

Domain walls from confining bubbles: SU(N) Yang-Mills theory at finite θ

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We study the confinement phase transition (PT) in $SU(N_c)$ pure Yang-Mills theory at finite $\theta \neq 0$ using the improved holographic QCD (IHQCD) model. We show that the maximal amount of supercooling is reduced in the confinement PT as a function of θ for large but fixed N_c . Upon completion of the confinement PT, a network of domain walls can be produced, owing to the multibranched vacuum structure of Yang-Mills theory at finite θ . We highlight the potential interplay between the produced domain walls and the confinement PT dynamics. We emphasize that domain wall production from bubble coalescence in strongly coupled nonconformal first order PTs is a dynamical process of vacuum assignment, hydrodynamics, and local reheating effects, all potentially affecting the approach toward the scaling regime. Lastly, we demonstrate the level of tuning necessary for potentially interesting imprints from gravitational waves through domain wall annihilation and its interplay with the confinement PT.

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I. INTRODUCTION

Spontaneous symmetry breaking is expected to happen in the early Universe, as fundamental symmetries are restored at high temperature. As the Universe cools down, the breaking may manifest itself as a first or second-order phase transition (FOPT/SOPT), leaving behind a population of topological defects such as monopoles [1–3], cosmic strings [4–6], or domain walls [7,8]. Crucially, FOPTs in the early Universe generate anisotropic stress in the energy-momentum tensor, thereby sourcing gravitational waves (GWs) that offer observational access to physics beyond the Standard Model [9–29]. While the electroweak and the QCD phase transitions are known to be smooth cross-overs [30,31], strongly coupled dark sectors may naturally exhibit FOPTs during confinement or chiral symmetry breaking. The GW signal from a $SU(N_c)$ Yang-Mills theory [32–39] is generally expected to be weak, due to the small amount of supercooling of the metastable deconfined phase. However, because complete

nonperturbative end-to-end computations of these FOPTs and their GWs are still lacking, this prediction is not conclusive yet [40,41]. Alternatively, one can envision strongly coupled quantum field theories (QFTs) that achieve substantially greater supercooling and, therefore, stronger GW signals in models with conformal, walking-type dynamics [42,43].

It is natural to extend previous studies of confining PTs to the case of nonzero θ -angle, to assess its quantitative effects on the PT phase diagram. Because the confinement PT is inherently strongly coupled, we employ holographic techniques in this work. In particular, we use the improved holographic QCD (IHQCD) model [44–47] with an axion to describe $SU(N_c)$ YM at finite θ . More recent studies of holographic models and setups to investigate the effects of θ have been carried out in Refs. [48–54]. These studies qualitatively find that the leading $\theta \neq 0$ effect scales as θ^2/N_c^2 in the limit $\theta \ll \pi$ with a negative coefficient, in agreement with the results of lattice QCD at imaginary $\theta \neq 0$ [55,56]. A similar finding was also obtained by looking at the topology $\mathbb{R}^3 \times S^1$ [57].

In this work, we focus on the confinement PTs of $SU(N_c)$ YM theories at $\theta \neq 0$, after inflation. The dark sector couples to the SM only gravitationally. It is well-known that, for fixed N_c , the critical temperature is reduced with the increase of θ [55,56,58]. We confirm this result using IHQCD and study the impact of θ on the maximum amount of supercooling in the confinement PT. Due to the multibranched vacuum structure of YM theory at finite θ ,

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a domain wall (DW) network can be formed upon the completion of the confining PT. We argue that DW formation in strongly coupled nonconformal FOPTs involves a hierarchy of hydrodynamic and cooling time-scales, going beyond the standard percolation picture. Finally, we elaborate on the interplay between the DWs and FOPT dynamics and discuss the imprints of $\theta \neq 0$ on the GW spectrum of annihilating DWs [59–64].

This paper is organized as follows: in Sec. II, we review the IHQCD model at finite θ , both at zero and finite temperature. In Sec. III we discuss the confinement FOPT with finite θ and N_c . Furthermore, in Sec. IV we explain how the DW network is produced after the confining PT is completed. We consider the GW signal from annihilating DWs in Sec. V. In Sec. VI we discuss and conclude our results.

II. REVIEW OF IMPROVED HOLOGRAPHIC QCD WITH FINITE θ

Holographic models arise as solutions of ten-dimensional string theory, and they have been successful in the description of confinement, IR dynamics, and thermal phase transitions of strongly coupled gauge theories (e.g., [65–72]). However, when applied to QCD, such top-down constructions typically contain extra Kaluza-Klein (KK) modes, not present in QCD. To avoid extra KK modes, we should consider string theories living in five dimensions, i.e., noncritical string theories. Meanwhile, still inspired by noncritical string theories, here we adopt a more phenomenological approach from [44–47], known as IHQCD. In this model, the effective gravitational theory is constructed bottom-up to reproduce general properties of a QCD-like theory, such as confinement, the glueball and meson spectrum, the finite-temperature phase diagram, etc. In this work, we focus on the gravity dual describing a pure $SU(N_c)$ Yang-Mills theory.

The spectrum of the dual gravity theory consists of:

- (i) Metric $g_{\mu\nu}$: dual to the energy-momentum tensor $T_{\mu\nu}$.
- (ii) Dilaton Φ : dual to $\text{Tr } F^2$. Asymptotically, it is related to the 't Hooft coupling of the Yang-Mills theory λ_{YM} .
- (iii) Axion a : dual to $\text{Tr } F \wedge F$. Its asymptotic value is equal to the UV θ -angle, i.e., θ_{UV} .

The action, written in the Einstein frame, is given by

$$S = -M_{\text{Pl}}^3 N_c^2 \times \int_{\mathcal{M}} d^5x \sqrt{g} \left[R - \frac{4}{3} (\partial\Phi)^2 - \frac{Z(\Phi)}{2N_c^2} (\partial a)^2 + V(\Phi) \right] + 2M_{\text{Pl}}^3 N_c^2 \int_{\partial\mathcal{M}} d^4x \sqrt{h} \mathcal{K}, \quad (1)$$

where M_{Pl} is the Planck mass, N_c is the number of colors and R is the 5D Ricci scalar. The boundary term is the

famous Gibbons Hawking York boundary term [73] and $\mathcal{K}$ is the extrinsic curvature

$$K_{\mu\nu} = \nabla_\mu n_\nu = \frac{1}{2} n^\rho \partial_\rho h_{\mu\nu} \quad \mathcal{K} = h^{ab} K_{ab}, \quad (2)$$

where h represents the induced metric on the boundary $\partial\mathcal{M}$. The effective 5D Newton constant is given by $G_5 = 1/(16\pi M_{\text{Pl}}^3 N_c^2)$.

The dilaton potential is determined from the requirement of having a confining background at zero temperature [44,45]. We will usually adopt the following potential ($\lambda = e^\Phi$) [33,47]

$$V(\lambda) = \frac{12}{\ell^2} (1 + V_0 \lambda + V_1 \lambda^{4/3} [\log(1 + V_2 \lambda^{4/3} + V_3 \lambda^2)]^{1/2}), \quad (3)$$

where ℓ is the anti-de Sitter (AdS) length and

$$V_0 = \frac{8}{9} b_0, \quad V_2 = b_0^4 \left(\frac{23 + 36 \frac{b_1}{b_0^2}}{81 V_1} \right)^2 \quad (4)$$

where $b_0 = 22/(3(4\pi)^2)$ and $b_1/b_0^2 = 51/121$. This choice comes from setting λ proportional to the 't Hooft coupling in the UV [33]. Then b_0 and b_1 are the coefficients of the asymptotic β -function

$$\beta(\lambda) \approx -b_0 \lambda - b_1 \lambda^2 + \mathcal{O}(\lambda^3), \quad \lambda \rightarrow 0, \quad (5)$$

up to two loop order. Note that, as $\lambda \rightarrow 0$, $V(\lambda) \rightarrow 12/\ell^2$ and the solutions to Einstein equations will be asymptotically AdS. In the case of pure YM, the β -function coefficients up to two-loop order are independent of N_c , see Appendix A in [44,45]. The coefficients V_1 and V_3 are determined by comparison with the lattice data¹ [47]

$$V_1 = 14, \quad V_3 = 170. \quad (6)$$

Similarly, the function $Z(\lambda)$ is determined from the lattice data and takes the form

$$Z(\lambda) = Z_0 (1 + c_a \lambda^4), \quad (7)$$

where $c_a = 0.26$ and $Z_0 = 33.25$ in order to match the pseudoscalar glueball spectrum and the topological susceptibility [47,74]. Note that the above matching was obtained for $N_c = 3$. However, we will keep N_c arbitrary in our expressions.

We will work in the large N_c limit, which means that the backreaction of the axion on the metric can be ignored.

¹In particular, by matching the glueball spectrum at zero temperature and the pressure and trace anomaly at some reference temperature.

Then, at finite temperature, obtained after compactifying Euclidean time on a circle of radius $1/T$ (i.e., $\tau \sim \tau + 1/T$), we have two solutions:

- (i) Thermal gas (TG) solution: The metric has the same form as in the zero-temperature solution, with the Euclidean time compactified

$$ds^2 = b_0(r)^2(dr^2 + d\tau^2 + dx_m dx^m), \quad \Phi = \Phi_0(r), \quad (8)$$

where r is a radial coordinate and $r \rightarrow 0$ represents the AdS boundary. This solution describes thermal excitations around the vacuum represented by the zero-temperature solution. That being said, it represents the confined phase of the gauge theory.

- (ii) Black hole (BH) solutions: These have the form

$$ds^2 = b(r)^2 \left[\frac{dr^2}{f(r)} + f(r) d\tau^2 + dx_m dx^m \right], \quad \Phi = \Phi(r), \quad (9)$$

and the presence of a solution with a regular horizon is characterized by

$$f(r_h) = 0, \quad \dot{f}(r_h) < 0, \quad (10)$$

where the dot represents the derivative with respect to r . The horizon is a regular surface if the Euclidean time is identified as

$$\tau \sim \tau + \frac{4\pi}{|\dot{f}(r_h)|}, \quad (11)$$

and the temperature is given by the usual Hawking temperature

$$T = \frac{|\dot{f}(r_h)|}{4\pi}. \quad (12)$$

This geometry corresponds to a deconfined phase in the dual gauge theory since the confining string tension is given by [75]

$$\min_r (\sqrt{g_{\tau\tau}(r)g_{xx}(r)}) \sim \sqrt{f(r_h)} = 0. \quad (13)$$

Since the solutions need to approach AdS asymptotically as $r \rightarrow 0$, in this limit we must have $b(r) \sim \ell/r$ and $f(r) \rightarrow 1$ up to logarithmic corrections. The typical solutions for $b(r)$ and $f(r)$, hence for the temperature T , are obtained numerically.² The holographic coordinate r and the dilaton

²In this case, it appears easier to work with scalar variables $X = \dot{\Phi}/(3\dot{A})$ and $Y = \dot{g}/(4\dot{A})$, where $b(r) = e^{A(r)}$. This procedure is summarized in the Appendix of [46].

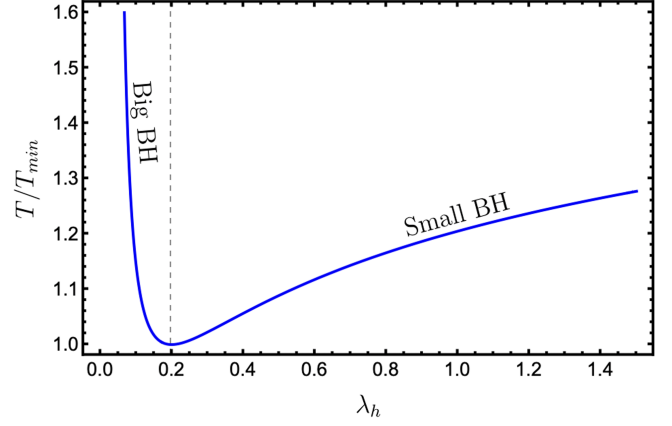


FIG. 1. Temperature curve $T/T_{\min}(\lambda_h)$.

potential $V(\lambda)$ are in one-to-one correspondence with the running energy scale and the β -function of the dual gauge theory, respectively [44,46]. Thus, intuitively, and as shown more precisely in [44,45] this means that one can use the dilaton λ as a coordinate to describe gravitational solutions. The temperature curve of the BH as a function of the dilaton value at the horizon λ_h is shown in Fig. 1. One observes the existence of a minimal temperature $T_{\min}$ ($\lambda_{h,\min} \approx 0.2$), below which the deconfined phase does not exist and the phase transition must be completed. For BHs with λ_h smaller (larger) than $\lambda_{h,\min}$, the horizon is closer to (further from) the AdS boundary and we refer to these as big (small) BHs.

The phase diagram is determined from the free energy curve. Namely, we look at the free energy difference between the black hole and the thermal gas solution

$$\mathcal{F} = T(S_{\text{BH}} - S_{\text{TG}}), \quad (14)$$

i.e., between the deconfined and confined phases. The action is generally divergent near the UV boundary $r = 0$ and a cutoff is placed at $r = \epsilon$ or, equivalently, at some $\lambda = \lambda_0$. Taking the difference in (14), we avoid the explicit computation of counterterms [46]. The phase diagram can also be computed with the entropy

$$\mathcal{F}(\lambda_h) = \int_{\lambda_h}^{\infty} s(\tilde{\lambda}_h) \frac{dT}{d\tilde{\lambda}_h} d\tilde{\lambda}_h. \quad (15)$$

The entropy is given by

$$s(\lambda_h) = \frac{\text{Area}}{4G_5} = 4\pi M_{\text{pl}}^3 N_c^2 V_3 b^3(\lambda_h), \quad (16)$$

where V_3 is the volume of the three-dimensional space spanned by the coordinates x^m . The typical phase diagram that one obtains is shown in Fig. 2. One can see the existence of a first-order phase transition at the critical temperature T_c . Moreover, the big BH represents the actual

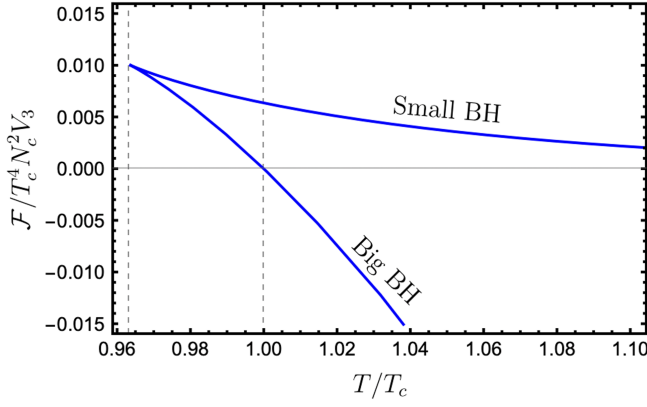


FIG. 2. Free energy diagram of the $SU(N_c)$ YM theory without the θ -term.

deconfined phase of the gauge theory, whereas the small BH shall be interpreted as the spinodal unstable phase, which in the case of large N_c asymptotes to the confined phase at $\lambda_h \rightarrow \infty$.

Note that this diagram was obtained without including the axion contribution to the free energy. In what follows, we discuss the axion profile in these backgrounds and its contribution to the phase diagram.

A. Axion background at zero and finite temperature

Even though we are working in the approximation where the axion does not backreact on the metric, it can still obtain a nontrivial background profile. Following [45,46], we will present the axion background at zero temperature and then extend it to nonzero temperature. As already mentioned, the only solution at zero temperature is the vacuum one, with the metric given in (8) and the Euclidean time uncompactified. The axion equation of motion is given by

$$\ddot{a} + \left(3\frac{\dot{b}}{b} + \partial_\lambda \log Z(\lambda)\dot{\lambda}\right)\dot{a} = 0. \quad (17)$$

The equation above has two independent solutions

$$a(r) = \theta_{UV} + 2\pi k + C a_1(r), \quad k \in \mathbb{Z}, \quad \theta_{UV} \in [0, 2\pi) \quad (18)$$

where $2\pi k$ was added to account for the 2π periodicity of the θ -angle in the UV. The nontrivial solution is given by

$$a_1(r) = \int_0^r \frac{d\tilde{r}}{\ell} \frac{1}{b^3(\tilde{r})Z(\lambda)} \quad (19)$$

and C is an integration constant proportional to the expectation value of the instanton density

$$C = Z_0 \ell^4 \frac{\langle F \wedge F \rangle}{32\pi^2}. \quad (20)$$

One can interpret the nontrivial axion profile as the running θ -angle, but this statement should be taken with some care. As the instanton density operator $\langle F \wedge F \rangle$ does not receive divergent UV corrections, the θ -angle does not get renormalized [76]. Instead, the point of view adopted in [45] is that the axion profile captures the nonperturbative corrections to the θ -angle in the IR.

One can calculate the θ -dependent vacuum energy

$$\mathcal{E}(\theta_{UV}) = \frac{M_{Pl}^3}{2} \int dr \sqrt{g} Z(\Phi) (\partial a)^2. \quad (21)$$

Using the above results, one can easily show that this is equal to [45]

$$\mathcal{E}(\theta_{UV}) = \frac{1}{2} \chi \min_k (\theta_{UV} + 2\pi k)^2, \quad (22)$$

where we had identified the topological susceptibility χ as

$$\chi = \frac{M_{Pl}^3}{\int_0^\infty \frac{dr}{b^3(r)Z(\lambda)}}. \quad (23)$$

and imposed the following IR boundary condition

$$\lim_{r \rightarrow \infty} a(r) = 0. \quad (24)$$

This boundary condition was adopted in [45] motivated by the observation that, in the spirit of holography, physical quantities are determined from the data at a single boundary. Moreover, it is motivated in certain string-theory models where the axion is identified with a Wilson loop on a disc [77]. As the disc shrinks to zero size in the IR, the axion profile $a(r)$ should also vanish there.

From (22) we can see that the vacuum structure of Yang-Mills is special. There is a single global minimum labeled by $k = k_{\min}$ which minimizes (22) for some value of θ_{UV} and all other integers $k \neq k_{\min}$ represent metastable vacua. As the value of θ_{UV} increases, the two neighboring minima, $k_{\min}$ and $k_{\min} - 1$, become degenerate at $\theta_{UV} = (1 - 2k_{\min})\pi$. Increasing θ_{UV} further causes the stable ($k_{\min}$) and the metastable vacuum ($k_{\min} - 1$) to change roles (see Fig. 3). Such a vacuum structure was initially proposed in [77] as it preserves the 2π -periodicity of θ at large N_c .

At finite temperature, the axion equation of motion changes by an additional term

$$\ddot{a} + \left(3\frac{\dot{b}}{b} + \frac{\dot{f}}{f} + \partial_\lambda \log Z(\lambda)\dot{\lambda}\right)\dot{a} = 0 \quad (25)$$

and this can be easily integrated as before to obtain

$$a(r) = \theta_{UV} + 2\pi k + C \int_0^r \frac{d\tilde{r}}{\ell} \frac{1}{f(\tilde{r})b^3(\tilde{r})Z(\lambda)}, \quad (26)$$

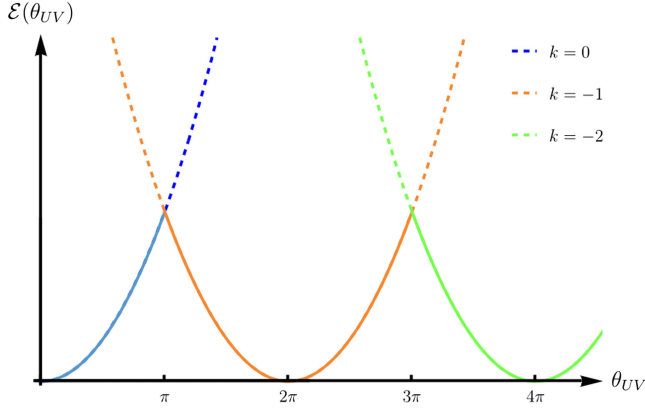


FIG. 3. Illustration of the branch structure of the Yang-Mills vacuum. Solid lines of different colors represent the true vacuum for that value of θ_{UV} , whereas dashed lines correspond to metastable vacua.

where C is again given by (20). In the thermal gas solution, the axion profile coincides with that of the zero-temperature background. In contrast, in the black hole background, since near the horizon $f(r) \sim \dot{f}(r_h)(r_h - r)$, the integral in (26) will be logarithmically divergent (b and Z are both finite near the horizon [46]). Since the action must be finite, we must impose that $C = 0$ and that

$$\langle F \wedge F \rangle_{\text{deconf.}} = 0, \quad (27)$$

as expected from lattice simulations at large N_c [76,78,79].

However, as we move the position of the horizon toward infinity $r_h \rightarrow \infty$, the exterior geometry is approaching the thermal gas solution. This limit is not reproduced within the axion profile if $C = 0$. This suggests that, at larger values of r_h , one should properly include the backreaction from the axion to account for the susceptibility in the SBH branch, which should approach the thermal gas one as $r_h \rightarrow \infty$. We will discuss the regime in which this approximation breaks down and the onset of significant axion backreaction.

III. CONFINEMENT PHASE TRANSITION WITH FINITE θ

We have shown the free energy diagram in Fig. 2, which signals a first-order transition between the deconfined and confined phase of Yang-Mills. It is illustrative to estimate the maximum amount of supercooling we can get from such phase transitions. Introducing the supercooling parameter [32,42]

$$\epsilon = 1 - \frac{T_n}{T_c}, \quad (28)$$

where T_n is the nucleation temperature, we see that the maximum supercooling one can obtain is

$$\epsilon_{\text{max}} = 1 - \frac{T_{\text{min}}}{T_c} \approx 0.04. \quad (29)$$

For small supercooling, the parameter $\beta/H \sim 1/\epsilon^3$ is larger, which suppresses the GW signal [32]. In this section, we investigate the impact of the axion on the supercooling of confining phase transitions in Yang-Mills theories.

We compute the contribution of the axion sector to the free energy density. In the black hole phase, this contribution vanishes since the axion profile is constant, and therefore we focus on the thermal gas solution. The axion part of the Euclidean action is given by

$$S_a^{\text{TG}} = \frac{M_{\text{Pl}}^3 V_3}{2 T} \int_0^\infty dr b_0(r)^3 Z(\lambda) (\partial a)^2. \quad (30)$$

The integral above is precisely the θ -dependent vacuum energy computed previously and it is easy to see that

$$S_a^{\text{TG}} = \frac{1}{2} \frac{V_3}{T} \chi \min_k (\theta_{UV} + 2\pi k)^2. \quad (31)$$

Since we will later consider cosmological estimates, it seems convenient to set the energy scale in terms of the critical temperature at $\theta_{UV} = 0$, i.e., $T_c^{\theta_{UV}=0}$. Since $Z(\lambda)$ is matched using the lattice data, we use the scaling from [55]

$$\chi \approx 0.0221 \sigma_{\text{conf}}^2, \quad T_c^{\theta_{UV}=0} \approx 0.597 \sqrt{\sigma_{\text{conf}}}, \quad (32)$$

where σ_{conf} is the confining string tension. Using this, the properly normalized contribution to the free energy from the axion is

$$\begin{aligned} \frac{\mathcal{F}_a}{N_c^2 V_3 (T_c^{\theta_{UV}=0})^4} &= -\frac{1}{2} \frac{(\theta_{UV} + 2\pi k)^2}{N_c^2} \frac{\chi}{(T_c^{\theta_{UV}=0})^4} \\ &\approx -\frac{(\theta_{UV} + 2\pi k)^2}{N_c^2} (0.09), \end{aligned} \quad (33)$$

where the minus sign comes from (14).

What is the minimal N_c for which the large N_c approximation is valid and the axion backreaction can be ignored? The free energy curve without the axion contribution evaluated at T_{min} is equal to

$$\frac{\mathcal{F}}{(T_c^{\theta_{UV}=0})^4 N_c^2 V_3} \left(\frac{T_{\text{min}}}{T_c^{\theta_{UV}=0}} \right) \approx 0.01. \quad (34)$$

One may ask what the minimal value of N_c is such that the contribution from the axion is equal to 0.01, since in this case it would spoil the metastability of the deconfined phase. In this case, one would need to include the full backreaction from the axion. That being said, the minimal value of N_c is given by

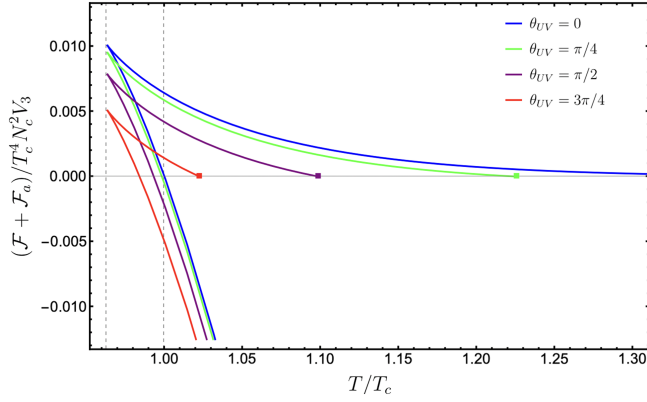


FIG. 4. Free energy curve with the axion contribution for various values of θ_{UV} . In this plot, we set $N_c = 10$ and $k = 0$. The squares indicate the value of λ_h where the axion backreaction should be taken into account.

$$(N_c)_{\min} \approx (\theta_{UV} + 2\pi k) \times 3 \quad (35)$$

Taking the $k = 0$ branch to be stable, and as a reference value, $\theta_{UV} = \pi$, where it becomes metastable, we obtain $(N_c)_{\min} \approx 9.4$. For some values of θ_{UV} and $N_c = 10$ in the $k = 0$ branch we find curves in Fig. 4. Additionally, as discussed in the previous section, we identify the breakdown of our calculation along the SBH branch with the points where the free energy crosses zero (indicated by squares in Fig. 4). This can be understood by interpreting the SBH as the sphaleron mediating tunneling to the confined phase. In this picture, we expect that including the full axion backreaction would smooth out the free energy curve, causing it to asymptotically approach zero along the SBH branch.

In 4D YM theory, the conventional order parameter for the deconfinement/confinement PT is the vacuum expectation value (VEV) of the Polyakov loop $\langle P \rangle$, which is a function of temperature. This can be computed in holography [80,81], and, in particular, we checked that lattice results are reproduced in IHQCD. Moreover, in the BH phase of IHQCD, one can relate both $\langle P \rangle$ and T to λ_h and use λ_h as the order parameter. In this picture, the restrictions above can also be understood from the perspective of the effective potential, i.e., $V_{\text{eff}}(\lambda_h, T)$. In the regimes discussed above, the contributions from the axion become comparable to the ones coming from the metric and the dilaton, requiring the full solution with the axion backreaction included. Furthermore, since the SBH corresponds to the local maximum in the effective potential, we expect the axion backreaction to become increasingly important as we vary λ_h from the local maximum to infinity.

From Fig. 4 we see that the addition of an axion reduces the amount of supercooling in the deconf./conf. phase transition of Yang-Mills theories, since the difference between T_c and $T_{\min}$ gets smaller and the GW spectrum

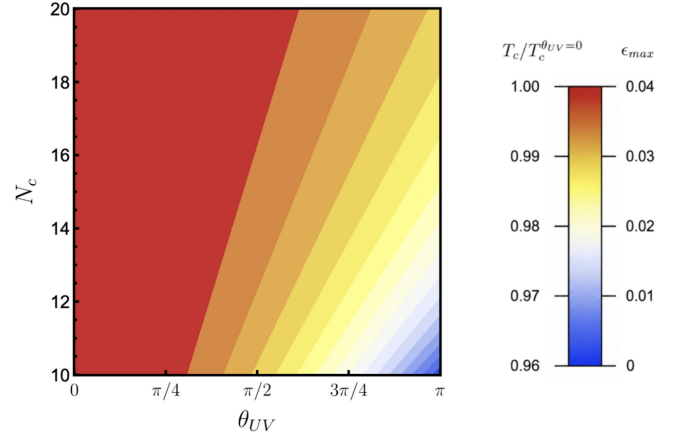


FIG. 5. Contours of $T_c/T_c^{\theta_{UV}=0}$ and the maximum supercooling $\epsilon_{\max}$ in the (N_c, θ_{UV}) plane.

gets more suppressed.³ The reduction in the critical temperature is, of course, in agreement with the lattice calculations [55,56]. This effect remains robust against moderate changes in the free parameters of the dilaton potential, even though the maximal amount of supercooling $\epsilon_{\max}$ has an $\mathcal{O}(1)$ uncertainty depending on which lattice data are considered for obtaining the free parameters V_1, V_3 [82].

However, the notion of minimal temperature $T_{\min}$, present in holography but harder to extract from lattice simulations, is crucial to estimate the maximum amount of supercooling. We show the contours of T_c at finite θ and N_c in Fig. 5.

In summary, it is clear from our discussion that a nonzero θ -angle makes the PT less supercooled, making the prospects for GW signals worse. In the next section, we will investigate the formation of domain walls due to the multibranch structure of the YM vacuum.

IV. DOMAIN WALL PRODUCTION BETWEEN DIFFERENT BRANCHES

In the previous section, we discussed how the presence of the θ -angle reduces the amount of supercooling in the confinement PT and, thus, the possible GW signal. However, there is an additional effect due to the multibranch structure of the YM vacuum. After the deconfinement/confinement phase transition in the early Universe has completed, different k -vacua can be populated,

³The statements above are made on the *maximum* supercooling available and not on the actual values of the bubble nucleation temperature T_n and β/H , which require a finite- θ bounce calculation and are important for the GW signal estimates. However, since the bounce calculation at $\theta = 0$ in [33] seems to indicate that their values are such that the GW signal is suppressed, it is only safe to assume that this continues to hold when θ is finite and the amount of available supercooling is smaller.

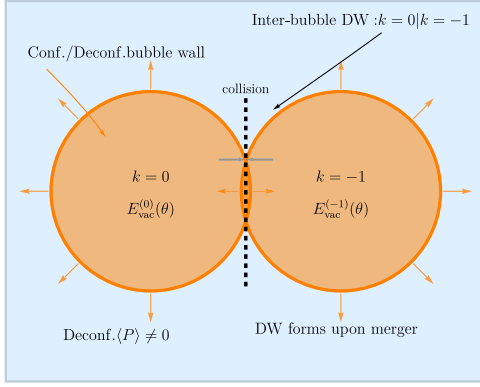


FIG. 6. Illustration of bubble coalescence for the two bubbles of different k branches, indicating that the domain walls can form within the collision region. $\langle P \rangle$ represents the VEV of the Polaykov loop.

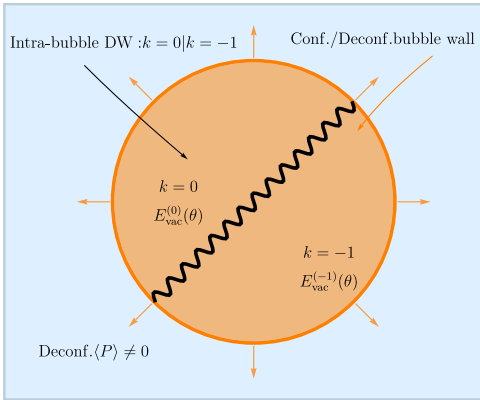


FIG. 7. Illustration of a domain wall forming within a confining bubble. As the domain wall breaks the spherical symmetry of the bubble, these configurations are expected to be suppressed.

producing domain walls across different patches in the Universe. Since the vacua are not degenerate, domain wall annihilation will occur,⁴ and this is expected to leave a GW signal [59,60].

In this section, we discuss the conditions under which DWs can form and later annihilate. We will consider for simplicity $\theta_{UV} \in [0, \pi)$, so that the $k = 0$ branch is stable and $k \neq 0$ ones are metastable. During the phase transition, the interior of confining bubbles will contain a stable/metastable vacuum in some branch k and domain walls will form at locations where the bubbles collide (Fig. 6). It can also happen that within the confining bubble, we have two vacua in different branches separated by a domain wall (Fig. 7). However, we expect nucleation of such

⁴Note that one can also tunnel between different branches via bubble nucleation [83,84]. The tunneling rate scales as $\Gamma \sim e^{-S}$, where $S \sim N_c^4$ and this process is suppressed at large N_c .

configurations to be suppressed as they break the spherical symmetry within the bubble.

A. Percolation theory

The bubble nucleation rate is computed from the bounce action, which is dominated by the $\theta = 0$ term. A population bias arises at $\theta \neq 0$ for different k branches, which can be estimated as follows. Once a bubble is nucleated, the relative probability that its interior is in the k th branch can be estimated as

$$p_k(\theta_{UV}) = \frac{e^{-\Delta F_k/T}}{\sum_{j \in \mathbb{Z}} e^{-\Delta F_j/T}}. \quad (36)$$

where $\Delta F_k = F_k - F_0$, and we indicate with capital F the total free energy inside the bubble. The energy F_k differs from $F_{\theta=0}$ because of the different energy density and the change in the bubble radius at nucleation. The former is expressed in Eq. (33), which gives

$$F_k \approx F_{\theta=0} - \frac{1}{2} \chi(\theta_{UV} + 2\pi k)^2 V_{\theta=0}. \quad (37)$$

The volume of the bubbles at nucleation must depend on the only scale at our disposal, which is the critical temperature T_c (or, equivalently, the strong coupling scale $\Lambda \approx 1.17 T_c$ [33]). We thus expect that the initial radius of the bubble at $\theta = 0$ will be $R_{\theta=0} \sim T_c^{-1}$, where the actual value can be obtained by solving the bounce equation. We find that typically [82]

$$R_{\theta=0} \approx \frac{\xi}{T_c} \quad \text{with} \quad \xi = \mathcal{O}(10). \quad (38)$$

The dependence of θ on the bubble volume can be estimated in the thin wall limit. The bubble radius scales as the inverse of the energy difference of the two vacua. The correction to F_k has the same form as that coming from the energy shift in Eq. (33), but with a smaller coefficient,⁵ and can be simply reabsorbed in the uncertainty of ξ . This means that

⁵In the thin-wall approximation [85,86], the bubble radius scales as the inverse of the energy difference of the two vacua. Thus, at $\theta \neq 0$ we expect a N_c^2 suppressed correction of the form

$$R_k \sim R_{\theta=0} \left(1 + c \frac{(\theta_{UV} + 2\pi k)^2}{N_c^2} \right)^{-1}, \quad (39)$$

where $c \sim \mathcal{O}(1)$. The correction to F_k is thus estimated as $3c(\theta_{UV} + 2\pi k)^2 F_{\theta=0}/N_c^2$, which is roughly a factor 3 smaller than the one coming from the energy shift in Eq. (33).

$$\frac{p_k(\theta_{UV})}{p_0(\theta_{UV})} = e^{-\Delta F_k/T} = \exp\left(-\frac{\chi}{T}(2k\pi)(\theta_{UV} + k\pi)V_{\theta=0}\right) \approx \exp[-4.58\xi^3 k(k\pi + \theta_{UV})]. \quad (40)$$

Depending on the value of θ_{UV} , there can be a finite probability of nucleating bubbles of the lowest-energy metastable branch $k = -1$. The nucleation of other metastable branches is significantly suppressed, and we will only consider domain walls between $k = 0$ and $k = -1$ vacua. Note that the critical temperature has disappeared from the expression in (40), since all dimensionful quantities in the theory scale with the critical temperature.

Although the probability for the nucleation of $k = -1$ bubbles can be finite, the question remains whether there is a significant number of $k = -1$ bubbles nucleating to form a domain wall network. We follow the reasoning given in [87], which rephrases this problem in the language of percolation theory [88–90]. In other words, we need to ask whether one can achieve connectivity of a single vacuum domain across large scales. We first assume that bubbles grow until they fill the Universe while satisfying Eq. (40). The critical probability for percolation in 3D is given by $p_c = 0.31$ [87]. Hence, if $p_k(\theta_{UV}) > p_c$ we can have a long-connected vacuum domain in branch k . In the opposite case, if $p_k(\theta_{UV}) < p_c$, we can have only finite-volume domains which collapse under their own surface tension. Applying the percolation bound for the $k = -1$ branch, we can place a lower limit on θ_{UV} for which we achieve a DW network

$$\pi - \theta_{UV} \equiv \epsilon_\theta \lesssim \frac{5.6 \times 10^{-2}}{\xi^3} \pi, \quad (41)$$

where we introduced the parameter ϵ_θ that indicates how close θ_{UV} is to π . Equation (41) has a strong dependence on ξ , the exact value of which depends on the parameters V_1 , V_3 of Eq. (6), the temperature and the form of the kinetic term in the effective action for λ_h . Under reasonable assumptions, we find that this number is always larger than 1, typically $\mathcal{O}(10)$, but can also be larger [82]. In any case, there must be a substantial amount of tuning in the angle θ_{UV} in order to achieve percolation.

For $\theta_{UV} \rightarrow \pi$, after the confining bubbles percolate and the phase transition completes, a domain wall network can form between them, as illustrated in Fig. 8. The typical domain length scale is given by the bubble radius at the end of the PT

$$R_* \approx (8\pi)^{1/3} v_w \left(\frac{1}{\beta/H} \right) \frac{1}{H} \ll \frac{1}{H}, \quad (42)$$

for low supercooling (typical values of β/H for the confinement PT are $\mathcal{O}(10^5)$ [33]), where v_w is the wall velocity.

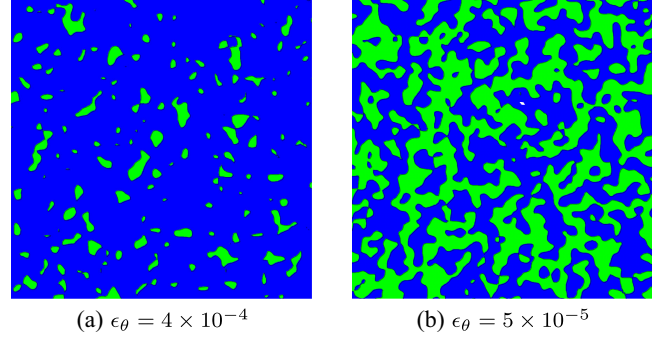


FIG. 8. Formation of a domain-wall network initially depends on whether θ_{UV} satisfies (41): (a) θ_{UV} does not satisfy (41) and the DW network does not form. (b) θ_{UV} satisfies (41) and the network does form. The domains of $k = 0$ and $k = -1$ vacua are shown in blue and green, respectively, and $\xi = 10$ in this figure.

Note that our criteria from percolation theory should be considered as an estimate, since the distribution of bubbles at the time of percolation and vacuum branch assignment are generated dynamically. Moreover, differences in bubble expansion velocities have been ignored. However, due to the large number of bubbles within a Hubble volume, i.e., large β/H , the percolation theory estimate (41) still provides us with a reasonable indication for which values of θ the DW network can form.

For now we have mainly considered the possibility of forming the DW network within the picture provided by percolation theory. We will now elaborate on the dynamics of bubble coalescence and DW formation.

B. Dynamics of bubble coalescence

1. Before bubble coalescence

The duration of the phase transition, from the moment of nucleation (T_n) to that of bubble percolation (T_p) is given by β^{-1} , where [91]

$$\beta = HT \left. \frac{dS_b}{dT} \right|_{T=T_p} \quad (43)$$

and S_b is the bounce action. The ratio β/H determines the average bubble separation R_* in (42). Initially, the bubble is nucleated in a region of space with either $k = 0$ or $k = -1$ and begins to expand to eventually coalesce with another bubble nucleated nearby, possibly with a different k . This is happening throughout the Hubble volume due to the large β/H . As a confining bubble nucleates and begins to expand, the topological susceptibility χ inside this region rapidly approaches its confined asymptotical value on a timescale much shorter than β^{-1} at percolation. This can be understood from the fact that localized equilibration in strongly coupled plasma dynamics is of the order $\tau_{eq} \sim \mathcal{O}(1/T_c)$.

The confinement FOPT is a deflagration arising from a strong phase transition, characterized by relatively large transition strengths, $\alpha \gtrsim 0.1$, and subsonic bubble wall velocities, $v_w \leq c_s(T_n)$. In this case, the substantial latent heat released can locally reheat the region surrounding the expanding true-vacuum bubbles above T_c , leading to the formation of remnant false-vacuum droplets and temporarily interrupting bubble coalescence [92–94]. One can estimate the locally reheated temperature as $T_{\text{re,loc}} \sim (1 + \alpha)^{1/4} T_n$, with α the amount of latent heat released. We have that $\alpha \sim 0.3\text{--}0.4$ [33,82] and $T_n \sim 0.99 T_c$, hence $T_{\text{re,loc}} \simeq 1.06 T_c$. This estimate is only valid right at the moment of nucleation and can be compared with the simulations conducted in Ref. [95] (see their Fig. 11), where they find that at bubble nucleation, the local reheated region has heated up to around 15% above the nucleation temperature.⁶ The key point is that false-vacuum droplets can still be present even after bubble coalescence, which can delay the formation of the DW network.

Therefore, in the subsequent subsection we want to elaborate on the dynamics as the initial bubble coalescence proceeds. We will discuss the subsequent evolution relying on macroscopic hydrodynamic relaxation toward equilibrium and use linearized hydrodynamics for qualitative estimates.

2. After bubble coalescence

In this section, we present the timescales relevant for the formation of the DW network after bubble coalescence. We use insights from the following collection of references regarding holographic thermalization, planar AdS/asymptotic AdS wall collisions, and strongly coupled equilibration dynamics [96–107]. We will summarize our findings below and highlight some more details in Appendix.

First, we will look at the set of timescales of the local collision region itself, which has a size $\ell_{\text{local}} \sim \mathcal{O}(1/T_c) \sim \mathcal{O}(1/\Lambda)$. This will give us the timescale for the emergence of local DW segments within the collision region. Following that, we will characterize the timescale when the thermal fluctuations of the system on larger scales $R_* \gg \ell_{\text{local}}$ become damped, denoted by τ_{damp} , which estimates the initial emergence of large connected DW network regions.

During bubble coalescence, the collision regions are initially driven far from equilibrium and undergo substantial local reheating, whereas remnant false-vacuum droplets still persist alongside the forming network. The postcollision system can be characterized by the effective temperature $T_{\text{eff}}(\tau, x)$, which immediately after the collision

satisfies $T_{\text{eff}}(\tau, x) > T_p$. We remind the reader that for strongly coupled FOPTs with little supercooling $T_p \sim T_c$. In order for domain walls to start to form locally, the remnant droplets must also cool below T_c , so that the surface tension of the domain wall is well defined. Then the relevant local timescales are the following:

- (i) At $\tau = \tau_{\text{hydro}}$ the system can be described by viscous hydrodynamics, even though the system is still out of equilibrium;
- (ii) At τ_{eq} the system approaches an effective local thermal equilibrium on a length scale ℓ_{local} ;
- (iii) $\tau_{\text{cool}}^{\text{loc}}$ is the time required for the locally reheated region to satisfy $T_{\text{eff}}(\tau, x) < T_c$; and
- (iv) Finally, at τ_{micro} nonhydrodynamical modes (large momenta) on microscopic scales thermalize, as reflected in the evolution of two-point correlation functions such as $\langle \mathcal{O}_\phi \mathcal{O}_\phi \rangle$ or $\langle T_{\mu\nu} T^{\mu\nu} \rangle$, toward their thermal equilibrium values.

Once $\tau \sim \tau_{\text{hydro}}$, the effective temperature of the system T_{eff} has already started settling toward T_c so that the subsequent evolution thereafter takes place with an effective temperature $T_{\text{eff}} \sim T_c$ [98,99]. The relevant local relaxation scales obey the hierarchy⁷

$$\tau_{\text{hydro}} \lesssim \tau_{\text{eq}} \sim \tau_{\text{micro}}. \quad (44)$$

All of these scales are of the order $\mathcal{O}(1/T_c) \sim \mathcal{O}(1/\Lambda)$ in strongly coupled plasmas [98,99].

The initial formation of local DW segments can occur when, at least within the effective local thermal equilibrium (LTE) regime, the associated temperature in a region drops below T_c . The local DW segment formation time can therefore be estimated as

$$\tau_{\text{DW}}^{\text{loc}} \simeq \max(\tau_{\text{eq}}, \tau_{\text{cool}}^{\text{loc}}). \quad (45)$$

Thus, $\tau_{\text{DW}}^{\text{loc}}$ refers to the DW formation within the local postcollision region, whose length scale is set by the wall thickness $\ell_{\text{DW}} \sim \ell_{\text{local}}$.

The next question is when these local DW segments can merge into a connected network. Even after approaching LTE within the collision region of two bubbles, large-scale temperature inhomogeneities persist over the mean bubble separation R_* . These fluctuations equilibrate on a timescale τ_{damp} , which should be shorter than a Hubble time to allow the formation of the DW network, i.e., $\tau_{\text{damp}} H < 1$. We estimate this damping timescale using linearized hydrodynamics. The amplitude of long-wavelength sound waves goes as

⁶The effect is more prominent when the bubble wall travels slowly. In such transitions, the GW spectrum can be further suppressed by the fact that the kinetic energy is converted into heat in the pocket regions [95].

⁷This hierarchy of timescales persists both in conformal and nonconformal strongly coupled plasmas, whereas the actual values differ by $\mathcal{O}(1)$ numbers, i.e., $\tau_{\text{nonCFT}}^{\text{loc}}/\tau_{\text{CFT}}^{\text{loc}} \sim \mathcal{O}(1)$. For details, see Appendix or [100,104,106–108].

$$\mathcal{A} \sim e^{-i\omega(k)t}, \quad (46)$$

where the dispersion relation for $\omega(k)$ is given by

$$\omega(k) = c_s k - \frac{i}{2} \Gamma_s k^2 + \mathcal{O}(k^3), \quad (47)$$

and we have chosen the branch with positive real part [109–111]. The negative imaginary part describes viscous attenuation. The expansion of Eq. (47) is only valid in the hydrodynamical regime at low momenta. The sound attenuation constant Γ_s is given by

$$\Gamma_s = \frac{1}{T} \left(\frac{4\eta(T)}{3s} + \frac{\zeta(T)}{s} \right), \quad (48)$$

where η is the shear viscosity and ζ is the bulk viscosity of the plasma. Hence, the attenuation of sound waves/thermal fluctuations evolves as

$$A(t) \simeq A_0 e^{-\frac{1}{2} k^2 \Gamma_s t}. \quad (49)$$

For the damping time $\tau_{\text{damp}} \sim 2/(k^2 \Gamma_s)$ over length scales of order $k^{-1} \sim R_*$ we get

$$\tau_{\text{damp}} \simeq \mathcal{O}(1) \frac{R_*^2}{\Gamma_s}, \quad (50)$$

where $T \simeq T_c$. From Appendix we know that in IHQCD

$$\Gamma_s = \frac{1}{sT_c} \left(\frac{7}{3} \eta \right), \quad (51)$$

where $\zeta/s \sim \eta/s = 1/(4\pi)$, at $T \simeq T_c$ see Fig. 10. Additionally, using $R_* H \simeq 10^{-5} v_w$, we find that the thermal damping timescale is roughly given by

$$H\tau_{\text{damp}} \sim \frac{12\pi 10^{-10} v_w^2}{7} \left(\frac{T_c}{H} \right), \quad (52)$$

where, assuming radiation domination, we have

$$H(T) \simeq \frac{\pi}{3} \left(\frac{g_*(T)}{10} \right)^{1/2} \frac{T^2}{M_{\text{Pl},4d}} \quad (53)$$

and $M_{\text{Pl},4d} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass, which is not to be confused with M_{Pl} used in holographic calculations above. Ultimately, we find that the time duration for damping of local thermal fluctuations is given by⁸ (using $g_*(T_c) \approx 100$)

⁸At large N_c and in the deconfined phase, our dark YM sector significantly contributes to $g_*(T)$. Here, the relevant scenario occurs after the confining PT has been completed, in which the spectrum consists of massive glueballs which do not contribute to $g_*(T)$.

$$H\tau_{\text{damp}} \sim \frac{12\pi}{7} v_w^2 10^{-10} \left(\frac{M_{\text{Pl},4d}}{T_c} \right). \quad (54)$$

The condition for fast damping of thermal fluctuations $\tau_{\text{damp}} H \leq 1$ gives us a lower bound on T_c

$$T_c \gtrsim 1.3 v_w^2 \times 10^9 \text{ GeV}. \quad (55)$$

As long as this bound on T_c is satisfied, the thermal fluctuations of size R_* produced in the surrounding plasma during bubble coalescence are damped in less than a Hubble time and will not obstruct the formation of long connected regions. Hence, the DW network is formed when

$$\tau_{\text{DW}}^{\text{net}} \sim \tau_{\text{damp}}. \quad (56)$$

The scenarios presented in Ref. [33], with $T_c = 50$ MeV, 100 GeV, and $v_w \sim 0.01$, violate Eq. (55), leading to a delay in DW formation, as we will now discuss.

If Eq. (55) is violated and $\tau_{\text{damp}} H > 1$, the connection of local DW segments is delayed due to large temperature fluctuations present in the plasma. There are two important consequences. On the one hand, as the Universe expands and cools, the typical length of DWs, R_* , has increased, and the initial population of DWs would be less overdense. On the other hand, the network may never form, since the potential bias δV breaking the degeneracy of the vacua may already dominate the surface tension σ leading to annihilation. In the following, we will assume that we are in the scenario where (55) is satisfied and the domain wall formation proceeds quickly.

C. Scaling regime

As we assume that the DW network forms quickly after the completion of the phase transition, the initial correlation length of the network is roughly given by R_* . This length scale will evolve until the scaling regime is reached. The scaling regime is an equilibrium attractorlike solution in which the correlation length of the network becomes comparable to the Hubble scale [61,112–116]. During their evolution, the energy density of domain walls is given by

$$\rho_{\text{DW}} \approx \frac{\sigma L^2}{L^3} \approx \sigma L^{-1}, \quad (57)$$

where L is the length scale at that instance in the evolution and σ is the surface tension of the walls. However, as discussed previously, the vacua considered here are not degenerate and there is a potential bias δV between them. This leads to annihilation, and DWs will annihilate when

$$\rho_{\text{DW}} = \delta V. \quad (58)$$

Since the GW spectrum is the strongest if the network has been in the scaling regime for some time [60,117], we need

to check whether the scaling regime can be reached at all in our framework.

To estimate whether our network can evolve to the scaling regime, one needs to find the surface tension of the domain walls σ . As this is not straightforward, we draw some intuition from the considerations in $\mathcal{N} = 1$ SYM [83,118–120]. In that theory, gaugino condensation spontaneously breaks the global $\mathbb{Z}_{2N_c}$ (the nonanomalous subgroup of $U(1)_R$) to $\mathbb{Z}_2$, leaving behind N_c degenerate vacua labeled by an integer k . The tension of the domain wall interpolating between the k th and k' th vacuum was found to be [83,119,120]

$$\sigma_{k,k'} = \frac{N_c^2}{4\pi^2} \left| \sin\left(\frac{\pi(k-k')}{N_c}\right) \right| \Lambda^3. \quad (59)$$

SUSY can be softly broken by adding a small gaugino mass m_g , which breaks the degeneracy between the vacua [83]. The SUSY-breaking term does not change the surface tension as long as $m_g \ll \Lambda$. For $m_g \gtrsim \Lambda$, the tension will change by an $\mathcal{O}(1)$ factor, but the overall scaling with N_c and Λ will not change. As there is no quantitative prediction of this surface tension computed with IHQCD, we adopt the SUSY result in our GW estimates.

As before, we will only consider domain walls separating the two lowest branches, i.e., $k = 0$ and $k = -1$. The domain-wall tension is given by

$$\sigma_{0,-1} \simeq \frac{N_c}{4\pi} \Lambda^3, \quad (60)$$

in the large N_c approximation. From Eq. (22), we can easily calculate the bias term for $\theta_{UV} \rightarrow \pi^-$

$$\delta V = 2\pi(\pi - \theta_{UV})\chi = 2\pi\epsilon_\theta\chi. \quad (61)$$

So, let us estimate whether our network evolves into the scaling regime or gets annihilated immediately after the completion of the phase transition ($T \approx T_c$). Since the length scale of domains after the phase transition is roughly given by R_* , the domain walls will survive annihilation if the surface tension wins over the bias

$$\sigma R_*^{-1} \gtrsim \delta V, \quad (62)$$

where $R_* \sim 10^{-5} H(T_c)^{-1}$ for typical parameters considered in [33]. Using the lattice inputs as before, we find

$$T_c \gtrsim \frac{\epsilon_\theta}{N_c} 2 \times 10^{14} \text{ GeV}. \quad (63)$$

If the critical temperature violates the inequality in (63), the domain wall annihilation will proceed immediately. This inequality is generally violated for the critical temperatures considered in [33], unless N_c is insanely large or $\epsilon_\theta \sim 10^{-12}$. For larger N_c , the range of validity in our

calculation improves and may be the most suitable parameter to vary over a wider range.

We assume that the inequality in (63) is satisfied and that the domain walls can reach the scaling regime. This raises the question of how long it takes for the formed domain wall network to reach the scaling regime. In simulations of domain wall networks, carried out mainly in weakly coupled theories, it takes the network roughly 3–7 Hubble times to reach the attractor scaling solution [59]. As discussed previously, the upshot here is that local microscopic equilibration in strongly coupled theories is a rapid process with relaxation timescales of order $\tau \sim T_c^{-1}$, thus removing one source of dynamical uncertainty. Hence, one could anticipate that approaching scaling in these theories would be at least equally fast as in weakly coupled theories, if not slightly faster. However, since the approach to scaling regime is a dynamical phenomenon, simulations are necessary to accurately quantify the claims above.

D. Summary

As the discussion on DWs was rather lengthy, we would like to provide a small summary with the most relevant results. During the phase transition of the $SU(N_c)$ YM theory, confining bubbles can occupy different vacuum branches which can lead to a DW network under certain conditions once the phase transition has completed. These conditions can be listed below by their chronological relevance:

- (i) The bound on θ_{UV} (or ϵ_θ) from percolation theory [Eq. (41)] tells us whether there are enough bubbles with vacuum branch index k nucleating to form a DW network at all.
- (ii) The bound on T_c in Eq. (55) comes from the requirement that the thermal fluctuations relax within a Hubble time. If this is not satisfied, the formation of a global DW network will be delayed, suppressing the possible GW signal, if a scaling network forms.
- (iii) The condition on T_c in Eq. (63) determines whether the DW network annihilates immediately after the completion of the PT or evolves into the scaling regime.

For the values of the critical temperatures considered in [33], the last two conditions are typically violated. Thus, even if the fluctuations manage to relax for the network to form, it will annihilate immediately. Instead, a large critical temperature $T_c \gtrsim 10^8 \text{ GeV}$ is needed. Assuming that all of the conditions above are satisfied, in the next section we estimate the GW signal from a DW network that reaches the scaling regime and then annihilates at later times before starting to dominate the energy budget of the Universe.

V. GRAVITATIONAL WAVES FROM ANNIHILATING DOMAIN WALLS

Thus, provided that the conditions for having metastable DWs are satisfied, the annihilation temperature is given by

equating the energy density of the DWs during scaling $\rho_{\text{DW}} \simeq \sigma_{\text{DW}} H(T_{\text{ann}})$ to the energy bias δV to find

$$T_{\text{ann}} \simeq 1.61 \sqrt{\frac{M_{\text{Pl},4d} T_c \epsilon_\theta}{N_c}}. \quad (64)$$

DWs are dangerous cosmological relics since their energy density scales as $\rho_{\text{DW}} \sim a^{-1}$, which is slower than radiation scaling with $\rho_{\text{rad}} \sim a^{-4}$. Therefore, one needs to ascertain that the network annihilates before it starts to become the dominant energy component of the Universe [121]. The temperature at which the produced DWs would start to dominate the energy density of the Universe is found by setting $\rho_{\text{DW}} = \rho_{\text{rad}}$ and one finds

$$T_{\text{dom}} \simeq 0.11 \sqrt{\frac{N_c T_c^3}{M_{\text{Pl},4d}}}. \quad (65)$$

Hence, the condition which we have to impose is

$$T_{\text{ann}} > T_{\text{dom}}. \quad (66)$$

It should be noted that the percolation theory condition (41) directly impacts the annihilation temperature, whereas the domination temperature is insensitive to θ_{UV} . In fact, reducing ϵ_θ allows us to delay annihilation for a fixed combination of N_c and T_c .

As we want to reach scaling, we parametrize the critical temperature with a dimensionless parameter x , namely

$$T_c = x \cdot \frac{\epsilon_\theta}{N_c} 2 \times 10^{14} \text{ GeV}. \quad (67)$$

Hence, to have a DW network which remains in the scaling regime for a sufficient time before annihilation, we require $T_{\text{ann}} \ll T_c$, leading to $x \gg 10^4$. This provides us with a lower bound on the critical temperature, reinterpreted as a prefactor x which needs to be sufficiently large. Interestingly enough, taking the extreme limit $T_{\text{ann}} \lesssim T_c$ one finds that the parameter x is independent of θ_{UV} and N_c . This implies that the separation between the critical temperature and the annihilation temperature can be made arbitrarily large by increasing x . The upper limit on x is set by requiring that we do not enter the DW domination era, i.e., $T_{\text{ann}} > T_{\text{dom}}$. From Eqs. (64) and (65) we get

$$\frac{T_{\text{ann}}}{T_{\text{dom}}} \approx 14.6 \frac{M_{\text{Pl},4d} \sqrt{\epsilon_\theta}}{T_c N_c} \approx \frac{1.78 \times 10^5}{x \sqrt{\epsilon_\theta}}. \quad (68)$$

Requiring $T_{\text{ann}} > T_{\text{dom}}$ leads to an upper bound on the critical temperature, i.e., on the parameter x . Thus, we obtain that the viable range of x is

$$10^4 \ll x \leq \frac{1.76 \times 10^5}{\sqrt{\epsilon_\theta}}. \quad (69)$$

Therefore, one needs to have sufficient tuning in θ (or ϵ_θ) both to have a sufficient separation of scales between the annihilation temperature and the critical temperature and to satisfy the percolation bound in (41). As an example, consider (41) with $\xi = 10$, then if we wish to have a separation of scales with $x \approx 10^7$, we need to have $\epsilon_\theta \lesssim 10^{-4}$.

As the DWs expand and eventually annihilate due to the volume pressure generated by the bias δV , they produce a direct source of anisotropic stress in the energy-momentum tensor, generating gravitational waves. The GWs are mainly emitted when the network annihilates, as that is when the total energy density ρ_{DW} is the largest. Our estimates bear some degree of uncertainty, as neither an explicit computation of the DW surface tension in IHQCD has been performed nor the explicit time to approach scaling. Hence, the estimates here should be thought of as an order-of-magnitude discussion. The magnitude and spectral shape of the GW spectrum from the annihilation of DWs are found from numerical simulations such as [59,60] with its peak amplitude and frequency given by

$$\Omega_{\text{GW}}^{\text{peak}} h^2 \simeq 2.8 \times 10^{-18} \left(\frac{\sigma}{\text{TeV}^3} \right)^2 \left(\frac{10 \text{ MeV}}{T_{\text{ann}}} \right)^4, \quad (70)$$

$$f_p \simeq 1.1 \times 10^{-9} \text{ Hz} \left(\frac{T_{\text{ann}}}{10 \text{ MeV}} \right). \quad (71)$$

and a broken power-law spectrum scaling as f^3 in the causal IR tail and as f^{-1} above the peak.

In our explicit model, the DW tension and annihilation temperature depend on T_c , N_c and $\epsilon_\theta = \pi - \theta_{\text{UV}}$. For the peak amplitude and peak frequency, we find

$$\Omega_{\text{GW}}^{\text{peak}} h^2 \simeq 1.2 \times 10^{-13} \left(\frac{T_c}{10^{12} \text{ GeV}} \right)^4 \left(\frac{10^{-7}}{\epsilon_\theta} \right)^2 \left(\frac{N_c}{100} \right)^4 \quad (72)$$

$$f_p \simeq 8.7 \text{ kHz} \left(\frac{T_c}{10^{12} \text{ GeV}} \right)^{1/2} \left(\frac{\epsilon_\theta}{10^{-7}} \right)^{1/2} \left(\frac{100}{N_c} \right)^{1/2}. \quad (73)$$

In terms of the prefactor x this can be written as

$$\Omega_{\text{GW}}^{\text{peak}} h^2 \simeq 1.9 \times 10^{-12} \left(\frac{x}{10^7} \right)^4 \left(\frac{\epsilon_\theta}{10^{-7}} \right)^2, \quad (74)$$

$$f_p \simeq 12 \text{ kHz} \left(\frac{x}{10^7} \right)^{1/2} \left(\frac{\epsilon_\theta}{10^{-7}} \right) \left(\frac{100}{N_c} \right),$$

where we stress that x has to satisfy Eq. (69). Replacing this upper limit in Eq. (74) we get the maximal allowed GW amplitude $\Omega_{\text{GW}}^{\text{peak}} h^2 \lesssim 1.8 \times 10^{-5}$. We show in Fig. 9

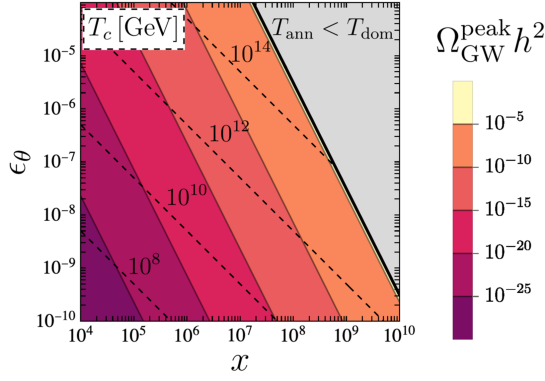


FIG. 9. Contours of the peak GW signal strength $\Omega_{\text{GW}}^{\text{peak}} h^2$ as a function of x , ϵ_θ . We show with dashed lines the contours of T_c in GeV, obtained for $N_c = 100$.

contours of the peak GW amplitude as a function of x and ϵ_θ , together with the corresponding values of T_c for N_c fixed to 100.

In order to maximize GW prospects, one needs to increase the factor x , which from Eq. (68) means that the domain wall network annihilates closer to the time when it would start dominating the energy density. This can be achieved by a larger critical temperature and a more severe fine tuning ϵ_θ .

Increasing T_c also shifts the peak toward higher frequencies. However, this can be compensated for by making N_c larger or getting ϵ_θ closer to 0, which would bring the signal to lower frequencies. In conclusion, to enter the range relevant for future GW experiments [9,122,123]

$$\Omega_{\text{GW}}^{\text{peak}} \sim 10^{-13}, \quad f_p \sim 10 - 100 \text{ Hz}, \quad (75)$$

one would need to achieve significant fine-tuning between ϵ_θ , N_c , and T_c (or x).⁹

In principle, a specific signature may appear in the GW signal if the split between the critical temperature and the annihilation temperature is maximized. Then two distinct peaks in the total GW spectrum appear, one from the PT and one from DW annihilation. The best combination is to consider $\epsilon_\theta \ll 1$, $N_c \gg 1$, and T_c as high as possible within the bounds to allow for the formation and adequate evolution of the DW network. Then, depending on how close ϵ_θ is to 0, it might be preferable to generate one large GW peak from DW annihilation, as the amplitude is

⁹We should also emphasize that the value of T_c needs to satisfy the bounds coming from reheating, regarding the maximal temperature of the Universe T_{max} . In single-field inflation with maximal reheating temperatures are given by $T_{\text{max}} \sim \mathcal{O}(3-5) \times 10^{15}$ GeV [124]. Interestingly enough, in the regime where the GW amplitudes are the largest T_{max} actually becomes a constraint of merit one needs to account for. For example, take again $\xi = 10$, in the case when $x = 10^6$ and $\epsilon_\theta = 1.71 \times 10^{-4}$, to satisfy $T_c < T_{\text{max}}$ one finds the bound $N_c \gtrsim 7$.

maximized and the peak frequency is lowered for fixed combinations of T_c and N_c . If the initial critical temperature is sufficiently low, one can try to have both GW peaks in the ET/DECIGO regime for correlated signal searches, but this seems severely tuned within our setup.

VI. DISCUSSION AND CONCLUSION

In this work, we showed that the inclusion of the θ -angle reduces the range of supercooling available in the confining phase transition of $\text{SU}(N_c)$ Yang-Mills theories. When studying the effects of finite θ on the YM phase diagram, we pinpoint the lower limit where large N_c approximations may be valid, and find the lower limit to be $N_{c,\text{min}} = 10$ where finite N_c effects are directly relevant. This bound may serve as a general insight when having large N_c arguments in mind, while working at moderately large but finite N_c .

Additionally, since different bubbles may occupy different branches of Yang-Mills vacua, the DW network can form if the θ -angle satisfies the bound from percolation theory. Provided the network forms, the bias is typically strong enough to annihilate the network immediately, effectively leaving no observational imprint in GWs unless significant fine-tuning is in place. In the future, one can attempt to compute the explicit surface tension of the domain walls in our holographic construction. For example, this can be done by assuming a planar wall and the axion profile which depends on an additional spatial coordinate, i.e., $a(r, z)$, where z is the coordinate perpendicular to the wall. However, as already discussed, we expect this to alter the surface tension by an $\mathcal{O}(1)$ factor, which appears to be irrelevant in our estimates.

Another effect that would be interesting to understand is how droplet formation would delay the initial completion of the PT and modify local DW formation, as surface tension would become spatially dependent. Our analysis suggests that DW formation in strongly coupled confinement transitions may be governed by a hierarchy of local equilibration and macroscopic damping scales. While local wall formation occurs rapidly, reheating and bulk-viscous effects may delay the emergence of a connected network. Quantifying these effects requires dedicated real-time simulations of planar walls in IHQCD with a θ -angle.

A natural improvement in our calculations would be to include the full backreaction of the axion on the metric and the dilaton.¹⁰ We have identified the values of N_c and θ_{UV} for which the backreaction should be included with the requirement that $T_c(\theta_{\text{UV}}) > T_{\text{min}}$. As the regular axion profile in the black hole background requires $C = 0$ [Eq. (26)], the axion does not backreact on the temperature curve $T(\lambda_h)$ (Fig. 1). Thus, the value of T_{min} will not change with the inclusion of the backreaction.

¹⁰Some general properties of axion RG flows were already considered in [125,126].

The backreaction becomes important far within the SBH branch, where it should make the free energy diagram smoothly approach zero. However, we do not expect our conclusions on the amount of supercooling to change significantly. The axion is different from the dilaton because it has a shift symmetry, i.e., $a \rightarrow a + \alpha$ for some constant α , and the conserved quantity in question is the axion charge. Thus, the full inclusion of the axion backreaction may lead to the appearance of new saddles charged under this symmetry, such as Giddings-Strominger wormholes [127] (see also, e.g., [128–139]). Whether axion wormholes are admitted in IHQCD and, if so, whether they contribute to the phase diagram of the dual gauge theory, remains, to our knowledge, an open question, which we leave for future work.

To summarize, the effect of a nonzero θ -angle on confining phase transitions in $SU(N_c)$ Yang-Mills theories is mainly a decrease of the critical temperature and, consequently, of the maximal amount of supercooling $\epsilon_{\max} = 1 - T_{\min}/T_c$. We computed this effect in the framework of IHQCD, obtaining results compatible with those in the literature. Although this requires a significant amount of tuning, our analysis shows that finite- θ confinement transitions can be accompanied by domain-wall production, governed by a hierarchy of dynamical processes. These include vacuum assignment during bubble nucleation, local equilibration and reheating during bubble coalescence, and the macroscopic damping of thermal fluctuations. Our work discusses the conditions under which such a domain-wall network can be generated and its subsequent evolution. Thus, in strongly coupled nonconformal FOPTs, domain-wall production should not be viewed as an instantaneous consequence of percolation alone. Moreover, we show how the dynamics of the domain wall network can lead to a distinctive GW signature, and highlight the conditions and the level of fine-tuning that are needed for realizing such a scenario.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX: RELEVANT TIMESCALES

In this appendix, we elaborate a bit further for the reader interested in the local equilibration timescales during the onset of bubble coalescence.

- (a) *Reaching τ_{hydro}* : Real-time simulations of planar shocks within the AdS/CFT framework in strongly coupled CFTs are mainly conducted in $\mathcal{N} = 4$ SUSY YM theory [98,99,105], where it was found that $\tau_{\text{hydro}} \sim \mathcal{O}(1/T)$ considering Bjorken flow. This is the timescale at which, from an initial state, the system’s temperature evolution is governed by viscous hydrodynamics. In particular, for $\tau \geq \tau_{\text{hydro}}$ the diffusion length of the system is governed by the shear viscosity η , which is related to the entropy density as $\eta/s = \frac{1}{4\pi}$. However, in nonconformal cases, τ_{hydro} is found to be increased [106,107] due to the presence of additional transport coefficients, primarily due to the presence of the bulk viscosity ζ . Hence $\tau_{\text{hydro}}^{\text{nonCFT}} \gtrsim (1.5\text{--}2)\tau_{\text{hydro}}^{\text{CFT}}$ roughly by the magnitude of the bulk viscosity ζ in relation to the shear viscosity. However, the studies reviewed in [100] indicate that $\tau_{\text{hydro}}^{\text{nonCFT}}$ is still within the same order of magnitude as in the CFT case. As bulk and shear viscosity affect different channels of relaxation, this is not a straightforward claim. However, when calculating ζ/s using IHQCD as in [103] one finds $\zeta/s \sim \eta/s$ in the range $T_c > T > T_{\min}$. This indicates that the sound attenuation constant which scales as

$$\Gamma_s \simeq \frac{1}{sT_c} \left(\frac{4}{3}\eta + \zeta \right) = \frac{1}{T_c} \frac{7}{3 \cdot 4\pi} = \frac{7}{12\pi T_c}, \quad (\text{A1})$$

is increased by a factor of $\frac{7}{4}$ compared to its conformal value $\Gamma_s = \frac{1}{3\pi T_c}$, validating the qualitative statements above for the macroscopic hydrodynamization timescale of thermal fluctuations.

For our purposes, the main implication is that the collision region quickly hydrodynamizes $\tau_{\text{hydro}} \sim \mathcal{O}(1/T_c)$, such that the subsequent evolution toward an effective LTE can be described by viscous hydrodynamics. When $\tau \sim \tau_{\text{eq}}$ the system is described by an effective LTE regime containing ideal hydro plus first order viscous hydro whose transport coefficients are calculated within IHQCD. To affirm our recent claims regarding the magnitude of the bulk viscosity ζ in relation to the shear viscosity η , we calculate the bulk viscosity coefficient ζ using the Eling-Oz formula [140]

$$\frac{\zeta}{\eta} = \left(s \frac{d\phi_h}{ds} \right)^2 = \frac{1}{9} A'(\phi_h)^{-2} = \frac{1}{9} (\lambda_h A'(\lambda_h))^{-2}. \quad (\text{A2})$$

This can be further simplified into an expression for the bulk viscosity to entropy ratio of the following form

$$\frac{\zeta}{s} = \frac{\eta}{s} \left(\frac{1}{9\lambda_h^2 A'(\lambda_h)^2} \right) = \frac{1}{4\pi} \left(\frac{1}{3\lambda_h A'(\lambda_h)} \right)^2, \quad (\text{A3})$$

see Fig. 10. In [102] it was shown that the Eling-Oz formula in IHQCD matches well with the conventional approach of computing the IR limit of the two-point function of the trace of the stress energy tensor [103,141].

- (b) *Reaching τ_{eq}* : We draw insights mainly from [106,107], which also showcases the timescale at which isotropization is achieved, in other words, when the different components of the pressure have become roughly equal. Interestingly enough, in [106–108,111,142] it was found that isotropization is often achieved more slowly even though the average pressure may have relaxed. Furthermore, the fact that the viscous hydrodynamic description is applicable before isotropization is achieved is quite remarkable. However, in our context of bubble collisions (assumed to be planar), this should instead translate into saying that the pressure gradient is becoming small. Nevertheless, for our purposes, it is more important to establish an estimate for when the system, after bubble coalescence, approaches an effective LTE regime, such that one can assign a local temperature and EoS sufficiently described by $T^{\mu\nu}(\bar{x}) \simeq (p + e)u^\mu u^\nu + \Pi^{\mu\nu}$ in which the viscous corrections $\frac{\eta}{p} < 1, \frac{\zeta}{p} < 1$. In the strongly coupled setting $\tau_{\text{eq}} \sim \mathcal{O}(5\text{--}10)\tau_{\text{hydro}}$ which, as we will see, is well within a Hubble time for our viable region of critical temperatures.

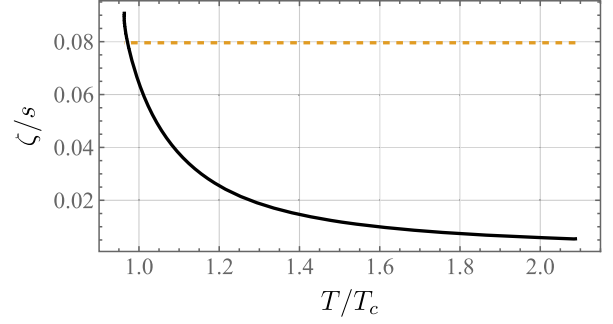


FIG. 10. Bulk viscosity-to-entropy density ratio ζ/s in black compared to the fixed value for the shear viscosity-to-entropy density bound $\eta/s = 1/4\pi$ in yellow dashed.

- (c) *Reaching τ_{micro}* : Microscopic thermalization of conformal strongly coupled QFTs was considered in [96,97], where they looked at the evolution of the vacuum expectation value of the energy-momentum tensor and scalar correlation functions. From the gravity picture one studies the equilibration of a black hole geometry by studying its perturbed evolution and decay of the quasinormal modes. In these scenarios, one finds that $\tau_{\text{micro}} \sim \mathcal{O}(1/T)$, in particular, the thermalization of modes with high frequency and small momenta is fastest. In contrast, modes with both large momenta and frequency thermalize more slowly. In nonconformal scenarios considered in [101,104], it was also found that the microscopic thermalization timescale still goes as $\tau_{\text{micro}}^{\text{nonCFT}} \sim \mathcal{O}(1/T_c)$. This has been further corroborated in actual computations of QNMs for IHQCD in [143], which showcased that the thermalization timescale is of the order $\mathcal{O}(1/T_c)$ near the confinement PT. The dominant QNM is the one stemming from the lightest glueball mass, whose QNM is the slowest to equilibrate. The main result of [143] was to show that in the PT regime it still decays on a timescale of $1/T_c$.

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