \right)^2. \tag{5}$$

In general, decay amplitudes admit several equivalent representations. Especially, the helicity amplitudes $\mathcal{A}^{0,+,-}$ relate to the spin amplitudes in the transversity basis ($\mathcal{A}^{L,\parallel,\perp}$) as follows:

$$\mathcal{A}_L = \mathcal{A}^0; \quad \mathcal{A}_\parallel = \frac{\mathcal{A}^+ + \mathcal{A}^-}{\sqrt{2}}; \quad \mathcal{A}_\perp = \frac{\mathcal{A}^+ - \mathcal{A}^-}{\sqrt{2}}. \tag{6}$$

In the transversity basis, the three amplitudes can be further simplified as

$$\begin{aligned}
\mathcal{A}_L &= i \frac{G_F}{\sqrt{2}} V_{\text{CKM}} f_2 m_D^2 A_0^{DV_1}(m_2^2), \\
\mathcal{A}_\parallel &= -i G_F V_{\text{CKM}} f_2 m_2 (m_D + m_1) A_1^{DV_1}(m_2^2), \\
\mathcal{A}_\perp &= i G_F V_{\text{CKM}} f_2 m_2 (m_D - m_1) V^{DV_1}(m_2^2).
\end{aligned} \tag{7}$$

As experimental data for $D \rightarrow VV$ decays are predominantly presented in terms of partial-wave (S , P , D) branching fractions, it is necessary to convert the transversity amplitudes into partial-wave amplitudes via

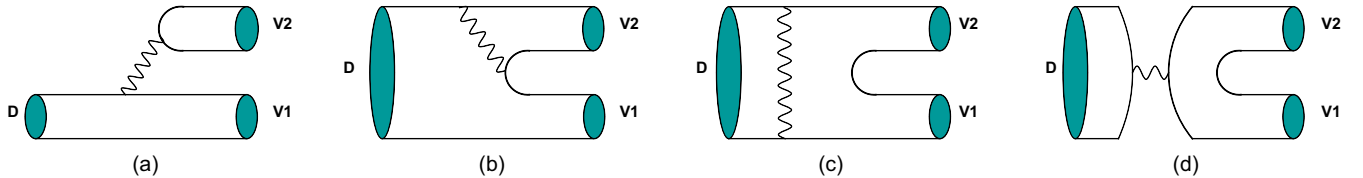


FIG. 1. Topological diagrams for $D \rightarrow V_1 V_2$ with the wave line representing a W boson: (a) the color-favored emission diagram T , (b) the color-suppressed emission diagram C , (c) the W -exchange diagram E , and (d) W -annihilation diagram A .

$$\begin{aligned}
S &= \frac{1}{\sqrt{3}} (-\mathcal{A}_L + \sqrt{2} \mathcal{A}_\parallel); & P &= \mathcal{A}_\perp; \\
D &= \frac{1}{\sqrt{3}} (\sqrt{2} \mathcal{A}_L + \mathcal{A}_\parallel).
\end{aligned} \tag{8}$$

Equations (7) and (8) reveal that, in the naive factorization approach, the polarization amplitude hierarchies satisfy $|\mathcal{A}_\parallel|^2 \geq |\mathcal{A}_L|^2 > |\mathcal{A}_\perp|^2$, and $|S|^2 \geq |P|^2 > |D|^2$ [17].

B. Polarization amplitudes within the FAT framework

The $D \rightarrow VV$ decay is a weak process induced primarily by the tree-level quark transition $c \rightarrow d(s)u\bar{d}(\bar{s})$. Although penguin contributions ($c \rightarrow uq\bar{q}$ with $q = u, d, s$) exist, they are neglected in branching fraction calculations due to suppression by both small Wilson coefficients and CKM matrix elements. Based on the weak interaction, the tree-level topological diagrams for $D \rightarrow V_1 V_2$ decays are conventionally classified into four categories: (a) color-favored emission diagram T , (b) color-suppressed emission diagram C , (c) W -exchange diagram E , and (d) W -annihilation diagram A , which are as illustrated in Fig. 1. Here, we factorize the weak interaction from strong interaction, thereby incorporating all perturbative and nonperturbative QCD corrections into the topological diagrams. While the conventional topological diagram approach treats these four topological amplitudes as unknown parameters fitted under $SU(3)$ symmetry, the FAT approach explicitly accounts for $SU(3)$ -breaking effects via factorization. In the following, we analyze the four topological amplitudes (T , C , E , and A) within the FAT framework.

First, the T diagram is typically treated as factorizable. To minimize the number of free parameters, we directly adopt the naive factorization results, analogous to Eq. (7),

including the relevant Wilson coefficients. The T amplitudes for the three polarization states are expressed as

$$\begin{aligned} T^0 &= i \frac{G_F}{\sqrt{2}} V_{\text{CKM}} a_1(\mu) m_D^2 f_2 A_0^{DV_1}(m_2^2), \\ T^\parallel &= -i G_F V_{\text{CKM}} a_1(\mu) m_2 (m_D + m_1) f_2 A_1^{DV_1}(m_2^2), \\ T^\perp &= i G_F V_{\text{CKM}} a_1(\mu) m_2 (m_D - m_1) f_2 V^{DV_1}(m_2^2), \end{aligned} \quad (9)$$

where $a_1(\mu) = C_2(\mu) + C_1(\mu)/3$ is the effective Wilson coefficient for four-quark operators. The scale parameter μ , representing the energy release, is treated as a single free parameter constrained to be below $m_c/2$. In contrast to the conventional diagrammatic approach, where describing the three decay amplitudes requires at least five free parameters (three magnitudes and two strong phase for T amplitudes) to be fitted from experimental data, our formulation introduces only one free parameter, μ , for all T amplitudes. Furthermore, SU(3)-breaking effects are naturally incorporated through the use of distinct decay constants and form factors for different decay modes.

The remaining three diagrams (C , E , and A) are dominated by nonfactorizable contributions. For the C diagram, after factorizing out form factor and decay constant to account for SU(3)-breaking effects, the residual contribution is expected to be universal across different processes. We introduce two sets of unknown parameters, $\chi_C^{0(\parallel, \perp)}$ and $\phi_C^{0(\parallel, \perp)}$, representing the magnitudes and associate strong phases for each polarization amplitude, respectively. These amplitudes are expressed as

$$\begin{aligned} C^0 &= i \frac{G_F}{\sqrt{2}} V_{\text{CKM}} \chi_C^0 e^{i\phi_C^0} m_D^2 f_2 A_0^{DV_1}(m_2^2), \\ C^\parallel &= -i G_F V_{\text{CKM}} \chi_C^\parallel e^{i\phi_C^\parallel} m_2 (m_D + m_1) f_2 A_1^{DV_1}(m_2^2), \\ C^\perp &= i G_F V_{\text{CKM}} \chi_C^\perp e^{i\phi_C^\perp} m_2 (m_D - m_1) f_2 V^{DV_1}(m_2^2). \end{aligned} \quad (10)$$

Similarly, for the E diagram, we factorize the relevant meson decay constants to characterize SU(3)-breaking effects. The polarized amplitudes are given by

$$\begin{aligned} E^0 &= i \frac{G_F}{\sqrt{2}} V_{\text{CKM}} \chi_E^0 e^{i\phi_E^0} m_D^2 f_D f_1 f_2 \frac{1}{f_\rho^2}, \\ E^\parallel &= -i G_F V_{\text{CKM}} \chi_E^\parallel e^{i\phi_E^\parallel} m_D (m_1 + m_2) f_D f_1 f_2 \frac{1}{f_\rho^2}, \\ E^\perp &= i G_F V_{\text{CKM}} \chi_E^\perp e^{i\phi_E^\perp} m_D^2 f_D f_1 f_2 \frac{1}{f_\rho^2}, \end{aligned} \quad (11)$$

where the parameters $\chi_E^{0, \parallel, \perp}$ are dimensionless quantities normalized to f_ρ^2 .

TABLE I. Masses m_V and decay constant of mesons (in units of MeV).

Meson	Mass	Decay constant
$D^\pm$	1869.66	212.0 ± 0.7
D^0	1864.84	212.0 ± 0.7
D_s	1968.35	249.9 ± 0.5
ρ	775.26	223.7 ± 11.2
ω	782.66	182.4 ± 9.1
ϕ	1019.46	236.3 ± 11.8
K^{*+}	891.67	214.7 ± 10.7
K^{*0}	895.55	214.7 ± 10.7

The A diagram amplitudes follow a parametrization similar to the E diagram, substituting the parameters $\chi_E^{0, \parallel, \perp}$ and $\phi_E^{0, \parallel, \perp}$ with $\chi_A^{0, \parallel, \perp}$ and $\phi_A^{0, \parallel, \perp}$:

$$\begin{aligned} A^0 &= i \frac{G_F}{\sqrt{2}} V_{\text{CKM}} \chi_A^0 e^{i\phi_A^0} m_D^2 f_D f_1 f_2 \frac{1}{f_\rho^2}, \\ A^\parallel &= -i G_F V_{\text{CKM}} \chi_A^\parallel e^{i\phi_A^\parallel} m_D (m_1 + m_2) f_D f_1 f_2 \frac{1}{f_\rho^2}, \\ A^\perp &= i G_F V_{\text{CKM}} \chi_A^\perp e^{i\phi_A^\perp} m_D^2 f_D f_1 f_2 \frac{1}{f_\rho^2}. \end{aligned} \quad (12)$$

Nevertheless, we neglect the A diagram in later analysis because its contribution is negligible. Attempting to include these parameters in our fitting program, we fail to obtain stable solutions due to the limited precision of current experimental data.

In total, we have nine magnitudes and nine strong phases, along with the scale parameter μ , to be fitted simultaneously to the experimental data. With these fitted parameters, we can predict the branching fractions and two out of the three polarization fractions (f_L , $f_\parallel$, and $f_\perp$) for all $D_{\bar{q}} \rightarrow VV$ ($\bar{q} = \bar{u}, \bar{d}, \bar{s}$) based on the standard definitions as follows:

$$\begin{aligned} \Gamma &\equiv \frac{|\mathbf{p}|}{8\pi m_D^2} \frac{|\mathcal{A}_L|^2 + |\mathcal{A}_\parallel|^2 + |\mathcal{A}_\perp|^2 + |\bar{\mathcal{A}}_L|^2 + |\bar{\mathcal{A}}_\parallel|^2 + |\bar{\mathcal{A}}_\perp|^2}{2} \\ &= \frac{|\mathbf{p}|}{8\pi m_D^2} \frac{|S|^2 + |P|^2 + |D|^2 + |\bar{S}|^2 + |\bar{P}|^2 + |\bar{D}|^2}{2} \end{aligned} \quad (13)$$

and

$$f_{L, \parallel, \perp}^D = \frac{\Gamma_{L, \parallel, \perp}}{\Gamma} = \frac{|\mathcal{A}_{L, \parallel, \perp}|^2}{|\mathcal{A}_L|^2 + |\mathcal{A}_\parallel|^2 + |\mathcal{A}_\perp|^2}, \quad (14)$$

respectively.

III. NUMERICAL RESULTS AND DISCUSSIONS

A. Input parameters

The input parameters are categorized into (i) CKM matrix elements and Wilson coefficients; (ii) meson masses and decay constants; and (iii) $D_{(s)} \rightarrow V$ transition form factors. The CKM matrix elements are taken from the PDG [40], and the Wilson coefficients, $C_{1(2)}(\mu)$, for D -meson decays follow Eqs. (B1) and (B2) in the Appendix in Ref. [26]. The masses and decay constants of the D mesons and vector mesons are listed in Table I. Specifically, all masses and $D_{(s)}$ meson decay constants are provided by the PDG [40]. The decay constants of vector mesons (ρ , ω , ϕ , and K^*) have not been measured experimentally but are calculated in several theoretical approaches, such as quark model [41], covariant light front approach [42], light-cone sum rules [43,44], QCD sum rules [45], etc. Given the variations among these theoretical results, we adopt the numerical values shown in Table I and keep a 5% uncertainty to them.

The transition form factors of $D \rightarrow V$ are also unmeasured. We adopt values from Ref. [10], calculated at zero recoil momentum ($q^2 = 0$) in Ref. [39] using relativistic oscillator wave functions. The corresponding values at $q^2 = 0$ are presented in Table II and a 10% uncertainty is assigned to these form factors. Their q^2 dependence is parametrized using the nearest pole dominance form [10,39] as follows:

$$F_i(q^2) = \frac{F_i(0)}{1 - q^2/m_{\text{pole}}^2}, \quad (15)$$

where F_i represents form factors A_0 , A_1 , or V and m_{pole} is the mass of the corresponding pole state, which are displayed in Table III from Refs. [10,39]. Specifically, we use $m(0^-, \bar{d}c)$ for $A_0^{D\rho, D\omega, D_s K^*}$ and $m(0^-, \bar{s}c)$ for $A_0^{DK^*, D_s \phi}$; $m(1^+, \bar{d}c)$ for $A_1^{D\rho, D\omega, D_s K^*}$ and $m(1^+, \bar{s}c)$ for $A_1^{DK^*, D_s \phi}$; and $m(1^-, \bar{d}c)$ for $V^{D\rho, D\omega, D_s K^*}$ and $m(1^-, \bar{s}c)$ for $V^{DK^*, D_s \phi}$.

B. The nonfactorizable parameters

The free parameters in the topological diagram amplitudes defined in Eqs. (9)–(12) consist of 19 parameters: $\chi_C^{0, \parallel, \perp}$, $\chi_E^{0, \parallel, \perp}$, and $\chi_A^{0, \parallel, \perp}$, along with their associated strong

TABLE II. Form factors at zero recoil momentum for $D \rightarrow V$ transitions [10].

	$V^{D \rightarrow \rho}$	$V^{D \rightarrow K^*}$	$V^{D_s \rightarrow K^*}$	$V^{D \rightarrow \omega}$	$V^{D_s \rightarrow \phi}$
$V(0)$	1.225	1.226	1.250	1.236	1.319
$A_0(0)$	0.669	0.733	0.634	0.669	0.700
$A_1(0)$	0.775	0.880	0.717	0.772	0.820

TABLE III. Values of pole masses used in Eq. (15) (in units of GeV).

Current	$m(0^-)$	$m(1^-)$	$m(0^+)$	$m(1^+)$
$\bar{d}c$	1.87	2.01	2.47	2.42
$\bar{s}c$	1.97	2.11	2.60	2.53

phases $\phi_E^{0, \parallel, \perp}$, $\phi_C^{0, \parallel, \perp}$, and $\phi_A^{0, \parallel, \perp}$, as well as the factorization scale μ [appearing in the Wilson coefficients $a_1(\mu)$ of the T diagram amplitude in Eq. (9)]. In the experimental sector, the total and partial branching fractions of $D \rightarrow VV$ have been measured by BESIII [4–6,46–49], LHCb [50,51], CLEO-c [8], FOCUS [7], and Mark III Collaborations [3], where all measurements of D^+ and D_s^+ decays are provided by BESIII. The data used in the global fit are listed in Tables IV–VI, taken from the PDG [40] and recent BESIII measurements. In total, 36 data points are available to constrain the 19 free parameters. However, we observe that the number of free parameters is too large to be precisely determined by the limited experimental data, thereby diminishing the predictive power of the FAT approach.

Given the very small values of $\chi_E^\perp$ found in preliminary fit, we neglect the parameter $\chi_E^\perp$ and its associated strong phase $\phi_E^\perp$. Moreover, as the A diagram amplitude is typically smaller than that of the E diagram, we ignore the A diagram contributions entirely. Consequently, by setting $\chi_E^\perp = \chi_A^\perp = \chi_A^\parallel = \chi_A^\perp = 0$ and $\phi_E^\perp = \phi_A^\perp = \phi_A^\parallel = \phi_A^\perp = 0$, the number of free parameters is reduced to 11. These remaining parameters are subsequently determined through a global fit to 36 experimental data points. The best-fitted values, together with their corresponding uncertainties, are list as follows:

$$\begin{aligned} \mu &= 465.14 \pm 0.12 \text{ MeV}, \\ \chi_C^0 &= 0.55 \pm 0.03, & \phi_C^0 &= 0.87 \pm 0.10, \\ \chi_C^\parallel &= 0.45 \pm 0.03, & \phi_C^\parallel &= 0.77 \pm 0.11, \\ \chi_C^\perp &= 0.37 \pm 0.02, & \phi_C^\perp &= 0.36 \pm 0.04, \\ \chi_E^0 &= 0.24 \pm 0.07, & \phi_E^0 &= 2.23 \pm 0.30, \\ \chi_E^\parallel &= 0.25 \pm 0.03, & \phi_E^\parallel &= 0.23 \pm 0.21, \end{aligned} \quad (16)$$

with $\chi^2/\text{d.o.f.} = 8.43$. These obtained nonfactorizable parameters are highly precise, with the exception of the strong phase $\phi_E^\parallel$. This large uncertainty arises because the E amplitude always appears alongside the T or C amplitude, and the transverse polarization branching fraction is smaller than that of longitude polarization. Consequently, no sufficiently precise data involving the E amplitude exist to constrain the strong phase $\phi_E^\parallel$.

At the factorization scale $\mu = 465.14 \text{ MeV}$, the Wilson coefficient is $a_1 = -0.47$ in the T amplitude, which corresponds to the factorizable contribution of the T

TABLE IV. Total and partial-wave (S , P , and D) branching fractions for Cabibbo-favored $D \rightarrow VV$ decays, given in units of percentage. Our theoretical predictions, denoted as $\mathcal{B}_{\text{FAT}}$, including uncertainties arising from the fitted parameters, form factors, and decay constants, respectively, are compared with experimental data, $\mathcal{B}_{\text{exp}}$. The second column indicates the topological diagram amplitudes, T , C , E , and A , contributing to each mode.

Modes	Amplitudes	$\mathcal{B}_{\text{exp}}$	$\mathcal{B}_{\text{FAT}}$
$D \rightarrow V_1 V_2$	$V_{cs}^* V_{ud}$		
$D^0 \rightarrow K^{*-} \rho^+$	$T + E$	6.5 ± 2.5	$4.23 \pm 0.82 \pm 0.50 \pm 0.28$
$D^0 \rightarrow K^{*-} \rho^+(S)$		1.4 ± 0.4	$2.07 \pm 0.51 \pm 0.36 \pm 0.30$
$D^0 \rightarrow K^{*-} \rho^+(P)$		0.9 ± 0.2	$0.46 \pm 0.02 \pm 0.09 \pm 0.05$
$D^0 \rightarrow K^{*-} \rho^+(D)$		2.9 ± 0.8	$1.70 \pm 0.50 \pm 0.34 \pm 0.20$
$D^0 \rightarrow \rho^0 \bar{K}^{*0}$	$\frac{1}{\sqrt{2}}(C - E)$	1.52 ± 0.08	$1.65 \pm 0.40 \pm 0.22 \pm 0.17$
$D^0 \rightarrow \rho^0 \bar{K}^{*0}(S)$		0.8 ± 0.12	$0.68 \pm 0.23 \pm 0.16 \pm 0.09$
$D^0 \rightarrow \rho^0 \bar{K}^{*0}(P)$		0.28 ± 0.03	$0.25 \pm 0.02 \pm 0.05 \pm 0.03$
$D^0 \rightarrow \rho^0 \bar{K}^{*0}(D)$		0.98 ± 0.1	$0.72 \pm 0.27 \pm 0.12 \pm 0.10$
$D^0 \rightarrow \omega \bar{K}^{*0}$	$\frac{1}{\sqrt{2}}(C + E)$	1.1 ± 0.5	$3.63 \pm 0.43 \pm 0.34 \pm 0.44$
$D^0 \rightarrow \omega \bar{K}^{*0}(S)$			$3.01 \pm 0.40 \pm 0.33 \pm 0.35$
$D^0 \rightarrow \omega \bar{K}^{*0}(P)$			$0.25 \pm 0.02 \pm 0.05 \pm 0.02$
$D^0 \rightarrow \omega \bar{K}^{*0}(D)$			$0.37 \pm 0.18 \pm 0.04 \pm 0.08$
$D^+ \rightarrow \bar{K}^{*0} \rho^+$	$T + C$	6.23 ± 0.27	$6.32 \pm 1.05 \pm 0.66 \pm 0.45$
$D^+ \rightarrow \bar{K}^{*0} \rho^+(S)$		6.05 ± 0.30	$5.61 \pm 0.99 \pm 0.57 \pm 0.40$
$D^+ \rightarrow \bar{K}^{*0} \rho^+(P)$		0.173 ± 0.026	$0.16 \pm 0.03 \pm 0.03 \pm 0.01$
$D^+ \rightarrow \bar{K}^{*0} \rho^+(D)$			$0.55 \pm 0.29 \pm 0.23 \pm 0.05$
$D_s^+ \rightarrow \phi \rho^+$	T	3.98 ± 0.39	$4.55 \pm 0.19 \pm 0.58 \pm 0.46$
$D_s^+ \rightarrow \phi \rho^+(S)$		3.32 ± 0.35	$3.85 \pm 0.16 \pm 0.55 \pm 0.38$
$D_s^+ \rightarrow \phi \rho^+(P)$		0.63 ± 0.13	$0.53 \pm 0.02 \pm 0.11 \pm 0.05$
$D_s^+ \rightarrow \phi \rho^+(D)$			$0.17 \pm 0.01 \pm 0.12 \pm 0.02$
$D_s^+ \rightarrow K^{*+} \bar{K}^{*0}$	$C + A$	5.93 ± 0.88	$5.07 \pm 0.36 \pm 0.67 \pm 0.51$
$D_s^+ \rightarrow K^{*+} \bar{K}^{*0}(S)$		5.01 ± 0.92	$4.12 \pm 0.34 \pm 0.58 \pm 0.41$
$D_s^+ \rightarrow K^{*+} \bar{K}^{*0}(P)$		1.10 ± 0.19	$0.56 \pm 0.06 \pm 0.11 \pm 0.06$
$D_s^+ \rightarrow K^{*+} \bar{K}^{*0}(D)$		0.65 ± 0.16	$0.40 \pm 0.11 \pm 0.20 \pm 0.04$

diagram. We find the hierarchies $|C^0| > |T^0| > |E^0|$ and $|T^{\parallel(\perp)}| \sim |C^{\parallel(\perp)}| > |E^{\parallel(\perp)}|$. This indicates that the non-factorizable contribution of the C diagram is comparable to the factorizable contributions of the T diagram. This suggests the potential need to include nonfactorizable contributions for the T diagram by inducing five additional parameters $\chi_T^{0,\parallel,\perp}$ and strong phases $\phi_T^{\parallel,\perp}$. However, obtaining reliable results for such a large number of parameters is difficult given the limited experimental data.

Using the specific decay modes $D^0 \rightarrow \bar{K}^{*-} \rho^+$ and $D^0 \rightarrow \rho^0 \bar{K}^{*0}$ as examples, we examine the hierarchies of polarization components (0, $\parallel$, and $\perp$) and partial-wave contributions (S , P , and D) in the T , C , and E amplitudes:

$$\begin{aligned}
|T_{D^0 \rightarrow K^{*-} \rho^+}^0| : |T_{D^0 \rightarrow K^{*-} \rho^+}^{\parallel}| : |T_{D^0 \rightarrow K^{*-} \rho^+}^{\perp}| &= 1:0.97:0.50, \\
|C_{D^0 \rightarrow \rho^0 \bar{K}^{*0}}^0| : |C_{D^0 \rightarrow \rho^0 \bar{K}^{*0}}^{\parallel}| : |C_{D^0 \rightarrow \rho^0 \bar{K}^{*0}}^{\perp}| &= 1:0.81:0.47, \\
|E_{D^0 \rightarrow K^{*-} \rho^+}^0| : |E_{D^0 \rightarrow K^{*-} \rho^+}^{\parallel}| &= 1:1.31 \quad (17)
\end{aligned}$$

and

$$\begin{aligned}
|T_{D^0 \rightarrow K^{*-} \rho^+}^S| : |T_{D^0 \rightarrow K^{*-} \rho^+}^P| : |T_{D^0 \rightarrow K^{*-} \rho^+}^D| &= 1:0.37:0.19, \\
|C_{D^0 \rightarrow \rho^0 \bar{K}^{*0}}^S| : |C_{D^0 \rightarrow \rho^0 \bar{K}^{*0}}^P| : |C_{D^0 \rightarrow \rho^0 \bar{K}^{*0}}^D| &= 1:0.37:0.28, \\
|E_{D^0 \rightarrow K^{*-} \rho^+}^S| : |E_{D^0 \rightarrow K^{*-} \rho^+}^D| &= 1:1.36, \quad (18)
\end{aligned}$$

respectively.

The hierarchies in Eq. (17) indicate that the polarization amplitudes satisfy $|\mathcal{A}^0| > |\mathcal{A}^{\parallel}| > |\mathcal{A}^{\perp}|$ for the T amplitude and, particularly, for the C amplitude. This differs from the naive factorization prediction, $|\mathcal{A}^{\parallel}| \geq |\mathcal{A}^0| > |\mathcal{A}^{\perp}|$. This discrepancy arises because the C diagram contribution is primarily nonfactorizable, which cannot be accurately calculated within the naive factorization approach. Our result implies that the longitudinal polarization f_L remains dominant in decay modes not involving the E amplitude. For decays involving interference between T and E , or C

TABLE V. The same as Table IV for singly Cabibbo-suppressed $D^0 \rightarrow VV$ decays in units of 10^{-3} .

Modes	Amplitudes	$\mathcal{B}_{\text{exp}}$	$\mathcal{B}_{\text{FAT}}$
$D \rightarrow V_1 V_2$	$V_{cd}^* V_{ud}$ or $V_{cs}^* V_{us}$		
$D^0 \rightarrow \rho^- \rho^+$	$T + E$	7.81 ± 1.14	$2.67 \pm 0.54 \pm 0.30 \pm 0.21$
$D^0 \rightarrow \rho^- \rho^+(S)$		1.20 ± 0.40	$1.08 \pm 0.31 \pm 0.18 \pm 0.13$
$D^0 \rightarrow \rho^- \rho^+(P)$		1.80 ± 0.30	$0.39 \pm 0.02 \pm 0.08 \pm 0.04$
$D^0 \rightarrow \rho^- \rho^+(D)$		3.30 ± 0.50	$1.20 \pm 0.35 \pm 0.21 \pm 0.09$
$D^0 \rightarrow \rho^0 \rho^0$	$\frac{1}{2}(E - C)$	1.33 ± 0.35	$0.49 \pm 0.13 \pm 0.07 \pm 0.04$
$D^0 \rightarrow \rho^0 \rho^0(S)$		0.18 ± 0.13	$0.18 \pm 0.07 \pm 0.04 \pm 0.03$
$D^0 \rightarrow \rho^0 \rho^0(P)$		0.53 ± 0.13	$0.061 \pm 0.006 \pm 0.012 \pm 0.006$
$D^0 \rightarrow \rho^0 \rho^0(D)$		0.62 ± 0.3	$0.26 \pm 0.09 \pm 0.04 \pm 0.02$
$D^0 \rightarrow K^{*-} K^{*+}$	$T + E$		$1.58 \pm 0.28 \pm 0.19 \pm 0.19$
$D^0 \rightarrow K^{*-} K^{*+}(S)$			$0.85 \pm 0.19 \pm 0.15 \pm 0.08$
$D^0 \rightarrow K^{*-} K^{*+}(P)$			$0.22 \pm 0.01 \pm 0.04 \pm 0.02$
$D^0 \rightarrow K^{*-} K^{*+}(D)$			$0.52 \pm 0.16 \pm 0.12 \pm 0.13$
$D^0 \rightarrow \rho^0 \phi$	$\frac{1}{\sqrt{2}}C$	1.56 ± 0.13	$1.13 \pm 0.08 \pm 0.14 \pm 0.11$
$D^0 \rightarrow \rho^0 \phi(S)$		1.40 ± 0.10	$0.92 \pm 0.08 \pm 0.13 \pm 0.09$
$D^0 \rightarrow \rho^0 \phi(P)$		0.081 ± 0.039	$0.149 \pm 0.015 \pm 0.030 \pm 0.015$
$D^0 \rightarrow \rho^0 \phi(D)$		0.085 ± 0.028	$0.053 \pm 0.019 \pm 0.032 \pm 0.005$
$D^0 \rightarrow \omega \phi$	$\frac{1}{\sqrt{2}}C$	0.648 ± 0.104	$1.066 \pm 0.074 \pm 0.135 \pm 0.107$
$D^0 \rightarrow \omega \phi(S)$			$0.873 \pm 0.073 \pm 0.124 \pm 0.087$
$D^0 \rightarrow \omega \phi(P)$			$0.141 \pm 0.014 \pm 0.028 \pm 0.014$
$D^0 \rightarrow \omega \phi(D)$			$0.051 \pm 0.018 \pm 0.031 \pm 0.005$
$D^0 \rightarrow \omega \rho^0$	$-E$		$0.77 \pm 0.19 \pm 0.08 \pm 0.08$
$D^0 \rightarrow \omega \rho^0(S)$			$0.35 \pm 0.14 \pm 0.08 \pm 0.03$
$D^0 \rightarrow \omega \rho^0(P)$			$0.0019 \pm 0.0002 \pm 0.0027 \pm 0.0014$
$D^0 \rightarrow \omega \rho^0(D)$			$0.42 \pm 0.17 \pm 0.02 \pm 0.07$
$D^0 \rightarrow \omega \omega$	$\frac{1}{2}(C + E)$		$0.71 \pm 0.09 \pm 0.07 \pm 0.10$
$D^0 \rightarrow \omega \omega(S)$			$0.58 \pm 0.08 \pm 0.06 \pm 0.08$
$D^0 \rightarrow \omega \omega(P)$			$0.041 \pm 0.004 \pm 0.008 \pm 0.004$
$D^0 \rightarrow \omega \omega(D)$			$0.09 \pm 0.041 \pm 0.01 \pm 0.02$

and E amplitudes, the strong phases $\phi_{E,C}^{0,\parallel}$ determine the interference pattern, potentially resulting in $f_L < f_{\parallel}$.

Regarding partial-wave amplitudes, Eq. (18) shows that $|S| > |P| > |D|$ for the T and C diagrams. Thus, the S -wave contribution is dominant. This is attributed to the definition in Eq. (8), where $\mathcal{A}^0$ and $\mathcal{A}^{\parallel}$ exhibit constructive interference for the S -wave amplitude but destructive interference for the D wave. However, the hierarchy is reversed for the E diagram (e.g., $|S| < |D|$) due to the large strong phase difference $|\phi_E^0 - \phi_E^{\parallel}|$. The hierarchy of $|S| < |D|$ for E amplitudes can lead to a larger D -wave branching fraction than that of the S wave in the decays involving the E diagram, contrary to the naive factorization prediction of S -wave dominance.

C. Branching fractions of $D \rightarrow VV$ decays

Our numerical predictions for the branching fractions of 28 decay modes of D meson are collected in Tables IV–VII,

organized by decay type: Cabibbo-favored (CF, Table IV), singly Cabibbo-suppressed (SCS, Tables V and VI), and doubly Cabibbo-suppressed (DCS, Table VII). In each table, the first column lists the decay modes, followed by the total branching fraction and its partial-wave components (S waves, P waves, and D waves). The second column identifies the relevant topological diagram amplitudes (T , C , E , and A) for the convenience of the subsequent discussion. The theoretical results, denoted as “ $\mathcal{B}_{\text{FAT}}$,” appear in the final column. The quoted uncertainties correspond to (i) variations in the nonperturbative parameters of Eq. (16); (ii) a 10% variation in the form factors; and (iii) a 5% variation in the unmeasured decay constants. Among these, the form factor uncertainty dominates. For comparison, we also list the experimental data (“ $\mathcal{B}_{\text{exp}}$ ”), which constitute the dataset used in our global fit. We explicitly exclude the $D^0 \rightarrow \bar{K}^{*0} K^{*0}$ channel from the fit. Including this mode would result in a χ^2 contribution of approximately 484 from this single mode alone,

TABLE VI. The same as Table IV for singly Cabibbo-suppressed $D_{(s)}^+ \rightarrow VV$ decays in units of 10^{-3} .

Modes	Amplitudes	$\mathcal{B}_{\text{exp}}$	$\mathcal{B}_{\text{FAT}}$
$D \rightarrow V_1 V_2$	$V_{cd}^* V_{ud}$ or $V_{cs}^* V_{us}$		
$D^+ \rightarrow \rho^0 \rho^+$	$-\frac{1}{\sqrt{2}}(T + C)$		$1.81 \pm 0.30 \pm 0.26 \pm 0.18$
$D^+ \rightarrow \rho^0 \rho^+(S)$			$1.53 \pm 0.27 \pm 0.22 \pm 0.15$
$D^+ \rightarrow \rho^0 \rho^+(P)$			$0.07 \pm 0.01 \pm 0.01 \pm 0.01$
$D^+ \rightarrow \rho^0 \rho^+(D)$			$0.21 \pm 0.10 \pm 0.09 \pm 0.02$
$D^+ \rightarrow \bar{K}^{*0} K^{*+}$	T		$4.22 \pm 0.18 \pm 0.53 \pm 0.42$
$D^+ \rightarrow \bar{K}^{*0} K^{*+}(S)$			$3.61 \pm 0.15 \pm 0.52 \pm 0.36$
$D^+ \rightarrow \bar{K}^{*0} K^{*+}(P)$			$0.54 \pm 0.02 \pm 0.11 \pm 0.05$
$D^+ \rightarrow \bar{K}^{*0} K^{*+}(D)$			$0.060 \pm 0.003 \pm 0.065 \pm 0.006$
$D^+ \rightarrow \omega \rho^+$	$\frac{1}{\sqrt{2}}(T + C + 2A)$		$1.52 \pm 0.24 \pm 0.16 \pm 0.12$
$D^+ \rightarrow \omega \rho^+(S)$			$1.27 \pm 0.21 \pm 0.14 \pm 0.10$
$D^+ \rightarrow \omega \rho^+(P)$			$0.10 \pm 0.02 \pm 0.04 \pm 0.02$
$D^+ \rightarrow \omega \rho^+(D)$			$0.15 \pm 0.07 \pm 0.05 \pm 0.01$
$D^+ \rightarrow \rho^+ \phi$	C		$5.88 \pm 0.41 \pm 0.74 \pm 0.59$
$D^+ \rightarrow \rho^+ \phi(S)$			$4.82 \pm 0.40 \pm 0.69 \pm 0.48$
$D^+ \rightarrow \rho^+ \phi(P)$			$0.78 \pm 0.08 \pm 0.16 \pm 0.08$
$D^+ \rightarrow \rho^+ \phi(D)$			$0.28 \pm 0.10 \pm 0.17 \pm 0.03$
$D_s^+ \rightarrow K^{*0} \rho^+$	$T + A$	3.95 ± 0.39	$2.63 \pm 0.11 \pm 0.33 \pm 0.26$
$D_s^+ \rightarrow K^{*0} \rho^+(S)$		1.41 ± 0.24	$2.06 \pm 0.09 \pm 0.29 \pm 0.21$
$D_s^+ \rightarrow K^{*0} \rho^+(P)$		2.53 ± 0.31	$0.43 \pm 0.02 \pm 0.09 \pm 0.04$
$D_s^+ \rightarrow K^{*0} \rho^+(D)$			$0.138 \pm 0.006 \pm 0.080 \pm 0.014$
$D_s^+ \rightarrow K^{*+} \rho^0$	$\frac{1}{\sqrt{2}}(A - C)$		$1.46 \pm 0.11 \pm 0.20 \pm 0.15$
$D_s^+ \rightarrow K^{*+} \rho^0(S)$			$1.17 \pm 0.09 \pm 0.17 \pm 0.12$
$D_s^+ \rightarrow K^{*+} \rho^0(P)$		0.42 ± 0.17	$0.14 \pm 0.01 \pm 0.03 \pm 0.01$
$D_s^+ \rightarrow K^{*+} \rho^0(D)$			$0.16 \pm 0.04 \pm 0.07 \pm 0.02$
$D_s^+ \rightarrow K^{*+} \omega$	$\frac{1}{\sqrt{2}}(C + A)$		$0.98 \pm 0.07 \pm 0.13 \pm 0.10$
$D_s^+ \rightarrow K^{*+} \omega(S)$			$0.78 \pm 0.06 \pm 0.11 \pm 0.08$
$D_s^+ \rightarrow K^{*+} \omega(P)$			$0.09 \pm 0.01 \pm 0.02 \pm 0.01$
$D_s^+ \rightarrow K^{*+} \omega(D)$			$0.10 \pm 0.03 \pm 0.05 \pm 0.01$
$D_s^+ \rightarrow \phi K^{*+}$	$T + C + A$		$1.26 \pm 0.20 \pm 0.15 \pm 0.10$
$D_s^+ \rightarrow \phi K^{*+}(S)$			$1.11 \pm 0.19 \pm 0.13 \pm 0.09$
$D_s^+ \rightarrow \phi K^{*+}(P)$			$0.04 \pm 0.01 \pm 0.01 \pm 0.01$
$D_s^+ \rightarrow \phi K^{*+}(D)$			$0.10 \pm 0.05 \pm 0.05 \pm 0.01$

significantly compromising the global fit quality. This discrepancy arises because, at the quark level, the decay amplitude is governed by the E diagram with a CKM structure of $V_{cd}^* V_{ud} + V_{cs}^* V_{us}$. Because of the unitarity of the CKM matrix, these two terms undergo significant cancellation, leading to a theoretically suppressed branching fraction that falls well short of the experimental measurement $\mathcal{B}(D^0 \rightarrow \bar{K}^{*0} K^{*0}) = (0.88 \pm 0.04) \times 10^{-3}$ [40]. With more precise data becoming available in the future, we will be able to incorporate additional SU(3)-breaking effects. For instance, by introducing two distinct sets of parameters for E amplitude (e.g., χ_q^E, χ_s^E and ϕ_q^E, ϕ_s^E), we can differentiate between strongly produced light-quark pair ($u\bar{u}$ or $d\bar{d}$) or strange quarks ($s\bar{s}$), as applied

to $D \rightarrow PP$ [26] and $D \rightarrow PV$ [19,20]. Consequently, the amplitude $E^i \propto V_{cd}^* V_{ud} \chi_s^i \phi_s^i + V_{cs}^* V_{us} \chi_q^i \phi_q^i$ (with $i = 0, \parallel, \perp$) will no longer be canceled due to the differences between χ_q^E, χ_s^E and ϕ_q^E, ϕ_s^E , potentially explaining the current experimental data.

A comparison of the values in the last two columns indicates that our results are consistent with the measured CF and SCS modes within uncertainties, with the exception of the P -wave branching fractions for $D_s^+ \rightarrow K^{*0} \rho^+$ and $D^0 \rightarrow \rho^- \rho^+$ decays. These two modes contribute significantly to the total χ^2 , yielding values of approximately 45 for $D_s^+ \rightarrow K^{*0} \rho^+$ and 22 for $D^0 \rightarrow \rho^- \rho^+$. Since the $E^\perp$ is not included in our analysis, these two modes receive contributions only from the T diagram, corresponding to

TABLE VII. The same as Table IV for doubly Cabibbo-suppressed $D \rightarrow VV$ decays in units of 10^{-4} .

Modes	Amplitudes	$\mathcal{B}_{\text{FAT}}$
$D \rightarrow V_1 V_2$	$V_{cd}^* V_{us}$	
$D^0 \rightarrow \rho^- K^{*+}$	$T + E$	$1.24 \pm 0.23 \pm 0.14 \pm 0.13$
$D^0 \rightarrow \rho^- K^{*+}(S)$		$0.55 \pm 0.14 \pm 0.09 \pm 0.06$
$D^0 \rightarrow \rho^- K^{*+}(P)$		$0.23 \pm 0.01 \pm 0.05 \pm 0.02$
$D^0 \rightarrow \rho^- K^{*+}(D)$		$0.46 \pm 0.14 \pm 0.09 \pm 0.07$
$D^0 \rightarrow K^{*0} \rho^0$	$\frac{1}{\sqrt{2}}(C - E)$	$0.47 \pm 0.11 \pm 0.06 \pm 0.05$
$D^0 \rightarrow K^{*0} \rho^0(S)$		$0.19 \pm 0.07 \pm 0.04 \pm 0.03$
$D^0 \rightarrow K^{*0} \rho^0(P)$		$0.07 \pm 0.01 \pm 0.01 \pm 0.01$
$D^0 \rightarrow K^{*0} \rho^0(D)$		$0.20 \pm 0.08 \pm 0.04 \pm 0.03$
$D^0 \rightarrow K^{*0} \omega$	$\frac{1}{\sqrt{2}}(C + E)$	$1.03 \pm 0.12 \pm 0.10 \pm 0.12$
$D^0 \rightarrow K^{*0} \omega(S)$		$0.86 \pm 0.12 \pm 0.09 \pm 0.10$
$D^0 \rightarrow K^{*0} \omega(P)$		$0.07 \pm 0.01 \pm 0.01 \pm 0.01$
$D^0 \rightarrow K^{*0} \omega(D)$		$0.11 \pm 0.05 \pm 0.01 \pm 0.02$
$D^+ \rightarrow K^{*0} \rho^+$	$C + A$	$3.17 \pm 0.22 \pm 0.41 \pm 0.32$
$D^+ \rightarrow K^{*0} \rho^+(S)$		$2.59 \pm 0.21 \pm 0.37 \pm 0.26$
$D^+ \rightarrow K^{*0} \rho^+(P)$		$0.37 \pm 0.04 \pm 0.07 \pm 0.04$
$D^+ \rightarrow K^{*0} \rho^+(D)$		$0.21 \pm 0.06 \pm 0.11 \pm 0.02$
$D^+ \rightarrow \rho^0 K^{*+}$	$\frac{1}{\sqrt{2}}(A - T)$	$1.50 \pm 0.06 \pm 0.18 \pm 0.15$
$D^+ \rightarrow \rho^0 K^{*+}(S)$		$1.17 \pm 0.05 \pm 0.17 \pm 0.12$
$D^+ \rightarrow \rho^0 K^{*+}(P)$		$0.29 \pm 0.01 \pm 0.06 \pm 0.03$
$D^+ \rightarrow \rho^0 K^{*+}(D)$		$0.037 \pm 0.002 \pm 0.030 \pm 0.004$
$D^+ \rightarrow \omega K^{*+}$	$\frac{1}{\sqrt{2}}(T + A)$	$1.47 \pm 0.06 \pm 0.18 \pm 0.15$
$D^+ \rightarrow \omega K^{*+}(S)$		$1.15 \pm 0.05 \pm 0.16 \pm 0.11$
$D^+ \rightarrow \omega K^{*+}(P)$		$0.29 \pm 0.01 \pm 0.06 \pm 0.03$
$D^+ \rightarrow \omega K^{*+}(D)$		$0.036 \pm 0.002 \pm 0.029 \pm 0.004$
$D_s^+ \rightarrow K^{*0} K^{*+}$	$T + C$	$0.82 \pm 0.13 \pm 0.12 \pm 0.08$
$D_s^+ \rightarrow K^{*0} K^{*+}(S)$		$0.70 \pm 0.12 \pm 0.10 \pm 0.07$
$D_s^+ \rightarrow K^{*0} K^{*+}(P)$		$0.037 \pm 0.007 \pm 0.007 \pm 0.004$
$D_s^+ \rightarrow K^{*0} K^{*+}(D)$		$0.08 \pm 0.04 \pm 0.04 \pm 0.01$

predictions made using naive factorization. Additionally, pure annihilation processes, $D_s^+ \rightarrow \rho^+ \omega$, $D^+ \rightarrow \bar{K}^{*0} K^{*+}$, and $D_s^+ \rightarrow K^{*+} \rho^0$, mediated exclusively by the A diagram are excluded from our predictions due to the neglect of A diagram contribution. Kinematically forbidden decays, $D^0 \rightarrow \bar{K}^{*0} \phi$, $K^{*0} \phi$, $\phi \phi$ and $D^+ \rightarrow K^{*+} \phi$, are also omitted. Predictions for unmeasured decay modes in the FAT approach remain to be tested against future experimental data.

In the following, we analyze the branching fraction values of specific modes listed in Tables IV–VII. The total branching fractions of these modes are dominated by contributions from the T and C diagrams. The partial-wave results are discussed explicitly for each mode.

We begin with modes where the D -wave branching fraction exceeds the S -wave, such as $D^0 \rightarrow \rho^0 \bar{K}^{*0}$, $D^0 \rightarrow \rho^0 \rho^0$, and $D^0 \rightarrow K^{*0} \rho^0$, which involve the amplitudes $\frac{1}{\sqrt{2}}(C - E)$, as well as $D^0 \rightarrow \omega \rho^0$ involving the E

amplitude. In these decays, the C and the E amplitudes interfere destructively for both the S -wave and the D -wave branching fractions. Because the D -wave amplitude for the E diagram is significantly larger than that for the C diagram, the combined D -wave term $\frac{1}{\sqrt{2}}(C - E)$ approximates $-\frac{1}{\sqrt{2}}E$. Conversely, strong destructive interference in the S -wave amplitude suppresses its branching fraction, which is slightly smaller than that of the D wave. It is obvious that $D^0 \rightarrow \omega \rho^0$, dominated by the E amplitude, follows the hierarchy $|S| < |D|$, yielding a larger D -wave branching fraction.

The situation is reversed for the modes $D^0 \rightarrow \omega \bar{K}^{*0}$, $D^0 \rightarrow \omega \omega$, and $D^0 \rightarrow K^{*0} \omega$ characterized by $\frac{1}{\sqrt{2}}(C + E)$ contributions, where the interference pattern is opposite. Specifically, constructive interference between the C and E amplitudes enhances the S -wave branching fractions, whereas destructive interference suppresses the D -wave component. These results are consistent with the naive factorization predictions, in which the S -wave component is dominant.

The third category of decay modes involving the E amplitude comprises those governed by $T + E$ amplitudes, including $D^0 \rightarrow K^{*-} \rho^+$, $D^0 \rightarrow \rho^- \rho^+$, $D^0 \rightarrow K^{*-} K^{*+}$, and $D^0 \rightarrow \rho^- K^{*+}$. Here, destructive interference between the T and E amplitudes in the S wave and constructive interference in the D wave result in comparable S -wave and D -wave branching fractions. Finally, the remaining decays precessing through T , C , or $T + C$ diagrams exhibit a dominant S -wave branching fraction.

As noted in Ref. [17], $D^0 \rightarrow \rho^0 \phi$ and $D^0 \rightarrow \omega \phi$ are expected to have identical branching fractions (both total and partial wave) as they share the same C topological amplitude. It is necessary to consider that final state rescattering (e.g., via $D^0 \rightarrow K^{*+} K^{*-}$) can contribute differently to these two modes [16]. In the FAT approach, the nonfactorizable parameters χ_C and ϕ_C effectively incorporate such FSIs. However, the SU(3)-breaking effects (specifically, those from the form factors $A^{D^0 \rightarrow \rho}$ and $A^{D^0 \rightarrow \omega}$, $V^{D^0 \rightarrow \rho}$ and $V^{D^0 \rightarrow \omega}$) are insufficient to account for the differing central values of total branching fractions for $D^0 \rightarrow \rho^0 \phi$ and $D^0 \rightarrow \omega \phi$. Nevertheless, within uncertainties, particularly given the large form factor errors, the branching fractions predicted by the FAT approach for both decays are consistent with experimental measurements. The P - and D -wave fractions of $D^0 \rightarrow \rho^0 \phi$ are also consistent with experimental data and align with the FSIs predictions of P -wave component in Ref. [16]. The partial-wave components of $D^0 \rightarrow \omega \phi$, however, await precise measurement by future experiments.

D. Polarization fractions of $D \rightarrow VV$ decays

Based on the fitted parameters in Eq. (16), we calculated the polarization fractions f_L and $f_{\parallel}$ for $D \rightarrow VV$ decays, as

TABLE VIII. The polarization fractions of $D \rightarrow V_1 V_2$ decays in units of percentage.

Modes	Amplitudes	f_L	$f_{\parallel}$
$D^0 \rightarrow \rho^0 \phi$	$\frac{1}{\sqrt{2}}C$	48.57 ± 7.25	38.22 ± 7.04
$D^0 \rightarrow \omega \phi$	$\frac{1}{\sqrt{2}}C$	48.58 ± 7.25	38.14 ± 7.04
$D^+ \rightarrow \bar{K}^{*0} \rho^+$	$T + C$	60.68 ± 9.29	36.75 ± 9.23
$D^+ \rightarrow \rho^0 \rho^+$	$-\frac{1}{\sqrt{2}}(T + C)$	64.57 ± 9.59	31.35 ± 9.35
$D^+ \rightarrow \bar{K}^{*0} K^{*+}$	T	39.91 ± 6.18	47.18 ± 6.36
$D^+ \rightarrow \omega \rho^+$	$\frac{1}{\sqrt{2}}(T + C)$	60.08 ± 8.96	33.15 ± 8.47
$D^+ \rightarrow \rho^+ \phi$	C	48.71 ± 7.25	38.07 ± 7.03
$D^+ \rightarrow K^{*0} \rho^+$	C	53.33 ± 7.25	35.13 ± 6.88
$D^+ \rightarrow \rho^0 K^{*+}$	$\frac{1}{\sqrt{2}}T$	40.65 ± 6.02	39.69 ± 5.98
$D^+ \rightarrow \omega K^{*+}$	$\frac{1}{\sqrt{2}}T$	40.65 ± 6.02	39.61 ± 5.98
$D_s^+ \rightarrow \phi \rho^+$	T	47.67 ± 6.42	40.68 ± 6.26
$D_s^+ \rightarrow K^{*+} \bar{K}^{*0}$	C	55.70 ± 7.19	33.28 ± 6.74
$D_s^+ \rightarrow K^{*0} \rho^+$	T	48.67 ± 6.25	34.82 ± 5.78
$D_s^+ \rightarrow K^{*+} \rho^0$	$\frac{1}{\sqrt{2}}C$	61.35 ± 6.92	29.34 ± 6.37
$D_s^+ \rightarrow K^{*+} \omega$	$\frac{1}{\sqrt{2}}C$	60.97 ± 6.95	29.61 ± 6.40
$D_s^+ \rightarrow \phi K^{*+}$	$T + C$	59.79 ± 9.50	36.91 ± 9.42
$D_s^+ \rightarrow K^{*0} K^{*+}$	$T + C$	61.69 ± 9.90	33.83 ± 9.70
$D^0 \rightarrow K^{*-} \rho^+$	$T + E$	84.80 ± 5.76	4.39 ± 4.51
$D^0 \rightarrow \rho^- \rho^+$	$T + E$	83.19 ± 5.22	2.16 ± 2.90
$D^0 \rightarrow K^{*-} K^{*+}$	$T + E$	79.05 ± 7.16	7.35 ± 5.93
$D^0 \rightarrow \rho^- K^{*+}$	$T + E$	77.69 ± 6.33	3.75 ± 3.82
$D^0 \rightarrow \rho^0 \bar{K}^{*0}$	$\frac{1}{\sqrt{2}}(C - E)$	72.23 ± 10.23	12.57 ± 8.88
$D^0 \rightarrow \rho^0 \rho^0$	$\frac{1}{2}(E - C)$	77.43 ± 9.46	10.32 ± 8.13
$D^0 \rightarrow K^{*0} \rho^0$	$\frac{1}{\sqrt{2}}(C - E)$	72.23 ± 10.23	12.57 ± 8.88
$D^0 \rightarrow \omega \bar{K}^{*0}$	$\frac{1}{\sqrt{2}}(C + E)$	41.36 ± 7.98	51.82 ± 7.79
$D^0 \rightarrow \omega \omega$	$\frac{1}{2}(C + E)$	45.43 ± 8.32	48.87 ± 8.06
$D^0 \rightarrow K^{*0} \omega$	$\frac{1}{\sqrt{2}}(C + E)$	41.36 ± 7.98	51.82 ± 7.79
$D^0 \rightarrow \omega \rho^0$	E	38.16 ± 14.21	61.59 ± 14.18

listed in Table VIII. The values for f_L and $f_{\parallel}$, accompanied by their uncertainties summed in quadrature, appear in the table's last two columns, respectively. For clarity in the following discussion, we categorize the 28 decay modes into three groups: those governed exclusively by T , C , or $T + C$ diagrams; those dominated by $T + E$ contributions; and those proceeding through C and E diagrams.

According to Eq. (17), for the first category of decay modes mediated by C or $T + C$ diagrams, their polarization fractions follow the same hierarchy as the nonfactorizable parameters, i.e., $\chi_C^0 > \chi_C^{\parallel}$, which implies $f_L > f_{\parallel}$. In contrast, for modes proceeding only via the T diagram, the longitudinal polarization f_L is comparable to or smaller than $f_{\parallel}$, as observed in the $D^+ \rightarrow \bar{K}^{*0} K^{*+}$ decay, based on the naive factorization framework.

For the second category of decay modes, where interference occurs between the T and the E amplitudes, their polarization fractions are governed by the strong phase

differences between the corresponding helicity amplitudes, $T^{0,\parallel}$ and $E^{0,\parallel}$. According to Eq. (16), the strong phase difference between the T^0 and E^0 amplitudes is less than $\pi/2$, leading to constructive interference that enhances f_L . Conversely, the strong phase difference between the $T^{\parallel}$ and $E^{\parallel}$ amplitudes exceeds $\pi/2$, resulting in destructive interference that suppresses $f_{\parallel}$. Consequently, the significant interference between the nonfactorizable E diagram and the T diagram drives the longitudinal polarization f_L to approximately 80%.

Finally, we turn to the third category of modes involving interference between the C and E amplitudes. Given that the strong phase difference satisfy $\phi_E^0 - \phi_C^0 > \pi/2$ while $\phi_E^{\parallel} - \phi_C^{\parallel} < \pi/2$, we observe destructive interference in the longitudinal component and constructive interference in the transverse component for decays with $C + E$ contributions. Consequently, f_L is suppressed while $f_{\parallel}$ is enhanced, yielding $f_{\parallel} > f_L$. This hierarchy also holds for the $D^0 \rightarrow \omega \rho^0$ decay, which is governed exclusively by the E amplitude. In contrast, for modes dominated by $C - E$ interference, f_L is enhanced and $f_{\parallel}$ is suppressed, resulting in a dominant longitudinal polarization of $f_L \sim 70\%$. Notably, our prediction for the longitudinal polarization f_L in $D^0 \rightarrow \rho^0 \rho^0$ decay agrees well with the experimental measurement of $(71 \pm 4 \pm 2)\%$ reported by the FOCUS Collaboration [7]. In contrast, our result for $D^0 \rightarrow K^{*-} K^{*+}$ deviates from the experimental value of $f_L = 0.468 \pm 0.046 \pm 0.011$ [52] by 3.8σ . This discrepancy is expected to be resolved by incorporating the nonfactorizable contribution into the T amplitude.

E. $D^0 - \bar{D}^0$ mixing parameter y in $D \rightarrow VV$ decays

$D^0 - \bar{D}^0$ mixing is conventionally characterized by the parameters $x \equiv \Delta m/\Gamma$ and $y \equiv \Delta\Gamma/2\Gamma$, where $\Delta m = m_1 - m_2$ and $\Delta\Gamma = \Gamma_1 - \Gamma_2$ denote the mass and width differences of the mass eigenstates $D_{1,2}$, respectively, and $\Gamma = (\Gamma_1 + \Gamma_2)/2$ is the average width. Assuming CP conservation, the mass eigenstates $D_{1,2}$ coincide with the CP eigenstates $|D_{\pm}\rangle = (|D^0\rangle \pm |\bar{D}^0\rangle)/\sqrt{2}$. Then, the parameter y can be formulated as [29]

$$y = \frac{1}{\Gamma} \sum_n \eta_{CP}(n) \rho_n \text{Re}[A(D^0 \rightarrow n) A^*(D^0 \rightarrow \bar{n})] \quad (19)$$

or as [17]

$$y = \sum_n \eta_{CKM}(n) \eta_{CP}(n) \cos \delta_n \sqrt{B(D^0 \rightarrow n) B(D^0 \rightarrow \bar{n})}, \quad (20)$$

where ρ_n is the phase-space factor for the decay into the final state n . For $D \rightarrow VV$ decays, the CP eigenvalue is given by $\eta_{CP}(n) = (-1)^L$, where L represents the relative

orbital angular momentum between the two vector mesons. The CKM factor is defined as $\eta_{\text{CKM}} = (-1)^{n_s}$, where n_s denotes the number of s and $\bar{s}$ quarks in the final state. The parameter δ_n corresponds to the strong phase difference between the $D^0 \rightarrow n$ and $\bar{D}^0 \rightarrow n$ amplitudes.

Using the fitted parameters given in Eq. (16), we calculate the mixing parameter y for $D \rightarrow VV$ decays via Eq. (20). The results for the individual S -, P -, and D -wave contributions, denoted as $y_{S,P,D}$, yield

$$\begin{aligned} y_{VV,S} &= (1.08 \pm 0.09)\%, \\ y_{VV,P} &= (-0.22 \pm 0.02)\%, \\ y_{VV,D} &= (0.55 \pm 0.08)\%, \end{aligned} \quad (21)$$

where the uncertainties are obtained by summing those from the nonfactorizable parameters, form factors, and decay constants in quadrature. Compared to the naive factorization predictions in Ref. [17], our results for $y_{VV,S,P}$ and $y_{VV,D}$ in Eq. (21) are larger by one and 2 orders of magnitude, respectively. Notably, the predictions for $y_{VV,S,P,D}$ in Ref. [17] are substantially modified when experimental data for measured modes are included, alongside naive factorization predictions for the unmeasured ones. In this case, their values of $y_{VV,P} = -0.0958\%$ and $y_{VV,D} = 0.361\%$ are in much better agreement with our results.

Regarding the longitudinal component y_L , Ref. [29] evaluated it using the nonfactorizable parameters χ_V^C, ϕ_V^C and $\chi_{q(s)}^E, \chi_{q(s)}^E$ fitted to data of $D \rightarrow PV$ decays as an approximation. Their result, $y_{VV,L} = (-0.42 \pm 0.34) \times 10^{-3}$, is significantly smaller than our value of $y_{VV,L} = (1.21 \pm 0.12)\%$, primarily due to the different nonfactorizable parameters employed.

IV. CONCLUSION

Within the FAT framework, we perform a systematical analysis of $D \rightarrow VV$ decays. Four types of topological diagrams, T , C , E , and A , contribute to these decays through weak interactions. The T amplitude is calculated using naive factorization. However, the nonfactorizable C and E amplitudes are described by only two sets of universal parameters, $\chi_C^{0,\parallel,\perp}, \phi_C^{0,\parallel,\perp}$ and $\chi_E^{0,\parallel}, \phi_E^{0,\parallel}$, respectively, for all $D \rightarrow VV$ modes after form factors and decay constants are factored out. This demonstrates that SU(3)-breaking effects are naturally incorporated in the FAT approach, which keeps the number of free parameters minimal. Specifically, these parameters characterizing the

nonfactorizable contributions of the C and E amplitudes, together with the factorization scale μ of the Wilson coefficient $a_1(\mu)$ for T amplitude, constitute 11 free parameters fitted globally to 36 experimental data points. The obtained nonfactorizable parameters are determined with high precision, except for $\phi_E^\parallel$.

The fitted parameters yield topological-amplitude hierarchies of $|C^0| > |T^0| > |E^0|$ and $|T^{\parallel(\perp)}| \sim |C^{\parallel(\perp)}| > |E^{\parallel(\perp)}|$, indicating that the nonfactorizable C contribution is comparable to the factorizable T contributions. Polarization amplitudes follow $|\mathcal{A}^0| > |\mathcal{A}^\parallel| > |\mathcal{A}^\perp|$ for the T and C amplitudes, contrary to the naive factorization prediction. Thus, the longitudinal polarization f_L dominates in modes governed by the T or C amplitudes. However, in decays with T and E , or C and E interference, strong phases $\phi_{E,C}^{0,\parallel}$ can possibly resulting in $f_L < f_\parallel$. Finally, the partial-wave hierarchy $|S| < |D|$ yields a dominant D -wave branching fraction in E diagram mediated decays, contrary to the naive S -wave dominance prediction.

With the fitted nonfactorizable parameters, we predict the branching fractions and polarization fractions for 28 decay modes of $D \rightarrow VV$. Our results are in agreement with the current experimental data within uncertainties. For modes involving the $C-E$ amplitudes, such as $D^0 \rightarrow \rho^0 \bar{K}^{*0}$, and modes governed by E , like $D^0 \rightarrow \omega \rho^0$, the S -wave branching fractions are slightly smaller than those of the D wave. This is contrary to naive factorization estimations but agrees with current observations. The polarization fractions f_L of $D^0 \rightarrow \rho^0 \rho^0$ is in well agreement with the experimental data. Predictions for unobserved decay modes await for test by upcoming experimental studies, especially those with branching fractions on the order of 10^{-3} – 10^{-2} , the D -wave-dominated modes, and modes exhibiting $f_\parallel > f_L$. We also calculate the $D^0 - \bar{D}^0$ mixing parameters $y_{S,P,D}$ and y_L , which are of the order of 10^{-3} – 10^{-2} .

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DATA AVAILABILITY

The data that support the findings of this article are openly available [40].

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