

$D^0 - \bar{D}^0$ mixing in the Dyson-Schwinger approachXiaotong Xie^{✉,*}, Hiroyuki Umeeda^{✉,†}, and Jinglong Zhu^{✉,‡}*Center for Theoretical Physics and College of Physics, Jilin University, Changchun, 130012, China*

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In view of difficulty to reproduce observables in the $D^0 - \bar{D}^0$ mixing via the operator product expansion, we discuss the Dyson-Schwinger approach to this process. Formulated by the parametrization of quark propagators, SU(3) breaking relevant to charm mixing is evaluated in such a way that properly takes account of dynamical chiral symmetry breaking. The $\bar{D}^0 \rightarrow D^0$ transition is discussed in the vacuum-insertion approximation with locality of the light valence-quark field, represented by the decay constant of D^0 meson as well as relevant momentum integrals. It is found that dimensionless mass-difference observable in this approach leads to $|x| = (1.3 - 2.9) \times 10^{-3}$, the order of magnitude comparable to the HFLAV data, and thereby offering a certain improvement as a theoretical framework.

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Owing to the notable development of heavy quark physics in the 1990s, strong interaction of the b quark system can be systematically handled, which enables us to analyze a variety of observables to date. Armed with those formulations, precise determinations of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements [1], as well as the search for physics beyond the standard model, have been implemented in recent flavor-factory experiments. In contrast to the mature status in B -meson system, however, there still exist difficulties in regards to the charm sector. In particular, dominance of long-distance effects for the $D^0 - \bar{D}^0$ mixing has been remarked [2], and therefore requiring dedicated analysis incorporating nonperturbative dynamics.

The conventional theoretical methods for the $D^0 - \bar{D}^0$ mixing are classified as two types: The exclusive and inclusive approaches. The former offers parametrization of decay amplitudes, with specific methods given by topological diagram approach [3,4], factorization-assisted topological approach [5] or flavor symmetry approach [6] and yields the data-driven formalism to predicts the observables in the $D^0 - \bar{D}^0$ mixing. This methodology takes finite sum over hadronic intermediate states of $\bar{D}^0 \rightarrow \text{hadrons} \rightarrow D^0$, reproducing approximately a half of the experimental data relevant to width difference [3–5].

As for the inclusive approach¹ [18,19] (a recent work is found in Ref. [20]), the primary formulation is based on the operator product expansion (OPE) [21]. In a sharp contrast to the mentioned status in the exclusive approach, the inclusive one gives predictions that are much smaller than the experimental data: If we take a dimensionless mass-difference observable as an example, the comparison between the OPE value at the next-to-leading order (NLO) in Ref. [18] and the experimental result from the heavy flavor averaging group (HFLAV) [22] reads,

$$x_{\text{Th.}} = 6 \times 10^{-7}, \quad x_{\text{Exp.}} = (4.07 \pm 0.44) \times 10^{-3}, \quad (1)$$

where the definition of x is properly introduced later in Sec. II A. As shown in Eq. (1), there exists a gap between the two results in four orders of magnitude.

The difficulty in reproducing the data for the inclusive approach is attributed to the extreme suppression that originates from the Glashow-Iliopoulos-Maiani (GIM) mechanism [23]. As a consequence of this, in the limit where the product of the CKM matrix elements ($V_{cb}V_{ub}^*$) vanishes, the observables in the $D^0 - \bar{D}^0$ mixing are proportional to SU(3) breaking as was remarked in Ref. [24]. In order to capture the relevant size of the observables, proper evaluation of *hadronic* (or nonperturbative) SU(3) breaking plays a crucial role. See Refs. [15,19,25,26] for further discussions.

¹See earlier discussions based on box diagrams in Refs. [7–10] and the ones in heavy quark effective theory (HQET) in Refs. [11,12]. The dispersive approach based on the inclusive analysis as input is found in Refs. [13,14]. Violation of quark-hadron duality is discussed in Refs. [15–17] in a generic context, the simplified model and the 't Hooft model, respectively.

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For the $c \rightarrow u\gamma$ decay, the difficulty similar to the $D^0 - \bar{D}^0$ mixing is also found: In Ref. [27], branching ratio of inclusive radiative decay, $\Delta\text{Br}[D^0 \xrightarrow{\Delta S=0} X\gamma]_{c \rightarrow u\gamma}$, is obtained as 2.5×10^{-8} , much smaller than the particle data group (PDG) value of one exclusive channel, $\text{Br}[D^0 \rightarrow \rho^0\gamma] = (1.82 \pm 0.32) \times 10^{-5}$ [28]. For this, robust enhancement of the result is confirmed [27], if one replaces the current-quark masses for s and d quarks by ones of the constituent-quark masses. Even though the process is different, a similar enhancement in a larger magnitude is expected for charm mixing. Furthermore, for the $D^0 - \bar{D}^0$ mixing in the OPE-based approaches, the condensate contributions [15], as well as nonlocal chiral condensates [20], are discussed as power-suppressed terms that can avoid severe GIM suppression, and thereby leading to large corrections. Those previous works generically imply the importance of chiral symmetry breaking in charm mixing.

In this work, we evaluate an observable in the $D^0 - \bar{D}^0$ mixing based on the Dyson-Schwinger equation (DSE) approach [29,30]. Formulated by the propagators for intermediate d and s quarks, the dimensionless mass-difference parameter is analyzed, with SU(3) breaking properly incorporated. To this end, the parametrization [31–35] (see Refs. [36,37] for recent works) accommodating confinement and dynamical chiral symmetry breaking (DCSB) as well as asymptotic freedom is adopted. This framework enables us to estimate the size of observables with nonperturbative SU(3) breaking via the relevant momentum integrals of the quark propagators.

This paper is organized as follows: In Sec. II A, theoretical formulas for the $D^0 - \bar{D}^0$ mixing, as well as definitions of mixing parameters, are introduced. In Sec. II B, the OPE approach to the $D^0 - \bar{D}^0$ mixing is briefly recapitulated. In Sec. II C, SU(3) breaking that originates from the GIM mechanism is discussed with some replacements of quark masses. In Sec. III, the mixing parameter is estimated in the DSE approach, giving numerical results in this work. The concluding remarks are addressed in Sec. IV.

II. $D^0 - \bar{D}^0$ MIXING

A. Mixing parameters

Here formulas for charm mixing are given in the CPT -conserving limit [22]. Taking the phase convention of CP transform as $\text{CP}|D^0\rangle = -|\bar{D}^0\rangle$, we introduce mass eigenstates,

$$|D_{1,2}\rangle = p|D^0\rangle \mp q|\bar{D}^0\rangle, \quad (2)$$

where $|p|^2 + |q|^2 = 1$. In the CP -conserving case of $p = q$, $D_1(D_2)$ is reduced to a CP -even (CP -odd) state. The matrix element for the $\bar{D}^0 \rightarrow D^0$ transition is divided into the dispersive and absorptive parts,

$$M_{12} - \frac{i}{2}\Gamma_{12} = \frac{\langle D^0 | \mathcal{H}_{\text{eff}} | \bar{D}^0 \rangle}{2M_{D^0}}, \quad \mathcal{H}_{\text{eff}} = \mathcal{H}_{\text{disp}} - \frac{i}{2}\mathcal{H}_{\text{abs}}, \quad (3)$$

or equivalently,

$$M_{12} = \frac{\langle D^0 | \mathcal{H}_{\text{disp}} | \bar{D}^0 \rangle}{2M_{D^0}}, \quad \Gamma_{12} = \frac{\langle D^0 | \mathcal{H}_{\text{abs}} | \bar{D}^0 \rangle}{2M_{D^0}}. \quad (4)$$

In our convention, D^0 and $\bar{D}^0$ states are relativistically normalized,

$$\langle D^0(\mathbf{p}) | D^0(\mathbf{p}') \rangle = \langle \bar{D}^0(\mathbf{p}) | \bar{D}^0(\mathbf{p}') \rangle = 2E_{\mathbf{p}}(2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{p}'). \quad (5)$$

The observables are defined by,

$$x = \frac{\Delta m}{\Gamma}, \quad y = \frac{\Delta \Gamma}{2\Gamma}, \quad (6)$$

$$\Delta m = m_1 - m_2, \quad \Delta \Gamma = \Gamma_1 - \Gamma_2, \quad \Gamma = \frac{\Gamma_1 + \Gamma_2}{2}. \quad (7)$$

In what follows, we take the CP -conserving limit, leading to,

$$M_{12} = \text{Re} \frac{\langle D^0 | \mathcal{H}_{\text{eff}} | \bar{D}^0 \rangle}{2M_{D^0}}, \quad \Gamma_{12} = -2\text{Im} \frac{\langle D^0 | \mathcal{H}_{\text{eff}} | \bar{D}^0 \rangle}{2M_{D^0}}, \quad (8)$$

$$\Delta m = 2M_{12}, \quad \Delta \Gamma = 2\Gamma_{12}. \quad (9)$$

The effective Hamiltonian for the $\Delta C = 1$ transition is defined by,

$$\mathcal{H}_{\text{eff}} = \frac{(-i)^2}{2!} i \int \text{T}[\mathcal{H}_{\Delta C=1}(x) \mathcal{H}_{\Delta C=1}(0)] d^4x. \quad (10)$$

In the case where CKM-suppressed contributions of b quark are neglected, the Hamiltonian reads,

$$\mathcal{H}_{\Delta C=1} = \sum_{i=d}^s \sum_{j=d}^s \mathcal{H}_{ij}, \quad (11)$$

$$\mathcal{H}_{ij} = \frac{4G_F}{\sqrt{2}} V_{ci} V_{uj}^* (C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2), \quad (12)$$

$$\mathcal{O}_1 = (\bar{c}^\alpha \gamma^\mu P_L q_i^\beta) (\bar{q}_j^\beta \gamma_\mu P_L u^\alpha), \quad (13)$$

$$\mathcal{O}_2 = (\bar{c}^\alpha \gamma^\mu P_L q_i^\alpha) (\bar{q}_j^\beta \gamma_\mu P_L u^\beta). \quad (14)$$

The sum over color indices denoted as α and β are understood in Eqs. (13) and (14). In what follows, the dispersive part, which is of our current interest, is analyzed.

B. OPE approach

We recapitulate the conventional inclusive approach [7–10,18,19] at the leading order (LO). One can implement

the replacement,

$$\mathcal{H}_{\Delta C=1}(x)\mathcal{H}_{\Delta C=1}(0) \rightarrow \sum_{i,j}^{d,s} \mathcal{H}_{ij}(x)\mathcal{H}_{ji}(0). \quad (15)$$

Furthermore, with $\lambda_i = V_{ci}V_{ui}^*$, the unitarity of the CKM matrix leads to,

$$\lambda_d + \lambda_s + \lambda_b = 0. \quad (16)$$

In the limit of $\lambda_b \simeq 0$, one can eliminate λ_d and write the result only in terms of λ_s . This procedure renders Eq. (10) recast into the form (see, e.g., Ref. [18]),

$$\mathcal{H}_{\text{disp}} \simeq -\frac{G_F^2 m_c^2}{8\pi^2} \lambda_s^2 [C_2^2 Q_1 + 2(C_2^2 - 2C_1 C_2 - N_c C_1^2) Q_2] \times \left(\frac{m_s^2 - m_d^2}{m_c^2} \right)^2, \quad (17)$$

$$Q_1 = [\bar{c}^\alpha \gamma^\mu (1 - \gamma^5) u^\alpha] [\bar{c}^\beta \gamma_\mu (1 - \gamma^5) u^\beta], \quad (18)$$

$$Q_2 = [\bar{c}^\alpha (1 - \gamma^5) u^\alpha] [\bar{c}^\beta (1 - \gamma^5) u^\beta], \quad (19)$$

where Q_i ($i = 1, 2$) is the $\Delta C = 2$ operator. The contribution of Q_2 in Eq. (17) is obtained by applying the equation of motion (see e.g., Ref. [17]). The matrix elements for Eqs. (18) and (19) are defined with bag parameters,

$$\langle D^0 | Q_1 | \bar{D}^0 \rangle = \frac{8}{3} M_{D^0}^2 f_{D^0}^2 B_1, \quad (20)$$

$$\langle D^0 | Q_2 | \bar{D}^0 \rangle = -\frac{5}{3} \left(\frac{M_{D^0}}{m_c + m_u} \right)^2 M_{D^0}^2 f_{D^0}^2 B_2, \quad (21)$$

leading to ($N_c = 3$),

$$M_{12} \simeq -\frac{G_F^2 f_{D^0}^2 M_{D^0}^2 m_c^2}{6\pi^2} \lambda_s^2 \left[C_2^2 B_1 - \frac{5}{4} (C_2^2 - 2C_1 C_2 - N_c C_1^2) \times \left(\frac{M_{D^0}}{m_c + m_u} \right)^2 B_2 \right] \left(\frac{m_s^2 - m_d^2}{m_c^2} \right)^2. \quad (22)$$

It is evident that M_{12} vanishes in the SU(3) limit, $m_s = m_d$, as a consequence of the GIM mechanism with $\lambda_b = 0$.

C. GIM suppression factor

One can estimate the size of the last factor in Eq. (22): If one fixes masses for s , d and c to the $\overline{\text{MS}}$ masses at the scale of charm quark mass, where renormalization group evolution (RGE) for m_s and m_d are evaluated by RunDec [38] with input parameters of $m_c(m_c)$, $m_d(2 \text{ GeV})$ and $m_s(2 \text{ GeV})$ in Ref. [28], we obtain,

$$\left(\frac{m_s^2 - m_d^2}{m_c^2} \right)^2 = 5.2 \times 10^{-5}, \quad (23)$$

giving the extreme suppression in Eq. (22).

Here we consider the replacement of the current-quark masses by the constituent-quark masses similarly to Ref. [27] for $c \rightarrow u\gamma$ decays. Specifically, the constituent-quark mass in the relativistic constituent quark model (RCQM) [39] and the Euclidean constituent-quark mass, obtained as a solution of $M^2(p_E^2) = p_E^2$ with $M(p_E^2)$ determined in the DSE approach [35], are adopted for illustrating the size of the factor. If we take $m_s^{\text{con}} = 0.49 \text{ GeV}$ and $m_d^{\text{con}} = 0.36 \text{ GeV}$ [35] as an example, the factor corresponding to Eq. (23) is,

$$\left[\frac{(m_s^{\text{con}})^2 - (m_d^{\text{con}})^2}{m_c^2} \right]^2 = 4.6 \times 10^{-3}, \quad (24)$$

which is two orders of magnitude larger than Eq. (23). The resulting observable is also enhanced by the mentioned factor.

In order to obtain the numerical result, we adopt $M_{D^0} = 1.86484 \text{ GeV}$ in PDG [28] and $f_{D^0} = 0.2120 \text{ GeV}$ in

TABLE I. The dimensionless mass-difference observable ($|x|$) with different values of quark-mass parameters: (a) $\overline{\text{MS}}$ mass. (b), (c), and (f) RCQM with the harmonic oscillator wave function. (d) constituent-quark mass adopted in $c \rightarrow u\gamma$ study. (e) and (g) the Euclidean constituent-quark mass in the DSE approach. For the constituent-quark masses, the isospin limit, $m_u^{\text{con}} = m_d^{\text{con}}$, is adopted.

	$ x $	$m_s^{(\text{con})}/\text{GeV}$	$m_d^{(\text{con})}/\text{GeV}$	Method/Model	References
(a)	2.5×10^{-6}	0.108	0.0055	$\overline{\text{MS}}$ mass	[28,38]
(b)	1.1×10^{-4}	0.38	0.26	RCQM	[39]
(c)	1.5×10^{-4}	0.40	0.267	RCQM	[39]
(d)	2.3×10^{-4}	0.45	0.30	Constituent-quark mass	[27]
(e)	2.3×10^{-4}	0.49	0.36	DSE	[35]
(f)	6.9×10^{-4}	0.55	0.33	RCQM	[39]
(g)	5.8×10^{-4}	0.70	0.56	DSE	[35]

lattice QCD [40] (see also [41]). As to the CKM matrix elements in the CP -conserving limit, we adopt the absolute values in PDG, $|V_{cs}| = 0.975$ and $|V_{us}| = 0.22431$ [28] for λ_s^2 in Eq. (22). For the bag parameters, they are computed via lattice QCD in Refs. [42–45], with central values of Ref. [44] in Eqs. (A1)–(A3) adopted for our analysis. The corrections from RGE to the bag parameters and the Wilson coefficients are given in the Appendix. The u quark mass in Eq. (22) is fixed to the $\overline{\text{MS}}$ mass at the scale of m_c via the RunDec [38].

In Table I, values of $|x|$ with different masses for s and d quarks are compared. As can be seen in Table I, the cases with constituent-quark masses give larger $|x|$ than that with the $\overline{\text{MS}}$ masses. The resulting enhancement is highly sensitive to the values of m_s^{con} and m_d^{con} .

It should be noted, however, that momentum dependence of the quark propagators is not fully incorporated in the simple replacement for the quark masses in Eq. (24) and also in the numerical results of Table I (b)–(g). In the next section, the integral over the relevant momenta is carried out using the DSE approach.

III. DSE APPROACH

A. Quark propagator

The quark propagator in the Minkowski notation is denoted as,

$$S_i(x, y) = \int \frac{d^4 p}{(2\pi)^4} i[p\sigma_V^i(-p^2) + \sigma_S^i(-p^2)]e^{-ip \cdot (x-y)}. \quad (25)$$

($i = d, s$)

In Eq. (25), $\sigma_V^i(-p^2)$ and $\sigma_S^i(-p^2)$ represent the vector and the scalar parts, respectively, with the sign of the argument taken as negative for the later convenience of the Euclidean notation. In Refs. [31–35], the parametrization of the $\sigma_{V,S}$ has been discussed. One can incorporate DCSB, which is not explicitly seen for the OPE at the leading power approximation, in this method. Color confinement is also accommodated in the sense that the parametrization does not possess the Lehmann representation [31]. The explicit forms for $\sigma_{V,S}$ are given by [31–35],

$$\sigma_V^i(p_E^2) = \frac{\bar{\sigma}_V^i(x)}{2D}, \quad \sigma_S^i(p_E^2) = \frac{\bar{\sigma}_S^i(x)}{\sqrt{2D}}, \quad (26)$$

$$\bar{\sigma}_V^i(x) = \frac{2(x + \bar{m}_i^2) - 1 + e^{-2(x + \bar{m}_i^2)}}{2(x + \bar{m}_i^2)^2}, \quad (27)$$

$$\begin{aligned} \bar{\sigma}_S^i(x) = & 2\bar{m}_i \mathcal{F}(2(x + \bar{m}_i^2)) \\ & + \mathcal{F}(b_1^i x) \mathcal{F}(b_3^i x) [b_0^i + b_2^i \mathcal{F}(ex)], \end{aligned} \quad (28)$$

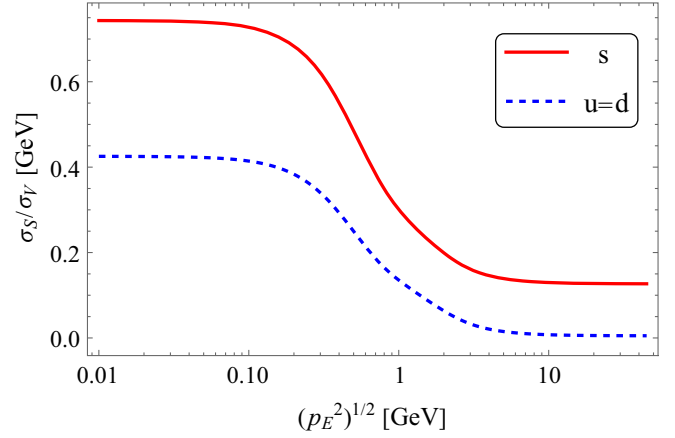


FIG. 1. Momentum dependence of σ_S/σ_V in the DSE approach based on the previous work [34].

where $p_E^2 = -p^2$, $x = p_E^2/(2D)$, $\bar{m}_i = \bar{m}_i/\sqrt{2D}$, and $\mathcal{F}(z) = (1 - e^{-z})/z$ while b_n^i ($n = 0, 1, 2, 3$), D and ϵ are parameters determined in the previous work [34].

In order to illustrate typical momentum dependence of the quark propagators, σ_S/σ_V , corresponding to the quark mass function, is plotted in Fig. 1, clearly showing DCSB. The parameters in Eqs. (26)–(28) are fixed by [34] $D = 0.160$ GeV, $\epsilon = 10^{-4}$ and,

$$\begin{aligned} \bar{m}_u = & 0.00897 \text{ GeV}, \quad b_0^u = 0.131, \\ b_1^u = & 2.90, \quad b_2^u = 0.603, \quad b_3^u = 0.185, \end{aligned} \quad (29)$$

$$\begin{aligned} \bar{m}_s = & 0.224 \text{ GeV}, \quad b_0^s = 0.105, \\ b_1^s = & 2.90, \quad b_2^s = 0.740, \quad b_3^s = 0.185. \end{aligned} \quad (30)$$

We take the isospin limit for Eq. (29), i.e., the equality of the parameters for u and d quarks.

B. Dispersive part of $\bar{D}^0 \rightarrow D^0$ transition

By implementing the Fierz transform, the $\Delta C = 1$ operator in Eq. (13) is rewritten by,

$$O_1 = (\bar{c}^\alpha \gamma^\mu P_L u^\alpha) (\bar{q}_j^\beta \gamma_\mu P_L q_i^\beta). \quad (31)$$

Hereafter, the operator basis in Eqs. (31) and (14) are used for analyzing the $D^0 - \bar{D}^0$ mixing. In this case, the color flow and the flavor flow are identical in a given diagram.

Contributions to the dispersive Hamiltonian are divided into four parts from the $\Delta C = 1$ operators (sum over $i, j = d, s$ is not taken),

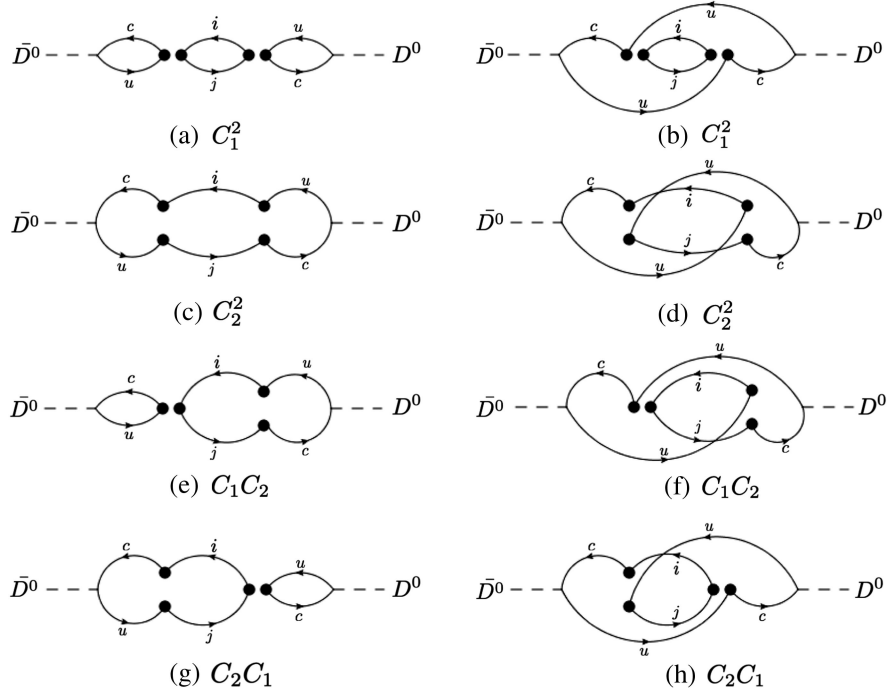


FIG. 2. Diagrams for $\bar{D}^0 \rightarrow D^0$ transition in the DSE approach with $i, j = d, s$. The black dots denote the Dirac matrix of the form, $\gamma^\mu P_L$. The fermion lines represent dressed quark propagators with the arrows denoting flavor and color flows. The similar diagrams where the roles of the left and right vertices are interchanged are not displayed.

$$M_{12}^{ij} = \frac{2iG_F^2}{M_D} \lambda_i \lambda_j \int d^4x \text{tr}[X_{\mu\nu}^{ij}(x)] \times \{ N_c C_1^2 \langle D^0 | T[\bar{c}^\alpha(x) \gamma^\mu P_L u^\alpha(x)] [\bar{c}^\beta(0) \gamma^\nu P_L u^\beta(0)] | \bar{D}^0 \rangle + C_2^2 \langle D^0 | T[\bar{c}^\alpha(x) \gamma^\mu P_L u^\beta(x)] [\bar{c}^\beta(0) \gamma^\nu P_L u^\alpha(0)] | \bar{D}^0 \rangle + C_1 C_2 \langle D^0 | T[\bar{c}^\alpha(x) \gamma^\mu P_L u^\alpha(x)] [\bar{c}^\beta(0) \gamma^\nu P_L u^\beta(0)] | \bar{D}^0 \rangle + C_2 C_1 \langle D^0 | T[\bar{c}^\alpha(x) \gamma^\mu P_L u^\alpha(x)] [\bar{c}^\beta(0) \gamma^\nu P_L u^\beta(0)] | \bar{D}^0 \rangle \}. \quad (32)$$

As shown above, the terms proportional to $C_1 C_2$ and $C_2 C_1$ yield identical contributions. In Eq. (32), the intermediate flavor dependence, represented by the indices of i and j , is included only in $\lambda_i \lambda_j$ and $X_{\mu\nu}^{ij}$ defined by,

$$X_{\mu\nu}^{ij}(x) = S_i(0, x) \gamma_\mu P_L S_j(x, 0) \gamma_\nu P_L. \quad (33)$$

In Eq. (32), the trace denoted as $\text{tr}[X_{\mu\nu}^{ij}(x)]$ runs over only the Dirac indices, not for color or flavor indices. It should be noted that σ_γ in Eq. (25) does not contribute to $\text{tr}[X_{\mu\nu}^{ij}(x)]$ due to its chirality structure, similarly to the conventional OPE analysis at LO.

In order to estimate Eq. (32), one inserts vacua in between the two currents for the matrix element, corresponding to the vacuum-insertion approximation (VIA). It is known [46] that this approximation works in the large N_c limit since gluon exchanges in between currents are suppressed in that limit while radiative corrections within one

current are included via decay constant. Since this procedure does not affect the primary SU(3) breaking, which we aim to evaluate in the present work, VIA can capture the order of magnitude of the observable as we shall see later. When inserting vacua, care must be taken for multiple ways of contracting fermion fields in the matrix element to valence quarks in the D -meson states, leading to

$$\begin{aligned} & \langle D^0 | T[\bar{c}^\alpha(x) \gamma^\mu P_L u^\alpha(x)] [\bar{c}^\beta(0) \gamma^\nu P_L u^\beta(0)] | \bar{D}^0 \rangle \\ & \simeq \langle D^0 | \bar{c}^\alpha(x) \gamma^\mu P_L u^\alpha(x) | 0 \rangle \langle 0 | \bar{c}^\beta(0) \gamma^\nu P_L u^\beta(0) | \bar{D}^0 \rangle \\ & + \langle D^0 | \bar{c}^\beta(0) \gamma^\nu P_L u^\beta(0) | 0 \rangle \langle 0 | \bar{c}^\alpha(x) \gamma^\mu P_L u^\alpha(x) | \bar{D}^0 \rangle \\ & - \langle D^0 | \bar{c}_a^\alpha(x) u_d^\beta(0) | 0 \rangle \langle 0 | \bar{c}_c^\beta(0) u_b^\alpha(x) | \bar{D}^0 \rangle (\gamma^\mu P_L)_{ab} (\gamma^\nu P_L)_{cd} \\ & - \langle D^0 | \bar{c}_c^\beta(0) u_b^\alpha(x) | 0 \rangle \langle 0 | \bar{c}_a^\alpha(x) u_d^\beta(0) | \bar{D}^0 \rangle (\gamma^\mu P_L)_{ab} (\gamma^\nu P_L)_{cd}, \end{aligned} \quad (34)$$

where a, b, c , and d represent the Dirac indices. For the third and fourth terms in Eq. (34), one can rearrange the color indices,

$$[\bar{c}^\alpha u^\beta] [\bar{c}^\beta u^\alpha] = \frac{1}{N_c} [\bar{c}^\alpha u^\alpha] [\bar{c}^\beta u^\beta] + 2 [\bar{c}^\alpha T_{\alpha\beta}^a u^\beta] [\bar{c}^\gamma T_{\gamma\delta}^a u^\delta], \quad (35)$$

which follows from $T_{\alpha\beta}^a T_{\gamma\delta}^a = (\delta_{\alpha\delta} \delta_{\gamma\beta} - \delta_{\alpha\beta} \delta_{\gamma\delta})/N_c$. Since the Bethe-Salpeter amplitude is diagonal in color indices, $\langle D^0 | u^\alpha \bar{c}^\beta | 0 \rangle \propto \delta^{\alpha\beta}$ (see, e.g., Ref. [47]), the matrix

element of the second term in Eq. (35) vanishes due to $\text{tr}(T^a) = 0$. Furthermore, for the third and fourth terms in Eq. (34), one can implement the Fierz transform,

$$(\gamma^\mu P_L)_{ab}(\gamma^\nu P_L)_{cd} = \frac{1}{2}(\gamma^\mu \gamma^\rho \gamma^\nu P_L)_{ad}(\gamma_\rho P_L)_{cb}. \quad (36)$$

By noting that parity must be preserved so that the matrix element of the form, $\langle D^0 | \bar{c} \gamma^\mu u | 0 \rangle$, vanishes, we rewrite Eq. (34),

$$\begin{aligned} & \langle D^0 | T[\bar{c}^\alpha(x) \gamma^\mu P_L u^\alpha(x)] [\bar{c}^\beta(0) \gamma^\nu P_L u^\beta(0)] | \bar{D}^0 \rangle \\ & \simeq \frac{1}{4} \langle D^0 | \bar{c}^\alpha(x) \gamma^\mu \gamma^5 u^\alpha(x) | 0 \rangle \langle 0 | \bar{c}^\beta(0) \gamma^\nu \gamma^5 u^\beta(0) | \bar{D}^0 \rangle \\ & + \frac{1}{4} \langle D^0 | \bar{c}^\beta(0) \gamma^\nu \gamma^5 u^\beta(0) | 0 \rangle \langle 0 | \bar{c}^\alpha(x) \gamma^\mu \gamma^5 u^\alpha(x) | \bar{D}^0 \rangle \\ & - \frac{1}{2N_c} \langle D^0 | \bar{c}^\alpha(x) \gamma^\mu \gamma^\rho \gamma^\nu P_L u^\alpha(0) | 0 \rangle \langle 0 | \bar{c}^\beta(0) \gamma_\rho P_L u^\beta(x) | \bar{D}^0 \rangle \\ & - \frac{1}{2N_c} \langle D^0 | \bar{c}^\alpha(0) \gamma_\rho P_L u^\alpha(x) | 0 \rangle \langle 0 | \bar{c}^\beta(x) \gamma^\mu \gamma^\rho \gamma^\nu P_L u^\beta(0) | \bar{D}^0 \rangle. \end{aligned} \quad (37)$$

Formulas similar to Eq. (37) are found for the terms proportional to C_2^2 , $C_1 C_2$, and $C_2 C_1$ in Eq. (32) due to the analogous procedure.

By using the decomposition in Eq. (37), one finds that the Hamiltonian in Eq. (10) includes diagrams in Fig. 2. As shown in the figure, there are eight types of diagrams, corresponding to terms proportional to C_1^2 , C_2^2 , $C_1 C_2$, and $C_2 C_1$, each of which has two types of different contractions, explicitly displayed in Fig. 2. N_c dependence of each diagram can be read off by counting the number of color traces. In addition to those diagrams, there are further eight diagrams, where the roles of the two $\Delta C = 1$ vertices are simply interchanged for each of A-H, which we denote $A' - H'$.

By evaluating the left-side of the diagrams plus corresponding primed ones in Fig. 2, partial contributions to M_{12} are,

$$M_{12}^{ij}|_{A+A'} = N_c C_1^2 N_I^{ij}, \quad M_{12}^{ij}|_{C+C'} = \frac{C_1^2}{N_c} N_I^{ij}, \quad (38)$$

$$M_{12}^{ij}|_{E+E'} = C_1 C_2 N_I^{ij}, \quad M_{12}^{ij}|_{G+G'} = C_2 C_1 N_I^{ij}. \quad (39)$$

where we introduced,

$$\begin{aligned} N_I^{ij} &= \frac{i G_F^2}{2 M_D} \lambda_i \lambda_j \int d^4 x \text{tr}[X_{\mu\nu}^{ij}(x)] [\langle D^0 | \bar{c}^\alpha(x) \gamma^\mu \gamma^5 u^\alpha(x) | 0 \rangle \\ & \times \langle 0 | \bar{c}^\beta(0) \gamma^\nu \gamma^5 u^\beta(0) | \bar{D}^0 \rangle + (x \leftrightarrow 0)]. \end{aligned} \quad (40)$$

In the above relation, the term denoted as $(x \leftrightarrow 0)$ represents a primed diagram. The matrix elements in Eq. (40) are written by the decay constant,

$$\begin{aligned} & \langle D^0 | \bar{c}^\alpha(x) \gamma^\mu \gamma^5 u^\alpha(x) | 0 \rangle \langle 0 | \bar{c}^\beta(0) \gamma^\nu \gamma^5 u^\beta(0) | \bar{D}^0 \rangle \\ & = f_{D^0}^2 p^\mu p^\nu e^{ip \cdot x}. \end{aligned} \quad (41)$$

In Eq. (41), p is the momentum of the D mesons satisfying the on-shell condition, $p^2 = M_{D^0}^2$.

One can similarly evaluate the right-side of the diagrams plus corresponding primed ones in Fig. 2. The third and fourth terms in Eq. (37) are simplified via the relation for gamma matrices ($\epsilon^{0123} = -\epsilon_{0123} = 1$),

$$\gamma^\mu \gamma^\rho \gamma^\nu = g^{\mu\rho} \gamma^\nu + g^{\rho\nu} \gamma^\mu - g^{\mu\nu} \gamma^\rho - i \epsilon^{\mu\nu\rho\sigma} \gamma_\sigma \gamma_5. \quad (42)$$

Using the mentioned algebra, we obtain,

$$M_{12}^{ij}|_{B+B'} = C_1^2 N_J^{ij}, \quad M_{12}^{ij}|_{D+D'} = C_2^2 N_J^{ij}, \quad (43)$$

$$M_{12}^{ij}|_{F+F'} = \frac{C_1 C_2}{N_c} N_J^{ij}, \quad M_{12}^{ij}|_{H+H'} = \frac{C_2 C_1}{N_c} N_J^{ij}. \quad (44)$$

In the above relations, the following quantity is introduced,

$$\begin{aligned} N_J^{ij} &= -i \frac{G_F^2}{4 M_D} \lambda_i \lambda_j \\ & \times \int d^4 x \text{tr}[X_{\mu\nu}^{ij}(x)] (g^{\mu\rho} g^{\nu\sigma} + g^{\nu\rho} g^{\mu\sigma} - g^{\mu\mu} g^{\rho\sigma}) \\ & \times [\langle D^0 | \bar{c}^\alpha(x) \gamma_\rho \gamma_5 u^\alpha(0) | 0 \rangle \langle 0 | \bar{c}^\beta(0) \gamma_\sigma \gamma_5 u^\beta(x) | \bar{D}^0 \rangle \\ & + (x \leftrightarrow 0)]. \end{aligned} \quad (45)$$

The matrix elements in Eq. (45) is evaluated in the approximation where light quark field is local, $u(x) \simeq u(0)$. This procedure leads to a relation similar to Eq. (41),

$$\begin{aligned} & \langle D^0 | \bar{c}^\alpha(x) \gamma_\rho \gamma_5 u^\alpha(0) | 0 \rangle \langle 0 | \bar{c}^\beta(0) \gamma_\sigma \gamma_5 u^\beta(x) | \bar{D}^0 \rangle \\ & \simeq f_{D^0}^2 p_\rho p_\sigma e^{ip \cdot x}. \end{aligned} \quad (46)$$

The contribution of A' to M_{12} is merely equal to the one from A , i.e., $M_{12}^{ij}|_A = M_{12}^{ij}|_{A'}$. The analogous relations are valid also for $B, C, \dots, H$ and the corresponding primed ones.

C. Expressions for M_{12}

By assembling the obtained results, the sum over the diagrams and intermediate flavors is carried out straightforwardly,

$$\begin{aligned} M_{12}^{(\alpha)} &\equiv M_{12}^{A+A'+C+C'+E+E'+G+G'} \\ &= 2 G_F^2 f_{D^0}^2 M_{D^0}^3 \lambda_s^2 \left(N_c C_1^2 + \frac{C_2^2}{N_c} + 2 C_1 C_2 \right) I, \end{aligned} \quad (47)$$

$$\begin{aligned} M_{12}^{(\beta)} &\equiv M_{12}^{B+B'+D+D'+F+F'+H+H'} \\ &= 4 G_F^2 f_{D^0}^2 M_{D^0}^3 \lambda_s^2 \left(C_1^2 + C_2^2 + \frac{2}{N_c} C_1 C_2 \right) J, \end{aligned} \quad (48)$$

TABLE II. $|x|$ in this work compared with previous results. The first to the sixth rows show the results in the present work based on Cases [A] and [B] (see main texts) and varying f_{D^0} and M_{D^0} . The seventh, eighth and ninth rows give the previous OPE-based results. For the seventh row, the result of $(\Delta m)_{\text{HQEFT}} \approx (0.9-3.5) \times 10^{-17}$ GeV [12] is converted to $|x|$ via the D^0 -meson lifetime data in Ref. [22]. The tenth row gives the experimental value.

This work	$ x = 1.7$	$\times 10^{-3}$	DSE ([A] $f_{D^0} = 0.212$ GeV, $M_{D^0} = 1.86484$ GeV)
This work	$ x = 1.3$	$\times 10^{-3}$	DSE ([B] $f_{D^0} = 0.212$ GeV, $M_{D^0} = 1.86484$ GeV)
This work	$ x = 2.0$	$\times 10^{-3}$	DSE ([A] $f_{D^0} = 0.227$ GeV, $M_{D^0} = 1.87$ GeV)
This work	$ x = 1.5$	$\times 10^{-3}$	DSE ([B] $f_{D^0} = 0.227$ GeV, $M_{D^0} = 1.87$ GeV)
This work	$ x = 2.9$	$\times 10^{-3}$	DSE ([A] $f_{D^0} = 0.272$ GeV, $M_{D^0} = 1.87$ GeV)
This work	$ x = 2.2$	$\times 10^{-3}$	DSE ([B] $f_{D^0} = 0.272$ GeV, $M_{D^0} = 1.87$ GeV)
Ref. [12]	$ x = (5.6-22)$	$\times 10^{-6}$	OPE (HQET)
Ref. [18]	$ x = 6$	$\times 10^{-7}$	OPE (NLO)
Ref. [20]	$ x^{\text{NLC}} = 7.7$	$\times 10^{-6}$	OPE (Nonlocal condensate)
HFLAV [22]	$x = (4.07 \pm 0.44)$	$\times 10^{-3}$	Experiment

where the unitarity of the CKM matrix is used in a way similar to the OPE analysis. In Eqs. (47) and (48), I and J are momentum integrals that consist of contributions which depend on intermediate flavors,

$$K = K_{dd} + K_{ss} - K_{sd} - K_{ds}. \quad (K = I, J) \quad (49)$$

It should be noted that I and J work as SU(3) breaking factors corresponding to $[(m_s^2 - m_d^2)/m_c^2]^2$ in Eq. (22) for the perturbative case, up to normalization. Each term in Eq. (49) is defined by,

$$I_{ij} = i \int \frac{d^4 l}{(2\pi)^4} \left[\frac{l^2 + p \cdot l}{M_{D^0}^2} - 2 \frac{(p \cdot l)^2}{M_{D^0}^4} \right] \times \sigma_V^i(-(l-p)^2) \sigma_V^j(-l^2), \quad (50)$$

$$J_{ij} = i \int \frac{d^4 l}{(2\pi)^4} \frac{p \cdot l}{M_{D^0}^2} \left(\frac{p \cdot l - M_{D^0}^2}{M_{D^0}^2} \right) \sigma_V^i(-(l-p)^2) \sigma_V^j(-l^2). \quad (51)$$

We evaluate the integrals in Eqs. (50) and (51) at the rest frame of the D meson, $p = (M_{D^0}, \mathbf{0})$, by performing the Wick rotation, where relevant Euclidean variables are introduced by, $l_0 = il_4$ and $l^2 = -l_E^2$ (see, e.g., Ref. [48]). The expressions read,

$$I_{ij} = \int_0^{\Lambda_{\text{cut}}} da \int_0^\pi d\omega \frac{a^3}{4\pi^3} \sin^2 \omega \left(\frac{a^2}{M_{D^0}^2} - \frac{2a^2}{M_{D^0}^2} \cos^2 \omega \right. \\ \left. - i \frac{a}{M_{D^0}} \cos \omega \right) \sigma_V^i(a^2 - M_{D^0}^2) \\ + 2iM_{D^0} a \cos \omega \sigma_V^j(a^2), \quad (52)$$

$$J_{ij} = \int_0^{\Lambda_{\text{cut}}} da \int_0^\pi d\omega \frac{a^3}{4\pi^3} \sin^2 \omega \frac{a \cos \omega}{M_{D^0}} \left(\frac{a \cos \omega}{M_{D^0}} + i \right) \\ \times \sigma_V^i(a^2 - M_{D^0}^2 + 2iM_{D^0} a \cos \omega) \sigma_V^j(a^2), \quad (53)$$

regulated by ultraviolet cutoff (Λ_{cut}). It should be noted that I and J are finite and do not depend on the cutoff for sufficiently large Λ_{cut} since contributions from large momenta cancel in Eq. (49). Equations (52) and (53) can be evaluated with the vector part of the quark propagators in Eqs. (26) and (27). The expression for the entire M_{12} is obtained by summing over Eqs. (47) and (48), i.e., $M_{12} = M_{12}^{(\alpha)} + M_{12}^{(\beta)}$.

D. Numerical result

For numerical input, we consider the following parameter set:

- (i) $f_{D^0} = 0.212$ GeV (lattice QCD [40]) and $M_{D^0} = 1.86484$ GeV (PDG [28]).
- (ii) $f_{D^0} = 0.227$ GeV and $M_{D^0} = 1.87$ GeV (DSE approach [34]).
- (iii) $f_{D^0} = 0.272$ GeV and $M_{D^0} = 1.87$ GeV (DSE approach [34]).

The decay constants in the second and third cases in the DSE approach are larger than the value in lattice QCD by 7% and 28%, respectively. As for the ultraviolet cutoff in Eqs. (52) and (53), we fix $\Lambda_{\text{cut}} = 10^2$ GeV. We verified that the numerical result is reasonably stable under the variation of Λ_{cut} . Furthermore, the following two cases are discussed,

Case[A]: VIA, local limit of $u(x)$,

$$\Delta C = 1 \text{ weak coupling: } C_1(m_c), \quad C_2(m_c)$$

Case[B]: VIA, local limit of $u(x)$,

$$\text{no QCD for } \Delta C = 1 \text{ weak coupling: } C_1 = 0, \quad C_2 = 1,$$

where specific values of $C_1(m_c)$ and $C_2(m_c)$ are fixed to the ones for the OPE analysis in Sec. II C. Other parameters such as CKM matrix elements are also fixed as the previous values.

The numerical results in the DSE approach are exhibited and compared with the previous works in Table II. One finds that the results in the DSE approach are

TABLE III. Partial contribution to x in this work separately taking the left-side ($x^{(\alpha)}$) and right-side ($x^{(\beta)}$) of the diagrams in Fig. 2.

$ x^{(\alpha)} = 1.2 \times 10^{-6}$	$ x^{(\beta)} = 1.7 \times 10^{-3}$	[A] $f_{D^0} = 0.212 \text{ GeV}, M_{D^0} = 1.86484 \text{ GeV}$
$ x^{(\alpha)} = 0.20 \times 10^{-3}$	$ x^{(\beta)} = 1.5 \times 10^{-3}$	[B] $f_{D^0} = 0.212 \text{ GeV}, M_{D^0} = 1.86484 \text{ GeV}$
$ x^{(\alpha)} = 1.4 \times 10^{-6}$	$ x^{(\beta)} = 2.0 \times 10^{-3}$	[A] $f_{D^0} = 0.227 \text{ GeV}, M_{D^0} = 1.87 \text{ GeV}$
$ x^{(\alpha)} = 0.24 \times 10^{-3}$	$ x^{(\beta)} = 1.8 \times 10^{-3}$	[B] $f_{D^0} = 0.227 \text{ GeV}, M_{D^0} = 1.87 \text{ GeV}$
$ x^{(\alpha)} = 2.1 \times 10^{-6}$	$ x^{(\beta)} = 2.9 \times 10^{-3}$	[A] $f_{D^0} = 0.272 \text{ GeV}, M_{D^0} = 1.87 \text{ GeV}$
$ x^{(\alpha)} = 0.34 \times 10^{-3}$	$ x^{(\beta)} = 2.5 \times 10^{-3}$	[B] $f_{D^0} = 0.272 \text{ GeV}, M_{D^0} = 1.87 \text{ GeV}$

$|x| = (1.3\text{--}2.9) \times 10^{-3}$, the order of magnitude comparable to the experimental value. This leads to a certain improvement compared to the OPE-based approaches, in which case the order of magnitude is far below the data. In addition, we find that the results in the DSE approach are sensitive to the parameters of f_{D^0} , C_1 and C_2 and slightly to M_{D^0} .

Numerical results which separately consider the left-side and right-side of the diagrams in Fig. 2 are respectively extracted by,

$$x^{(\alpha)} = \frac{2M_{12}^{(\alpha)}}{\Gamma}, \quad x^{(\beta)} = \frac{2M_{12}^{(\beta)}}{\Gamma}, \quad (54)$$

where $M_{12}^{(\alpha)}$ and $M_{12}^{(\beta)}$ are defined in Eqs. (47) and (48). The results of $x^{(\alpha)}$ and $x^{(\beta)}$ are exhibited in Table III with different input parameters for f_{D^0} , M_{D^0} , C_1 and C_2 , as well as Table II. We find that for Case [A], the value of $|x^{(\alpha)}|$ is of $\mathcal{O}(10^{-6})$, much smaller than $|x^{(\beta)}| = \mathcal{O}(10^{-3})$. This is caused by the accidental cancellation for the Wilson coefficients in Eq. (47),

$$N_c C_1^2 + \frac{C_2^2}{N_c} + 2C_1 C_2 \approx 0.002 \ll 2 \left(C_1^2 + C_2^2 + \frac{2}{N_c} C_1 C_2 \right) \approx 2.3. \quad (55)$$

As for Case [B] in Table III, one finds that $|x^{(\alpha)}| = \mathcal{O}(10^{-4})$ and $|x^{(\beta)}| = \mathcal{O}(10^{-3})$. The slightly small value of $|x^{(\alpha)}|$ can be partially understood by taking $C_2 \rightarrow 1$ and $C_1 \rightarrow 0$ in Eq. (55), leading to $N_c^{-1} < 2$.

IV. CONCLUSION

In this work, we have discussed the $D^0 - \bar{D}^0$ mixing in such a way that chiral symmetry breaking is properly taken into account. In the conventional OPE approach, the factor proportional to SU(3) breaking, which originates from the GIM mechanism, gives the extreme suppression so that x at the lowest order is significantly smaller than the HFLAV data. We have assessed how this result is altered when the current-quark masses are replaced by the constituent-quark masses. It is found that the order of the magnitude is

enhanced typically by $\mathcal{O}(10^2)$ although the detailed values are sensitive to the constituent-quark masses.

In order to incorporate the entire momentum dependence of the quark propagators, not just replacing the quark masses, the DSE approach is discussed to evaluate the mixing parameter. On the basis of the parametrization for quark propagators in the previous works [31–35], DCSB from primary SU(3) breaking is appropriately taken into account. With the $\Delta C = 1$ local operators, the $\bar{D}^0 \rightarrow D^0$ transition is evaluated in the VIA, formulated by the decay constant of D^0 meson, as well as relevant momentum integrals.

In the local limit of the fermion field corresponding to the valence light quark of D meson, the dimensionless mass-difference observable is numerically investigated. It is shown that the result is sensitive to the values of the decay constant, $\Delta C = 1$ Wilson coefficients and (slightly) D^0 -meson mass. The contributions of the two different types of diagrams, denoted as $x^{(\alpha)}$ and $x^{(\beta)}$, are discussed separately. We have found that for $C_1 = C_1(m_c)$ and $C_2 = C_2(m_c)$, the accidental cancellation occurs between some of the diagrams, resulting in suppression for $x^{(\alpha)}$. For $C_1 = 0$ and $C_2 = 1$, $|x^{(\alpha)}| < |x^{(\beta)}|$ is still realized, partially understandable via the large- N_c counting.

The obtained numerical results for the observable read $|x| = (1.3\text{--}2.9) \times 10^{-3}$, where the uncertainty range comes from the variation of f_{D^0} , M_{D^0} , C_1 and C_2 , while experimental 95% confidence-level interval is given by $x = [3.20, 4.93] \times 10^{-3}$ [22]. As such, the order of magnitude of x is comparable to the data in the DSE approach, which leads to a certain improvement from the OPE-based approaches. Further scrutinization of the result is foreseen [49] in such a way that takes account of nonlocality for the light quark field by means of the Bethe-Salpeter approach.

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DATA AVAILABILITY

The data supporting this study's findings are available within the article.

APPENDIX: RGE AT LO IN OPE

Here we denote $\mu_L = 3$ GeV, $\mu_c = m_c(m_c)$ and $\mu_b = m_b(m_c)$, with the masses given by the $\overline{\text{MS}}$ scheme at the scale of the charm-quark mass. The bag parameters from the ETM Collaboration [44] are,

$$B_1(3 \text{ GeV}) = 0.757(27)(4), \quad (\text{A1})$$

$$B_2(3 \text{ GeV}) = 0.65(3)(2), \quad (\text{A2})$$

$$B_3(3 \text{ GeV}) = 0.96(8)(2). \quad (\text{A3})$$

In order to evaluate Eq. (22), $B_1(\mu_c)$ and $B_2(\mu_c)$ are required. These quantities can be straightforwardly obtainable from the discussion in Ref. [50], resulting in at LO,

$$B_1(\mu_c) = B_1(\mu_L) \left[\frac{\alpha_s(\mu_c)}{\alpha_s(\mu_L)} \right]^{\frac{\gamma_{\text{VLL}}}{2\beta_0(4)}},$$

$$B_2(\mu_c) = (U_{11}^{\text{SLL}} + 4U_{21}^{\text{SLL}})B_2(\mu_L) - \frac{8}{5}U_{21}^{\text{SLL}}B_3(\mu_L), \quad (\text{A4})$$

where the factors in regards to the RGE are [50],

$$\beta_0(N_f) = 11 - \frac{2}{3}N_f, \quad (\text{A5})$$

$$\gamma^{\text{VLL}} = 4, \quad (\text{A6})$$

$$U^{\text{SLL}} = V^{\text{SLL}} \text{diag} \left\{ \left[\frac{\alpha_s(\mu_c)}{\alpha_s(\mu_L)} \right]^{\frac{\gamma^{\text{SLL}}}{2\beta_0(4)}} \right\} (V^{\text{SLL}})^{-1}, \quad (\text{A7})$$

$$\tilde{\gamma}^{\text{SLL}} = \left(\frac{2}{3} + \frac{2}{3}\sqrt{241}, \frac{2}{3} - \frac{2}{3}\sqrt{241} \right), \quad (\text{A8})$$

$$V^{\text{SLL}} = \begin{pmatrix} -60 & -60 \\ 16 + \sqrt{241} & 16 - \sqrt{241} \end{pmatrix}. \quad (\text{A9})$$

In Eq. (A5), N_f represents the number of quarks.

As for the RGE for the Wilson coefficients of the current-current operators, these are obtained in a standard way in Ref. [51] and references therein: By introducing,

$$C_{\pm}(\mu) = C_2(\mu) \pm C_1(\mu), \quad (\text{A10})$$

and using $C_1(M_W) = 0$ and $C_2(M_W) = 1$ at LO,

$$C_{\pm}(\mu_c) = \left[\frac{\alpha_s(\mu_b)}{\alpha_s(\mu_c)} \right]^{\frac{\gamma_{\pm}}{2\beta_0(4)}} \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu_b)} \right]^{\frac{\gamma_{\pm}}{2\beta_0(5)}}, \quad (\text{A11})$$

$$\gamma_{\pm} = \pm 6 \frac{N_c \mp 1}{N_c}. \quad (\text{A12})$$

The values of $C_1(\mu_c)$ and $C_2(\mu_c)$ are determined by solving Eq. (A10) with Eq. (A11). Numerically, those LO values are $C_1(\mu_c) \approx -0.35$ and $C_2(\mu_c) \approx 1.13$.

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