

Spin polarization of holographic baryon in strongly coupled fluid

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The spin polarization for a baryon in a hydrodynamic medium has been extensively studied in the weakly coupled regime using quantum kinetic theory. As a first study of this problem in the strongly coupled regime, we investigate holographically the spectral function of a probe baryon in the fluid-gravity background. This is done by carefully performing gradient expansion of the Dirac equation in the fluid-gravity background. Different contributions in the expansion are understood in terms of density matrices of the probe baryon and the medium. The resulting spectral functions indicate that the holographic baryon is polarized as responses to fluid acceleration, shear stress, and vorticity. The structures of the responses are similar to those found in weakly coupled studies.

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I. INTRODUCTION

Spin polarization is one of the key observables in particle physics, which is a probe on the interaction among particles. Recently there has been growing interest in spin polarization in relativistic heavy ion collisions: based on the early idea of spin-orbit coupling, baryons and vector mesons are predicted to have spin polarization and spin alignment respectively [1,2]. These have indeed been confirmed by heavy ion experiments [3]. A distinguishing feature of heavy ion experiments is the presence of rapid spinning quark-gluon plasma (QGP), which encodes the collective motion of quarks and gluons and adopts a hydrodynamic description in terms of vorticity. The measurements of global polarization of the Λ hyperon have been well understood with spin-vorticity coupling [4–9]. More recently, it has been realized that spin is also coupled to other hydrodynamic gradients [10–12], which is crucial for understanding measurements of local polarization [13–15].

A notable framework in understanding spin polarization is the quantum kinetic theory (QKT), which is based on quasiparticles with spin degrees of freedom and incorporates a collisional effect systematically; see [16] for a recent review. Useful insights have been gained by studies within the QKT framework: we can separate the spin polarization

into a local equilibrium contribution and a steady state contribution. The first one is induced through a spin response to hydrodynamic gradients. The free theory limit of this contribution is widely known [4,11,12] and extensively implemented in phenomenological studies [13–15]. The second one is unique to interacting theories. In the presence of a dissipative gradient, such as shear and temperature gradient but not vorticity, the fermions will reach a steady state, in which the nonvanishing collision with other particles in the medium balances the effective force exerted by the dissipative gradient. The deviation of a steady state from a local equilibrium state, as well as the resulting contribution to spin polarization, depends on details of the interaction [17–20]. In many phenomenology motivated theoretical studies, a strange quark is often considered as a probe to off-equilibrium QGP. This allows us to separate the steady state effect of the medium and the probe [17,19,20]. More recently, a general interaction effect has been classified into modifications to the spectral function, Kubo-Martin-Schwinger relation, and the distribution function [21]. Radiative correction to spectral function has been considered in [21,22].

Despite success of the QKT framework, the effectiveness of a quasiparticle picture in QGP should not be taken for granted: phenomenological studies of QGP point to a low viscosity, which is close to the viscosity lower bound [23]. Such a low viscosity suggests QGP might be strongly coupled, for which quasiparticles might not exist. The purpose of this work is to provide a study of spin polarization in the regime of strong coupling using gauge-gravity duality [24–26]. While there is no quasiparticle in the strongly coupled regime, we will see it is still possible to understand contributions from different origins in terms of a density matrix, which can be further separated into

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density matrices of the probe and medium in the probe limit. We will find the spectral function of a holographic baryon in the fluid-gravity background. The correction to the spectral function in the background encodes the spin polarization of the baryon in response to hydrodynamic gradients. We will see the corresponding spectral function splits naturally into a local equilibrium state contribution and steady state one. A similar study has been performed in the weakly coupled case, but for the local equilibrium state only [21]. This work also provides a first example, to the best of our knowledge, of a steady state contribution to a spectral function.¹

The paper is structured as follows: We first present in Sec. II a general analysis on the structure of a correlation function and constraint from parity; we then discuss how to perform a consistent gradient expansion in the fluid gravity background in Sec. III. Complete calculations of the retarded and spectral functions are presented in Sec. IV. The spectral function is found to be spin polarized in responses to fluid acceleration, shear stress, and vorticity. A conclusion and outlook are devoted to Sec. V. Notations and some explicit expressions are collected in two Appendixes.

II. GENERAL STRUCTURE OF FERMIONIC CORRELATION FUNCTION

Since the holographic baryon we study consists of two Weyl fermions of opposite helicities, we first discuss correlation functions of Weyl fermion with one helicity. The retarded and advanced fermionic correlation function are defined respectively as

$$\begin{aligned} G_{\alpha\beta}^R(X, p) &= i \int d^4(x-y) e^{ip \cdot (x-y)} \theta(x^0 - y^0) \\ &\quad \times \langle \Psi_\alpha(x) \Psi_\beta^\dagger(y) + \Psi_\beta^\dagger(y) \Psi_\alpha(x) \rangle, \\ G_{\alpha\beta}^A(X, p) &= -i \int d^4(x-y) e^{ip \cdot (x-y)} \theta(-x^0 + y^0) \\ &\quad \times \langle \Psi_\alpha(x) \Psi_\beta^\dagger(y) + \Psi_\beta^\dagger(y) \Psi_\alpha(x) \rangle, \end{aligned} \quad (2.1)$$

with Ψ being a composite gauge invariant Weyl operator. $X = \frac{x+y}{2}$ and $p^\mu = (\omega, \vec{p})$. The fermionic correlator is to be evaluated in a fluid background characterized by gradients of temperature and fluid velocity (to be denoted by ∂_X). We require $\omega, p \gg \partial_X$ such that we can interpret X and p as the coarse-grained position and momentum of the baryon. α, β denote Weyl indices. G_R is a 2×2 matrix. The expectation values are to be understood as $\text{Tr}[D \cdots]$ with D being the density matrix. We will consider a probe baryon in a fluid background, in which the density matrix can be factorized

as $D = D_{\text{probe}} \otimes D_{\text{fluid}}$. Since D is Hermitian, we can easily show

$$G^R(\omega, \vec{p})^\dagger = G^A(\omega, \vec{p}). \quad (2.2)$$

A closely related quantity is the spectral function defined similarly as

$$\begin{aligned} \rho_{\alpha\beta} \left(X = \frac{x+y}{2}, p \right) \\ = \int d^4(x-y) e^{ip \cdot (x-y)} \langle \Psi_\alpha(x) \Psi_\beta^\dagger(y) + \Psi_\beta^\dagger(y) \Psi_\alpha(x) \rangle. \end{aligned} \quad (2.3)$$

By definition, we have

$$i\rho(X, p) = G^R(X, p) - G^A(X, p) = G^R(X, p) - G^R(X, p)^\dagger, \quad (2.4)$$

where we have used (2.2) in the last equality. It follows that $\rho = \rho^\dagger$.

To model a baryon field, we shall combine the contribution from both helicities. We shall perform explicit calculations for one helicity only. The contribution from the other helicity will be obtained by parity transformation. We now derive the precise relation. We start with the following identity:

$$\langle \Omega_b | \pi^\dagger \pi O \pi^\dagger \pi | \Omega_a \rangle = \langle \tilde{\Omega}_b | \pi O \pi^\dagger | \tilde{\Omega}_a \rangle, \quad (2.5)$$

where π is the parity operator and $|\tilde{\Omega}_{a/b}\rangle = \pi |\Omega_{a/b}\rangle$. The matrix element is readily generalizable to the expectation value, in which the in and out states are replaced by a trace over the density matrix. To be specific, let us take the Weyl fermion to be left-handed, and denote the corresponding density matrix to be $D_L \otimes D_{\text{fluid}}$. Parity transformation turns the fermion to be right-handed and also acts on the fluid background, with the transformed density matrix denoted by $D_R \otimes \tilde{D}_{\text{fluid}}$. We take the operator to be

$$O_{\alpha\beta}(x; y) = \Psi_\alpha^L(x) \Psi_\beta^{L\dagger}(y) + \Psi_\beta^{L\dagger}(y) \Psi_\alpha^L(x), \quad (2.6)$$

where we have used the superscript L to indicate the Weyl field being left-handed. The parity transformation of $O_{\alpha\beta}$ is given by

$$\begin{aligned} \pi O_{\alpha\beta}(x^0, \vec{x}; y^0, \vec{y}) \pi^\dagger \\ = \Psi_\alpha^R(x^0, -\vec{x}) \Psi_\beta^{R\dagger}(y^0, -\vec{y}) + \Psi_\beta^{R\dagger}(y^0, \vec{y}) \Psi_\alpha^R(x^0, -\vec{x}). \end{aligned} \quad (2.7)$$

The net effect is the following: the operator remains the same but with the field replaced by the right-handed counterpart. The argument is spatially inverted. Applying the Wigner

¹The steady state effect in the QKT framework corrects the spin polarization, but not the spectral function.

transform, we easily find

$$\rho_{\alpha\beta}^L(X, \omega, \vec{p}) = \tilde{\rho}_{\alpha\beta}^R(X, \omega, -\vec{p}). \quad (2.8)$$

Here the tilde indicates the parity transformation on the fluid background. A similar relation can be derived for retarded and advanced correlators, which we do not elaborate.

III. HOLOGRAPHIC PRESCRIPTION AND TRANSFORMATIONS

In heavy ion collisions, the baryons form at freeze-out as an end point of QGP evolution. The hydrodynamic gradients at freeze-out leave imprints on the spectral function of baryons in the form of gradient corrections, contributing a part of the spin polarization of baryons. Motivated by this picture, we consider holographic baryons in QGP fluid. The former is modeled by bulk Dirac fermions in a bottom-up approach; see also [27,28] for top-down approaches. The latter is modeled by the fluid-gravity background. The holographic prescription for the retarded function of fermionic operators has been established in [29–31]. We will follow the notation of [32] below. However, the formulation in the Fourier component assumes translational invariance, which does not hold in the fluid background we use. We shall generalize the prescription to coordinate space. For pedagogical purposes, we start with the Dirac action in an AdS-Schwarzschild background,

$$ds^2 = \frac{-f(z)dt^2 + d\vec{x}^2 + dz^2/f(z)}{z^2}, \quad (3.1)$$

with $f(z) = 1 - \frac{z^4}{b^4}$. By using integrations by part and the bulk Dirac equation $(\Gamma^M \nabla_M - m)\psi = 0$, we obtain the boundary term as²

$$\begin{aligned} S &= \int d^5x \sqrt{-g} \bar{\psi}(x) i(\Gamma^M \nabla_M - m) \psi(x) \\ &= i \int d^4x \sqrt{-g} g_{zz}^{-1/2} \bar{\psi}(x) \gamma^{\hat{z}} \psi(x)|_{z \rightarrow 0} + \dots \end{aligned} \quad (3.2)$$

where $\Gamma^{\hat{z}} = e_{\hat{z}}^z \gamma^{\hat{z}} = g_{zz}^{-1/2} \gamma^{\hat{z}}$. We shall not need the explicit form of the Dirac equation and representation of gamma matrices, which are collected in Appendix A. For now, we need

²We have set the prefactor of the Dirac action to be unity as in [32]. The prefactor cancels out in the measure of polarization, which is the ratio of an off-equilibrium correction to a spectral function to its equilibrium counterpart.

$$\gamma^{\hat{z}} = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix} \equiv (-gg^{zz})^{-1/4} \begin{pmatrix} \chi_R \\ \chi_L \end{pmatrix}. \quad (3.3)$$

Asymptotic analysis gives [32]

$$\chi_L = Az^{-m} + Bz^{1+m}, \quad \chi_R = Cz^{1-m} + Dz^m. \quad (3.4)$$

For $m \geq 0$ ($m \leq 0$), χ_L (χ_R) should be chosen as a source to the fermionic operator Ψ with scaling dimension $\Delta = |m| + 2$. We shall use two Weyl fermions with opposite helicity to make a holographic baryon. To be specific, we choose $m = 0$.³ Plugging (3.3) into (3.2), we obtain

$$S = \int d^4x \left(-\chi_L^{b\dagger}(x) \chi_R^b(x) + \chi_R^{b\dagger}(x) \chi_L^b(x) \right) \quad (3.5)$$

with $\chi_{R/L}^b = \chi_{R/L}(z \rightarrow 0)$. Clearly, quantization with either χ_L^b or χ_R^b is allowed. To proceed, we choose χ_L^b as the source, which leads to a correlator of the left-handed Weyl operator. The counterpart for the right-handed Weyl fermion can in principle be obtained with an alternative quantization [32]. We will instead deduce the corresponding correlator by (2.8). Imposing the infalling boundary condition at the horizon, we have the following retarded correlator:

$$G_R(x, y) = -\frac{\delta^2 S}{\delta \chi_L^{b\dagger}(x) \delta \chi_L^b(y)} = \frac{\delta \chi_R^b(x)}{\delta \chi_L^b(y)}. \quad (3.6)$$

The prescription will be generalized to the fluid gravity background, which is given by [33]

$$\begin{aligned} ds^2 &= -2u_\mu dx^\mu dr - r^2 f(br) u_\mu u_\nu dx^\mu dx^\nu + r^2 P_{\mu\nu} dx^\mu dx^\nu \\ &\quad + 2r^2 bF(br) \sigma_{\mu\nu} dx^\mu dx^\nu + \frac{2}{3} r u_\mu u_\nu \partial_\lambda u^\lambda dx^\mu dx^\nu \\ &\quad - r u^\lambda \partial_\lambda (u_\nu u_\mu) dx^\mu dx^\nu, \end{aligned} \quad (3.7)$$

with

$$\begin{aligned} F(r) &= \int_r^\infty dx \frac{x^2 + x + 1}{x(x+1)(x^2+1)} \\ &= \frac{1}{4} \left[\ln \left(\frac{(1+r)^2(1+r^2)}{r^4} \right) - 2 \arctan(r) + \pi \right]. \end{aligned} \quad (3.8)$$

The first line of (3.7) is simply a boosted Schwarzschild background written in Eddington-Finkelstein (EF)

³This choice leads to $\Delta = 2$ as opposed to $\Delta = 3/2$ for a free Dirac fermion. We believe results obtained with this choice can still provide insights into the polarization phenomenon in an off-equilibrium medium.

coordinates with $u_\mu(x)$ and $b(x)$ being the slow-varying velocity and inverse temperature. The explicit relations are given by

$$r = \frac{1}{z}, \quad dt = d\bar{t} - \frac{dz}{f(z)}, \quad dx^i = d\bar{x}^i, \quad (3.9)$$

with the barred coordinate corresponding to the Schwarzschild coordinate. The second line of (3.7) is the gradient correction to the boosted metric, with the three terms corresponding to shear, bulk stress, and acceleration in the local rest frame of the fluid respectively. The explicit form of the metric in the local rest frame of the fluid is collected in Appendix B for completeness. The two lines have clear physical interpretation: the first line corresponds to the local equilibrium state of the QGP, while the second line characterizes the steady state deviation from the local equilibrium one. They encode the local equilibrium part and gradient correction part of D_{fluid} . Since the bulk stress carries no spatial index, it cannot couple to spin polarization. We shall not consider bulk stress in this paper.

Before making the generalization, it is useful to discuss symmetry transformations of the correlator. The bulk action is invariant under two symmetries: local Lorentz transformation (LLT) and diffeomorphism by construction, but the boundary correlator is not invariant. Under the LLT, the bulk fermion transforms as

$$\psi_L(x) \rightarrow M_L(x)\psi_L(x), \quad \psi_L^\dagger(y) \rightarrow \psi_L^\dagger(y)M_R(y)^{-1}, \quad (3.10)$$

where $M_L(x)$ and $M_R(y)$ are Lorentz transformations on the 5D fermion. On the boundary, the LLT reduces to

$$\chi_L^b(x) \rightarrow M_L^b(x)\chi_L^b(x), \quad \chi_L^{b\dagger}(y) \rightarrow \chi_L^{b\dagger}(y)M_R^b(y)^{-1}, \quad (3.11)$$

with $M_L^b = M_L(z \rightarrow 0)$ and $M_R^b = M_R(z \rightarrow 0)$ in the above. It follows that

$$G_R(x, y) \rightarrow M_R^b(x)^{-1}G_R(x, y)M_L^b(y). \quad (3.12)$$

In order to fix the ambiguity, we need to choose a frame. A natural choice is to require the boundary fermion to be in the same frame as the bulk fermion. This is most clearly written for the AdS-Schwarzschild background as

$$\bar{e}_t^i \rightarrow \frac{1}{z}, \quad \bar{e}_i^i \rightarrow \frac{1}{z}, \quad \bar{e}_z^z \rightarrow \frac{1}{z}, \quad (3.13)$$

as $z \rightarrow 0$. We have used barred vielbein to indicate they correspond to the Schwarzschild coordinate. Hatted and unhatted coordinates correspond to flat and curved coordinates respectively. Note that $M_{L/R}^b$ include both 4D Lorentz transformation and special conformal transformation. The choice (3.13) eliminates both freedoms. The frame choice is readily generalized to the fluid gravity

background. We first note that the background at $O(\partial^0)$ is nothing but a boosted AdS-Schwarzschild metric. Note that it corresponds to a 4D boost on the boundary, which brings a static fluid element to a moving fluid element. If we choose a local rest frame, the local metric is a simple AdS-Schwarzschild, then the appropriate frame choice is obtained by transforming (3.13) to EF coordinates using $e_M^a = \bar{e}_N^a \frac{\partial \bar{x}^N}{\partial x^M}$. We obtain

$$e_t^i \rightarrow \frac{1}{z}, \quad e_i^i \rightarrow \frac{1}{z}, \quad e_i^i \rightarrow \frac{1}{z}, \quad e_z^z \rightarrow \frac{1}{z}. \quad (3.14)$$

Furthermore, the gradient correction to the boost metric is subleading as $z \rightarrow 0$, which means the asymptotic behavior (3.14) fixing the frame choice is unaffected by the subleading correction.

Turning to diffeomorphism, we note that the bulk spinor is invariant under diffeomorphism. Although the definitions of $\chi_{R/L}$ in (3.3) do seem to introduce dependence on choice of the coordinate through the factor $(-gg^{zz})^{-1/4}$, the factor is introduced merely to remove extra powers of z at the boundary. The powers can be related to the scaling dimension of the boundary operators that are uniquely determined from asymptotic AdS metric. We may insist on using the same numerical factor in defining $\chi_{R/L}$ in a fluid gravity background. This way the bulk diffeomorphism leaves no freedom on the boundary correlator.⁴

IV. GRADIENT EXPANSION AND RETARDED CORRELATOR

In this section, we study the first-order gradient correction to the retarded correlator, which arises from the following gradient correction to density matrix $D_{\text{fluid}}^{(0)} \otimes D_{\text{baryon}}^{(1)}$ and $D_{\text{fluid}}^{(1)} \otimes D_{\text{baryon}}^{(0)}$, with superscripts denoting orders of the gradient. The gradient expansion of the Dirac equation involves expansions of the Dirac field and the Dirac operator separately, which are to be identified with $D_{\text{fluid}}^{(0)} \otimes D_{\text{baryon}}^{(1)}$ and $D_{\text{fluid}}^{(1)} \otimes D_{\text{baryon}}^{(0)}$ respectively. We shall first perform an expansion of the Dirac field in the boosted AdS-Schwarzschild background, then an expansion of the Dirac operator.

A. Gradient correction to the Dirac field

In a translational background like AdS-Schwarzschild, we can apply a Fourier transform to (3.6) to obtain

$$\chi_R^b(p) = G_R(p)\chi_L^b(p). \quad (4.1)$$

⁴In principle, the factor we use instead of powers of z could bring in possible contact terms to the boundary correlator. This does not occur for the choice $m = 0$.

This allows us to solve the bulk EOM for $\chi_{R/L}(z, p)$ and extract $G_R(p)$ from the boundary behavior. In a fluid gravity background, $G_R(x, y)$ is to be Wigner transformed as in (2.2) and to be calculated using gradient expansion. The tricky point is that we can do gradient expansion for $G_R(x, y)$, but not $\chi_R(x)$ and $\chi_L(y)$ separately, as the gradient corresponds to variation with respect to X . We instead perform gradient expansion to the bulk extension of $G_R(x, y)$, which is the boundary-bulk propagator $G_R(x, z|y)$ satisfying

$$\chi_R(x, z) = \int_y G(x, z|y) \chi_L^b(y), \quad (4.2)$$

with $\int_y = \int d^4y$. Note that $G(x, z|y)$ satisfies the same EOM as $\chi_R(x, z)$. We proceed with the boosted AdS-Schwarzschild background in an EF coordinate and choose the following vielbein:

$$e_t^{\hat{t}} = \frac{f^{1/2}}{z}, \quad e_t^{\hat{z}} = \frac{1}{zf^{1/2}}, \quad e_i^{\hat{t}} = \frac{1}{z}, \quad e_i^{\hat{z}} = \frac{1}{f^{1/2}}. \quad (4.3)$$

All other components vanish. The resulting Dirac equation in terms of components $\chi_{R/L}$ reads

$$\begin{aligned} f^{1/2} \left(\partial_z - \frac{\partial_0^x}{f} \right) \chi_R(x, z) + i D_\mu^x \sigma^\mu \chi_L(x, z) &= 0, \\ f^{1/2} \left(\partial_z - \frac{\partial_0^x}{f} \right) \chi_L(x, z) - i D_\mu^x \bar{\sigma}^\mu \chi_R(x, z) &= 0, \end{aligned} \quad (4.4)$$

with $D_\mu^x = (f^{-1/2} \partial_0^x, \partial_i^x)$. We can derive a second-order EOM for χ_R from the two first-order equations

$$\chi_R'' + \frac{f'}{2f} \chi_R' - \frac{2D_0^x}{f^{1/2}} \chi_R' + \frac{D_0^x f'}{2f^{3/2}} \chi_R + \frac{\partial_i^x \partial_i^x}{f} \chi_R - \frac{i D_0^x f'}{2f^{3/2}} \chi_L = 0. \quad (4.5)$$

We have suppressed the common argument (x, z) and denoted ∂_z by a prime. To express χ_L in the last term in terms of χ_R , we act on the first line of (4.4) by $D_\mu^x \bar{\sigma}^\mu$ to obtain

$$D^x \cdot \bar{\sigma} (f^{1/2} \chi_R' - D_0^x \chi_R) - i D_z^2 \chi_L = 0. \quad (4.6)$$

We can solve for χ_L by formally dividing D_x^2 and plug into (4.5) to arrive at

$$\begin{aligned} \chi_R'' + \frac{f'}{2f} \chi_R' - \frac{2D_0^x}{f^{1/2}} \chi_R' + \frac{D_0^x f'}{2f^{3/2}} \chi_R + \frac{\partial_i^x \partial_i^x}{f} \chi_R \\ + \frac{D_0^x f'}{2f^{3/2}} \frac{D^x \cdot \bar{\sigma} f' (f^{1/2} \chi_R' - D_0^x \chi_R)}{-D_x^2} = 0. \end{aligned} \quad (4.7)$$

The same equation is satisfied by $G(x, z|y)$. The Wigner transform of $G(x, z|y)$ is defined as

$$G(X, p, z) = \int_s e^{-ip \cdot s} G(x, z|y), \quad (4.8)$$

with $s = x - y$. We can then perform a gradient expansion using

$$\partial_\mu^x = \partial_\mu^s + \frac{1}{2} \partial_\mu^X \rightarrow ip_\mu + \frac{1}{2} \partial_\mu^X. \quad (4.9)$$

Denoting $P^\mu = (f^{-1/2} \omega, p_i)$ and $D_\mu^X = (f^{-1/2} \partial_0^X, \partial_i^X)$, we easily find the following EOM expanded up to $O(\partial^X)$:

$$\begin{aligned} G'' + 2i \frac{\omega}{f} G' + \frac{f'}{2f} G' - i \omega \frac{f'}{2f^2} G - \frac{p^2}{f} G \\ + \frac{f'}{2f^{3/2}} \omega \frac{P \cdot \bar{\sigma}}{P^2} \left(G' + \frac{i \omega}{f} G \right) + \left[-\frac{\partial_0^X}{f} G' + \frac{f'}{4f^2} \partial_0^X G \right. \\ + ip_i \frac{\partial_i^X}{f} G + \frac{f'}{4f^{3/2}} \frac{P \cdot \bar{\sigma}}{P^2} i \partial_0^X \left(G' + \frac{2i \omega}{f} G \right) \\ \left. + \frac{f'}{2f^{3/2}} i \omega \left(\frac{D^X \cdot \bar{\sigma}}{-2P^2} + \frac{P \cdot D_X P \cdot \bar{\sigma}}{(P^2)^2} \right) \left(G' + \frac{i \omega}{f} G \right) \right] \\ = 0. \end{aligned} \quad (4.10)$$

In arriving at the equation above, we assume the operator D_x^2 is invertible. It would be violated if $P^2 = -\omega^2/f(z) + p^2$ vanished in the bulk. This does not occur for timelike momenta, which is what we need for baryonic excitation. It is possible to consider excitation with spacelike momenta by introducing the $i\epsilon$ prescription. We shall restrict ourselves to timelike momenta below. The boundary condition can be deduced from the first line of (4.4) and (4.2) as

$$\begin{aligned} -i D_\mu^x \sigma^\mu \chi_L(x, z) &= (f^{1/2} \partial_z \chi_R(x, z) - D_0^x \chi_R) \\ &= \int_y (f^{1/2} \partial_z - D_0^x) G(x, z|y) \chi_L^b(y). \end{aligned} \quad (4.11)$$

In the limit $z \rightarrow 0$, we have

$$(\partial_z - \partial_0^x) G(x, z|y) \rightarrow -i \partial^x \cdot \sigma \delta(x - y). \quad (4.12)$$

Its Wigner transform gives the boundary condition for $G(X, p, z)$,

$$\left(\partial_z + i \omega - \frac{1}{2} \partial_0^X \right) G(X, p, z \rightarrow 0) \rightarrow p \cdot \sigma. \quad (4.13)$$

The boundary correlator is given by $G_R(x, y) = G(x, z \rightarrow 0|y)$, or its Wigner transformed version

$$G_R(X, p) = G_R(X, p, z \rightarrow 0). \quad (4.14)$$

B. Retarded correlator from gradient correction to the probe

We now solve (4.10) in gradient expansion as

$$G = G_0 + G_1 + \dots, \quad (4.15)$$

with the subscript indicating the order of gradient. G_0 satisfies

$$G_0'' + 2i\frac{\omega}{f}G_0' + \frac{f'}{2f}G_0' - i\omega\frac{f'}{2f^2}G_0 - \frac{p^2}{f}G_0 + \frac{f'}{2f^{3/2}}\omega\frac{P \cdot \bar{\sigma}}{P^2}\left(G_0' + \frac{i\omega}{f}G_0\right) = 0. \quad (4.16)$$

By rotational invariance, we can decompose $G_0 = A(\omega, p) + B(\omega, p)\hat{p} \cdot \sigma$. Plugging this into (4.16), we obtain the following coupled EOM from the coefficients of 1 and $\hat{p} \cdot \sigma$:

$$\begin{aligned} & -\frac{p^2 A}{f} + \frac{2i\omega A'}{f} - \frac{i\omega^3 A f'}{2(p^2 - \frac{\omega^2}{f})f^3} + \frac{i p \omega^2 B f'}{2(p^2 - \frac{\omega^2}{f})f^{5/2}} \\ & - \frac{i\omega A f'}{2f^2} - \frac{\omega^2 A' f'}{2(p^2 - \frac{\omega^2}{f})f^2} + \frac{A' f'}{2f} \\ & + \frac{p\omega B' f'}{2(p^2 - \frac{\omega^2}{f})f^{3/2}} + A'' = 0, \end{aligned} \quad (4.17)$$

$$\begin{aligned} & -\frac{p^2 B}{f} + \frac{2i\omega B'}{f} - \frac{i\omega^3 B f'}{2(p^2 - \frac{\omega^2}{f})f^3} + \frac{i p \omega^2 A f'}{2(p^2 - \frac{\omega^2}{f})f^{5/2}} \\ & - \frac{i\omega B f'}{2f^2} - \frac{\omega^2 B' f'}{2(p^2 - \frac{\omega^2}{f})f^2} + \frac{B' f'}{2f} \\ & + \frac{p\omega A' f'}{2(p^2 - \frac{\omega^2}{f})f^{3/2}} + B'' = 0. \end{aligned} \quad (4.18)$$

We can easily decouple the EOM with the linear combination $A_{\pm}(\omega, p) = A(\omega, p) \pm B(\omega, p)$, which satisfies

$$\begin{aligned} & -\frac{p^2 A_{\pm}}{f} + \frac{2i\omega A'_{\pm}}{f} - \frac{i\omega^3 A_{\pm} f'}{2(p^2 - \frac{\omega^2}{f})f^3} \pm \frac{i p \omega^2 A_{\pm} f'}{2(p^2 - \frac{\omega^2}{f})f^{5/2}} \\ & - \frac{i\omega A_{\pm} f'}{2f^2} - \frac{\omega^2 A'_{\pm} f'}{2(p^2 - \frac{\omega^2}{f})f^2} + \frac{A'_{\pm} f'}{2f} \\ & \pm \frac{p\omega A'_{\pm} f'}{2(p^2 - \frac{\omega^2}{f})f^{3/2}} + A''_{\pm} = 0. \end{aligned} \quad (4.19)$$

The boundary condition from (4.13) reduces at $O(\partial^0)$ to

$$(\partial_z + i\omega)A_{\pm}(z \rightarrow 0) \rightarrow -\omega \mp p. \quad (4.20)$$

The other boundary condition at the horizon is a regularity condition for $A_{\pm}$. Without actually solving the equation, we can easily deduce the symmetry property $A_+(\omega, p, z) = A_-(\omega, -p, z)$. Note that the boundary correlator at $O(\partial^0)$ is given by

$$G_R^{(0)}(\omega, p) = A_+(\omega, p, z \rightarrow 0) \frac{1 + \hat{p} \cdot \sigma}{2} + A_-(\omega, p, z \rightarrow 0) \frac{1 - \hat{p} \cdot \sigma}{2}. \quad (4.21)$$

Equation (4.19) is solved numerically and the spectral function is extracted as

$$\begin{aligned} \rho^L &= \text{Im}[A_+(\omega, p, z \rightarrow 0)] \frac{1 + \hat{p} \cdot \sigma}{2} \\ &+ \text{Im}[A_-(\omega, p, z \rightarrow 0)] \frac{1 - \hat{p} \cdot \sigma}{2} \\ &\equiv \rho_+ \frac{1 + \hat{p} \cdot \sigma}{2} + \rho_- \frac{1 - \hat{p} \cdot \sigma}{2}. \end{aligned} \quad (4.22)$$

We have reinstated the superscript L for the left-handed fermionic operator. The structures $\frac{1 \pm \hat{p} \cdot \sigma}{2}$ have positive and negative helicity over chirality ratio, thus ρ_+ and ρ_- correspond to spectral functions of particle and plasmino modes respectively [34]. The particle mode will be used to model the left-handed half of the baryon. As an example, we show in Fig. 1 the particle spectral function ρ_+ as a function of ω for a given p for timelike momenta. This is to be compared to the vacuum counterpart [32]: $\rho_+ = 2\text{sgn}(\omega)\theta(\omega^2 - p^2)(\frac{\omega+p}{\omega-p})^{1/2}$, which shows a nonanalytic singularity at $\omega = p$. The presence of a medium softens the singularity. The polarization of the particle

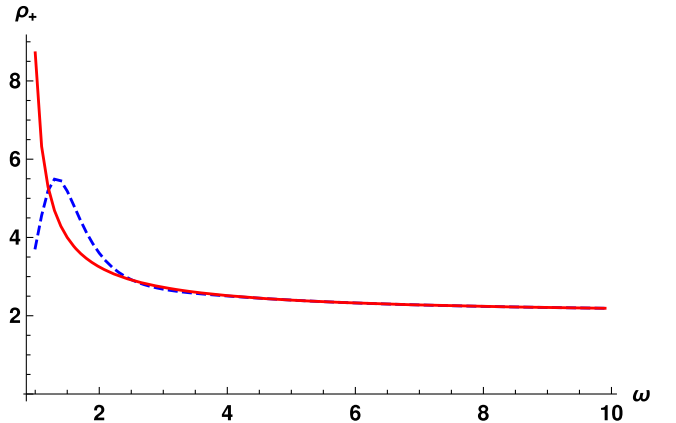


FIG. 1. ρ_+ versus ω for $p = 0.9$ plotted in dashed blue. The unit is set by $\pi T = 1$. We also plot the counterpart from vacuum spectral function in solid red, whose analytic form is known [32]: $\rho_+ = 2\text{sgn}(\omega)\theta(\omega^2 - p^2)(\frac{\omega+p}{\omega-p})^{1/2}$. The effect of the medium softens the singularity in the vacuum spectral function.

mode can be obtained by tracing with the Pauli matrices

$$\text{tr} \left[\rho_+ \frac{1 + \hat{p} \cdot \vec{\sigma}}{2} \sigma^i \right] = \rho_+ \hat{p}_i. \quad (4.23)$$

This is of course not surprising as it comes from spin-momentum enslavement for left-handed fermionic excitation. The right-handed counterpart is obtained by (2.8). Using that the equilibrium medium is invariant under parity; we have $\rho^L(\omega, \vec{p}) = \rho^R(\omega, -\vec{p})$. Noting that $\rho_{\pm}$ are also invariant under parity, we then have

$$\rho^R = \rho_+ \frac{1 - \hat{p} \cdot \vec{\sigma}}{2} + \rho_- \frac{1 + \hat{p} \cdot \vec{\sigma}}{2}. \quad (4.24)$$

It follows that there is no net polarization when we combine two opposite helicities to form a baryon.

Now we turn to G_1 , which satisfies

$$\begin{aligned} G_1'' + 2i \frac{\omega}{f} G_1' + \frac{f'}{2f} G_1' - i\omega \frac{f'}{2f^2} G_1 - \frac{p^2}{f} G_1 \\ + \frac{f'}{2f^{3/2}} \omega \frac{P \cdot \vec{\sigma}}{P^2} \left(G_1' + \frac{i\omega}{f} G_1 \right) + \left[-\frac{\partial_0^X}{f} G_0' + \frac{f'}{4f^2} \partial_0^X G_0 \right. \\ \left. + ip_i \frac{\partial_0^X}{f} G_0 + \frac{f'}{4f^{3/2}} \frac{P \cdot \vec{\sigma}}{P^2} i \partial_0^X \left(G_0' + \frac{2i\omega}{f} G_0 \right) \right. \\ \left. + \frac{f'}{2f^{3/2}} i\omega \left(\frac{D^X \cdot \vec{\sigma}}{-2P^2} + \frac{P \cdot D_X P \cdot \vec{\sigma}}{(P^2)^2} \right) \left(G_0' + \frac{i\omega}{f} G_0 \right) \right] \\ = 0. \end{aligned} \quad (4.25)$$

This is an inhomogeneous ordinary differential equation (ODE). The source terms involve X derivatives on G_0 . To calculate the derivatives, we first note that G_0 is the local bulk-boundary propagator. In the boosted AdS-Schwarzschild background, tubes of fluid velocities are related by a slow-varying boost. We would obtain the same G_0 if we choose a local rest frame in each tube. To take derivatives in a local tube, we need to express G_0 in the neighboring tubes in the frame of a local tube, to be denoted as G_0^u . Obviously G_0^u and G_0 are related by an infinitesimal boost. Denoting the boost velocity by u_i , we have the Lorentz boost up to $O(u)$,

$$dt' = dt - u_i dx_i, \quad dx_i' = dx_i - u_i dt, \quad (4.26)$$

with primed and unprimed coordinates corresponding to local rest frames of the neighboring tube and local tube. The Lorentz boost for 4-momentum is given by

$$\omega' = \omega - p_i u_i, \quad p_i' = p_i - u_i \omega. \quad (4.27)$$

We then have

$$\begin{aligned} G_0^u(\omega, p, z) &= G_0(\omega', p', z) \\ &= G_0(\omega, p, z) + \frac{\partial G_0}{\partial \omega} (-p_i u_i) + \frac{\partial G_0}{\partial p_i} (-u_i \omega). \end{aligned} \quad (4.28)$$

Apart from variation of fluid velocity, there is also variation of temperature. Since local temperature is fixed by $\pi T = b^{-1} = z_h^{-1}$, the variation of temperature can be achieved by scaling of coordinate $t \rightarrow \lambda t$, $x_i \rightarrow \lambda x_i$ and $z \rightarrow \lambda z$. The only effect of the scaling is to shift the horizon location. It follows that G_0 in the neighboring tube with different temperature, denoted as G_0^b , can be expressed as an infinitesimal scaling of G_0 in a local tube,

$$\begin{aligned} G_0^b(\omega, p, z) &= G_0 \left(\frac{\omega}{\lambda}, \frac{p}{\lambda}, \lambda z \right) \\ &= G_0(\omega, p, z) + \delta b \left(-\omega \frac{\partial}{\partial \omega} - p_i \frac{\partial}{\partial p_i} + \frac{\partial}{\partial z} \right) \\ &\quad \times G_0(\omega, p, z). \end{aligned} \quad (4.29)$$

In the second line, we have specialized to infinitesimal variation and used $\delta \lambda = \frac{\delta b}{b} = \delta b$ for our choice $b = 1$ in the local tube.

Now we are ready to calculate the derivative terms. Using

$$\begin{aligned} \frac{\partial G_0}{\partial \omega} &= \frac{\partial A}{\partial \omega} + \frac{\partial B}{\partial \omega} \hat{p} \cdot \vec{\sigma}, \\ \frac{\partial G_0}{\partial p_i} &= \frac{\partial A}{\partial p} \hat{p}_i + \left(\frac{\partial B}{\partial p} - \frac{B}{p} \right) \hat{p}_i \hat{p} \cdot \vec{\sigma} + \frac{B}{p} \sigma_i, \end{aligned} \quad (4.30)$$

we obtain the following derivative terms:

$$\begin{aligned} \partial_\mu^X G_0^u &= \left(\frac{\partial A}{\partial \omega} + \frac{\partial B}{\partial \omega} \hat{p} \cdot \vec{\sigma} \right) (-p_i \partial_\mu u_i) \\ &\quad + \left(\frac{\partial A}{\partial p} \hat{p}_i + \left(\frac{\partial B}{\partial p} - \frac{B}{p} \right) \hat{p}_i \hat{p} \cdot \vec{\sigma} + \frac{B}{p} \sigma_i \right) (-\omega \partial_\mu u_i) \\ &= \left(\left(\frac{\partial A}{\partial \omega} + \frac{\partial A}{\partial p} \frac{\omega}{p} \right) + \left(\frac{\partial B}{\partial \omega} + \left(\frac{\partial B}{\partial p} - \frac{B}{p} \right) \frac{\omega}{p} \right) \hat{p} \cdot \vec{\sigma} \right) \\ &\quad \times (-p_i \partial_\mu u_i) + \frac{B}{p} \sigma_i (-\omega \partial_\mu u_i). \end{aligned} \quad (4.31)$$

$$\begin{aligned} \partial_\mu^X G_0^b &= \partial_\mu b \left(-\omega \left(\frac{\partial A}{\partial \omega} + \frac{\partial B}{\partial \omega} \hat{p} \cdot \vec{\sigma} \right) \right. \\ &\quad \left. - p_i \left(\frac{\partial A}{\partial p} \hat{p}_i + \left(\frac{\partial B}{\partial p} - \frac{B}{p} \right) \hat{p}_i \hat{p} \cdot \vec{\sigma} + \frac{B}{p} \sigma_i \right) \right. \\ &\quad \left. + z(A' + B' \hat{p} \cdot \vec{\sigma}) \right) \\ &= \left(\left(\frac{\partial A}{\partial \omega} + \frac{\partial A}{\partial p} \frac{p}{\omega} - \frac{zA'}{\omega} \right) \right. \\ &\quad \left. + \left(\frac{\partial B}{\partial \omega} + \frac{\partial B}{\partial p} \frac{p}{\omega} - \frac{zB'}{\omega} \right) \hat{p} \cdot \vec{\sigma} \right) (-\omega \partial_\mu b). \end{aligned} \quad (4.32)$$

We have split the terms into those with the same structure as G_0 and those with new structures. The possible gradients

appearing in the new structure are $\partial_0 u_i$, $\partial_j u_i = \epsilon^{ijk} \omega_k + \sigma_{ij} + \frac{1}{3} \delta_{ij} \partial_k u_k$. The first one is acceleration, which induces simultaneously the temperature gradient by hydrodynamic equation $\partial_i b = \partial_0 u_i$ [33]. The second one includes vorticity, shear, and bulk stress. We again turn off bulk stress to focus on spin polarization. The derivative terms also enter the boundary condition as

$$\begin{aligned} (\partial_z + i\omega)G_1^a &= \frac{1}{2} \partial_0^X G_0^a = [(\cdots)1 + (\cdots)\hat{p} \cdot \vec{\sigma}] \hat{p}_i \partial_0 u_i \\ &\quad + \frac{B}{2p} \sigma_i (-\omega \partial_0 u_i), \\ (\partial_z + i\omega)G_1^b &= \frac{1}{2} \partial_0^X G_0^b = 0. \end{aligned} \quad (4.33)$$

The second line vanishes because we have turned off the bulk stress $\partial_0 b = \frac{1}{3} \partial_i u^i = 0$ [33]. From the Dirac structure of EOM (4.25) and boundary condition (4.33), we can deduce the Dirac structure of G_1 for different sources separately. For acceleration, the EOM and boundary condition are modified by a new structure, which can be written schematically as

$$\begin{aligned} \square G_1 &= [(\cdots)1 + (\cdots)\hat{p} \cdot \vec{\sigma}] \hat{p}_i \partial_0 u_i \\ &\quad + [C_1^a + C_2^a \hat{p} \cdot \vec{\sigma}] \partial_0 u_i \sigma_i, \end{aligned} \quad (4.34)$$

with $\square$ being shorthand notation for the operator acting on G_1 in (4.25). $C_{1,2}^a$ are to be evaluated from the source term in (4.25) by using (4.31). Equation (4.34) can be solved by the following ansatz:

$$G_1 = [(\cdots)1 + (\cdots)\hat{p} \cdot \vec{\sigma}] \hat{p}_i \partial_0 u_i + [D_1^a + D_2^a \hat{p} \cdot \vec{\sigma}] \partial_0 u_i \sigma_i. \quad (4.35)$$

Note that the operator $\square$ closes on the two structures $[(\cdots)1 + (\cdots)\hat{p} \cdot \vec{\sigma}]$ and $[D_1^a + D_2^a \hat{p} \cdot \vec{\sigma}] \partial_0 u_i \sigma_i$ separately. It follows that we can solve for the two parts separately. The boundary condition for this case is determined by merging the two derivatives from acceleration and temperature gradient in (4.31) as

$$(\partial_z + i\omega)G_1 = [(\cdots)1 + (\cdots)\hat{p} \cdot \vec{\sigma}] \hat{p}_i \partial_0 u_i - \frac{\bar{B}}{2} \partial_0 u_i \sigma_i, \quad (4.36)$$

with $\bar{B} = \frac{\omega}{p} B$.

We will not solve for the structure $[(\cdots)1 + (\cdots)\hat{p} \cdot \vec{\sigma}] \hat{p}_i \partial_0 u_i$ in G_1 for the following reason: the spin dependent term $(\cdots)\hat{p} \cdot \vec{\sigma} \hat{p}_i \partial_0 u_i$ is parity odd. By using (2.8), we deduce an opposite contribution from the right-handed counterpart. It follows that the correction cancels out in spin polarization of the baryon. In contrast, the spin independent term $\hat{p}_i \partial_0 u_i$ does add up. We will

focus on the spin dependent contributions. This allows us to drop the structure $[(\cdots)1 + (\cdots)\hat{p} \cdot \vec{\sigma}] \hat{p}_i \partial_0 u_i$ and work with a simplified EOM

$$\square G_1 = [C_1^a + C_2^a \hat{p} \cdot \vec{\sigma}] \partial_0 u_i \sigma_i, \quad (4.37)$$

with

$$(\partial_z + i\omega)G_1 = -\frac{\bar{B}}{2} \partial_0 u_i \sigma_i. \quad (4.38)$$

G_1 can be solved by

$$G_1 = [D_1^a + D_2^a \hat{p} \cdot \vec{\sigma}] \partial_0 u_i \sigma_i. \quad (4.39)$$

To proceed, we split the differential operator as $\square = \square_1 + \square_2 \hat{p} \cdot \vec{\sigma}$. Plugging (4.39) into (4.37), we obtain

$$\begin{aligned} \square_1 D_1^a + \square_2 D_2^a &= C_1^a, \\ \square_1 D_2^a + \square_2 D_1^a &= C_2^a. \end{aligned} \quad (4.40)$$

The boundary condition from (4.38) reduces to

$$(\partial_z + i\omega)D_1^a = -\frac{\bar{B}}{2}, \quad (\partial_z + i\omega)D_2^a = 0. \quad (4.41)$$

The other boundary condition at the horizon is fixed as follows: by analyzing the asymptotic behavior of G_1 , we find two independent series in the form of $(1-z)^{-i\omega}$ and powers of $(1-z)$ (including also logarithms). The former corresponds to the outfalling solution and the latter is a variation of the regular solution. We keep the latter. This fixes the solution up to two independent integration constants to be fixed by the boundary condition (4.41) at $z=0$. In fact we do not have to tune the integration constants: a new inhomogeneous solution can be obtained by adding homogeneous solutions, which can be chosen as $A \partial_0 u_i \sigma_i + B \hat{p} \cdot \vec{\sigma} \partial_0 u_i \sigma_i$. Crucially, unlike in the solution of G_0 , here A and B can satisfy arbitrary boundary condition at $z=0$, so the two independent homogeneous solutions precisely match the number of integration constants. In practice, we integrate (4.40) using arbitrary integration constants from the horizon, then find a suitable homogeneous solution to be added such that (4.41) is satisfied.

The cases of shear is analyzed similarly. The counterpart of (4.34) is given by

$$\begin{aligned} \square G_1 &= [(\cdots)1 + (\cdots)\hat{p} \cdot \vec{\sigma}] \hat{p}_i \hat{p}_j \sigma_{ij} \\ &\quad + [C_1^a + C_2^a \hat{p} \cdot \vec{\sigma}] \hat{p}_j \sigma_{ij} \sigma_i. \end{aligned} \quad (4.42)$$

Since shear arises from $\partial_i^X G_0^a$, the boundary condition for the shear case follows from (4.33) trivially as

$$(\partial_z + i\omega)G_1 = 0. \quad (4.43)$$

We can use the same argument as in the acceleration case. The spin dependent structure $\hat{p} \cdot \vec{\sigma} \hat{p}_i \hat{p}_j \sigma_{ij}$ is parity odd, so its contribution to spin polarization will be canceled by an

opposite contribution from a right-handed counterpart due to (2.8). Thus we can drop the structure $[(\cdots)1 + (\cdots)\hat{p} \cdot \vec{\sigma}] \hat{p}_i \hat{p}_j \sigma_{ij}$ and use a simplified ansatz for G_1 as

$$G_1 = [D_1^\sigma + D_2^\sigma \hat{p} \cdot \vec{\sigma}] \hat{p}_j \sigma_{ij} \sigma_i. \quad (4.44)$$

One might hope similar simplification can be used for the case of vorticity. It turns out not to be the case. The source terms in (4.25) can be written as

$$\begin{aligned} & E_1 \epsilon^{ijk} \omega_i \hat{p}_j \sigma_k + E_2 \hat{p} \cdot \vec{\sigma} \epsilon^{ijk} \omega_i \hat{p}_j \sigma_k \\ & + E_3 \sigma_j \epsilon^{ijk} \omega_k \sigma_i + E_4 \hat{p} \cdot \vec{\sigma} \sigma_j \epsilon^{ijk} \omega_k \sigma_i \\ & = i[(E_1 - 2E_4) \hat{p} \cdot \vec{\sigma} \omega \cdot \vec{\sigma} - E_1 \hat{p} \cdot \omega \\ & + (E_2 - 2E_3) \omega \cdot \vec{\sigma} - E_2 \hat{p} \cdot \vec{\sigma} \hat{p} \cdot \omega] \\ & \equiv [C_1^\omega + C_2^\omega \hat{p} \cdot \vec{\sigma}] \omega_i \sigma_i + [C_1^{\omega\parallel} + C_2^{\omega\parallel} \hat{p} \cdot \vec{\sigma}] \hat{p} \cdot \vec{\omega}. \end{aligned} \quad (4.45)$$

The resulting EOM is given by

$$\square G_1 = [C_1^\omega + C_2^\omega \hat{p} \cdot \vec{\sigma}] \omega_i \sigma_i + [C_1^{\omega\parallel} + C_2^{\omega\parallel} \hat{p} \cdot \vec{\sigma}] \hat{p} \cdot \vec{\omega}, \quad (4.46)$$

with the same boundary condition as (4.43). This is to be solved by

$$G_1 = [D_1^\omega + D_2^\omega \hat{p} \cdot \vec{\sigma}] \omega_i \sigma_i + [D_1^{\omega\parallel} + D_2^{\omega\parallel} \hat{p} \cdot \vec{\sigma}] \hat{p} \cdot \vec{\omega}. \quad (4.47)$$

Crucially, $\hat{p} \cdot \vec{\sigma} \hat{p} \cdot \vec{\omega}$ is parity even, unlike the other two cases. It follows that both structures contribute to the spin polarization of the baryon upon combining the left- and right-handed components.

With G_1 solved, the resulting boundary correlator is given by

$$\begin{aligned} \delta G_R = & [D_1^a + D_2^a \hat{p} \cdot \vec{\sigma}] \partial_0 u_i \sigma_i + [D_1^\sigma + D_2^\sigma \hat{p} \cdot \vec{\sigma}] \hat{p}_j \sigma_{ij} \sigma_i \\ & + [D_1^\omega + D_2^\omega \hat{p} \cdot \vec{\sigma}] \omega_i \sigma_i + [D_1^{\omega\parallel} + D_2^{\omega\parallel} \hat{p} \cdot \vec{\sigma}] \hat{p} \cdot \vec{\omega}, \end{aligned} \quad (4.48)$$

where it is understood that the limit $z \rightarrow 0$ is taken. It is instructive to extract the polarization dependence by tracing (4.48) with σ_k to give polarization of the spectral function from (2.4)

$$\begin{aligned} \frac{1}{2} \text{tr}[\delta \rho^L \sigma_k] = & 2 \left[(\text{Im}[D_1^a] \partial_0 u_k - \text{Re}[D_2^a] \epsilon^{ijk} \hat{p}_j \partial_0 u_i) \right. \\ & + (\text{Im}[D_1^\sigma] \hat{p}_j \sigma_{kj} - \text{Re}[D_2^\sigma] \epsilon^{ijk} \hat{p}_j \hat{p}_l \sigma_{il}) \\ & + (\text{Im}[D_1^\omega] \omega_k - \text{Re}[D_2^\omega] \epsilon^{ijk} \hat{p}_j \omega_i) \\ & \left. + \text{Im}[D_1^{\omega\parallel}] \omega_k^\parallel \right], \end{aligned} \quad (4.49)$$

with $\omega_k^\parallel \equiv \hat{p}_k \hat{p} \cdot \omega$. We have reinstated the superscript L for the left-handed fermionic operator. The counterpart for the right-handed operator is obtained by parity transformation (2.8). Note that $\delta \rho^{L/R}$ arise from $D_{\text{probe}}^{(1)}$, which ultimately reduces to the structures $\partial_0 u_k$, $\epsilon^{ijk} \hat{p}_j \partial_0 u_i$ etc. Applying parity transformation to these structures, we obtain

$$\begin{aligned} \frac{1}{2} \text{tr}[\delta \rho^R \sigma_k] = & 2 \left[(-\text{Im}[D_1^a] \partial_0 u_k - \text{Re}[D_2^a] \epsilon^{ijk} \hat{p}_j \partial_0 u_i) \right. \\ & + (-\text{Im}[D_1^\sigma] \hat{p}_j \sigma_{kj} - \text{Re}[D_2^\sigma] \epsilon^{ijk} \hat{p}_j \hat{p}_l \sigma_{il}) \\ & + (\text{Im}[D_1^\omega] \omega_k + \text{Re}[D_2^\omega] \epsilon^{ijk} \hat{p}_j \omega_i) \\ & \left. + \text{Im}[D_1^{\omega\parallel}] \omega_k^\parallel \right]. \end{aligned} \quad (4.50)$$

Combining contributions from both helicities, we obtain

$$\begin{aligned} \frac{1}{2} \text{tr}[(\delta \rho^L + \delta \rho^R) \sigma_k] = & 4 \left(-\text{Re}[D_2^a] \epsilon^{ijk} \hat{p}_j \partial_0 u_i - \text{Re}[D_2^\sigma] \epsilon^{ijk} \hat{p}_j \hat{p}_l \sigma_{il} \right. \\ & \left. + \text{Im}[D_1^\omega] \omega_k + \text{Im}[D_1^{\omega\parallel}] \omega_k^\parallel \right). \end{aligned} \quad (4.51)$$

We show in Fig. 2 numerical results for the coefficients in (4.51) for timelike momenta for three values of p .

C. Gradient correction to the Dirac operator

Now we consider the gradient correction to the Dirac operator, which encodes gradient correction to the medium density matrix $D_{\text{fluid}}^{(1)}$,

$$\Gamma^M \nabla_M \psi = \Gamma^M \left(\partial_M + \frac{1}{4} \omega_{Mab} \gamma^{ab} \right) \psi = 0, \quad (4.52)$$

with

$$\omega_{Mab} = \eta_{cb} \left(e_a^N \partial_M e_N^c - e_a^N \Gamma_{MN}^K e_K^c \right). \quad (4.53)$$

There are two possible corrections to the Dirac operator at $O(\partial_X)$: one from the obvious $O(\partial_X)$ metric correction, which can be attributed to a steady state effect of the medium, and the other less obvious one from the derivative term $\eta_{cb} e_a^N \partial_M e_N^c$ on $O(\partial_X^0)$ vielbein in ω_{Mab} . The form of the correction in the latter case suggests it originates from the choice of the local Lorentz frame, which exists in a local equilibrium state of the medium. Following the logic, we shall denote the corresponding medium density matrices as $D_{\text{fluid}}^{(1)\text{ss}}$ and $D_{\text{fluid}}^{(1)\text{le}}$ respectively. We start with the calculation of the latter contribution. We define $\delta \omega_{ab} = \eta_{cb} e_a^N \partial_M \delta e_N^c$ with δe_N^c coming from variation of either fluid velocity

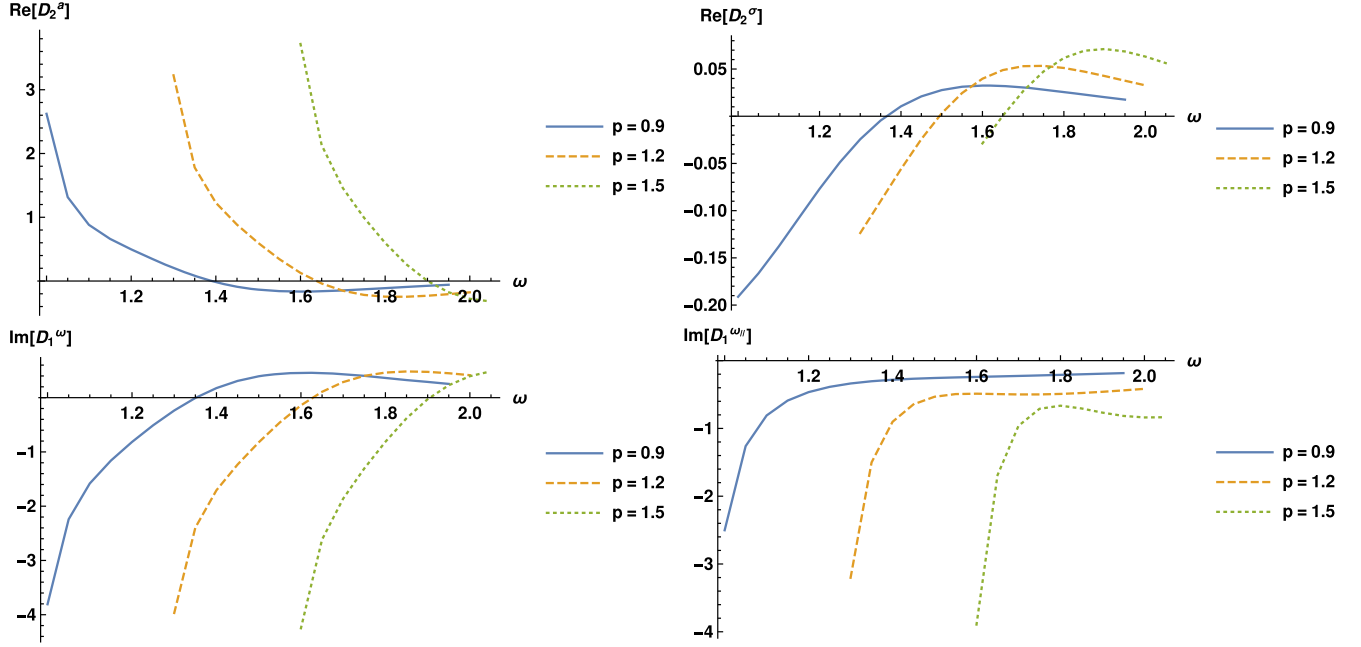


FIG. 2. The coefficients of gradient correction defined in (4.51) for local equilibrium state versus ω for three values of p . The unit is set by $\pi T = 1$. The magnitude of coefficients generically grow as the light cone is approached $\omega \rightarrow p$, which is reminiscent of the singular behavior in a vacuum correlator. The coefficients are suppressed at large ω .

or temperature. This way we have $\eta_{cb} e_a^N \partial_M e_N^c = \frac{\delta \omega_{ab}}{\delta x^M}$ with M taking only boundary coordinates. The vielbeins are chosen as⁵

$$\begin{aligned} e_i^t &= \frac{z}{f^{1/2}}, & e_{\hat{z}}^t &= -\frac{z}{f^{1/2}}, & e_i^z &= z, & e_{\hat{z}}^z &= z f^{1/2}, & e_i^t &= u_i z, & e_{\hat{z}}^t &= -e_{\hat{z}}^i = u_i \frac{z}{f^{1/2}}, \\ e_i^{\hat{t}} &= \frac{f^{1/2}}{z}, & e_{\hat{z}}^{\hat{t}} &= \frac{1}{z f^{1/2}}, & e_i^{\hat{z}} &= \frac{1}{z}, & e_{\hat{z}}^{\hat{z}} &= \frac{1}{z f^{1/2}}, & e_i^{\hat{t}} &= -u_i \frac{f^{1/2}}{z}, & e_{\hat{z}}^{\hat{t}} &= -\frac{u_i}{z}, \end{aligned} \quad (4.54)$$

where we have only kept up to $O(u_i)$ for our purposes. We then have the following $\delta \omega_{ab}$:

$$\delta \omega_{i\hat{t}} = u_i f^{1/2}, \quad \delta \omega_{\hat{z}\hat{t}} = -\frac{u_i}{f^{1/2}}, \quad \delta \omega_{z\hat{t}} = \frac{u_i}{f^{1/2}}, \quad \delta \omega_{\hat{z}\hat{z}} = \frac{\delta f}{f}. \quad (4.55)$$

The gradient correction to the Dirac operator $\delta(\Gamma^M \nabla_M)$ is then given by

$$\begin{aligned} & \frac{1}{4} \left[z \partial_0 u_i \sigma_i \begin{pmatrix} -1 & -i \\ i & -1 \end{pmatrix} + z \sigma_j \sigma_i \partial_j u_i \begin{pmatrix} -1 & i(1+f) \\ i(1+f) & 1 \end{pmatrix} \frac{1}{f^{1/2}} - z^2 \frac{f'}{f} \frac{\partial_i b}{b} \begin{pmatrix} -\sigma_i & 0 \\ 0 & -\sigma_i \end{pmatrix} \right] \\ &= \frac{1}{4} \left[z \partial_0 u_i \sigma_i \begin{pmatrix} -1 & -i \\ i & -1 \end{pmatrix} + z 2i \omega_i \sigma_i \begin{pmatrix} -1 & i(1+f) \\ i(1+f) & 1 \end{pmatrix} \frac{1}{f^{1/2}} - z^2 \frac{f'}{f} \partial_0 u_i \begin{pmatrix} -\sigma_i & 0 \\ 0 & -\sigma_i \end{pmatrix} \right], \end{aligned} \quad (4.56)$$

where we have used $\frac{\partial_i b}{b} = \partial_0 u_i$ and $\partial_j u_i \sigma_j \sigma_i = -2i \omega_k \sigma_k$ from the vanishing of bulk stress. Note that the contribution from shear stress drops out at this step. We have also dropped the temporal derivative of temperature in (4.56) as it is related to bulk stress as $\partial_0 b = \partial_k u_k$ [33]. Note that the correction can be treated as a constant in a given fluid element, so that the Wigner transform essentially reduces to the Fourier transform. We then easily obtain the following EOM:

$$-P \cdot \sigma \chi_L + \frac{i\omega}{f^{1/2}} \chi_R + f^{1/2} \chi'_R - \frac{i}{4} \partial_0 u_i \sigma_i z \chi_L - \frac{1}{4} \partial_0 u_i \sigma_i \left(z - z^2 \frac{f'}{f} \right) \chi_R - \frac{z}{2f^{1/2}} \omega_i \sigma_i (\chi_L + i \chi_R) - \frac{z f^{1/2}}{2} \omega_i \sigma_i \chi_L = 0, \quad (4.57)$$

⁵We stress that the following choice is not contradictory with the asymptotic condition of vielbein (3.14) written in a fluid tube. The condition is satisfied in the local rest frame of a given tube.

$$-P \cdot \vec{\sigma} \chi_R - \frac{i\omega}{f^{1/2}} \chi_L - f^{1/2} \chi'_L + \frac{1}{4} \partial_0 u_i \sigma_i z \chi_R - \frac{1}{4} \partial_0 u_i \sigma_i \left(z - z^2 \frac{f'}{f} \right) \chi_L - \frac{z}{2f^{1/2}} \omega_i \sigma_i (-i\chi_L + \chi_R) - \frac{zf^{1/2}}{2} \omega_i \sigma_i \chi_R = 0. \quad (4.58)$$

As before, we shall derive one second-order ODE from the two first-order ODEs. We do so separately for the sources of acceleration and vorticity. For acceleration, we find the following EOM for G (from the counterpart for χ_R) as

$$\square G_1 + \left[\frac{ip_i \partial_0 u_i z G_0}{2f} - \frac{\partial_0 u_i \sigma_i G_0}{4\sqrt{f}} + \frac{\partial_0 u_i \sigma_i z G_0 f'}{2f^{3/2}} - \frac{\partial_0 u_i \sigma_i \sigma_i z G_0 f'}{2f^{3/2}} + \frac{i\partial_0 u_i \sigma_i P \cdot \vec{\sigma} (i\omega G_0 + fG'_0)}{4fP^2} \right. \\ \left. - \omega \frac{f'}{2f^2} \left(\frac{-P \cdot \vec{\sigma} \left(-\frac{1}{4} \partial_0 u_i \sigma_i \left(z - z^2 \frac{f'}{f} \right) G_0 \right)}{P^2} + \frac{\left(i \frac{\partial_0 u_i \sigma_i z}{4P^2} - \frac{ip_i \partial_0 u_i z P \cdot \vec{\sigma}}{2P^4} \right) (i\omega G_0 + fG'_0)}{\sqrt{f}} \right) \right] = 0. \quad (4.59)$$

Following the procedure as the previous sections, we do not keep terms with the same Dirac structure as G_0 in the source. These include terms proportional to $p_i \partial_0 u_i$. Using

$$\partial_0 u_i \sigma_i \hat{p} \cdot \vec{\sigma} = 2\hat{p}_i \partial_0 u_i - \hat{p} \cdot \vec{\sigma} \partial_0 u_i \sigma_i, \quad (4.60)$$

and again dropping terms proportional to $\hat{p}_i \partial_0 u_i$, the EOM can be written as

$$\square G_1 = [C_3^a + C_4^a \hat{p} \cdot \vec{\sigma}] \partial_0 u_i \sigma_i. \quad (4.61)$$

It is formally the same as (4.37), but with a slightly different boundary condition, which we now derive. Applying Fourier transform to (4.2), and using (4.57), we obtain

$$\left(P \cdot \sigma + \frac{i}{4} \partial_0 u_i \sigma_i z \right) \chi_L(\omega, \vec{p}, z) = \left(f^{1/2} \partial_z + \frac{i\omega}{f^{1/2}} - \frac{1}{4} \partial_0 u_i \sigma_i \left(z - z^2 \frac{f'}{f} \right) \right) G(\omega, \vec{p}, z) \chi_L^b(\omega, \vec{p}). \quad (4.62)$$

Taking $z \rightarrow 0$, we simply find $P \cdot \sigma = (\partial_z + i\omega)G(\omega, \vec{p}, z \rightarrow 0)$. The same boundary condition has been satisfied by G_0 , so that we have

$$(\partial_z + i\omega)G_1(\omega, \vec{p}, z \rightarrow 0) = 0. \quad (4.63)$$

We will not repeat the derivation for the case of vorticity. The resulting EOM is given by

$$\square G_1 + \left[\frac{z\omega\omega_i \sigma_i G_0}{f^2} + \frac{i\epsilon^{ijk} \omega_i p_j \sigma_k G_0}{f^{3/2}} (1+f) - \frac{i\omega_i \sigma_i G_0}{2f} + \frac{iz\omega_i \sigma_i G_0 f'}{4f^2} - \frac{iz\omega_i \sigma_i G'_0}{f} \right. \\ \left. + \frac{\omega_i \sigma_i P \cdot \vec{\sigma} (i\omega G_0 + fG'_0)}{2f^{3/2} P^2} + \frac{\omega_i \sigma_i P \cdot \vec{\sigma} (i\omega G_0 + fG'_0)}{2\sqrt{f} P^2} - \frac{z\omega_i \sigma_i P \cdot \vec{\sigma} (i\omega G_0 + fG'_0) f'}{4f^{5/2} P^2} (1-f) \right. \\ \left. - \omega \frac{f'}{2f^2} \left(\frac{izP \cdot \vec{\sigma} \omega_i \sigma_i G_0}{2\sqrt{f} P^2} + \frac{1}{\sqrt{f}} \left(-\frac{z\omega_i \sigma_i (1+f)}{2\sqrt{f} P^2} + \frac{iz\epsilon^{ijk} \omega_i p_j \sigma_k (1+f) P \cdot \vec{\sigma}}{\sqrt{f} P^4} \right) \right) \right] = 0. \quad (4.64)$$

Again we need to treat the vorticity case carefully, which is expected to lead to two independent contributions to polarization. Using $i\epsilon^{ijk} \omega_i p_j \sigma_k = \vec{p} \cdot \vec{\omega} - \vec{p} \cdot \vec{\sigma} \omega_i \sigma_i = -\vec{p} \cdot \vec{\omega} + \omega_i \sigma_i \vec{p} \cdot \vec{\sigma}$, we can rewrite the EOM as

$$\square G_1 = [C_3^\omega + C_4^\omega \hat{p} \cdot \vec{\sigma}] \omega_i \sigma_i + [C_3^{\omega_\parallel} + C_4^{\omega_\parallel} \hat{p} \cdot \vec{\sigma}] \hat{p}_i \omega_i. \quad (4.65)$$

The boundary condition is derived similarly as (4.62) to give

$$(\partial_z + i\omega)G_1(\omega, \vec{p}, z \rightarrow 0) = 0. \quad (4.66)$$

Finally we consider the steady state correction from metric at $O(\partial_X)$, which can modify the spin connection through either the vielbein or the affine connection. The following components of vielbein will be corrected:

$$\delta e_j^i = \frac{F}{z} \sigma_{ij}, \quad \delta e_i^{\hat{t}} = \delta e_i^{\hat{z}} = -\frac{1}{f^{1/2}} \partial_0 u_i, \quad (4.67)$$

which give rise to the following coupled EOM:

$$\begin{aligned} & - \left(P \cdot \sigma + \frac{1}{2} p_j \sigma_{ij} \sigma_i F + \left(\frac{i}{2} - i \frac{zf'}{4f} \right) \partial_0 u_i \sigma_i \right) \chi_L \\ & + \left(\frac{1}{4} - \frac{zf'}{4f} \right) \partial_0 u_i \sigma_i \chi_R - iz \partial_0 u_i \sigma_i \chi'_L \\ & + i \frac{\omega}{f^{1/2}} \chi_R + f^{1/2} \chi'_R = 0, \\ & - \left(P \cdot \bar{\sigma} - \frac{1}{2} p_j \sigma_{ij} \sigma_i F - \left(\frac{i}{2} - i \frac{zf'}{4f} \right) \partial_0 u_i \sigma_i \right) \chi_R \\ & + \left(\frac{1}{4} - \frac{zf'}{4f} \right) \partial_0 u_i \sigma_i \chi_L + iz \partial_0 u_i \sigma_i \chi'_R \\ & - i \frac{\omega}{f^{1/2}} \chi_L - f^{1/2} \chi'_L = 0. \end{aligned} \quad (4.68)$$

We will decouple the equations for the source of shear and acceleration separately. For the shear, we find

$$\begin{aligned} \square G_1 + \frac{\sigma_{ij} \sigma_i}{4f^{5/2} P^2} p_j (-i\omega^2 F G_0 f' + 2i\omega f^{3/2} P \cdot \bar{\sigma} G_0 F' \\ - \omega f F f' G'_0 + 2f^{5/2} P \cdot \bar{\sigma} F' G'_0) = 0, \end{aligned} \quad (4.69)$$

with the following boundary condition:

$$(\partial_z + i\omega) G_1(\omega, \vec{p}, z \rightarrow 0) = 0. \quad (4.70)$$

For the acceleration, the decoupling is complicated by the presence of terms $iz \partial_0 u_i \sigma_i \chi'_{L/R}$ in (4.68). Nevertheless the decoupling can be done perturbatively in the source. The resulting EOM is lengthy and not shown explicitly. The corresponding boundary condition is derived as

$$\begin{aligned} & \left(p \cdot \sigma + \frac{i}{2} \partial_0 u_i \sigma_i \right) \\ & = \left(\partial_z + i\omega + \frac{1}{4} \partial_0 u_i \sigma_i \right) G(\omega, \vec{p}, z \rightarrow 0), \\ & \Rightarrow \frac{i}{2} \partial_0 u_i \sigma_i \\ & = (\partial_z + i\omega) G_1(\omega, \vec{p}, z \rightarrow 0) + \frac{1}{4} \partial_0 u_i \sigma_i G_0(\omega, \vec{p}, z \rightarrow 0). \end{aligned} \quad (4.71)$$

D. Retarded correlator from gradient correction to the medium

Now we solve the corresponding EOMs and extract the boundary correlator. The procedure is similar to Sec. IV B. The following ansatz will be used:

$$\begin{aligned} G_1 &= (D_3^a + \hat{p} \cdot \vec{\sigma} D_4^a) \partial_0 u_i \sigma_i, \\ G_1 &= (D_3^\omega + \hat{p} \cdot \vec{\sigma} D_4^\omega) \omega_i \sigma_i + (D_3^{\omega\parallel} + \hat{p} \cdot \vec{\sigma} D_4^{\omega\parallel}) \hat{p}_i \omega_i, \\ G_1 &= (D_5^a + \hat{p} \cdot \vec{\sigma} D_6^a) \partial_0 u_i \sigma_i, \\ G_1 &= (D_5^\sigma + \hat{p} \cdot \vec{\sigma} D_6^\sigma) \hat{p}_j \sigma_{ij} \sigma_i, \end{aligned} \quad (4.72)$$

with the subscripts 3,4 and 5,6 corresponding to local equilibrium and steady state effect respectively as argued below (4.53). Collecting all gradient corrections to the Dirac operator, we can extract the polarization dependence of the spectral function as

$$\begin{aligned} \frac{1}{2} \text{tr}[\delta \rho^L \sigma_k] &= 2 \left[\text{Im}[D_3^a + D_5^a] \partial_0 u_k - \text{Re}[D_4^a + D_6^a] \epsilon^{ijk} \hat{p}_j \partial_0 u_i \right. \\ &\quad + \text{Im}[D_5^\sigma] \hat{p}_j \sigma_{kj} - \text{Re}[D_6^\sigma] \epsilon^{ijk} \hat{p}_j \hat{p}_l \sigma_{il} \\ &\quad + \text{Im}[D_3^\omega] \omega_k - \text{Re}[D_4^\omega] \epsilon^{ijk} \hat{p}_j \omega_i \\ &\quad \left. + \text{Im}[D_3^{\omega\parallel}] \omega_k^\parallel \right]. \end{aligned} \quad (4.73)$$

We have reinstated the superscript L for the left-handed operator. The counterpart for the right-handed operator is obtained by (2.8) as before,

$$\begin{aligned} \frac{1}{2} \text{tr}[\delta \rho^R \sigma_k] &= 2 \left[-\text{Im}[D_3^a + D_5^a] \partial_0 u_k - \text{Re}[D_4^a + D_6^a] \epsilon^{ijk} \hat{p}_j \partial_0 u_i \right. \\ &\quad - \text{Im}[D_5^\sigma] \hat{p}_j \sigma_{kj} - \text{Re}[D_6^\sigma] \epsilon^{ijk} \hat{p}_j \hat{p}_l \sigma_{il} \\ &\quad \left. + \text{Im}[D_3^\omega] \omega_k + \text{Re}[D_4^\omega] \epsilon^{ijk} \hat{p}_j \omega_i + \text{Im}[D_3^{\omega\parallel}] \omega_k^\parallel \right]. \end{aligned} \quad (4.74)$$

The combined spectral function for the baryon is given by

$$\begin{aligned} & \frac{1}{2} \text{tr}[(\delta \rho^L + \delta \rho^R) \sigma_k] \\ &= 4 \left[-\text{Re}[D_4^a + D_6^a] \epsilon^{ijk} \hat{p}_j \partial_0 u_i - \text{Re}[D_6^\sigma] \epsilon^{ijk} \hat{p}_j \hat{p}_l \sigma_{il} \right. \\ &\quad \left. + \text{Im}[D_3^\omega] \omega_k + \text{Im}[D_3^{\omega\parallel}] \omega_k^\parallel \right]. \end{aligned} \quad (4.75)$$

We show in Fig. 3 the coefficient functions versus ω for three values of p .

E. Summary of gradient corrections and discussion

We summarize gradient contributions to the polarization dependence of the spectral function in Table I. We show

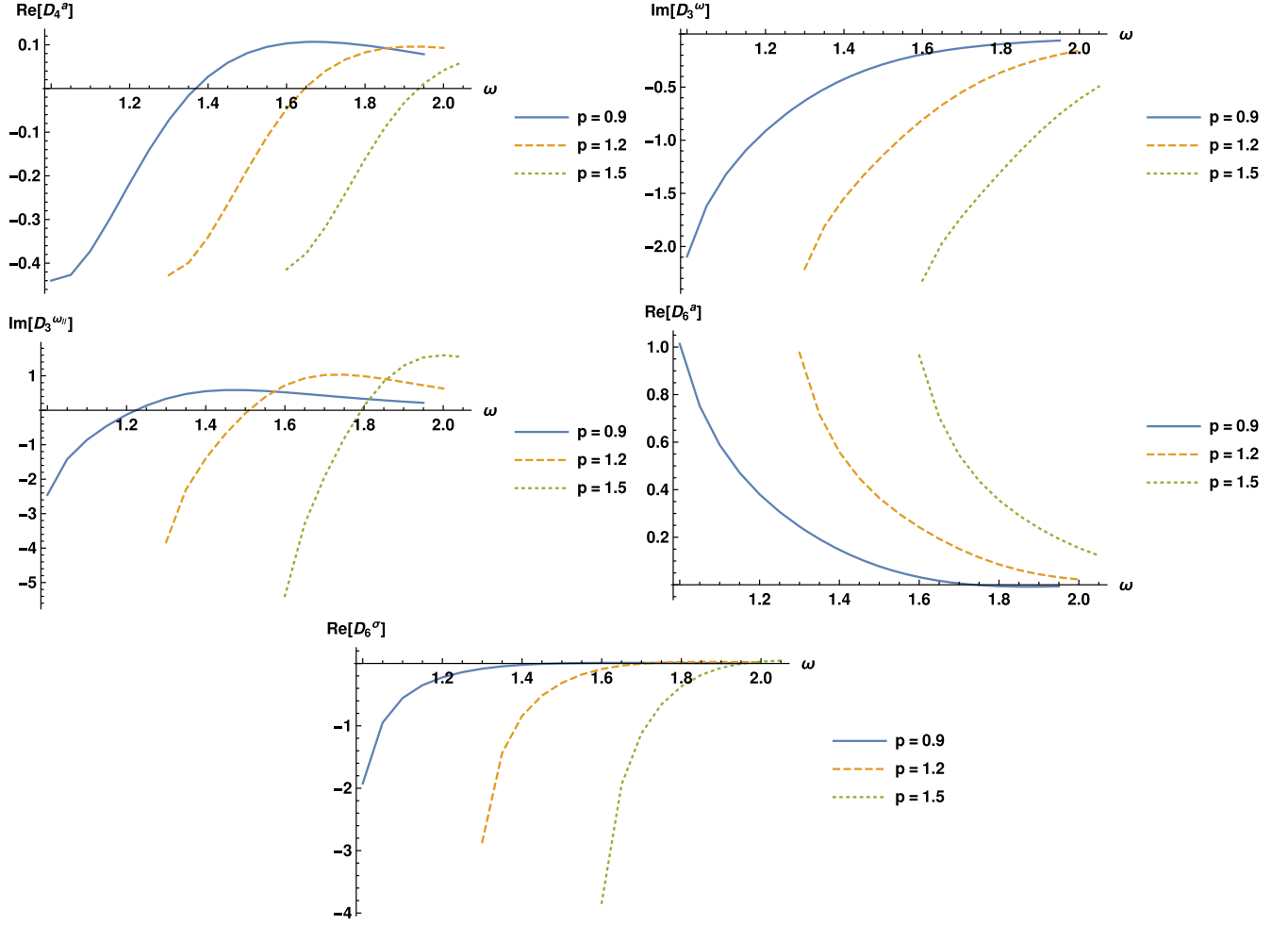


FIG. 3. The coefficients of gradient correction defined in (4.72) versus ω for three values of p . The unit is set by $\pi T = 1$. The first three and last two plots arise from $D_{\text{fluid}}^{(1)\text{le}}$ and $D_{\text{fluid}}^{(1)\text{ss}}$ respectively, corresponding to gradient correction to local equilibrium state and steady state effect of the fluid. The spectral function is enhanced toward the light cone and suppressed at large ω .

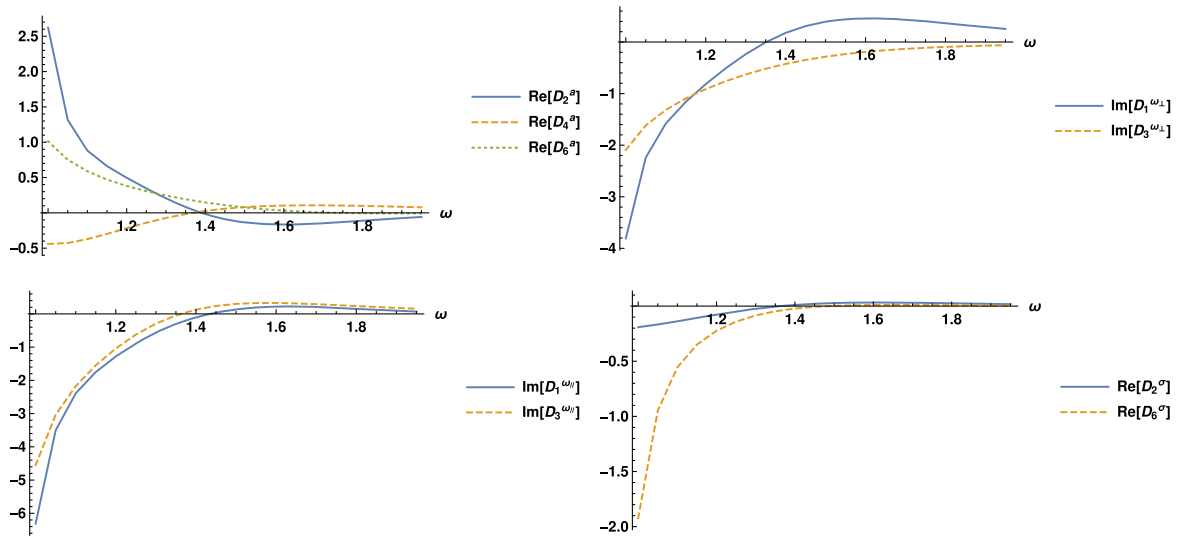


FIG. 4. Comparisons of contributions from the same hydrodynamic gradients listed in Table I as a function of ω at $p = 0.9$. The unit is set by $\pi T = 1$. We have defined $\omega_k^\perp = \omega_k - \omega_k^\parallel$ and switched to the basis $\omega_\perp$ and $\omega_\parallel$.

TABLE I. Summary of gradient correction. The leftmost column denotes the origin of the polarized spectral function. $D_{\text{probe}}^{(1)}$ comes from gradient correction to density matrix of the probe baryon. $D_{\text{fluid}}^{(1)\text{le}}$ and $D_{\text{fluid}}^{(1)\text{ss}}$ come from gradient correction to density matrices in local equilibrium state and steady state of the fluid respectively. We have defined $\omega_k^\perp = \omega_k - \omega_k^\parallel$ and switched to the basis $\omega_\perp$ and $\omega_\parallel$.

$\text{tr}[(\delta\rho^L + \delta\rho^R)\sigma_k]$	$\epsilon^{ijk}\hat{p}_j\partial_0 u_i$	$\epsilon^{ijk}\hat{p}_j\hat{p}_l\sigma_{il}$	$\omega_k^\perp$	$\omega_k^\parallel$
$D_{\text{probe}}^{(1)}$	$-4\text{Re}[D_2^a]$	$-4\text{Re}[D_2^s]$	$4\text{Im}[D_1^\omega]$	$4\text{Im}[D_1^\omega + D_1^{\omega_\parallel}]$
$D_{\text{fluid}}^{(1)\text{le}}$	$-4\text{Re}[D_4^a]$		$4\text{Im}[D_3^\omega]$	$4\text{Im}[D_3^\omega + D_3^{\omega_\parallel}]$
$D_{\text{fluid}}^{(1)\text{ss}}$	$-4\text{Re}[D_6^a]$	$-4\text{Re}[D_6^s]$		

in Fig. 4 comparisons of contributions from the same hydrodynamic gradients listed in Table I as a function of ω for a fixed p .

It is instructive to compare the present results with the counterpart obtained in the weak coupling regime. In [21], the polarized spectral function for quarks from gradient corrections has been obtained based on the quantum kinetic approach appropriate for weak coupling. The main difference is in the domain of gradient correction to spectral function: in the weak coupling case, the domain of corrections remains localized at $\omega = p$ as in the equilibrium spectral function; in the strong coupling case, the equilibrium spectral function extends as a wide domain starting from $\omega = p$. Gradient corrections have a similar wide domain. Interestingly the gradient corrections generically grow toward the light cone, which seems to be reminiscent of the vacuum spectral function rather than equilibrium spectral function; see Fig. 1. A second difference of the gradient corrections lies in their dependencies on the coupling constant. The corrections at weak coupling scales as $O(g^2 C_F) \sim O(\lambda)$ as $N_c \rightarrow \infty$, whereas the counterpart at strong coupling scales as $O(\lambda^0)$ from the holographic model. We stress that the scaling comparison above is meaningful as the gradient corrections to the spectral function have been normalized by the equilibrium counterpart, and so are independent of being from quarks or baryons.

With possible gradient corrections classified in terms of density matrices, we may try to make a closer comparison. We first note that the polarized part of the spectral function found in the weak coupling regime includes $D_{\text{probe}}^{(1)}$ and $D_{\text{fluid}}^{(1)\text{le}}$, but not $D_{\text{fluid}}^{(1)\text{ss}}$ ⁶. Recall the weakly coupled results arise from [21]: (i) $O(\partial_X^0)$ local equilibrium self-energy combined with $O(\partial_X)$ correction to the resummation equation (Kadanoff-Baym equation); (ii) $O(\partial_X)$ off-equilibrium self-energy, combined with the ordinary $O(\partial_X^0)$ resummation equation. (i) and (ii) naturally

fit into $D_{\text{probe}}^{(1)}$ and $D_{\text{fluid}}^{(1)\text{le}}$ respectively. While the weak coupling studies show polarized spectral function in response to acceleration, shear stress and vorticity respectively [21], the strong coupling study has vanishing response to the shear stress in the row of $D_{\text{fluid}}^{(1)\text{le}}$ in Table I.

V. CONCLUSION AND OUTLOOK

We have studied the spectral function of a probe holographic baryon in the fluid-gravity background, with the baryon modeled by a combination of left- and right-handed Weyl fermions. The spectral function in a slow-varying background is meaningful when the frequency and momentum of the baryon are much larger than the gradient of the background. We show how to perform gradient expansion for the probe baryon consistently, which includes expansions of the Dirac field and the Dirac operator. The complete gradient correction to the spectral function has been understood as a gradient correction to density matrices of the probe baryon and the medium, including $D_{\text{probe}}^{(1)}$, $D_{\text{fluid}}^{(1)\text{le}}$, and $D_{\text{fluid}}^{(1)\text{ss}}$. The corresponding baryon is found to be spin polarized in the presence of fluid acceleration, vorticity, and shear stress up to $O(\partial_X)$.

As mentioned in the Introduction, the modified spectral function found in this study only leads to one contribution to spin polarization. A more relevant quantity is the lesser function of the baryon, which directly connects to what is measured experimentally [4]. The lesser function involves solving the Dirac equation in the Schwinger-Keldysh extended fluid-gravity background, which can help reveal more structure in the contribution to spin polarization.

While modeling a baryon with a bulk Dirac fermion is an obvious first step, the choice is known to violate the single-particle spectral sum rule [35]. A real world baryon in vacuum is expected to satisfy the sum rule. It is therefore necessary to adopt a more realistic baryon model. The prescription for restoring the sum rule is known in holography [35]. Studies with more realistic baryon models would help identify generic features in spin polarization.

⁶ $D_{\text{fluid}}^{(1)\text{ss}}$ in the kinetic description involves a shifted distribution function in the steady state that is considered in [17,18], but the corresponding correction to the spectral function has not been worked out.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [36]; embargo periods may apply.

APPENDIX A: THE NOTATIONS

In this work, the capital letters $L, M, N \dots$ refer to the indices of the bulk coordinate as x^M , the lowercase letters a, b, c refer to the corresponding indices of the coordinate in tangent space. The gamma matrices γ^a , Γ^M as the generators of the Clifford algebra with elfbein e_M^a are related as

$$\{\gamma^a, \gamma^b\} = 2\eta^{ab}, \quad \{\Gamma^M, \Gamma^N\} = 2g^{MN},$$

$$\Gamma^M = e_a^M \gamma^a, \quad g_{MN} = e_M^a \eta_{ab} e_N^a, \quad (\text{A1})$$

where g_{MN} refers to the metric on a manifold and η^{ab} refers to the Minkowskian metric. The five gamma matrices are chosen as

$$\gamma^\mu = i \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad \gamma^4 = \gamma = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (\text{A2})$$

with the Pauli matrices $\sigma^\mu = (1, -\sigma^i)$, $\bar{\sigma}^\mu = (1, \sigma^i)$, $i = 1, 2, 3$. The spin connection is defined as

$$\omega_{\text{Mab}} = \eta_{cb} \left(e_a^N \partial_M e_N^c - e_a^N \Gamma_{MN}^K e_K^c \right), \quad (\text{A3})$$

where the affine connection Γ_{MN}^K is given by

$$\Gamma_{MN}^K = \frac{1}{2} g^{KL} (\partial_M g_{LN} + \partial_N g_{ML} - \partial_L g_{MN}). \quad (\text{A4})$$

The Greek letters as $\mu, \nu \dots$ are used to refer to the indices of coordinate as x^μ at the holographic boundary. The lowercase letters i, j, k, l run over the spatial directions of the holographic boundary i.e. $i, j, k, l = 1, 2, 3$.

APPENDIX B: THE FLUID-GRAVITY BACKGROUND

We collect an explicit metric of the fluid-gravity background [33] and hydrodynamic relations following from it. The bulk metric up to first order in gradient of the fluid velocity and temperature in the local rest frame is given as

$$ds^2 = ds_{(0)}^2 + ds_{(1)}^2,$$

$$ds_{(0)}^2 = 2dt dr - r^2 f(r) dt^2 + r^2 \delta_{ij} dx^i dx^j,$$

$$ds_{(1)}^2 = -2x^\mu \partial_\mu u_i dr dx^i - 2x^\mu \partial_\mu u_i r^2 (1-f) dt dx^i$$

$$- \frac{4x^\mu \partial_\mu b}{r^2} dt^2 + 2r^2 b F(r) \sigma_{ij} dx^i dx^j$$

$$+ \frac{2}{3} r \partial_i u_i dt^2 + 2r \partial_0 u_i dt dx^i, \quad (\text{B1})$$

with $x^\mu = \{t, x^i\}$ in the EF coordinates. Here we have written the metric in the unit of $R = 1$ (the radius of bulk). The relevant functions presented in (B1) are given as

$$\sigma_{ij} = \partial_i u_j + \partial_j u_i - \frac{2}{3} \delta_{ij} \partial^k u_k,$$

$$f(r) = 1 - \frac{1}{b^4 r^4}, \quad b = \frac{1}{\pi T} = r_H^{-1},$$

$$F(r) = \int_{br}^\infty dx \frac{x^2 + x + 1}{x(x+1)(x^2+1)}$$

$$= \frac{1}{4} \left[\ln \left[\frac{(1+br)^2(1+b^2r^2)}{b^4 r^4} \right] - 2 \arctan(br) + \pi \right]. \quad (\text{B2})$$

The following hydrodynamic relations follow from the background

$$\partial_0 b = \frac{1}{3} \partial_i u^i, \quad \partial_i b = \partial_0 u_i. \quad (\text{B3})$$

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