

Stability of baryonic black membranes

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Near-extremal black membranes with topological (baryonic) $U(1)_B$ charge of M theory compactified on the coset space $M^{1,1,0}$ are stable. $M^{1,1,0}$ coset is a $\mathbb{Z}_2$ -invariant truncation of a larger $Q^{1,1,1}$ coset, with diagonal $U(1)_B \equiv U(1)_{B,+} \subset U(1)_B^2$ symmetry of the latter. We show that the baryonic black membranes of M-theory $M^{1,1,0}$ compactifications are unstable to $\mathbb{Z}_2$ -odd gravitational bulk gauge and scalar fluctuations, but only if this bulk scalar is identified with the holographically dual $2+1$ -dimensional superconformal gauge theory operator of conformal dimension $\Delta = 1$. The instability is associated with the unstable charge transport of the off-diagonal $U(1)_{B,-} \subset U(1)_B^2$ symmetry.

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I. INTRODUCTION AND SUMMARY

Near-extremal black branes with finite entropy density in the limit of vanishing temperature $T \rightarrow 0$, ubiquitous in the holographic correspondence [1,2], recently gained renewed interest as laboratories of quantum gravity [3]. The best explored holographic example is that of the strongly coupled $\mathcal{N} = 4$ supersymmetric Yang-Mills plasma in four space-time dimensions. Here, the equilibrium states of the gauge theory plasma, with the same chemical potential μ for all $U(1)$ factors of the maximal Abelian subgroup $U(1)^3 \subset SU(4)$ R symmetry, reach the quantum critical regime as $\frac{T}{\mu} \rightarrow 0$. In the gravitational dual, such states are represented by a Reissner-Nordström black brane in asymptotically AdS_5 spacetime. Unfortunately, in the extremal limit, the black branes typically suffer from a variety of instabilities: the nonperturbative “Fermi-seasickness” instability [4], the perturbative “superconducting” instability [5], or the perturbative “charge-clumping” instability [6]—any one of which precludes reaching the interesting quantum critical regime.

With the goal of constructing reliable (and stable) extremal horizons, the authors of [7,8] focused on holographic models from compactifications of string theory and M theory on $AdS_{p+2} \times Y$ manifolds with nonzero p th Betti number b_p , leading to $U(1)^{b_p}$ “baryonic” global symmetry. Nonsupersymmetric extremal quantum states supported by the baryonic $U(1)^{b_p}$ chemical potentials do not have superconducting instabilities. As an example, consider strongly coupled $\mathcal{N} = 1$ $SU(N) \times SU(N)$ gauge theory

in four spacetime dimensions, the Klebanov-Witten (KW) model [9]. The theory has $U(1)_R \times U(1)_B$ global symmetry, which supports quantum critical states charged under either of the $U(1)$ ’s. The R -symmetry charged quantum critical states are unstable due to the condensation of the chiral primary $\mathcal{O}_F \equiv \text{Tr}(W_1^2 + W_2^2)$, where W_i are the gauge superfields corresponding to the two gauge group factors of $SU(N) \times SU(N)$ quiver [10]. The gauge-invariant operators of the KW theory charged under $U(1)_B$ have conformal dimensions of order¹ N , with the charge-to-mass ratio too small to trigger the superconducting instability [7]. Nonetheless, quantum critical states with a baryonic charge of the KW theory are unstable [11]: even though such states have zero R -symmetry charge density, at low temperatures R charge starts “clumping,” breaking the homogeneity of $U(1)_B$ charged thermal equilibrium state.²

So far, the only known example of the stable non-supersymmetric extremal horizon of string theory and M theory is realized in a membrane theory of Klebanov, Pufu, and Tesileanu (KPT) [8]. The KPT model is a holographic example of a three-dimensional superconformal gauge theory arising from compactification of M theory on regular seven-dimensional Sasaki-Einstein manifold— $\frac{SU(3) \times SU(2)}{SU(2) \times U(1)}$ coset, known as $M^{1,1,0}$. Much like the KW theory, the holographic membrane model of M theory on $M^{1,1,0}$ has $U(1)_R \times U(1)_B$ global symmetry. Here, there are three distinct near-extremal regimes: one supported by the $U(1)_R$ charge density and the other two

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¹The smallest such operators involve determinants of the bifundamental matter fields of the KW quiver gauge theory. This justifies the nomenclature “baryonic symmetry.”

²This is a direct consequence of the thermodynamic instabilities of the underlying thermal states [12]. For charged plasma this was originally explained in [6,13].

supported by the $U(1)_B$ charge density. The reason for the distinct baryonic near-criticality comes from the fact that the dual gravitational backgrounds have nontrivial support from the bulk scalar with $m^2 L^2 = -2$, corresponding to an operator of conformal dimension $\Delta = (2, 1)$. Depending on whether one uses a normal or an alternative quantization [14], one obtains either of two field theory duals, each with a near-extremal regime. It was shown in [15] that only the baryonic black membranes are stable: extremal horizons supported by $U(1)_R$ charge density suffer from both the (threshold) superconducting and $U(1)_B$ charge clumping instabilities.³

In this paper we further explore instabilities of the KPT baryonic membranes. There is a larger consistent truncation of M theory on $\frac{SU(2)^3}{U(1)^2}$ coset, known as $Q^{1,1,1}$, which is a $U(1)$ fibration over $\mathbb{CP}^1 \times \mathbb{CP}^1 \times \mathbb{CP}^1$. This manifold has the second Betti number $b_2 = 2$, so that the corresponding boundary superconformal theory has $U(1)_R \times U(1)_B^2$ global symmetry. The KPT model is a consistent truncation of $Q^{1,1,1}$ M-theory compactification where the two Betti vector multiples of $Q^{1,1,1}$ are identified. From this perspective the KPT model is a $\mathbb{Z}_2$ -even sector of $Q^{1,1,1}$ model under the interchange of the Betti multiples, and its baryonic symmetry $U(1)_B \equiv U_{B,+}$ is a diagonal subgroup of $U(1)_B^2$ of this larger model. We present detailed analysis of the hydrodynamic transport of the off-diagonal $U(1)_{B,-} \subset U(1)_B^2$ charge density fluctuations. This $\mathbb{Z}_2$ -odd sector of the gravitational dual fluctuations includes a massless Betti vector A_- and a scalar v_- with $m^2 L^2 = -2$, corresponding to an operator $\mathcal{O}_-$ of conformal dimension $\Delta = (2, 1)$. Here, once again we can use either a normal $\dim \mathcal{O}_- = 2$ or an alternative $\dim \mathcal{O}_- = 1$ quantization. We find that the diffusion coefficient $D_{B,-}$ of the charge density fluctuations associated with $U(1)_{B,-}$ symmetry becomes negative below some critical temperature T_{crit} , relative to the $U(1)_{B,+}$ -charge chemical potential μ_B of the near-critical thermal equilibrium states of KPT plasma, provided $\dim \mathcal{O}_- = 1$; $D_{B,-} > 0$ at any temperature if $\dim \mathcal{O}_- = 2$:

$$\begin{cases} D_{B,-} > 0, & \frac{T}{\mu_B} > \frac{T}{\mu_B}|_{\text{crit}}, & \dim \mathcal{O}_- = 1, \\ D_{B,-} < 0, & \frac{T}{\mu_B} < \frac{T}{\mu_B}|_{\text{crit}}, & \dim \mathcal{O}_- = 1, \\ D_{B,-} > 0, & \frac{T}{\mu_B} \text{ is any,} & \dim \mathcal{O}_- = 2. \end{cases} \quad (1.1)$$

Whenever $D_{B,-} < 0$, the $U(1)_{B,-}$ charge density is unstable to clumping: indeed, in the hydrodynamic⁴ $q \rightarrow 0$ limit, the dispersion relation characterizing the charge diffusion is

³The $U(1)_B$ charge transport instability sets in at higher temperatures.

⁴We use notations $\mathfrak{w} \equiv \frac{w}{2\pi T}$ and $\mathfrak{q} \equiv \frac{|\vec{k}|}{2\pi T}$ where $e^{-i\omega t + i\vec{k} \cdot \vec{x}}$ is the profile of the hydrodynamic perturbation.

$$\mathfrak{w} = -iD_{B,-}q^2 + \mathcal{O}(q^2), \quad (1.2)$$

and, thus,

$$D_{B,-} < 0 \Leftrightarrow \text{Im}[\mathfrak{w}] > 0. \quad (1.3)$$

The precise value of the critical temperature in (1.1) depends on the quantization of the $m^2 L^2 = -2$ scalar supporting the background geometry.

The rest of the paper is organized as follows. In the next section we summarize⁵ the relevant effective action for the M-theory flux compactifications on $Q^{1,1,1}$. We review the background geometry describing baryonic black branes. In Sec. III we compute $D_{B,-}$ for different quantizations of the background scalar supporting the near-extremal baryonic black branes and for different quantizations of the $\mathbb{Z}_2$ -odd bulk scalar v_- . Additionally, in Sec. IV, we argue that there are no homogeneous and isotropic equilibrium phases of the baryonic black branes with spontaneously broken $\mathbb{Z}_2$ symmetry. Appendixes A and B collect technical details necessary to reproduce the claims of the paper.

The main conceptual point of the paper is that the onset of instabilities in the extremal limit of the boundary CFTs, with the same global symmetries and the conserved currents, depends on the details of their operator spectra, i.e., on the identification of the bulk scalar v_- with the operator $\mathcal{O}_-$ either of the conformal dimension $\dim \mathcal{O}_- \equiv \dim \mathcal{O}_-^{(-)} = 1$ or $\dim \mathcal{O}_- \equiv \dim \mathcal{O}_-^{(+)} = 2$. The CFT with a $\mathcal{O}_-^{(-)}$ operator is unstable in the extremal limit, while a CFT with a $\mathcal{O}_-^{(+)}$ operator is stable in the extremal limit. Interestingly,⁶ both CFTs are related by a holographic RG flow, triggered by a relevant double-trace deformation [16–19] $\int f(\mathcal{O}_-^{(-)})^2$. Thus, there is a potential for a new critical phenomenon, where the onset of the dynamical instability depends on how large the double-trace mass scale f is compare to the $U(1)_{B,+}$ baryonic chemical potential μ_B . We will present such analysis elsewhere.

We leave open for the future work the study of the end point of the identified instability in the CFT with a $\mathcal{O}_-^{(-)}$ operator in the spectrum: while there are multiple occurrences of the related instabilities in other models [6,11,13,15], their ultimate fate is unknown (and, unfortunately, likely to be difficult to analyze). There is also currently no understanding what underlying physics makes some models stable in the extremal limit and the other ones not.

II. M THEORY ON $Q^{1,1,1}$ AND NEAR-EXTREMAL BARYONIC BLACK BRANES

Effective four-dimensional action of $\mathcal{N} = 2$ gauged supergravity describing flux compactifications of M theory

⁵See Ref. [15] for additional details.

⁶I am grateful to a referee for raising this issue.

on $Q^{1,1,1}$ is given by [15,20]

$$S_{Q^{1,1,1}} = \frac{1}{\kappa_4^2} \int_{\mathcal{M}_4} \left[\frac{1}{2} R_4 \star 1 - \{(\partial\phi)^2 + g_{ij} \partial t^i \partial \bar{t}^j\} \star 1 - \frac{1}{4} e^{-4\phi} dB \wedge \star dB \right. \\ \left. + \frac{1}{4} \text{Im} \mathcal{N}_{IJ} F^I \wedge \star F^J + \frac{1}{4} \text{Re} \mathcal{N}_{IJ} F^I \wedge F^J - \frac{1}{2} e_0 dB \wedge A^0 - V_{Q^{1,1,1}} \star 1 \right], \\ V_{Q^{1,1,1}} = e^{4\phi} \mathcal{K} \cdot \sum_i v_i^{-2} - 8e^{2\phi} \cdot \sum_i v_i^{-1} + \frac{e^{4\phi}}{4} \mathcal{K}^{-1} \cdot \sum_k \left[\sum_{ij} \mathcal{K}_{ijk} b_i m_j v_k \right]^2 + \frac{e^{4\phi}}{4} \mathcal{K}^{-1} \cdot \left[e_0 + \frac{1}{2} \sum_{i,j,k} \mathcal{K}_{ijk} b_i b_j m_k \right]^2, \quad (2.1)$$

with $t^i \equiv v_i + i b_i$. Here,

- (i) $\{I, J\} = \{0, 1, 2, 3\}$, $\{i, j, k\} = \{1, 2, 3\}$, and $m^I = \{0, 2, 2, 2\}$. The constant e_0 sets the radius L of the asymptotic AdS_4 spacetime; in what follows we will choose $e_0 = 6 \Rightarrow L = \frac{1}{2}$.

- (ii) B is a 2-form on $\mathcal{M}_4$; A^I are the 1-forms on $\mathcal{M}_4$ with the field strength $\mathcal{F}^I \equiv dA^I$, and the generalized field strengths F^I are defined as $F^I = \mathcal{F}^I - m^I B$. t^i and ϕ are 0-forms on $\mathcal{M}_4$.
(iii) Explicit expression for the gauge kinetic matrix is

$$\begin{aligned} \text{Re} \mathcal{N}_{00} &= -\frac{1}{3} \mathcal{K}_{ijk} b_i b_j b_k, & \text{Re} \mathcal{N}_{0i} &= \frac{1}{2} \mathcal{K}_{ijk} b_j b_k, & \text{Re} \mathcal{N}_{ij} &= -\mathcal{K}_{ijk} b_k, \\ \text{Im} \mathcal{N}_{00} &= -\mathcal{K}(1 + 4g_{ij} b_i b_j), & \text{Im} \mathcal{N}_{0i} &= 4\mathcal{K} g_{ij} b_j, & \text{Im} \mathcal{N}_{ij} &= -4\mathcal{K} g_{ij}, \end{aligned} \quad (2.2)$$

where $\mathcal{K}_{ijk} = 1$ for $i \neq j \neq k$ and 0 otherwise, $\mathcal{K} = \prod_i v_i$, and $g_{ij} = \frac{1}{4} v_i^{-2} \delta_{ij}$.

Consistent subtruncation of the effective action (2.1) $S_{Q^{1,1,1}} \rightarrow S_{M^{1,1,0}}$ identifies the Betti vector multiples

$$\{A^3, t^3\} \equiv \{A^1, t^1\}. \quad (2.3)$$

Near-extremal black membranes with a $U(1)_B$ chemical potential are homogeneous and isotropic solutions of the effective action $S_{M^{1,1,0}}$ with [8,15]

$$A^0 \equiv 0, \quad b_i \equiv 0, \quad B \equiv 0, \quad (2.4)$$

the background 4D metric on $\mathcal{M}_4$ and the remaining 2-form field strengths $\{\mathcal{F}^1, \mathcal{F}^2\}$ as

$$ds_4^2 = -\frac{4\alpha^2 f}{r^2} dt^2 + \frac{4\alpha^2}{r^2} dx^2 + \frac{s^2}{4r^2 f} dr^2, \\ \mathcal{F}^1 = \frac{q\alpha s}{v_2} dr \wedge dt, \quad \mathcal{F}^2 = -\frac{2v_2^2}{v_1^2} \mathcal{F}^1, \quad (2.5)$$

where α and q are constant coefficients and $f, s, v_1, v_2, g \equiv e^\phi$ are all functions of the radial coordinate

$$r \in (0, 1). \quad (2.6)$$

The asymptotic AdS_4 boundary is located as $r \rightarrow 0_+$, requiring

$$\lim_{r \rightarrow 0_+} \{f, s, v_1, v_2, g\}(r) = 1, \quad (2.7)$$

and the regular Schwarzschild horizon is located at a simple root of the blackening factor f , with all the other bulk fields being finite. Using a constant rescaling of a radial coordinate $r \rightarrow \lambda r$ we can always assume that the horizon is as $r \rightarrow 1_-$, thus requiring

$$\lim_{r \rightarrow 1_-} f(r) = 0, \quad \lim_{r \rightarrow 1_-} \{s, v_1, v_2, g\}(r) = \text{finite}. \quad (2.8)$$

The constant α in (2.5) is a scale resulting from fixing the horizon location as in (2.8); it is necessary to define the temperature $T \propto |\alpha|$ and the chemical potential $\mu_B \propto \alpha$ of the boundary superconformal theory thermal state, dual to a baryonic black membrane geometry (2.5). This constant will drop out from all the dimensionless thermodynamic ratios, e.g., $\frac{T}{\mu_B}$. The dimensionless parameter q in (2.5) is related to a baryonic chemical potential: specifically, the conserved $U(1)_B$ current of the boundary 2 + 1-dimensional superconformal gauge theory is holographically dual to a bulk 1-form gauge potential A^1 ,

$$dA^1 = \mathcal{F}^1 = \frac{q\alpha s}{v_2} dr \wedge dt \Rightarrow \frac{dA^1}{dr} = \frac{q\alpha s}{v_2}, \quad (2.9)$$

and, thus, we require

$$\lim_{r \rightarrow 0_+} A^1(r) = \mu_B, \quad \lim_{r \rightarrow 1_-} A^1(r) = 0. \quad (2.10)$$

TABLE I. Holographic spectroscopy of the bulk scalars supporting baryonic black membranes [20].

Mass eigenstate	$m^2 L^2$	Δ
$\ln[v_1 v_2^{-1}]$	-2	(2, 1)
$\ln[v_1^2 v_2 g^3]$	4	4
$\ln[v_1^2 v_2 g^{-4}]$	18	6

The holographic spectroscopy relates the bulk scalars $\{v_1, v_2, g\}$ to the boundary gauge theory operators $\mathcal{O}_\Delta$ of conformal dimension Δ as in Table I.

The bulk scalar $\ln[v_1 v_2^{-1}]$ can be identified either with the operator $\mathcal{O}_2$, the *normal quantization*, or with the operator $\mathcal{O}_1$, the *alternative quantization*. Each of the identifications allows for a nonsingular extremal limit of the baryonic black membrane (2.5) $\mathcal{M}_4 \rightarrow \text{AdS}_2 \times \mathbb{R}^2$, i.e., the limit of vanishing of its Hawking temperature $T \rightarrow 0$. Notice that at extremality the Bekenstein entropy density s of the membrane remains finite:

$$s = \frac{2\pi}{\kappa_4^2} \lim_{r \rightarrow 1_-} \frac{4\alpha^2}{r^2} = \frac{8\pi\alpha^2}{\kappa_4^2}, \quad (2.11)$$

while the dimensionless α -independent ratio $\frac{s}{T^2} \rightarrow \infty$.

The equations of motion for the baryonic black membrane background fields $\{f, s, v_1, v_2, g\}$ derived from the effective action $S_{M^{1,1,0}}$ are collected in Appendix A, along with the near-boundary $r \rightarrow 0_+$ and the near-horizon $r \rightarrow 1_-$ asymptotic expansions, enforcing the boundary conditions (2.7) and (2.8). Explicit expressions for T and μ_B are given by (A19) and (A20). As the baryonic black membrane temperature varies as $\frac{T}{\mu_B} \in (0, \infty)$, parameter $q \in (0, q_{\text{crit}} = 2^{15/4}/3^{5/4})$, with [15]

$$\lim_{q \rightarrow 0} \frac{T}{\mu_B} = \infty, \quad \lim_{q \rightarrow q_{\text{crit}}} \frac{T}{\mu_B} \propto \left(1 - \frac{q}{q_{\text{crit}}}\right) \rightarrow 0. \quad (2.12)$$

III. $\mathbb{Z}_2$ -ODD FLUCTUATIONS AND THE $U(1)_{B,-}$ CHARGE TRANSPORT

In this section we consider fluctuations about the baryonic black membrane solution (2.4) and (2.5) that are odd with respect to the interchange of the Betti vector multiples of the effective action $S_{Q^{1,1,1}}$,

$$\{A^3, t^3\} \leftrightarrow \{A^1, t^1\}. \quad (3.1)$$

Specifically, we introduce linearized fluctuation $\{\delta v_-, \delta b_-, \delta A_-\}$ as

TABLE II. Holographic spectroscopy of $\mathbb{Z}_2$ -odd (pseudo)scalars.

Mass eigenstate	$m^2 L^2$	Δ
$b_1 - b_3$	-2	(2, 1)
$\ln[v_1 v_3^{-1}]$	-2	(2, 1)

$$t_1 = v_1 e^{\frac{1}{2}\delta v_-} + i\frac{1}{2}\delta b_-, \quad t_3 = v_1^{-\frac{1}{2}\delta v_-} - i\frac{1}{2}\delta b_-,$$

$$A^1 \rightarrow A^1 + \frac{1}{2}\delta A_-, \quad A^3 \rightarrow A^1 - \frac{1}{2}\delta A_-, \quad (3.2)$$

so that, under (3.1),

$$\{\delta v_-, \delta b_-, \delta A_-\} \rightarrow -\{\delta v_-, \delta b_-, \delta A_-\}. \quad (3.3)$$

As the fluctuations of all the other fields of the baryonic black membranes within $S_{Q^{1,1,1}}$ are even⁷ under (3.1), $\mathbb{Z}_2$ -odd modes will decouple, governed by the quadratic action $S_- \{\delta v_-, \delta b_-, \delta A_-\} \equiv S_{Q^{1,1,1}} - S_{M^{1,1,0}}$:

$$S_- = \frac{1}{\kappa_4^2} \int_{\mathcal{M}_4} \left[-\left\{ \frac{1}{8v_1^2} (\partial\delta b_-)^2 + \frac{1}{8} (\partial\delta v_-)^2 \right\} \star 1 \right. \\ \left. - \frac{v_2}{8} \delta\mathcal{F}_- \wedge \star \delta\mathcal{F}_- - \frac{v_2}{4} (\delta v_-)^2 \mathcal{F}^1 \wedge \star \mathcal{F}^1 \right. \\ \left. + \frac{v_2}{2} \delta v_- \delta\mathcal{F}_- \wedge \star \mathcal{F}^1 + \frac{1}{4} \delta b_- \delta\mathcal{F}_- \wedge \mathcal{F}^2 - V_- \star 1 \right],$$

$$V_- = \frac{g^2(g^2 v_1 v_2 - 2)}{v_1} (\delta v_-)^2 + \frac{g^4(v_1^2 - 3)}{2v_2 v_1^2} (\delta b_-)^2,$$

$$\delta\mathcal{F}_- \equiv dA_-. \quad (3.4)$$

Within the effective action (3.4), we further consider fluctuations to be functions of t , x_2 , and r as follows:

$$\delta A_- = e^{-i\omega t + i k x_2} (\mathcal{A}_r dt + \mathcal{A}_2 dx_2 + \mathcal{A}_r dr),$$

$$\{\delta v_-, \delta b_-\} = e^{-i\omega t + i k x_2} \{\mathcal{V}, \mathcal{B}\}, \quad (3.5)$$

where $\{\mathcal{A}_{t,2,r}, \mathcal{V}, \mathcal{B}\}$ are functions of the radial coordinate r . We use the bulk gauge transformations of Betti vectors A^1 and A^3 to set⁸

$$\mathcal{A}_r = 0. \quad (3.6)$$

The equations of motion for the fluctuations are collected in Appendix B. Following [20], the holographic spectroscopy relates the (pseudo)scalar modes $\{\mathcal{V}, \mathcal{B}\}$ to the boundary gauge theory operators $\delta\mathcal{O}_\Delta^\mathcal{V}, \delta\mathcal{O}_\Delta^\mathcal{B}$, of conformal dimension Δ as in Table II. Here again, we have the choice to quantize the fluctuations so that they correspond either to boundary

⁷These modes were studied in detail in [15].

⁸This would lead to the first-order constraint (B1).

CFT operators of dimension 2 (the *normal quantization*) or to operators of dimension 1 (the *alternative quantization*). This choice is independent from the choice of quantization for the background solution.

We find that the fluctuations $\{\mathcal{A}_t, \mathcal{A}_2, \mathcal{V}\}$ decouple from $\mathcal{B}$ —the former describe the $U(1)_{B,-}$ charge transport, while the latter can lead to potential threshold instabilities (to be further discussed in Sec. IV). To proceed with the $U(1)_{B,-}$ charge transport we introduce

$$Z \equiv \mathfrak{q}\mathcal{A}_t + \mathfrak{w}\mathcal{A}_2. \quad (3.7)$$

We use the constraint (B1) to obtain from (B2)–(B4) a decoupled set of the second-order equations for

$$\{Z, \mathcal{V}\}. \quad (3.8)$$

Solutions of the resulting equations with appropriate boundary conditions determine the spectrum of $U(1)_{B,-}$ charged quasinormal modes of the baryonic black membranes—equivalently the $\mathbb{Z}_2$ -odd subsector of physical spectrum of linearized fluctuations in membrane gauge theory plasma with a baryonic chemical potential. Following [21,22] we impose the incoming-wave boundary conditions at the black membrane horizon and “normalizability” at the asymptotic AdS_4 boundary. Focusing on the $\text{Re}[\mathfrak{w}] = 0$ diffusive branch and introducing

$$Z = (1-r)^{-i\mathfrak{w}/2} z, \quad \mathcal{V} = (1-r)^{-i\mathfrak{w}/2} u, \quad \mathfrak{w} = -iv\mathfrak{q}, \quad (3.9)$$

we solve the fluctuation equations subject to the asymptotics:

In the UV, i.e., as $r \rightarrow 0_+$, and with the identifications⁹ $\ln[v_1 v_2^{-1}] \Leftrightarrow \mathcal{O}_2$ and $u \Leftrightarrow \delta\mathcal{O}_2^\mathcal{V}$,

$$\begin{aligned} z &= \mathfrak{q}r - \frac{1}{2}\mathfrak{q}^2 v r^2 + \mathcal{O}(r^3), \\ u &= u_2 r^2 - \frac{1}{2}\mathfrak{q} v u_2 r^3 + \mathcal{O}(r^4), \end{aligned} \quad (3.10)$$

specified, for a fixed background and a momentum $\mathfrak{q}$, by

$$\{v, u_2\}; \quad (3.11)$$

In the IR, i.e., as $y \equiv 1-r \rightarrow 0_+$,

$$z = z_0^h + \mathcal{O}(y), \quad u = u_0^h + \mathcal{O}(y), \quad (3.12)$$

specified by

⁹Likewise, we develop the UV expansions for the alternative quantization of either the background, $\ln[v_1 v_2^{-1}]$, or the fluctuation scalar, u : $\{\mathcal{O}_2, \delta\mathcal{O}_1^\mathcal{V}\}$, $\{\mathcal{O}_1, \delta\mathcal{O}_2^\mathcal{V}\}$, and $\{\mathcal{O}_1, \delta\mathcal{O}_1^\mathcal{V}\}$.

$$\{z_0^h, u_0^h\}. \quad (3.13)$$

Note that in total we have $2 + 2 = 4$ parameters—see (3.11) and (3.13)—which is precisely what is necessary to identify a solution of a coupled system of two second-order ordinary differential equations for $\{z, u\}$. Furthermore, since the equations are linear in the fluctuations, we can, without loss of generality, normalize the solutions so that

$$\lim_{r \rightarrow 0} \frac{dz}{dr} = \mathfrak{q}. \quad (3.14)$$

Once we fix the background and solve the fluctuation equations of motion, we obtain $v = v(\mathfrak{q})$. Given v we extract the $U(1)_{B,-}$ -charge diffusion coefficient $\mathcal{D}_{B,-}$, as

$$\mathfrak{w} = -i \cdot \underbrace{2\pi T \mathcal{D}_{B,-}}_{\equiv \mathcal{D}_{B,-}} \cdot \mathfrak{q}^2 + \mathcal{O}(\mathfrak{q}^3), \quad \mathcal{D}_{B,-} \equiv \left. \frac{dv}{d\mathfrak{q}} \right|_{\mathfrak{q}=0}. \quad (3.15)$$

For general values of q we have to solve the fluctuation equations numerically. At $q = 0$, an analytic solution is possible in the limit $\mathfrak{q} \rightarrow 0$ —which is precisely what is needed to extract $\mathcal{D}_{B,-}$ [15]:

$$\mathcal{D}_{B,-} \Big|_{q=0} = \frac{3}{2}. \quad (3.16)$$

For $q \in (0, q_{\text{crit}})$ the $U(1)_{B,-}$ -charge diffusion coefficient of the baryonic membrane theory plasma is computed numerically; see Fig. 1. Black curves correspond to the background scalar quantization as $\ln[v_1 v_2^{-1}] \Leftrightarrow \mathcal{O}_2$, while the blue curves correspond to the quantization $\ln[v_1 v_2^{-1}] \Leftrightarrow \mathcal{O}_1$. Furthermore, the solid curves represent $\mathbb{Z}_2$ -odd scalar $\ln[v_1 v_3^{-1}]$ quantization as $\delta \ln[v_1 v_3^{-1}] \Leftrightarrow \delta\mathcal{O}_2^\mathcal{V}$, while the dashed curves represent $\delta \ln[v_1 v_3^{-1}] \Leftrightarrow \delta\mathcal{O}_1^\mathcal{V}$. In the latter case (the dashed curves), there is the $U(1)_{B,-}$ charge clumping instability for $q > q_{\text{unstable}}$ (correspondingly $\frac{T}{\mu_B} < \frac{T}{\mu_B} \Big|_{\text{crit}}$), represented by vertical red lines:

$\{\mathcal{O}, \delta\mathcal{O}^\mathcal{V}\}$	Fig.1 curve style	$q_{\text{unstable}}/q_{\text{crit}}$	$T/\mu_B \Big _{\text{crit}}$
$\{\mathcal{O}_2, \delta\mathcal{O}_1^\mathcal{V}\}$	(black, dashed)	0.597(6)	0.251(5)
$\{\mathcal{O}_1, \delta\mathcal{O}_1^\mathcal{V}\}$	(blue, dashed)	0.547(1)	0.276(2)

(3.17)

To compute $\frac{T}{\mu_B} \Big|_{\text{crit}}$ for a given value of $\frac{q_{\text{unstable}}}{q_{\text{crit}}}$ we use (A19) and (A20).

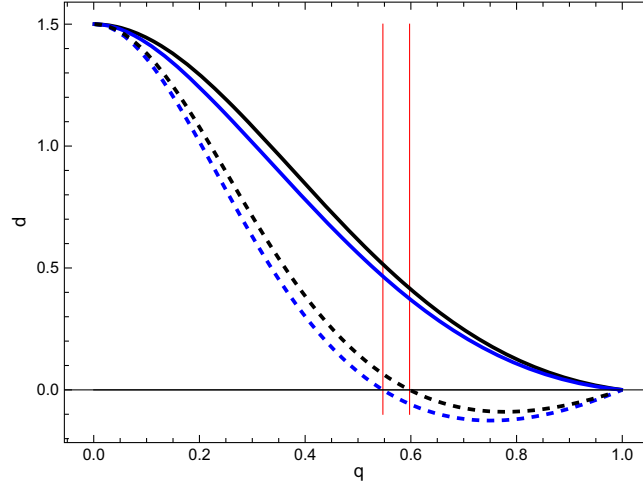


FIG. 1. $U(1)_{B,-}$ -charge dimensionless diffusion coefficient $\mathcal{D}_{B,-} = 2\pi T D_{B,-}$ of the baryonic membrane theory plasma for different quantizations of the gravitational dual scalars $\{\ln[v_1 v_2^{-1}], \delta \ln[v_1 v_3^{-1}]\}$: $\{\mathcal{O}_2, \delta \mathcal{O}_2^\mathcal{V}\}$ (black, solid), $\{\mathcal{O}_2, \delta \mathcal{O}_1^\mathcal{V}\}$ (black, dashed), $\{\mathcal{O}_1, \delta \mathcal{O}_2^\mathcal{V}\}$ (blue, solid), and $\{\mathcal{O}_1, \delta \mathcal{O}_1^\mathcal{V}\}$ (blue, dashed). At $q = 0$, $\mathcal{D}_{B,-} = \frac{3}{2}$ (3.16), while it vanishes in the quantum critical regime $q \rightarrow q_{\text{crit}}$, $\mathcal{D}_{B,-} \propto \frac{T}{\mu_B} \rightarrow 0$. Independent of the background scalar $\ln[v_1 v_2^{-1}]$ quantization, there is an onset of the $U(1)_{B,-}$ charge clumping instability for $\delta \ln[v_1 v_3^{-1}] \Leftrightarrow \delta \mathcal{O}_1^\mathcal{V}$ quantization (the dashed curves), represented by vertical red lines.

IV. THRESHOLD INSTABILITIES FROM CONDENSATION OF $\{\delta v_-, \delta b_-\}$

Consider spatially homogeneous and isotropic fluctuations of the bulk (pseudo)scalars $\mathcal{V}$ and $\mathcal{B}$ about baryonic black membrane of Sec. II. The corresponding equations of motion can be obtained from (B1)–(B5) in the limit

$$\{w, k\} \rightarrow 0, \quad (4.1)$$

provided we set $\mathcal{A}_2 = 0$. We find two decoupled sets.

(i) $\{\mathcal{V}, a \equiv \mathcal{A}'_t\}$:

$$0 = a' - \frac{2qs}{v_2} \mathcal{V}' + \left(\frac{(v'_2)^2 r}{v_2^2} + \frac{2(v'_1)^2 r}{v_1^2} + \frac{4(g')^2 r}{g^2} + \frac{4v'_2}{v_2} \right) \frac{a}{4}, \quad (4.2)$$

$$0 = \mathcal{V}'' + \left(\frac{s^2(v_1^4 + 2v_1^2 v_2^2 + 9)}{4rfv_2 v_1^2} g^4 - \frac{2s^2(v_1 + 2v_2)}{v_1 r f v_2} g^2 + \frac{s^2 q^2 r^3 (v_1^2 + 2v_2^2)}{8f v_2 v_1^2} + \frac{1}{r} \right) \mathcal{V}' + \left(-\frac{2s^2 v_2}{f r^2} g^4 + \frac{4s^2}{v_1 f r^2} g^2 + \frac{s^2 q^2 r^2}{2f v_2} \right) \mathcal{V} - \frac{s a q r^2}{2f}; \quad (4.3)$$

(ii) $\{\mathcal{B}\}$:

$$0 = \mathcal{B}'' + \left(\frac{s^2(v_1^4 + 2v_1^2 v_2^2 + 9)}{4rfv_2 v_1^2} g^4 - \frac{2s^2(v_1 + 2v_2)}{v_1 r f v_2} g^2 + \frac{s^2 q^2 r^3 (v_1^2 + 2v_2^2)}{8f v_2 v_1^2} - \frac{2v'_1}{v_1} + \frac{1}{r} \right) \mathcal{B}' - \frac{s^2 \mathcal{B} g^4 (v_1^2 - 3)}{v_2 f r^2}. \quad (4.4)$$

In the UV, i.e., as $r \rightarrow 0_+$, and with the identification¹⁰ $\ln[v_1 v_2^{-1}] \Leftrightarrow \mathcal{O}_2$,

$$\begin{aligned} a &= 2qu_1 r + 2qu_2 r^2 + \mathcal{O}(r^3), & \mathcal{V} &= u_1 r + u_2 r^2 + \mathcal{O}(r^4), \\ \mathcal{B} &= \mathcal{B}_1 r + \mathcal{B}_2 r^2 + \mathcal{O}(r^4). \end{aligned} \quad (4.5)$$

¹⁰Likewise, we develop the UV expansions for the alternative quantization of the background scalar $\ln[v_1 v_2^{-1}] \Leftrightarrow \mathcal{O}_1$.

Notice that $\lim_{r \rightarrow 0} a = 0$ —this ensures that the fluctuations $\{\mathcal{V}, a\}$ have the vanishing $U(1)_{B,-}$ charge. In the quantization where $\mathcal{V}$ (or $\mathcal{B}$) is identified with the boundary gauge theory operator $\delta\mathcal{O}_2^\mathcal{V}$ (correspondingly $\delta\mathcal{O}_2^\mathcal{B}$) the coefficient u_1 (correspondingly $\mathcal{B}_1$) is the source, while in the identification $\mathcal{V} \Leftrightarrow \delta\mathcal{O}_1^\mathcal{V}$ (or $\mathcal{B} \Leftrightarrow \delta\mathcal{O}_1^\mathcal{B}$) the source term is u_2 (correspondingly $\mathcal{B}_2$). In the IR, i.e., as $y \equiv 1 - r \rightarrow 0$,

$$\begin{aligned}\mathcal{V} &= u_0^h + \mathcal{O}(y), & a &= a_0^h + \mathcal{O}(y), \\ \mathcal{B} &= \mathcal{B}_0^h + \mathcal{O}(y).\end{aligned}\quad (4.6)$$

Following [23], e.g., to identify the onset of the instability associated with the condensation of $\delta\mathcal{O}_2^\mathcal{V}$ we keep fixed the source term of the operator, $u_1 = 1$, and scan q (correspondingly $\frac{T}{\mu_B}$), looking for a divergence of the expectation value of the corresponding

operator $\langle \delta\mathcal{O}_2^\mathcal{V} \rangle \propto u_2$. A divergence signals the presence of a homogeneous and isotropic normalizable mode of the fluctuations of $\mathcal{V}$ —the threshold for the instability. We performed all such scans, for both quantizations of the background scalar $\ln[v_1 v_2^{-1}]$ and independently for both quantizations of the $\mathbb{Z}_2$ -odd (pseudo)scalars $\mathcal{V}$ and $\mathcal{B}$ —there are no divergences of the expectation values of the corresponding operators.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [24]; embargo periods may apply.

APPENDIX A: BACKGROUND EQUATIONS OF MOTION AND THE ASYMPTOTIC EXPANSIONS

$$0 = f' + f \left(\frac{rv_2'^2}{4v_2^2} + \frac{rv_1'^2}{2v_1^2} + \frac{rg'^2}{g^2} - \frac{3}{r} \right) - \frac{s^2 r^3 (2v_2^2 + v_1^2) q^2}{8v_2 v_1^2} - \frac{s^2 g^4 (2v_2^2 v_1^2 + v_1^4 + 9)}{4v_2 v_1^2 r} + \frac{2g^2 s^2 (2v_2 + v_1)}{v_2 v_1 r}, \quad (\text{A1})$$

$$0 = s' + \frac{sr}{4} \left(\frac{v_2'^2}{v_2^2} + \frac{2v_1'^2}{v_1^2} + \frac{4g'^2}{g^2} \right), \quad (\text{A2})$$

$$\begin{aligned}0 = v_1'' - \frac{v_1'^2}{v_1} + v_1' & \left(\frac{s^2 g^4 (2v_2^2 v_1^2 + v_1^4 + 9)}{4f v_2 v_1^2 r} - \frac{2s^2 g^2 (2v_2 + v_1)}{v_1 f v_2 r} + \frac{s^2 r^3 q^2 (2v_2^2 + v_1^2)}{8f v_2 v_1^2} + \frac{1}{r} \right) \\ & - \frac{s^2 g^4 (v_1^4 - 9)}{2v_1 f v_2 r^2} + \frac{s^2 r^2 v_2 q^2}{2v_1 f} - \frac{4s^2 g^2}{f r^2},\end{aligned}\quad (\text{A3})$$

$$\begin{aligned}0 = v_2'' - \frac{v_2'^2}{v_2} + v_2' & \left(\frac{s^2 g^4 (2v_2^2 v_1^2 + v_1^4 + 9)}{4f v_2 v_1^2 r} - \frac{2s^2 g^2 (2v_2 + v_1)}{v_1 f v_2 r} + \frac{s^2 r^3 q^2 (2v_2^2 + v_1^2)}{8f v_2 v_1^2} + \frac{1}{r} \right) \\ & - \frac{s^2 (2v_2^2 v_1^2 - v_1^4 - 9) g^4}{2f v_1^2 r^2} - \frac{4s^2 g^2}{f r^2} - \frac{s^2 q^2 r^2 (2v_2^2 - v_1^2)}{4f v_1^2},\end{aligned}\quad (\text{A4})$$

$$\begin{aligned}0 = g'' - \frac{g'^2}{g} + g' & \left(\frac{g^4 s^2 (2v_2^2 v_1^2 + v_1^4 + 9)}{4f v_2 v_1^2 r} - \frac{2g^2 s^2 (2v_2 + v_1)}{v_1 f v_2 r} + \frac{s^2 r^3 q^2 (2v_2^2 + v_1^2)}{8f v_2 v_1^2} + \frac{1}{r} \right) \\ & - \frac{s^2 g^5 (2v_2^2 v_1^2 + v_1^4 + 9)}{2f v_2 v_1^2 r^2} + \frac{2g^3 s^2 (2v_2 + v_1)}{v_1 f v_2 r^2}.\end{aligned}\quad (\text{A5})$$

Equations (A1)–(A5) should be solved numerically, subject to the following asymptotic expansion. In the UV, i.e., as $r \rightarrow 0$, and with the identification $\ln[v_1 v_2^{-1}] \Leftrightarrow \mathcal{O}_2$, we have

$$f = 1 + f_3 r^3 + \frac{3}{8} q^2 r^4 - \frac{1}{6} v_{1,2} q^2 r^6 + \mathcal{O}(r^7), \quad s = 1 - \frac{3}{2} v_{1,2}^2 r^4 + \frac{1}{6} v_{1,2} q^2 r^6 + \mathcal{O}(r^7), \quad (\text{A6})$$

$$v_1 = 1 + v_{1,2}r^2 + \left(v_{1,4} + \left(\frac{24}{35}v_{1,2}^2 - \frac{1}{35}q^2 \right) \ln r \right) r^4 - \frac{1}{3}f_3v_{1,2}r^5 \\ + \left(v_{1,6} + \left(-\frac{13}{350}v_{1,2}q^2 + \frac{156}{175}v_{1,2}^3 \right) \ln r \right) r^6 + \mathcal{O}(r^7 \ln r), \quad (\text{A7})$$

$$v_2 = 1 - 2v_{1,2}r^2 + \left(\frac{3}{2}v_{1,2}^2 + v_{1,4} + \frac{1}{8}q^2 + \left(\frac{24}{35}v_{1,2}^2 - \frac{1}{35}q^2 \right) \ln r \right) r^4 + \frac{2}{3}f_3v_{1,2}r^5 \\ + \left(v_{1,6} - \frac{39}{10}v_{1,2}v_{1,4} + \frac{4647}{3500}v_{1,2}^3 - \frac{653}{3500}v_{1,2}q^2 + \left(\frac{13}{175}v_{1,2}q^2 - \frac{312}{175}v_{1,2}^3 \right) \ln r \right) r^6 + \mathcal{O}(r^7 \ln r), \quad (\text{A8})$$

$$g = 1 + \left(-\frac{3}{56}v_{1,2}^2 + \frac{3}{4}v_{1,4} + \frac{1}{56}q^2 + \left(\frac{18}{35}v_{1,2}^2 - \frac{3}{140}q^2 \right) \ln r \right) r^4 \\ + \left(-v_{1,6} + \frac{13}{10}v_{1,2}v_{1,4} - \frac{1549}{3500}v_{1,2}^3 - \frac{37}{1750}v_{1,2}q^2 \right) r^6 + \mathcal{O}(r^7 \ln r); \quad (\text{A9})$$

i.e., the UV part of the solution is characterized (given q) by

$$\{f_3, v_{1,2}, v_{1,4}, v_{1,6}\}. \quad (\text{A10})$$

In the UV, i.e., as $r \rightarrow 0$, and instead with the identification $\ln[v_1 v_2^{-1}] \Leftrightarrow \mathcal{O}_1$, we have

$$f = 1 + f_3r^3 + \frac{3}{8}q^2r^4 + \left(-\frac{9}{20}v_{1,1}^2f_3 - \frac{3}{10}v_{1,1}q^2 \right) r^5 + \frac{37}{120}v_{1,1}^2q^2r^6 + \mathcal{O}(r^7), \quad (\text{A11})$$

$$s = 1 - \frac{3}{4}v_{1,1}^2r^2 + \frac{489}{800}v_{1,1}^4r^4 + \left(v_{1,1}^5 + \frac{2}{5}v_{1,1}^2f_3 + \frac{1}{10}v_{1,1}q^2 \right) r^5 \\ + \left(\frac{5661}{22400}v_{1,1}^6 + \frac{1}{8}v_{1,1}^3f_3 - \frac{269}{1680}v_{1,1}^2q^2 + \frac{3}{4}v_{1,1}v_{1,4} + \left(-\frac{51}{70}v_{1,1}^6 - \frac{3}{140}v_{1,1}^2q^2 \right) \ln r \right) r^6 + \mathcal{O}(r^7 \ln r), \quad (\text{A12})$$

$$v_1 = 1 + v_{1,1}r - \frac{1}{5}v_{1,1}^2r^2 - \frac{31}{20}v_{1,1}^3r^3 + \left(v_{1,4} + \left(-\frac{34}{35}v_{1,1}^4 - \frac{1}{35}q^2 \right) \ln r \right) r^4 \\ + \left(-\frac{103}{800}v_{1,1}^5 + \frac{19}{60}v_{1,1}^2f_3 + \frac{11}{120}v_{1,1}q^2 + \frac{3}{2}v_{1,1}v_{1,4} + \left(-\frac{3}{70}v_{1,1}q^2 - \frac{51}{35}v_{1,1}^5 \right) \ln r \right) r^5 \\ + \left(v_{1,6} + \left(-\frac{51}{70}v_{1,1}^6 - \frac{3}{140}v_{1,1}^2q^2 \right) \ln r \right) r^6 + \mathcal{O}(r^7 \ln r), \quad (\text{A13})$$

$$v_2 = 1 - 2v_{1,1}r + \frac{13}{10}v_{1,1}^2r^2 + \frac{1}{10}v_{1,1}^3r^3 + \left(\frac{131}{40}v_{1,1}^4 + \frac{1}{2}v_{1,1}f_3 + v_{1,4} + \frac{1}{8}q^2 \right) r^4 \\ + \left(-\frac{34}{35}v_{1,1}^5 - \frac{1}{35}q^2 \right) \ln r + \left(-\frac{4597}{400}v_{1,1}^5 - \frac{14}{15}v_{1,1}^2f_3 - \frac{13}{30}v_{1,1}q^2 - 3v_{1,1}v_{1,4} \right. \\ \left. + \left(\frac{3}{35}v_{1,1}q^2 + \frac{102}{35}v_{1,1}^5 \right) \ln r \right) r^5 + \left(\frac{166743}{14000}v_{1,1}^6 - \frac{29}{40}v_{1,1}^3f_3 + \frac{8061}{14000}v_{1,1}^2q^2 + \frac{39}{20}v_{1,1}^2v_{1,4} + v_{1,6} \right. \\ \left. + \left(-\frac{459}{175}v_{1,1}^6 - \frac{27}{350}v_{1,1}^2q^2 \right) \ln r \right) r^6 + \mathcal{O}(r^7 \ln r), \quad (\text{A14})$$

$$\begin{aligned}
g = & 1 - \frac{3}{10} v_{1,1}^2 r^2 - \frac{1}{2} v_{1,1}^3 r^3 + \left(\frac{2047}{1400} v_{1,1}^4 + \frac{1}{8} v_{1,1} f_3 + \frac{3}{4} v_{1,4} + \frac{1}{56} q^2 \right. \\
& + \left(-\frac{51}{70} v_{1,1}^4 - \frac{3}{140} q^2 \right) \ln r \Big) r^4 + \left(-\frac{73}{40} v_{1,1}^5 + \frac{1}{10} v_{1,1}^2 f_3 \right) r^5 + \left(-\frac{6761}{14000} v_{1,1}^6 + \frac{283}{240} v_{1,1}^3 f_3 \right. \\
& + \frac{2837}{21000} v_{1,1}^2 q^2 + \frac{19}{40} v_{1,1}^2 v_{1,4} - v_{1,6} + \left. \left(\frac{187}{700} v_{1,1}^6 + \frac{11}{1400} v_{1,1}^2 q^2 \right) \ln r \right) r^6 + \mathcal{O}(r^7 \ln r),
\end{aligned} \tag{A15}$$

characterized (given q) by

$$\{v_{1,1}, f_3, v_{1,4}, v_{1,6}\}. \tag{A16}$$

In the IR, i.e., as $y \equiv 1 - r \rightarrow 0$, we have

$$\begin{aligned}
f = & -\frac{(s_0^h)^2}{8v_{2,0}^h(v_{1,0}^h)^2} \left(2(g_0^h)^4((v_{1,0}^h)^4 + 2(v_{1,0}^h)^2(v_{2,0}^h)^2 + 9) - 16(g_0^h)^2 v_{1,0}^h(v_{1,0}^h + 2v_{2,0}^h) \right. \\
& + \left. q^2((v_{1,0}^h)^2 + 2(v_{2,0}^h)^2) \right) y + \mathcal{O}(y^2), \\
s = & s_0^h + \mathcal{O}(y), \quad v_i = v_{i,0}^h + \mathcal{O}(y), \quad g = g_0^h + \mathcal{O}(y),
\end{aligned} \tag{A17}$$

characterized (given q) by

$$\{s_0^h, v_{1,0}^h, v_{2,0}^h, g_0^h\}. \tag{A18}$$

Given q , a numerical solution is characterized by (A10) [or (A16)] and (A18), which determine the black membrane Hawking temperature T , and the baryonic chemical potential μ_B ,

$$\frac{T}{|\alpha|} = \frac{s_0^h}{8\pi v_{2,0}^h(v_{1,0}^h)^2} \left(2(g_0^h)^2(8v_{1,0}^h(v_{1,0}^h + 2v_{2,0}^h) - (g_0^h)^2((v_{1,0}^h)^4 + 2(v_{1,0}^h)^2(v_{2,0}^h)^2 + 9)) - ((v_{1,0}^h)^2 + 2(v_{2,0}^h)^2)q^2 \right), \tag{A19}$$

and [see (2.5)]

$$\frac{\mu_B}{\alpha} = \frac{1}{\alpha} A_t^1 \Big|_{r=0} = - \int_0^1 \frac{q s}{v_2} dr. \tag{A20}$$

APPENDIX B: EQUATIONS OF MOTION FOR $\mathbb{Z}_2$ -ODD FLUCTUATIONS OF THE BARYONIC BLACK MEMBRANES

$$0 = \mathcal{A}_2' + \frac{c_2^2 w}{c_1^2 k} \mathcal{A}_t' - \frac{2F c_2^2 w}{c_1^2 k} \mathcal{V}, \tag{B1}$$

$$0 = \mathcal{A}_t'' + \left(-\frac{c_3'}{c_3} - \frac{c_1'}{c_1} + \frac{v_2'}{v_2} + 2\frac{c_2'}{c_2} \right) \mathcal{A}_t' - \frac{c_3^2 k}{c_2^2} (\mathcal{A}_t k + \mathcal{A}_2 w) - 2(F\mathcal{V})' + 2\mathcal{V}F \left(\frac{c_3'}{c_3} + \frac{c_1'}{c_1} - \frac{2c_2'}{c_2} - \frac{v_2'}{v_2} \right), \tag{B2}$$

$$0 = \mathcal{A}_2'' + \left(-\frac{c_3'}{c_3} + \frac{c_1'}{c_1} + \frac{v_2'}{v_2} \right) \mathcal{A}_2' + \frac{c_3^2 w}{c_1^2} (\mathcal{A}_t k + \mathcal{A}_2 w), \tag{B3}$$

$$0 = \mathcal{V}'' + \left(-\frac{c_3'}{c_3} + \frac{c_1'}{c_1} + \frac{2c_2'}{c_2} \right) \mathcal{V}' - \frac{2F v_2}{c_1^2} \mathcal{A}_t' + \left(\frac{2v_2 F^2}{c_1^2} - \frac{c_3^2 (c_1^2 k^2 - c_2^2 w^2)}{c_1^2 c_2^2} - \frac{8g^2 c_3^2 (g^2 v_1 v_2 - 2)}{v_1} \right) \mathcal{V}, \tag{B4}$$

$$0 = \mathcal{B}'' + \left(\frac{c'_1}{c_1} + \frac{2c'_2}{c_2} - \frac{2v'_1}{v_1} - \frac{c'_3}{c_3} \right) \mathcal{B}' - \left(\frac{4c_3^2 g^4 (v_1^2 - 3)}{v_2} + \frac{c_3^2 (c_1^2 k^2 - c_2^2 w^2)}{c_1^2 c_2^2} \right) \mathcal{B}, \quad (\text{B5})$$

where [compare with (2.5)]

$$c_1 = \frac{2\alpha\sqrt{f}}{r}, \quad c_2 = \frac{2\alpha}{r}, \quad c_3 = \frac{s}{2r\sqrt{f}}, \quad F = \frac{q\alpha s}{v_2}. \quad (\text{B6})$$

We explicitly verified that (B1) is consistent with (B2)–(B4).

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- [1] J. M. Maldacena, The large N limit of superconformal field theories and supergravity, *Adv. Theor. Math. Phys.* **2**, 231 (1998).
 - [2] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri, and Y. Oz, Large N field theories, string theory and gravity, *Phys. Rep.* **323**, 183 (2000).
 - [3] G. J. Turiaci, New insights on near-extremal black holes, [arXiv:2307.10423](https://arxiv.org/abs/2307.10423).
 - [4] S. A. Hartnoll, J. Polchinski, E. Silverstein, and D. Tong, Towards strange metallic holography, *J. High Energy Phys.* **04** (2010) 120.
 - [5] S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz, Building a holographic superconductor, *Phys. Rev. Lett.* **101**, 031601 (2008).
 - [6] L. Gladden, V. Ivo, P. Kovtun, and A. O. Starinets, Instability in $N = 4$ supersymmetric Yang-Mills theory at finite density, *Phys. Rev. D* **111**, 086030 (2025).
 - [7] C. P. Herzog, I. R. Klebanov, S. S. Pufu, and T. Tesileanu, Emergent quantum near-criticality from baryonic black branes, *J. High Energy Phys.* **03** (2010) 093.
 - [8] I. R. Klebanov, S. S. Pufu, and T. Tesileanu, Membranes with topological charge and $\text{AdS}_4/\text{CFT}_3$ correspondence, *Phys. Rev. D* **81**, 125011 (2010).
 - [9] I. R. Klebanov and E. Witten, Superconformal field theory on three-branes at a Calabi-Yau singularity, *Nucl. Phys.* **B536**, 199 (1998).
 - [10] A. Buchel, Fluxification and scalarization of the conifold black holes, *Phys. Rev. D* **111**, 046010 (2025).
 - [11] A. Buchel, Instability of baryonic black branes, *J. High Energy Phys.* **05** (2025) 215.
 - [12] A. Buchel, A holographic perspective on Gubser-Mitra conjecture, *Nucl. Phys.* **B731**, 109 (2005).
 - [13] L. Gladden, V. Ivo, P. K. Kovtun, and A. O. Starinets, Hydrodynamics with multiple charges and holography, *J. High Energy Phys.* **12** (2025) 173.
 - [14] I. R. Klebanov and E. Witten, AdS/CFT correspondence and symmetry breaking, *Nucl. Phys.* **B556**, 89 (1999).
 - [15] A. Buchel and R. Monten, Near-extremal membranes in M-theory, *J. High Energy Phys.* **02** (2026) 231.
 - [16] O. Aharony, M. Berkooz, and E. Silverstein, Multiple trace operators and nonlocal string theories, *J. High Energy Phys.* **08** (2001) 006.
 - [17] E. Witten, Multitrace operators, boundary conditions, and AdS/CFT correspondence, [arXiv:hep-th/0112258](https://arxiv.org/abs/hep-th/0112258).
 - [18] M. Berkooz, A. Sever, and A. Shomer, ‘Double trace’ deformations, boundary conditions and space-time singularities, *J. High Energy Phys.* **05** (2002) 034.
 - [19] S. S. Gubser and I. R. Klebanov, A universal result on central charges in the presence of double trace deformations, *Nucl. Phys.* **B656**, 23 (2003).
 - [20] D. Cassani, P. Koerber, and O. Varela, All homogeneous $N = 2$ M-theory truncations with supersymmetric AdS_4 vacua, *J. High Energy Phys.* **11** (2012) 173.
 - [21] P. K. Kovtun and A. O. Starinets, Quasinormal modes and holography, *Phys. Rev. D* **72**, 086009 (2005).
 - [22] D. T. Son and A. O. Starinets, Minkowski space correlators in AdS/CFT correspondence: Recipe and applications, *J. High Energy Phys.* **09** (2002) 042.
 - [23] A. Buchel, χ SB of cascading gauge theory in de Sitter, *J. High Energy Phys.* **05** (2020) 035.
 - [24] https://github.com/abuchel-hepht/data_arXiv-2601.01548.