

η_w -meson from topological properties of the electroweak vacuumGia Dvali,^{1,2} Archil Kobakhidze^{1,3}, and Otari Sakhelashvili²¹*Arnold Sommerfeld Center, Ludwig-Maximilians-Universität, Theresienstraße 37, 80333 München, Germany*²*Max-Planck-Institut für Physik, Boltzmannstr. 8, 85748 Garching, Germany*³*Sydney Consortium for Particle Physics and Cosmology, School of Physics, The University of Sydney, NSW 2006, Australia*

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We further scrutinize the evidence for a recently suggested pseudoscalar particle, the electroweak η_w meson. Its existence is demanded by matching the removal of the weak vacuum angle θ_w by the anomalous $B + L$ symmetry with a massive pole in the topological susceptibility of the vacuum. We specifically focus on the possibility of the emergence of η_w as a collective excitation of the phase of the condensate of the 't Hooft fermion determinant, generated by the electroweak instantons, which breaks the $B + L$ symmetry spontaneously. We argue that the generation of the 't Hooft vertex is in one-to-one correspondence with its nonzero vacuum expectation value which is cutoff insensitive. We outline certain puzzles about the nature of the emergent η_w which require further investigation.

DOI: [10.1103/jc2g-q3nh](https://doi.org/10.1103/jc2g-q3nh)**I. INTRODUCTION**

Recently [1], we have pointed out that the electroweak sector of the Standard Model (SM) contains a new degree of freedom, the η_w meson. The emergence of η_w is correlated with the elimination of the electroweak vacuum angle θ_w by the anomalous $B + L$ symmetry of the SM. The connection between anomalous symmetry and nullification of topological susceptibility of the vacuum (TSV) demands the existence of a particle which gets its mass from TSV [2]. In the present context, this particle is sourced by the anomalous $B + L$ current and gets its mass from the TSV of the electroweak gauge theory [1]. In this sense, the η_w meson of the electroweak theory plays a role analogous to the η' meson of QCD, which is sourced by the anomalous axial current and gets its mass from the TSV of QCD.

Although the main indication for the existence of η_w comes from the gauge structure of the electroweak theory, for certain shortcut conclusions in [1], we used gravity as an external monitoring tool. At the basic level of reasoning leading to the existence of η_w , gravity was used as a spectator device without any assumptions beyond the properties defined by the ordinary general covariance. These features show that η_w is a necessary

ingredient of the SM, the least, upon its coupling to gravity. This demand becomes even stronger within the S matrix formulation of gravity, which is generically inconsistent with θ vacua [3–6].

In the present paper, we wish to ignore gravity and focus on the evidence for θ_w provided by the electroweak sector of the SM. The main question is whether η_w emerges as the phase of the condensate of the fermion 't Hooft determinant. This order parameter is generated by the electroweak instantons and breaks the anomalous $B + L$ symmetry spontaneously. We show that the non-zero vacuum expectation value (VEV) of the fermion determinant is intrinsically linked with the generation of the same structure 't Hooft vertex, which breaks $B + L$ symmetry explicitly. As a byproduct of our analysis, we clarify the issue about the seeming cutoff-sensitivity of the fermion condensate that is exponentially growing in fermion flavors. We argue that in the correct treatment this sensitivity is absent, and the enhancement of the instanton amplitude boils down to the exponential degeneracy factor by the fermion zero modes.

At the level of our analysis, certain puzzles remain about the nature of the η_w as of the collective excitation of the condensate phase. In particular, the relation between the validity domains of the effective field theoretic (EFT) descriptions of η_w and the SM fermions require a further clarification. These are necessary for understanding whether η_w is a collective excitation delivered by the electroweak sector or rather it represents a new elementary degree of freedom demanded by consistency.

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II. EVIDENCE FROM TSV AND ANOMALOUS $B+L$ SYMMETRY

The main argument for the existence of η_w presented in [1] comes from applying to the Standard Model the general correspondence developed in a series of articles [2,4,5,7–10]; for a detailed discussion and summary, see [9].

This correspondence says that if a nonzero TSV is made zero by a deformation of the theory, this deformation must produce a massive pseudoscalar particle. In particular, if the physics that makes TSV zero is an anomalous $U(1)$ symmetry, the theory must possess a pseudoscalar particle shifting under this symmetry as well as a nonzero fermion condensate that breaks $U(1)$ spontaneously.

In [2], it was argued that the classic example realizing this connection is provided by QCD with massless quarks. In this case, the anomalous symmetry that eliminates TSV is the axial symmetry. The emerging particle is the η' meson, which is the phase of the chiral quark condensate and gets its mass from the instantons [11].

It was further suggested that the same reasoning must apply to gravity [2]. Moreover it was argued [8,9,12] that in a theory with gravitational TSV and a chiral gravitational anomaly of fermions that kills it, the fermion condensate must form, giving rise to a composite pseudoscalar. More recently [6], we have clarified that due to the specifics of gravitational Eguchi-Hanson instantons [13,14], the fermions that eliminate TSV via gravitational anomaly must have spin-3/2 (rather than spin-1/2) since the spin-1/2 fermions do not deposit the zero modes in the Eguchi-Hanson background. Furthermore, the same analysis restricts the fermion content in the presence of gravity, requiring cancellation of the total spin-1/2 gravitational anomaly [6].

In the present paper, we focus on the application of this criterion to the electroweak sector of the SM offered in [1]. There we have argued that, despite the fact that the theory is in the Higgs phase, the correspondence is fully effective, leading to the existence of a new particle. In particular, the role of anomalous symmetry that eliminates TSV is played by $B+L$. We have shown that the $B+L$ violating fermion condensate forms due to electroweak instantons. Although the instantons get constrained due to the Higgs vacuum expectation value (VEV), the zero-mode structure is effective since the fermion masses fully respect the $B+L$ symmetry. We now discuss this in more detail.

A. TSV in theory without fermions

Let us consider the reduced version of the Standard Model from which we temporarily exclude all fermions. We focus on the electroweak sector. The $SU(2)_W \times U(1)_Y$ gauge symmetry is in the Higgs phase. This breaking, however, maintains the topological structure of the vacuum, since the instantons are not abolished but rather only become constrained. The $U(1)_Y$ part of the theory is

irrelevant for the further discussion. Correspondingly, the vacuum has an ordinary θ vacuum structure [15–17]. The choice of the vacuum is accounted by the boundary term,

$$S_\theta = \frac{\theta_w}{16\pi^2} \int_{3+1} W_{\mu\nu} \tilde{W}^{\mu\nu},$$

$$W_{\mu\nu} \tilde{W}^{\mu\nu} \equiv \epsilon^{\mu\nu\alpha\beta} \text{tr} W_{\mu\nu} W_{\alpha\beta}, \quad (1)$$

where $W_{\mu\nu}$ is the $SU(2)_W$ field strength of the 2×2 gauge field matrix $A_\mu \equiv A_\mu^c \tau^c$, with τ^c , $c = 1, 2, 3$ being the three Pauli matrices.

This term is a total derivative,

$$W \tilde{W} = \epsilon^{\mu\nu\alpha\beta} \partial_\mu C_{\nu\alpha\beta}, \quad (2)$$

where

$$C_{\mu\nu\alpha} \equiv \text{tr} \left(A_{[\mu} \partial_\nu A_{\alpha]} + \frac{2}{3} A_{[\mu} A_\nu A_{\alpha]} \right), \quad (3)$$

is the Chern-Simons 3 form.

The physicality of the θ_w term is directly linked with the TSV correlator,

$$FT \langle W \tilde{W}, W \tilde{W} \rangle_{p \rightarrow 0} \equiv$$

$$\equiv \lim_{p \rightarrow 0} \int d^4x e^{ipx} \langle T[W \tilde{W}(x), W \tilde{W}(0)] \rangle = \text{const}, \quad (4)$$

where T stands for time-ordering, FT stands for the Fourier transformation, and p is a four-momentum. The θ term is physical if the above correlator is nonzero and vice versa.

We now come to the following key point: the expression (4) implies that the Källén-Lehmann spectral representation of the Chern-Simons correlator includes a physical pole at $p^2 = 0$ [2,4,5,9],

$$FT \langle C, C \rangle = \frac{1}{p^2} + \sum_{m \neq 0} \frac{\rho(m^2)}{p^2 - m^2}, \quad (5)$$

where $\rho(m^2)$ is a properly normalized spectral function. Notice that the presence of a massless pole $p^2 = 0$ is a gauge-invariant statement. Since this pole appears in a 3 form, it does not contain any propagating degrees of freedom.

Now, at nonzero coupling, the only gauge-invariant way of making TSV zero is to shift the pole towards a nonzero mass $p^2 = m^2 \neq 0$. However, a massive 3 form propagates one pseudoscalar degree of freedom. This implies the presence of a physical particle in the spectrum. Thus, the same physics that renders the TSV zero will bring the propagating particle in the spectrum of the theory.

B. $B + L$ symmetry

Next, let us restore the fermion content of the Standard Model. The theory now has the global $U(1)_{B+L}$ symmetry, under which fermions transform as

$$\begin{aligned} (q_L, u_R, d_R) &\rightarrow e^{i\frac{1}{6}\alpha}(q_L, u_R, d_R), \\ (\ell_L, e_R, \nu_R) &\rightarrow e^{i\frac{1}{2}\alpha}(\ell_L, e_R, \nu_R), \end{aligned} \quad (6)$$

where $q_L \equiv (u_L, d_L)$ and $\ell_L \equiv (\nu_L, e_L)$ denote $SU(2)_W$ doublets of lefthanded quarks and leptons, respectively.

It is well known that $U(1)_{B+L}$ renders the electroweak θ_w unphysical [15–17]. This can be seen from the fact that, due to anomalous nonconservation of $B + L$ current

$$\partial^\mu J_\mu^{(B+L)} \propto W\tilde{W}. \quad (7)$$

Under the $B + L$ transformation (6), the θ_w term shifts as follows:

$$\theta_w \rightarrow \theta_w + N_f \alpha, \quad (8)$$

where N_f is a number of flavors in the Standard Model.

Alternatively, the vanishing of TSV can be understood from the fact that the fermions deposit zero modes in the instanton background.

C. Emergence of η_w

We thus observe the following scenario. In a fermion-deprived version of the Standard Model, the TSV is nonzero. This implies the presence of a massless pole in the Chern-Simons 3 form. Once the fermions are introduced, the TSV vanishes. This implies that the pole gets shifted to a nonzero value $p^2 = m_\eta^2$. A nonzero pole in the 3 form correlator corresponds to a pseudoscalar particle, since a massive 3 form propagates exactly one physical degree of freedom. We are thus led to the conclusion that the electroweak sector must contain a new particle, η_w meson.

In order to make this connection explicit at the level of the effective theory, let us first outline the general consistency arguments [2, 7] indicating that the removal of TSV must be accompanied by the emergence of a pseudoscalar. Although this part of the story is rather general and applicable to an arbitrary gauge sector with θ vacua, we shall use notations specific to the electroweak theory, which is our main target.

First, following [2], we argue that the existence of a massless pole in TSV fixes the low-energy EFT in the following way:

$$L = \Lambda^4 \mathcal{K}(E/\Lambda^2) + \dots \quad (9)$$

Here, $\mathcal{K}(E/\Lambda^2)$ is an even algebraic function of its argument, and the ellipses stand for all possible high-derivative

terms. Due to the periodicity of physics in θ_w , the first derivative of $\mathcal{K}$, must be an inverse of the periodic function.

When applied to electroweak θ_w , the equation (9) gives the first surprise. In order to match both the proper periodicity in θ_w , as well as the TSV, the scale, Λ , must be set by a proper power of the instanton rate. Therefore, it is exponentially suppressed relative to the scale v below which the instantons are constrained. At the same time, the scale v sets the perturbative mass gap in the theory.

Thus, the equation (9) creates an impression that the effective cutoff of EFT of the 3 form is well below the gap. The resolution of this seeming puzzle is that the massless 3 form contains no propagating degrees of freedom. Its sole contribution to physics is the existence of a constant “electric” field, E , which parameterizes different vacuum states.

Notice that for a constant E , all high-derivative terms drop out of the equation of motion, which takes the form

$$\partial_\mu \left(\frac{\partial \mathcal{K}(E/\Lambda^2)}{\partial E} \right) = 0, \quad (10)$$

and is solved by an arbitrary constant E .

In the absence of fermions, the theory contains no massless mobile sources for E . Correspondingly, each value of E marks an exact vacuum state. The Hilbert space of the theory thus splits into an infinite set of super-selection sectors parameterized by E . These are the θ_w vacua of the electroweak theory described in the 3 form language. This description is exact. The correspondence between the two parameterizations (for a small angle) is $\theta_w \sim E/\Lambda^2$.

In order to eliminate this vacuum structure, one must remove the massless pole from the correlator (5). That is, one must put the 3 form in the corresponding “Higgs” phase. The massive 3 form propagates a single pseudo-scalar degree of freedom. Correspondingly, the necessary condition is the existence of a pseudoscalar η_w such that, up to a boundary term, the theory is invariant under the following shift symmetry

$$\eta_w \rightarrow \eta_w + \text{const}. \quad (11)$$

Then, modulo the irrelevant high derivative terms, the Lagrangian is uniquely fixed as

$$L = \Lambda^4 \mathcal{K}(E/\Lambda^2) + \frac{1}{2} (\partial \eta_w)^2 + \frac{\eta_w}{f_\eta} \Lambda^2 E, \quad (12)$$

where f_η is a scale. It is easy to see that, after integrating out the 3 form, the theory reduces to a theory of a pseudoscalar,

$$L = \frac{1}{2} (\partial \eta_w)^2 - V(\eta_w), \quad (13)$$

with the potential satisfying the relation,

$$\frac{\partial V(\eta_w)}{\partial \eta_w} \propto E(\eta_w), \quad (14)$$

where $E(\eta_w)$ as a function of η_w is the solution of the following algebraic equation:

$$\frac{\partial \mathcal{K}}{\partial E} \propto \frac{\eta_w}{f_\eta} + \theta_w. \quad (15)$$

Notice that in this equation, the θ_w angle enters as an arbitrary integration constant. It is obvious that the η_w boson makes θ_w unobservable. It suffices to notice that, as indicated by the equation (14), at any extremum of the potential, the CP violating electric field E is zero.

Next, as shown in [7], whenever there exists an anomalous current J_μ satisfying

$$\partial^\mu J_\mu = \Lambda^2 E, \quad (16)$$

the generation of the mass gap can be deduced without an explicit reference to an elementary η_w . Indeed, including in the effective action the anomalous coupling,

$$L = \Lambda^4 \mathcal{K}(E/\Lambda^2) + \frac{\Lambda^2}{f_\eta^2} E \frac{1}{\square} \partial^\mu J_\mu, \quad (17)$$

and taking into account the anomalous divergence (16), it is easy to see that E propagates a massive pseudoscalar.

Finally, it has been suggested in [8,12] and systematically argued in [9] that, if J_μ represents a fermionic Noether current of some anomalous $U(1)$ symmetry, the story reduces to (12) with the pseudoscalar η_w emerging as the pseudo-Goldstone phase of the fermion condensate of the corresponding 't Hooft determinant. In other words, very general consistency arguments indicate that whenever anomalous $U(1)$ removes TSV, the corresponding 't Hooft determinant must condense and its pseudo-Goldstone phase must dynamically relax the θ term.

As already noticed in [7], a classic realization of the above scenario is the elimination of θ_{QCD} by the chiral symmetry of a massless quark. The pseudo-Goldstone degree of freedom in this case is the phase of the chiral quark condensate described by the η' meson.

Applying the same general reasoning to the electroweak sector of the SM, we are led to a conclusion that, on top of the $B + L$ violating 't Hooft vertex, there must exist its condensate. The elimination of θ_w then can be understood as its dynamical relaxation by the phase of the condensate, described by the η_w meson.

In the subsequent chapters, we shall support the existence of $B + L$ —violating condensate by an explicit standard instanton calculation as well as by a new approach of resolving the instanton contribution in terms of multi-particle amplitudes.

III. FERMION CONDENSATE

We shall now illustrate the existence of a fermion condensate by an explicit computation extending the

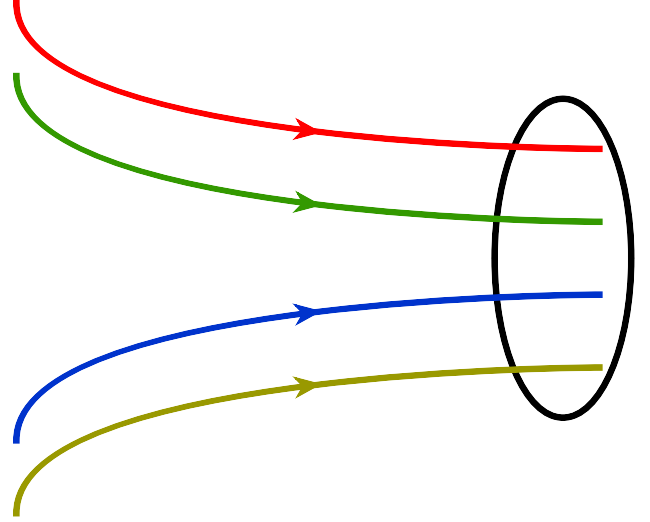


FIG. 1. Graphical representation of the 't Hooft vertex.

previous analysis [1].¹ The instanton index is correlated with the change in the $B + L$ number via the anomaly. The index is given by the equation,

$$\Delta Q_{B+L} = N_f \nu, \quad (18)$$

where Q_{B+L} is the charge corresponding to the J_{B+L} current, ν is the instanton index, and N_f is the number of Standard Model fermion generations. Without loss of generality, let us consider $\nu = 1$ and $N_f = 1$. The anomaly equation relates $B + L$ violating processes to instanton transition probabilities, which scale as $\sim e^{-\frac{2\pi}{\alpha}}$. Thus, the 't Hooft vertex is generated:

$$\mathcal{L} \sim e^{-\frac{2\pi}{\alpha}} qqql. \quad (19)$$

The diagrammatic representation of the 't Hooft vertex (see Fig. 1) is given by a merger of all $SU(2)$ doublet fermion lines.

We shall later argue that the existence of the 't Hooft vertex implies the existence of the condensate and vice versa. Since in the electroweak theory, the 't Hooft vertex is a well-understood construct, the above relation provides a shortcut argument for the presence of the condensate.

However, we shall first compute the condensate via the standard instanton calculus, and later rethink it in the diagrammatic language that shall make the connection with the 't Hooft vertex very transparent.

A. Fermion condensate from instantons

In the Standard Model, a unit instanton has $2N_f$ zero modes. Therefore, the instanton measure [19] takes the form

¹More extended analysis shall be given in [18].

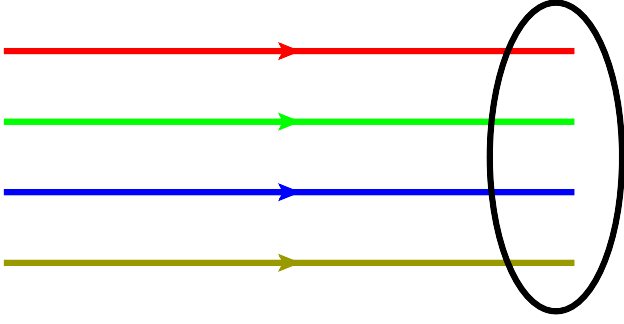


FIG. 2. Graphical representation of the condensate.

$$\left(\frac{2\pi}{\alpha_2(v)}\right)^4 \int d^4z \int \frac{d\rho}{\rho^5} e^{-\frac{2\pi}{\alpha_2(\rho)} - 2\pi^2 v^2 \rho^2} (\mu\rho)^{2N_f}, \quad (20)$$

where μ is an IR regulator, introduced in the Lagrangian via the terms $\mu q q$ and $\mu q l$. At the end of the calculation, it should be set to zero.

In the instanton background, the propagator is expressed via the zero modes (Ψ):

$$\langle x | \frac{1}{\hat{D} + i\mu} | y \rangle = \frac{\Psi_0^\dagger(x-z)\Psi_0(y-z)}{i\mu} + \Delta(x-y) + \mathcal{O}(\mu). \quad (21)$$

Using the expression above, we can compute the 't Hooft vertex by inserting the propagators at different points, or compute the condensate by considering all external legs inserted at the same point, see Fig. 2.

Thus, we get

$$\langle (qqql)^{N_f} \rangle \sim \left(\frac{2\pi}{\alpha_2(v)}\right)^4 \int d^4z \int \frac{d\rho}{\rho^5} e^{-\frac{2\pi}{\alpha_2(\rho)} - 2\pi^2 v^2 \rho^2} \times \left(\langle 0 | \frac{\mu\rho}{\hat{D} + i\mu} | 0 \rangle \right)^{2N_f}. \quad (22)$$

We can integrate this expression. Since for the Yukawa couplings $y \ll 1$ we can ignore the Dirac masses, the only scale the integral can produce is $\frac{1}{\rho}$. Therefore, we obtain

$$\langle (qqql)^{N_f} \rangle \sim \left(\frac{2\pi}{\alpha_2(v)}\right)^4 e^{-\frac{2\pi}{\alpha_2(v)}} v^{6N_f} \int \frac{d(\rho v)}{\rho v} \times e^{-2\pi^2 v^2 \rho^2} (\rho v)^{-\frac{43-8N_f}{6}} (\rho v)^{-6N_f} \times C(N_f), \quad (23)$$

where

$$C(N_f) = (-1)^{N_f} \left(\frac{8}{\pi^2}\right)^{2N_f-1} \frac{1}{(3N_f-1)(6N_f-1)}. \quad (24)$$

This expression is convergent for the toy model with $N_f = \frac{1}{2}$, but is divergent for realistic models with more flavors. This divergence creates a (false) impression that for

a large number of fermion flavors the fermion condensate is sensitive to the UV cutoff of the SM. Already the following commonsense EFT argument tells us that this cannot be the case.

Indeed, the sources of the fermion condensate are the electroweak instanton effects. These instantons are constrained in IR by the length $\rho \sim v^{-1}$. Correspondingly, the dominant ones are the instantons of this size since they possess the smallest Euclidean actions. For more compact instantons, $\rho \ll v^{-1}$, the action is suppressed due to the lower value of the running gauge coupling $\alpha_2(\rho)$ at the corresponding scale.

With this understanding, it looks natural that the physical effects of instantons cannot be exponentially UV-sensitive in the number of condensed fermion flavors. One argument supporting this view comes from the analytic continuation.

A more precise form of the integral is

$$\langle (qqql)^{N_f} \rangle \sim \left(\frac{2\pi}{\alpha_2(v)}\right)^4 e^{-\frac{2\pi}{\alpha_2(v)}} v^{6N_f} \times C(N_f) \times \frac{1}{2} (2\pi^2)^{\frac{44N_f-43}{12}} \Gamma\left(\frac{43-44N_f}{12}\right). \quad (25)$$

Analytically continuing the above expression, we get

$$\langle (qqql)^{N_f} \rangle \sim \left(\frac{2\pi}{\alpha_2(v)}\right)^4 e^{-\frac{2\pi}{\alpha_2(v)}} v^{6N_f} \times F(N_f), \quad (26)$$

where

$$F(N_f) = \frac{(-1)^{N_f}}{32(2\pi^2)^{\frac{31}{12}}} \frac{2^{8N_f} (2\pi^2)^{\frac{5N_f}{3}}}{(6N_f-1)(3N_f-1)} \Gamma\left(\frac{43-44N_f}{12}\right), \quad (27)$$

N_f	$ F(N_f) $
$\frac{1}{2}$	0.00248592
1	0.658208
2	93.4076
3	1433.89
4	25610.4

where the table indicates absolute values of the function $F(N_f)$ for different numbers of flavors.

In the next section, we shall support the statement about the UV-insensitivity of the condensate from a different angle. Namely, we shall argue that the effect is of the same nature as the seeming divergence of multiparticle amplitudes with the particle number. This divergence is unphysical and is removable by a proper resummation.

Before concluding this section, we wish to clarify a potentially puzzling point concerning the fermion

condensate in the effective low-energy theory. At first glance, one might expect that integrating out heavy fermions (such as the top quark) would allow instantons to generate 't Hooft vertices—and hence condensates—with fewer fermion fields. This, however, would appear to contradict the topological index theorem, suggesting the possibility of nonperturbative processes within the SM that violate $B + L$ by other than 3ν units. A more careful analysis shows that this does not occur: once heavy fermion fields are integrated out, no new 't Hooft vertices or associated fermion condensates emerge at low energies, because the action of the electroweak constrained instantons diverges. For further technical discussion, see, e.g., [20].

B. Resolving instantons as multi- W processes

In this section, we shall argue for the UV-insensitivity of the fermion condensate from the point of view of resolving the instantons as multiparticle processes. Our reasoning incorporates the following two points:

- (i) The resolution of a nonperturbative instanton process in terms of a multiparticle amplitude [21]; and
- (ii) The bound on the enhancement of the process due to the microstate degeneracy [22,23].

The first argument we shall use is a part of a wider program in which the nonperturbative effects are understood as the perturbative multiparticle processes; see [21] and references therein. This correspondence has been explicitly demonstrated by reproducing some known semiclassical results of nonperturbative processes by perturbative computation of multiparticle scattering.

For example, in gravity, the famous Hawking's black hole evaporation has been described as a perturbative quantum rescattering of a large number of gravitons [24]. Likewise, the semiclassical amplitude of black hole formation in high-energy particle scattering has been reproduced as a quantum process of scattering of two highly energetic gravitons into many soft ones [25,26]. Next, a semiclassical Schwinger effect has been reproduced as a fully perturbative multiphoton process [21]. Also, an instanton in the $(1+1)$ -dimensional electrodynamics was resolved via coherent state construction [27].

Based on the above, we adopt the map between a nonperturbative vacuum instanton process and a perturbative multiparticle scattering. The key is to think about the semiclassical instanton transition as a quantum transition process with a high number of intermediate quanta (see schematic diagram in Fig. 3). This correspondence allows us to view the instanton process diagrammatically, as a vacuum transition process with a large number of intermediate W bosons. Regardless of the technical difficulty of the actual computation, this gives a powerful handle on the problem since it allows us to disentangle the real physical effects from fictional effects emerging from the breakdown of perturbation theory.

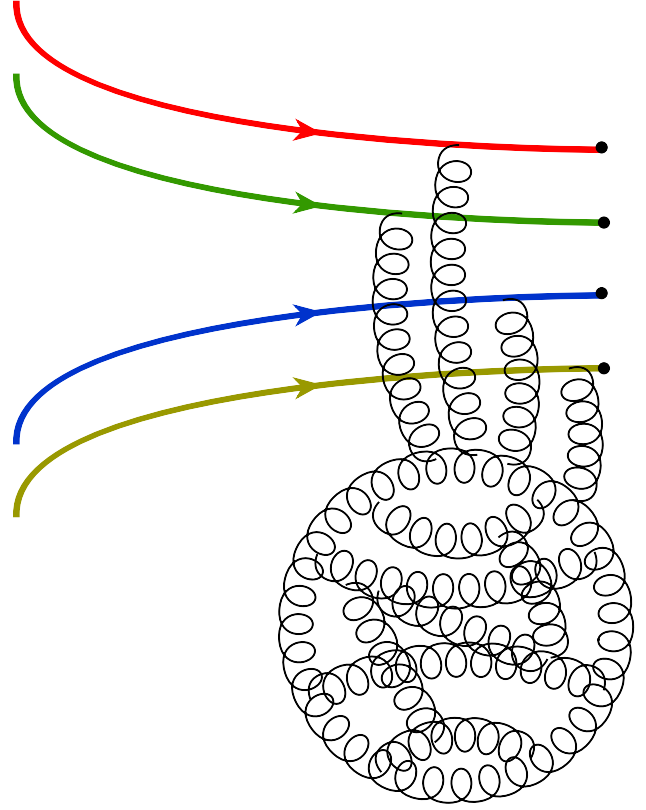


FIG. 3. Representation of the instanton part of the condensate via W bosons.

The second part of the story is the upper bound on the microstate degeneracy of the instanton. It has been argued [22,23,28] that the microstate entropy of any physical entity (a state or a process) with a localization length-scale ρ is bounded by

$$S_{\max} \sim \frac{1}{\alpha(\rho)}, \quad (28)$$

where $\alpha(\rho)$ is the relevant running coupling evaluated at the scale ρ . In [23], the entities saturating this bound were called “saturons.” Most importantly for the present discussion, the above bound constrains the instantons [22]. The instanton degeneracy factor cannot exceed $e^{S_{\max}}$ without invalidating the EFT description. In the latter case, the theory must change the regime.

A particular source of the microstate degeneracy is provided by the fermionic zero modes. In the present case their number is given by $2N_f$. The saturation criterion, therefore, restricts the number of instanton fermionic zero modes as $2N_f < 2\pi/\alpha$ [22]. Physically, this bound is rather transparent, since the presence of fermion zero modes enhances the instanton contribution into a process by a factor e^{2N_f} , thereby promoting the point of saturation into an unsuppressed instanton process. That is, at the saturation

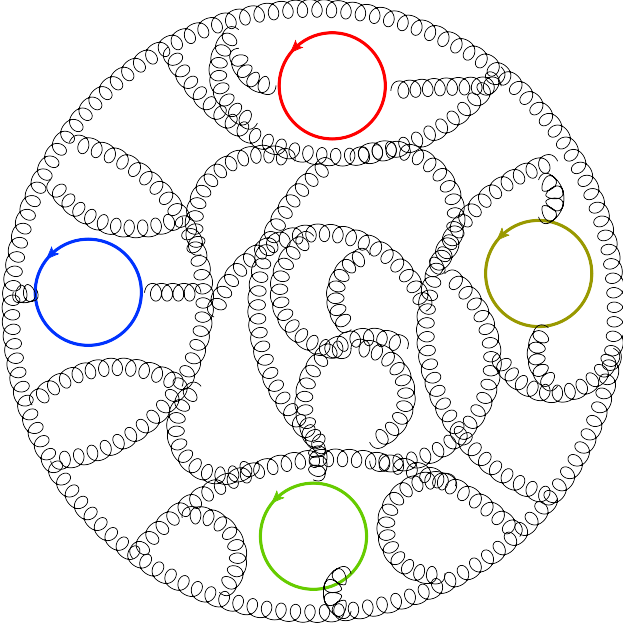


FIG. 4. An example of a multiparticle process, with many virtual W bosons and intermediate fermion loops. Even for an undersaturated value of the 't Hooft coupling, $\alpha N \ll 1$, the multiplicity of diagrams grows factorially with the number of participating quanta n . However, this enhancement signals the breakdown of perturbation theory and cannot be trusted beyond the point of optimal truncation $n \sim 1/\alpha$.

point, the collective ('t Hooft) coupling αN_f becomes critical, and instanton rates are unsuppressed. Beyond this point, the would-be-strong instanton effects invalidate the description, and the theory must change the regime.

Notice that this saturation is correlated with the breakdown of the fermionic loop expansion, which contributes as the power series in the flavor 't Hooft coupling αN_f [22].

This is not the case for electroweak instantons. Indeed, given the number of $SU(2)_W$ fermion flavors in the SM, $N_f = 3$, the electroweak instantons are well below the saturation. Correspondingly, no change of regime of the theory is expected by the instantons. This is a fully nonperturbative statement.

Now, naively, even for undersaturated collective coupling $\alpha N_f \ll 1$, for the processes with a large number of participating particles $n \gg 1$ (see, e.g., Fig. 4), the combinatorics grow factorially with n , leading to a fictional enhancement of the effect. This enhancement is, however, related to the breakdown of perturbation theory rather than with a real physical enhancement of the effect.

Such divergences cannot be trusted beyond the point of the optimal truncation. This point is reached when the number of quanta n participating in the process reaches the critical value [23],

$$n \simeq \frac{1}{\alpha}. \quad (29)$$

Following the above paper, the origin of the optimal truncation point can be understood by the following counting. Consider a process that involves a transition between a few (say two) to $n \gg 1$ quanta. For example, the n participating quanta can appear as on-shell final (or initial) state particles in $2 \rightarrow n$ tree-level scattering process, or as a virtual intermediate quanta of n -loop diagram describing the $2 \rightarrow 2$ transition. In the latter case, the $2 \rightarrow n$ transition rate can be extracted as the imaginary part of the amplitude.

This rate scales as $\sim n! \alpha^n$, where the factorial enhancement comes from the combinatorics of diagrams and the suppression comes from n interaction vertices. The perturbative expansion breaks down when $n + 1$ order effect catches up with the one coming from the order n . This happens for (29), which defines the point of optimal truncation. Using Stirling's approximation, it is easy to see that the corresponding transition rate scales as

$$\Gamma \sim e^n \simeq e^{-\frac{1}{\alpha}}. \quad (30)$$

Beyond this point, the series has to be re-summed with higher-order effects, giving more suppression. The leading order effect is thus given by (30). Thus, the exponential suppression characteristic of nonperturbative semiclassical effects is reproduced by a perturbative multiparticle process.

Notice that there exist general nonperturbative arguments [23] indicating that within the validity of given degrees of freedom, the matrix element of any process of the type $\text{few} \rightarrow n$ is bounded from above by e^{-n} modulo the microstate degeneracy factor e^S . The total rate thus scales as

$$\Gamma \sim e^{-\frac{1}{\alpha} + S}. \quad (31)$$

Thus, the nonperturbative physics is telling us that the enhancement factor due to the divergent integral in (22) cannot exceed the enhancement factor by the instanton's microstate entropy due to the fermionic zero modes. Moreover, the fermion enhancement of the microstate entropy is essentially independent of the instanton scale. We thus reach a conclusion that the final enhancement factor due to fermion zero modes cannot exceed the exponential factor in N_f .

A schematic diagrammatic representation of the process responsible for the creation of the condensate as well as of the 't Hooft vertex is given in Fig. 3. The solid colored lines represent different flavors of the fermion doublets, whereas the wavy lines describe the virtual W bosons. The multiloop blob of virtual W bosons describes the instanton process. This instanton represents a process with the participation of $n \simeq 2\pi/\alpha$ coherent W bosons with characteristic momentum $1/\rho$. The emergence of the

zero modes that are the source of the 't Hooft vertex as well as of the fermion condensate is due to the connection between the instanton blob and each fermion line via virtual W lines. To the leading order, the number of connector W lines is the same as the number of fermion doublets, $3N_f$.

Now, increasing the virtual momenta in the connector W lines can increase the constituency of the instanton blob by populating it with a larger number of virtual quanta, $n \gg 2\pi/\alpha$, including the additional fermionic loops (see Fig. 4). As we already discussed, this leads to a fictional factorial growth of the contribution, which, in reality, signals the breakdown of perturbation theory and goes away after the proper re-summation. The enhancement cannot exceed an exponential factor in N_f .

This settles the issue of cutoff-(in)sensitivity of the fermion condensate. In a full-fledged calculation, the integral over ρ in (22) must be effectively cut off around the scale v . The enhancement factor emerges after the re-summation beyond the point of optimal truncation and boils down to an exponential factor in N_f .

The diagrammatic visualization of the instanton process allows us to establish an immediate one-to-one correspondence between the generation of the vertex of the 't Hooft determinant and its expectation value. Indeed, in the diagrammatic language, the 't Hooft vertex absorbs $3N_f$ fermion lines. Since the Lagrangian possesses no flavor-changing vertexes, the fermion lines can only end on the condensate.

IV. WHERE IS η_w ?

In understanding the features of η_w , it is illuminating to confront it with the situation with the η' meson of QCD in case of a massless (or light) quark. It was already pointed out in [16] that the way $B + L$ makes θ_w unphysical is very similar to how the axial $U(1)_A$ symmetry of a massless quark eliminates θ_{QCD} . However, another important part of the parallel has been overlooked.

First, in QCD, there exists an associated degree of freedom in the form of the η' meson which gets rid of the massless pole in TSV and simultaneously relaxes θ_{QCD} to zero. This degree of freedom emerges as the phase of the fermion condensate of the 't Hooft determinant. A small explicit breaking of $U(1)_A$ symmetry by the quark mass makes the cancellation of θ_{QCD} imprecise, but does not jeopardize the existence of the η' meson.

The point made by us in [1] and further justified in the present work is that there must exist an analogous boson in the electroweak case. The following parallels exist between the two systems.

In both cases, in the pure gauge sector without fermions the vacuum has a topological structure. Instantons induce a nonzero TSV and correspondingly produce the physical θ vacua. In both theories, upon

introduction of (good quality) anomalous symmetries acting on fermions ($U(1)_A$ in QCD and $U(1)_{B+L}$ in electroweak theory), the respective θ terms become unphysical. This is in full agreement with the emergence of the fermionic zero modes that, on one hand, suppress the instanton transitions and, on the other hand, lead to the generation of the fermionic 't Hooft determinant vertexes. These vertices break the respective anomalous global symmetries explicitly down to their anomaly-free discrete subgroups.

In addition, in both cases, the corresponding 't Hooft determinants acquire the vacuum expectation values, breaking the respective anomalous symmetries spontaneously. This was well-known for QCD. We have argued that a similar effect takes place in the electroweak theory. The condensation of the fermion determinant is in accordance with the previously suggested general connection between the elimination of topological susceptibility by anomalous symmetry and the existence of a fermion condensate that breaks the same symmetry spontaneously [9].

Now, in QCD, the fluctuation of the phase of the condensate of the fermion determinant is a physical pseudo-Goldstone degree of freedom, η' . The vanishing of the θ terms can be understood as the dynamical relaxation of the pseudo-Goldstone. However, this degree of freedom is also necessary for explaining the removal of the massless pole in TSV. This is the source of the mass of η' [29,30]. As already discussed, the phenomenon of η' mass-generation from TSV represents a 3 form analog of the ordinary Higgs effect: η' is “eaten-up” by the Chern-Simons 3 form, thereby shifting the pole to a massive value [2]. In other words, the presence of a pseudoscalar degree of freedom is the only gauge-invariant way for making a 3 form massive.

Our point is that the presence of an analogous degree of freedom is necessary in order to explain the removal of the electroweak TSV by the anomalous $B + L$ symmetry. However, the question is about its origin.

Of course, there exists an option that the η_w particle is an “external” degree of freedom accompanying the SM by consistency. However, the existence of the fermion condensate with the right quantum numbers suggests that, instead of seeking external help, the SM itself generates the right degree of freedom dynamically. This indicates that η_w can emerge as the collective excitation describing the fluctuation of the phase of the fermion condensate. Such a scenario would be in close correspondence with the story of η' in QCD. The similarities between the above theories can be summarized in Table I.

However, there exist certain important differences between the two theories. Taking these differences into account leaves some open questions about the features of the η_w particle. Let us briefly outline the puzzle.

TABLE I. Parallels between QCD and Electroweak theories.

Theory:	QCD	Electroweak
TSV in pure gauge sector (physical θ):	Physical θ_{QCD}	Physical θ_w
TSV removed by anomalous symmetry (θ rendered unphysical):	$U(1)_A$	$U(1)_{B+L}$
Fermion condensate (flavor-det):	$\langle \det_{\text{QCD}} \bar{q}q \rangle$	$\langle \det_{\text{Weak}} qqql \rangle$
pseudo-Goldstone:	η'	η_w

The drastic difference between the two scenarios is that in QCD, due to the confinement, quarks and mesons represent the valid degrees of freedom in the separate domains of the description. In particular, quarks cannot coexist with the η' meson. This is not *a priori* the case in the electroweak sector. From the first glance, it appears that, without further precautions, the emergence of the η_w meson does not necessarily invalidate fermions as the acceptable asymptotic states.²

In such a case, the η_w meson and fermions would coexist within the same energy domain. Notice that, since the condensate is formed by the effects operating around the scale v , the η_w meson must emerge as the collective excitation at the same scale. At the same time the fermions can be taken to be much lighter (as this is the case for all fermions in the SM except the top). Thus, η_w and fermions can have the overlapping domains of validity of their EFT descriptions.

In other words, the dramatic distinction between the two cases (η' versus η_w) is the hierarchy of underlying scales. In QCD, the scale of the condensate formation (the QCD scale) and the value of the condensate are roughly of the same order.³ That is, the scale of physics that creates the condensate is not hierarchically larger than the scale of the condensate. The unusual thing about the electroweak case is that the two scales are exponentially separated. That is, the instanton physics that form the condensate operate at the scale v , whereas the value of the resulting condensate is exponentially smaller. This raises the question about the domain of validity of the EFT of the η_w meson.

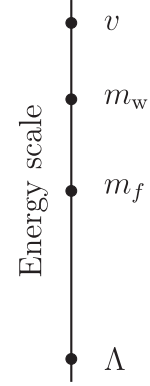
To sharpen the question, let us choose the fermion Yukawa couplings in such a way that the fermion masses, m_f , are well below the instanton scale v but are much above the scale of the topological susceptibility Λ (9):

$$v \gg m_f \gg \Lambda. \quad (32)$$

²Of course, due to QCD dynamics, the quarks are confined. However, this effect is independent from the electroweak instantons that form the fermion condensate and produce the η_w meson.

³In QCD with a large number of colors N_c , the value of the condensate grows with N_c and thus is even higher than the QCD scale.

The hierarchy has the following form:



Let the UV cutoff (upper bound on the validity scale) of EFT of the η_w meson be f_η and ask where the place of f_η is in the above hierarchy.

Putting $f_\eta \ll m_f$ creates a naive problem of UV completion of the theory of η_w above the scale f_η . Indeed, in such a case, there would exist an “unpopulated” energy gap between the two scales.

On the other hand, putting $f_\eta \gg m_f$ is equally puzzling as this degree of freedom would coexist with elementary fermions within a finite interval of scales between f_η and m_f , and make it impossible to match anomalies between high and low energy scales. Such an arrangement would also be in tension with the *a* theorem, since the number of degrees of freedom would increase below the scale f_η .

From the above considerations, the minimalistic consistent case emerges to be $f_\eta \sim m_f$. This would avoid both of the above issues since fermions and η_w would exist in separate domains of validity, and the theory of η_w would be UV-completed by fermions.

However, this still leaves the following question. In our calculation of the condensate, the fermion masses were subdominant. The scale of the physics that forms the condensate is v . It is therefore unclear why the cutoff of η_w must be sensitive to the fermion mass scale. Above the scale m_f , the 't Hooft vertex is kinematically accessible, and we could think that all the anomalous processes can be described via fermions. Following this, existence of the η_w in this domain is not possible, so the condensate should start to evaporate, or at least it should not deliver a Goldstone boson anymore.

One may argue that within the realistic SM, the fermion masses do not obey the hierarchy (32) due to the mass of the top quark, and this may affect the story. However, this does not appear to be a fully satisfactory argument, since we would expect that the SM must stay a consistent theory even under the deformed Yukawa couplings that push the top mass well below v .

In summary, the validity scale f_η and its connection with the scale of instantons v and the fermion masses m_f remains to be understood.

A. Parameters from first principles

In principle, it should be possible to compute from first principles, via the instanton calculus, the mass, decay constant, and effective cutoff of the η_w . Let us consider an operator $\hat{O}(x)$, which, after a $B + L$ rotation, gives the quark multilinear operator that enters in the 't Hooft vertex; in other words,

$$\delta_{B+L} O = qqql(x), \quad (33)$$

where, for simplicity and without loss of generality, we take $N_f = 1$ (generalization to $N_f > 1$ is straightforward). Then, the following correlator,

$$\text{FT}(|O(x)O(0)|)_p = \frac{\rho(m_{\eta_w})}{p^2 - m_{\eta_w}^2} + \dots, \quad (34)$$

contains the η_w meson as the lightest degree of freedom in the spectral representation. The process described by the above correlator requires a $\nu = 2$ change in winding, corresponding to its total charge $B + L = 2$. Thus, we can deduce that the spectral weight is proportional to

$$\rho(m_{\eta_w}^2) \propto (qqql)^2. \quad (35)$$

Since this correlator is represented via the η_w 's propagator, we can extract the mass from it. Using the Witten-Veneziano relation [29,30]

$$m_{\eta'}^2 f_\eta^2 \sim \Lambda^4,$$

where Λ is the EFT scale (9), we have the possibility to compute the decay constant f_η .

Also, the above correlator carries the total $B + L$ charge corresponding to the winding number $\nu = 2$. Therefore, the correlator must be computed via the $\nu = 2$ correlated instantons. Due to this selection rule, the computation should be well defined (due to the absence of mixing with $\nu = 0$). Some of the tools necessary for this computation were developed in the context of QCD in [31]. The correlator should contain information about the fermion masses. Unlike the computation of the condensate, this correlator is highly sensitive to the form of the zero modes,

since the correlation is given by their overlap on different instanton centers; see the nonlocal part of the computation from [31]. In other words, the zero modes for the $\nu = 2$ instanton configuration should be computed while carefully retaining their dependence on the Dirac masses. Profiles for the zero modes in the $\nu = 1$ instanton background in the small-mass approximation were derived in [32]. Following a similar procedure, one can obtain the mass-dependent zero mode solutions for $\nu = 2$, insert them into the correlator expression in (34), then subsequently integrate over the instanton moduli space and perform the Fourier transform. This procedure yields the mass of η_w as a function of the average instanton separation in the dilute gas approximation, as well as the fermion masses.

Moreover, the structure of the correlator (34) effectively encodes the cutoff scale of the low-energy effective field theory. This is manifested through the emergence of a branch cut in the correlator. Its scale corresponds to the breakdown of the effective description. This can determine the scale f_η , the cutoff of the theory, and their relation.

V. OUTLOOK

In this article, following [1], we have reiterated that the physics of the electroweak vacuum points towards the existence of a new degree of freedom, the η_w meson. This boson is sourced by the anomalous $B + L$ current and gets its mass from the TSV of the electroweak vacuum. In the previous study, we have also used gravity as a “spectator” for making certain statements exact. However, even decoupling gravity, the structure of the electroweak vacuum offers nontrivial evidence for the existence of η_w .

The purpose of the present work was to clearly identify and separate the evidence for η_w that is provided by the pure standard model without any external help.

The main guideline is a general criterion consisting of two parts. First, the removal of TSV implies the existence of a pseudoscalar particle that gets its mass from shifting the would-be massless pole in TSV to a nonzero value [2,7]. Moreover, if TSV is removed by an anomalous symmetry acting on fermions, the corresponding fermionic 't Hooft determinant acquires a nonzero expectation value [8,9,12]. The required pseudoscalar then emerges as the fluctuation of the phase of the fermion condensate; for a detailed unified argument, see [9].

The explicit example of this phenomenon is provided by the η' meson in QCD with a massless quark. Another, less known example [6] is the removal of the gravitational TSV, generated by Eguchi-Hanson instantons [13,14] by the anomalous symmetry of a spin-3/2 fermion (gravitino). According to the index theorem [33], a spin-3/2 particle is the only fermion possessing the zero modes in the Eguchi-Hanson background. The generation of the gravitino condensate by the instanton zero modes has been demonstrated explicitly [34,35].

In the present paper, following our recent work [1], we have applied a similar reasoning to the electroweak vacuum. It is well understood [15,16] that θ_w is rendered unphysical by the $U(1)_{B+L}$ symmetry of the SM. We thereby expect that, by consistency, the theory must contain a pseudoscalar that removes TSV. We showed that some crucial ingredients supporting the existence of such a scalar are in place. Namely, the instanton zero modes generate the VEV of the 't Hooft fermionic determinant (25).

As an important byproduct, we have clarified the issue of the (seeming) exponential cutoff-sensitivity to the number of fermion flavors appearing in the leading-order calculation from integration over the instanton size (23). This sensitivity has caused uncertainties in the previous literature, creating an impression that for a large number of flavors the fermion condensate may be strongly enhanced by the cutoff physics.

Adopting the reasoning of [23], we have argued that this divergence is an artefact of extending the multiparticle amplitudes (representing the instanton process) beyond the point of the optimal truncation. In reality, the maximal enhancement by the zero-mode degeneracy of the instanton action cannot exceed (31). Correspondingly, the UV-sensitivity of the fermion condensate is absent, and the leading-order result is given by (25).

In addition, by mapping the nonperturbative semiclassical instanton process on a multiparticle transition amplitude, in the spirit of [21,23], we established a one-to-one correspondence between the existence of the 't Hooft vertex and its vacuum expectation value.

Thus, our analysis demonstrates the existence of the fermionic condensate. This leads us to the following situation. On one hand, the elimination of the TSV by $B+L$ symmetry demands the presence of the η_w boson realizing the anomalous symmetry nonlinearly. On the other hand, the theory provides a fermion condensate that breaks the same symmetry spontaneously. Given this evidence, it is reasonable to expect that the required boson originates from the phase fluctuation of the fermion condensate.

Although the story is similar to the emergence of η' meson in QCD, there are some important differences which leave some questions about the nature of the η_w boson open. The main puzzle is the relation between the EFT cutoff (f_η) of the theory of η_w and the masses of fermions.

The EFT intuition implies that η_w must emerge at the scale below the masses of (at least some of) the fermions. However, our calculations indicate that in the case of light fermions, the condensate is insensitive to the fermion masses. This puzzle requires further clarification. It is possible that it gets resolved by taking into account the mass of the top quark.

We must stress that, with our current understanding, it remains an option that gravity is essential for having η_w as an elementary degree of freedom, implying that in pure SM

the θ_w is neutralized by a broad resonance composed of SM fermions that cannot be regarded as a well-defined asymptotic state. This would imply that in the limit of zero gravity, $\rho(0) = 0$ in (5). Despite the opposite evidence, discussed in the main body of the paper as well as in [1], we were not able to exclude such an option. Of course, since gravity is part of nature, one way or the other η_w is expected to be a physical state.

Independently, whether the η_w meson is emergent as a collective excitation from SM physics or has to be introduced by consistency as a new elementary degree of freedom, it is expected to be an extremely weakly interacting light particle. This creates a substantial difference from the case of simply unphysical θ_w .

Indeed, if $U(1)_{B+L}$ could remove θ_w without making it dynamical, no physical effects of θ_w can exist in principle. The situation is dramatically different if θ_w is relaxed dynamically through η_w . In such a case, the off-vacuum configurations with oscillating θ_w must exist. Obviously, these are physically distinct from the state of a non-dynamical and constant $\theta_w = 0$.

Moreover, cosmologically, the states with oscillating θ_w are maximally probable. Indeed, since the relaxation time of the η_w boson is extremely long, with a high likelihood, cosmologically, we are expected to live in a background with $\frac{\eta_w}{f_\eta} \sim 1$. Notice that possible high-dimensional operators generated by physics beyond the SM can shift neither the minimum of η_w nor its mass substantially.

Note added. Shortly before submitting this paper, we received a draft by Giacomo Cacciapaglia, Francesco Sannino, and Jessica Turner, in which they identify the electroweak η_w with a CP -odd combination of the hydrogen and antihydrogen atoms. We are grateful to these authors for sharing their preliminary results.

While looking for the candidates for η_w among the existing atomic states is a natural minimalistic approach, in light of our analysis, the validity of such identification is not fully clear. Putting aside the question of the contribution of the heavy fermions and focusing on a toy version of the SM with a single light generation, the condensate of 't Hooft determinant (22), $\langle qqql \rangle$, from which the η_w meson emerges as a collective excitation, does have the quantum numbers of a proper hydrogen state. In this respect one can say that the author's proposal is aligned with our setup.

However, it is doubtful whether an on-shell state of hydrogen-antihydrogen, even with the correct quantum number, is the right match for a one-particle η_w state. For example, the mass gap of the state must be exponentially suppressed, as it must be generated by the electroweak instantons. This is not the case for the states constructed out of on-shell (anti)hydrogen.

Even if a degree of freedom with such a high mass gap can be identified, it is unclear how it can Higgs the 3 form.

The latter then shall remain massless. Upon taking into account gravity, which excludes the decoupling of the 3 form, η_w must enter as a different degree of freedom from that which Higgses it.

To summarize, the role of a pseudo-Goldstone that is required for Higgsing the 3 form is unlikely to be played by a broad resonance composed out of the heavy modes. This, however, does not exclude the mixing between the two types of degrees of freedom (hydrogen-type heavy modes and the “genuine” η_w) which can lead to potentially interesting physical effects.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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