

# Invisible neutron decay and light BSM particles

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In Standard Model (SM) effective field theory, invisible neutron decay arises from  $d = 12$  operators. Adding new, light particles to the field content of the SM, such as right-handed neutrinos, allows to construct operators for invisible neutron decay at much lower dimensions. Observing invisible neutron decay, if nucleon decays with charged leptons remain absent, would therefore point towards the existence of new neutral degrees of freedom. Here, we discuss four cases: (i) Adding right-handed neutrinos to the SM; (ii) a right-handed neutrino and an axionlike particle; (iii) a right-handed neutrino and a (nearly) massless singlet scalar; and (iv) a right-handed neutrino and a light  $Z'$ . We give the general tree-level decomposition for the resulting  $d = (7-9)$  operators for invisible neutron decay. We also briefly discuss LHC searches related to the exotic states found in these UV completions.

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## I. INTRODUCTION

Searches for nucleon decays by Super-Kamiokande have pushed limits to  $10^{34}$  yr and above for several decay channels. The best-known examples are probably  $p \rightarrow \ell^+ \pi^0$  ( $\ell = e, \mu$ ) [1] and  $n \rightarrow \bar{\nu} K^0$  [2], which are motivated by grand unified theories (GUTs). Less known are decays to three charged leptons, where Super-Kamiokande now also gives limits in excess of  $10^{34}$  yr [3]. However, limits on many other channels are less stringent, sometimes by orders of magnitude; see [4,5]. On the other hand, nucleon decay searches form an important part of the agenda of next-generation underground detectors, such as JUNO [6], Hyper-Kamiokande [7], or DUNE [8], and significant improvements in sensitivity on many nucleon decay modes are expected from these experiments in the foreseeable future.

Traditionally, searches for proton decay were motivated by the predictions from GUTs, such as the classical  $SU(5)$  [9]; for a review, see, for example, [10]. More recently, however, many theoretical papers have taken a more agnostic view and studied nucleon decays from the point of view of effective field theory (EFT) [11–27].<sup>1</sup> In Standard Model EFT (SMEFT), one can classify different decay modes according to the dimensionality of the corresponding operators. For example, two-body decays with  $\Delta(B + L) = 2$  are described by  $d = 6$  operators,  $\Delta(B - L) = 2$  are  $d = 7$ , and *genuine* three-body decays need at least  $d = 9$  operators [17].

In this paper, we are interested in invisible neutron decay. Considering only SM particles, the only possibility for a neutron to decay into “nothing” is the channel  $n \rightarrow 3\nu$ .<sup>2</sup> In SMEFT, one has to go up to  $d = 12$  to find for the first time an operator capable of generating invisible neutron decay, without the corresponding decays to charged leptons, as already realized by Weinberg more than 40 years ago [30]. Weinberg gave as an example the operator [30]

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<sup>1</sup>Here we also cite some pioneering works on nucleon decays in SMEFT [28–33] and the studies on the decompositions of the relevant effective operators [34–39].

<sup>2</sup>Here both  $\nu$  and  $\bar{\nu}$  are mentioned as  $\nu$ .

$$\mathcal{L}_{d=12}^W = \frac{c_{12}^W}{\Lambda^8} \epsilon_{IJK} \epsilon^{ij} \epsilon^{kl} \epsilon^{mn} (\bar{u}_R^I L_i) (\bar{d}_R^J L_k) (\bar{d}_R^K L_m) H_j H_l H_n, \quad (1)$$

where  $\Lambda$  is the typical scale of the origin of this effective operator and  $c_{12}^W$  is the Wilson coefficient. In Eq. (1), we have suppressed generation indices for simplicity, and  $\epsilon_{IJK}$  and  $\epsilon^{ij}$  are antisymmetric tensors for the contraction of  $SU(3)_c$  and  $SU(2)_L$  indices. For this  $d = 12$  operator, one can estimate the nucleon decay width for the three-body final state  $n \rightarrow 3\nu$  roughly as [17]

$$\Gamma \sim \frac{\beta_h^2 m_N^5 \langle H^0 \rangle^6}{6144 \pi^3 \Lambda^{16}} \Rightarrow \frac{1}{9 \times 10^{29} \text{ yr}} \left( \frac{13.4 \text{ TeV}}{\Lambda} \right)^{16}. \quad (2)$$

Here,  $\beta_h \simeq 0.014 \text{ GeV}^3$  is the nuclear matrix element [40],  $m_N$  is the nucleon mass,  $\langle H^0 \rangle \simeq 174 \text{ GeV}$  is the vacuum expectation value of the SM Higgs field, and  $c_{12}^W$  is taken to be unity.

Since the operator in Eq. (1) by itself will not generate final states with charged leptons at any measurable level, one might be tempted to assume that just having a low enough scale  $\Lambda$  would be sufficient to create an observable rate for the invisible neutron decay. However, this statement neglects that there are lower-dimensional operators that will generate neutron decays with charged lepton final states. SMEFT operators with three leptons were discussed in [13]. In SMEFT, the lowest-dimensional operator with three leptons has  $d = 9$ ; one example is  $\mathcal{O} \propto (QL)(\bar{L}\bar{L})(d_R d_R)$ . Reference [13] lists a total of 16 operators of this type at  $d = 9$ . Some of these  $d = 9$  operators can generate the decay  $n \rightarrow 3\nu$ . However, all the operators that can do so always also generate decays to charged leptons with very similar rates. Since nucleon decay searches are much more sensitive to final states with charged leptons [3] than to  $n \rightarrow 3\nu$  [5], none of the  $d = 9$  operators can generate invisible neutron decay at observable rates. Moreover, if any of these  $d = 9$  operators is present in the theory, one expects it to dominate the nucleon decay rate by many orders of magnitude with respect to the  $d = 12$  operator in Eq. (1).

To avoid this conclusion, one could argue that all  $d = 9$  operators are actually forbidden by some symmetry, such that the  $d = 12$  operator in Eq. (1) is the lowest dimension at which neutron decay is possible. However, this is not possible. While some specific symmetry could indeed eliminate all  $d = 9$  operators, at  $d = 10$  one finds an operator with three leptons [30]:  $\mathcal{O} \propto d^c d^c d^c LLLH$ .<sup>3</sup> This operator induces the decay  $n \rightarrow e^- \pi^+ \nu \nu$  [14] and will dominate the decay rate unless its Wilson coefficient is fine-tuned to be very close to zero. As we show in

Appendix B, one cannot forbid this operator by any symmetry if the  $d = 12$  operator is to be allowed. Thus, within pure SMEFT, one expects that invisible neutron decay is always accompanied by neutron decays with charged lepton final states.

For invisible neutron decay, the Particle Data Group [5] cites an old Kamiokande result (from 1993) [41] ( $4.9 \times 10^{26} \text{ yr}$ ), but there are more recent and stronger bounds set by SNO [42] ( $2 \times 10^{29}$ ), KamLAND [43] ( $5.8 \times 10^{29} \text{ yr}$ , cf. also [44]), and two analyses by the SNO + Collaboration [45] ( $2.5 \times 10^{29} \text{ yr}$ ) and [46] ( $9.0 \times 10^{29} \text{ yr}$ ). For the nearer and farther future sensitivity, estimates have been given by JUNO [47] ( $5 \times 10^{31} \text{ yr}$ ) and for THEIA (80 kt), which could ultimately reach  $\sim 4 \times 10^{32} \text{ yr}$  according to their proposal [48].

However, there exists a simple possibility to generate invisible neutron decay with lower-dimensional operators *and without generating sizable rates into final states with charged leptons*: We can extend the SM particle content with new light degrees of freedom. In this paper, we discuss four different possibilities. First, we add (i) (at least two) right-handed neutrinos  $N_R$ , also sometimes called sterile neutrinos in the literature, cf. [49–55] for reviews and current bounds, motivated from various neutrino experiments [56–62]. In addition, the introduction of such light degrees of freedom is often discussed in the context of cosmology, e.g., the stringent bound on neutrino masses from recent cosmological observations [63–66] may be alleviated by the mixing between active neutrinos and massless sterile neutrinos [67–70]. In  $N_R$ SMEFT (for a list of basis operators, see [71–73]), invisible neutron decay operators appear at  $d = 9$ .

Second, we add (ii) a right-handed neutrino and an axion-like particle (ALP),  $a$ . The axion is one of the best-motivated light new physics fields [74,75]. It arises as a consequence of the breaking of the Peccei-Quinn symmetry introduced to solve the strong  $CP$  problem [76,77]. For experimental searches and phenomenology of ALPs, cf., e.g., [78–85] and for cosmological aspects of ALPs, cf., e.g., [86–91]. The basis operators of  $a$ SMEFT, the effective theory with the SM particle content plus an ALP, are listed in [92–94]. In  $aN_R$ SMEFT, i.e., SMEFT with  $N_R$ 's and ALPs, invisible neutron decay operators appear at  $d = 8$ .<sup>4</sup>

Finally, we add a right-handed neutrino and (iii) a (nearly) massless scalar,  $\phi$ , or (iv) a light vector boson,  $Z'$ . An ultralight scalar is an attractive candidate for dark matter, aka “fuzzy” dark matter [97]. For experimental searches and cosmological effects of the light vector boson, aka dark photon, cf., e.g., [98–101]. The basis operators of  $\phi$ SMEFT (SMEFT with a SM singlet scalar) and  $Z'$ SMEFT

<sup>3</sup>This operator is nonzero only for two or more generations of down quarks *or* leptons.

<sup>4</sup>Basis operators with a given particle content can be constructed up to a given mass dimension  $d$  by AutoEFT [95] or Sym2Int [96].

(SMEFT with a dark photon) are listed in [92,93].<sup>5</sup> The invisible decay of neutrons is generated from  $d = 7$  operators in both  $\phi N_R$ SMEFT (SMEFT with  $\phi$  and  $N_R$ ) and  $Z' N_R$ SMEFT (SMEFT with  $Z'$  and  $N_R$ ) [102].

In this paper, we give the operators for invisible neutron decay for all four cases discussed above, estimate the neutron decay half-lives, and give the complete list of the possible UV completions for these operators at tree level. For the different beyond-the-SM (BSM) particles found in these UV models, we also briefly discuss current LHC limits.

This introduction would not be complete without mentioning that invisible neutron decay was discussed in some earlier papers. For example, in [103] the authors speculated about “decays to nothing” in models with extra time dimensions. The authors of [104] considered universal extra dimensions and an extended gauge group, to have the neutron decay dominantly into  $n \rightarrow \nu \bar{\nu}_s \bar{\nu}_s$ , where  $\nu_s$  is some sterile neutrino. The effective operators relevant to  $n \rightarrow \bar{\nu}_s \bar{\nu}_s' \bar{\nu}_s''$  are listed in [105] in the context of an extra-dimension model. In [106], invisible neutron decay was studied in a left-right symmetric model with large extra dimensions. Some recent works have also considered the possibility of a “dark” (invisible) neutron decay [107–111] in relation to the neutron lifetime anomaly [112]. And, finally, a particular UV model for invisible dineutron decay to two singlet fermions was studied in [113], where the authors also discussed possible constraints from neutron stars.

The rest of this paper is organized as follows. In Sec. II we introduce the different model variants in a bit more detail, discuss some generalities, define the operators, and estimate the neutron decay half-lives for the different cases. In Sec. III we then decompose the operators to find the possible UV models at tree level. Section IV discusses a number of different LHC searches and how they can be used or reinterpreted to derive limits on the BSM states found in Sec. III. We then close with a short discussion and summary. Some tables for the decompositions are relegated to the appendices for the sake of keeping the main text more compact.

## II. SETUP: MODELS AND EFFECTIVE OPERATORS

In this section, we describe the theoretical setup. We consider four different extensions of the SM particle content. For each case, we briefly describe the model. We then define the operators relevant for invisible neutron decay. We give an estimate of the neutron decay half-life in

<sup>5</sup>Later, we will use the symbol  $S$  for the scalars that mediate the effective operators, and therefore, to avoid confusion, we use  $\phi$  for the light scalar field in the final state of the invisible neutron decay process, although the effective theory with  $\phi$  is called sometimes  $s$ SMEFT in the literature.

each case. Finally, in an additional subsection, we discuss a possible caveat. The decomposition of the operators defined here is discussed in Sec. III.

### A. Variant I: Right-handed neutrinos

In the SM, neutrinos are massless. While there are many extensions of the SM that can explain neutrino masses, as observed in oscillation experiments, maybe the simplest possibility is to add right-handed neutrinos to the SM particle content.

Massive neutrinos can be either Dirac or Majorana particles. In the classical type-I seesaw model [114–117], right-handed neutrinos have a Majorana mass term and the Lagrangian can be written as :

$$\mathcal{L}_Y = (Y_\nu^*)_{\alpha j} \bar{N}_{Rj} L_\alpha \cdot H + \frac{1}{2} (M_M)_{jj} \bar{N}_{Rj}^c N_{Rj} + \text{H.c.} \quad (3)$$

Note that  $M_M$  can be taken diagonal without loss of generality. Majorana neutrinos, on the other hand, violate lepton number by two units. Whether such a  $\Delta L = 2$  term is allowed or not will depend on the UV completion of the operators defined below. We note, however, that the invisible neutron decay  $n \rightarrow 3\bar{N}$  violates  $L$  by three units, and thus Majorana masses need not to be allowed in general and neutrinos could be Dirac particles, as far as invisible neutron decay is concerned.

For the fit to current neutrino data, at least two copies of  $N_R$  are needed and both Dirac and Majorana neutrinos can explain existing data. We consider three copies of  $N_R$ , to mirror the three SM generations, but doing an exact fit to oscillation data is irrelevant for the neutron decay we are interested in. The only condition for the decay to occur is that the right-handed neutrino masses are smaller than  $m_n/3$ .

Two operators can be constructed at level  $d = 9$  that contribute to invisible neutron decay:

$$\mathcal{O}_1^N = \frac{c_{\alpha\beta\gamma ijk}^{(1)}}{\Lambda^5} (\bar{u}_{R\alpha}^c d_{R\beta}) (\bar{d}_{R\gamma}^c N_{Ri}) (\bar{N}_{Rj}^c N_{Rk}), \quad (4)$$

$$\mathcal{O}_2^N = \frac{c_{\alpha\beta\gamma ijk}^{(2)}}{\Lambda^5} (\bar{Q}_\alpha^c Q_\beta) (\bar{d}_{R\gamma}^c N_{Ri}) (\bar{N}_{Rj}^c N_{Rk}). \quad (5)$$

Here,  $\alpha, \beta, \gamma$  are SM generation indices, while  $i, j, k$  run over the generation of  $N_R$ . To keep the expressions simpler, we have suppressed  $SU(3)_c$  [and  $SU(2)_L$ ] indices. Here and in all cases discussed below, the color of the three quarks must be contracted with a completely antisymmetric tensor,  $\epsilon_{IJK}$ . It is straightforward to show that both of these operators vanish in case there is only one copy of  $N_R$ , and we have also checked this with Sym2Int [96].

Assuming the mass of the right-handed neutrinos to be negligible and one of the  $c_{111ijk}^{(1/2)} = 1$ , we can estimate the

partial half-life for the invisible neutron decay as

$$\Gamma(n \rightarrow \bar{N}_i \bar{N}_j \bar{N}_k) \simeq \frac{\beta_h^2 m_n^5}{6144 \pi^3 \Lambda^{10}} \simeq \frac{1}{9 \times 10^{29} \text{ yr}} \left( \frac{180 \text{ TeV}}{\Lambda} \right)^{10}. \quad (6)$$

Here again,  $\beta_h \simeq 0.014 \text{ GeV}^3$  is the nuclear matrix element [40] and the half-life is normalized to the current bound from SNO+ [46]. We can apply Eq. (6) to the different existing and projected half-life limits and obtain estimates for the sensitivity to the new physics scale  $\Lambda$ :

$$\Gamma \sim \begin{cases} \frac{1}{4.9 \times 10^{26} \text{ yr}} \left( \frac{84 \text{ TeV}}{\Lambda} \right)^{10} \text{ Kamikande (PDG),} \\ \frac{1}{5.0 \times 10^{31} \text{ yr}} \left( \frac{270 \text{ TeV}}{\Lambda} \right)^{10} \text{ JUNO (future),} \\ \frac{1}{4.0 \times 10^{32} \text{ yr}} \left( \frac{330 \text{ TeV}}{\Lambda} \right)^{10} \text{ THEIA (future).} \end{cases} \quad (7)$$

## B. Variant II: A right-handed neutrino and an ALP

ALPs appear in many extensions of the SM. While the ALP mass is a free parameter, the couplings of the ALPs with SM fields are protected by an approximate classical shift symmetry, as is the case for the classical axion. Many different studies of properties of and searches for ALPs have been published; see, for example, [79,80,118,119]. Here we only briefly mention that ALPs could also explain the dark matter [120,121] and/or be long-lived particles [121,122].

The ALP Lagrangian up to  $d = 5$  contains the following terms [123]:

$$\begin{aligned} \mathcal{L}_a = & \frac{1}{2} \partial_\mu a \partial^\mu a - \frac{1}{2} m_a^2 a^2 - \sum_X \frac{c_{X\tilde{X}a}}{\Lambda} a X^{\mu\nu} \tilde{X}_{\mu\nu} \\ & - \sum_\psi \frac{c_{\psi a}}{\Lambda} \partial_\mu a (\bar{\psi} \gamma^\mu \psi). \end{aligned} \quad (8)$$

Here  $X^{\mu\nu}$  stands for any of the field-strength tensors of the SM, i.e.,  $X = B, W, G$ , and  $\tilde{X}_{\mu\nu}$  is its dual.  $m_a$  is the ALP mass, in principle a free parameter. Note, however, that one usually assumes the ALP to be the pseudo-Goldstones of a spontaneously broken global symmetry, and thus  $m_a$  can be naturally small compared with the scale of symmetry breaking.  $\psi$  are the SM fermions, and for the case we are interested in also  $\psi = N_R$ . It is important to note that the ALP couples only derivatively to fermions.

There are two  $d = 8$   $B$ -violating operators containing a right-handed singlet fermion,  $N_R$ , and an ALP,  $a$ :

$$\begin{aligned} \mathcal{O}_1^a &= \frac{c_{\alpha\beta\gamma}^{(1,a)}}{\Lambda^4} (\partial_\mu a) (\bar{N}_R \gamma^\mu d_{R\alpha}) (\bar{u}_{R\beta}^c d_{R\gamma}), \\ \mathcal{O}_2^a &= \frac{c_{\alpha\beta\gamma}^{(2,a)}}{\Lambda^4} (\partial_\mu a) (\bar{N}_R \gamma^\mu d_{R\alpha}) (\bar{Q}_\beta^c Q_\gamma). \end{aligned} \quad (9)$$

One copy of  $N_R$  is enough to form these operators; thus, for simplicity, we have not added a generation index to  $N_R$ . However, two  $N_R$  would still be needed if one wants to explain neutrino data.

There are two more  $d = 8$   $B$ -violating operators, containing an ALP, that can be written down with only SM fields:

$$\begin{aligned} \mathcal{O}_3^a &= \frac{c_{\alpha\beta\gamma\delta}^{(3,a)}}{\Lambda^4} (\partial_\mu a) (\bar{L}_\alpha d_{R\beta}) (\bar{Q}_\gamma^c \gamma^\mu d_{R\delta}), \\ \mathcal{O}_4^a &= \frac{c_{\alpha\beta\gamma\delta}^{(4,a)}}{\Lambda^4} (\partial_\mu a) (\bar{e}_{R\alpha} \gamma^\mu d_{R\beta}) (\bar{d}_{R\gamma}^c d_{R\delta}). \end{aligned} \quad (10)$$

The operators in Eq. (9) can generate invisible neutron decays without accompanying charged lepton final states, while  $\mathcal{O}_3^a$  will always also have charged leptons in the nucleon decays. We have listed  $\mathcal{O}_4^a$  for completeness, but it does not give a contribution to invisible neutron decay.

The operators in Eqs. (4) and (5) violate  $(B + L) = 4$ . Thus, it is easy to argue that the standard  $d = 6$  two-body  $(B + L) = 2$  decays are absent on symmetry grounds for the invisible neutron decay to three  $N_R$ 's. However, the situation is different for the case of an ALP. ALPs do not usually carry either  $B$  or  $L$ , as can be seen from Eq. (8). For an ALP with  $(B, L) = (0, 0)$ , however,  $\mathcal{O}_{1,2}^a$  both have  $(B - L) = 2$ . One can write down operators at  $d = 7$  with  $(B - L) = 2$ , such as, for example,  $\mathcal{O} \propto (\bar{N}_R \gamma^\mu d_{R\alpha}) (\bar{Q}_\beta^c (iD_\mu) Q_\gamma)$ . Since this type of operators leads to two-body decays, such as  $n \rightarrow \pi^0 + \cancel{e}$  and  $p \rightarrow \pi^+ + \cancel{e}$ , one would expect the invisible neutron decay to be a very subdominant decay mode in this case.

A possible way out of this conclusion is based on the following argument. If we assign  $a$  the quantum numbers  $(B, L) = (-1, 1)$ ,  $\mathcal{O}_{1,2}^a$  both conserve  $(B + L)$  and  $(B - L)$ , and thus none of the  $(B - L) = 2$  operators need to be present in the theory. For this to be possible, however, the “standard”  $d = 5$  ALP interaction terms in Eq. (8) need to be absent. It is possible to write down UV completions that fulfill these assignments.

For the operators  $\mathcal{O}_{1,2}^a$ , the width for the two-body decay  $n \rightarrow Na$  can be estimated to be roughly

$$\Gamma(n \rightarrow Na) \simeq \frac{\beta_h^2 m_n^3}{32 \pi \Lambda^8} \simeq \frac{1}{9.0 \times 10^{29} \text{ yr}} \left( \frac{9.6 \times 10^6 \text{ GeV}}{\Lambda} \right)^8. \quad (11)$$

The much larger scale found here, compared with the case  $n \rightarrow 3\bar{N}$  [see Eq. (6)], makes it unlikely that the UV

completions for these operators can ever be tested in accelerator experiments.

### C. Variant III: A right-handed neutrino and a light scalar

The third possibility for generating operators for the invisible neutron decay with BSM fields is to add a right-handed neutrino and a light scalar to the SM. We discuss this case only briefly, since we believe it to be less motivated and add it only for completeness of the discussion.

For a model with a light singlet scalar,  $\phi$ , and right-handed neutrinos there are two  $d = 7$  operators contributing to invisible neutron decay:

$$\begin{aligned}\mathcal{O}_1^\phi &= \frac{c_{\alpha\beta\gamma}^{(1,\phi)}}{\Lambda^3} \phi (\bar{N}_R^c d_{R\alpha}) (\bar{u}_{R\beta}^c d_{R\gamma}), \\ \mathcal{O}_2^\phi &= \frac{c_{\alpha\beta\gamma}^{(2,\phi)}}{\Lambda^3} \phi (\bar{N}_R^c d_{R\alpha}) (\bar{Q}_\beta^c Q_\gamma).\end{aligned}\quad (12)$$

Similar to the ALP case discussed above, if  $\phi$  has  $(B, L) = (0, 0)$ , the  $d = 6$  operators that one can construct from Eq. (12), eliminating  $\phi$ , would lead to dominant two-body decays, rendering invisible neutron decay uninteresting. Assigning nonzero baryon and lepton number to  $\phi$  allows to eliminate this problem technically.

The width  $n \rightarrow \bar{N}\phi$  can be estimated to be roughly

$$\Gamma(n \rightarrow \bar{N}\phi) \simeq \frac{\beta_h^2 m_n}{32\pi\Lambda^6} \simeq \frac{1}{9.0 \times 10^{29} \text{ yr}} \left( \frac{2.1 \times 10^9 \text{ GeV}}{\Lambda} \right)^6. \quad (13)$$

### D. Variant IV: A right-handed neutrino and a light vector

Finally, we briefly mention the possibility where a neutron decays to a right-handed neutrino and an SM-singlet vector boson  $Z'$ , which is prompted by the following  $d = 7$  effective operators:

$$\begin{aligned}\mathcal{O}_1^{Z'} &= \frac{c_{\alpha\beta\gamma}^{(1,Z')}}{\Lambda^3} Z'_\mu (\bar{N}_R \gamma^\mu d_{R\alpha}) (\bar{u}_{R\beta}^c d_{R\gamma}), \\ \mathcal{O}_2^{Z'} &= \frac{c_{\alpha\beta\gamma}^{(2,Z')}}{\Lambda^3} Z'_\mu (\bar{N}_R \gamma^\mu d_{R\alpha}) (\bar{Q}_\beta^c Q_\gamma).\end{aligned}\quad (14)$$

To avoid the possible  $d = 6$  nucleon decays and make the  $d = 7$  invisible neutron decay dominant, we need to assign charges (which may be the  $B$  and  $L$  numbers) to  $Z'$ . The argument is the same as for the ALP case, cf. Sec. II B.

Assuming  $Z'$  is a massless gauge boson under a new gauge symmetry, the decay rate is calculated to be

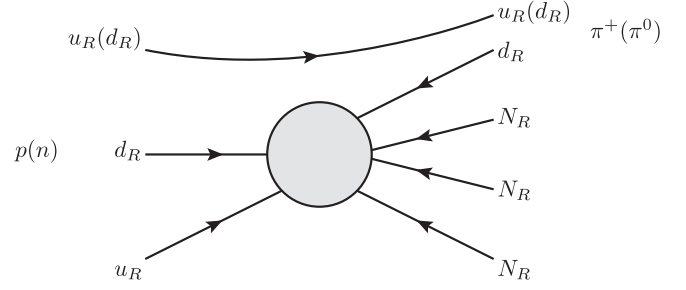


FIG. 1.  $p(n) \rightarrow \pi^+(\pi^0) + \text{missing}$  induced by the  $d = 9$   $u_R d_R d_R N_R N_R N_R$  operator.

$$\Gamma(n \rightarrow NZ') \simeq \frac{\beta_h^2 m_n}{16\pi\Lambda^6} \simeq \frac{1}{9.0 \times 10^{29} \text{ yr}} \left( \frac{2.3 \times 10^9 \text{ GeV}}{\Lambda} \right)^6. \quad (15)$$

When  $Z'$  acquires a mass through the spontaneous breaking of some new gauge symmetry, the decay rate changes to

$$\Gamma \simeq \frac{\beta_h^2 m_n^3}{32\pi\Lambda^6 M_{Z'}^2} \left( 1 - \frac{M_{Z'}^2}{m_n^2} \right)^2 \left( 1 + \frac{2M_{Z'}^2}{m_n^2} \right). \quad (16)$$

This equation seems to diverge in the limit of  $M_{Z'} \rightarrow 0$ . However, for a gauge boson, such divergencies are known to be unphysical. In a full-fledged model for the massive  $Z'$ , one would also need to specify the symmetry-breaking sector of the theory. The full calculation then includes the correct treatment of the would-be Goldstone boson of the theory, and in the limit of  $M_{Z'} \ll m_n$ , Eq. (16) will reduce to the same expression as for the scalar case Eq. (13), up to a coefficient. However, constructing such a complete model for the  $Z'$  is beyond the scope of this paper.

### E. A caveat

There is one important caveat to the above discussion: For any operator that annihilates three quarks to invisible particles, it is possible to replace an initial-state quark with a final-state antiquark. An example is shown in Fig. 1. Thus, four-body final states with a (neutral or charged) pion will always accompany three-body invisible decays. And, similarly for the two-body final states  $N_R + a$  and  $N_R + S$ , three-body final states with one additional pion will be generated.<sup>6</sup>

No search for such three- and four-body final states exists. However, Super-Kamiokande has a search for  $n \rightarrow \pi^0 \bar{\nu}$  and  $p \rightarrow \pi^+ \bar{\nu}$ . The current bounds are [124]

$$\Gamma(p \rightarrow \pi^+ \bar{\nu}) > 3.9 \times 10^{32} \text{ yr}, \quad (17)$$

<sup>6</sup>Since one can replace an initial  $d$  quark by a final state  $\bar{s}$ , producing a final-state kaon, this also allows, in principle, sensitivity to second-generation indices in the operators.

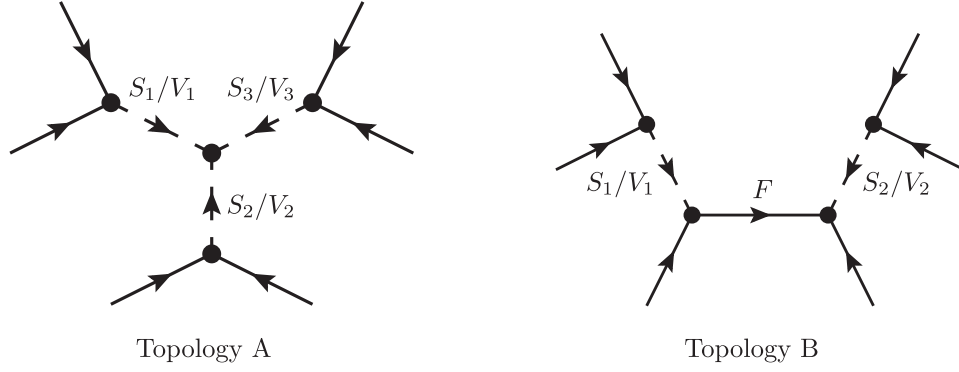


FIG. 2. Topologies for tree-level realizations of the  $d = 9$  neutron decay operators; see Eqs. (4) and (5). Depending on the chiralities of the outer fermion fields, the dashed lines can be either a scalar,  $S$ , or vector,  $V$ . The mediator  $F$  in Topology B must be introduced as a vector-like fermion, but when it is an SM singlet, it can also be a Majorana fermion.

$$\Gamma(n \rightarrow \pi^0 \bar{\nu}) > 1.1 \times 10^{33} \text{ yr}. \quad (18)$$

These are two-body decays, and thus the pion momentum is fixed at roughly  $p(\pi) \sim 460$  MeV. Super-Kamiokande uses this constraint in setting the limits. For the four-body decays, on the other hand, the pion momentum takes a distribution, which depends in addition on the mass of the final-state particles. Nevertheless, Super-Kamiokande provides sufficient information such that we can make a rough estimate of the limit for the four-body decay  $n \rightarrow \pi^0 + 3\bar{N}$ .

The limit on  $\Gamma(n \rightarrow \pi^0 \bar{\nu})$  is set by Super-Kamiokande [124] excluding 19.1 signal events in the bin  $p(\pi^0) = [400, 500]$  MeV. From Fig. 3 of [124], we can estimate that the total number of background events in the range  $p(\pi^0) = [0, 500]$  MeV sums to about 850 events, and about 80 events in the bin with  $p(\pi) = [400, 500]$  MeV. Thus, we estimate that a limit of  $19.1 \times \sqrt{850/80} \simeq 62$  events could be set from the total data. This would lower the exclusion limit by a factor of roughly 3.3 and the limit on  $\Gamma(n \rightarrow \pi^0 + 3\bar{N})$  should roughly be  $T_{1/2} \gtrsim 3.3 \times 10^{32}$  yr. We stress that this is only a rough estimate and also that the Super-Kamiokande [124] result is based on only 172.8 kton. yr of data. A reanalysis of all Super-Kamiokande data should be able to provide a limit, which could be up to a factor of 2 better than our naive estimate.

For the four-body decays of Fig. 1, we can estimate the partial half-life, using results from [17], with appropriate replacements:

$$\begin{aligned} \Gamma(n \rightarrow \pi^0 \bar{N}_i \bar{N}_j \bar{N}_k) &\simeq \frac{m_n^7 W_0(\pi)^2}{737280 \pi^5 \Lambda^{10}} \\ &\simeq \frac{1}{3.3 \times 10^{32} \text{ yr}} \left( \frac{245 \text{ TeV}}{\Lambda} \right)^{10}. \end{aligned} \quad (19)$$

Here the matrix element is taken to be  $|W_0(\pi)| = |\langle \pi^0 | (ud)_R d_R | n \rangle| = 0.134 \text{ GeV}^2$  [40]. And again, we assume the Wilson coefficient is equal to one.

Comparing this estimate with Eq. (7), we can see that the sensitivity of this pion mode to the new physics scale  $\Lambda$  is actually better than the sensitivity of the invisible mode at SNO+ (180 TeV), and only moderately lower than the future sensitivity estimated for JUNO (270 TeV). Given these numbers, one must conclude that searches for  $n \rightarrow \pi^0 + \cancel{E}$  provide important constraints on searches for invisible neutron decay. Therefore, we encourage experimentalists to study the  $n \rightarrow \pi^0 + \text{“multiple missing”}$  mode (independently from  $n \rightarrow \pi^0 \bar{\nu}$ ) and provide the bound. We stress again that this is true not only for the case of the  $d = 9$  operator discussed here, but for all operators generating invisible neutron decay.

### III. HIGH-ENERGY COMPLETIONS

In the previous section, we defined the effective operators for invisible neutron decay for the four possible different model variants. In this section, we discuss how these operators can be generated in the UV, after integrating out heavy particles. Our aim here is to systematically find *all* possible UV models via the diagrammatic method. Since, however, the basic procedure is the same for all decompositions, we discuss here only one decomposition of the operator  $\mathcal{O}_1^N \propto u_R d_R d_R N_R N_R N_R$  in detail. Full tables for all other operators and decompositions are given in the appendices.

For six-fermion operators, there are only two possible topologies at tree level [125], using only renormalizable interactions. They are shown in Fig. 2. Note that here the arrows indicate the flow of the particle number. Internal particles can be either scalars, fermions, or vectors. For Topology A, there are exactly three possibilities to distribute the fermions of  $\mathcal{O}_1^N$  to the outer legs of the diagram. We list these in Table I.

For  $\mathcal{O}_1^N$  and Topology A, all mediator fields are Lorentz scalars (indicated with the symbol  $S$ ). Their charges under the SM gauge symmetries, in the order  $(SU(3)_c, SU(2)_L, U(1)_Y)$ , are uniquely determined, as given in the table.

TABLE I. Decompositions for the operator  $u_R d_R d_R N_{Ri} N_{Rj} N_{Rk}$  allowed with Topology A. The fields in parentheses form an interaction with the corresponding mediator field,  $S_{1,2,3}$ . For the explicit form of the interaction Lagrangians, cf. Eqs. (20) and (21).

| Decomposition                          | $S_1$                                  | $S_2$                                  | $S_3$                            | Comment                                   | Eq.  |
|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------|-------------------------------------------|------|
| $(u_R d_R)(d_R N_{Ri})(N_{Rj} N_{Rk})$ | $(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$ | $(\mathbf{3}, \mathbf{1}, -1/3)$       | $(\mathbf{1}, \mathbf{1}, 0)$    | $S_2 \neq S_1^\dagger$ to avoid $d = 6$   | (20) |
| $(u_R N_{Ri})(d_R d_R)(N_{Rj} N_{Rk})$ | $(\mathbf{3}, \mathbf{1}, +2/3)$       | $(\bar{\mathbf{3}}, \mathbf{1}, -2/3)$ | $(\mathbf{1}, \mathbf{1}, 0)$    | $d_R d_R S_2^\dagger = 0$                 |      |
| $(u_R N_{Ri})(d_R N_{Rj})(d_R N_{Rk})$ | $(\mathbf{3}, \mathbf{1}, +2/3)$       | $(\mathbf{3}, \mathbf{1}, -1/3)$       | $(\mathbf{3}, \mathbf{1}, -1/3)$ | $S_3 \neq S_2$ to avoid $S_1 S_2 S_3 = 0$ | (21) |

The interaction Lagrangians for the different cases can be written as

$$\mathcal{L}_{A-1} = y_{ud} \epsilon^{IJK} (\overline{u_R^c})_{I\dot{a}} (d_R)_{J\dot{a}} (S_1^\dagger)_K + y_{dN} (\overline{d_R^c})_{I\dot{a}} (N_{Ri})_{\dot{a}} (S_2^\dagger)^I + y_{NN} (\overline{N_{Rj}^c})_{\dot{a}} (N_{Rk})_{\dot{a}} S_3^\dagger + \mu (S_1)^I (S_2)_I S_3, \quad (20)$$

$$\mathcal{L}_{A-3} = y_{uN} (\overline{u_R^c})_{I\dot{a}} (N_{Ri})_{\dot{a}} (S_1^\dagger)^I + y_{dS_2} (\overline{d_R^c})_{I\dot{a}} (N_{Rj})_{\dot{a}} (S_2^\dagger)^I + y_{dS_3} (\overline{d_R^c})_{I\dot{a}} (N_{Rk})_{\dot{a}} (S_3^\dagger)^I + \mu \epsilon^{IJK} (S_1)_I (S_2)_J (S_3)_K, \quad (21)$$

where the indices  $I, J, K$  in the lower (upper) position are for  $\mathbf{3}$  ( $\bar{\mathbf{3}}$ ) in  $SU(3)_c$ , and  $\dot{a}$  is for a right-handed 2-spinor. For case A-2 the interaction  $d_R d_R S_2^\dagger$  vanishes identically, and thus it is not a valid decomposition for  $\mathcal{O}_1^N$ .<sup>7</sup> For a more compact notation, we have suppressed generation indices in these Lagrangians. The different Yukawas are to be understood as matrices of either dimensions  $(3, 3)$ ,  $(3, n)$ , or  $(n, n)$ , where  $n$  is the number of copies of right-handed neutrinos.

A few more comments are in order. In case A-1, described by the Lagrangian (20), the diquark,  $S_1$ , has the same SM charges as the leptoquark,  $S_2^\dagger$ . However, these two fields have to be distinct states. If they were identical, one could construct from  $S_1$  a  $d = 6$  effective operator  $\mathcal{O} \propto u_R d_R d_R N_R$ . This operator generates the two-body decays  $n(p) \rightarrow \pi^0(\pi^+) \bar{N}$  with a significantly larger rate than that of the invisible neutron decay. To make  $n \rightarrow 3 \bar{N}$  the dominant decay mode, one must avoid all  $d = 6$  nucleon decays, i.e.,  $S_1$  and  $S_2$  must be two different fields. This can be realized by assigning for case A-1 all  $S_i$  definite baryon and lepton numbers. With the assignments  $S_1 = (2/3, 0)$ ,  $S_2 = (1/3, 1)$  and  $S_3 = (0, 2)$ , where the brackets stand for  $(B, L)$ , all Yukawa couplings conserve baryon and lepton number trivially. The only  $B$ - and  $L$ -violating coupling would then be the soft parameter  $\mu$ . Since  $\mu$  is a soft parameter, one can actually argue that it can be technically

<sup>7</sup>However, the interaction  $d_R S_R S_2^\dagger$  does not vanish. Since operators are usually given in the flavor basis, while in the neutron the quarks are in the mass eigenstate basis, decomposition A-2 can contribute to the invisible neutron decay rate. The rate, however, will be suppressed by a factor  $\sin^2 \theta_C$ , where  $\theta_C$  is the Cabibbo angle. We do not discuss this possibility in further detail.

small in the sense of 't Hooft's naturalness, since in the limit of  $\mu \rightarrow 0$  baryon and lepton numbers are conserved. A small value of  $\mu$  would change the expected mass scale for the  $S_i$  drastically. Instead of Eq. (6), for topology-A one can express the decay width as

$$\Gamma(n \rightarrow \bar{N}_i \bar{N}_j \bar{N}_k) \simeq \frac{\beta_h^2 m_n^5 \mu^2}{6144 \pi^3 \Lambda^{12}} \simeq \frac{1}{9 \times 10^{29} \text{ yr}} \left( \frac{1 \text{ keV}}{\mu} \right)^2 \left( \frac{2.4 \text{ TeV}}{\Lambda} \right)^{10}. \quad (22)$$

Here, it is assumed that  $m_{S_1} \simeq m_{S_2} \simeq m_{S_3} \simeq \Lambda$ . Putting  $\mu = \Lambda$ , Eq. (22) reduces to Eq. (6), of course. However, this argument shows that it might not be completely hopeless to search for the BSM particles that appear in the decomposition of a  $d = 9$  operator at the LHC or the future FCC-hh.

The third case, case A-3, is similar to case A-1. However [see Eq. (21)], case A-3 contains three leptoquarks, and two of them,  $S_2$  and  $S_3$ , interact with  $d_R$  and  $N_R$ . They therefore have the same SM charges. However, also in this case they must be two different fields; otherwise, the triple scalar interaction vanishes. In this case, a simple lepton number assignment, as discussed for case A-1, is not sufficient to make  $S_2$  and  $S_3$  distinct. This problem can, on the other hand, be cured by adding a discrete symmetry, for example,  $\mathbb{Z}_3$ , with the charge  $\omega \equiv e^{i2\pi/3}$ . We can assign  $\omega$  to  $N_{Rj}$  and  $S_2$  and  $\omega^2$  to  $N_{Rk}$  and  $S_3$ . Under this assignment, both the triple scalar interaction and the Yukawas are allowed under the discrete symmetry. However, the  $\mu$ -term will still break lepton and baryon numbers softly, thus guaranteeing that the dangerous two-body nucleon decay modes are absent.

In the appendices, we list all mediator fields that appear in the high-energy completions of the  $u_R d_R d_R N_R N_R N_R$  operator with Topology B in Fig. 2. There we also give the decompositions of the  $QQ d_R N_R N_R N_R$  operator (5) with both the topologies. We note that also for topology B it is, of course, possible to lower the expected mass scale. In EFT, one usually estimates the scale  $\Lambda$  for a Wilson coefficient  $c \simeq 1$ . However, for a  $d = 9$  operator,  $c \propto Y^4$ , where  $Y$  stands symbolically for any Yukawa coupling (see Fig. 2). Thus, even moderately small Yukawa couplings will drastically change the expectations given in Eq. (6):

TABLE II. List of the mediators that appear in the decompositions of the effective operators relevant for the invisible neutron decay.

| Mediator                          | $n \rightarrow 3\bar{N}$ |          | $n \rightarrow Na(NZ')$                       |                                               | $n \rightarrow \bar{N}\phi$ |                |
|-----------------------------------|--------------------------|----------|-----------------------------------------------|-----------------------------------------------|-----------------------------|----------------|
|                                   | $uudNNN$                 | $QQdNNN$ | $\partial a u d d \bar{N} (Z' u d d \bar{N})$ | $\partial a Q Q d \bar{N} (Z' Q Q d \bar{N})$ | $u u d N \phi$              | $Q Q d N \phi$ |
| $S(\mathbf{1}, \mathbf{1}, 0)$    | ✓                        | ✓        |                                               |                                               |                             |                |
| $S(\mathbf{3}, \mathbf{1}, -1/3)$ | ✓                        | ✓        | ✓                                             | ✓                                             | ✓                           | ✓              |
| $S(\mathbf{3}, \mathbf{1}, +2/3)$ | ✓                        |          | ✓                                             |                                               | ✓                           |                |
| $S(\mathbf{3}, \mathbf{2}, +1/6)$ |                          |          |                                               | ✓                                             |                             |                |
| $V(\mathbf{3}, \mathbf{1}, -1/3)$ |                          |          | ✓                                             | ✓                                             |                             |                |
| $V(\mathbf{3}, \mathbf{1}, +2/3)$ |                          |          | ✓                                             |                                               |                             |                |
| $V(\mathbf{3}, \mathbf{2}, +1/6)$ |                          | ✓        |                                               | ✓                                             |                             | ✓              |
| $F(\mathbf{1}, \mathbf{1}, 0)$    | ✓                        | ✓        | ✓                                             | ✓                                             | ✓                           | ✓              |
| $F(\mathbf{3}, \mathbf{1}, -1/3)$ | ✓                        | ✓        | ✓                                             | ✓                                             | ✓                           | ✓              |
| $F(\mathbf{3}, \mathbf{1}, +2/3)$ | ✓                        |          | ✓                                             |                                               | ✓                           |                |
| $F(\mathbf{3}, \mathbf{2}, +1/6)$ |                          | ✓        |                                               | ✓                                             |                             | ✓              |

$$\Gamma(n \rightarrow \bar{N}_i \bar{N}_j \bar{N}_k) \simeq \frac{\beta_h^2 Y^8 m_n^5}{6144 \pi^3 \Lambda^{10}} \simeq \frac{1}{9 \times 10^{29} \text{ yr}} \left( \frac{Y}{0.01} \right)^8 \left( \frac{4.5 \text{ TeV}}{\Lambda} \right)^{10}. \quad (23)$$

In addition, the appendices contain tables of the decompositions for the effective operators in the other variants, which were presented in Secs. II B–II D. Similar to cases A-1 and A-3 discussed here for  $\mathcal{O}_1^N$ , in all cases one has to make sure that the dangerous two-body nucleon decays are absent by the use of some symmetry; otherwise, the rate for the invisible neutron decay will be negligible.

Table II lists all mediator fields that appear in the decompositions of the  $d = (7-9)$  operators. There are a total of 11 fields, four scalars, four fermions, and three vectors. Some fields appear in all operators and in all four model variants ( $S_{3,1,-1/3}$ ,  $F_{1,1,0}$ , and  $F_{3,1,-1/3}$ ), while some others appear only for a particular model variant ( $S_{1,1,0}$ ,  $V_{3,1,-1/3}$ ) or even only one particular operator ( $S_{3,2,1/6}$  and  $V_{3,1,2/3}$ ). Note that the fields appearing in the decompositions of the effective operator with a  $Z'$  are the same as those for the operators with an ALP, and the corresponding column applies to both fields.

#### IV. LHC PHENOMENOLOGY

In this section, we briefly discuss a variety of LHC searches that can be reinterpreted as limits for fields listed in Table II. Naive expectations for the scale,  $\Lambda$ , of the  $d = (7-9)$  operators discussed in Sec. II make it look quite unlikely that the LHC can find any positive signals for these states. However, as discussed around Eqs. (22) and (23), the estimates derived using EFT in Sec. II might vastly overestimate the masses of the BSM fields in a full-fledged UV model. We therefore feel motivated to at least briefly discuss the current experimental status.

The decomposition of the  $d = 9$  operators for the  $N_R$  SMEFT model, discussed in Sec. III, generates a set of eight different BSM fields: Four vector-like fermions ( $F_{1,1,0}$ ,  $F_{3,1,-1/3}$ ,  $F_{3,1,2/3}$ , and  $F_{3,2,1/6}$ ), three types of scalars ( $S_{1,1,0}$ ,  $S_{3,1,-1/3}$ , and  $S_{3,1,2/3}$ ), and one vector ( $V_{3,2,1/6}$ ). Three more fields appear only in the ALP/ $Z'$  variants ( $S_{3,2,1/6}$ ,  $V_{3,1,-1/3}$ , and  $V_{3,1,2/3}$ ). We mention these only for the sake of completeness here, since an LHC discovery for the fields from the  $d = 7, 8$  operator seems (even) less likely than those appearing in models for the  $d = 9$  operators.

First, we mention that for the singlet fields, there is no production mode at the LHC other than decays of the colored BSM states. Moreover,  $F_{1,1,0}$  and  $S_{1,1,0}$  will decay to  $N_R$ 's and thus be invisible if they are lighter than the colored states. Thus, there are no constraints on these two fields from LHC searches.

Let us then concentrate on the colored fermions. Here, we follow largely the discussion in [126]. All three colored fermions can have interactions with SM quarks and the Higgs:

$$\mathcal{L}_Y = Y_{U_R} \bar{Q} F_{3,1,2/3} H^\dagger + Y_{D_R} \bar{Q} F_{3,1,-1/3} H + Y_{Q_u} \bar{F}_{3,2,1/6} u_R H^\dagger + Y_{Q_d} \bar{F}_{3,2,1/6} d_R H + \text{H.c.} \quad (24)$$

In Eq. (24), we have suppressed generation indices. The fermions  $F_{3,1,-1/3}$ ,  $F_{3,1,2/3}$ , and  $F_{3,2,1/6}$  must also have vector-like mass terms. After electroweak symmetry breaking, Eq. (24) leads to mixing between heavy and SM quarks. This mixing will dominate their decay widths unless the corresponding Yukawa couplings are tiny. If, on the other hand, some Yukawa couplings are large, this mixing may also dominate vector-like quark (VLQ) production at the LHC; see the left panel of Fig. 3. In addition, VLQs can always be pair produced (see the right panel

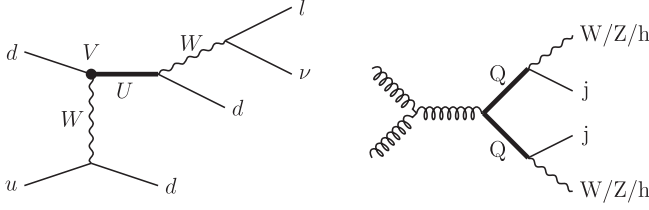


FIG. 3. Example Feynman diagrams for single and pair production of VLQs and their decays.

of Fig. 3), independent of the size of the mixing. Note that in this figure we show the VLQs decaying to “j” to indicate any final-state quark, i.e., including also third-generation quarks.

Both ATLAS and CMS have searched for such VLQs in a number of publications. For example, ATLAS has searched for pair-produced VLQs decaying to first-generation quarks in [127]. Limits are given as functions of the VLQ mass for different branching ratios  $\text{Br}(Q \rightarrow W + j)$  versus  $\text{Br}(Q \rightarrow H + j)$ , assuming  $\text{Br}(Q \rightarrow W + j) + \text{Br}(Q \rightarrow H + j) + \text{Br}(Q \rightarrow Z + j) = 1$  [127]. Derived limits are in the range (900–1500) GeV for  $\text{Br}(Q \rightarrow W + j)$  in the range (10–100)%. In [128], ATLAS presented the results of a search for pair-produced VLQs decaying to third-generation quarks. Limits range from (1–1.7) TeV depending mostly on the branching ratio  $\text{Br}(Q \rightarrow W + t/b)$ . These limits are slightly stronger than the first-generation quark limits, mostly due to lower backgrounds.

There are also searches for singly produced VLQs. However, in the search reported in [129] ATLAS assumed that the VLQ decays to the final state  $t + h$  or  $W + b$ , where the top/bottom is tagged in order to reduce backgrounds. Similarly, CMS [130] searched for single VLQ production in the final state  $t + h$  and  $t + Z$ . For  $\kappa_T = 1$ , where  $\kappa_T$  measures the strength of the interaction of the VLQ with electroweak gauge bosons relative to standard electroweak couplings, a lower limit on VLQs decaying to top quarks of around  $m_{\text{VLQ}} \geq 1.9$  TeV is found in [129]. There are no searches yet published for singly produced VLQs decaying to second- or first-generation quarks. However, due to much larger backgrounds in such searches, one can expect limits to be considerably weaker.

However, we need to stress that all the VLQ searches discussed up to now depend on the presence of mixing. The neutron decay diagrams that we study here, however, do not require any of the Yukawa coupling in Eq. (24) to be nonzero. Instead, couplings of the form  $F$ - $q$ - $S$ , where  $F$ ,  $q$ , and  $S$  stand for a VLQ, an SM quark, and one of the BSM scalars, must be present. If the VLQs are pair produced at the LHC they can then decay to final states containing  $q + N_R + N_R$ , i.e., jet plus missing energy. This signal is equivalent to the one used in the standard squark search at the LHC. For example, ATLAS [131] has searched for pair-produced squarks with  $\mathcal{L} = 139/\text{fb}$ . ATLAS derives limits for one generation of squarks (roughly  $m_{\tilde{q}} \geq 1.2$  TeV) and

eight degenerate squarks (roughly  $m_{\tilde{q}} \geq 1.8$  TeV) for light neutralinos. Since VLQs (being fermions) have cross sections at least  $\sim 4$  times larger than a color triplet scalar of the same mass, this supersymmetry search [131] could be reinterpreted to yield a lower limit on the VLQ mass. We estimate this limit to be roughly  $m_{\text{VLQ}} \geq 1.5$  TeV for any of the three colored VLQs under consideration.

The colored scalar states appear as either leptoquarks or diquarks in the different decompositions discussed in the previous sections. The leptoquark couplings are

$$\begin{aligned} \mathcal{L}^{\text{LQ}} = & Y_{Nu} \overline{u_R^c} N_R S_{3,1,2/3}^\dagger + Y_{de} \overline{d_R^c} e_R S_{3,1,2/3} \\ & + Y_{Nd} \overline{d_R^c} N_R S_{3,1,-1/3}^\dagger + Y_{eu} \overline{u_R^c} e_R S_{3,1,-1/3}^\dagger \\ & + Y_{LQ} \overline{Q^c} L S_{3,1,-1/3}^\dagger + \text{H.c.}, \end{aligned} \quad (25)$$

while the diquark couplings can be written as

$$\begin{aligned} \mathcal{L}^{\text{DQ}} = & Y_{dd} \overline{d_R^c} d_R S_{3,1,2/3} + Y_{QQ} \overline{Q^c} Q S_{3,1,-1/3} \\ & + Y_{ud} \overline{u_R^c} d_R S_{3,1,-1/3} + \text{H.c.} \end{aligned} \quad (26)$$

Again, we have suppressed generation indices. We stress, however, that the flavor-diagonal entries in  $Y_{dd}$  are zero; see Sec. III. Importantly, while the leptoquarks and diquarks appear with the same SM quantum numbers, they *must* be different states; otherwise, one can generate  $d = 6$  proton decay tree-level diagrams, which would lead to lower limits on their masses near the GUT scale and render invisible neutron decay completely negligible, as discussed in Sec. III. Note that only ( $Y_{Nu}$ ,  $Y_{Nd}$ , and  $Y_{ud}$ ) of the eight couplings given in Eqs. (25) and (26) appear in the decompositions for the neutron decay. This is important, since none of these couplings will generate final states with charged leptons.

Leptoquarks, both pair and singly produced, have been searched for at the LHC in a number of different final states. The couplings  $Y_{eu}$  and  $Y_{LQ}$  do not appear in the decompositions, but will lead to final states with charged leptons, which are more easily constrained at the LHC. For example, [132] searched for pair-produced leptoquarks decaying to third-generation quarks. Lower limits on leptoquark masses in the range (1.2–1.7) TeV are derived, depending on leptoquark decay branching ratios. ATLAS also searched for leptoquarks decaying to light quarks [133]. For a branching ratio of  $\text{Br}(\text{LQ} \rightarrow l + q) = 1$ , limits of  $m_{\text{LQ}} \geq 1.8$  TeV and  $m_{\text{LQ}} \geq 1.7$  TeV are excluded in the electron and muon channels. These limits weaken to roughly (700–900) GeV for  $\text{Br}(\text{LQ} \rightarrow l + q) = 0.1$ .

Single or resonant production of leptoquarks requires large Yukawa couplings to give sizable cross sections. ATLAS searched for resonant leptoquarks [134] via a lepton-jet signature, with either one or two leptons in the final state. For  $Y_{de} = 1.0$ , leptoquarks with masses below  $m_{\text{LQ}} = 3.4$  TeV are excluded by this search. CMS

searched for single leptoquarks in  $t$ -channel diagrams [135], leading to dilepton final states. Both scalar and vector leptoquarks were considered. For scalar [vector] leptoquarks with masses between (1–5) TeV, Yukawa couplings larger than (0.3–1.0) [(0.1–1.4)] have been excluded.

Again, the searches just discussed require charged lepton final states and the corresponding couplings are not required by the decompositions for invisible neutron decay. Leptoquarks that decay only to quarks and invisible final states are less constrained. For example, the supersymmetry search by ATLAS [131] will give a limit of roughly  $m_{LQ} \gtrsim 1.2$  TeV for pair-produced leptoquarks. A CMS paper [136] gave limits for pair-produced leptoquarks decaying to neutrinos. Limits are  $m_{LQ} \geq 1140$  GeV for scalar leptoquarks and  $m_{LQ} \geq (1560\text{--}1980)$  GeV for vector leptoquarks, depending on the strength of the leptoquark gluon coupling. The more stringent limits for vector leptoquarks simply reflect the larger cross sections for vectors compared with scalars. These limits apply to the state  $V_{3,2,1/6}$  that appears in the decomposition of  $\mathcal{O}_2^N$ .

Finally, diquarks are stringently constrained from dijet searches, since the production is  $s$ -channel enhanced. For example, CMS [137] gave a lower limit on the mass of a color triplet scalar of  $m_{DQ} \simeq 7.5$  TeV assuming a Yukawa coupling to two quarks of electromagnetic strength,  $e$ . The limits derived by CMS are strictly speaking valid only for a scalar diquark coupling to both up-type and down-type quarks with the same strength. Thus, they apply only to  $S_{3,1,-1/3}$ .

The searches discussed above are based on luminosities up to 140/fb. Moderate improvements can be expected from the high-luminosity LHC. However, the future FCC-hh [138] would considerably improve sensitivities. For example, [139] quoted sensitivities up to  $m_{LQ} \sim 8$  (15) TeV for pair (singly) produced leptoquarks, decaying to charged leptons. Many other search channels, discussed above, should improve by similar factors.

We close this section with a short discussion of the  $d = 12$  operator given in Eq. (1). In the beginning of this section, we argued that a Wilson coefficient of  $c_W = 1$  is often unrealistic in UV models, and thus EFT tends to overestimate the masses of the BSM states. This is certainly also true for a  $d = 12$  operator.<sup>8</sup> We used an automated code for operator decomposition, based on the diagrammatic method and described in [140,141], to decompose Eq. (1) at tree level. At such a large dimension, there is a proliferation of models. We count 39 topologies, 150 different diagrams, and 12 713 model variants.

Despite this large number of possible model variations, all model variants can be described with just 38 different fields (scalars and fermions, no vectors). Twenty-six of

these are triplets of color, either diquarks, leptoquarks, or heavy vector-like quarks. All scalars and fermions discussed above appear in this list too and the constraints we discussed above also apply to this case. The remaining colored states not covered in the above discussion will have bounds that are at least as strong, since they are all either larger  $SU(2)$  multiplets or states with larger hypercharge. There is, however, one important difference between the UV states of the  $d = 12$  operator and those of the  $d = (7\text{--}9)$  operators. Whereas for the  $d = 9$  operators, strictly speaking, only right-handed neutrino final states are required by the couplings appearing in the decomposition, in the case of the  $d = 12$  operator, similar decay rates of the leptoquarks to final states with missing energy and final states with charged leptons are expected. As discussed above, final states with charged leptons are a better signal than missing energy at the LHC and thus give generally more stringent limits. In other words, limits on states from the decompositions of the  $d = 12$  operator from LHC will in general be more stringent than those for the  $d = 9$  case, which motivated the discussion given in this section.

## V. SUMMARY

In this paper, we studied invisible neutron decay. Limits on nucleon decay modes with three charged leptons [3] are much stronger than those of invisible neutron decay [5]. Invisible neutron decay with observable rates can therefore be generated only by operators that do not also generate charged lepton final states. In SMEFT, one has to go up to  $d = 12$  to find such an operator. Current limits on invisible neutron decay correspond then to scales of order  $\Lambda \simeq 13$  TeV. An observation of invisible neutron decay in the next round of experiments might then be accompanied by new physics at the future FCC-hh [139].

However, this conclusion changes if one adds new light degrees of freedom to the SM particle content. We discussed four different extensions of the SM for which invisible neutron decay with observable rates can be induced by operators of dimension  $d = (7\text{--}9)$ . We have estimated the half-lives in each case, and for each model we also gave all possible UV completions at tree level. Current limits on the scales of these operators range from  $\Lambda \simeq 180$  TeV for the variant model with only right-handed neutrinos ( $d = 9$  operator) to  $\Lambda \simeq 2 \times 10^9$  GeV ( $d = 7$  operator).

A discovery of invisible neutron decay, therefore, may point towards the existence of new, light BSM states. In particular, we stress that in all possible models a neutral fermion state with the quantum numbers of a right-handed neutrino must exist. Thus, these models will also be able to explain neutrino masses.

We also discussed constraints on the allowed rate for invisible neutron decay, which could be placed by a search for  $n \rightarrow \pi^0 + \cancel{e}$ . Hyper-Kamiokande [7] should be able to improve on the limit of  $n \rightarrow \pi^0 + \cancel{e}$  from Super-Kamiokande [124] by a considerable factor. We note that

<sup>8</sup>Note that in the decomposition of Eq. (1) up to seven different Yukawa couplings appear.

the Super-Kamiokande analysis [124] gave only a limit on a two-body final state, but the same type of search can also be used to constrain decays with more than one invisible particle in the final state. Nonobservation of  $n \rightarrow \pi^0 + 3\bar{N}$  at Hyper-Kamiokande might rule out the possibility of observing invisible neutron decay in the foreseeable future.

However, to end this discussion on a more positive note, we mention that the sensitivity of Hyper-Kamiokande and the future JUNO and (the proposed) THEIA experiment to the scales of the invisible neutron decay operators will be rather similar. Observation of invisible neutron decay in JUNO should lead to an excess in the Hyper-Kamiokande search for  $n \rightarrow \pi^0 + \cancel{E}$  and vice versa. The question of whether invisible neutron decay or the decay  $n \rightarrow \pi^0 + \cancel{E}$  would be *discovered first*, however, is not so easy to answer. While the future Hyper-Kamiokande experiment [7] is much larger than Super-Kamiokande, and thus allows to probe much larger half-lives, already the Super-Kamiokande limit [124] is background dominated. To claim a discovery in this case requires both hundreds of signal events and a detailed understanding of backgrounds. JUNO's search for the invisible neutron decay [47], on the other hand, is based on a characteristic triple coincidence arising from the invisible decays of  $s$ -shell neutrons in  $^{12}\text{C}$ , which leaves very few background events, making a discovery with just a handful of events possible.

Finally, one can speculate that adding the two gammas from the  $\pi^0$  decay to the triple coincidence for the invisible neutron decay would eliminate all remaining background in JUNO's search. We do not know, however, whether this advantage is (over)compensated by the loss of signal extraction efficiency or not. Only the experimental collaboration can perform an analysis sufficiently sophisticated to decide whether  $n \rightarrow \pi^0 + \cancel{E}$  or the invisible neutron decay itself would be more sensitive to the different models (and the corresponding operators) that we discussed in this paper.

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## DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

## APPENDIX A: LIST OF THE HIGH-ENERGY COMPLETIONS

Here we list the decompositions of the effective operators and the charges under the SM gauge symmetries of the necessary mediator fields in each decomposition.

- (1) Variant I with light right-handed neutrinos  $N_R$ : Tables I, III, IV, and V, and Fig. 2 for the topologies.
- (2) Variant II with an  $N_R$  and an axion-like particle  $a$ : Tables VI and VII, and Fig. 4 for the topology.
- (3) Variant III with an  $N_R$  and a scalar field  $\phi$ : Tables VIII and IX, and Fig. 4 for the topology.
- (4) Variant IV with an  $N_R$  and a vector boson  $Z'$ : The mediators appearing in this variant are the same as those listed in Tables VI and VII for Variant II.

The “Decomposition” column shows how the outer fields are distributed to the vertices in the corresponding topology. The fields in parentheses are in the same vertex. Let us take the first line of Table III,

$$(u_R d_R)(N_{Ri})(N_{Rj})(d_R N_{Rk}), \quad (\text{A1})$$

as an example. “ $(u_R d_R)$ ” means that  $u_R$  and  $d_R$  correspond to the two outer fields in the leftmost vertex in Topology B given in Fig. 2. The mediator field between the  $(u_R d_R)$  vertex and the second-to-left vertex with  $N_{Ri}$  is  $S_1(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$ . The direction of the  $S_1$  particle number is indicated with an arrow in Fig. 2. The two middle vertices with  $N_{Ri}$  and  $N_{Rj}$  are mediated by the fermion  $F(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$ . The outer fields in the rightmost vertex are  $d_R$  and  $N_{Rk}$ , and the mediator between the  $(N_{Rj})$  vertex and the  $(d_R N_{Rk})$  vertex is  $S_2(\mathbf{3}, \mathbf{1}, -1/3)$ . In short, the high-energy completion of this decomposition is determined as

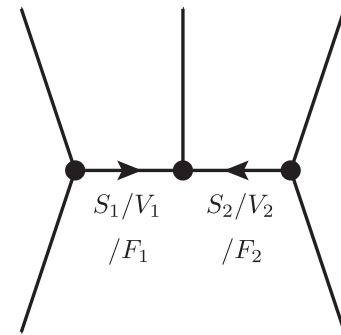


FIG. 4. Topology for the operators with four fermions and one scalar, such as  $u_R d_R d_R N_R \phi$  and  $u_R d_R d_R \bar{N}_R a$ , i.e., one of the outer legs should be a scalar,  $\phi$  or  $a$ . The directions of arrows on the outer legs depend on the decomposition. Although the mediators are given with solid lines in this topology, they can be scalars  $S$ , vectors  $V$ , or fermions  $F$ , depending on the distribution of outer fields.

TABLE III. Decompositions and the necessary mediator fields of the  $d = 9$  effective operator  $u_R d_R d_R N_R N_R N_R$  with Topology B in Fig. 2.

| Decomposition                           | $S_1$                                  | $F$                                    | $S_2$                                  | Comment                                                                                                                    |
|-----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------------------------------------------------------------------------------------------|
| $(u_R d_R)(N_{Ri})(N_{Rj})(d_R N_{Rk})$ | $(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$ | $(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$ | $(\mathbf{3}, \mathbf{1}, -1/3)$       | $S_2 \neq S_1^\dagger$ to avoid $d = 6$<br>$F$ can be Majorana                                                             |
| $(u_R d_R)(d_R)(N_{Ri})(N_{Rj} N_{Rk})$ | $(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$ | $(\mathbf{1}, \mathbf{1}, 0)$          | $(\mathbf{1}, \mathbf{1}, 0)$          |                                                                                                                            |
| $(u_R d_R)(N_{Ri})(d_R)(N_{Rj} N_{Rk})$ | $(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$ | $(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$ | $(\mathbf{1}, \mathbf{1}, 0)$          |                                                                                                                            |
| $(d_R N_{Ri})(u_R)(d_R)(N_{Rj} N_{Rk})$ | $(\mathbf{3}, \mathbf{1}, -1/3)$       | $(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$ | $(\mathbf{1}, \mathbf{1}, 0)$          |                                                                                                                            |
| $(d_R N_{Ri})(d_R)(u_R)(N_{Rj} N_{Rk})$ | $(\mathbf{3}, \mathbf{1}, -1/3)$       | $(\bar{\mathbf{3}}, \mathbf{1}, -2/3)$ | $(\mathbf{1}, \mathbf{1}, 0)$          |                                                                                                                            |
| $(u_R N_{Ri})(N_{Rj})(N_{Rk})(d_R d_R)$ | $(\mathbf{3}, \mathbf{1}, +2/3)$       | $(\mathbf{3}, \mathbf{1}, +2/3)$       | $(\bar{\mathbf{3}}, \mathbf{1}, -2/3)$ | $d_R d_R S_2^\dagger = 0$<br>$\epsilon^{IJK}(d_{Ri} d_{Rj}) = 0$<br>$d_R d_R S_1^\dagger = 0$<br>$d_R d_R S_1^\dagger = 0$ |
| $(u_R N_{Ri})(d_R)(d_R)(N_{Rj} N_{Rk})$ | $(\mathbf{3}, \mathbf{1}, +2/3)$       | $(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$ | $(\mathbf{1}, \mathbf{1}, 0)$          |                                                                                                                            |
| $(d_R d_R)(u_R)(N_{Ri})(N_{Rj} N_{Rk})$ | $(\bar{\mathbf{3}}, \mathbf{1}, -2/3)$ | $(\mathbf{1}, \mathbf{1}, 0)$          | $(\mathbf{1}, \mathbf{1}, 0)$          |                                                                                                                            |
| $(d_R d_R)(N_{Ri})(u_R)(N_{Rj} N_{Rk})$ | $(\bar{\mathbf{3}}, \mathbf{1}, -2/3)$ | $(\bar{\mathbf{3}}, \mathbf{1}, -2/3)$ | $(\mathbf{1}, \mathbf{1}, 0)$          |                                                                                                                            |
| $(u_R N_{Ri})(d_R)(N_{Rj})(d_R N_{Rk})$ | $(\mathbf{3}, \mathbf{1}, +2/3)$       | $(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$ | $(\mathbf{3}, \mathbf{1}, -1/3)$       |                                                                                                                            |
| $(u_R N_{Ri})(N_{Rj})(d_R)(d_R N_{Rk})$ | $(\mathbf{3}, \mathbf{1}, +2/3)$       | $(\mathbf{3}, \mathbf{1}, +2/3)$       | $(\mathbf{3}, \mathbf{1}, -1/3)$       |                                                                                                                            |
| $(d_R N_{Ri})(u_R)(N_{Rj})(d_R N_{Rk})$ | $(\mathbf{3}, \mathbf{1}, -1/3)$       | $(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$ | $(\mathbf{3}, \mathbf{1}, -1/3)$       |                                                                                                                            |

TABLE IV. Decompositions of the  $QQ d_R N_R N_R N_R$  operator with Topology A in Fig. 2.

| Decomposition                     | $S_1/V_1$                                 | $S_2/V_2$                           | $S_3/V_3$                           | Comment     |
|-----------------------------------|-------------------------------------------|-------------------------------------|-------------------------------------|-------------|
| $(QQ)(d_R N_{Ri})(N_{Rj} N_{Rk})$ | $S_1(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$ | $S_2(\mathbf{3}, \mathbf{1}, -1/3)$ | $S_3(\mathbf{1}, \mathbf{1}, 0)$    | $V_1 = V_2$ |
| $(Qd_R)(QN_{Ri})(N_{Rj} N_{Rk})$  | $V_1(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$ | $V_2(\mathbf{3}, \mathbf{2}, +1/6)$ | $S_3(\mathbf{1}, \mathbf{1}, 0)$    |             |
| $(QN_{Ri})(QN_{Rj})(d_R N_{Rk})$  | $V_1(\mathbf{3}, \mathbf{2}, +1/6)$       | $V_2(\mathbf{3}, \mathbf{2}, +1/6)$ | $S_3(\mathbf{3}, \mathbf{1}, -1/3)$ |             |

$$\begin{aligned}
\mathcal{L}_{\text{Tab.III-1}} = & y_{ud} \epsilon^{IJK} (\overline{u_R^c})_{I\dot{a}} (d_R)_{\dot{J}}^\dagger (S_1^\dagger)_K \\
& + y_{dN} (\overline{d_R^c})_{I\dot{a}} (N_R)_{\dot{J}}^\dagger (S_2^\dagger)^I \\
& + y_{\bar{F}NS_1} (\overline{F_L})_{I\dot{a}} (N_R)_{\dot{J}}^\dagger (S_1)^I \\
& + y_{NFS_2} (\overline{N_R^c})_{\dot{a}} (F_R)^{I\dot{a}} (S_2)_I. \quad (\text{A2})
\end{aligned}$$

$$\mathcal{L}_{d=6} = \frac{y_{ud} y_{dN}}{M_{S_1}^2} \epsilon^{IJK} (\overline{u_R^c})_{I\dot{a}} (d_R)_{\dot{J}}^\dagger (\overline{d_R^c})_{K\dot{b}} (N_R)^{\dot{b}}, \quad (\text{A3})$$

The charges of  $S_2$  and  $S_1^\dagger$  are the same. However, if they are an identical field, it mediates the  $d = 6$  effective operator

which induces  $p \rightarrow \pi^+ \bar{N}$  with a larger decay rate than  $d = 9$  induced invisible neutron decay, i.e., the strong bound from  $p \rightarrow \pi^+ \bar{N}$  constrains the invisible neutron decay to an unreachable level. In short, if one wants to have this decomposition for the invisible neutron decay, the mediator  $S_2$  must be an independent field from  $S_1^\dagger$ , although the

TABLE V. Decompositions of the  $QQ d_R N_R N_R N_R$  operator with Topology B in Fig. 2.

| Decomposition                       | $S_1/V_1$                                 | $F$                                    | $S_2/V_2$                           | Comment             |
|-------------------------------------|-------------------------------------------|----------------------------------------|-------------------------------------|---------------------|
| $(QQ)(N_{Ri})(N_{Rj})(d_R N_{Rk})$  | $S_1(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$ | $(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$ | $S_2(\mathbf{3}, \mathbf{1}, -1/3)$ | $F$ can be Majorana |
| $(QQ)(d_R)(N_{Ri})(N_{Rj} N_{Rk})$  | $S_1(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$ | $(\mathbf{1}, \mathbf{1}, 0)$          | $S_2(\mathbf{1}, \mathbf{1}, 0)$    |                     |
| $(QQ)(N_{Ri})(d_R)(N_{Rj} N_{Rk})$  | $S_1(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$ | $(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$ | $S_2(\mathbf{1}, \mathbf{1}, 0)$    |                     |
| $(d_R N_{Ri})(Q)(Q)(N_{Rj} N_{Rk})$ | $S_1(\mathbf{3}, \mathbf{1}, -1/3)$       | $(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$ | $S_2(\mathbf{1}, \mathbf{1}, 0)$    |                     |
| $(Qd_R)(N_{Ri})(N_{Rj})(QN_{Rk})$   | $V_1(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$ | $(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$ | $V_2(\mathbf{3}, \mathbf{2}, +1/6)$ | $F$ can be Majorana |
| $(Qd_R)(Q)(N_{Ri})(N_{Rj} N_{Rk})$  | $V_1(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$ | $(\mathbf{1}, \mathbf{1}, 0)$          | $S_2(\mathbf{1}, \mathbf{1}, 0)$    |                     |
| $(Qd_R)(N_{Ri})(Q)(N_{Rj} N_{Rk})$  | $V_1(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$ | $(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$ | $S_2(\mathbf{1}, \mathbf{1}, 0)$    |                     |
| $(QN_{Ri})(Q)(d_R)(N_{Rj} N_{Rk})$  | $V_1(\mathbf{3}, \mathbf{2}, +1/6)$       | $(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$ | $S_2(\mathbf{1}, \mathbf{1}, 0)$    |                     |
| $(QN_{Ri})(d_R)(Q)(N_{Rj} N_{Rk})$  | $V_1(\mathbf{3}, \mathbf{2}, +1/6)$       | $(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$ | $S_2(\mathbf{1}, \mathbf{1}, 0)$    |                     |
| $(QN_{Ri})(d_R)(N_{Rj})(QN_{Rk})$   | $V_1(\mathbf{3}, \mathbf{2}, +1/6)$       | $(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$ | $V_2(\mathbf{3}, \mathbf{2}, +1/6)$ | $V_1 = V_2$         |
| $(QN_{Ri})(Q)(N_{Rj})(d_R N_{Rk})$  | $V_1(\mathbf{3}, \mathbf{2}, +1/6)$       | $(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$ | $S_2(\mathbf{3}, \mathbf{1}, -1/3)$ |                     |
| $(QN_{Ri})(N_{Rj})(Q)(d_R N_{Rk})$  | $V_1(\mathbf{3}, \mathbf{2}, +1/6)$       | $(\mathbf{3}, \mathbf{2}, +1/6)$       | $S_2(\mathbf{3}, \mathbf{1}, -1/3)$ |                     |

TABLE VI. Decompositions of  $u_R d_R d_R \overline{N_R} a$ , which result in the  $d = 8$  effective operator with a derivative operator,  $\partial u_R d_R d_R \overline{N_R}$ . The operator  $u_R d_R d_R \overline{N_R} Z'$  with a vector boson is decomposed with the same mediators.

| Decomposition                                | $S_1/V_1/F_1$                                  | $S_2/V_2/F_2$                                  | Comment                   |
|----------------------------------------------|------------------------------------------------|------------------------------------------------|---------------------------|
| $[\partial](d_R d_R)(u_R)(\overline{N_R} a)$ | $S_1(\overline{\mathbf{3}}, \mathbf{1}, -2/3)$ | $F_2(\mathbf{1}, \mathbf{1}, 0)$               | $d_R d_R S_1^\dagger = 0$ |
| $[\partial](d_R \overline{N_R})(u_R)(d_R a)$ | $V_1(\mathbf{3}, \mathbf{1}, -1/3)$            | $F_2(\mathbf{3}, \mathbf{1}, -1/3)$            |                           |
| $[\partial](u_R d_R)(d_R)(\overline{N_R} a)$ | $S_1(\overline{\mathbf{3}}, \mathbf{1}, +1/3)$ | $F_2(\mathbf{1}, \mathbf{1}, 0)$               | $F_2$ can be Majorana     |
| $[\partial](u_R \overline{N_R})(d_R)(d_R a)$ | $V_1(\mathbf{3}, \mathbf{1}, +2/3)$            | $F_2(\mathbf{3}, \mathbf{1}, -1/3)$            |                           |
| $[\partial](u_R a)(d_R)(d_R \overline{N_R})$ | $F_1(\mathbf{3}, \mathbf{1}, +2/3)$            | $V_2(\mathbf{3}, \mathbf{1}, -1/3)$            |                           |
| $[\partial](u_R d_R)(\overline{N_R})(d_R a)$ | $S_1(\overline{\mathbf{3}}, \mathbf{1}, +1/3)$ | $F_2(\mathbf{3}, \mathbf{1}, -1/3)$            |                           |
| $[\partial](u_R a)(\overline{N_R})(d_R d_R)$ | $F_1(\mathbf{3}, \mathbf{1}, +2/3)$            | $S_2(\overline{\mathbf{3}}, \mathbf{1}, -2/3)$ | $d_R d_R S_2^\dagger = 0$ |
| $[\partial](u_R d_R)(a)(d_R \overline{N_R})$ | $S_1(\overline{\mathbf{3}}, \mathbf{1}, +1/3)$ | $V_2(\mathbf{3}, \mathbf{1}, -1/3)$            |                           |
| $[\partial](u_R \overline{N_R})(a)(d_R d_R)$ | $V_1(\mathbf{3}, \mathbf{1}, +2/3)$            | $S_2(\overline{\mathbf{3}}, \mathbf{1}, -2/3)$ | $d_R d_R S_2^\dagger = 0$ |

TABLE VII. Decompositions of  $QQ d_R \overline{N_R} a$ , which result in the  $d = 8$  effective operator with a derivative operator,  $\partial a QQ d_R \overline{N_R}$ . The operator  $QQ d_R \overline{N_R} Z'$  is decomposed with the same mediators.

| Decomposition                            | $S_1/V_1/F_1$                                  | $S_2/V_2/F_2$                                  | Comment             |
|------------------------------------------|------------------------------------------------|------------------------------------------------|---------------------|
| $[\partial](QQ)(d_R)(\overline{N_R} a)$  | $S_1(\overline{\mathbf{3}}, \mathbf{1}, +1/3)$ | $F_2(\mathbf{1}, \mathbf{1}, 0)$               | $F$ can be Majorana |
| $[\partial](Q \overline{N_R})(d_R)(Q a)$ | $S_1(\mathbf{3}, \mathbf{2}, +1/6)$            | $F_2(\mathbf{3}, \mathbf{2}, +1/6)$            |                     |
| $[\partial](Q d_R)(Q)(\overline{N_R} a)$ | $V_1(\overline{\mathbf{3}}, \mathbf{2}, -1/6)$ | $F_2(\mathbf{1}, \mathbf{1}, 0)$               | $F$ can be Majorana |
| $[\partial](Q \overline{N_R})(Q)(d_R a)$ | $S_1(\mathbf{3}, \mathbf{2}, +1/6)$            | $F_2(\mathbf{3}, \mathbf{1}, -1/3)$            |                     |
| $[\partial](Q a)(Q)(d_R \overline{N_R})$ | $F_1(\mathbf{3}, \mathbf{2}, +1/6)$            | $V_2(\mathbf{3}, \mathbf{1}, -1/3)$            |                     |
| $[\partial](Q d_R)(\overline{N_R})(Q a)$ | $V_1(\overline{\mathbf{3}}, \mathbf{2}, -1/6)$ | $F_2(\mathbf{3}, \mathbf{2}, +1/6)$            |                     |
| $[\partial](d_R a)(\overline{N_R})(QQ)$  | $F_1(\mathbf{3}, \mathbf{1}, -1/3)$            | $S_2(\overline{\mathbf{3}}, \mathbf{1}, +1/3)$ |                     |
| $[\partial](Q d_R)(a)(Q \overline{N_R})$ | $V_1(\overline{\mathbf{3}}, \mathbf{2}, -1/6)$ | $S_2(\mathbf{3}, \mathbf{2}, +1/6)$            |                     |
| $[\partial](d_R \overline{N_R})(a)(QQ)$  | $V_1(\mathbf{3}, \mathbf{1}, -1/3)$            | $S_2(\overline{\mathbf{3}}, \mathbf{1}, +1/3)$ |                     |

SM charges are the same. This is mentioned in the “Comment” column as “ $S_2 \neq S_1^\dagger$  to avoid  $d = 6$ .”

The decompositions with the comment “ $d_R d_R S^\dagger = 0$ ” contain the interaction

$$\mathcal{L} = y \epsilon^{IJK} (\overline{d_R^c})_{I\dot{a}} (d_R)_{\dot{J}} (S^\dagger)_{K\dot{c}}, \quad (\text{A4})$$

which vanishes because of the color antisymmetric nature of the two  $d_R$ 's forming a scalar. The decomposition

TABLE VIII. Decompositions of  $u_R d_R d_R N_R \phi$ .

| Decomposition              | $S_1/F_1$                                      | $S_2/F_2$                                      | Comment                                                   |
|----------------------------|------------------------------------------------|------------------------------------------------|-----------------------------------------------------------|
| $(d_R d_R)(u_R)(N_R \phi)$ | $S_1(\overline{\mathbf{3}}, \mathbf{1}, -2/3)$ | $F_2(\mathbf{1}, \mathbf{1}, 0)$               | $d_R d_R S_1^\dagger = 0$                                 |
| $(d_R N_R)(u_R)(d_R \phi)$ | $S_1(\mathbf{3}, \mathbf{1}, -1/3)$            | $F_2(\mathbf{3}, \mathbf{1}, -1/3)$            |                                                           |
| $(u_R d_R)(d_R)(N_R \phi)$ | $S_1(\overline{\mathbf{3}}, \mathbf{1}, +1/3)$ | $F_2(\mathbf{1}, \mathbf{1}, 0)$               | $F_2$ can be Majorana                                     |
| $(u_R N_R)(d_R)(d_R \phi)$ | $S_1(\mathbf{3}, \mathbf{1}, +2/3)$            | $F_2(\mathbf{3}, \mathbf{1}, -1/3)$            |                                                           |
| $(u_R \phi)(d_R)(d_R N_R)$ | $F_1(\mathbf{3}, \mathbf{1}, +2/3)$            | $S_2(\mathbf{3}, \mathbf{1}, -1/3)$            |                                                           |
| $(u_R d_R)(N_R)(d_R \phi)$ | $S_1(\overline{\mathbf{3}}, \mathbf{1}, +1/3)$ | $F_2(\mathbf{3}, \mathbf{1}, -1/3)$            |                                                           |
| $(u_R \phi)(N_R)(d_R d_R)$ | $F_1(\mathbf{3}, \mathbf{1}, +2/3)$            | $S_2(\overline{\mathbf{3}}, \mathbf{1}, -2/3)$ | $d_R d_R S_2^\dagger = 0$                                 |
| $(u_R d_R)(\phi)(d_R N_R)$ | $S_1(\overline{\mathbf{3}}, \mathbf{1}, +1/3)$ | $S_2(\mathbf{3}, \mathbf{1}, -1/3)$            | $S_2^\dagger \neq S_1$ to avoid $d = 6$ $u_R d_R d_R N_R$ |
| $(u_R N_R)(\phi)(d_R d_R)$ | $S_1(\mathbf{3}, \mathbf{1}, +2/3)$            | $S_2(\overline{\mathbf{3}}, \mathbf{1}, -2/3)$ | $d_R d_R S_2^\dagger = 0$                                 |

TABLE IX. Decompositions of  $QQd_R N_R \phi$ .

| Decomposition         | $S_1/V_1/F_1$                             | $S_2/V_2/F_2$                       | Comment                                             |
|-----------------------|-------------------------------------------|-------------------------------------|-----------------------------------------------------|
| $(Qd_R)(Q)(N_R\phi)$  | $V_1(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$ | $F_2(\mathbf{1}, \mathbf{1}, 0)$    | $F_2$ can be Majorana                               |
| $(QN_R)(Q)(d_R\phi)$  | $V_1(\mathbf{3}, \mathbf{2}, +1/6)$       | $F_2(\mathbf{3}, \mathbf{1}, -1/3)$ |                                                     |
| $(Q\phi)(Q)(d_R N_R)$ | $F_1(\mathbf{3}, \mathbf{2}, +1/6)$       | $S_2(\mathbf{3}, \mathbf{1}, -1/3)$ |                                                     |
| $(QQ)(d_R)(N_R\phi)$  | $S_1(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$ | $F_2(\mathbf{1}, \mathbf{1}, 0)$    | $F_2$ can be Majorana                               |
| $(QN_R)(d_R)(Q\phi)$  | $V_1(\mathbf{3}, \mathbf{2}, +1/6)$       | $F_2(\mathbf{3}, \mathbf{2}, +1/6)$ |                                                     |
| $(QQ)(N_R)(d_R\phi)$  | $S_1(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$ | $F_2(\mathbf{3}, \mathbf{1}, -1/3)$ |                                                     |
| $(Qd_R)(N_R)(Q\phi)$  | $V_1(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$ | $F_2(\mathbf{3}, \mathbf{2}, +1/6)$ |                                                     |
| $(QQ)(\phi)(d_R N_R)$ | $S_1(\bar{\mathbf{3}}, \mathbf{1}, +1/3)$ | $S_2(\mathbf{3}, \mathbf{1}, -1/3)$ | $S_2^\dagger \neq S_1$ to avoid $d = 6$ $QQd_R N_R$ |
| $(Qd_R)(\phi)(QN_R)$  | $V_1(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$ | $V_2(\mathbf{3}, \mathbf{2}, +1/6)$ | $V_2^\dagger \neq V_1$ to avoid $d = 6$ $QQd_R N_R$ |

$(u_R N_{Ri})(d_R)(d_R)(N_{Rj} N_{Rk})$  does not contain the interaction (A4), but the effective operator induced from this decomposition results in the same structure  $\epsilon^{IJK}(\bar{d}_R^c)_{I\dot{a}}(d_R)_{\dot{a}}^j$  that vanishes.

For all fermion mediators that appear in the decompositions, we assume vector-like fermions (left and right 2-spinors with the same charges form a four-component Dirac fermion). However, the mediator of the SM singlet field can be a Majorana fermion (only two components out of four are independent). The decompositions with the SM singlet fermion mediator are indicated with the comment “ $F$  can be Majorana.”

In the last decomposition in Table IV, the two vector mediators have the same SM charges. Since they are antisymmetric under both  $SU(3)_c$  and  $SU(2)_L$ , the interaction  $V_1 V_2 S_3$

$$\mathcal{L}_{\text{Tab.IV-3}} = \epsilon^{IJK}(V_1)_{Ii\rho}(\mathrm{i}\tau^2)^{ij}(V_2)_{Jj}^\rho(S_3)_K + \dots \quad (\text{A5})$$

does not vanish even if they are identical fields. This is different from the last decomposition in Table I, where  $S_3 \neq S_2$  is required so that the  $S_1 S_2 S_3$  interaction does not vanish. The comment “ $V_1 = V_2$ ” indicates this fact.

## APPENDIX B: A “BLACK BOX THEOREM” FOR $\Delta(L)=3$ OPERATORS

In the Introduction, we argue that with only the particle content of the SM it is highly unlikely that invisible neutron decay can exist with measurable rates, without also having nucleon decays with charged leptons in the final state with at least comparable rates. In this appendix, we discuss the basis of this claim in more detail.

As we mention in the Introduction, it is possible to forbid all  $d = 9$  operators by some symmetry. However, as we now proceed to show, this is not possible at  $d = 10$ . Concretely, we show that if the  $d = 12$  SMEFT operator of Eq. (1) exists, then the  $d = 10$   $\Delta(L) = 3$  operator,  $\mathcal{O}_{d=10} \propto d^c d^c d^c LLLH$ , must have a nonzero Wilson coefficient. This proof is modeled on the “black box theorem”

for neutrinoless double-beta decay [142,143]. This “theorem” is a proof by contradiction. Given two particular operators, one assumes that one of them exists, while the other is forbidden by some symmetry. One then proceeds to show that no such symmetry can exist and that either both operators or none of them will be present in the Lagrangian.

For the formulation of this proof, in addition to the two operators of interest,  $\mathcal{O}_{d=10}$  and  $\mathcal{O}_{d=12}^W$  [see Eq. (1)], we need the following SM Yukawa interactions for up and down quarks:

$$\mathcal{L}^{\text{Yuk}} = Y_d \bar{Q}_L \cdot H d_R + Y_u \bar{Q}_L \cdot H^\dagger u_R + \text{H.c.} \quad (\text{B1})$$

Then, assume there exists a global symmetry, which we denote as  $G_\eta$ , under which the SM fields transform as

$$\begin{aligned} H &\xrightarrow{G_\eta} \eta_H \cdot H, \\ L &\xrightarrow{G_\eta} \eta_L \cdot L, & Q_L &\xrightarrow{G_\eta} \eta_Q \cdot Q_L, \\ u_R &\xrightarrow{G_\eta} \eta_u \cdot u_R, & d_R &\xrightarrow{G_\eta} \eta_d \cdot d_R \end{aligned} \quad (\text{B2})$$

The Yukawa interactions in Eq. (B1) then give the constraints

$$\eta_Q^\dagger \eta_d \eta_H = 1 \quad \text{and} \quad \eta_Q^\dagger \eta_u \eta_H^\dagger = 1. \quad (\text{B3})$$

Now, assume  $\mathcal{O}_{d=12}^W$  is allowed by this symmetry, while  $\mathcal{O}_{d=10}$  is forbidden. This leads to the constraints

$$\begin{aligned} \eta(\mathcal{O}_{d=12}^W) &\rightarrow \eta_u^\dagger (\eta_d^\dagger)^2 (\eta_L)^3 (\eta_H)^3 = 1 \\ \text{and } \eta(\mathcal{O}_{d=10}) &\rightarrow (\eta_d^\dagger)^3 (\eta_L)^3 \eta_H \neq 1. \end{aligned} \quad (\text{B4})$$

Using Eq. (B3), one finds  $\eta_u = \eta_d \eta_H^2$ . Inserting this relation into Eq. (B4) yields

$$\eta(\mathcal{O}_{d=12}^W) = \eta(\mathcal{O}_{d=10}), \quad (\text{B5})$$

which contradicts our assumption. Thus, either both operators  $\mathcal{O}_{d=12}^W$  and  $\mathcal{O}_{d=10}$  are allowed, or both are forbidden.

This symmetry argument makes it clear why invisible neutron decay cannot exist in SMEFT without nucleon decays with charged leptons in the final state, in principle. However, this kind of “proof” based on symmetries strictly speaking only guarantees that the Wilson coefficients of the operators of interest are both nonzero; it does not require them to be equal. We stress, however, that even if  $c_{10}$  were zero at a high scale, renormalization group equation running would generate a nonzero  $c_{10}$  at the low scale precisely because  $c_{10}$  cannot be protected by a symmetry.

Since the renormalization group equations for these high-dimensional operators are not known, we cannot quantify the effect. Instead, we ask the question: How much fine-tuning would one need to accept in order to suppress decays generated from  $\mathcal{O}_{d=10}$  to an acceptable rate? To make a quantitative statement, we therefore compare the predicted half-lives for the  $\mathcal{O}_{d=12}^W$  and  $\mathcal{O}_{d=10}$  operators with the corresponding experimental sensitivities. For  $\mathcal{O}_{d=12}^W$ , Eq. (2) gives our estimate. For  $\mathcal{O}_{d=10}$ , we estimate the partial width for the decay  $n \rightarrow e^- \pi^+ \nu \nu$  as

$$\Gamma \sim \frac{W_0(\pi)^2 m_N^7 \langle H^0 \rangle^2}{589824 \pi^5 \Lambda^{12}} \Rightarrow \frac{1}{3.3 \times 10^{23} \text{ yr}} \left( \frac{13.4 \text{ TeV}}{\Lambda} \right)^{12}. \quad (\text{B6})$$

Both Eqs. (2) and (B6) assume the Wilson coefficient to be equal to one, and the widths scale as the Wilson coefficients squared.

To the best of our knowledge, there is no dedicated search for  $n \rightarrow e^- \pi^+ \nu \nu$ . However, we can use the Super-Kamiokande search for  $n \rightarrow e^+ \pi^-$  [144] to derive a conservative limit on  $n \rightarrow e^- \pi^+ \nu \nu$ . Based on a statistics of  $0.316 \text{ megaton} \cdot \text{years}$  and a background estimate of  $0.41 \pm 0.13$  events, Super-Kamiokande gives a limit of  $5.3 \times 10^{33} \text{ years}$  [144] on  $n \rightarrow e^+ \pi^-$ . In their Fig. 18, Super-Kamiokande provides the distribution of  $e + \pi$  events as a function of the total momentum. The experimental collaboration used the Monte Carlo expectation for the signal events of this distribution to cut backgrounds. Since the final state  $n \rightarrow e^- \pi^+ \nu \nu$  involves missing momentum, we can use the results of their Fig. 18 to give a very rough, but also very conservative limit on  $n \rightarrow e^- \pi^+ \nu \nu$  by simply assuming that the number of events for  $n \rightarrow e^- \pi^+ \nu \nu$  does not exceed the number of *all* background events. This simple-minded but conservative procedure leads to an estimate for the lower limit on the half-life of  $n \rightarrow e^- \pi^+ \nu \nu$  of roughly  $(1.5\text{--}3.5) \times 10^{31} \text{ years}$ .<sup>9</sup>

For the  $\mathcal{O}_{d=10}$  to generate a decay rate obeying this bound, while using  $\Lambda = 13.4 \text{ TeV}$ , the Wilson coefficient  $c_{10}$  must be smaller than  $c_{10} \lesssim 10^{-4}$ . Recall that in the estimate (2), we use  $c_{12}^W = 1$ . Thus, at least four orders of fine-tuning in the Wilson coefficient  $c_{10}$  would be necessary to avoid the constraints coming from searches with charged lepton final states.

<sup>9</sup>The sum of all events in Fig. 18 of [144] is roughly  $\sim 300$ . For a half-life as given in Eq. (B6), one would expect roughly  $10^8$  events.

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