Final-state rescattering mechanism of doubly-charmed baryon decays: $\mathcal{B}_{cc} \to \mathcal{B}_c V$

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We study the nonleptonic weak decays of doubly charmed baryons ($\mathcal{B}_{cc}$) into singly charmed baryons ($\mathcal{B}_c$) and vector mesons (V), denoted as $\mathcal{B}_{cc} \to \mathcal{B}_c V$. The short-distance contributions are calculated within the naive factorization hypothesis, while the long-distance final-state interaction effects are modeled via hadronic triangle diagrams. Unlike previous approaches, which compute only the imaginary part using the Cutkosky cutting rule, we evaluate the complete loop integrals to obtain both the real and imaginary parts of the amplitudes. These provide the nontrivial strong phases essential for CP violation. The model parameters are determined using experimental data. With this improved calculation method, we predict the branching ratios and decay asymmetry parameters for various decay channels, as well as CP violations for short-distance dominated and singly Cabibbo-suppressed channels. This strengthens our theoretical framework for future study of doubly charmed baryons. Certain decays, primarily driven by long-distance effects, have been calculated, and their observation in future experiments could help clarify the role of final-state interactions in charm baryon decays. Therefore, our calculation of $\mathcal{B}_{cc} \to \mathcal{B}_c V$ provides crucial predictions for branching ratios, decay asymmetry parameters, and CP violation, which are essential for guiding experimental study at LHCb.

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I. INTRODUCTION

The doubly heavy baryons with two heavy quarks (b or c quark), predicted by the quark model, have attracted extensive attention in both theoretical and experimental studies. In 2002, SELEX Collaboration initially reported the detection of Ξ_{cc}^+ with an unexpected short lifetime and a relatively large production rate [1,2]. However, subsequent experiments, including FOCUS [3], *BABAR* [4], Belle [5], and LHCb [6], have not confirmed this discovery, which has remained a longstanding puzzle in experiments. In 2017, LHCb Collaboration announced the discovery of $\Xi_{cc}^{++}(3621)$ in the invariant mass spectrum of $\Lambda_c^+ K^- \pi^+ \pi^+$ [7]. The existence of Ξ_{cc}^{++} was further confirmed through the decay channel $\Xi_{cc}^{++} \to \Xi_c^+ \pi^+$ [8]. LHCb has performed precise measurements of its mass and lifetime [9,10], as well as the branching fraction ratio $\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c'^+ \pi^+)/\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c^+ \pi^+) = (1.41 \pm 0.17 \pm 0.10)$ [11]. Through the continuous search by LHCb Collaboration [12–14], the first observation of the singly charged doubly charmed baryon Ξ_{cc}^+ has been reported through its decay $\Xi_{cc}^+ \to \Lambda_c^+ K^- \pi^+$ by LHCb [15] this year, with a statistical significance exceeding seven standard deviations. The study of Ω_{cc}^+ is also ongoing, with LHCb searching for its decay signals [16]. Other doubly heavy baryons, such as Ξ_{bc}^+ [17], Ξ_{bc}^0 [18,19], and Ω_{bc}^0 [19], have also been investigated by LHCb Collaboration, though no signals have been detected to date. The doubly charmed tetraquark $T_{cc}^+(3875)$ was identified by LHCb Collaboration [20,21], which may provide insights into the nature of doubly heavy systems.

Prior to the experimental discovery of Ξ_{cc}^{++}, theoretical studies had already suggested that the most likely discovery channels for these doubly charmed baryons would be $\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+$ and $\Xi_{cc}^{++} \to \Xi_c^+ \pi^+$ [22]. Therefore, pretheoretical studies of doubly charmed baryon decays are crucial for experimental searches. Theoretical investigations on doubly heavy baryons have been extensive, covering production mechanisms, mass spectra, and both

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strong and weak decays [22–46]. However, studies on nonleptonic weak decays remain relatively limited compared to semileptonic decays. This is mainly due to the large number of nonleptonic decay channels and the presence of nonfactorizable contributions that are difficult to calculate using QCD methods.

In our previous work, we investigated the decays $\mathcal{B}_{cc} \to \mathcal{B}_c P$ in the final-state interaction approach. In order to study the weak decay of the doubly charmed baryon system and select the golden mode for the research of doubly charmed baryons, in this work, we investigate the decay $\mathcal{B}_{cc} \to \mathcal{B}_c V$ with vector mesons in the final state. The vector meson final states are especially important for several reasons: (i) vector mesons can be reconstructed efficiently through their decay products, such as $\rho \to \pi^+\pi^-$, $K^* \to K^+\pi^-$, $\omega \to \pi^+\pi^-\pi^0$, and $\phi \to K^+K^-$, which have high detection efficiencies in experiments, (ii) the angular distributions of vector mesons provide additional information about the decay dynamics through decay asymmetry parameters, and (iii) understanding the $\mathcal{B}_{cc} \to \mathcal{B}_c V$ decays is essential for predicting the discovery channels of yet-unobserved doubly charmed baryons Ω_{cc}^+.

In this paper, we study the nonleptonic weak decays $\mathcal{B}_{cc} \to \mathcal{B}_c V$ for doubly charmed baryons $\mathcal{B}_{cc} = (\Xi_{cc}^{++}, \Xi_{cc}^+, \Omega_{cc}^+)$, where $\mathcal{B}_c$ denotes singly charmed baryons, including both antitriplet $(\Lambda_c,\ \Xi_c)$ and sextet $(\Sigma_c, \Xi_c', \Omega_c)$ states, and V represents light vector mesons $(\rho, K^*, \omega, \phi)$. We calculate branching ratios and decay asymmetry parameters for 49 decay channels. For the specific decay mode $\mathcal{B}_{cc} \to \mathcal{B}_c V$, there is currently one dedicated theoretical study in the literature [47]. Compared with the previous study [47], we improve the calculation method for long-distance contributions by utilizing the complete analytical expression of loop integrals rather than only the imaginary parts from the Cutkosky cutting rules. This allows us to obtain both the magnitude and strong phase of the triangle diagrams. The strong phases are essential for calculating decay asymmetry parameters. We adopt the Pauli-Villars scheme with a momentum cutoff to regularize the divergence in loop integrals.

This paper is organized as follows. In Sec. II, we present the topological analysis, the short-distance factorization calculation, and the long-distance rescattering mechanism. Section III contains the numerical results and discussions, including input parameters, branching ratios, and decay asymmetry parameters. A summary is given in Sec. IV. Details of the decay amplitudes and strong couplings are provided in the Appendixes.

II. THEORETICAL FRAMEWORK

In this section, we will present a topological analysis of each nonleptonic weak decay channel of doubly charmed baryons, as detailed in Sec. II A. Subsequently, we present the general framework for calculating the short-distance and long-distance dynamics of all topological diagrams in Sec. II B and Sec. II C, respectively.

A. Topological analysis

In this work, we will investigate the exclusive channels of nonleptonic weak decay of doubly charmed baryons $\mathcal{B}_{cc} \to \mathcal{B}_c V$. The initial state is characterized by a doubly charmed baryon triplet $\mathcal{B}_{cc} = (\Xi_{cc}^{++}, \Xi_{cc}^+, \Omega_{cc}^+)$, while the final state encompasses the singly charmed baryon $\mathcal{B}_c = (\mathcal{B}_{\bar{3}}, \mathcal{B}_6)$, and the light vector mesons $V = (\rho, K^*, \omega, \phi)$, as shown in Appendix C. Based on the symmetry analysis of the initial and final states of particles, we will study 49 decay processes.

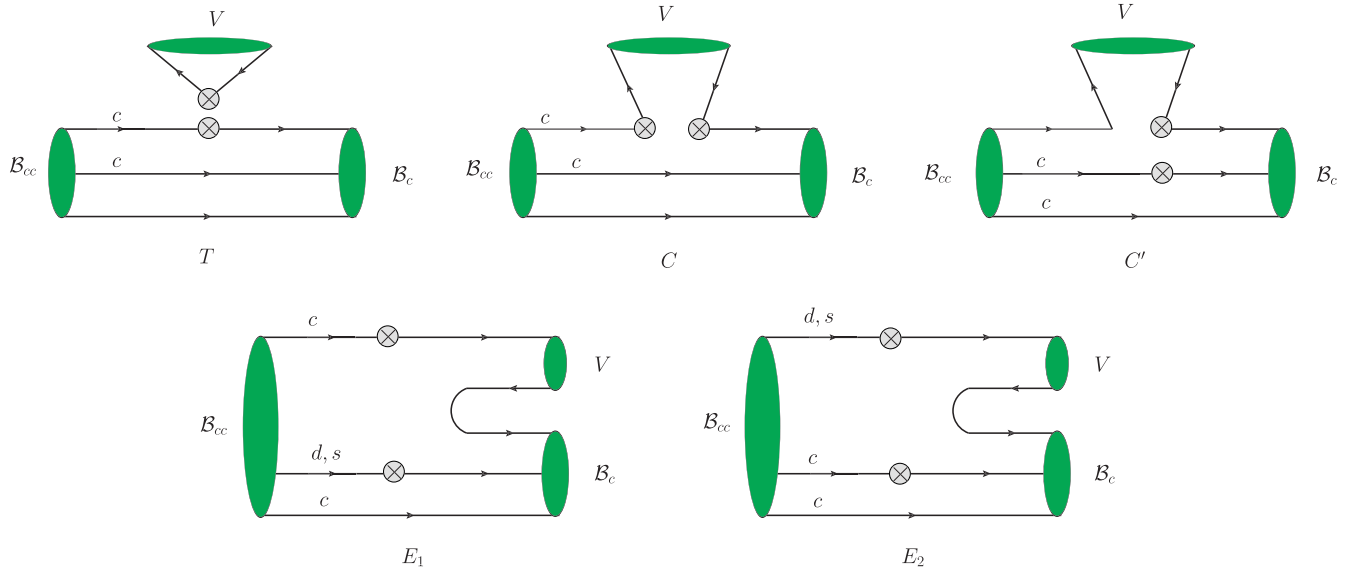


FIG. 1. The five tree-level topological diagrams for two-body nonleptonic decays $\mathcal{B}_{cc} \to \mathcal{B}_c V$ of the doubly charmed baryons $\mathcal{B}_{cc} = (\Xi_{cc}^{++}, \Xi_{cc}^+, \Omega_{cc}^+)$.

The tree-level contributions to $\mathcal{B}_{cc} \to \mathcal{B}_c V$ decays (Fig. 1) consist of both short-distance and long-distance parts. The five diagrams are classified into three types: T (color-favored external W-emission), C and C' (color-suppressed internal W-emission), and E_1 and E_2 (two W-exchange topologies). The distinction between C and C' lies in whether both quarks of the final light meson originate from the weak vertex (only in C). In E_1, the light quark from the charmed quark decay goes to the final light meson, and in E_2, it goes to the final charmed baryon.

The topological amplitudes for each channel are summarized in Table I. Those involving sextet single-charmed baryons are denoted with a tilde (e.g., $\tilde{T}$). Short-distance dominated modes are listed first, and the remaining three parts correspond to long-distance dominated modes, grouped by Cabibbo-Kobayashi-Maskawa (CKM) matrix elements: Cabibbo-favored ($c \to su\bar{d}$), singly Cabibbo-suppressed ($c \to du\bar{d}$ or $c \to su\bar{s}$), and doubly Cabibbo-suppressed ($c \to du\bar{s}$). The T diagram, dominated by factorizable short-distance contributions [48], can be computed using factorization. The $C(C')$ diagram is strongly suppressed at the charm scale due to the color factor $a_2(m_c) = C_1(m_c) + C_2(m_c)/N_c$ [22]. For $E_1(E_2)$, long-distance contributions exceed short-distance ones, which are suppressed by at least an order of magnitude [48]. However, nonfactorizable long-distance effects may significantly influence $C(C')$ and thus the branching ratios, warranting further study. Our model incorporates such effects via final-state interactions mediated by loop mechanisms, allowing improved agreement with experiment. The calculation of factorizable short-distance and nonfactorizable long-distance contributions is detailed in the following subsections.

B. Calculation of short-distance amplitudes under the factorization hypothesis

At the tree level, the nonleptonic weak decay of doubly charmed baryons is induced by the decay of the charm quark. The effective Hamiltonian can be represented as

TABLE I. The topological analysis of each nonleptonic weak decay channel.

Decay	Topology	Decay	Topology	Decay	Topology
$\Xi_{cc}^{++} \to \Xi_c^+ \rho^+$	$\lambda_{sd}(T + C')$	$\Xi_{cc}^{+} \to \Xi_c^0 \rho^+$	$\lambda_{sd}(T - E_2)$	$\Omega_{cc}^{+} \to \Omega_c^0 \rho^+$	$\lambda_{sd}\tilde{T}$
$\Xi_{cc}^{++} \to \Xi_c'^+ \rho^+$	$\frac{1}{\sqrt{2}}\lambda_{sd}(\tilde{T} + \tilde{C}')$	$\Xi_{cc}^{+} \to \Xi_c'^0 \rho^+$	$\frac{1}{\sqrt{2}}\lambda_{sd}(\tilde{T} + \tilde{E}_2)$	$\Omega_{cc}^{+} \to \Xi_c^0 \rho^+$	$-\lambda_d T - \lambda_s E_2$
$\Xi_{cc}^{++} \to \Sigma_c^+ \rho^+$	$\frac{1}{\sqrt{2}}\lambda_d(\tilde{T} + \tilde{C}')$	$\Xi_{cc}^{+} \to \Sigma_c^0 \rho^+$	$\lambda_d(\tilde{T} + \tilde{E}_2)$	$\Omega_{cc}^{+} \to \Xi_c'^0 \rho^+$	$\frac{1}{\sqrt{2}}(\lambda_d\tilde{T} + \lambda_s\tilde{E}_2)$
$\Xi_{cc}^{++} \to \Lambda_c^+ \rho^+$	$\lambda_d(T + C')$	$\Xi_{cc}^{+} \to \Xi_c'^0 K^{*+}$	$\frac{1}{\sqrt{2}}(\lambda_s\tilde{T} + \lambda_d\tilde{E}_2)$	$\Omega_{cc}^{+} \to \Omega_c^0 K^{*+}$	$\lambda_s(\tilde{T} + \tilde{E}_2)$
$\Xi_{cc}^{++} \to \Xi_c'^+ K^{*+}$	$\frac{1}{\sqrt{2}}\lambda_s(\tilde{T} + \tilde{C}')$	$\Xi_{cc}^{+} \to \Xi_c^0 K^{*+}$	$\lambda_s T + \lambda_d E_2$	$\Omega_{cc}^{+} \to \Xi_c^0 K^{*+}$	$\lambda_{ds}(-T + E_2)$
$\Xi_{cc}^{++} \to \Xi_c^+ K^{*+}$	$\lambda_s(T + C')$	$\Xi_{cc}^{+} \to \Sigma_c^0 K^{*+}$	$\lambda_{ds}\tilde{T}$	$\Omega_{cc}^{+} \to \Xi_c'^0 K^{*+}$	$\frac{1}{\sqrt{2}}\lambda_{ds}(\tilde{T} + \tilde{E}_2)$
$\Xi_{cc}^{++} \to \Lambda_c^+ K^{*+}$	$\lambda_{ds}(T + C')$	$\Xi_{cc}^{++} \to \Sigma_c^+ K^{*+}$	$\frac{1}{\sqrt{2}}\lambda_{ds}(\tilde{T} + \tilde{C}')$		
$\Xi_{cc}^{++} \to \Sigma_c^{++} \bar{K}^{*0}$	$\tilde{C}$	$\Xi_{cc}^{+} \to \Xi_c'^+ \rho^0$	$\frac{1}{2}(-\tilde{C}' + \tilde{E}_2)$	$\Xi_{cc}^{+} \to \Xi_c^+ \omega$	$\frac{1}{\sqrt{6}}(C' - 2E_1 - E_2)$
$\Xi_{cc}^{+} \to \Omega_c^0 K^{*+}$	$\tilde{E}_2$	$\Xi_{cc}^{+} \to \Xi_c'^+ \phi$	$\frac{1}{\sqrt{6}}(\tilde{C}' + \tilde{E}_1 + \tilde{E}_2)$	$\Omega_{cc}^{+} \to \Xi_c'^+ \bar{K}^{*0}$	$\frac{1}{\sqrt{2}}(\tilde{C} + \tilde{C}')$
$\Xi_{cc}^{+} \to \Sigma_c^+ \bar{K}^{*0}$	$\frac{1}{\sqrt{2}}(\tilde{C} + \tilde{E}_1)$	$\Xi_{cc}^{+} \to \Xi_c'^+ \omega$	$\frac{1}{2\sqrt{3}}(\tilde{C}' - 2\tilde{E}_1 + \tilde{E}_2)$	$\Omega_{cc}^{+} \to \Xi_c^+ \bar{K}^{*0}$	$-C + C'$
$\Xi_{cc}^{+} \to \Lambda_c^+ \bar{K}^{*0}$	$-C + E_1$	$\Xi_{cc}^{+} \to \Xi_c^+ \rho^0$	$-\frac{1}{\sqrt{2}}(C' + E_2)$		
$\Xi_{cc}^{+} \to \Sigma_c^{++} K^{*-}$	$\tilde{E}_1$	$\Xi_{cc}^{+} \to \Xi_c^+ \phi$	$\frac{1}{\sqrt{3}}(C' + E_1 - E_2)$		
$\Xi_{cc}^{++} \to \Sigma_c^{++} \rho^0$	$-\frac{1}{\sqrt{2}}\tilde{C}$	$\Xi_{cc}^{+} \to \Xi_c'^+ K^{*0}$	$\frac{1}{\sqrt{2}}(\lambda_s\tilde{C}' + \lambda_d\tilde{E}_1)$	$\Omega_{cc}^{+} \to \Sigma_c^{++} K^{*-}$	$\lambda_s\tilde{E}_1$
$\Xi_{cc}^{++} \to \Sigma_c^{++} \phi$	$\frac{1}{\sqrt{3}}(\lambda_d + \lambda_s)\tilde{C}$	$\Xi_{cc}^{+} \to \Xi_c^+ K^{*0}$	$\lambda_s C' + \lambda_d E_1$	$\Omega_{cc}^{+} \to \Xi_c'^+ \rho^0$	$\frac{1}{2}(-\lambda_d\tilde{C} + \lambda_s\tilde{E}_2)$
$\Xi_{cc}^{++} \to \Sigma_c^{++} \omega$	$\frac{1}{\sqrt{6}}(\lambda_d - 2\lambda_s)\tilde{C}$	$\Xi_{cc}^{+} \to \Lambda_c^+ \rho^0$	$\frac{\lambda_d}{\sqrt{2}}(C - C' - E_1 - E_2)$	$\Omega_{cc}^{+} \to \Xi_c^+ \rho^0$	$\frac{1}{\sqrt{2}}(\lambda_d C - \lambda_s E_2)$
$\Xi_{cc}^{+} \to \Sigma_c^+ \rho^0$	$\frac{\lambda_d}{2}(-\tilde{C} - \tilde{C}' + \tilde{E}_1 + \tilde{E}_2)$	$\Xi_{cc}^{+} \to \Lambda_c^+ \phi$	$\frac{1}{\sqrt{3}}[\lambda_d(C' - C + E_1 - E_2) - \lambda_s C]$	$\Omega_{cc}^{+} \to \Xi_c'^+ \phi$	$\frac{1}{\sqrt{6}}[\lambda_s(\tilde{C} + \tilde{C}' + \tilde{E}_1 + \tilde{E}_2) + \lambda_d\tilde{C}]$
$\Xi_{cc}^{+} \to \Sigma_c^+ \phi$	$\frac{1}{\sqrt{6}}[\lambda_d(\tilde{C} + \tilde{C}' + \tilde{E}_1 + \tilde{E}_2) + \lambda_s\tilde{C}]$	$\Xi_{cc}^{+} \to \Lambda_c^+ \omega$	$\frac{1}{\sqrt{6}}[\lambda_d(C' - C + E_1 - E_2) + 2\lambda_s C]$	$\Omega_{cc}^{+} \to \Xi_c'^+ \omega$	$\frac{1}{2\sqrt{3}}[\lambda_s(\tilde{E}_2 - 2\tilde{C} - 2\tilde{C}' - 2\tilde{E}_1) + \lambda_d\tilde{C}]$
$\Xi_{cc}^{+} \to \Sigma_c^+ \omega$	$\frac{1}{2\sqrt{3}}[\lambda_d(\tilde{C} + \tilde{C}' + \tilde{E}_1 + \tilde{E}_2) - 2\lambda_s\tilde{C}]$	$\Omega_{cc}^{+} \to \Xi_c^+ \omega$	$\frac{1}{\sqrt{6}}[\lambda_s(2C - 2C' + 2E_1 - E_2) - \lambda_d C]$	$\Omega_{cc}^{+} \to \Xi_c^+ \phi$	$\frac{1}{\sqrt{3}}[\lambda_s(C' - C + E_1 - E_2) - \lambda_d C]$
$\Xi_{cc}^{+} \to \Sigma_c^{++} \rho^-$	$\lambda_d\tilde{E}_1$	$\Omega_{cc}^{+} \to \Sigma_c^+ \bar{K}^{*0}$	$\frac{1}{\sqrt{2}}(\lambda_d\tilde{C}' + \lambda_s\tilde{E}_1)$	$\Omega_{cc}^{+} \to \Lambda_c^+ \bar{K}^{*0}$	$\lambda_d C' + \lambda_s E_1$
$\Xi_{cc}^{++} \to \Sigma_c^{++} K^{*0}$	$\tilde{C}$	$\Omega_{cc}^{+} \to \Sigma_c^+ \omega$	$\frac{1}{2\sqrt{3}}(-2\tilde{C}' + \tilde{E}_1 + \tilde{E}_2)$	$\Omega_{cc}^{+} \to \Sigma_c^{++} \rho^-$	$\tilde{E}_1$
$\Xi_{cc}^{+} \to \Sigma_c^+ K^{*0}$	$\frac{1}{\sqrt{2}}(\tilde{C} + \tilde{C}')$	$\Omega_{cc}^{+} \to \Lambda_c^+ \rho^0$	$-\frac{1}{\sqrt{2}}(E_1 + E_2)$	$\Omega_{cc}^{+} \to \Xi_c'^+ K^{*0}$	$\frac{1}{\sqrt{2}}(\tilde{C} + \tilde{E}_1)$
$\Xi_{cc}^{+} \to \Lambda_c^+ K^{*0}$	$-C + C'$	$\Omega_{cc}^{+} \to \Lambda_c^+ \phi$	$\frac{1}{\sqrt{3}}(C' + E_1 - E_2)$	$\Omega_{cc}^{+} \to \Xi_c^+ K^{*0}$	$-C + E_1$
$\Omega_{cc}^{+} \to \Sigma_c^+ \rho^0$	$\frac{1}{2}(-\tilde{E}_1 + \tilde{E}_2)$	$\Omega_{cc}^{+} \to \Lambda_c^+ \omega$	$-\frac{1}{\sqrt{6}}(2C' - E_1 + E_2)$		
$\Omega_{cc}^{+} \to \Sigma_c^+ \phi$	$\frac{1}{\sqrt{6}}(\tilde{C}' + \tilde{E}_1 + \tilde{E}_2)$	$\Omega_{cc}^{+} \to \Sigma_c^0 \rho^+$	$\tilde{E}_2$		

follows:

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{q'=d,s} V^*_{cq'} V_{uq} [C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu)] + \text{H.c.} \quad (1)$$

Here, $V_{cq'}$ and V_{uq} represent the CKM matrix elements. The four-fermion operators O_1 and O_2 can be expressed as

$$O_1 = (\bar{u}_\alpha q_\alpha)_{V-A} (\bar{q}'_\beta c_\beta)_{V-A},$$
$$O_2 = (\bar{u}_\alpha q_\beta)_{V-A} (\bar{q}'_\beta c_\alpha)_{V-A}, \quad (2)$$

with the color indices α and β, and $C_{1,2}(\mu)$ are the relevant Wilson coefficients. After inserting the effective Hamiltonian as Eq. (1), the amplitude of the nonleptonic weak decay of doubly charmed baryons $\mathcal{B}_{cc} \to \mathcal{B}_c V$ can be evaluated with the hadronic matrix element,

$$\langle \mathcal{B}_c V | \mathcal{H}_{\rm eff} | \mathcal{B}_{cc} \rangle = \frac{G_F}{\sqrt{2}} V^*_{cq'} V_{uq} \sum_{i=1,2} C_i \langle \mathcal{B}_c V | O_i | \mathcal{B}_{cc} \rangle. \quad (3)$$

In this work, we neglect the penguin operators for the strong suppression of CKM matrix elements in charmed hadron decays. According to the factorization hypothesis, the matrix elements $\langle \mathcal{B}_c M | O_i | \mathcal{B}_{cc} \rangle$ in Eq. (3) can be factorized into two parts. The first part is parametrized with the decay constant of the emitted meson, while the second part can be evaluated using the heavy-light transition form factors. The short-distance contribution of the T diagram can be expressed as

$$\langle \mathcal{B}_c M | \mathcal{H}_{\rm eff} | \mathcal{B}_{cc} \rangle^T_{SD} = \frac{G_F}{\sqrt{2}} V^*_{cq'} V_{uq} a_1(\mu) \langle M | \bar{u} \gamma^\mu (1-\gamma_5) q | 0 \rangle \times \langle \mathcal{B}_c | \bar{q}' \gamma_\mu (1-\gamma_5) c | \mathcal{B}_{cc} \rangle, \quad (4)$$

where $a_1(\mu) = C_1(\mu) + C_2(\mu)/3$ is the effective Wilson coefficients. For the color-suppressed C diagram, its short-distance contribution can be given via its relation to the T diagram after Fierz transformation. Under the charm scale $\mu = m_c$, the Wilson coefficients are taken as $C_1(\mu) = 1.21$ and $C_2(\mu) = -0.42$ [49]. M represents both the pseudoscalar meson (P) and the vector meson (V). The vector meson also contributes to long-distance dynamics as an intermediate state in the next subsections.

By utilizing the heavy-light transition form factors $f_{1,2,3}$ and $g_{1,2,3}$, the transition matrix elements of $\mathcal{B}_{cc} \to \mathcal{B}_c$ can be effectively parametrized as

$$\langle \mathcal{B}_c(p', s'_z) | \bar{q}' \gamma_\mu (1-\gamma_5) c | \mathcal{B}_{cc}(p, s_z) \rangle = \bar{u}(p', s'_z) \left[\gamma_\mu f_1(q^2) + i\sigma_{\mu\nu} \frac{q^\nu}{M_{\mathcal{B}_{cc}}} f_2(q^2) + \frac{q^\mu}{M_{\mathcal{B}_{cc}}} f_3(q^2) \right] u(p, s_z) - \bar{u}(p', s'_z) \left[\gamma_\mu g_1(q^2) + i\sigma_{\mu\nu} \frac{q^\nu}{M_{\mathcal{B}_{cc}}} g_2(q^2) + \frac{q^\mu}{M_{\mathcal{B}_{cc}}} g_3(q^2) \right] \gamma_5 u(p, s_z). \quad (5)$$

Here, $q = p - p'$, and $M_{\mathcal{B}_{cc}}$ is the mass of doubly charmed baryons.

The first matrix elements in Eq. (4) are defined using the decay constants of the emitted mesons P and V, denoted by f_P and f_V, respectively,

$$\langle P(p) | \bar{u} \gamma^\mu (1-\gamma_5) q | 0 \rangle = -i f_P p^\mu, \quad (6)$$

$$\langle V(p) | \bar{u} \gamma^\mu (1-\gamma_5) q | 0 \rangle = m_V f_V \epsilon^{*\mu}, \quad (7)$$

where ϵ^μ represents the polarization vector of the vector meson.

Conventionally, the amplitude of two-body nonleptonic charmed baryon decays is given in terms of the partial wave scheme,

$$\mathcal{A}(\mathcal{B}_{cc} \to \mathcal{B}_c P) = i \bar{u}_f (A + B\gamma_5) u_i, \quad (8)$$

specifically, the invariant amplitudes A and B are expressed as the following factorization form by using the form factors f_i and g_i:

$$A = \lambda f_P (m_i - m_f) f_1(m^2),$$
$$B = \lambda f_P (m_i + m_f) g_1(m^2). \quad (9)$$

For the vector meson, the amplitudes are

$$\mathcal{A}(\mathcal{B}_{cc} \to \mathcal{B}_c V) = \bar{u}_f \left(A_1 \gamma_\mu \gamma_5 + A_2 \frac{p_{f\mu}}{m_i} \gamma_5 + B_1 \gamma_\mu + B_2 \frac{p_{f\mu}}{m_i} \right) \varepsilon^{*\mu} u_i, \quad (10)$$

where

$$A_1 = -\lambda f_V m \left(g_1(m^2) + g_2(m^2) \frac{m_i - m_f}{m_i} \right),$$
$$A_2 = -2\lambda f_V m g_2(m^2), \quad (11)$$

$$B_1 = \lambda f_V m \left(f_1(m^2) - f_2(m^2) \frac{m_i + m_f}{m_i} \right),$$
$$B_2 = 2\lambda f_V m f_2(m^2), \quad (12)$$

with $\lambda = \frac{G_F}{\sqrt{2}} V_{cq'}^* V_{uq} a_{1,2}(\mu)$ for simplicity, and m is the mass of the final meson. As we mentioned before, the factorization formulas in Eqs. (8) and (10) are also essential ingredients for calculating the long-distance final-state interaction effects.

C. Calculation of long-distance contributions using the final-state rescattering mechanism

The long-distance contributions are large and difficult to evaluate. In this work, we employ the final-state rescattering mechanism, as in Ref. [22], to compute these contributions. This mechanism proceeds via the rescattering of two intermediate particles, illustrated in Fig. 2. We detail our calculation of the nonleptonic decay amplitudes $\mathcal{B}_{cc} \to \mathcal{B}_c V$. At the quark level, the first weak vertex in the triangle diagram is induced by the external W-emission diagram T, dominated by the factorizable contribution. To avoid double counting, the two-body weak transition amplitudes of doubly charmed baryons include only these short-distance factorizable parts (introduced in the previous section). The rescattering can occur via s-channel or t/u-channel exchange. The s-channel would be significant only if the exchanged particle mass was close to $m_{\mathcal{B}_{cc}}$. However, the heaviest known singly charmed baryon is ∼500 MeV lighter than $m_{\Xi_{cc}^+}$, thus the s-channel is strongly suppressed by off-shell effects and neglected. We, therefore, consider only the t/u-channel triangle diagram. Several methods exist for evaluating triangle amplitudes [50–56], differing mainly in the treatment of loop integrations. For instance, Refs. [55,57] apply the optical theorem and Cutkosky cutting rules to compute only the absorptive (imaginary) part—the real part, obtainable via dispersion relations, suffers from large ambiguities and is often omitted. To obtain the complete amplitude reliably, we directly evaluate the loop integrals. We use the Passarino-Veltman method to reduce tensor integrals and integration by parts to express results in terms of master integrals.

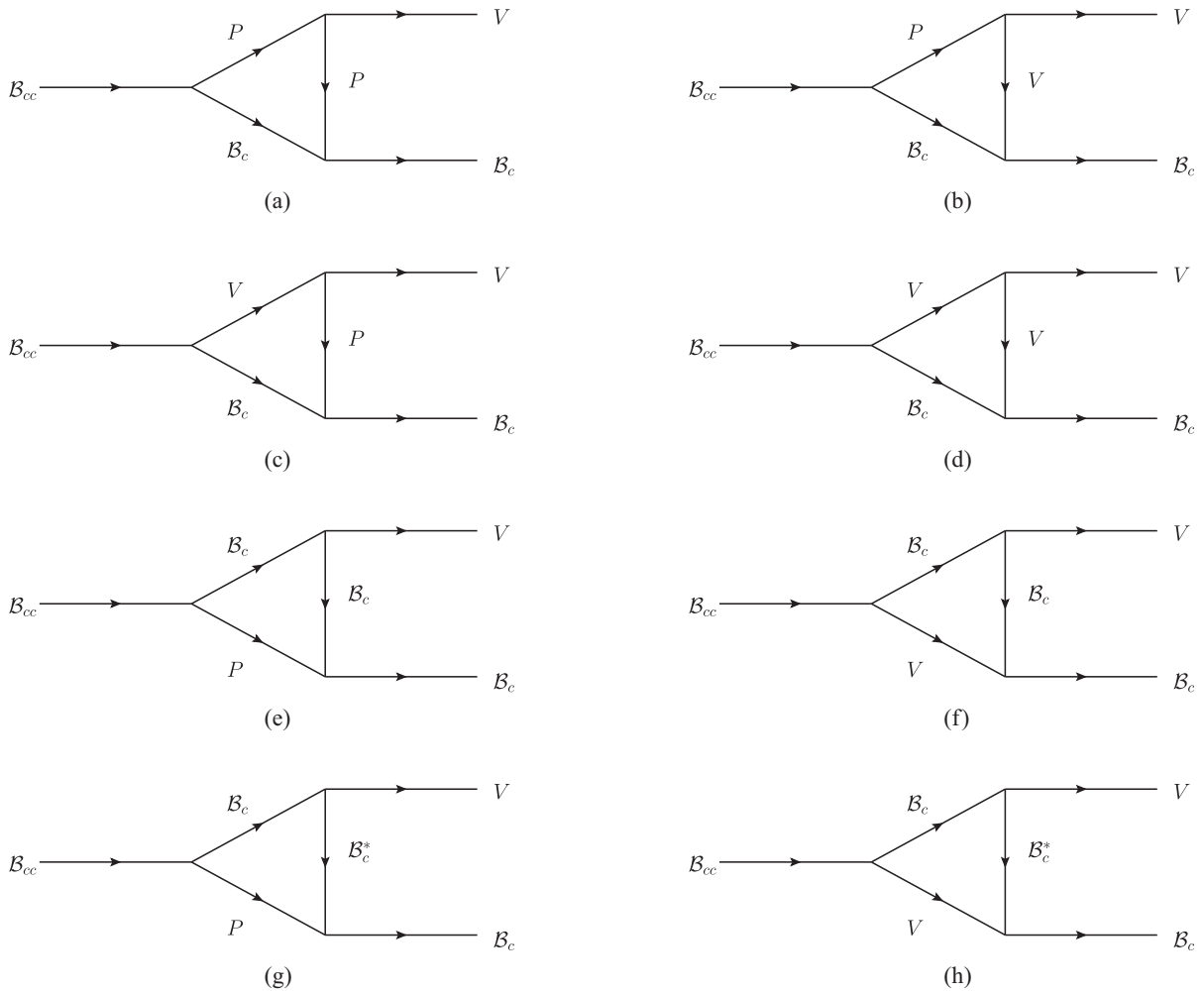


FIG. 2. The long-distance rescattering contributions to $\mathcal{B}_{cc} \to \mathcal{B}_c P$ manifested at hadron level via single particle exchange, generating distinct triangle diagrams with intermediate states: (a) $\{P, \mathcal{B}_c; P\}$, (b) $\{P, \mathcal{B}_c; V\}$, (c) $\{V, \mathcal{B}_c; P\}$, (d) $\{V, \mathcal{B}_c; V\}$, (e) $\{\mathcal{B}_c, P; \mathcal{B}_c\}$, (f) $\{\mathcal{B}_c, V; \mathcal{B}_c\}$, (g) $\{\mathcal{B}_c, P; \mathcal{B}_c^*\}$, and (h) $\{\mathcal{B}_c, V; \mathcal{B}_c^*\}$. The corresponding amplitudes of these triangle diagrams have been given by Eqs. (13)–(20).

To compute the triangle diagrams, we combine the weak transition vertex (weak effective Hamiltonian with naive factorization) with the hadronic scattering amplitudes. The two strong-interaction vertices in that diagram are evaluated using the hadronic effective Lagrangians [58–63] given in Appendix B. Specifically, we determine the amplitude for $\mathcal{B}_{cc}(p_i) \to \mathcal{B}_c(p_4)V(p_3)$. Figure 2(a) shows a triangle diagram with intermediate states $\{P, \mathcal{B}_c; P\}$: a weak vertex $\mathcal{B}_{cc}(p_i) \to \mathcal{B}_c(p_2)P(p_1)$ and a rescattering amplitude $\mathcal{B}_c(p_2)P(p_1) \to \mathcal{B}_c(p_4)V(p_3)$. We denote a triangle amplitude by $\mathcal{M}[P_1, P_2; P_k]$. We unify momentum and spin-polarization notation as follows: $\mathcal{B}_{cc}(p_i, s_i) \to \mathcal{B}_c(p_3, s_3)V(p_4, \varepsilon_4)$ with rescattering particles $\mathcal{B}_c(p_1, s_1)P(p_2)/V(p_2, \varepsilon_2)$ and exchanged particles $P(k), V(k, \varepsilon_k), \mathcal{B}_c/\mathcal{B}_c^*(k, s_k)$.

Under this convention and applying the factorization formula and the hadronic Feynman rules derived accordingly (Appendix B), we obtain analytical expressions for the eight hadronic loop diagrams shown in Fig. 2, with the loop integral variable p_1,

$$\mathcal{M}[P, \mathcal{B}_c; P] = -i \int \frac{d^4p_1}{(2\pi)^4} g_{P\mathcal{B}_c\mathcal{B}_c} \cdot g_{VPP} \bar{u}(p_4, s_4)\gamma_5(\not{p}_2 + m_2)(A + B\gamma_5)u(p_i, s_i)\varepsilon_\alpha^*(p_3, \lambda_3)(p_1 + k)^\alpha \mathcal{PF}, \tag{13}$$

$$\begin{aligned}\mathcal{M}[P, \mathcal{B}_c; V] = i \int \frac{d^4p_1}{(2\pi)^4} \frac{g_{VVP}}{\sqrt{m_{V_1} m_{V_2}}} \bar{u}(p_4, s_4) \left[\left(f_{1V\mathcal{B}_c\mathcal{B}_c}\gamma_\nu - \frac{if_{2V\mathcal{B}_c\mathcal{B}_c}}{m_2 + m_4}\sigma_{\mu\nu}k^\mu \right) \varepsilon^\nu(k, \lambda_k) \right] (\not{p}_2 + m_2) \\ \times (A + B\gamma_5)u(p_i, s_i)\varepsilon^{\alpha\beta\rho\tau}\varepsilon_\beta^*(p_3, \lambda_3)\epsilon_\tau^*(k, \lambda_k)p_{3\alpha}p_{1\rho}\mathcal{PF},\end{aligned} \tag{14}$$

$$\begin{aligned}\mathcal{M}[V, \mathcal{B}_c; P] = \int \frac{d^4p_1}{(2\pi)^4} \frac{g_{VVP}}{\sqrt{m_{V_1} m_{V_2}}} g_{P\mathcal{B}_c\mathcal{B}_c} \bar{u}(p_4, s_4)\gamma_5(\not{p}_2 + m_2) \left[A_1\gamma_\mu\gamma_5 + A_2\frac{p_{2\mu}}{m_i}\gamma_5 + B_1\gamma_\mu + B_2\frac{p_{2\mu}}{m_i} \right] u(p_i, s_i) \\ \times \epsilon^{\alpha\beta\rho\tau}\epsilon_\beta^*(p_3)\epsilon_\tau(p_1)p_{3\alpha}k_\rho\epsilon^{*\mu}(p_1)\mathcal{PF},\end{aligned} \tag{15}$$

$$\begin{aligned}\mathcal{M}[V, \mathcal{B}_c; V] = \int \frac{d^4p_1}{(2\pi)^4} \frac{g_{VVV}}{\sqrt{2}} [\epsilon^\alpha(p_1)\epsilon^{*\beta}(p_3)\epsilon_\beta^*(k)(p_{3\alpha} - k_\alpha) - \epsilon^{*\alpha}(k)\epsilon^\beta(p_1)\epsilon_\beta^*(p_3)(p_{1\alpha} + p_{3\alpha}) \\ + \epsilon^{*\alpha}(p_3)\epsilon^\beta(p_1)\epsilon_\beta^*(k)(k_\alpha + p_{1\alpha})]\bar{u}(p_4, s_4) \left(f_{1V\mathcal{B}_c\mathcal{B}_c}\gamma_\nu - \frac{if_{2V\mathcal{B}_c\mathcal{B}_c}}{m_2 + m_4}\sigma_{\mu\nu}k^\mu \right) \varepsilon^\nu(k, \lambda_k)(\not{p}_2 + m_2) \\ \times \epsilon^\rho(p_1) \left[A_1\gamma_\rho\gamma_5 + A_2\frac{p_{2\rho}}{m_i}\gamma_5 + B_1\gamma_\rho + B_2\frac{p_{2\rho}}{m_i} \right] u(p_i, s_i) \times \mathcal{PF},\end{aligned} \tag{16}$$

$$\begin{aligned}\mathcal{M}[\mathcal{B}_c, P; \mathcal{B}_c] = -\int \frac{d^4p_1}{(2\pi)^4} g_{P\mathcal{B}_c\mathcal{B}_c} \bar{u}(p_4, s_4)\gamma_5(\not{k} + m_k) \left(f_{1V\mathcal{B}_c\mathcal{B}_c}\gamma_\nu + \frac{if_{2V\mathcal{B}_c\mathcal{B}_c}}{m_1 + m_k}\sigma_{\mu\nu}p_3^\mu \right) \\ \times \varepsilon^{\nu*}(p_3, \lambda_3)(\not{p}_1 + m_1)(A + B\gamma_5)u(p_i, s_i) \times \mathcal{PF},\end{aligned} \tag{17}$$

$$\begin{aligned}\mathcal{M}[\mathcal{B}_c, V; \mathcal{B}_c] = -i \int \frac{d^4p_1}{(2\pi)^4} \bar{u}(p_4, s_4) \left(f_{1V\mathcal{B}_c\mathcal{B}_c}\gamma_\nu - \frac{if_{2V\mathcal{B}_c\mathcal{B}_c}}{m_4 + m_k}\sigma_{\mu\nu}p_2^\mu \right) \varepsilon^{*\nu}(p_2, \lambda_2)(\not{k} + m_k) \\ \times \left(f_{1V\mathcal{B}_c\mathcal{B}_c}\gamma_\beta + \frac{if_{2V\mathcal{B}_c\mathcal{B}_c}}{m_1 + m_k}\sigma_{\alpha\beta}p_3^\alpha \right) \varepsilon^{*\nu}(p_3, \lambda_3)(\not{p}_1 + m_1) \\ \times \epsilon^\rho(p_2, \lambda_2) \left(A_1\gamma_\rho\gamma_5 + A_2\frac{p_{1\rho}}{m_i}\gamma_5 + B_1\gamma_\rho + B_2\frac{p_{1\rho}}{m_i} \right) u(p_i, s_i) \times \mathcal{PF},\end{aligned} \tag{18}$$

$$\begin{aligned}\mathcal{M}[\mathcal{B}_c, P; \mathcal{B}_c^*] = ig_{P\mathcal{B}_c\mathcal{B}_c^*}g_{P\mathcal{B}_c\mathcal{B}_c^*} \int \frac{d^4p_1}{(2\pi)^4} \bar{u}(p_4, \lambda_4)p_{2\mu}u_\nu(k, \lambda_k)\bar{u}^\rho(k, \lambda_k) \times \mathcal{PF}, \\ \times \gamma_5\gamma^\sigma[p_{3\rho}\varepsilon_\sigma^*(p_3, \lambda_3) - p_{3\sigma}\varepsilon_\rho^*(p_3, \lambda_3)](\not{p}_1 + m_1)(A + B\gamma_5)u(p_i, s_i)\end{aligned} \tag{19}$$

$$\begin{aligned}\mathcal{M}[\mathcal{B}_c, V; \mathcal{B}_c^*] = -ig_{V\mathcal{B}_c\mathcal{B}_c^*}g_{V\mathcal{B}_c\mathcal{B}_c^*} \int \frac{d^4p_1}{(2\pi)^4} \bar{u}(p_4, \lambda_4)\gamma_5\gamma^\beta[p_{2\alpha}\varepsilon_\beta^*(p_2, \lambda_2) - p_{2\beta}\varepsilon_\alpha^*(p_2, \lambda_2)] \\ \times u^\alpha(k, \lambda_k)\bar{u}^\rho(k, \lambda_k)\gamma_5\gamma^\sigma[p_{3\rho}\varepsilon_\sigma^*(p_3, \lambda_3) - p_{3\sigma}\varepsilon_\rho^*(p_3, \lambda_3)](\not{p}_1 + m_1) \\ \times \epsilon^\mu(p_2, \lambda_2) \left[A_1\gamma_\mu\gamma_5 + A_2\frac{p_{1\mu}}{m_i}\gamma_5 + B_1\gamma_\mu + B_2\frac{p_{1\mu}}{m_i} \right] u(p_i, s_i) \times \mathcal{PF},\end{aligned} \tag{20}$$

where the note $\mathcal{PF}$ represents the multiplication of the propagators and form factor,

$$\mathcal{PF} = \frac{1}{(p_1^2 - m_1^2 + i\epsilon)(p_2^2 - m_2^2 + i\epsilon)(p_k^2 - m_k^2 + i\epsilon)} \times \left(\frac{\Lambda_k^2 - m_k^2}{\Lambda_k^2 - p_k^2}\right)^n. \tag{21}$$

The amplitudes for the weak vertices $\mathcal{B}_{cc}(p_i) \to \mathcal{B}_c(p_2)P/V(p_1)$ can be calculated using Eqs. (8) and (10). Accordingly, the definitions of A, B, $A_{1,2}$, and $B_{1,2}$ appearing in Eqs. (13)–(20) are given in Eqs. (9) and (12). The strong coupling constants (e.g., g_{VPP}, $f_{1V\mathcal{B}_c\mathcal{B}_c}$, and $f_{2V\mathcal{B}_c\mathcal{B}_c}$) in the equation are calculated on-shell, but their reliability is compromised because the exchanged particle is generally off-shell. To account for off-shell effects and ensure theoretical self-consistency, a form factor $F(p_j, m_j)$ is introduced [55]. Moreover, a proper regularization scheme is required to handle the divergences in the triangle diagram master integrals. The form factor in Eq. (21) is consistent with the Pauli-Villars regularization scheme,

$$F(p_k, m_k) = \left(\frac{\Lambda_k^2 - m_k^2}{\Lambda_k^2 - p_k^2}\right)^n. \tag{22}$$

The cutoff Λ_k can be given as

$$\Lambda_k = m_k + \eta\Lambda_{\rm QCD}. \tag{23}$$

With $\Lambda_{\rm QCD} = 330$ MeV for charm quark decays, the phenomenological parameter η is determined by experimental data, not from first principles. Multiple strong vertices would require extensive data to determine η individually, but in our previous work on $\mathcal{B}_{cc} \to \mathcal{B}_c V$ [64] we fixed η using the experimental ratio $\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c'^{+}\pi^+)/\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c^+\pi^+) = (1.41 \pm 0.17 \pm 0.10)$ [11]. Since the same charm quark decay is involved, we adopt that value. The form factor in Eq. (22) typically exhibits monopole or dipole behavior with exponent $n = 1$ or 2. As the branching ratios of B meson decays [55] are similar for both choices, we adopt $n = 2$.

After gathering all the fragments, the amplitude of decay $\mathcal{B}_{cc} \to \mathcal{B}_c V$ can be expressed as

$$\begin{aligned}\mathcal{A}(\mathcal{B}_{cc} \to \mathcal{B}_c V) &= \mathcal{M}_{SD}(\mathcal{B}_{cc} \to \mathcal{B}_c V) + \mathcal{M}[P, \mathcal{B}_c; P] + \mathcal{M}[P, \mathcal{B}_c; V] \\ &+ \mathcal{M}[V, \mathcal{B}_c; P] + \mathcal{M}[V, \mathcal{B}_c; V] + \mathcal{M}[\mathcal{B}_c, P; \mathcal{B}_c] \\ &+ \mathcal{M}[\mathcal{B}_c, V; \mathcal{B}_c] + \mathcal{M}[\mathcal{B}_c, P; \mathcal{B}_c^*] + \mathcal{M}[\mathcal{B}_c, V; \mathcal{B}_c^*],\end{aligned} \tag{24}$$

where $\mathcal{M}_{SD}$ labels its short-distance contributions, including the factorization contributions of T and C diagrams. The amplitudes for all channels can be found in Appendix A.

D. Decay asymmetry parameters and *CP* violation

As the amplitude for the two-body nonleptonic weak decay of doubly charmed baryon $\mathcal{B}_{cc} \to \mathcal{B}_c V$ can be deduced as

$$\mathcal{M}(\mathcal{B}_{cc} \to \mathcal{B}_c V) = \epsilon^{*\mu}\bar{u}_{\mathcal{B}_c}\left[A_1\gamma_\mu\gamma_5 + A_2\frac{p_{f\mu}}{m_i}\gamma_5 + B_1\gamma_\mu + B_2\frac{p_{f\mu}}{m_i}\right]u_{\mathcal{B}_{cc}}, \tag{25}$$

here, $p_{f\mu}$ is the momentum of singly charmed baryon $\mathcal{B}_c$ in the rest frame of the mother particle $\mathcal{B}_{cc}$. The $\mathcal{B}_{cc}$ spin has not yet been measured, but the quark model prediction is spin 1/2. The $\mathcal{B}_{cc} \to \mathcal{B}_c V$ mode is, therefore, the decay of a spin 1/2 particle into a spin 1 and a spin 1/2 particle. In the helicity formalism, the decay can be described by four $H^{\lambda_i}_{\lambda_f,\lambda_V}$ helicity amplitudes ($H^{\frac{1}{2}}_{-\frac{1}{2},0}$, $H^{-\frac{1}{2}}_{\frac{1}{2},0}$, $H^{-\frac{1}{2}}_{-\frac{1}{2},-1}$, and $H^{\frac{1}{2}}_{\frac{1}{2},1}$), where λ_i, λ_f, and λ_V are the helicities of $\mathcal{B}_{cc}$, $\mathcal{B}_c$, and V, respectively. In this decay process, the helicity amplitudes can be obtained in the Pauli-Dirac representation,

$$\begin{aligned}HV^{\lambda_i}_{\lambda_f,\lambda_V} &= \langle\mathcal{B}_f(\lambda_f)|\bar{c}\gamma^\mu b|\mathcal{B}_i(\lambda_i)\rangle\epsilon^*_\mu(\lambda_V), \\ HA^{\lambda_i}_{\lambda_f,\lambda_V} &= \langle\mathcal{B}_f(\lambda_f)|\bar{c}\gamma^\mu\gamma_5 b|\mathcal{B}_i(\lambda_i)\rangle\epsilon^*_\mu(\lambda_V), \\ H^{\lambda_i}_{\lambda_f,\lambda_V} &= HV^{\lambda_i}_{\lambda_f,\lambda_V} - HA^{\lambda_i}_{\lambda_f,\lambda_V}.\end{aligned} \tag{26}$$

The helicity amplitudes are related to the transition form factors as follows:

$$HV^{\frac{1}{2}}_{-\frac{1}{2},0} = HV^{-\frac{1}{2}}_{\frac{1}{2},0} = -i\sqrt{\frac{Q_-}{q^2}}\left((m_i + m_f)f_1 - \frac{q^2}{m_i}f_2\right), \tag{27}$$

$$HV^{\frac{1}{2}}_{\frac{1}{2},1} = HV^{-\frac{1}{2}}_{-\frac{1}{2},-1} = i\sqrt{2Q_-}\left(-f_1 + \frac{m_i + m_f}{m_i}f_2\right), \tag{28}$$

$$HA^{\frac{1}{2}}_{-\frac{1}{2},0} = -HA^{-\frac{1}{2}}_{\frac{1}{2},0} = -i\sqrt{\frac{Q_+}{q^2}}\left((m_i - m_f)g_1 + \frac{q^2}{m_i}g_2\right), \tag{29}$$

$$HA^{\frac{1}{2}}_{\frac{1}{2},1} = -HA^{-\frac{1}{2}}_{-\frac{1}{2},-1} = i\sqrt{2Q_+}\left(-g_1 - \frac{m_i - m_f}{m_i}g_2\right). \tag{30}$$

Then, the decay width can be given as follows:

$$\begin{aligned}\Gamma(\mathcal{B}_{cc} \to \mathcal{B}_c V) &= \frac{|\vec{p}_c|(E_f + m_f)}{4\pi m_i}\left[2(|\mathcal{S}|^2 + |\mathcal{P}_2|^2) + \frac{E_V^2}{m_V^2}(|\mathcal{S} + \mathcal{D}|^2 + |\mathcal{P}_1|^2)\right] \\ &= \frac{|\vec{p}_c|}{8\pi m_i^2}\frac{1}{2}\sum_{\lambda_i,\lambda_f,\lambda_V}|H^{\lambda_i}_{\lambda_f,\lambda_V}(\mathcal{B}_{cc} \to \mathcal{B}_c V)|^2,\end{aligned} \tag{31}$$

where $|\vec{p}_c|$ is the magnitude of three-momentum of $\mathcal{B}_c$ in the rest frame of $\mathcal{B}_{cc}$. The relations between A_1, A_2, B_1, B_2 and $\mathcal{S},\mathcal{P}_1,\mathcal{P}_2,\mathcal{D}$ can be given as

$$\begin{aligned}\mathcal{S} &= -A_1,\\ \mathcal{P}_1 &= -\frac{|\vec{p}_c|}{E_V}\left(\frac{m_i+m_f}{E_f+m_f}B_1+B_2\right),\\ \mathcal{P}_2 &= \frac{|\vec{p}_c|}{E_f+m_f}B_1,\\ \mathcal{D} &= -\frac{|\vec{p}_c|^2}{E_V(E_f+m_f)}(A_1-A_2).\end{aligned} \tag{32}$$

Specifically, the decay process $\mathcal{B}_{cc}\to\mathcal{B}_cV$ involves four distinct nonzero helicity amplitudes ($H^{\frac{1}{2}}_{-\frac{1}{2},0}$, $H^{-\frac{1}{2}}_{\frac{1}{2},0}$, $H^{-\frac{1}{2}}_{-\frac{1}{2},-1}$, and $H^{\frac{1}{2}}_{\frac{1}{2},1}$). Armed with these helicity amplitudes given in Eq. (26), we can also proceed to perform the calculations of the decay branching ratios and various asymmetry parameters, which are defined as [65,66]

$$\begin{aligned}\mathcal{BR} &= \Gamma(\mathcal{B}_{cc}\to\mathcal{B}_cV)\cdot\tau_{\Xi_{bc}},\\ \alpha_b &= -|\hat{H}^{\frac{1}{2}}_{\frac{1}{2},1}|^2+|\hat{H}^{-\frac{1}{2}}_{-\frac{1}{2},-1}|^2+|\hat{H}^{-\frac{1}{2}}_{\frac{1}{2},0}|^2-|\hat{H}^{\frac{1}{2}}_{-\frac{1}{2},0}|^2,\\ \alpha_2 &= |\hat{H}^{\frac{1}{2}}_{\frac{1}{2},1}|^2-|\hat{H}^{-\frac{1}{2}}_{-\frac{1}{2},-1}|^2+|\hat{H}^{-\frac{1}{2}}_{\frac{1}{2},0}|^2-|\hat{H}^{\frac{1}{2}}_{-\frac{1}{2},0}|^2,\\ r_0 &= |\hat{H}^{\frac{1}{2}}_{\frac{1}{2},1}|^2+|\hat{H}^{\frac{1}{2}}_{-\frac{1}{2},0}|^2,\\ r_1 &= |\hat{H}^{\frac{1}{2}}_{\frac{1}{2},1}|^2-|\hat{H}^{\frac{1}{2}}_{-\frac{1}{2},0}|^2.\end{aligned} \tag{33}$$

Here, we introduce the moduli squared of normalized helicity amplitudes $|\hat{H}^{\lambda_i}_{\lambda_f,\lambda_V}|^2=|H^{\lambda_i}_{\lambda_f,\lambda_V}|^2/H_N$, as in Ref. [66], with

$$\begin{aligned}H_N &= \sum_{\lambda_i,\lambda_f,\lambda_V}|H^{\lambda_i}_{\lambda_f,\lambda_V}(\Xi_{bc}\to\Xi_cJ/\Psi)|^2,\\ &= |H^{\frac{1}{2}}_{\frac{1}{2},1}|^2+|H^{-\frac{1}{2}}_{-\frac{1}{2},-1}|^2+|H^{-\frac{1}{2}}_{\frac{1}{2},0}|^2+|H^{\frac{1}{2}}_{-\frac{1}{2},0}|^2,\end{aligned} \tag{34}$$

and α_b is the parity-violating asymmetry parameter, α_2 represents the longitudinal polarization of the daughter Ξ_c baryon, and $r_0(r_1)$ corresponds to the longitudinal unpolarized (polarized) parameter.

As our calculation can give the strong phase of the decay amplitudes, the information of CP violation can be derived. The direct CP asymmetry is defined as

$$\begin{aligned}A^{\rm dir}_{CP}(\mathcal{B}_{cc}\to\mathcal{B}_cV) &= \frac{\Gamma(\mathcal{B}_{cc}\to\mathcal{B}_cV)-\bar{\Gamma}(\mathcal{B}_{cc}\to\mathcal{B}_cV)}{\Gamma(\mathcal{B}_{cc}\to\mathcal{B}_cV)+\bar{\Gamma}(\mathcal{B}_{cc}\to\mathcal{B}_cV)}\\ &= \frac{2r\sin\Delta\delta\sin\Delta\phi}{1+r^2+2r\cos\Delta\delta\cos\Delta\phi},\end{aligned} \tag{35}$$

where r denotes the amplitude ratio. Furthermore, $\Delta\delta$ and $\Delta\phi$ symbolize the strong and weak phase differences, respectively.

The violation of CP arises from the parameters associated with decay asymmetry,

$$\alpha_{CP}=\frac{\alpha-\bar{\alpha}}{\alpha+\bar{\alpha}},\qquad r_{CP}=\frac{r-\bar{r}}{r+\bar{r}}. \tag{36}$$

III. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we present the numerical results for the branching ratios and decay asymmetry parameters of the $\mathcal{B}_{cc}\to\mathcal{B}_cV$ decays.

A. Input parameters

The calculation requires inputs, including initial and final-state masses, decay constants of pseudoscalar and vector mesons, strong couplings, transition form factors, and the lifetimes of the doubly charmed baryons. In the following, we will classify and discuss the input parameter values used in this work:

(i) LHCb Collaboration has measured the mass and lifetime of the Ξ_{cc}^{++} baryon as $m_{\Xi_{cc}^{++}}=3.621$ GeV and $\tau_{\Xi_{cc}^{++}}=256$ fs [9,10], along with the mass of the Ξ_{cc}^{+} baryon: $m_{\Xi_{cc}^{+}}=3.619$ GeV [15]. From Refs. [67–69], we adopt $\tau_{\Xi_{cc}^{+}}=47$ fs, as well as $m_{\Omega_{cc}^{+}}=3.738$ GeV and $\tau_{\Omega_{cc}^{+}}=179$ fs. The masses of the final states, including singly heavy baryons

TABLE II. Decay constants (in units of MeV) of the light mesons [70–72] and strong couplings [55,58–63,74,75,77] used in this work.

f_π	f_{η_1}	f_ρ	f_ω	f_{η_8}	f_ϕ	f_K	f_{K^*}
130.2 ± 1.2	151 ± 2.6	216 ± 5	195 ± 3	169 ± 2.6	233 ± 4.6	155.7 ± 3	217 ± 7

$g_{\rho\rho\rho}$	$g_{\rho\pi\pi}$	$g_{\Xi_c^+\Xi_c^+\pi^0}$	$g_{\Xi_c'^+\Xi_c^+\pi^0}$	$g_{\Sigma_c^+\Sigma_c^0\pi^+}$	$g_{\Sigma_c^{*0}\Lambda_c^+\pi^-}$	$g_{\Sigma_c^{*+}\Sigma_c^0\pi^+}$
4.17 ± 1	6.05 ± 0.02	0.70 ± 0.22	3.1 ± 1.1	8.0 ± 2.8	3.9 ± 0.6	4.3 ± 0.4

$g_{\omega\rho\pi}$	$g_{\Sigma_c^{*+}\Lambda_c^+\rho^0}$	$g_{\Sigma_c^{*0}\Sigma_c^+\rho^-}$	$g_{\Xi_c^0\Lambda_c^+K^{*-}}$	$g_{\Sigma_c^0\Lambda_c^+\rho^-}$	$g_{\Sigma_c^+\Sigma_c^0\rho^+}$
-10 ± 1	10 ± 1.8	5.77 ± 0.5	$\{4.6\pm1.5,6\pm2\}$	$\{2.6\pm0.9,16\pm5.3\}$	$\{4\pm1.3,27\pm9\}$

TABLE III. Branching ratios and decay asymmetry parameters for the short-distance dynamics dominated modes. The "CF," "SCS," and "DCS," represent CKM-favored, singly CKM-suppressed, and doubly CKM-suppressed processes, respectively, and $\mathcal{B}_{SD}$, $\mathcal{B}_{LD}$, and $\mathcal{B}_{\rm tot}$ denote the branching ratio of short-distance contribution, long-distance contribution, and the total one, respectively. The uncertainty is due to the variation of the phenomenological parameter $\eta = 0.3 \pm 0.05$.

Channels	$\mathcal{B}_{SD}[10^{-2}]$	$\mathcal{B}_{LD}[10^{-2}]$	$\mathcal{B}_{Tot}[10^{-2}]$	α_b	α_2	r_0	r_1	$H^{1/2}_{1/2,1}$	$H^{-1/2}_{-1/2,-1}$	$H^{-1/2}_{1/2,0}$	$H^{1/2}_{-1/2,0}$
$\Xi_{cc}^{++} \to \Xi_c^+ \rho^+$	11.8	$1.08^{+1.06}_{-0.63}$	$12.4^{+0.8}_{-0.4}$	$-0.58^{+0.02}_{-0.02}$	$-0.36^{+0.05}_{-0.04}$	$0.89^{+0.01}_{-0.01}$	$-0.47^{+0.04}_{-0.03}$	$0.11^{+0.01}_{-0.01}$	$0.0021^{+0.0013}_{-0.0008}$	$0.21^{+0.01}_{-0.01}$	$0.68^{+0.02}_{-0.02}$
$\Xi_{cc}^{++} \to \Xi_c'^+ \rho^+$	15.9	$0.201^{+0.169}_{-0.109}$	$18.9^{+1.1}_{-1.0}$	$-0.75^{+0.01}_{-0.01}$	$-0.100^{+0.009}_{-0.010}$	$0.51^{+0.00}_{-0.00}$	$-0.43^{+0.00}_{-0.00}$	$0.41^{+0.01}_{-0.01}$	$0.084^{+0.007}_{-0.006}$	$0.040^{+0.001}_{-0.000}$	$0.47^{+0.00}_{-0.00}$
$\Xi_{cc}^{++} \to \Sigma_c^+ \rho^+$	0.962	$0.0303^{+0.0280}_{-0.0173}$	$1.18^{+0.09}_{-0.07}$	$-0.82^{+0.04}_{-0.03}$	$-0.14^{+0.01}_{-0.01}$	$0.55^{+0.00}_{-0.00}$	$-0.48^{+0.01}_{-0.01}$	$0.39^{+0.01}_{-0.01}$	$0.052^{+0.011}_{-0.009}$	$0.036^{+0.006}_{-0.004}$	$0.52^{+0.00}_{-0.00}$
$\Xi_{cc}^{++} \to \Lambda_c^+ \rho^+$	0.901	$0.0931^{+0.0938}_{-0.0550}$	$0.850^{+0.025}_{-0.001}$	$-0.61^{+0.02}_{-0.00}$	$-0.42^{+0.02}_{-0.01}$	$0.88^{+0.02}_{-0.03}$	$-0.52^{+0.02}_{-0.01}$	$0.11^{+0.01}_{-0.01}$	$0.015^{+0.013}_{-0.008}$	$0.18^{+0.01}_{-0.00}$	$0.70^{+0.01}_{-0.03}$
$\Xi_{cc}^{++} \to \Xi_c'^+ K^{*+}$	0.770	$0.00552^{+0.00476}_{-0.00306}$	$0.793^{+0.012}_{-0.006}$	$-0.69^{+0.01}_{-0.01}$	$-0.0073^{+0.0153}_{-0.0172}$	$0.43^{+0.00}_{-0.00}$	$-0.35^{+0.01}_{-0.01}$	$0.45^{+0.00}_{-0.00}$	$0.11^{+0.00}_{-0.00}$	$0.042^{+0.005}_{-0.005}$	$0.39^{+0.01}_{-0.01}$
$\Xi_{cc}^{++} \to \Xi_c^+ K^{*+}$	0.496	$0.0301^{+0.0289}_{-0.0175}$	$0.717^{+0.099}_{-0.082}$	$-0.71^{+0.01}_{-0.01}$	$-0.40^{+0.02}_{-0.01}$	$0.82^{+0.01}_{-0.01}$	$-0.56^{+0.00}_{-0.00}$	$0.17^{+0.01}_{-0.01}$	$0.011^{+0.000}_{-0.000}$	$0.13^{+0.01}_{-0.01}$	$0.69^{+0.01}_{-0.01}$
$\Xi_{cc}^{++} \to \Lambda_c^+ K^{*+}$	0.0454	$0.00156^{+0.00163}_{-0.00093}$	$0.0592^{+0.0064}_{-0.0052}$	$-0.69^{+0.02}_{-0.02}$	$-0.42^{+0.01}_{-0.00}$	$0.86^{+0.01}_{-0.01}$	$-0.55^{+0.01}_{-0.01}$	$0.14^{+0.01}_{-0.01}$	$0.0019^{+0.0008}_{0.0001}$	$0.15^{+0.01}_{-0.01}$	$0.71^{+0.00}_{-0.00}$
$\Xi_{cc}^{++} \to \Sigma_c^+ K^{*+}$	0.0532	$0.000831^{+0.000720}_{-0.000484}$	$0.0515^{+0.0008}_{-0.0006}$	$-0.85^{+0.00}_{-0.00}$	$-0.042^{+0.025}_{-0.031}$	$0.49^{+0.01}_{-0.00}$	$-0.44^{+0.01}_{-0.01}$	$0.46^{+0.01}_{-0.01}$	$0.055^{+0.005}_{-0.003}$	$0.022^{+0.004}_{-0.004}$	$0.47^{+0.01}_{-0.01}$
$\Xi_{cc}^{+} \to \Xi_c^0 \rho^+$	2.13	$0.0822^{+0.0768}_{-0.0471}$	$2.78^{+0.31}_{-0.25}$	$-0.70^{+0.02}_{-0.03}$	$-0.48^{+0.01}_{-0.01}$	$0.88^{+0.01}_{-0.01}$	$-0.59^{+0.02}_{-0.02}$	$0.11^{+0.01}_{-0.01}$	$0.0088^{+0.0009}_{-0.0008}$	$0.14^{+0.01}_{-0.01}$	$0.73^{+0.00}_{-0.00}$
$\Xi_{cc}^{+} \to \Xi_c'^0 \rho^+$	2.91	$0.0253^{+0.0251}_{-0.0148}$	$2.54^{+0.13}_{-0.14}$	$-0.81^{+0.01}_{-0.01}$	$-0.10^{+0.01}_{-0.01}$	$0.52^{+0.00}_{-0.00}$	$-0.46^{+0.01}_{-0.01}$	$0.42^{+0.00}_{-0.00}$	$0.066^{+0.001}_{-0.002}$	$0.030^{+0.004}_{-0.004}$	$0.49^{+0.01}_{-0.01}$
$\Xi_{cc}^{+} \to \Sigma_c^0 \rho^+$	0.352	$0.00745^{+0.00705}_{-0.00429}$	$0.343^{+0.005}_{-0.006}$	$-0.89^{+0.00}_{-0.00}$	$-0.12^{+0.01}_{-0.01}$	$0.55^{+0.00}_{-0.00}$	$-0.50^{+0.00}_{-0.01}$	$0.42^{+0.00}_{-0.00}$	$0.034^{+0.001}_{-0.001}$	$0.022^{+0.002}_{-0.003}$	$0.53^{+0.00}_{-0.00}$
$\Xi_{cc}^{+} \to \Xi_c'^0 K^{*+}$	0.140	$0.00686^{+0.00694}_{-0.00404}$	$0.111^{+0.010}_{-0.011}$	$-0.70^{+0.01}_{-0.01}$	$0.0022^{+0.0145}_{-0.0168}$	$0.43^{+0.00}_{-0.00}$	$-0.35^{+0.01}_{-0.01}$	$0.46^{+0.00}_{-0.00}$	$0.11^{+0.00}_{-0.00}$	$0.042^{+0.005}_{-0.005}$	$0.39^{+0.01}_{-0.01}$
$\Xi_{cc}^{+} \to \Xi_c^0 K^{*+}$	0.0893	$0.000921^{+0.000953}_{-0.000546}$	$0.0802^{+0.0031}_{-0.0032}$	$-0.61^{+0.04}_{-0.03}$	$-0.41^{+0.02}_{-0.01}$	$0.88^{+0.01}_{-0.01}$	$-0.51^{+0.03}_{-0.02}$	$0.11^{+0.01}_{-0.01}$	$0.012^{+0.001}_{-0.001}$	$0.18^{+0.02}_{-0.02}$	$0.69^{+0.01}_{-0.01}$
$\Xi_{cc}^{+} \to \Sigma_c^0 K^{*+}$	0.364	⋯	0.364	−0.84	0.020	0.48	−0.41	0.48	0.048	0.034	0.44
$\Omega_{cc}^{+} \to \Omega_c^0 \rho^+$	22.5	⋯	22.5	−0.79	−0.073	0.51	−0.43	0.42	0.066	0.039	0.47
$\Omega_{cc}^{+} \to \Xi_c^0 \rho^+$	0.661	$0.0186^{+0.0178}_{-0.0108}$	$0.786^{+0.053}_{-0.045}$	$-0.76^{+0.05}_{-0.05}$	$-0.30^{+0.02}_{-0.02}$	$0.75^{+0.01}_{-0.01}$	$-0.53^{+0.03}_{-0.04}$	$0.24^{+0.01}_{-0.01}$	$0.010^{+0.004}_{-0.003}$	$0.11^{+0.02}_{-0.02}$	$0.64^{+0.01}_{-0.01}$
$\Omega_{cc}^{+} \to \Xi_c'^0 \rho^+$	0.610	$0.0271^{+0.0265}_{-0.0158}$	$0.804^{+0.086}_{-0.071}$	$-0.66^{+0.06}_{-0.05}$	$-0.19^{+0.03}_{-0.02}$	$0.56^{+0.01}_{-0.01}$	$-0.43^{+0.05}_{-0.04}$	$0.34^{+0.00}_{-0.01}$	$0.10^{+0.01}_{-0.01}$	$0.066^{+0.018}_{-0.014}$	$0.49^{+0.02}_{-0.03}$
$\Omega_{cc}^{+} \to \Omega_c^0 K^{*+}$	1.09	$0.00509^{+0.00484}_{-0.00294}$	$1.10^{+0.02}_{-0.01}$	$-0.71^{+0.01}_{-0.02}$	$0.031^{+0.000}_{-0.001}$	$0.43^{+0.00}_{-0.00}$	$-0.34^{+0.01}_{-0.01}$	$0.47^{+0.00}_{-0.00}$	$0.100^{+0.004}_{-0.004}$	$0.047^{+0.003}_{-0.003}$	$0.38^{+0.00}_{-0.00}$
$\Omega_{cc}^{+} \to \Xi_c^0 K^{*+}$	0.0338	$0.0000687^{+0.0000647}_{-0.0000389}$	$0.0359^{+0.0008}_{-0.0007}$	$-0.68^{+0.01}_{-0.02}$	$-0.20^{+0.01}_{-0.01}$	$0.71^{+0.00}_{-0.00}$	$-0.44^{+0.01}_{-0.01}$	$0.26^{+0.00}_{-0.00}$	$0.024^{+0.000}_{-0.001}$	$0.13^{+0.01}_{-0.01}$	$0.58^{+0.01}_{-0.00}$
$\Omega_{cc}^{+} \to \Xi_c'^0 K^{*+}$	0.0337	$0.000118^{+0.000112}_{-0.000073}$	$0.0327^{+0.0005}_{-0.0004}$	$-0.76^{+0.02}_{-0.02}$	$-0.13^{+0.01}_{-0.01}$	$0.51^{+0.00}_{-0.00}$	$-0.44^{+0.01}_{-0.01}$	$0.40^{+0.01}_{-0.01}$	$0.089^{+0.008}_{-0.008}$	$0.033^{+0.003}_{-0.003}$	$0.48^{+0.01}_{-0.01}$

TABLE IV. Branching ratios and decay asymmetry parameters for the long-distance dominated Cabibbo-favored (λ_{sd}) modes.

Channels	$\mathcal{B}_{SD}[10^{-5}]$	$\mathcal{B}_{LD}[10^{-2}]$	$\mathcal{B}_{Tot}[10^{-2}]$	α_b	α_2	r_0	r_1	$H^{1/2}_{1/2,1}$	$H^{-1/2}_{1/2,-1}$	$H^{-1/2}_{1/2,0}$	$H^{1/2}_{1/2,0}$
$\Xi_{cc}^{++}\to\Sigma_c^{++}\bar{K}^{*0}$	10.9	$3.27^{+3.31}_{-1.91}$	$3.54^{+3.44}_{-2.01}$	$-0.23^{+0.04}_{-0.04}$	$-0.023^{+0.003}_{-0.005}$	$0.35^{+0.00}_{-0.00}$	$-0.13^{+0.02}_{-0.02}$	$0.38^{+0.01}_{-0.01}$	$0.27^{+0.01}_{-0.01}$	$0.11^{+0.01}_{-0.01}$	$0.24^{+0.01}_{-0.01}$
$\Xi_{cc}^{+}\to\Omega_c^{0}K^{*+}$	...	$0.0364^{+0.0355}_{-0.0213}$	$0.0364^{+0.0355}_{-0.0213}$	$0.028^{+0.018}_{-0.015}$	$0.100^{+0.002}_{-0.003}$	$0.35^{+0.00}_{-0.00}$	$0.064^{+0.008}_{-0.006}$	$0.34^{+0.00}_{-0.01}$	$0.31^{+0.00}_{-0.00}$	$0.21^{+0.00}_{-0.00}$	$0.15^{+0.00}_{-0.00}$
$\Xi_{cc}^{+}\to\Sigma_c^{+}\bar{K}^{*0}$	0.999	$0.919^{+0.923}_{-0.539}$	$0.951^{+0.938}_{-0.551}$	$-0.26^{+0.04}_{-0.03}$	$-0.036^{+0.009}_{-0.010}$	$0.32^{+0.00}_{-0.00}$	$-0.15^{+0.01}_{-0.01}$	$0.40^{+0.01}_{-0.01}$	$0.29^{+0.01}_{-0.01}$	$0.085^{+0.007}_{-0.006}$	$0.23^{+0.01}_{-0.01}$
$\Xi_{cc}^{+}\to\Lambda_c^{+}\bar{K}^{*0}$	0.493	$1.22^{+1.27}_{-0.73}$	$1.23^{+1.28}_{-0.73}$	$0.28^{+0.03}_{-0.04}$	$-0.14^{+0.02}_{-0.02}$	$0.21^{+0.00}_{-0.00}$	$0.070^{+0.007}_{-0.009}$	$0.29^{+0.02}_{-0.01}$	$0.50^{+0.01}_{-0.01}$	$0.14^{+0.00}_{-0.01}$	$0.069^{+0.003}_{-0.003}$
$\Xi_{cc}^{+}\to\Sigma_c^{++}K^{*-}$	...	$0.433^{+0.417}_{-0.251}$	$0.433^{+0.417}_{-0.251}$	$-0.45^{+0.04}_{-0.04}$	$-0.050^{+0.010}_{-0.010}$	$0.34^{+0.00}_{-0.00}$	$-0.25^{+0.01}_{-0.01}$	$0.43^{+0.01}_{-0.01}$	$0.23^{+0.01}_{-0.01}$	$0.044^{+0.006}_{-0.007}$	$0.29^{+0.01}_{-0.01}$
$\Xi_{cc}^{+}\to\Xi_c^{+}\rho^{0}$	...	$0.0475^{+0.0448}_{-0.0272}$	$0.0475^{+0.0448}_{-0.0272}$	$0.19^{+0.02}_{-0.02}$	$-0.21^{+0.02}_{-0.02}$	$0.45^{+0.01}_{-0.01}$	$-0.0070^{+0.0213}_{-0.0197}$	$0.17^{+0.00}_{-0.00}$	$0.38^{+0.00}_{-0.00}$	$0.22^{+0.01}_{-0.01}$	$0.23^{+0.01}_{-0.01}$
$\Xi_{cc}^{+}\to\Xi_c^{\prime+}\phi$	...	$0.0422^{+0.0410}_{-0.0245}$	$0.0422^{+0.0410}_{-0.0245}$	$-0.16^{+0.01}_{-0.01}$	$-0.017^{+0.004}_{-0.004}$	$0.33^{+0.00}_{-0.00}$	$-0.090^{+0.004}_{-0.004}$	$0.37^{+0.00}_{-0.00}$	$0.30^{+0.00}_{-0.00}$	$0.12^{+0.00}_{-0.00}$	$0.21^{+0.00}_{-0.00}$
$\Xi_{cc}^{+}\to\Xi_c^{+}\omega$	...	$0.0468^{+0.0440}_{-0.0268}$	$0.0468^{+0.0440}_{-0.0268}$	$0.19^{+0.02}_{-0.02}$	$-0.21^{+0.02}_{-0.02}$	$0.45^{+0.01}_{-0.01}$	$-0.0061^{+0.0208}_{-0.0191}$	$0.18^{+0.00}_{-0.00}$	$0.38^{+0.00}_{-0.00}$	$0.22^{+0.01}_{-0.01}$	$0.23^{+0.01}_{-0.01}$
$\Xi_{cc}^{+}\to\Xi_c^{+}\rho^{0}$	...	$0.0801^{+0.0797}_{-0.0470}$	$0.0801^{+0.0797}_{-0.0470}$	$-0.55^{+0.00}_{-0.00}$	$-0.24^{+0.01}_{-0.01}$	$0.69^{+0.00}_{-0.00}$	$-0.39^{+0.01}_{-0.01}$	$0.23^{+0.00}_{-0.00}$	$0.077^{+0.001}_{-0.000}$	$0.15^{+0.00}_{-0.00}$	$0.54^{+0.01}_{-0.01}$
$\Xi_{cc}^{+}\to\Xi_c^{+}\phi$	...	$0.108^{+0.111}_{-0.064}$	$0.108^{+0.111}_{-0.064}$	$0.35^{+0.01}_{-0.01}$	$-0.19^{+0.01}_{-0.01}$	$0.29^{+0.00}_{-0.00}$	$0.077^{+0.002}_{-0.002}$	$0.22^{+0.01}_{-0.01}$	$0.49^{+0.01}_{-0.01}$	$0.18^{+0.00}_{-0.00}$	$0.11^{+0.00}_{-0.00}$
$\Xi_{cc}^{+}\to\Xi_c^{+}\omega$	...	$0.0779^{+0.0775}_{-0.0457}$	$0.0779^{+0.0775}_{-0.0457}$	$-0.55^{+0.00}_{-0.00}$	$-0.24^{+0.01}_{-0.01}$	$0.69^{+0.00}_{-0.00}$	$-0.39^{+0.01}_{-0.01}$	$0.23^{+0.00}_{-0.00}$	$0.078^{+0.001}_{-0.001}$	$0.15^{+0.00}_{-0.00}$	$0.54^{+0.01}_{-0.01}$
$\Omega_{cc}^{+}\to\Xi_c^{\prime+}\bar{K}^{*0}$	3.68	$2.29^{+2.30}_{-1.34}$	$2.40^{+2.36}_{-1.39}$	$-0.044^{+0.038}_{-0.036}$	$-0.042^{+0.003}_{-0.003}$	$0.37^{+0.00}_{-0.00}$	$-0.043^{+0.021}_{-0.019}$	$0.31^{+0.01}_{-0.01}$	$0.31^{+0.01}_{-0.01}$	$0.16^{+0.01}_{-0.01}$	$0.21^{+0.01}_{-0.01}$
$\Omega_{cc}^{+}\to\Xi_c^{+}\bar{K}^{*0}$	2.24	$2.04^{+2.10}_{-1.20}$	$2.14^{+2.14}_{-1.24}$	$0.066^{+0.034}_{-0.029}$	$-0.073^{+0.006}_{-0.006}$	$0.34^{+0.00}_{-0.00}$	$-0.0034^{+0.0140}_{-0.0119}$	$0.29^{+0.01}_{-0.01}$	$0.36^{+0.01}_{-0.01}$	$0.17^{+0.01}_{-0.01}$	$0.17^{+0.01}_{-0.01}$

and light mesons, are taken from the Particle Data Group [70].

(ii) Decay constants of pseudoscalar and vector mesons are obtained from Refs. [70–72] and are summarized in Table II. The form factors for the transition $\mathcal{B}_{cc}\to\mathcal{B}_c$ are taken from the light-front quark model calculations [73], which have been successfully used to predict the discovery channel of Ξ_{cc}^{++} in Ref. [22].

(iii) The nonperturbative strong coupling constants are crucial for our calculation. Most of them are taken from theoretical works [55,58–63,74–76], which have been listed in Table II. The remaining couplings are inferred from $SU(3)$ flavor symmetry.

B. Branching ratios and decay asymmetry parameters

We present our numerical results for the branching ratios, helicity amplitudes, and four decay asymmetry parameters for all 49 decay channels. The results are summarized in four tables: Table III for short-distance dominated modes, Table IV for long-distance dominated Cabibbo-favored modes, Table V for singly Cabibbo-suppressed modes, and Table VI for doubly Cabibbo-suppressed modes. In our previous study of the $\mathcal{B}_{cc}\to\mathcal{B}_cP$ process [64], the model parameter η in the form factor Eq. (22) was determined to be $\eta=0.9\pm0.2$ by fitting to the experimental data $\mathcal{B}(\Xi_{cc}^{++}\to\Xi_c'^{+}\pi^{+})/\mathcal{B}(\Xi_{cc}^{++}\to\Xi_c^{+}\pi^{+})=(1.41\pm0.17\pm0.10)$ [11]. Since the process $\mathcal{B}_{cc}\to\mathcal{B}_cV$ also involves a charm quark decay, we adopt the same value as in our previous work [64]. We also analyze the dependence of the branching ratios and decay parameters on the model parameter η, as shown in Fig. 3.

The main features of our results are as follows:

(i) The branching ratios for the short-distance dominated modes are typically at the level of 10^{-4} to 10^{-2}.

(ii) The long-distance contributions are significant for the nonfactorizable channels, often comparable to or even larger than the short-distance contributions.

(iii) The decay asymmetry parameters α_b, α_2, r_0, and r_1 show rich patterns across different channels, reflecting the underlying dynamics of the decay processes.

(iv) One can see that the branching ratios are moderately sensitive to this parameter, while the decay asymmetry parameters show relatively smaller sensitivity. This feature is advantageous because the asymmetry parameters can be measured more precisely in experiments, providing stronger constraints on the theoretical framework.

C. *CP* violations

We present our numerical results for *CP* violations. The results are summarized in two tables: Table VII for short-distance dominated modes and Table VIII for singly Cabibbo-suppressed modes. We also analyze the

TABLE V. Same as Table IV but for the long-distance dominated singly Cabibbo-suppressed modes.

Channels	$\mathcal{B}_{SD}[10^{-7}]$	$\mathcal{B}_{LD}[10^{-3}]$	$\mathcal{B}_{Tot}[10^{-3}]$	α_b	α_2	r_0	r_1	$H^{1/2}_{1/2,1}$	$H^{-1/2}_{-1/2,-1}$	$H^{-1/2}_{1/2,0}$	$H^{1/2}_{-1/2,0}$
$\Xi_{cc}^{++} \to \Sigma_c^{++}\rho^0$	50.0	$1.24^{+1.30}_{-0.74}$	$1.14^{+1.25}_{-0.70}$	$-0.25^{+0.05}_{-0.05}$	$-0.097^{+0.013}_{-0.011}$	$0.49^{+0.00}_{-0.00}$	$-0.17^{+0.03}_{-0.03}$	$0.29^{+0.01}_{-0.01}$	$0.21^{+0.01}_{-0.01}$	$0.16^{+0.02}_{-0.02}$	$0.33^{+0.01}_{-0.02}$
$\Xi_{cc}^{++} \to \Sigma_c^{++}\phi$	69.9	$1.64^{+1.64}_{-0.97}$	$1.81^{+1.71}_{-1.03}$	$-0.14^{+0.03}_{-0.04}$	$-0.025^{+0.011}_{-0.007}$	$0.33^{+0.00}_{-0.00}$	$-0.084^{+0.013}_{-0.014}$	$0.36^{+0.01}_{-0.01}$	$0.30^{+0.01}_{-0.01}$	$0.12^{+0.01}_{-0.01}$	$0.21^{+0.01}_{-0.01}$
$\Xi_{cc}^{++} \to \Sigma_c^{++}\omega$	50.5	$0.422^{+0.386}_{-0.238}$	$0.381^{+0.366}_{-0.222}$	$-0.64^{+0.03}_{-0.02}$	$0.059^{+0.014}_{-0.014}$	$0.32^{+0.00}_{-0.01}$	$-0.29^{+0.01}_{-0.01}$	$0.52^{+0.01}_{-0.01}$	$0.17^{+0.01}_{-0.01}$	$0.013^{+0.00}_{-0.00}$	$0.30^{+0.01}_{-0.01}$
$\Xi_{cc}^{+} \to \Sigma_c^{+}\rho^0$	4.59	$0.538^{+0.533}_{-0.315}$	$0.522^{+0.525}_{-0.308}$	$-0.27^{+0.04}_{-0.03}$	$-0.11^{+0.01}_{-0.01}$	$0.44^{+0.00}_{-0.00}$	$-0.19^{+0.02}_{-0.02}$	$0.32^{+0.01}_{-0.01}$	$0.24^{+0.01}_{-0.00}$	$0.13^{+0.01}_{-0.01}$	$0.32^{+0.01}_{-0.01}$
$\Xi_{cc}^{+} \to \Sigma_c^{+}\phi$	6.42	$0.151^{+0.150}_{-0.089}$	$0.167^{+0.157}_{-0.095}$	$-0.14^{+0.03}_{-0.04}$	$-0.026^{+0.011}_{-0.007}$	$0.33^{+0.00}_{-0.00}$	$-0.083^{+0.013}_{-0.014}$	$0.36^{+0.01}_{-0.01}$	$0.30^{+0.01}_{-0.01}$	$0.13^{+0.01}_{-0.01}$	$0.21^{+0.01}_{-0.01}$
$\Xi_{cc}^{+} \to \Sigma_c^{+}\omega$	4.63	$0.294^{+0.279}_{-0.169}$	$0.289^{+0.276}_{-0.167}$	$-0.41^{+0.03}_{-0.03}$	$-0.11^{+0.00}_{-0.00}$	$0.39^{+0.00}_{-0.00}$	$-0.26^{+0.01}_{-0.02}$	$0.38^{+0.01}_{-0.01}$	$0.23^{+0.01}_{-0.01}$	$0.063^{+0.007}_{-0.008}$	$0.32^{+0.01}_{-0.01}$
$\Xi_{cc}^{+} \to \Lambda_c^{+}\rho^0$	2.59	$0.468^{+0.488}_{-0.279}$	$0.464^{+0.486}_{-0.278}$	$0.50^{+0.02}_{-0.02}$	$-0.045^{+0.015}_{-0.011}$	$0.31^{+0.00}_{-0.00}$	$0.23^{+0.00}_{-0.00}$	$0.21^{+0.01}_{-0.01}$	$0.48^{+0.01}_{-0.01}$	$0.27^{+0.00}_{-0.00}$	$0.044^{+0.001}_{-0.001}$
$\Xi_{cc}^{+} \to \Lambda_c^{+}\phi$	2.98	$0.239^{+0.252}_{-0.144}$	$0.244^{+0.254}_{-0.145}$	$0.23^{+0.03}_{-0.04}$	$-0.033^{+0.016}_{-0.015}$	$0.28^{+0.00}_{-0.00}$	$0.096^{+0.006}_{-0.010}$	$0.30^{+0.01}_{-0.01}$	$0.42^{+0.01}_{-0.01}$	$0.19^{+0.00}_{-0.01}$	$0.092^{+0.003}_{-0.001}$
$\Xi_{cc}^{+} \to \Lambda_c^{+}\omega$	2.59	$0.211^{+0.216}_{-0.125}$	$0.204^{+0.212}_{-0.122}$	$0.14^{+0.02}_{-0.02}$	$-0.56^{+0.01}_{-0.01}$	$0.28^{+0.00}_{-0.00}$	$-0.21^{+0.01}_{-0.00}$	$0.18^{+0.01}_{-0.01}$	$0.54^{+0.01}_{-0.01}$	$0.034^{+0.002}_{-0.001}$	$0.24^{+0.00}_{-0.00}$
$\Xi_{cc}^{+} \to \Sigma_c^{++}\rho^-$	⋯	$0.251^{+0.244}_{-0.146}$	$0.251^{+0.244}_{-0.146}$	$-0.40^{+0.02}_{-0.02}$	$-0.13^{+0.01}_{-0.01}$	$0.35^{+0.00}_{-0.00}$	$-0.26^{+0.01}_{-0.00}$	$0.39^{+0.01}_{-0.01}$	$0.26^{+0.01}_{-0.01}$	$0.045^{+0.004}_{-0.003}$	$0.31^{+0.00}_{-0.00}$
$\Xi_{cc}^{+} \to \Xi_c'^{+}K^{*0}$	⋯	$0.0117^{+0.0122}_{-0.0070}$	$0.0117^{+0.0122}_{-0.0070}$	$-0.14^{+0.02}_{-0.04}$	$0.26^{+0.03}_{-0.02}$	$0.37^{+0.03}_{-0.03}$	$0.058^{+0.025}_{-0.029}$	$0.41^{+0.02}_{-0.01}$	$0.22^{+0.01}_{-0.02}$	$0.21^{+0.03}_{-0.03}$	$0.16^{+0.00}_{-0.00}$
$\Xi_{cc}^{+} \to \Xi_c^{+}K^{*0}$	⋯	$0.0515^{+0.0511}_{-0.0307}$	$0.0515^{+0.0511}_{-0.0307}$	$-0.14^{+0.05}_{-0.08}$	$-0.20^{+0.02}_{-0.01}$	$0.33^{+0.03}_{-0.02}$	$-0.17^{+0.02}_{-0.03}$	$0.32^{+0.01}_{-0.01}$	$0.35^{+0.02}_{-0.04}$	$0.083^{+0.002}_{-0.002}$	$0.25^{+0.03}_{-0.02}$
$\Omega_{cc}^{+} \to \Sigma_c^{+}\bar{K}^{*0}$	⋯	$0.967^{+1.010}_{-0.578}$	$0.967^{+1.010}_{-0.578}$	$0.40^{+0.02}_{-0.02}$	$-0.39^{+0.01}_{-0.01}$	$0.28^{+0.00}_{-0.00}$	$0.0065^{+0.0047}_{-0.0047}$	$0.16^{+0.00}_{-0.01}$	$0.56^{+0.01}_{-0.01}$	$0.14^{+0.00}_{-0.00}$	$0.14^{+0.00}_{-0.00}$
$\Omega_{cc}^{+} \to \Lambda_c^{+}\bar{K}^{*0}$	⋯	$1.49^{+1.54}_{-0.88}$	$1.49^{+1.54}_{-0.88}$	$0.23^{+0.05}_{-0.05}$	$-0.72^{+0.01}_{-0.01}$	$0.31^{+0.01}_{-0.01}$	$-0.25^{+0.02}_{-0.02}$	$0.11^{+0.01}_{-0.01}$	$0.58^{+0.02}_{-0.02}$	$0.030^{+0.004}_{-0.004}$	$0.28^{+0.01}_{-0.01}$
$\Omega_{cc}^{+} \to \Sigma_c^{++}K^{*-}$	⋯	$1.04^{+1.06}_{-0.62}$	$1.04^{+1.06}_{-0.62}$	$0.067^{+0.036}_{-0.043}$	$-0.44^{+0.00}_{-0.00}$	$0.31^{+0.00}_{-0.00}$	$-0.19^{+0.02}_{-0.02}$	$0.22^{+0.01}_{-0.01}$	$0.47^{+0.01}_{-0.01}$	$0.059^{+0.006}_{-0.008}$	$0.25^{+0.01}_{-0.01}$
$\Omega_{cc}^{+} \to \Xi_c'^{+}\rho^0$	⋯	$0.981^{+0.990}_{-0.578}$	$0.981^{+0.990}_{-0.578}$	$-0.15^{+0.05}_{-0.05}$	$-0.037^{+0.010}_{-0.006}$	$0.47^{+0.00}_{-0.00}$	$-0.093^{+0.030}_{-0.027}$	$0.29^{+0.01}_{-0.01}$	$0.24^{+0.01}_{-0.01}$	$0.19^{+0.02}_{-0.02}$	$0.28^{+0.01}_{-0.01}$
$\Omega_{cc}^{+} \to \Xi_c'^{+}\phi$	⋯	$2.47^{+2.44}_{-1.45}$	$2.47^{+2.44}_{-1.45}$	$0.048^{+0.020}_{-0.021}$	$-0.12^{+0.00}_{-0.00}$	$0.33^{+0.00}_{-0.00}$	$-0.037^{+0.010}_{-0.009}$	$0.29^{+0.01}_{-0.01}$	$0.38^{+0.01}_{-0.01}$	$0.15^{+0.00}_{-0.00}$	$0.18^{+0.00}_{-0.00}$
$\Omega_{cc}^{+} \to \Xi_c'^{+}\omega$	⋯	$0.0880^{+0.0766}_{-0.0488}$	$0.0880^{+0.0766}_{-0.0488}$	$-0.53^{+0.05}_{-0.04}$	$0.45^{+0.02}_{-0.02}$	$0.50^{+0.01}_{-0.01}$	$-0.040^{+0.035}_{-0.031}$	$0.50^{+0.01}_{-0.01}$	$0.0055^{+0.0022}_{-0.0017}$	$0.23^{+0.02}_{-0.02}$	$0.27^{+0.01}_{-0.01}$
$\Omega_{cc}^{+} \to \Xi_c^{+}\rho^0$	⋯	$0.785^{+0.831}_{-0.472}$	$0.785^{+0.831}_{-0.472}$	$0.61^{+0.02}_{-0.01}$	$-0.22^{+0.00}_{-0.00}$	$0.35^{+0.01}_{-0.01}$	$0.20^{+0.01}_{-0.01}$	$0.12^{+0.01}_{-0.01}$	$0.53^{+0.00}_{-0.00}$	$0.27^{+0.01}_{-0.01}$	$0.078^{+0.001}_{-0.001}$
$\Omega_{cc}^{+} \to \Xi_c^{+}\phi$	⋯	$3.09^{+3.16}_{-1.84}$	$3.09^{+3.16}_{-1.84}$	$0.26^{+0.02}_{-0.02}$	$-0.26^{+0.01}_{-0.01}$	$0.28^{+0.00}_{-0.00}$	$-0.013^{+0.0066}_{-0.0064}$	$0.23^{+0.01}_{-0.01}$	$0.49^{+0.01}_{-0.01}$	$0.14^{+0.00}_{-0.00}$	$0.14^{+0.00}_{-0.00}$
$\Omega_{cc}^{+} \to \Xi_c^{+}\omega$	⋯	$0.165^{+0.162}_{-0.097}$	$0.165^{+0.162}_{-0.097}$	$0.030^{+0.042}_{-0.043}$	$-0.47^{+0.03}_{-0.03}$	$0.25^{+0.01}_{-0.01}$	$-0.22^{+0.01}_{-0.01}$	$0.25^{+0.01}_{-0.01}$	$0.50^{+0.02}_{-0.02}$	$0.016^{+0.002}_{-0.002}$	$0.24^{+0.01}_{-0.01}$

TABLE VI. Same as Table IV but for the long-distance dominated doubly Cabibbo-suppressed modes.

Channels	$\mathcal{B}_{SD}[10^{-8}]$	$\mathcal{B}_{LD}[10^{-5}]$	$\mathcal{B}_{Tot}[10^{-5}]$	a_b	a_2	r_0	r_1	$H^{1/2}_{1/2,1}$	$H^{-1/2}_{1/2,-1}$	$H^{-1/2}_{1/2,0}$	$H^{1/2}_{1/2,0}$
$\Xi_{cc}^{++}\to\Sigma_c^{++}K^{*0}$	31.0	$7.14^{+7.37}_{-4.06}$	$7.85^{+7.67}_{-4.30}$	$-0.43^{+0.06}_{-0.03}$	$0.011^{+0.005}_{-0.023}$	$0.39^{+0.00}_{-0.00}$	$-0.21^{+0.02}_{-0.01}$	$0.42^{+0.01}_{-0.02}$	$0.20^{+0.02}_{-0.01}$	$0.089^{+0.007}_{-0.007}$	$0.30^{+0.01}_{-0.01}$
$\Xi_{cc}^{+}\to\Sigma_c^{+}K^{*0}$	2.84	$1.68^{+1.68}_{-0.96}$	$1.76^{+1.71}_{-0.99}$	$-0.23^{+0.05}_{-0.03}$	$0.017^{+0.000}_{-0.018}$	$0.41^{+0.00}_{-0.00}$	$-0.11^{+0.01}_{-0.01}$	$0.36^{+0.01}_{-0.02}$	$0.23^{+0.02}_{-0.01}$	$0.15^{+0.01}_{-0.01}$	$0.26^{+0.01}_{-0.01}$
$\Xi_{cc}^{+}\to\Lambda_c^{+}K^{*0}$	1.40	$1.56^{+1.61}_{-0.91}$	$1.60^{+1.63}_{-0.92}$	$-0.060^{+0.052}_{-0.025}$	$-0.036^{+0.012}_{-0.016}$	$0.35^{+0.00}_{-0.00}$	$-0.048^{+0.018}_{-0.019}$	$0.33^{+0.00}_{-0.02}$	$0.32^{+0.02}_{-0.00}$	$0.15^{+0.01}_{-0.01}$	$0.20^{+0.01}_{-0.01}$
$\Omega_{cc}^{+}\to\Sigma_c^{+}\rho^{0}$	...	$0.939^{+0.921}_{-0.549}$	$0.939^{+0.921}_{-0.549}$	$-0.37^{+0.02}_{-0.02}$	$-0.30^{+0.01}_{-0.01}$	$0.50^{+0.00}_{-0.00}$	$-0.33^{+0.01}_{-0.01}$	$0.27^{+0.01}_{-0.01}$	$0.23^{+0.01}_{-0.01}$	$0.086^{+0.002}_{-0.003}$	$0.42^{+0.00}_{-0.00}$
$\Omega_{cc}^{+}\to\Sigma_c^{+}\phi$	...	$0.786^{+0.731}_{-0.450}$	$0.786^{+0.731}_{-0.450}$	$0.038^{+0.001}_{-0.001}$	$0.081^{+0.013}_{-0.012}$	$0.42^{+0.00}_{-0.00}$	$0.059^{+0.006}_{-0.006}$	$0.30^{+0.00}_{-0.00}$	$0.28^{+0.01}_{-0.01}$	$0.24^{+0.01}_{-0.01}$	$0.18^{+0.00}_{-0.00}$
$\Omega_{cc}^{+}\to\Sigma_c^{+}\omega$	...	$2.92^{+2.97}_{-1.73}$	$2.92^{+2.97}_{-1.73}$	$0.12^{+0.02}_{-0.02}$	$-0.27^{+0.01}_{-0.01}$	$0.34^{+0.00}_{-0.00}$	$-0.078^{+0.007}_{-0.008}$	$0.23^{+0.01}_{-0.01}$	$0.43^{+0.01}_{-0.01}$	$0.13^{+0.00}_{-0.00}$	$0.21^{+0.00}_{-0.00}$
$\Omega_{cc}^{+}\to\Lambda_c^{+}\rho^{0}$	...	$1.78^{+1.85}_{-1.06}$	$1.78^{+1.85}_{-1.06}$	$0.68^{+0.02}_{-0.02}$	$-0.58^{+0.02}_{-0.02}$	$0.14^{+0.01}_{-0.01}$	$0.050^{+0.000}_{-0.001}$	$0.11^{+0.00}_{-0.00}$	$0.74^{+0.01}_{-0.01}$	$0.096^{+0.005}_{-0.005}$	$0.046^{+0.004}_{-0.004}$
$\Omega_{cc}^{+}\to\Lambda_c^{+}\phi$	...	$1.37^{+1.35}_{-0.80}$	$1.37^{+1.35}_{-0.80}$	$-0.77^{+0.01}_{-0.01}$	$0.16^{+0.01}_{-0.01}$	$0.39^{+0.01}_{-0.01}$	$-0.30^{+0.01}_{-0.01}$	$0.53^{+0.00}_{-0.00}$	$0.071^{+0.003}_{-0.002}$	$0.045^{+0.002}_{-0.002}$	$0.35^{+0.01}_{-0.01}$
$\Omega_{cc}^{+}\to\Lambda_c^{+}\omega$	...	$2.94^{+3.00}_{-1.74}$	$2.94^{+3.00}_{-1.74}$	$-0.020^{+0.041}_{-0.041}$	$-0.43^{+0.02}_{-0.02}$	$0.32^{+0.00}_{-0.00}$	$-0.23^{+0.01}_{-0.01}$	$0.24^{+0.01}_{-0.01}$	$0.44^{+0.02}_{-0.02}$	$0.046^{+0.004}_{-0.004}$	$0.27^{+0.01}_{-0.01}$
$\Omega_{cc}^{+}\to\Sigma_c^{0}\rho^{+}$	...	$1.32^{+1.36}_{-0.78}$	$1.32^{+1.36}_{-0.78}$	$0.50^{+0.00}_{-0.00}$	$0.062^{+0.007}_{-0.006}$	$0.55^{+0.00}_{-0.00}$	$0.28^{+0.00}_{-0.00}$	$0.11^{+0.00}_{-0.00}$	$0.33^{+0.00}_{-0.00}$	$0.42^{+0.00}_{-0.00}$	$0.14^{+0.00}_{-0.00}$
$\Omega_{cc}^{+}\to\Sigma_c^{++}\rho^{-}$	...	$6.44^{+6.46}_{-3.80}$	$6.44^{+6.46}_{-3.80}$	$-0.10^{+0.03}_{-0.03}$	$-0.35^{+0.01}_{-0.01}$	$0.35^{+0.00}_{-0.00}$	$-0.23^{+0.01}_{-0.01}$	$0.26^{+0.01}_{-0.01}$	$0.39^{+0.01}_{-0.01}$	$0.059^{+0.004}_{-0.004}$	$0.29^{+0.01}_{-0.00}$
$\Omega_{cc}^{+}\to\Xi_c^{\prime+}K^{*0}$	10.5	$8.17^{+8.33}_{-4.72}$	$8.53^{+8.50}_{-4.85}$	$-0.24^{+0.04}_{-0.03}$	$-0.14^{+0.01}_{-0.02}$	$0.35^{+0.00}_{-0.00}$	$-0.19^{+0.01}_{-0.01}$	$0.35^{+0.01}_{-0.01}$	$0.30^{+0.02}_{-0.01}$	$0.080^{+0.005}_{-0.004}$	$0.27^{+0.00}_{-0.01}$
$\Omega_{cc}^{+}\to\Xi_c^{+}K^{*0}$	6.37	$11.9^{+12.6}_{-7.1}$	$12.2^{+12.7}_{-7.2}$	$0.59^{+0.01}_{-0.01}$	$-0.41^{+0.01}_{-0.02}$	$0.21^{+0.00}_{-0.00}$	$0.093^{+0.002}_{-0.002}$	$0.14^{+0.01}_{-0.01}$	$0.65^{+0.01}_{-0.01}$	$0.15^{+0.00}_{-0.00}$	$0.059^{+0.000}_{-0.000}$

dependence of CP violations on the model parameter η, as shown in Fig. 4.

The main features of our results are as follows:

(i) The CP violations for the short-distance dominated modes are typically at the level of 10^{-5} to 10^{-4}, and the ones of singly Cabibbo-suppressed modes are at the level of 10^{-7} to 10^{-3}.

(ii) One can see that the CP violations are relatively smaller, sensitive to this parameter η.

D. Analysis of Ω_{cc}

As shown in Tables III and IV, there are three decays, all of which have large decay branching ratios,

$$\mathcal{B}(\Omega_{cc}^{+}\to\Omega_c^{0}\rho^{+}) = 22.52\%, \tag{37}$$

$$\mathcal{B}(\Omega_{cc}^{+}\to\Xi_c^{\prime+}\bar{K}^{*0}) = (2.40^{+2.36}_{-1.39})\%, \tag{38}$$

$$\mathcal{B}(\Omega_{cc}^{+}\to\Xi_c^{+}\bar{K}^{*0}) = (2.14^{+2.14}_{-1.24})\%. \tag{39}$$

In this work, the value of the lifetime of Ω_{cc} is taken as 176 fs [69]. In Ref. [47], the decay widths of these decays of Ω_{cc} have been calculated and, in order to compare with them, we calculate the decay widths of them and list them in Table IX.

For the first process $\Omega_{cc}^{+}\to\Omega_c^{0}\rho^{+}$, the two results are similar because this process only involves tree-level contributions (ignoring the s-channel contribution). Therefore, the slight difference between the two studies arises from the input parameters, such as decay constants or form factors.

For the other two processes, $\Omega_{cc}^{+}\to\Xi_c^{\prime+}\bar{K}^{*0}$ and $\Omega_{cc}^{+}\to\Xi_c^{+}\bar{K}^{*0}$, we can see our results are smaller than the ones of Ref. [47]. This is because, different from Ref. [47], for the parameter in Eq. (22), we take the value $n=2$. This choice is more effective at suppressing the contribution of loop diagrams, leading to more reasonable results. Different from Ref. [47], which computes loop diagrams using the cutting rule and only evaluates the imaginary part, we have calculated the complete loop diagrams. Furthermore, we have additionally taken into account the contributions from two other kinds of loop diagrams, $\{\mathcal{B}_c, P; \mathcal{B}_c^*\}$ and $\{\mathcal{B}_c, V; \mathcal{B}_c^*\}$, as shown by Figs. 2(g) and (h).

At LHCb Run 3, an integrated luminosity of 23 fb^{-1} is anticipated [37], with a projected Ω_{cc} production cross section of 60–1800 nb in the LHCb detector [16,23,78–83]. The corresponding production event of Ω_{cc}, $N_p(\Omega_{cc})$, is approximately 10^9–10^{10}. Considering the subsequent decay chain involving $\mathrm{Br}(\Xi_c^{\prime+}\to\Xi_c^{+}\gamma)=100\%$, $\mathrm{Br}(\Xi_c^{+}\to pK^{-}\pi^{+}) = 6.2\times10^{-3}$ [84,85], $\mathrm{Br}(\bar{K}^{*0}\to K^{+}\pi^{-})=100\%$ [70], $\mathrm{Br}(\Omega_c^{0}\to\Omega^{-}\pi^{+}) = 5.1\times10^{-3}\sim3.43\times10^{-2}$ [86,87], $\mathrm{Br}(\Omega^{-}\to\Lambda K^{-}) = 67.8\%$ [70], $\mathrm{Br}(\Lambda\to p\pi^{-}) = 63.9\%$ [70], and $\mathrm{Br}(\rho^{+}\to\pi^{+}\pi^{0})=100\%$ [70], combined with detection efficiencies for final states $\epsilon_{p,K^{\pm},\pi^{\pm}} = 90\sim95\%$ [88,89], $\epsilon_{\pi^0} = 20\sim25\%$ [90], and $\epsilon_{\gamma}\sim1\%$ [91], the

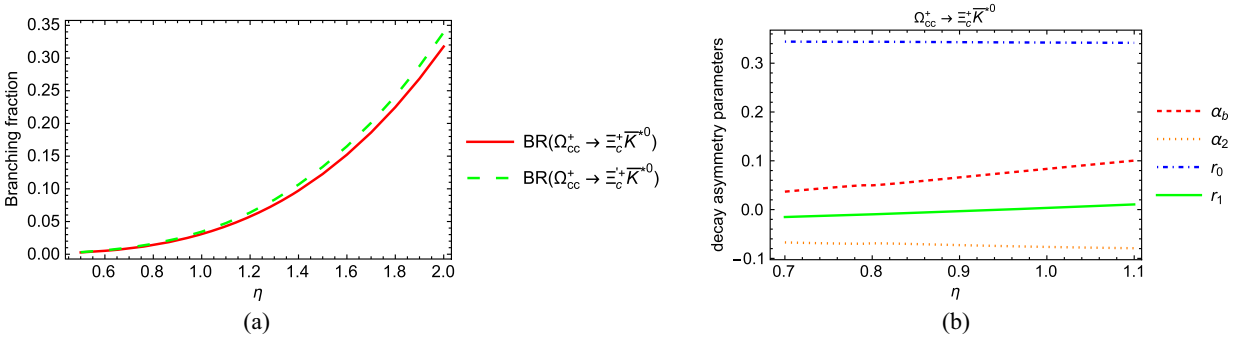


FIG. 3. The dependence of the branching ratios and decay parameters on the model parameter η.

TABLE VII. *CP* violations for short-distance dominated modes.

Channels	$A_{CP}^{\rm dir}[10^{-5}]$	$\alpha_{bCP}[10^{-5}]$	$\alpha_{2CP}[10^{-5}]$	$r_{0CP}[10^{-5}]$	$r_{1CP}[10^{-5}]$
$\Xi_{cc}^{++} \to \Sigma_c^+ \rho^+$	$0.91^{+0.23}_{-0.27}$	$0.86^{+0.69}_{-0.42}$	$-2.1^{+0.6}_{-0.2}$	$-0.31^{+0.10}_{0.10}$	$0.29^{+0.26}_{-0.15}$
$\Xi_{cc}^{++} \to \Lambda_c^+ \rho^+$	$2.1^{+0.8}_{-0.7}$	$-2.2^{+0.9}_{-1.2}$	$-1.1^{+0.3}_{-0.3}$	$0.35^{+0.25}_{-0.16}$	$-1.7^{+0.7}_{-0.8}$
$\Xi_{cc}^{++} \to \Xi_c'^+ K^{*+}$	$-0.14^{+0.07}_{-0.09}$	$-0.39^{+0.17}_{-0.22}$	26^{+14}_{-40}	$0.18^{+0.06}_{-0.09}$	$-0.11^{+0.06}_{-0.07}$
$\Xi_{cc}^{++} \to \Xi_c^+ K^{*+}$	$-1.1^{+0.3}_{-0.3}$	$1.00^{+0.32}_{-0.31}$	$0.60^{+0.07}_{-0.32}$	$-0.44^{+0.15}_{-0.32}$	$0.85^{+0.24}_{-0.25}$
$\Xi_{cc}^{+} \to \Sigma_c^0 \rho^+$	$-0.68^{+0.26}_{-0.34}$	$0.24^{+0.08}_{-0.08}$	$-1.4^{+0.4}_{-0.3}$	$-0.16^{+0.06}_{-0.34}$	$0.043^{+0.022}_{-0.015}$
$\Xi_{cc}^{+} \to \Xi_c'^0 K^{*+}$	$0.38^{+0.13}_{-0.14}$	$-0.017^{+0.027}_{-0.007}$	64^{+59}_{-59}	$-0.17^{+0.07}_{-0.07}$	$-0.22^{+0.07}_{-0.04}$
$\Xi_{cc}^{+} \to \Xi_c^0 K^{*+}$	$-0.30^{+0.10}_{-0.11}$	$-0.57^{+0.22}_{-0.31}$	$-0.42^{+0.17}_{-0.11}$	$0.23^{+0.09}_{-0.11}$	$-0.51^{+0.20}_{-0.29}$
$\Omega_{cc}^{+} \to \Xi_c^0 \rho^+$	$0.0094^{+0.0153}_{-0.0115}$	$-0.80^{+0.18}_{-0.13}$	$-0.82^{+0.21}_{-0.02}$	$-0.0077^{+0.0100}_{0.0100}$	$-0.81^{+0.19}_{-0.13}$
$\Omega_{cc}^{+} \to \Xi_c'^0 \rho^+$	$-1.1^{+0.3}_{-0.3}$	$0.51^{+0.03}_{-0.09}$	$-0.16^{+0.13}_{-0.29}$	$0.071^{+0.001}_{-0.291}$	$0.36^{+0.04}_{-0.07}$
$\Omega_{cc}^{+} \to \Omega_c^0 K^{*+}$	$0.80^{+0.24}_{-0.25}$	$-0.060^{+0.048}_{-0.079}$	$-3.1^{+1.3}_{-0.2}$	$0.12^{+0.05}_{-0.04}$	$0.080^{+0.008}_{-0.010}$

TABLE VIII. *CP* violations for singly Cabibbo-suppressed modes.

Channels	$A_{CP}^{\rm dir}[10^{-4}]$	$\alpha_{bCP}[10^{-4}]$	$\alpha_{2CP}[10^{-4}]$	$r_{0CP}[10^{-4}]$	$r_{1CP}[10^{-4}]$
$\Xi_{cc}^{++} \to \Sigma_c^{++} \rho^0$	$0.46^{+0.05}_{-0.04}$	$1.6^{+0.5}_{-0.3}$	$-0.26^{+0.01}_{-0.04}$	$-0.038^{+0.009}_{-0.039}$	$1.1^{+0.3}_{-0.2}$
$\Xi_{cc}^{++} \to \Sigma_c^{++} \omega$	$-1.1^{+0.0}_{-0.1}$	$-0.41^{+0.07}_{-0.07}$	$-1.2^{+0.7}_{-0.0}$	$-0.36^{+0.00}_{-0.01}$	$-0.33^{+0.00}_{-0.00}$
$\Xi_{cc}^{+} \to \Sigma_c^{+} \rho^0$	$0.040^{+0.006}_{-0.004}$	$-0.072^{+0.013}_{-0.015}$	$0.24^{+0.04}_{-0.04}$	$-0.0080^{+0.0005}_{-0.0041}$	$0.0098^{+0.0020}_{-0.0025}$
$\Xi_{cc}^{+} \to \Sigma_c^{+} \omega$	$-0.30^{+0.02}_{-0.03}$	$-0.40^{+0.06}_{-0.07}$	$0.25^{+0.02}_{-0.02}$	$-0.017^{+0.009}_{-0.009}$	$-0.26^{+0.04}_{-0.04}$
$\Xi_{cc}^{+} \to \Lambda_c^{+} \rho^0$	$-0.0032^{+0.0002}_{-0.0015}$	$-0.29^{+0.04}_{-0.04}$	$1.2^{+0.1}_{-0.0}$	$-0.037^{+0.001}_{-0.002}$	$-0.89^{+0.12}_{-0.09}$
$\Xi_{cc}^{+} \to \Lambda_c^{+} \omega$	$0.068^{+0.007}_{-0.015}$	$-1.3^{+0.1}_{-0.1}$	$0.25^{+0.01}_{0.01}$	$0.19^{+0.00}_{-0.00}$	$0.78^{+0.05}_{-0.04}$
$\Xi_{cc}^{+} \to \Sigma_c^{++} \rho^-$	$-0.066^{+0.005}_{-0.006}$	$-0.37^{+0.05}_{-0.05}$	$0.61^{+0.00}_{-0.00}$	$0.012^{+0.001}_{-0.001}$	$-0.13^{+0.02}_{-0.02}$
$\Xi_{cc}^{+} \to \Xi_c'^{+} K^{*0}$	$-0.13^{+0.02}_{-0.02}$	$-8.4^{+1.7}_{-1.4}$	$-2.7^{+0.4}_{-0.0}$	$-0.26^{+0.03}_{0.02}$	$4.3^{+3.7}_{-1.1}$
$\Xi_{cc}^{+} \to \Xi_c^{+} K^{*0}$	$-0.41^{+0.04}_{-0.06}$	$-4.1^{+1.8}_{-2.4}$	$1.2^{+0.1}_{-0.0}$	$-0.29^{+0.06}_{0.04}$	$-0.93^{+0.19}_{-0.16}$
$\Omega_{cc}^{+} \to \Sigma_c^{+} \bar{K}^{*0}$	$-0.068^{+0.001}_{-0.001}$	$-0.31^{+0.02}_{-0.02}$	$-0.12^{+0.00}_{-0.00}$	$0.047^{+0.001}_{-0.001}$	$-6.0^{+2.6}_{-16.1}$
$\Omega_{cc}^{+} \to \Lambda_c^{+} \bar{K}^{*0}$	$-0.00085^{+0.00422}_{-0.00412}$	$-0.54^{+0.10}_{-0.14}$	$-0.11^{+0.01}_{-0.01}$	$0.038^{+0.004}_{-0.004}$	$0.089^{+0.012}_{-0.011}$
$\Omega_{cc}^{+} \to \Sigma_c^{++} K^{*-}$	$-0.28^{+0.03}_{-0.04}$	$2.7^{+4.7}_{-0.9}$	$0.17^{+0.02}_{-0.02}$	$0.0040^{+0.0070}_{-0.0070}$	$-0.29^{+0.07}_{-0.06}$
$\Omega_{cc}^{+} \to \Xi_c'^{+} \rho^0$	$0.13^{+0.01}_{-0.01}$	$0.57^{+0.28}_{-0.14}$	$-0.81^{+0.08}_{-0.01}$	$-0.0084^{+0.0003}_{-0.0102}$	$0.30^{+0.15}_{-0.08}$

(Table continued)

TABLE VIII. *(Continued)*

Channels	$A_{CP}^{\rm dir}[10^{-4}]$	$\alpha_{bCP}[10^{-4}]$	$\alpha_{2CP}[10^{-4}]$	$r_{0CP}[10^{-4}]$	$r_{1CP}[10^{-4}]$
$\Omega_{cc}^+ \to \Xi_c'^+\phi$	$0.10^{+0.01}_{-0.01}$	$-1.3^{+0.4}_{-1.0}$	$-0.037^{+0.007}_{-0.008}$	$0.0087^{+0.0009}_{-0.0075}$	$0.78^{+0.29}_{-0.17}$
$\Omega_{cc}^+ \to \Xi_c'^+\omega$	$-1.5^{+0.0}_{-0.0}$	$-0.73^{+0.17}_{-0.20}$	$0.83^{+0.01}_{-0.01}$	$0.17^{+0.04}_{-0.04}$	$-9.6^{+4.8}_{-67.7}$
$\Omega_{cc}^+ \to \Xi_c^+\rho^0$	$0.096^{+0.007}_{-0.006}$	$-0.23^{+0.02}_{-0.02}$	$-0.043^{+0.016}_{-0.020}$	$-0.0015^{+0.0023}_{-0.0064}$	$-0.33^{+0.02}_{-0.02}$
$\Omega_{cc}^+ \to \Xi_c^+\phi$	$0.0028^{+0.0005}_{-0.0003}$	$0.24^{+0.03}_{-0.02}$	$-0.17^{+0.00}_{-0.01}$	$0.0037^{+0.0008}_{-0.0007}$	-40^{+50}_{-33}
$\Omega_{cc}^+ \to \Xi_c^+\omega$	$-0.45^{+0.01}_{-0.01}$	13^{+8}_{-41}	$0.36^{+0.04}_{-0.01}$	$-0.18^{+0.02}_{-0.01}$	$-0.50^{+0.07}_{-0.06}$

expected signal events Ns for the three decay channels can be estimated as follows:

$$Ns = Np(\Omega_{cc})\mathrm{Br}(\Omega_{cc}^+ \to \Omega_c^0\rho^+)\mathrm{Br}(\Omega_c^0 \to \Omega^-\pi^+)\mathrm{Br}(\rho^+ \to \pi^+\pi^0) \times \mathrm{Br}(\Omega^- \to \Lambda K^-)\mathrm{Br}(\Lambda \to p\pi^-)(90\%)^5(25\%) \sim (7.2\times10^4 - 4.7\times10^6), \tag{40}$$

$$Ns = Np(\Omega_{cc})\mathrm{Br}(\Omega_{cc}^+ \to \Xi_c'^+\bar{K}^{*0})\mathrm{Br}(\Xi_c'^+ \to \Xi_c^+\gamma)\mathrm{Br}(\Xi_c^+ \to pK^-\pi^+) \times \mathrm{Br}(\bar{K}^{*0} \to K^+\pi^-)(90\%)^5(1\%) = (8.8\times10^2 \sim 8.8\times10^3), \tag{41}$$

$$Ns = Np(\Omega_{cc})\mathrm{Br}(\Omega_{cc}^+ \to \Xi_c^+\bar{K}^{*0})\mathrm{Br}(\Xi_c^+ \to pK^-\pi^+)\mathrm{Br}(\bar{K}^{*0} \to K^+\pi^-)(90\%)^5 = (7.8\times10^4 - 7.8\times10^5). \tag{42}$$

Before the summary, we briefly discuss the contributions beyond the naive factorization approximation. The short-distance α_s corrections to the factorizable amplitudes are typically of order $\alpha_s(m_c)C_F/(4\pi) \sim 10\%$, with additional power suppression of $\Lambda_{\rm QCD}/m_c \sim 0.2$–$0.3$ [92,93]. These corrections are perturbatively calculable and can introduce strong phases via the imaginary parts of loop integrals. The penguin operators $O_3 - O_6$, neglected in our calculation, also contribute at the short-distance level [94]. Their effects are CKM-suppressed by λ^2 in Cabibbo-favored modes and thus remain at the percent level, though they may become more important in Cabibbo-suppressed channels.

In contrast, the long-distance final-state rescattering effects via triangular diagrams, which are the main focus of this work, are nonperturbative and free from short-distance suppression. They can be resonantly enhanced through intermediate hadronic states and may produce large strong phases, making them particularly relevant for decay asymmetry parameters. These effects are complementary to the short-distance α_s corrections and penguin contributions, rather than being alternatives. A complete treatment incorporating both types of contributions is desirable but beyond the present scope. Our results should, therefore, be viewed as estimates within the adopted approximation, providing a baseline for future more complete calculations in the QCD factorization framework.

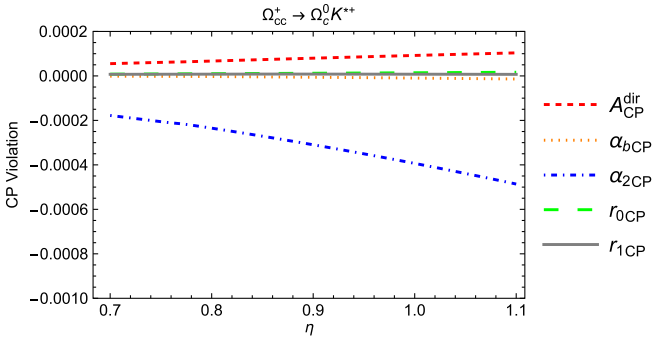


FIG. 4. The dependence of the *CP* violations on the model parameter η.

TABLE IX. Comparison of decay widths for three decays $\mathcal{B}_{cc} \to \mathcal{B}_c V$ ($\Gamma_{Tot}[10^{-14}]$).

Channels	This work	Ref. [47]
$\Omega_{cc}^+ \to \Omega_c^0\rho^+$	82.8	87.5
$\Omega_{cc}^+ \to \Xi_c'^+\bar{K}^{*0}$	$8.82^{+8.66}_{-5.09}$	$26.4^{+27.2}_{-17.9}$
$\Omega_{cc}^+ \to \Xi_c^+\bar{K}^{*0}$	$7.86^{+7.88}_{-4.57}$	$13.8^{+14.9}_{-9.5}$

IV. SUMMARY

In this work, we study the two-body nonleptonic weak decays $\mathcal{B}_{cc} \to \mathcal{B}_c V$ [with $\mathcal{B}_{cc} = (\Xi_{cc}^{++}, \Xi_{cc}^{+}, \Omega_{cc}^{+})$, $\mathcal{B}_c = (\mathcal{B}_{\bar{3}}, \mathcal{B}_6)$, and $V = (\rho, K^*, \omega, \phi)$] under the final-state interaction mechanism. As shown in Table III, the factorizable diagram T dominates over the long-distance contributions from C' and E_2. Conversely, from Tables IV–VI, long-distance effects dominate because the short-distance amplitude C_{SD} is suppressed by the effective Wilson coefficients at the charm scale, where $a_2(m_c) = -0.017 < a_1(m_c) = 1.07$—the latter is used for the weak decay vertex. No absolute branching fraction for $\mathcal{B}_{cc} \to \mathcal{B}_c V$ has yet been measured directly. Since this process involves a charm quark decay, we adopt $\eta = 0.9 \pm 0.2$, as determined in our previous work [64], by fitting the ratio $\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c'^{+}\pi^+)/\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c^{+}\pi^+) = (1.41 \pm 0.17 \pm 0.10)$ from LHCb [11]. Further findings are as follows:

(1) Using $\eta = 0.9 \pm 0.2$, we predict the branching ratios, decay asymmetry parameters, and CP violation for all $\mathcal{B}_{cc} \to \mathcal{B}_c V$ decays.
(2) The decay asymmetry parameters are less sensitive to the model parameter η than the branching ratios, as shown in Fig. 3, regardless of whether short-distance or long-distance contributions dominated.
(3) After the calculation of the decay widths of each decay mode, we conduct a comparative analysis with other theoretical works and analyze the production events in the LHCb experiment. Different from other studies, our work includes both real and imaginary contributions of the amplitudes.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: AMPLITUDES OF EACH MODE

The expressions of amplitudes for all 47 $\mathcal{B}_{cc} \to \mathcal{B}_c V$ decays considered in this paper are collected in this appendix. Twenty amplitudes for the short-distance dominated modes are given as follows:

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{++} \to \Xi_c^{+}\rho^+) &= T_{SD}(\Xi_{cc}^{++} \to \Xi_c^{+}\rho^+) \\ &+ \mathcal{M}(\pi^+, \Xi_c^{(\prime)+}; \pi^0) + \mathcal{M}(\pi^+, \Xi_c^{(\prime)+}; \rho^0) + \mathcal{M}(\rho^+, \Xi_c^{(\prime)+}; \pi^0) + \mathcal{M}(\rho^+, \Xi_c^{(\prime)+}; \rho^0) \\ &+ \mathcal{M}(\Xi_c^{(\prime)+}, \pi^+; \Xi_c^{(\prime)0}) + \mathcal{M}(\Xi_c^{(\prime)+}, \rho^+; \Xi_c^{(\prime)0}) + \mathcal{M}(\Xi_c^{(\prime)+}, \pi^+; \Xi_c^{*\prime 0}) + \mathcal{M}(\Xi_c^{(\prime)+}, \rho^+; \Xi_c^{*\prime 0}), \end{aligned} \tag{A1}$$

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{++} \to \Xi_c'^{+}\rho^+) &= T_{SD}(\Xi_{cc}^{++} \to \Xi_c'^{+}\rho^+) \\ &+ \mathcal{M}(\pi^+, \Xi_c^{(\prime)+}; \pi^0) + \mathcal{M}(\pi^+, \Xi_c^{(\prime)+}; \rho^0) + \mathcal{M}(\rho^+, \Xi_c^{(\prime)+}; \pi^0) + \mathcal{M}(\rho^+, \Xi_c^{(\prime)+}; \rho^0) \\ &+ \mathcal{M}(\Xi_c^{(\prime)+}, \pi^+; \Xi_c^{(\prime)0}) + \mathcal{M}(\Xi_c^{(\prime)+}, \rho^+; \Xi_c^{(\prime)0}) + \mathcal{M}(\Xi_c^{(\prime)+}, \pi^+; \Xi_c^{*\prime 0}) + \mathcal{M}(\Xi_c^{(\prime)+}, \rho^+; \Xi_c^{*\prime 0}), \end{aligned} \tag{A2}$$

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{++} \to \Sigma_c^{+}\rho^+) &= T_{SD}(\Xi_{cc}^{++} \to \Sigma_c^{+}\rho^+) \\ &+ \mathcal{M}(\pi^+, \Lambda_c^{+}; \pi^0) + \mathcal{M}(K^+, \Xi_c^{(\prime)+}; K^0) + \mathcal{M}(\pi^+, \Lambda_c^{+}; \rho^0) + \mathcal{M}(K^+, \Xi_c^{(\prime)+}; K^{*0}) \\ &+ \mathcal{M}(\rho^+, \Lambda_c^{+}; \pi^0) + \mathcal{M}(K^{*+}, \Xi_c^{(\prime)+}; K^0) + \mathcal{M}(\rho^+, \Lambda_c^{+}; \rho^0) + \mathcal{M}(K^{*+}, \Xi_c^{(\prime)+}; K^{*0}) \\ &+ \mathcal{M}(\Lambda_c^{+}/\Sigma_c^{+}, \pi^+; \Sigma_c^{0}) + \mathcal{M}(\Xi_c^{(\prime)+}, K^+; \Xi_c^{(\prime)0}) + \mathcal{M}(\Lambda_c^{+}/\Sigma_c^{+}, \rho^+; \Sigma_c^{0}) + \mathcal{M}(\Xi_c^{(\prime)+}, K^{*+}; \Xi_c^{(\prime)0}) \\ &+ \mathcal{M}(\Lambda_c^{+}/\Sigma_c^{+}, \pi^+; \Sigma_c^{*0}) + \mathcal{M}(\Xi_c^{(\prime)+}, K^+; \Xi_c^{*\prime 0}) + \mathcal{M}(\Lambda_c^{+}/\Sigma_c^{+}, \rho^+; \Sigma_c^{*0}) + \mathcal{M}(\Xi_c^{(\prime)+}, K^{*+}; \Xi_c^{*\prime 0}), \end{aligned} \tag{A3}$$

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{++}\to\Lambda_c^+\rho^+) &= T_{SD}(\Xi_{cc}^{++}\to\Lambda_c^+\rho^+)\\ &+\mathcal{M}(\pi^+,\Sigma_c^+;\pi^0)+\mathcal{M}(K^+,\Xi_c^{(\prime)+};K^0)+\mathcal{M}(\pi^+,\Sigma_c^+;\rho^0)+\mathcal{M}(K^+,\Xi_c^{(\prime)+};K^{*0})\\ &+\mathcal{M}(\rho^+,\Sigma_c^+;\pi^0)+\mathcal{M}(K^{*+},\Xi_c^{(\prime)+};K^0)+\mathcal{M}(\rho^+,\Sigma_c^+;\rho^0)+\mathcal{M}(K^{*+},\Xi_c^{(\prime)+};K^{*0})\\ &+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,\pi^+;\Sigma_c^0)+\mathcal{M}(\Xi_c^{(\prime)+},K^+;\Xi_c^{(\prime)0})+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,\rho^+;\Sigma_c^0)+\mathcal{M}(\Xi_c^{(\prime)+},K^{*+};\Xi_c^{(\prime)0})\\ &+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,\pi^+;\Sigma_c^{*0})+\mathcal{M}(\Xi_c^{(\prime)+},K^+;\Xi_c^{*\prime 0})+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,\rho^+;\Sigma_c^{*0})+\mathcal{M}(\Xi_c^{(\prime)+},K^{*+};\Xi_c^{*\prime 0}),\end{aligned}\tag{A4}$$

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{++}\to\Xi_c^{\prime+}K^{*+}) &= T_{SD}(\Xi_{cc}^{++}\to\Xi_c^{\prime+}K^{*+})\\ &+\mathcal{M}(K^+,\Xi_c^{(\prime)+};\pi^0)+\mathcal{M}(K^+,\Xi_c^{(\prime)+};\eta_8)+\mathcal{M}(K^+,\Xi_c^{\prime+};\eta_1)+\mathcal{M}(\pi^+,\Lambda_c^+/\Sigma_c^+;\bar{K}^0)\\ &+\mathcal{M}(K^+,\Xi_c^{(\prime)+};\rho^0)+\mathcal{M}(K^+,\Xi_c^{(\prime)+};\omega)+\mathcal{M}(K^+,\Xi_c^{(\prime)+};\phi)+\mathcal{M}(\pi^+,\Lambda_c^+/\Sigma_c^+;\bar{K}^{*0})\\ &+\mathcal{M}(K^{*+},\Xi_c^{(\prime)+};\pi^0)+\mathcal{M}(K^{*+},\Xi_c^{(\prime)+};\eta_8)+\mathcal{M}(K^{*+},\Xi_c^{\prime+};\eta_1)+\mathcal{M}(\rho^+,\Lambda_c^+/\Sigma_c^+;\bar{K}^0)\\ &+\mathcal{M}(K^{*+},\Xi_c^{(\prime)+};\rho^0)+\mathcal{M}(K^{*+},\Xi_c^{(\prime)+};\omega)+\mathcal{M}(K^{*+},\Xi_c^{(\prime)+};\phi)+\mathcal{M}(\rho^+,\Lambda_c^+/\Sigma_c^+;\bar{K}^{*0})\\ &+\mathcal{M}(\Xi_c^{(\prime)+},K^+;\Omega_c^0)+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,\pi^+;\Xi_c^{(\prime)0})+\mathcal{M}(\Xi_c^{(\prime)+},K^{*+};\Omega_c^0)+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,\rho^+;\Xi_c^{(\prime)0})\\ &+\mathcal{M}(\Xi_c^{(\prime)+},K^+;\Omega_c^{*0})+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,\pi^+;\Xi_c^{*\prime 0})+\mathcal{M}(\Xi_c^{(\prime)+},K^{*+};\Omega_c^{*0})+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,\rho^+;\Xi_c^{*\prime 0}),\end{aligned}\tag{A5}$$

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{++}\to\Xi_c^{+}K^{*+}) &= T_{SD}(\Xi_{cc}^{++}\to\Xi_c^{+}K^{*+})\\ &+\mathcal{M}(K^+,\Xi_c^{(\prime)+};\pi^0)+\mathcal{M}(K^+,\Xi_c^{(\prime)+};\eta_8)+\mathcal{M}(K^+,\Xi_c^{+};\eta_1)+\mathcal{M}(\pi^+,\Lambda_c^+/\Sigma_c^+;\bar{K}^0)\\ &+\mathcal{M}(K^+,\Xi_c^{(\prime)+};\rho^0)+\mathcal{M}(K^+,\Xi_c^{(\prime)+};\omega)+\mathcal{M}(K^+,\Xi_c^{(\prime)+};\phi)+\mathcal{M}(\pi^+,\Lambda_c^+/\Sigma_c^+;\bar{K}^{*0})\\ &+\mathcal{M}(K^{*+},\Xi_c^{(\prime)+};\pi^0)+\mathcal{M}(K^{*+},\Xi_c^{(\prime)+};\eta_8)+\mathcal{M}(K^{*+},\Xi_c^{+};\eta_1)+\mathcal{M}(\rho^+,\Lambda_c^+/\Sigma_c^+;\bar{K}^0)\\ &+\mathcal{M}(K^{*+},\Xi_c^{(\prime)+};\rho^0)+\mathcal{M}(K^{*+},\Xi_c^{(\prime)+};\omega)+\mathcal{M}(K^{*+},\Xi_c^{(\prime)+};\phi)+\mathcal{M}(\rho^+,\Lambda_c^+/\Sigma_c^+;\bar{K}^{*0})\\ &+\mathcal{M}(\Xi_c^{(\prime)+},K^+;\Omega_c^0)+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,\pi^+;\Xi_c^{(\prime)0})+\mathcal{M}(\Xi_c^{(\prime)+},K^{*+};\Omega_c^0)+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,\rho^+;\Xi_c^{(\prime)0})\\ &+\mathcal{M}(\Xi_c^{(\prime)+},K^+;\Omega_c^{*0})+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,\pi^+;\Xi_c^{*\prime 0})+\mathcal{M}(\Xi_c^{(\prime)+},K^{*+};\Omega_c^{*0})+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,\rho^+;\Xi_c^{*\prime 0}),\end{aligned}\tag{A6}$$

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{++}\to\Sigma_c^{+}K^{*+}) &= T_{SD}(\Xi_{cc}^{++}\to\Sigma_c^{+}K^{*+})\\ &+\mathcal{M}(K^+,\Lambda_c^+;\pi^0)+\mathcal{M}(K^+,\Sigma_c^+;\eta_8)+\mathcal{M}(K^+,\Sigma_c^+;\eta_1)+\mathcal{M}(K^+,\Lambda_c^+;\rho^0)\\ &+\mathcal{M}(K^+,\Sigma_c^+;\omega)+\mathcal{M}(K^{*+},\Lambda_c^+;\pi^0)+\mathcal{M}(K^{*+},\Sigma_c^+;\eta_8)+\mathcal{M}(K^{*+},\Sigma_c^+;\eta_1)\\ &+\mathcal{M}(K^{*+},\Lambda_c^+;\rho^0)+\mathcal{M}(K^{*+},\Sigma_c^+;\omega)+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,K^+;\Xi_c^{(\prime)0})+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,K^{*+};\Xi_c^{(\prime)0})\\ &+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,K^+;\Xi_c^{*\prime 0})+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,K^{*+};\Xi_c^{*\prime 0}),\end{aligned}\tag{A7}$$

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{++}\to\Lambda_c^{+}K^{*+}) &= T_{SD}(\Xi_{cc}^{++}\to\Lambda_c^{+}K^{*+})\\ &+\mathcal{M}(K^+,\Sigma_c^+;\pi^0)+\mathcal{M}(K^+,\Lambda_c^+;\eta_8)+\mathcal{M}(K^+,\Lambda_c^+;\eta_1)+\mathcal{M}(K^+,\Sigma_c^+;\rho^0)\\ &+\mathcal{M}(K^+,\Lambda_c^+;\omega)+\mathcal{M}(K^{*+},\Sigma_c^+;\pi^0)+\mathcal{M}(K^{*+},\Lambda_c^+;\eta_8)+\mathcal{M}(K^{*+},\Lambda_c^+;\eta_1)\\ &+\mathcal{M}(K^{*+},\Sigma_c^+;\rho^0)+\mathcal{M}(K^{*+},\Lambda_c^+;\omega)+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,K^+;\Xi_c^{(\prime)0})+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,K^{*+};\Xi_c^{(\prime)0})\\ &+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,K^+;\Xi_c^{*\prime 0})+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,K^{*+};\Xi_c^{*\prime 0}),\end{aligned}\tag{A8}$$

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{+}\to\Xi_c^{0}\rho^+) &= T_{SD}(\Xi_{cc}^{+}\to\Xi_c^{0}\rho^+)+\mathcal{M}(\pi^+,\Xi_c^{(\prime)0};\pi^0)+\mathcal{M}(\pi^+,\Xi_c^{(\prime)0};\eta_8)+\mathcal{M}(\pi^+,\Xi_c^{0};\eta_1)+\mathcal{M}(\pi^+,\Xi_c^{(\prime)0};\rho^0)\\ &+\mathcal{M}(\pi^+,\Xi_c^{(\prime)0};\omega)+\mathcal{M}(\rho^+,\Xi_c^{(\prime)0};\pi^0)+\mathcal{M}(\rho^+,\Xi_c^{(\prime)0};\eta_8)+\mathcal{M}(\rho^+,\Xi_c^{0};\eta_1)\\ &+\mathcal{M}(\rho^+,\Xi_c^{(\prime)0};\rho^0)+\mathcal{M}(\rho^+,\Xi_c^{(\prime)0};\omega),\end{aligned}\tag{A9}$$

$$\mathcal{A}(\Xi_{cc}^{+}\to\Xi_c^{\prime 0}\rho^{+}) = T_{SD}(\Xi_{cc}^{+}\to\Xi_c^{\prime 0}\rho^{+}) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\pi^{0}) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\eta_8) + \mathcal{M}(\pi^{+},\Xi_c^{\prime 0};\eta_1) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\rho^{0}) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\omega) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\pi^{0}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\eta_8) + \mathcal{M}(\rho^{+},\Xi_c^{\prime 0};\eta_1) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\rho^{0}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\omega), \tag{A10}$$

$$\mathcal{A}(\Xi_{cc}^{+}\to\Sigma_c^{0}\rho^{+}) = T_{SD}(\Xi_{cc}^{+}\to\Sigma_c^{0}\rho^{+}) + \mathcal{M}(\pi^{+},\Sigma_c^{0};\pi^{0}) + \mathcal{M}(\pi^{+},\Sigma_c^{0};\eta_8) + \mathcal{M}(\pi^{+},\Sigma_c^{0};\eta_1) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};K^{0}) + \mathcal{M}(\pi^{+},\Sigma_c^{0};\rho^{0}) + \mathcal{M}(\pi^{+},\Sigma_c^{0};\omega) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};K^{*0}) + \mathcal{M}(\rho^{+},\Sigma_c^{0};\pi^{0}) + \mathcal{M}(\rho^{+},\Sigma_c^{0};\eta_8) + \mathcal{M}(\rho^{+},\Sigma_c^{0};\eta_1) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{0}) + \mathcal{M}(\rho^{+},\Sigma_c^{0};\rho^{0}) + \mathcal{M}(\rho^{+},\Sigma_c^{0};\omega) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{*0}), \tag{A11}$$

$$\mathcal{A}(\Xi_{cc}^{+}\to\Xi_c^{0}K^{*+}) = T_{SD}(\Xi_{cc}^{+}\to\Xi_c^{0}K^{*+}) + \mathcal{M}(\pi^{+},\Sigma_c^{0};\bar{K}^{0}) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};\pi^{0}) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};\eta_8) + \mathcal{M}(K^{+},\Xi_c^{0};\eta_1) + \mathcal{M}(\pi^{+},\Sigma_c^{0};\bar{K}^{*0}) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};\rho^{0}) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};\omega) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};\phi) + \mathcal{M}(\rho^{+},\Sigma_c^{0};\bar{K}^{0}) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\pi^{0}) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\eta_8) + \mathcal{M}(K^{*+},\Xi_c^{0};\eta_1) + \mathcal{M}(\rho^{+},\Sigma_c^{0};\bar{K}^{*0}) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\rho^{0}) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\omega) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\phi), \tag{A12}$$

$$\mathcal{A}(\Xi_{cc}^{+}\to\Xi_c^{\prime 0}K^{*+}) = T_{SD}(\Xi_{cc}^{+}\to\Xi_c^{\prime 0}K^{*+}) + \mathcal{M}(\pi^{+},\Sigma_c^{0};\bar{K}^{0}) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};\pi^{0}) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};\eta_8) + \mathcal{M}(K^{+},\Xi_c^{\prime 0};\eta_1) + \mathcal{M}(\pi^{+},\Sigma_c^{0};\bar{K}^{*0}) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};\rho^{0}) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};\omega) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};\phi) + \mathcal{M}(\rho^{+},\Sigma_c^{0};\bar{K}^{0}) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\pi^{0}) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\eta_8) + \mathcal{M}(K^{*+},\Xi_c^{\prime 0};\eta_1) + \mathcal{M}(\rho^{+},\Sigma_c^{0};\bar{K}^{*0}) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\rho^{0}) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\omega) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\phi), \tag{A13}$$

$$\mathcal{A}(\Xi_{cc}^{+}\to\Sigma_c^{0}K^{*+}) = \mathcal{T}_{SD}(\Xi_{cc}^{+}\to\Sigma_c^{0}K^{*+}), \qquad \mathcal{A}(\Omega_{cc}^{+}\to\Omega_c^{0}\rho^{+}) = \mathcal{T}_{SD}(\Omega_{cc}^{+}\to\Omega_c^{0}\rho^{+}), \tag{A14}$$

$$\mathcal{A}(\Omega_{cc}^{+}\to\Xi_c^{0}\rho^{+}) = T_{SD}(\Omega_{cc}^{+}\to\Xi_c^{0}\rho^{+}) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\pi^{0}) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\eta_8) + \mathcal{M}(\pi^{+},\Xi_c^{0};\eta_1) + \mathcal{M}(K^{+},\Omega_c^{0};K^{0}) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\rho^{0}) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\omega) + \mathcal{M}(K^{+},\Omega_c^{0};K^{*0}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\pi^{0}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\eta_8) + \mathcal{M}(\rho^{+},\Xi_c^{0};\eta_1) + \mathcal{M}(K^{*+},\Omega_c^{0};K^{0}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\rho^{0}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\omega) + \mathcal{M}(K^{*+},\Omega_c^{0};K^{*0}), \tag{A15}$$

$$\mathcal{A}(\Omega_{cc}^{+}\to\Xi_c^{\prime 0}\rho^{+}) = T_{SD}(\Omega_{cc}^{+}\to\Xi_c^{\prime 0}\rho^{+}) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\pi^{0}) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\eta_8) + \mathcal{M}(\pi^{+},\Xi_c^{\prime 0};\eta_1) + \mathcal{M}(K^{+},\Omega_c^{0};K^{0}) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\rho^{0}) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\omega) + \mathcal{M}(K^{+},\Omega_c^{0};K^{*0}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\pi^{0}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\eta_8) + \mathcal{M}(\rho^{+},\Xi_c^{\prime 0};\eta_1) + \mathcal{M}(K^{*+},\Omega_c^{0};K^{0}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\rho^{0}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\omega) + \mathcal{M}(K^{*+},\Omega_c^{0};K^{*0}), \tag{A16}$$

$$\mathcal{A}(\Omega_{cc}^{+}\to\Omega_c^{0}K^{*+}) = T_{SD}(\Omega_{cc}^{+}\to\Omega_c^{0}K^{*+}) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\bar{K}^{0}) + \mathcal{M}(K^{+},\Omega_c^{0};\eta_8) + \mathcal{M}(K^{+},\Omega_c^{0};\eta_1) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\bar{K}^{*0}) + \mathcal{M}(K^{+},\Omega_c^{0};\phi) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\bar{K}^{0}) + \mathcal{M}(K^{*+},\Omega_c^{0};\eta_8) + \mathcal{M}(K^{*+},\Omega_c^{0};\eta_1) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\bar{K}^{*0}) + \mathcal{M}(K^{*+},\Omega_c^{0};\phi), \tag{A17}$$

$$\mathcal{A}(\Omega_{cc}^{+}\to\Xi_c^{0}K^{*+}) = T_{SD}(\Omega_{cc}^{+}\to\Xi_c^{0}K^{*+}) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};\pi^{0}) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};\eta_8) + \mathcal{M}(K^{+},\Xi_c^{0};\eta_1) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};\rho^{0}) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};\omega) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};\phi) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\pi^{0}) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\eta_8) + \mathcal{M}(K^{*+},\Xi_c^{0};\eta_1) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\rho^{0}) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\omega) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\phi), \tag{A18}$$

$$\mathcal{A}(\Omega_{cc}^{+}\to\Xi_c^{\prime 0}K^{*+}) = T_{SD}(\Omega_{cc}^{+}\to\Xi_c^{\prime 0}K^{*+}) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};\pi^0) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};\eta_8) + \mathcal{M}(K^{+},\Xi_c^{\prime 0};\eta_1) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};\rho^0) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};\omega) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};\phi) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\pi^0) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\eta_8) + \mathcal{M}(K^{*+},\Xi_c^{\prime 0};\eta_1) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\rho^0) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\omega) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\phi). \tag{A19}$$

The amplitudes for long-distance dominated and Cabibbo-flavored modes can be given as follows:

$$\mathcal{A}(\Xi_{cc}^{++}\to\Sigma_c^{++}\bar{K}^{*0}) = C_{SD}(\Xi_{cc}^{++}\to\Sigma_c^{++}\bar{K}^{*0}) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)+};K^{+}) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)+};K^{*+}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)+};K^{+}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)+};K^{*+}) + \mathcal{M}(\Xi_c^{(\prime)+},\pi^{+};\Lambda_c^{+}/\Sigma_c^{+}) + \mathcal{M}(\Xi_c^{(\prime)+},\rho^{+};\Lambda_c^{+}/\Sigma_c^{+}) + \mathcal{M}(\Xi_c^{(\prime)+},\pi^{+};\Sigma_c^{*+}) + \mathcal{M}(\Xi_c^{(\prime)+},\rho^{+};\Sigma_c^{*+}), \tag{A20}$$

$$\mathcal{A}(\Xi_{cc}^{+}\to\Omega_c^{0}K^{*+}) = \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\bar{K}^{0}) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\bar{K}^{*0}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\bar{K}^{0}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\bar{K}^{*0}), \tag{A21}$$

$$\mathcal{A}(\Xi_{cc}^{+}\to\Sigma_c^{+}\bar{K}^{*0}) = C_{SD}(\Xi_{cc}^{+}\to\Sigma_c^{+}\bar{K}^{*0}) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};K^{+}) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};K^{*+}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};K^{+}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};K^{*+}) + \mathcal{M}(\Xi_c^{(\prime)0},\pi^{+};\Sigma_c^{0}) + \mathcal{M}(\Xi_c^{(\prime)0},\rho^{+};\Sigma_c^{0}) + \mathcal{M}(\Xi_c^{(\prime)0},\pi^{+};\Sigma_c^{*0}) + \mathcal{M}(\Xi_c^{(\prime)0},\rho^{+};\Sigma_c^{*0}), \tag{A22}$$

$$\mathcal{A}(\Xi_{cc}^{+}\to\Lambda_c^{+}\bar{K}^{*0}) = C_{SD}(\Xi_{cc}^{+}\to\Lambda_c^{+}\bar{K}^{*0}) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};K^{+}) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};K^{*+}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};K^{+}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};K^{*+}) + \mathcal{M}(\Xi_c^{(\prime)0},\pi^{+};\Sigma_c^{0}) + \mathcal{M}(\Xi_c^{(\prime)0},\rho^{+};\Sigma_c^{0}) + \mathcal{M}(\Xi_c^{(\prime)0},\pi^{+};\Sigma_c^{*0}) + \mathcal{M}(\Xi_c^{(\prime)0},\rho^{+};\Sigma_c^{*0}), \tag{A23}$$

$$\mathcal{A}(\Xi_{cc}^{+}\to\Sigma_c^{++}K^{*-}) = \mathcal{M}(\Xi_c^{(\prime)0},\pi^{+};\Lambda_c^{+}/\Sigma_c^{+}) + \mathcal{M}(\Xi_c^{(\prime)0},\rho^{+};\Lambda_c^{+}/\Sigma_c^{+}) + \mathcal{M}(\Xi_c^{(\prime)0},\pi^{+};\Sigma_c^{*+}) + \mathcal{M}(\Xi_c^{(\prime)0},\rho^{+};\Sigma_c^{*+}), \tag{A24}$$

$$\mathcal{A}(\Xi_{cc}^{+}\to\Xi_c^{\prime+}\rho^{0}) = \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\pi^{+}) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\rho^{+}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\pi^{+}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\rho^{+}) + \mathcal{M}(\Xi_c^{(\prime)0},\pi^{+};\Xi_c^{(\prime)0}) + \mathcal{M}(\Xi_c^{(\prime)0},\rho^{+};\Xi_c^{(\prime)0}) + \mathcal{M}(\Xi_c^{(\prime)0},\pi^{+};\Xi_c^{*\prime 0}) + \mathcal{M}(\Xi_c^{(\prime)0},\rho^{+};\Xi_c^{*\prime 0}), \tag{A25}$$

$$\mathcal{A}(\Xi_{cc}^{+}\to\Xi_c^{\prime+}\phi) = \mathcal{M}(\Xi_c^{(\prime)0},\pi^{+};\Xi_c^{(\prime)0}) + \mathcal{M}(\Xi_c^{(\prime)0},\rho^{+};\Xi_c^{(\prime)0}) + \mathcal{M}(\Xi_c^{(\prime)0},\pi^{+};\Xi_c^{*\prime 0}) + \mathcal{M}(\Xi_c^{(\prime)0},\rho^{+};\Xi_c^{*\prime 0}), \tag{A26}$$

$$\mathcal{A}(\Xi_{cc}^{+}\to\Xi_c^{\prime+}\omega) = \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\pi^{+}) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\rho^{+}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\pi^{+}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\rho^{+}) + \mathcal{M}(\Xi_c^{(\prime)0},\pi^{+};\Xi_c^{(\prime)0}) + \mathcal{M}(\Xi_c^{(\prime)0},\rho^{+};\Xi_c^{(\prime)0}) + \mathcal{M}(\Xi_c^{(\prime)0},\pi^{+};\Xi_c^{*\prime 0}) + \mathcal{M}(\Xi_c^{(\prime)0},\rho^{+};\Xi_c^{*\prime 0}), \tag{A27}$$

$$\mathcal{A}(\Xi_{cc}^{+}\to\Xi_c^{+}\rho^{0}) = \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\pi^{+}) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\rho^{+}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\pi^{+}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\rho^{+}) + \mathcal{M}(\Xi_c^{(\prime)0},\pi^{+};\Xi_c^{(\prime)0}) + \mathcal{M}(\Xi_c^{(\prime)0},\rho^{+};\Xi_c^{(\prime)0}) + \mathcal{M}(\Xi_c^{(\prime)0},\pi^{+};\Xi_c^{*\prime 0}) + \mathcal{M}(\Xi_c^{(\prime)0},\rho^{+};\Xi_c^{*\prime 0}), \tag{A28}$$

$$\mathcal{A}(\Xi_{cc}^{+}\to\Xi_c^{+}\phi) = \mathcal{M}(\Xi_c^{(\prime)0},\pi^{+};\Xi_c^{(\prime)0}) + \mathcal{M}(\Xi_c^{(\prime)0},\rho^{+};\Xi_c^{(\prime)0}) + \mathcal{M}(\Xi_c^{(\prime)0},\pi^{+};\Xi_c^{*\prime 0}) + \mathcal{M}(\Xi_c^{(\prime)0},\rho^{+};\Xi_c^{*\prime 0}), \tag{A29}$$

$$\mathcal{A}(\Xi_{cc}^{+}\to\Xi_c^{+}\omega) = \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\pi^{+}) + \mathcal{M}(\pi^{+},\Xi_c^{(\prime)0};\rho^{+}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\pi^{+}) + \mathcal{M}(\rho^{+},\Xi_c^{(\prime)0};\rho^{+}) + \mathcal{M}(\Xi_c^{(\prime)0},\pi^{+};\Xi_c^{(\prime)0}) + \mathcal{M}(\Xi_c^{(\prime)0},\rho^{+};\Xi_c^{(\prime)0}) + \mathcal{M}(\Xi_c^{(\prime)0},\pi^{+};\Xi_c^{*\prime 0}) + \mathcal{M}(\Xi_c^{(\prime)0},\rho^{+};\Xi_c^{*\prime 0}), \tag{A30}$$

$$\mathcal{A}(\Omega_{cc}^{+}\to\Xi_c^{\prime+}\bar{K}^{*0}) = C_{SD}(\Omega_{cc}^{+}\to\Xi_c^{\prime+}\bar{K}^{*0}) + \mathcal{M}(\pi^{+},\Omega_c^{0};K^{+}) + \mathcal{M}(\pi^{+},\Omega_c^{0};K^{*+}) + \mathcal{M}(\rho^{+},\Omega_c^{0};K^{+}) + \mathcal{M}(\rho^{+},\Omega_c^{0};K^{*+}) + \mathcal{M}(\Omega_c^{0},\pi^{+};\Xi_c^{(\prime)0}) + \mathcal{M}(\Omega_c^{0},\rho^{+};\Xi_c^{(\prime)0}) + \mathcal{M}(\Omega_c^{0},\pi^{+};\Xi_c^{*\prime 0}) + \mathcal{M}(\Omega_c^{0},\rho^{+};\Xi_c^{*\prime 0}), \tag{A31}$$

$$\mathcal{A}(\Omega_{cc}^{+}\to\Xi_c^{+}\bar{K}^{*0}) = C_{SD}(\Omega_{cc}^{+}\to\Xi_c^{+}\bar{K}^{*0}) + \mathcal{M}(\pi^{+},\Omega_c^{0};K^{+}) + \mathcal{M}(\pi^{+},\Omega_c^{0};K^{*+}) + \mathcal{M}(\rho^{+},\Omega_c^{0};K^{+}) + \mathcal{M}(\rho^{+},\Omega_c^{0};K^{*+}) + \mathcal{M}(\Omega_c^{0},\pi^{+};\Xi_c^{(\prime)0}) + \mathcal{M}(\Omega_c^{0},\rho^{+};\Xi_c^{(\prime)0}) + \mathcal{M}(\Omega_c^{0},\pi^{+};\Xi_c^{*\prime 0}) + \mathcal{M}(\Omega_c^{0},\rho^{+};\Xi_c^{*\prime 0}). \tag{A32}$$

The amplitudes for long-distance dominated singly Cabibbo-suppressed modes can be given as follows:

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{++}\to\Sigma_c^{++}\rho^{0}) &= C_{SD}(\Xi_{cc}^{++}\to\Sigma_c^{++}\rho^{0})\\ &+ \mathcal{M}(\pi^{+},\Lambda_c^{+}/\Sigma_c^{+};\pi^{+}) + \mathcal{M}(K^{+},\Xi_c^{(\prime)+};K^{+}) + \mathcal{M}(\pi^{+},\Lambda_c^{+}/\Sigma_c^{+};\rho^{+}) + \mathcal{M}(K^{+},\Xi_c^{(\prime)+};K^{*+})\\ &+ \mathcal{M}(\rho^{+},\Lambda_c^{+}/\Sigma_c^{+};\pi^{+}) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)+};K^{+}) + \mathcal{M}(\rho^{+},\Lambda_c^{+}/\Sigma_c^{+};\rho^{+}) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)+};K^{*+})\\ &+ \mathcal{M}(\Lambda_c^{+},\pi^{+};\Sigma_c^{+}) + \mathcal{M}(\Sigma_c^{+},\pi^{+};\Lambda_c^{+}) + \mathcal{M}(\Xi_c^{(\prime)+},K^{+};\Xi_c^{(\prime)+}) + \mathcal{M}(\Lambda_c^{+},\rho^{+};\Sigma_c^{+})\\ &+ \mathcal{M}(\Sigma_c^{+},\rho^{+};\Lambda_c^{+}) + \mathcal{M}(\Xi_c^{(\prime)+},K^{*+};\Xi_c^{(\prime)+}) + \mathcal{M}(\Lambda_c^{+},\pi^{+};\Sigma_c^{*+}) + \mathcal{M}(\Xi_c^{(\prime)+},K^{+};\Xi_c^{*\prime+})\\ &+ \mathcal{M}(\Lambda_c^{+},\rho^{+};\Sigma_c^{*+}) + \mathcal{M}(\Xi_c^{(\prime)+},K^{*+};\Xi_c^{*\prime+}),\end{aligned} \tag{A33}$$

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{++}\to\Sigma_c^{++}\phi) &= C_{SD}(\Xi_{cc}^{++}\to\Sigma_c^{++}\phi)\\ &+ \mathcal{M}(K^{+},\Xi_c^{(\prime)+};K^{+}) + \mathcal{M}(K^{+},\Xi_c^{(\prime)+};K^{*+}) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)+};K^{+}) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)+};K^{*+})\\ &+ \mathcal{M}(\Xi_c^{(\prime)+},K^{+};\Xi_c^{(\prime)+}) + \mathcal{M}(\Xi_c^{(\prime)+},K^{*+};\Xi_c^{(\prime)+}) + \mathcal{M}(\Xi_c^{(\prime)+},K^{+};\Xi_c^{*\prime+}) + \mathcal{M}(\Xi_c^{(\prime)+},K^{*+};\Xi_c^{*\prime+}),\end{aligned} \tag{A34}$$

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{++}\to\Sigma_c^{++}\omega) &= C_{SD}(\Xi_{cc}^{++}\to\Sigma_c^{++}\omega)\\ &+ \mathcal{M}(\pi^{+},\Lambda_c^{+}/\Sigma_c^{+};\pi^{+}) + \mathcal{M}(K^{+},\Xi_c^{(\prime)+};K^{+}) + \mathcal{M}(\pi^{+},\Lambda_c^{+}/\Sigma_c^{+};\rho^{+}) + \mathcal{M}(K^{+},\Xi_c^{(\prime)+};K^{*+})\\ &+ \mathcal{M}(\rho^{+},\Lambda_c^{+}/\Sigma_c^{+};\pi^{+}) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)+};K^{+}) + \mathcal{M}(\rho^{+},\Lambda_c^{+}/\Sigma_c^{+};\rho^{+}) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)+};K^{*+})\\ &+ \mathcal{M}(\Lambda_c^{+},\pi^{+};\Lambda_c^{+}) + \mathcal{M}(\Sigma_c^{+},\pi^{+};\Sigma_c^{+}) + \mathcal{M}(\Xi_c^{(\prime)+},K^{+};\Xi_c^{(\prime)+})\\ &+ \mathcal{M}(\Lambda_c^{+},\rho^{+};\Lambda_c^{+}) + \mathcal{M}(\Sigma_c^{+},\rho^{+};\Sigma_c^{+}) + \mathcal{M}(\Xi_c^{(\prime)+},K^{*+};\Xi_c^{(\prime)+})\\ &+ \mathcal{M}(\Sigma_c^{+},\pi^{+};\Sigma_c^{*+}) + \mathcal{M}(\Xi_c^{(\prime)+},K^{+};\Xi_c^{*\prime+}) + \mathcal{M}(\Sigma_c^{+},\rho^{+};\Sigma_c^{*+}) + \mathcal{M}(\Xi_c^{(\prime)+},K^{*+};\Xi_c^{*\prime+}),\end{aligned} \tag{A35}$$

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{+}\to\Sigma_c^{+}\rho^{0}) &= C_{SD}(\Xi_{cc}^{+}\to\Sigma_c^{+}\rho^{0})\\ &+ \mathcal{M}(\pi^{+},\Sigma_c^{0};\pi^{+}) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};K^{+}) + \mathcal{M}(\pi^{+},\Sigma_c^{0};\rho^{+}) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};K^{*+})\\ &+ \mathcal{M}(\rho^{+},\Sigma_c^{0};\pi^{+}) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{+}) + \mathcal{M}(\rho^{+},\Sigma_c^{0};\rho^{+}) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{*+})\\ &+ \mathcal{M}(\Sigma_c^{0},\pi^{+};\Sigma_c^{0}) + \mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Xi_c^{(\prime)0}) + \mathcal{M}(\Sigma_c^{0},\rho^{+};\Sigma_c^{0}) + \mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{(\prime)0})\\ &+ \mathcal{M}(\Sigma_c^{0},\pi^{+};\Sigma_c^{*0}) + \mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Xi_c^{*\prime 0}) + \mathcal{M}(\Sigma_c^{0},\rho^{+};\Sigma_c^{*0}) + \mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{*\prime 0}),\end{aligned} \tag{A36}$$

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{+}\to\Sigma_c^{+}\phi) &= C_{SD}(\Xi_{cc}^{+}\to\Sigma_c^{+}\phi)\\ &+ \mathcal{M}(K^{+},\Xi_c^{(\prime)0};K^{+}) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};K^{*+}) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{+}) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{*+})\\ &+ \mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Xi_c^{(\prime)0}) + \mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{(\prime)0}) + \mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Xi_c^{*\prime 0}) + \mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{*\prime 0}),\end{aligned} \tag{A37}$$

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{+}\to\Sigma_c^{+}\omega) &= C_{SD}(\Xi_{cc}^{+}\to\Sigma_c^{+}\omega)\\ &+ \mathcal{M}(\pi^{+},\Sigma_c^{0};\pi^{+}) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};K^{+}) + \mathcal{M}(\pi^{+},\Sigma_c^{0};\rho^{+}) + \mathcal{M}(K^{+},\Xi_c^{(\prime)0};K^{*+})\\ &+ \mathcal{M}(\rho^{+},\Sigma_c^{0};\pi^{+}) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{+}) + \mathcal{M}(\rho^{+},\Sigma_c^{0};\rho^{+}) + \mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{*+})\\ &+ \mathcal{M}(\Sigma_c^{0},\pi^{+};\Sigma_c^{0}) + \mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Xi_c^{(\prime)0}) + \mathcal{M}(\Sigma_c^{0},\rho^{+};\Sigma_c^{0}) + \mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{(\prime)0})\\ &+ \mathcal{M}(\Sigma_c^{0},\pi^{+};\Sigma_c^{*0}) + \mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Xi_c^{*\prime 0}) + \mathcal{M}(\Sigma_c^{0},\rho^{+};\Sigma_c^{*0}) + \mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{*\prime 0}),\end{aligned} \tag{A38}$$

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{+}\to\Lambda_c^{+}\rho^0) &= C_{SD}(\Xi_{cc}^{+}\to\Lambda_c^{+}\rho^0)\\ &+\mathcal{M}(\pi^+,\Sigma_c^0;\pi^+)+\mathcal{M}(K^+,\Xi_c^{(\prime)0};K^+)+\mathcal{M}(\pi^+,\Sigma_c^0;\rho^+)+\mathcal{M}(K^+,\Xi_c^{(\prime)0};K^{*+})\\ &+\mathcal{M}(\rho^+,\Sigma_c^0;\pi^+)+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^+)+\mathcal{M}(\rho^+,\Sigma_c^0;\rho^+)+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{*+})\\ &+\mathcal{M}(\Sigma_c^0,\pi^+;\Sigma_c^0)+\mathcal{M}(\Xi_c^{(\prime)0},K^+;\Xi_c^{(\prime)0})+\mathcal{M}(\Sigma_c^0,\rho^+;\Sigma_c^0)+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{(\prime)0})\\ &+\mathcal{M}(\Sigma_c^0,\pi^+;\Sigma_c^{*0})+\mathcal{M}(\Xi_c^{(\prime)0},K^+;\Xi_c^{*\prime 0})+\mathcal{M}(\Sigma_c^0,\rho^+;\Sigma_c^{*0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{*\prime 0}),\end{aligned}\tag{A39}$$

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{+}\to\Lambda_c^{+}\phi) &= C_{SD}(\Xi_{cc}^{+}\to\Lambda_c^{+}\phi)\\ &+\mathcal{M}(K^+,\Xi_c^{(\prime)0};K^+)+\mathcal{M}(K^+,\Xi_c^{(\prime)0};K^{*+})+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^+)+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{*+})\\ &+\mathcal{M}(\Xi_c^{(\prime)0},K^+;\Xi_c^{(\prime)0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{(\prime)0})+\mathcal{M}(\Xi_c^{(\prime)0},K^+;\Xi_c^{*\prime 0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{*\prime 0}),\end{aligned}\tag{A40}$$

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{+}\to\Lambda_c^{+}\omega) &= C_{SD}(\Xi_{cc}^{+}\to\Lambda_c^{+}\omega)\\ &+\mathcal{M}(\pi^+,\Sigma_c^0;\pi^+)+\mathcal{M}(K^+,\Xi_c^{(\prime)0};K^+)+\mathcal{M}(\pi^+,\Sigma_c^0;\rho^+)+\mathcal{M}(K^+,\Xi_c^{(\prime)0};K^{*+})\\ &+\mathcal{M}(\rho^+,\Sigma_c^0;\pi^+)+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^+)+\mathcal{M}(\rho^+,\Sigma_c^0;\rho^+)+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{*+})\\ &+\mathcal{M}(\Sigma_c^0,\pi^+;\Sigma_c^0)+\mathcal{M}(\Xi_c^{(\prime)0},K^+;\Xi_c^{(\prime)0})+\mathcal{M}(\Sigma_c^0,\rho^+;\Sigma_c^0)+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{(\prime)0})\\ &+\mathcal{M}(\Sigma_c^0,\pi^+;\Sigma_c^{*0})+\mathcal{M}(\Xi_c^{(\prime)0},K^+;\Xi_c^{*\prime 0})+\mathcal{M}(\Sigma_c^0,\rho^+;\Sigma_c^{*0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{*\prime 0}),\end{aligned}\tag{A41}$$

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{+}\to\Sigma_c^{++}\rho^-) &= \mathcal{M}(\Sigma_c^0,\pi^+;\Lambda_c^+/\Sigma_c^+)+\mathcal{M}(\Xi_c^{(\prime)0},K^+;\Xi_c^{(\prime)+})+\mathcal{M}(\Sigma_c^0,\rho^+;\Lambda_c^+/\Sigma_c^+)+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{(\prime)+})\\ &+\mathcal{M}(\Sigma_c^0,\pi^+;\Sigma_c^{*+})+\mathcal{M}(\Xi_c^{(\prime)0},K^+;\Xi_c^{*\prime+})+\mathcal{M}(\Sigma_c^0,\rho^+;\Sigma_c^{*+})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{*\prime+}),\end{aligned}\tag{A42}$$

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{+}\to\Xi_c^{\prime+}K^{*0}) &= \mathcal{M}(K^+,\Xi_c^{(\prime)0};\pi^+)+\mathcal{M}(K^+,\Xi_c^{(\prime)0};\rho^+)+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\pi^+)+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\rho^+)\\ &+\mathcal{M}(\Xi_c^{(\prime)0},K^+;\Omega_c^0)+\mathcal{M}(\Sigma_c^0,\pi^+;\Xi_c^{(\prime)0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Omega_c^0)+\mathcal{M}(\Sigma_c^0,\rho^+;\Xi_c^{(\prime)0})\\ &+\mathcal{M}(\Xi_c^{(\prime)0},K^+;\Omega_c^{*0})+\mathcal{M}(\Sigma_c^0,\pi^+;\Xi_c^{*\prime 0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Omega_c^{*0})+\mathcal{M}(\Sigma_c^0,\rho^+;\Xi_c^{*\prime 0}),\end{aligned}\tag{A43}$$

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{+}\to\Xi_c^{+}K^{*0}) &= \mathcal{M}(K^+,\Xi_c^{(\prime)0};\pi^+)+\mathcal{M}(K^+,\Xi_c^{(\prime)0};\rho^+)+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\pi^+)+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\rho^+)\\ &+\mathcal{M}(\Xi_c^{(\prime)0},K^+;\Omega_c^0)+\mathcal{M}(\Sigma_c^0,\pi^+;\Xi_c^{(\prime)0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Omega_c^0)+\mathcal{M}(\Sigma_c^0,\rho^+;\Xi_c^{(\prime)0})\\ &+\mathcal{M}(\Xi_c^{(\prime)0},K^+;\Omega_c^{*0})+\mathcal{M}(\Sigma_c^0,\pi^+;\Xi_c^{*\prime 0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Omega_c^{*0})+\mathcal{M}(\Sigma_c^0,\rho^+;\Xi_c^{*\prime 0}),\end{aligned}\tag{A44}$$

$$\begin{aligned}\mathcal{A}(\Omega_{cc}^{+}\to\Sigma_c^{+}\bar{K}^{*0}) &= \mathcal{M}(\pi^+,\Xi_c^{(\prime)0};K^+)+\mathcal{M}(\pi^+,\Xi_c^{(\prime)0};K^{*+})+\mathcal{M}(\rho^+,\Xi_c^{(\prime)0};K^+)+\mathcal{M}(\rho^+,\Xi_c^{(\prime)0};K^{*+})\\ &+\mathcal{M}(\Omega_c^0,K^+;\Xi_c^{(\prime)0})+\mathcal{M}(\Xi_c^{(\prime)0},\rho^+;\Sigma_c^0)+\mathcal{M}(\Omega_c^0,K^{*+};\Xi_c^{(\prime)0})+\mathcal{M}(\Xi_c^{(\prime)0},\rho^+;\Sigma_c^0)\\ &+\mathcal{M}(\Omega_c^0,K^+;\Xi_c^{*\prime 0})+\mathcal{M}(\Xi_c^{(\prime)0},\pi^+;\Sigma_c^{*0})+\mathcal{M}(\Omega_c^0,K^{*+};\Xi_c^{*\prime 0})+\mathcal{M}(\Xi_c^{(\prime)0},\rho^+;\Sigma_c^{*0}),\end{aligned}\tag{A45}$$

$$\begin{aligned}\mathcal{A}(\Omega_{cc}^{+}\to\Lambda_c^{+}\bar{K}^{*0}) &= \mathcal{M}(\pi^+,\Xi_c^{(\prime)0};K^+)+\mathcal{M}(\pi^+,\Xi_c^{(\prime)0};K^{*+})+\mathcal{M}(\rho^+,\Xi_c^{(\prime)0};K^+)+\mathcal{M}(\rho^+,\Xi_c^{(\prime)0};K^{*+})\\ &+\mathcal{M}(\Omega_c^0,K^+;\Xi_c^{(\prime)0})+\mathcal{M}(\Xi_c^{(\prime)0},\rho^+;\Sigma_c^0)+\mathcal{M}(\Omega_c^0,K^{*+};\Xi_c^{(\prime)0})+\mathcal{M}(\Xi_c^{(\prime)0},\rho^+;\Sigma_c^0)\\ &+\mathcal{M}(\Omega_c^0,K^+;\Xi_c^{*\prime 0})+\mathcal{M}(\Xi_c^{(\prime)0},\pi^+;\Sigma_c^{*0})+\mathcal{M}(\Omega_c^0,K^{*+};\Xi_c^{*\prime 0})+\mathcal{M}(\Xi_c^{(\prime)0},\rho^+;\Sigma_c^{*0}),\end{aligned}\tag{A46}$$

$$\begin{aligned}\mathcal{A}(\Omega_{cc}^{+}\to\Sigma_c^{++}K^{*-}) &= \mathcal{M}(\Omega_c^0,K^+;\Xi_c^{(\prime)+})+\mathcal{M}(\Xi_c^{(\prime)0},\pi^+;\Lambda_c^+/\Sigma_c^+)+\mathcal{M}(\Omega_c^0,K^{*+};\Xi_c^{\Sigma_c^+ +})+\mathcal{M}(\Xi_c^{\Sigma_c^+ 0},\rho^+;\Lambda_c^+/\Sigma_c^+)\\ &+\mathcal{M}(\Omega_c^0,K^+;\Xi_c^{*\prime+})+\mathcal{M}(\Xi_c^{(\prime)0},\pi^+;\Sigma_c^{*+})+\mathcal{M}(\Omega_c^0,K^{*+};\Xi_c^{*\prime+})+\mathcal{M}(\Xi_c^{(\prime)0},\rho^+;\Sigma_c^{*+}),\end{aligned}\tag{A47}$$

$$\begin{aligned}\mathcal{A}(\Omega_{cc}^{+}\to\Xi_c^{\prime+}\rho^0)=&\mathcal{M}(\pi^+,\Xi_c^{(\prime)0};\pi^+)+\mathcal{M}(K^+,\Omega_c^0;K^+)+\mathcal{M}(\pi^+,\Xi_c^{(\prime)0};\rho^+)+\mathcal{M}(K^+,\Omega_c^0;K^{*+})\\&+\mathcal{M}(\rho^+,\Xi_c^{(\prime)0};\pi^+)+\mathcal{M}(K^{*+},\Omega_c^0;K^+)+\mathcal{M}(\rho^+,\Xi_c^{(\prime)0};\rho^+)+\mathcal{M}(K^{*+},\Omega_c^0;K^{*+})\\&+\mathcal{M}(\Xi_c^{(\prime)0},\pi^+;\Xi_c^{(\prime)0})+\mathcal{M}(\Omega_c^0,K^+;\Omega_c^0)+\mathcal{M}(\Xi_c^{(\prime)0},\rho^+;\Xi_c^{(\prime)0})+\mathcal{M}(\Omega_c^0,K^{*+};\Omega_c^0)\\&+\mathcal{M}(\Xi_c^{(\prime)0},\pi^+;\Xi_c^{*\prime0})+\mathcal{M}(\Xi_c^{(\prime)0},\rho^+;\Xi_c^{*\prime0}),\end{aligned}\tag{A48}$$

$$\begin{aligned}\mathcal{A}(\Omega_{cc}^{+}\to\Xi_c^{\prime+}\phi)=&\mathcal{M}(K^+,\Omega_c^0;K^+)+\mathcal{M}(K^+,\Omega_c^0;K^{*+})+\mathcal{M}(K^{*+},\Omega_c^0;K^+)+\mathcal{M}(K^{*+},\Omega_c^0;K^{*+})\\&+\mathcal{M}(\Xi_c^{(\prime)0},\pi^+;\Xi_c^{(\prime)0})+\mathcal{M}(\Omega_c^0,K^+;\Omega_c^0)+\mathcal{M}(\Xi_c^{(\prime)0},\rho^+;\Xi_c^{(\prime)0})+\mathcal{M}(\Omega_c^0,K^{*+};\Omega_c^0)\\&+\mathcal{M}(\Xi_c^{(\prime)0},\pi^+;\Xi_c^{*\prime0})+\mathcal{M}(\Omega_c^0,K^+;\Omega_c^{*0})+\mathcal{M}(\Xi_c^{(\prime)0},\rho^+;\Xi_c^{*\prime0})+\mathcal{M}(\Omega_c^0,K^{*+};\Omega_c^{*0}),\end{aligned}\tag{A49}$$

$$\begin{aligned}\mathcal{A}(\Omega_{cc}^{+}\to\Xi_c^{\prime+}\omega)=&\mathcal{M}(\pi^+,\Xi_c^0;\pi^+)+\mathcal{M}(\pi^+,\Xi_c^{\prime0};\pi^+)+\mathcal{M}(K^+,\Omega_c^0;K^+)+\mathcal{M}(\pi^+,\Xi_c^0;\rho^+)\\&+\mathcal{M}(\pi^+,\Xi_c^{\prime0};\rho^+)+\mathcal{M}(K^+,\Omega_c^0;K^{*+})+\mathcal{M}(\rho^+,\Xi_c^0;\pi^+)+\mathcal{M}(\rho^+,\Xi_c^{\prime0};\pi^+)\\&+\mathcal{M}(K^{*+},\Omega_c^0;K^+)+\mathcal{M}(\rho^+,\Xi_c^0;\rho^+)+\mathcal{M}(\rho^+,\Xi_c^{\prime0};\rho^+)+\mathcal{M}(K^{*+},\Omega_c^0;K^{*+})\\&+\mathcal{M}(\Xi_c^0,\pi^+;\Xi_c^0)+\mathcal{M}(\Xi_c^{\prime0},\pi^+;\Xi_c^0)+\mathcal{M}(\Xi_c^0,\pi^+;\Xi_c^{\prime0})+\mathcal{M}(\Xi_c^{\prime0},\pi^+;\Xi_c^{\prime0})\\&+\mathcal{M}(\Xi_c^0,\rho^+;\Xi_c^0)+\mathcal{M}(\Xi_c^{\prime0},\rho^+;\Xi_c^0)+\mathcal{M}(\Xi_c^0,\rho^+;\Xi_c^{\prime0})\\&+\mathcal{M}(\Xi_c^{\prime0},\rho^+;\Xi_c^{\prime0})+\mathcal{M}(\Xi_c^0,\pi^+;\Xi_c^{*\prime0})+\mathcal{M}(\Xi_c^{\prime0},\pi^+;\Xi_c^{*\prime0})\\&+\mathcal{M}(\Xi_c^0,\rho^+;\Xi_c^{*\prime0})+\mathcal{M}(\Xi_c^{\prime0},\rho^+;\Xi_c^{*\prime0}),\end{aligned}\tag{A50}$$

$$\begin{aligned}\mathcal{A}(\Omega_{cc}^{+}\to\Xi_c^{+}\rho^0)=&\mathcal{M}(\pi^+,\Xi_c^{(\prime)0};\pi^+)+\mathcal{M}(K^+,\Omega_c^0;K^+)+\mathcal{M}(\pi^+,\Xi_c^{(\prime)0};\rho^+)+\mathcal{M}(K^+,\Omega_c^0;K^{*+})\\&+\mathcal{M}(\rho^+,\Xi_c^{(\prime)0};\pi^+)+\mathcal{M}(K^{*+},\Omega_c^0;K^+)+\mathcal{M}(\rho^+,\Xi_c^{(\prime)0};\rho^+)+\mathcal{M}(K^{*+},\Omega_c^0;K^{*+})\\&+\mathcal{M}(\Xi_c^{(\prime)0},\pi^+;\Xi_c^{(\prime)0})+\mathcal{M}(\Omega_c^0,K^+;\Omega_c^0)+\mathcal{M}(\Xi_c^{(\prime)0},\rho^+;\Xi_c^{(\prime)0})+\mathcal{M}(\Omega_c^0,K^{*+};\Omega_c^0)\\&+\mathcal{M}(\Xi_c^{(\prime)0},\pi^+;\Xi_c^{*\prime0})+\mathcal{M}(\Xi_c^{(\prime)0},\rho^+;\Xi_c^{*\prime0}),\end{aligned}\tag{A51}$$

$$\begin{aligned}\mathcal{A}(\Omega_{cc}^{+}\to\Xi_c^{+}\phi)=&\mathcal{M}(K^+,\Omega_c^0;K^+)+\mathcal{M}(K^+,\Omega_c^0;K^{*+})+\mathcal{M}(K^{*+},\Omega_c^0;K^+)+\mathcal{M}(K^{*+},\Omega_c^0;K^{*+})\\&+\mathcal{M}(\Xi_c^{(\prime)0},\pi^+;\Xi_c^{(\prime)0})+\mathcal{M}(\Omega_c^0,K^+;\Omega_c^0)+\mathcal{M}(\Xi_c^{(\prime)0},\rho^+;\Xi_c^{(\prime)0})+\mathcal{M}(\Omega_c^0,K^{*+};\Omega_c^0)\\&+\mathcal{M}(\Xi_c^{(\prime)0},\pi^+;\Xi_c^{*\prime0})+\mathcal{M}(\Omega_c^0,K^+;\Omega_c^{*0})+\mathcal{M}(\Xi_c^{(\prime)0},\rho^+;\Xi_c^{*\prime0})+\mathcal{M}(\Omega_c^0,K^{*+};\Omega_c^{*0}),\end{aligned}\tag{A52}$$

$$\begin{aligned}\mathcal{A}(\Omega_{cc}^{+}\to\Xi_c^{+}\omega)=&\mathcal{M}(\pi^+,\Xi_c^{(\prime)0};\pi^+)+\mathcal{M}(K^+,\Omega_c^0;K^+)+\mathcal{M}(\pi^+,\Xi_c^{(\prime)0};\rho^+)+\mathcal{M}(K^+,\Omega_c^0;K^{*+})\\&+\mathcal{M}(\rho^+,\Xi_c^{(\prime)0};\pi^+)+\mathcal{M}(K^{*+},\Omega_c^0;K^+)+\mathcal{M}(\rho^+,\Xi_c^{(\prime)0};\rho^+)+\mathcal{M}(K^{*+},\Omega_c^0;K^{*+})\\&+\mathcal{M}(\Xi_c^{(\prime)0},\pi^+;\Xi_c^{(\prime)0})+\mathcal{M}(\Omega_c^0,K^+;\Omega_c^0)+\mathcal{M}(\Xi_c^{(\prime)0},\rho^+;\Xi_c^{(\prime)0})+\mathcal{M}(\Omega_c^0,K^{*+};\Omega_c^0)\\&+\mathcal{M}(\Xi_c^{(\prime)0},\pi^+;\Xi_c^{*\prime0})+\mathcal{M}(\Xi_c^{(\prime)0},\rho^+;\Xi_c^{*\prime0}).\end{aligned}\tag{A53}$$

The amplitudes for long-distance dominated doubly Cabibbo-suppressed modes can be given as follows:

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{++}\to\Sigma_c^{++}K^{*0})=&C_{SD}(\Xi_{cc}^{++}\to\Sigma_c^{++}K^{*0})\\&+\mathcal{M}(K^+,\Lambda_c^+/\Sigma_c^+;\pi^+)+\mathcal{M}(K^+,\Lambda_c^+/\Sigma_c^+;\rho^+)+\mathcal{M}(K^{*+},\Lambda_c^+/\Sigma_c^+;\pi^+)+\mathcal{M}(K^{*+},\Lambda_c^+/\Sigma_c^+;\rho^+)\\&+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,K^+;\Xi_c^{(\prime)+})+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,K^{*+};\Xi_c^{(\prime)+})\\&+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,K^+;\Xi_c^{*\prime+})+\mathcal{M}(\Lambda_c^+/\Sigma_c^+,K^{*+};\Xi_c^{*\prime+}),\end{aligned}\tag{A54}$$

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{+}\to\Sigma_c^{+}K^{*0}) &= C_{SD}(\Xi_{cc}^{+}\to\Sigma_c^{+}K^{*0})\\ &+\mathcal{M}(K^{+},\Sigma_c^{0};\pi^{+})+\mathcal{M}(K^{+},\Sigma_c^{0};\rho^{+})+\mathcal{M}(K^{*+},\Sigma_c^{0};\pi^{+})+\mathcal{M}(K^{*+},\Sigma_c^{0};\rho^{+})\\ &+\mathcal{M}(\Sigma_c^{0},K^{+};\Xi_c^{(\prime)0})+\mathcal{M}(\Sigma_c^{0},K^{*+};\Xi_c^{(\prime)0})+\mathcal{M}(\Sigma_c^{0},K^{+};\Xi_c^{*\prime0})+\mathcal{M}(\Sigma_c^{0},K^{*+};\Xi_c^{*\prime0}),\end{aligned}\tag{A55}$$

$$\begin{aligned}\mathcal{A}(\Xi_{cc}^{+}\to\Lambda_c^{+}K^{*0}) &= C_{SD}(\Xi_{cc}^{+}\to\Lambda_c^{+}K^{*0})\\ &+\mathcal{M}(K^{+},\Sigma_c^{0};\pi^{+})+\mathcal{M}(K^{+},\Sigma_c^{0};\rho^{+})+\mathcal{M}(K^{*+},\Sigma_c^{0};\pi^{+})+\mathcal{M}(K^{*+},\Sigma_c^{0};\rho^{+})\\ &+\mathcal{M}(\Sigma_c^{0},K^{+};\Xi_c^{(\prime)0})+\mathcal{M}(\Sigma_c^{0},K^{*+};\Xi_c^{(\prime)0})+\mathcal{M}(\Sigma_c^{0},K^{+};\Xi_c^{*\prime0})+\mathcal{M}(\Sigma_c^{0},K^{*+};\Xi_c^{*\prime0}),\end{aligned}\tag{A56}$$

$$\begin{aligned}\mathcal{A}(\Omega_{cc}^{+}\to\Sigma_c^{+}\rho^{0}) &= \mathcal{M}(K^{+},\Xi_c^{(\prime)0};K^{+})+\mathcal{M}(K^{+},\Xi_c^{(\prime)0};K^{*+})+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{+})+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{*+})\\ &+\mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Xi_c^{(\prime)0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{(\prime)0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Xi_c^{*\prime0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{*\prime0}),\end{aligned}\tag{A57}$$

$$\begin{aligned}\mathcal{A}(\Omega_{cc}^{+}\to\Sigma_c^{+}\phi) &= \mathcal{M}(K^{+},\Xi_c^{(\prime)0};K^{+})+\mathcal{M}(K^{+},\Xi_c^{(\prime)0};K^{*+})+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{+})+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{*+})\\ &+\mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Xi_c^{(\prime)0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{(\prime)0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Xi_c^{*\prime0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{*\prime0}),\end{aligned}\tag{A58}$$

$$\begin{aligned}\mathcal{A}(\Omega_{cc}^{+}\to\Sigma_c^{+}\omega) &= \mathcal{M}(K^{+},\Xi_c^{(\prime)0};K^{+})+\mathcal{M}(K^{+},\Xi_c^{(\prime)0};K^{*+})+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{+})+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{*+})\\ &+\mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Xi_c^{(\prime)0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{(\prime)0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Xi_c^{*\prime0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{*\prime0}),\end{aligned}\tag{A59}$$

$$\begin{aligned}\mathcal{A}(\Omega_{cc}^{+}\to\Lambda_c^{+}\rho^{0}) &= \mathcal{M}(K^{+},\Xi_c^{(\prime)0};K^{+})+\mathcal{M}(K^{+},\Xi_c^{(\prime)0};K^{*+})+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{+})+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{*+})\\ &+\mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Xi_c^{(\prime)0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{(\prime)0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Xi_c^{*\prime0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{*\prime0}),\end{aligned}\tag{A60}$$

$$\begin{aligned}\mathcal{A}(\Omega_{cc}^{+}\to\Lambda_c^{+}\phi) &= \mathcal{M}(K^{+},\Xi_c^{(\prime)0};K^{+})+\mathcal{M}(K^{+},\Xi_c^{(\prime)0};K^{*+})+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{+})+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{*+})\\ &+\mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Xi_c^{(\prime)0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{(\prime)0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Xi_c^{*\prime0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{*\prime0}),\end{aligned}\tag{A61}$$

$$\begin{aligned}\mathcal{A}(\Omega_{cc}^{+}\to\Lambda_c^{+}\omega) &= \mathcal{M}(K^{+},\Xi_c^{(\prime)0};K^{+})+\mathcal{M}(K^{+},\Xi_c^{(\prime)0};K^{*+})+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{+})+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{*+})\\ &+\mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Xi_c^{(\prime)0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{(\prime)0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Xi_c^{*\prime0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{*\prime0}),\end{aligned}\tag{A62}$$

$$\mathcal{A}(\Omega_{cc}^{+}\to\Sigma_c^{0}\rho^{+}) = \mathcal{M}(K^{+},\Xi_c^{(\prime)0};K^{0})+\mathcal{M}(K^{+},\Xi_c^{(\prime)0};K^{*0})+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{0})+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};K^{*0}),\tag{A63}$$

$$\mathcal{A}(\Omega_{cc}^{+}\to\Sigma_c^{++}\rho^{-}) = \mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Xi_c^{(\prime)+})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{(\prime)+})+\mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Xi_c^{*\prime+})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Xi_c^{*\prime+}),\tag{A64}$$

$$\begin{aligned}\mathcal{A}(\Omega_{cc}^{+}\to\Xi_c^{\prime+}K^{*0}) &= C_{SD}(\Omega_{cc}^{+}\to\Xi_c^{\prime+}K^{*0})\\ &+\mathcal{M}(K^{+},\Xi_c^{(\prime)0};\pi^{+})+\mathcal{M}(K^{+},\Xi_c^{(\prime)0};\rho^{+})+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\pi^{+})+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\rho^{+})\\ &+\mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Omega_c^{0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Omega_c^{0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Omega_c^{*0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Omega_c^{*0}),\end{aligned}\tag{A65}$$

$$\begin{aligned}\mathcal{A}(\Omega_{cc}^{+}\to\Xi_c^{+}K^{*0}) &= C_{SD}(\Omega_{cc}^{+}\to\Xi_c^{+}K^{*0})\\ &+\mathcal{M}(K^{+},\Xi_c^{(\prime)0};\pi^{+})+\mathcal{M}(K^{+},\Xi_c^{(\prime)0};\rho^{+})+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\pi^{+})+\mathcal{M}(K^{*+},\Xi_c^{(\prime)0};\rho^{+})\\ &+\mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Omega_c^{0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Omega_c^{0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{+};\Omega_c^{*0})+\mathcal{M}(\Xi_c^{(\prime)0},K^{*+};\Omega_c^{*0}).\end{aligned}\tag{A66}$$

APPENDIX B: LAGRANGIAN COUPLINGS

The hadronic strong interaction Lagrangians [58–63] can be presented as follows:

$$\mathcal{L}_{VPP} = ig_{VP_1P_2}[V^{\mu}P_1\partial_{\mu}P_2 - V^{\mu}P_2\partial_{\mu}P_1],\tag{B1}$$

$$\mathcal{L}_{VVP} = \frac{g_{V_1V_2P}}{\sqrt{m_{V_1}m_{V_2}}}\varepsilon_{\mu\nu\alpha\beta}\mathrm{Tr}[\partial^\mu V_1^\nu \partial^\alpha P V_2^\beta], \tag{B2}$$

$$\mathcal{L}_{V_1V_2V_3} = i\frac{g_{V_1V_2V_3}}{\sqrt{2}}[V_1^\alpha(\partial_\alpha V_2^\beta V_{3\beta} - V_2^\beta\partial_\alpha V_{3\beta}) + (\partial_\alpha V_1^\beta V_{2\beta} - V_1^\beta\partial_\alpha V_{2\beta})V_3^\alpha + V_2^\alpha(V_1^\beta\partial_\alpha V_{3\beta} - \partial_\alpha V_1^\beta V_{3\beta})], \tag{B3}$$

$$\mathcal{L}_{P\mathcal{B}_c\mathcal{B}_c} = g_{P\mathcal{B}_c\mathcal{B}_c}\mathrm{Tr}[\bar{\mathcal{B}}_c i\gamma_5 P\mathcal{B}_c], \tag{B4}$$

$$\mathcal{L}_{V\mathcal{B}_c\mathcal{B}_c} = f_{1V\mathcal{B}_c\mathcal{B}_c}\mathrm{Tr}[\bar{\mathcal{B}}_c\gamma_\mu V^\mu\mathcal{B}_c] + \frac{f_{2V\mathcal{B}_c\mathcal{B}_c}}{m_{\mathcal{B}_c}+m_{\mathcal{B}_c}}\mathrm{Tr}[\bar{\mathcal{B}}_c\sigma_{\mu\nu}\partial^\mu V^\nu\mathcal{B}_c], \tag{B5}$$

$$\mathcal{L}_{P\mathcal{B}_c\mathcal{B}_c^*} = g_{P\mathcal{B}_c\mathcal{B}_c^*}\epsilon^{ijk}(\bar{\mathcal{B}}_c)_i^j(\mathcal{B}_c^*)_\mu^{mkl}\partial^\mu P_m^i, \tag{B6}$$

$$\mathcal{L}_{V\mathcal{B}_c\mathcal{B}_c^*} = -ig_{V\mathcal{B}_c\bar{\mathcal{B}}_c^*}[\bar{\mathcal{B}}_c^{*\mu}\gamma^5\gamma^\nu\mathcal{B}_c + \bar{\mathcal{B}}_c\gamma^5\gamma^\nu\mathcal{B}_c^*](\partial_\mu V_\nu - \partial_\nu V_\mu). \tag{B7}$$

For estimating the hadronic scattering amplitudes, we need the following Feynman rules derived from the matrix element of the effective hadronic Lagrangian with definite initial and final states:

$$\langle V(k,\lambda_k)P(p_2)|i\mathcal{L}_{VPP}|P(p_1)\rangle = ig_{VVP}\varepsilon^{*\mu}(k,\lambda_k)(p_1+p_2)_\mu, \tag{B8}$$

$$\langle \mathcal{B}_2(p_2)P(q)|i\mathcal{L}_{PBB}|\mathcal{B}_1(p_1)\rangle = ig_{BBP}\bar{u}(p_2)i\gamma_5 u(p_1), \tag{B9}$$

$$\langle \mathcal{B}_2(p_2)V(q,\lambda_q)|i\mathcal{L}_{VBB}|\mathcal{B}_1(p_1)\rangle = i\bar{u}(p_2)\left[f_1\gamma_\nu + f_2\frac{i}{m_1+m_2}\sigma_{\mu\nu}q^\mu\right]\varepsilon^{*\nu}(q,\lambda_q)u(p_1), \tag{B10}$$

$$\langle V(p_3,\lambda_3)V(k,\lambda_k)|i\mathcal{L}_{VVP}|P(p_1)\rangle = i\frac{g_{V_1V_2P}}{\sqrt{m_{V_1}m_{V_2}}}\epsilon^{\mu\nu\alpha\beta}p_{3\mu}\varepsilon_\nu^*(\lambda_3,p_3)p_{1\alpha}\varepsilon_\beta^*(k,\lambda_k), \tag{B11}$$

$$\langle B(p_4,\lambda_4)|i\mathcal{L}_{VBB^*}|B^*(k,\lambda_k)V(p_1,\lambda_1)\rangle = -ig_{V\mathcal{B}_c\mathcal{B}_c^*}\bar{u}(p_4,\lambda_4)\gamma^5\gamma^\nu u^\mu(k,\lambda_k)[p_{1\mu}\varepsilon_\nu(p_1,\lambda_1) - p_{1\nu}\varepsilon_\mu(p_1,\lambda_1)], \tag{B12}$$

$$\langle B(p_{4,\lambda_4})|i\mathcal{L}_{PBB^*}|B^*(k,\lambda_k)P(p_1,\lambda_1)\rangle = g_{P\mathcal{B}_c\mathcal{B}_c^*}\bar{u}(p_4,\lambda_4)p_{1\mu}u^\mu(k,\lambda_k), \tag{B13}$$

$$\begin{aligned}\langle V(p_3,\lambda_3)V(k,\lambda_k)|i\mathcal{L}_{VVV}|V(p_1,\lambda_1)\rangle = &-\frac{ig_{VVV}}{\sqrt{2}}\varepsilon_\mu(p_1,\lambda_1)\varepsilon^{\mu*}(p_3,\lambda_3)\varepsilon_\nu^*(k,\lambda_k)(p_3^\nu+p_1^\nu)\\ &-\frac{ig_{VVV}}{\sqrt{2}}\varepsilon_\mu^*(k,\lambda_k)\varepsilon^\mu(p_1,\lambda_1)\varepsilon_\nu^*(p_3,\lambda_3)(-p_1^\nu-p_k^\nu)\\ &-\frac{ig_{VVV}}{\sqrt{2}}\varepsilon_\mu^*(p_3,\lambda_3)\varepsilon^{*\mu}(k,\lambda_k)\varepsilon_\nu(p_1,\lambda_1)(p_k^\nu-p_3^\nu),\end{aligned} \tag{B14}$$

where $\varepsilon^{\alpha\beta\rho\tau}$ is the Levi-Civita tensor.

APPENDIX C: STRONG COUPLINGS

In general, meson-baryon and meson-meson strong couplings can be obtained from the SU(3)-invariant strong Hamiltonian. In the present work, we use the SU(3) flavor symmetry to analyze the vector-vector-pseudoscalar (VVP), vector-pseudoscalar-pseudoscalar (VPP), baryon-baryon-pseudoscalar (BBP/BB^*P), and baryon-baryon-vector (BBV/BB^*V) couplings. Under SU(3) flavor symmetry, the pseudoscalar and vector meson can be written as follows:

$$P(J^P=0^-) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta_8 \end{pmatrix} + \frac{1}{\sqrt{3}}\begin{pmatrix} \eta_1 & 0 & 0 \\ 0 & \eta_1 & 0 \\ 0 & 0 & \eta_1 \end{pmatrix}, \tag{C1}$$

$$V(J^P=1^-)=\begin{pmatrix}\frac{\rho^0}{\sqrt{2}}+\frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+}\\ \rho^- & -\frac{\rho^0}{\sqrt{2}}+\frac{\omega}{\sqrt{2}} & K^{*0}\\ K^{*-} & \bar{K}^{*0} & \phi\end{pmatrix}. \tag{C2}$$

The spin-parity $1/2^+$ singly charmed baryons include antitriplet $\mathcal{B}_{\bar{\mathbf{3}}}$ and sextet $\mathcal{B}_{\mathbf{6}}$, which can be given as follows:

$$B_6\left(J^P=\frac{1^+}{2}\right)=\begin{pmatrix}\Sigma_c^{++} & \frac{\Sigma_c^+}{\sqrt{2}} & \frac{\Xi_c^{\prime+}}{\sqrt{2}}\\ \frac{\Sigma_c^+}{\sqrt{2}} & \Sigma_c^0 & \frac{\Xi_c^{\prime0}}{\sqrt{2}}\\ \frac{\Xi_c^{\prime+}}{\sqrt{2}} & \frac{\Xi_c^{\prime0}}{\sqrt{2}} & \Omega_c^0\end{pmatrix},\qquad B_{\bar{3}}\left(J^P=\frac{1^+}{2}\right)=\begin{pmatrix}0 & \Lambda_c^+ & \Xi_c^+\\ -\Lambda_c^+ & 0 & \Xi_c^0\\ -\Xi_c^+ & -\Xi_c^0 & 0\end{pmatrix}. \tag{C3}$$

In addition, the spin-parity $3/2^+$ singly charmed baryons sextet $\mathcal{B}_{\mathbf{6}}$ can be given as follows:

$$B_6^*\left(J^P=\frac{3^+}{2}\right)=\begin{pmatrix}\Sigma_c^{*++} & \frac{\Sigma_c^{*+}}{\sqrt{2}} & \frac{\Xi_c^{*\prime+}}{\sqrt{2}}\\ \frac{\Sigma_c^{*+}}{\sqrt{2}} & \Sigma_c^{*0} & \frac{\Xi_c^{*\prime0}}{\sqrt{2}}\\ \frac{\Xi_c^{*\prime+}}{\sqrt{2}} & \frac{\Xi_c^{*\prime0}}{\sqrt{2}} & \Omega_c^{*0}\end{pmatrix}. \tag{C4}$$

Then, with the help of Eqs. (C2)–(C4), we can obtain the relationships of these strong couplings, as shown in Tables X–XIV.

TABLE X. The strong coupling constants between the vector mesons and two pseudoscalar mesons g_{VPP} can be obtained by using the coupling $g_{\rho\pi\pi}=6.05=-\frac{g_{VPP}}{\sqrt{2}}$ [55] and SU(3) relations.

Vertex	g	Vertex	g	Vertex	g	Vertex	g
$\rho^0\pi^+\pi^-$	$-\frac{g_{VPP}}{\sqrt{2}}$	$\rho^0\pi^0\eta_8$	$\frac{g_{VPP}}{\sqrt{6}}$	$\rho^0\pi^0\eta_1$	$\frac{g_{VPP}}{\sqrt{3}}$	$\rho^0\pi^-\pi^+$	$\frac{g_{VPP}}{\sqrt{2}}$
$\rho^0\bar{K}^0K^0$	$-\frac{g_{VPP}}{\sqrt{2}}$	$\rho^0K^-K^+$	$\frac{g_{VPP}}{\sqrt{2}}$	$\rho^0\eta_8\pi^0$	$\frac{g_{VPP}}{\sqrt{6}}$	$\rho^0\eta_1\pi^0$	$\frac{g_{VPP}}{\sqrt{3}}$
$\omega\pi^+\pi^-$	$\frac{g_{VPP}}{\sqrt{2}}$	$\omega\pi^0\pi^0$	$\frac{g_{VPP}}{\sqrt{2}}$	$\omega\pi^-\pi^+$	$\frac{g_{VPP}}{\sqrt{2}}$	$\omega\bar{K}^0K^0$	$\frac{g_{VPP}}{\sqrt{2}}$
ωK^-K^+	$\frac{g_{VPP}}{\sqrt{2}}$	$\omega\eta_8\eta_8$	$\frac{g_{VPP}}{3\sqrt{2}}$	$\omega\eta_8\eta_1$	$\frac{g_{VPP}}{3}$	$\omega\eta_1\eta_8$	$\frac{g_{VPP}}{3}$
$\omega\eta_1\eta_1$	$\frac{\sqrt{2}g_{VPP}}{3}$	$\rho^+\pi^+\pi^0$	$\frac{g_{VPP}}{\sqrt{2}}$	$\rho^+\pi^+\eta_8$	$\frac{g_{VPP}}{\sqrt{6}}$	$\rho^+\pi^+\eta_1$	$\frac{g_{VPP}}{\sqrt{3}}$
$\rho^+\pi^0\pi^+$	$-\frac{g_{VPP}}{\sqrt{2}}$	$\rho^+\bar{K}^0K^+$	g_{VPP}	$\rho^+\eta_8\pi^+$	$\frac{g_{VPP}}{\sqrt{6}}$	$\rho^+\eta_1\pi^+$	$\frac{g_{VPP}}{\sqrt{3}}$
$K^{*+}K^+\pi^0$	$\frac{g_{VPP}}{\sqrt{2}}$	$K^{*+}K^+\eta_8$	$\frac{g_{VPP}}{\sqrt{6}}$	$K^{*+}K^+\eta_1$	$\frac{g_{VPP}}{\sqrt{3}}$	$K^{*+}K^0\pi^+$	g_{VPP}
$K^{*+}\eta_8K^+$	$-\sqrt{\frac{2}{3}}g_{VPP}$	$K^{*+}\eta_1K^+$	$\frac{g_{VPP}}{\sqrt{3}}$	$\rho^-\pi^0\pi^-$	$\frac{g_{VPP}}{\sqrt{2}}$	$\rho^-\pi^-\pi^0$	$-\frac{g_{VPP}}{\sqrt{2}}$
$\rho^-\pi^-\eta_8$	$\frac{g_{VPP}}{\sqrt{6}}$	$\rho^-\pi^-\eta_1$	$\frac{g_{VPP}}{\sqrt{3}}$	$\rho^-K^-K^0$	g_{VPP}	$\rho^-\eta_8\pi^-$	$\frac{g_{VPP}}{\sqrt{6}}$
$\rho^-\eta_1\pi^-$	$\frac{g_{VPP}}{\sqrt{3}}$	$K^{*0}K^+\pi^-$	g_{VPP}	$K^{*0}K^0\pi^0$	$-\frac{g_{VPP}}{\sqrt{2}}$	$K^{*0}K^0\eta_8$	$\frac{g_{VPP}}{\sqrt{6}}$
$K^{*0}K^0\eta_1$	$\frac{g_{VPP}}{\sqrt{3}}$	$K^{*0}\eta_8K^0$	$-\sqrt{\frac{2}{3}}g_{VPP}$	$K^{*0}\eta_1K^0$	$\frac{g_{VPP}}{\sqrt{3}}$	$K^{*-}\pi^0K^-$	$\frac{g_{VPP}}{\sqrt{2}}$
$K^{*-}\pi^-\bar{K}^0$	g_{VPP}	$K^{*-}K^-\eta_8$	$-\sqrt{\frac{2}{3}}g_{VPP}$	$K^{*-}K^-\eta_1$	$\frac{g_{VPP}}{\sqrt{3}}$	$K^{*-}\eta_8K^-$	$\frac{g_{VPP}}{\sqrt{6}}$
$K^{*-}\eta_1K^-$	$\frac{g_{VPP}}{\sqrt{3}}$	$\bar{K}^{*0}\pi^+K^-$	g_{VPP}	$\bar{K}^{*0}\pi^0\bar{K}^0$	$-\frac{g_{VPP}}{\sqrt{2}}$	$\bar{K}^{*0}\bar{K}^0\eta_8$	$-\sqrt{\frac{2}{3}}g_{VPP}$
$\bar{K}^{*0}\bar{K}^0\eta_1$	$\frac{g_{VPP}}{\sqrt{3}}$	$\bar{K}^{*0}\eta_8\bar{K}^0$	$\frac{g_{VPP}}{\sqrt{6}}$	$\bar{K}^{*0}\eta_1\bar{K}^0$	$\frac{g_{VPP}}{\sqrt{3}}$	ϕK^+K^-	g_{VPP}
$\phi K^0\bar{K}^0$	g_{VPP}	$\phi\eta_8\eta_8$	$\frac{2g_{VPP}}{3}$	$\phi\eta_8\eta_1$	$-\frac{1}{3}\sqrt{2}g_{VPP}$	$\phi\eta_1\eta_8$	$-\frac{1}{3}\sqrt{2}g_{VPP}$
$\phi\eta_1\eta_1$	$\frac{g_{VPP}}{3}$						

TABLE XI. The strong couplings between the pseudoscalar meson (P) and two vector mesons (V) can be obtained by using the coupling constant $g_{\omega\rho\pi} = g_{VVP}/\sqrt{2} = -10$ [74] and SU(3) relationships.

Vertex	g	Vertex	g	Vertex	g	Vertex	g
$\rho^0\rho^0\eta_8$	$\frac{g_{VVP}}{\sqrt{6}}$	$\rho^0\rho^0\eta_1$	$\frac{g_{VVP}}{\sqrt{3}}$	$\rho^0\omega\pi^0$	$\frac{g_{VVP}}{\sqrt{2}}$	$\rho^0\rho^+\pi^-$	$-\frac{g_{VVP}}{\sqrt{2}}$
$\rho^0\rho^-\pi^+$	$\frac{g_{VVP}}{\sqrt{2}}$	$\rho^0 K^{*-}K^+$	$\frac{g_{VVP}}{\sqrt{2}}$	$\rho^0\bar{K}^{*0}K^0$	$-\frac{g_{VVP}}{\sqrt{2}}$	$\omega\rho^0\pi^0$	$\frac{g_{VVP}}{\sqrt{2}}$
$\omega\omega\eta_8$	$\frac{g_{VVP}}{\sqrt{6}}$	$\omega\omega\eta_1$	$\frac{g_{VVP}}{\sqrt{3}}$	$\omega\rho^+\pi^-$	$\frac{g_{VVP}}{\sqrt{2}}$	$\omega\rho^-\pi^+$	$\frac{g_{VVP}}{\sqrt{2}}$
$\omega K^{*-}K^+$	$\frac{g_{VVP}}{\sqrt{2}}$	$\omega\bar{K}^{*0}K^0$	$\frac{g_{VVP}}{\sqrt{2}}$	$\rho^+\rho^0\pi^+$	$-\frac{g_{VVP}}{\sqrt{2}}$	$\rho^+\omega\pi^+$	$\frac{g_{VVP}}{\sqrt{2}}$
$\rho^+\rho^+\pi^0$	$\frac{g_{VVP}}{\sqrt{2}}$	$\rho^+\rho^+\eta_8$	$\frac{g_{VVP}}{\sqrt{6}}$	$\rho^+\rho^+\eta_1$	$\frac{g_{VVP}}{\sqrt{3}}$	$\rho^+\bar{K}^{*0}K^+$	g_{VVP}
$K^{*+}K^{*+}\pi^0$	$\frac{g_{VVP}}{\sqrt{2}}$	$K^{*+}K^{*+}\eta_8$	$\frac{g_{VVP}}{\sqrt{6}}$	$K^{*+}K^{*+}\eta_1$	$\frac{g_{VVP}}{\sqrt{3}}$	$K^{*+}K^{*0}\pi^+$	g_{VVP}
$K^{*+}\phi K^+$	g_{VVP}	$\rho^-\rho^0\pi^-$	$\frac{g_{VVP}}{\sqrt{2}}$	$\rho^-\omega\pi^-$	$\frac{g_{VVP}}{\sqrt{2}}$	$\rho^-\rho^-\pi^0$	$-\frac{g_{VVP}}{\sqrt{2}}$
$\rho^-\rho^-\eta_8$	$\frac{g_{VVP}}{\sqrt{6}}$	$\rho^-\rho^-\eta_1$	$\frac{g_{VVP}}{\sqrt{3}}$	$\rho^- K^{*-}K^0$	g_{VVP}	$K^{*0}K^{*+}\pi^-$	g_{VVP}
$K^{*0}K^{*0}\pi^0$	$-\frac{g_{VVP}}{\sqrt{2}}$	$K^{*0}K^{*0}\eta_8$	$\frac{g_{VVP}}{\sqrt{6}}$	$K^{*0}K^{*0}\eta_1$	$\frac{g_{VVP}}{\sqrt{3}}$	$K^{*0}\phi K^0$	g_{VVP}
$K^{*-}\rho^0K^-$	$\frac{g_{VVP}}{\sqrt{2}}$	$K^{*-}\omega K^-$	$\frac{g_{VVP}}{\sqrt{2}}$	$K^{*-}\rho^-\bar{K}^0$	g_{VVP}	$K^{*-}K^{*-}\eta_8$	$-\sqrt{\frac{2}{3}}g_{VVP}$
$K^{*-}K^{*-}\eta_1$	$\frac{g_{VVP}}{\sqrt{3}}$	$\bar{K}^{*0}\rho^0\bar{K}^0$	$-\frac{g_{VVP}}{\sqrt{2}}$	$\bar{K}^{*0}\omega\bar{K}^0$	$\frac{g_{VVP}}{\sqrt{2}}$	$\bar{K}^{*0}\rho^+K^-$	g_{VVP}
$\bar{K}^{*0}\bar{K}^{*0}\eta_8$	$-\sqrt{\frac{2}{3}}g_{VVP}$	$\bar{K}^{*0}\bar{K}^{*0}\eta_1$	$\frac{g_{VVP}}{\sqrt{3}}$	$\phi K^{*+}K^-$	g_{VVP}	$\phi K^{*0}\bar{K}^0$	g_{VVP}
$\phi\phi\eta_8$	$-\sqrt{\frac{2}{3}}g_{VVP}$	$\phi\phi\eta_1$	$\frac{g_{VVP}}{\sqrt{3}}$				

TABLE XII. The strong coupling constants between three vector mesons g_{VVV} can be obtained by using the coupling $g_{\rho\rho\rho} = 4.17 = -\frac{g_{VVV}}{\sqrt{2}}$ [77] and SU(3) relations.

Vertex	g	Vertex	g	Vertex	g	Vertex	g
$\rho^0\rho^0\omega$	$\frac{g_{VVV}}{\sqrt{2}}$	$\rho^0\omega\rho^0$	$\frac{g_{VVV}}{\sqrt{2}}$	$\rho^0\rho^+\rho^-$	$-\frac{g_{VVV}}{\sqrt{2}}$	$\rho^0\rho^-\rho^+$	$\frac{g_{VVV}}{\sqrt{2}}$
$\rho^0 K^{*-}K^{*+}$	$\frac{g_{VVV}}{\sqrt{2}}$	$\rho^0\bar{K}^{*0}K^{*0}$	$-\frac{g_{VVV}}{\sqrt{2}}$	$\omega\rho^0\rho^0$	$\frac{g_{VVV}}{\sqrt{2}}$	$\omega\omega\omega$	$\frac{g_{VVV}}{\sqrt{2}}$
$\omega\rho^+\rho^-$	$\frac{g_{VVV}}{\sqrt{2}}$	$\omega\rho^-\rho^+$	$\frac{g_{VVV}}{\sqrt{2}}$	$\omega K^{*-}K^{*+}$	$\frac{g_{VVV}}{\sqrt{2}}$	$\omega\bar{K}^{*0}K^{*0}$	$\frac{g_{VVV}}{\sqrt{2}}$
$\rho^+\rho^0\rho^+$	$-\frac{g_{VVV}}{\sqrt{2}}$	$\rho^+\omega\rho^+$	$\frac{g_{VVV}}{\sqrt{2}}$	$\rho^+\rho^+\rho^0$	$\frac{g_{VVV}}{\sqrt{2}}$	$\rho^+\rho^+\omega$	$\frac{g_{VVV}}{\sqrt{2}}$
$\rho^+\bar{K}^{*0}K^{*+}$	g_{VVV}	$K^{*+}K^{*+}\rho^0$	$\frac{g_{VVV}}{\sqrt{2}}$	$K^{*+}K^{*+}\omega$	$\frac{g_{VVV}}{\sqrt{2}}$	$K^{*+}K^{*0}\rho^+$	g_{VVV}
$K^{*+}\phi K^{*+}$	g_{VVV}	$\rho^-\rho^0\rho^-$	$\frac{g_{VVV}}{\sqrt{2}}$	$\rho^-\omega\rho^-$	$\frac{g_{VVV}}{\sqrt{2}}$	$\rho^-\rho^-\rho^0$	$-\frac{g_{VVV}}{\sqrt{2}}$
$\rho^-\rho^-\omega$	$\frac{g_{VVV}}{\sqrt{2}}$	$\rho^- K^{*-}K^{*0}$	g_{VVV}	$K^{*0}K^{*+}\rho^-$	g_{VVV}	$K^{*0}K^{*0}\rho^0$	$-\frac{g_{VVV}}{\sqrt{2}}$
$K^{*0}K^{*0}\omega$	$\frac{g_{VVV}}{\sqrt{2}}$	$K^{*0}\phi K^{*0}$	g_{VVV}	$K^{*-}\rho^0K^{*-}$	$\frac{g_{VVV}}{\sqrt{2}}$	$K^{*-}\omega K^{*-}$	$\frac{g_{VVV}}{\sqrt{2}}$
$K^{*-}\rho^-\bar{K}^{*0}$	g_{VVV}	$K^{*-}K^{*-}\phi$	g_{VVV}	$\bar{K}^{*0}\rho^0\bar{K}^{*0}$	$-\frac{g_{VVV}}{\sqrt{2}}$	$\bar{K}^{*0}\omega\bar{K}^{*0}$	$\frac{g_{VVV}}{\sqrt{2}}$
$\bar{K}^{*0}\rho^+K^{*-}$	g_{VVV}	$\bar{K}^{*0}\bar{K}^{*0}\phi$	g_{VVV}	$\phi K^{*+}K^{*-}$	g_{VVV}	$\phi K^{*0}\bar{K}^{*0}$	g_{VVV}
$\phi\phi\phi$	g_{VVV}						

TABLE XIII. The spin $1/2$ singly charmed baryon ground states include antitriplet $\mathcal{B}_{\bar{3}}$ and sextet $\mathcal{B}_{6}$. The strong couplings of $\mathcal{B}_{\bar{3}}\mathcal{B}_{\bar{3}}P$, $\mathcal{B}_{6}\mathcal{B}_{\bar{3}}P$, and $\mathcal{B}_{6}\mathcal{B}_{6}P$ can be obtained by using the coupling constants $g_{\Xi_c^+\Xi_c^+\pi^0}=0.7=\frac{g_{PBB1}}{\sqrt{2}}$, $g_{\Xi_c'^+\Xi_c^+\pi^0}=3.1=\frac{g_{PBB2}}{2}$, and $\Sigma_c^+\Sigma_c^0\pi^+=8.0=\frac{g_{PBB3}}{\sqrt{2}}$ [58], respectively. Since the matrix forms of the spin $3/2$ singly charmed baryon ground state $\mathcal{B}_{6}^{*}$ and the $1/2$ sextet $\mathcal{B}_{6}$ under symmetry are similar, the derived relations are identical. Then, the couplings $\mathcal{B}_{6}^{*}\mathcal{B}_{\bar{3}}P$ and $\mathcal{B}_{6}^{*}\mathcal{B}_{6}P$ can be obtained from $g_{\Sigma_c^{*0}\Lambda_c^+\pi^-}=3.9=g_{PB^*B2}$ and $g_{\Sigma_c^{*+}\Sigma_c^0\pi^+}=4.3=\frac{g_{PB^*B3}}{\sqrt{2}}$ [75], respectively.

Vertex	g	Vertex	g	Vertex	g	Vertex	g
$\Lambda_c^+\Lambda_c^+\eta_8$	$\sqrt{\frac{2}{3}}g_{PBB1}$	$\Lambda_c^+\Lambda_c^+\eta_1$	$\frac{2g_{PBB1}}{\sqrt{3}}$	$\Lambda_c^+\Xi_c^+K^0$	g_{PBB1}	$\Lambda_c^+\Xi_c^0K^+$	$-g_{PBB1}$
$\Xi_c^+\Lambda_c^+\bar{K}^0$	g_{PBB1}	$\Xi_c^+\Xi_c^+\pi^0$	$\frac{g_{PBB1}}{\sqrt{2}}$	$\Xi_c^+\Xi_c^+\eta_8$	$-\frac{g_{PBB1}}{\sqrt{6}}$	$\Xi_c^+\Xi_c^+\eta_1$	$\frac{2g_{PBB1}}{\sqrt{3}}$
$\Xi_c^+\Xi_c^0\pi^+$	g_{PBB1}	$\Xi_c^0\Lambda_c^+K^-$	$-g_{PBB1}$	$\Xi_c^0\Xi_c^+\pi^-$	g_{PBB1}	$\Xi_c^0\Xi_c^0\pi^0$	$-\frac{g_{PBB1}}{\sqrt{2}}$
$\Xi_c^0\Xi_c^0\eta_8$	$-\frac{g_{PBB1}}{\sqrt{6}}$	$\Xi_c^0\Xi_c^0\eta_1$	$\frac{2g_{PBB1}}{\sqrt{3}}$				
$\Sigma_c^{++}\Lambda_c^+\pi^+$	$-g_{PBB2}$	$\Sigma_c^{++}\Xi_c^+K^+$	$-g_{PBB2}$	$\Sigma_c^+\Lambda_c^+\pi^0$	g_{PBB2}	$\Sigma_c^+\Xi_c^+K^0$	$-\frac{g_{PBB2}}{\sqrt{2}}$
$\Sigma_c^+\Xi_c^0K^+$	$-\frac{g_{PBB2}}{\sqrt{2}}$	$\Sigma_c^0\Lambda_c^+\pi^-$	g_{PBB2}	$\Sigma_c^0\Xi_c^0K^0$	$-g_{PBB2}$	$\Xi_c'^+\Lambda_c^+\bar{K}^0$	$-\frac{g_{PBB2}}{\sqrt{2}}$
$\Xi_c'^+\Xi_c^+\pi^0$	$\frac{g_{PBB2}}{2}$	$\Xi_c'^+\Xi_c^+\eta_8$	$\frac{\sqrt{3}g_{PBB2}}{2}$	$\Xi_c'^+\Xi_c^0\pi^+$	$\frac{g_{PBB2}}{\sqrt{2}}$	$\Xi_c'^0\Lambda_c^+K^-$	$\frac{g_{PBB2}}{\sqrt{2}}$
$\Xi_c'^0\Xi_c^+\pi^-$	$\frac{g_{PBB2}}{\sqrt{2}}$	$\Xi_c'^0\Xi_c^0\pi^0$	$-\frac{g_{PBB2}}{2}$	$\Xi_c'^0\Xi_c^0\eta_8$	$\frac{\sqrt{3}g_{PBB2}}{2}$	$\Omega_c^0\Xi_c^+K^-$	g_{PBB2}
$\Omega_c^0\Xi_c^0\bar{K}^0$	g_{PBB2}						
$\Sigma_c^{++}\Sigma_c^{++}\pi^0$	$\frac{g_{PBB3}}{\sqrt{2}}$	$\Sigma_c^{++}\Sigma_c^{++}\eta_8$	$\frac{g_{PBB3}}{\sqrt{6}}$	$\Sigma_c^{++}\Sigma_c^{++}\eta_1$	$\frac{g_{PBB3}}{\sqrt{3}}$	$\Sigma_c^{++}\Sigma_c^+\pi^+$	$\frac{g_{PBB3}}{\sqrt{2}}$
$\Sigma_c^{++}\Xi_c'^+K^+$	$\frac{g_{PBB3}}{\sqrt{2}}$	$\Sigma_c^+\Sigma_c^{++}\pi^-$	$\frac{g_{PBB3}}{\sqrt{2}}$	$\Sigma_c^+\Sigma_c^+\eta_8$	$\frac{g_{PBB3}}{\sqrt{6}}$	$\Sigma_c^+\Sigma_c^+\eta_1$	$\frac{g_{PBB3}}{\sqrt{3}}$
$\Sigma_c^+\Sigma_c^0\pi^+$	$\frac{g_{PBB3}}{\sqrt{2}}$	$\Sigma_c^+\Xi_c'^0K^+$	$\frac{g_{PBB3}}{2}$	$\Sigma_c^+\Xi_c'^0K^+$	$\frac{g_{PBB3}}{2}$	$\Sigma_c^0\Sigma_c^+\pi^-$	$\frac{g_{PBB3}}{\sqrt{2}}$
$\Sigma_c^0\Sigma_c^0\pi^0$	$-\frac{g_{PBB3}}{\sqrt{2}}$	$\Sigma_c^0\Sigma_c^0\eta_8$	$\frac{g_{PBB3}}{\sqrt{6}}$	$\Sigma_c^0\Sigma_c^0\eta_1$	$\frac{g_{PBB3}}{\sqrt{3}}$	$\Sigma_c^0\Xi_c'^0K^0$	$\frac{g_{PBB3}}{\sqrt{2}}$
$\Xi_c'^+\Sigma_c^{++}K^-$	$\frac{g_{PBB3}}{\sqrt{2}}$	$\Xi_c'^+\Sigma_c^+\bar{K}^0$	$\frac{g_{PBB3}}{2}$	$\Xi_c'^+\Xi_c'^+\pi^0$	$\frac{g_{PBB3}}{2\sqrt{2}}$	$\Xi_c'^+\Xi_c'^+\eta_8$	$-\frac{g_{PBB3}}{2\sqrt{6}}$
$\Xi_c'^+\Xi_c'^+\eta_1$	$\frac{g_{PBB3}}{\sqrt{3}}$	$\Xi_c'^+\Xi_c'^0\pi^+$	$\frac{g_{PBB3}}{2}$	$\Xi_c'^+\Omega_c^0K^+$	$\frac{g_{PBB3}}{\sqrt{2}}$	$\Xi_c'^0\Sigma_c^+K^-$	$\frac{g_{PBB3}}{2}$
$\Xi_c'^0\Sigma_c^0\bar{K}^0$	$\frac{g_{PBB3}}{\sqrt{2}}$	$\Xi_c'^0\Xi_c'^+\pi^-$	$\frac{g_{PBB3}}{2}$	$\Xi_c'^0\Xi_c'^0\pi^0$	$-\frac{g_{PBB3}}{2\sqrt{2}}$	$\Xi_c'^0\Xi_c'^0\eta_8$	$-\frac{g_{PBB3}}{2\sqrt{6}}$
$\Xi_c'^0\Xi_c'^0\eta_1$	$\frac{g_{PBB3}}{\sqrt{3}}$	$\Xi_c'^0\Omega_c^0K^0$	$\frac{g_{PBB3}}{\sqrt{2}}$	$\Omega_c^0\Xi_c'^+K^-$	$\frac{g_{PBB3}}{\sqrt{2}}$	$\Omega_c^0\Xi_c'^0\bar{K}^0$	$\frac{g_{PBB3}}{\sqrt{2}}$
$\Omega_c^0\Omega_c^0\eta_8$	$-\sqrt{\frac{2}{3}}g_{PBB3}$	$\Omega_c^0\Omega_c^0\eta_1$	$\frac{g_{PBB3}}{\sqrt{3}}$				

TABLE XIV. Similar to Table XIII, the strong couplings of $\mathcal{B}_{\bar{3}}\mathcal{B}_{\bar{3}}V$, $\mathcal{B}_{6}\mathcal{B}_{\bar{3}}V$, and $\mathcal{B}_{6}\mathcal{B}_{6}V$ can be obtained by using the coupling constants $g_{\Xi_c^0\Lambda_c^+K^{*-}}=\{4.6,6\}=g_{VBB1}$, $g_{\Sigma_c^0\Lambda_c^+\rho^-}=\{2.6,16\}=g_{VBB2}$, and $g_{\Sigma_c^+\Sigma_c^0\rho^+}=\{4.0,27\}=\frac{g_{VBB3}}{\sqrt{2}}$ [63], respectively. Then, the couplings $\mathcal{B}_{6}^{*}\mathcal{B}_{\bar{3}}V$ and $\mathcal{B}_{6}^{*}\mathcal{B}_{6}V$ can be obtained from $g_{\Sigma_c^{*+}\Lambda_c^+\rho^0}=10=g_{VB^*B2}$ and $g_{\Sigma_c^{*0}\Sigma_c^+\rho^-}=5.77=\frac{g_{VB^*B3}}{\sqrt{2}}$ [76], respectively.

Vertex	g	Vertex	g	Vertex	g	Vertex	g
$\Lambda_c^+\Lambda_c^+\omega$	$-\sqrt{2}g_{VBB1}$	$\Lambda_c^+\Xi_c^+K^{*0}$	$-g_{VBB1}$	$\Lambda_c^+\Xi_c^0K^{*+}$	g_{VBB1}	$\Xi_c^+\Lambda_c^+\bar{K}^{*0}$	$-g_{VBB1}$
$\Xi_c^+\Xi_c^+\rho^0$	$-\frac{g_{VBB1}}{\sqrt{2}}$	$\Xi_c^+\Xi_c^+\omega$	$-\frac{g_{VBB1}}{\sqrt{2}}$	$\Xi_c^+\Xi_c^+\phi$	$-g_{VBB1}$	$\Xi_c^+\Xi_c^0\rho^+$	$-g_{VBB1}$
$\Xi_c^0\Lambda_c^+K^{*-}$	g_{VBB1}	$\Xi_c^0\Xi_c^+\rho^-$	$-g_{VBB1}$	$\Xi_c^0\Xi_c^0\rho^0$	$\frac{g_{VBB1}}{\sqrt{2}}$	$\Xi_c^0\Xi_c^0\omega$	$-\frac{g_{VBB1}}{\sqrt{2}}$
$\Xi_c^0\Xi_c^0\phi$	$-g_{VBB1}$						
$\Sigma_c^{++}\Lambda_c^+\rho^+$	$-g_{VBB2}$	$\Sigma_c^{++}\Xi_c^+K^{*+}$	$-g_{VBB2}$	$\Sigma_c^+\Lambda_c^+\rho^0$	g_{VBB2}	$\Sigma_c^+\Xi_c^+K^{*0}$	$-\frac{g_{VBB2}}{\sqrt{2}}$
$\Sigma_c^+\Xi_c^0K^{*+}$	$-\frac{g_{VBB2}}{\sqrt{2}}$	$\Sigma_c^0\Lambda_c^+\rho^-$	g_{VBB2}	$\Sigma_c^0\Xi_c^0K^{*0}$	$-g_{VBB2}$	$\Xi_c'^+\Lambda_c^+\bar{K}^{*0}$	$-\frac{g_{VBB2}}{\sqrt{2}}$
$\Xi_c'^+\Xi_c^+\rho^0$	$\frac{g_{VBB2}}{2}$	$\Xi_c'^+\Xi_c^+\omega$	$\frac{g_{VBB2}}{2}$	$\Xi_c'^+\Xi_c^+\phi$	$-\frac{g_{VBB2}}{\sqrt{2}}$	$\Xi_c'^+\Xi_c^0\rho^+$	$\frac{g_{VBB2}}{\sqrt{2}}$
$\Xi_c'^0\Lambda_c^+K^{*-}$	$\frac{g_{VBB2}}{\sqrt{2}}$	$\Xi_c'^0\Xi_c^+\rho^-$	$\frac{g_{VBB2}}{\sqrt{2}}$	$\Xi_c'^0\Xi_c^0\rho^0$	$-\frac{g_{VBB2}}{2}$	$\Xi_c'^0\Xi_c^0\omega$	$\frac{g_{VBB2}}{2}$
$\Xi_c'^0\Xi_c^0\phi$	$-\frac{g_{VBB2}}{\sqrt{2}}$	$\Omega_c^0\Xi_c^+K^{*-}$	g_{VBB2}	$\Omega_c^0\Xi_c^0\bar{K}^{*0}$	g_{VBB2}		

(Table continued)

TABLE XIV. *(Continued)*

Vertex	g	Vertex	g	Vertex	g	Vertex	g
$\Sigma_c^{++}\Sigma_c^{++}\rho^0$	$\frac{g_{VBB3}}{\sqrt{2}}$	$\Sigma_c^{++}\Sigma_c^{++}\omega$	$\frac{g_{VBB3}}{\sqrt{2}}$	$\Sigma_c^{++}\Sigma_c^{+}\rho^+$	$\frac{g_{VBB3}}{\sqrt{2}}$	$\Sigma_c^{++}\Xi_c'^{+}K^{*+}$	$\frac{g_{VBB3}}{\sqrt{2}}$
$\Sigma_c^{+}\Sigma_c^{++}\rho^-$	$\frac{g_{VBB3}}{\sqrt{2}}$	$\Sigma_c^{+}\Sigma_c^{+}\omega$	$\frac{g_{VBB3}}{\sqrt{2}}$	$\Sigma_c^{+}\Sigma_c^{0}\rho^+$	$\frac{g_{VBB3}}{\sqrt{2}}$	$\Sigma_c^{+}\Xi_c'^{+}K^{*0}$	$\frac{g_{VBB3}}{2}$
$\Sigma_c^{+}\Xi_c'^{0}K^{*+}$	$\frac{g_{VBB3}}{2}$	$\Sigma_c^{0}\Sigma_c^{+}\rho^-$	$\frac{g_{VBB3}}{\sqrt{2}}$	$\Sigma_c^{0}\Sigma_c^{0}\rho^0$	$-\frac{g_{VBB3}}{\sqrt{2}}$	$\Sigma_c^{0}\Sigma_c^{0}\omega$	$\frac{g_{VBB3}}{\sqrt{2}}$
$\Sigma_c^{0}\Xi_c'^{0}K^{*0}$	$\frac{g_{VBB3}}{\sqrt{2}}$	$\Xi_c'^{+}\Sigma_c^{++}K^{*-}$	$\frac{g_{VBB3}}{\sqrt{2}}$	$\Xi_c'^{+}\Sigma_c^{+}\bar{K}^{*0}$	$\frac{g_{VBB3}}{2}$	$\Xi_c'^{+}\Xi_c'^{+}\rho^0$	$\frac{g_{VBB3}}{2\sqrt{2}}$
$\Xi_c'^{+}\Xi_c'^{+}\omega$	$\frac{g_{VBB3}}{2\sqrt{2}}$	$\Xi_c'^{+}\Xi_c'^{+}\phi$	$\frac{g_{VBB3}}{2}$	$\Xi_c'^{+}\Xi_c'^{0}\rho^+$	$\frac{g_{VBB3}}{2}$	$\Xi_c'^{+}\Omega_c^{0}K^{*+}$	$\frac{g_{VBB3}}{\sqrt{2}}$
$\Xi_c'^{0}\Sigma_c^{+}K^{*-}$	$\frac{g_{VBB3}}{2}$	$\Xi_c'^{0}\Sigma_c^{0}\bar{K}^{*0}$	$\frac{g_{VBB3}}{\sqrt{2}}$	$\Xi_c'^{0}\Xi_c'^{+}\rho^-$	$\frac{g_{VBB3}}{2}$	$\Xi_c'^{0}\Xi_c'^{0}\rho^0$	$-\frac{g_{VBB3}}{2\sqrt{2}}$
$\Xi_c'^{0}\Xi_c'^{0}\omega$	$\frac{g_{VBB3}}{2\sqrt{2}}$	$\Xi_c'^{0}\Xi_c'^{0}\phi$	$\frac{g_{VBB3}}{2}$	$\Xi_c'^{0}\Omega_c^{0}K^{*0}$	$\frac{g_{VBB3}}{\sqrt{2}}$	$\Omega_c^{0}\Xi_c'^{+}K^{*-}$	$\frac{g_{VBB3}}{\sqrt{2}}$
$\Omega_c^{0}\Xi_c'^{0}\bar{K}^{*0}$	$\frac{g_{VBB3}}{\sqrt{2}}$	$\Omega_c^{0}\Omega_c^{0}\phi$	g_{VBB3}				

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