

Real-time evolution of critical modes in the QCD phase diagram

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We develop a QCD-assisted relaxation dynamic model for the critical mode of the critical end point (CEP) in the QCD phase diagram, which allows us to investigate the critical slowing-down effect quantitatively in the QCD phase diagram, especially in the proximity of the CEP, without any phenomenological parameters. The relaxation time from nonequilibrium to equilibrium in the QCD phase diagram is extracted from the Langevin simulations of the QCD-assisted relaxation dynamic model. It is found that in a narrow region along the phase boundary radiated from the CEP, the relaxation time is enhanced significantly. Outside this narrow region, the relaxation time drops drastically, which implies that the dynamic critical region is small in the QCD phase diagram. Within the present setup in this work, we further find that the relaxation time is not strongly enhanced along the considered chemical freeze-out curves, suggesting that the effects of critical slowing down may be limited there.

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Introduction. Studying the QCD phase diagram plays a pivotal role in exploring properties of strongly interacting matter under extreme conditions [1–8]. The structure of the QCD phase diagram and the nature of its phase transitions are central frontiers in modern nuclear and particle physics, with implications for understanding the early Universe, neutron star interiors, and other fundamental questions. The QCD phase diagram is characterized by the critical end point (CEP) of possible existence [9,10], that is, the end point of the first-order phase transition line in the regime of high baryon chemical potential or, equivalently, high baryon densities. Recent years have seen significant progress in the study of the QCD phase diagram, e.g., the estimate of the location of the CEP in the phase diagram from direct computations of functional QCD [11–13] or extrapolations of lattice QCD based on Yang-Lee edge singularities [14,15]. Fluctuation observables, such as the high-order cumulants of net-proton (baryon) number distributions, are employed to search for the CEP in heavy-ion collision experiments [16–27], where a peak structure of the kurtosis of baryon number fluctuations as a function of the collision energy is expected, as the fireball in heavy-ion collisions evolves near the CEP [21,26,27].

Another intricacy arises from an effect called the critical slowing down [28]. As the system is evolving in the proximity of CEP, the correlation length ξ increases significantly due to the nature of second-order phase transitions for the CEP. Since the curvature of the effective potential of the critical mode, i.e., the sigma mode, behaves as $\sim 1/\xi^2$, the relaxation time from nonequilibrium to equilibrium would be enhanced remarkably near the CEP. Consequently, the nonequilibrium effects would come into play, for instance, drastically modifying the high-order fluctuations due to the memory effects [29].

To resolve the problems mentioned above, in principle, one needs to simulate the real-time dynamics from microscopic interactions, e.g., QCD for a many-body system, which is, however, a formidable task. It turns out that one can simplify the situations by starting from mesoscopic instead of microscopic dynamics. For instance, the dynamic models summarized in [30] have provided us with valuable information about the real-time dynamics of critical phenomena in phase transitions. The $O(4)$ chiral phase transition of QCD can be classified as Model G of dynamic universality classes [31], and the CEP as Model H [32].

The mesoscopic dynamic models are powerful in studying dynamic universal properties of critical phenomena inside the critical region, e.g., the dynamic critical exponent [30]. However, this is not the case outside the critical region when nonuniversal properties are concerned since dynamic models usually rely on phenomenological parameters, such as various transport coefficients, which cannot be computed in the models themselves but rather have to be calculated from microscopic interactions. Therefore, it is a goal of

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real-time dynamics to combine the microscopic and mesoscopic models, or even macroscopic approaches, e.g., hydrodynamics.

The renormalization group (RG) is a tailor-made method of resolving the physics of different scales. In particular, the functional renormalization group (fRG) has been successfully used in the study of both QCD at finite temperature and densities [7,8,11,33–35] and dynamic models [36–46]. In this letter, we combine the microscopic QCD and the mesoscopic dynamic models within the fRG approach, which allows us to develop a QCD-assisted relaxation dynamics for the critical mode of CEP. In the QCD-assisted relaxation dynamics, the phenomenological parts are completely computed from QCD at finite temperature and densities. Therefore, one is able to investigate the properties of critical slowing down quantitatively in the QCD phase diagram, specifically near the CEP.

We first describe the relaxation dynamics of the critical mode with the Schwinger-Keldysh (SK) effective action. The QCD-assisted Langevin transport model is then discussed in detail. Next, we simulate the QCD-assisted Langevin transport in $3 + 1$ dimensions, from which the relaxation time of the critical mode in the QCD phase diagram can be extracted. More details about the QCD-assisted Langevin transport model and its simulations are presented in the Supplemental Material [47]. Our results offer a novel perspective on the critical slowing down near the CEP in the QCD phase diagram.

Relaxation dynamics of the critical mode. The nonequilibrium relaxation evolution of a critical mode, e.g., the σ mode in QCD phase transitions, can be well described by Model A, which is a purely dissipative relaxation model classified in the seminal review about dynamical models in [30]. Note that the QCD critical point is expected to belong to the Model H universality class [32], in which the conserved order parameter is reversibly coupled to the momentum density. The Model A description adopted here captures the dominant slowing down of the order parameter but neglects this mode coupling; we will come back to this limitation when discussing the phenomenological implications below. The related SK effective action reads

$$\Gamma[\phi_c, \phi_q] = \int d^4x (Z_\phi^{(t)} \sigma_q \partial_t \sigma_c - Z_\phi^{(i)} \sigma_q \partial_i^2 \sigma_c + 2U'(\sigma_c) \sigma_q - 2Z_\phi^{(t)} T \sigma_q^2), \quad (1)$$

with the field $\phi = \{\sigma\}$, $\partial_t \equiv \partial/\partial t$ and $\partial_i \equiv \partial/\partial x^i$ ($i = 1, 2, \dots, d$). Here, the spatial dimension $d = 3$ is chosen, and a summation for i in ∂_i^2 is assumed. The subscripts c and q stand for the classical and quantum fields, respectively; see, e.g., Refs. [41,43,58] for more details. Discussions about the functional renormalization group with the Schwinger-Keldysh path integral can be found

in, e.g., Refs. [58–60]. In effective action Eq. (1), $Z_\phi^{(t)}$ and $Z_\phi^{(i)}$ are the temporal and spatial wave functions, respectively. Here, $U(\sigma_c)$ denotes the effective potential and $U'(\sigma_c)$ the derivative, which arises from the difference between the effective action on the two different time branches on the SK contour. The last term quadratic in σ_q represents a Gaussian white noise with temperature T . Note that the temporal wave function $Z_\phi^{(t)}$ appears as the coefficient of the noise term due to the fluctuation-dissipation theorem; see [43] for more details.

The effective action in Eq. (1) is expanded up to the quadratic term in the ϕ_q field, which is also well known as the semiclassical approximation. The semiclassical effective action is equivalent to a description of Langevin transport. The counterpart of Eq. (1) in terms of the Langevin equation reads

$$Z_\phi^{(t)} \partial_t \sigma - Z_\phi^{(i)} \partial_i^2 \sigma + U'(\sigma) = \xi, \quad (2a)$$

with the correlation of the Gaussian white noise

$$\langle \xi(t, \mathbf{x}) \xi(t', \mathbf{x}') \rangle = 2Z_\phi^{(t)} T \delta(t - t') \delta(\mathbf{x} - \mathbf{x}'). \quad (2b)$$

In Eq. (2) we have used $\sigma = \sigma_c / \sqrt{2}$ for simplicity.

QCD-assisted Langevin transport model. In order to simulate the Langevin transport equation (2), one needs the input of three components: the potential $U(\sigma)$ or $U'(\sigma)$, the spatial wave function of the σ mode $Z_\phi^{(i)}$, and the temporal wave function $Z_\phi^{(t)}$, respectively. Note that the first two quantities describe static properties of the system and thus can be calculated directly from, e.g., Euclidean field theory. In contradistinction, the temporal wave function is essentially a dissipation coefficient; it depicts the dynamic properties, so it can only be computed from real-time correlation functions in Minkowski field theories, e.g., the SK field theory, or, equivalently, from the real-time correlation functions continued analytically from the Euclidean ones.

In this work, we adopt the three ingredients necessary in Eq. (2), calculated from QCD at finite temperature and densities within the fRG approach [11,35]. This provides us with a unified method to obtain both the static and dynamic properties of hot QCD matter. In the fRG approach to QCD, quantum fluctuations of quarks, gluons, and ghosts, where the Landau gauge is usually chosen in functional QCD, are integrated in successively with the evolution of the RG scale from the ultraviolet (UV) to the infrared (IR). Moreover, the RG also provides us with a tailor-made approach to deal with transition of degrees of freedom (d.o.f.) in the low-energy QCD. When the RG scale is lowered such that the system enters into the regime

of nonperturbative QCD, e.g., $\lesssim 1$ GeV, a finite gluon mass develops, which entails the glue sector, including the gluons and ghosts, decoupling from the system [61,62]. In addition, as the chiral symmetry is broken spontaneously, collective composite d.o.f., e.g., the pion mesons and the sigma mode $\phi = \{\sigma, \pi\}$, take over the fundamental d.o.f. of $\varphi = \{A, c, \bar{c}, q, \bar{q}\}$ and behave as the most active d.o.f. in the low-energy QCD. Here, $A, c, \bar{c}, q, \bar{q}$ stand for the gluon, ghost, antighost, quark, and antiquark, respectively. This transition of d.o.f. is well described in the fRG approach by virtue of a mature technique called dynamical hadronization [11,63–65]. It is found that there is no double counting with dynamical hadronization, which has been investigated in detail in [48].

Consequently, one is able to compute correlation functions not only for the fundamental fields φ but also the composite fields, e.g., ϕ , in the fRG approach to QCD. All these correlation functions can be derived from a unified effective action $\Gamma[\Phi]$ with $\Phi = \{\varphi, \phi\}$. For instance, the effective potential in Eq. (1) is obtained as follows:

$$U'(\sigma) = \frac{\delta\Gamma[\Phi]}{\delta\sigma} \Big|_{\substack{\sigma(x)=\sigma \\ \Phi=\Phi_{\text{EOM}}}}, \quad (3)$$

where we consider a coordinate-independent potential and $\tilde{\Phi}$ denotes all the other fields except σ , which are on their respective equations of motion (EOM). In the same way, the wave functions can be obtained from the two-point correlation function of mesons, i.e.,

$$\Gamma_{\sigma\sigma}^{(2)} = \frac{\delta^2\Gamma[\Phi]}{\delta\sigma\delta\sigma} \Big|_{\Phi=\Phi_{\text{EOM}}}. \quad (4)$$

Then, for the spatial wave function, one finds

$$Z_{\phi}^{(i)} = \frac{\partial\Gamma_{\sigma\sigma}^{(2)}(p_0, \mathbf{p})}{\partial\mathbf{p}^2} \Big|_{\substack{p_0=0 \\ \mathbf{p}=0}}. \quad (5)$$

As discussed above, the computation of the temporal wave function is more involved. The Euclidean correlation function in Eq. (4) has to be continued analytically to the retarded correlation function in Minkowski spacetime, that is,

$$\Gamma_{\sigma\sigma, \text{R}}^{(2)}(\omega, \mathbf{p}) = \lim_{\epsilon \rightarrow 0^+} \Gamma_{\sigma\sigma}^{(2)}(p_0 = -i(\omega + i\epsilon), \mathbf{p}). \quad (6)$$

Note that in the fRG approach to QCD, the analytic continuation is done on the level of the flow equations, which allows us to perform numerical computation in Minkowski spacetime directly and thus does not suffer from the subtlety of inverse problems if the continuation is done on, e.g., numerical data. Details about the calculations of real-time correlations within the fRG approach to QCD can be found in Ref. [35]. Consequently, the

temporal wave function can be obtained from the imaginary part of the retarded two-point correlation function of the sigma mode, i.e.,

$$Z_{\phi}^{(i)} = \lim_{\omega \rightarrow 0} \frac{\partial}{\partial\omega} (-) \text{Im} \Gamma_{\sigma\sigma, \text{R}}^{(2)}(\omega, |\mathbf{p}| = p_L), \quad (7)$$

where the long time limit $\omega \rightarrow 0$ is implemented. In Eq. (7) the spatial momentum is chosen to be a value $|\mathbf{p}| = p_L$, corresponding to the size of the system. A typical value $p_L = 10$ MeV corresponds to the length scale $1/p_L \sim 20$ fm, which is comparable to the typical size of a fireball in heavy-ion collisions. We also slightly change the value of p_L , and we are interested in the relative change of the relaxation time in the QCD phase diagram, which is found to be insensitive to the specific value of p_L . Note that one is not allowed to adopt the limit $|\mathbf{p}| = p_L \rightarrow 0$ since it is found that, for the sigma mode in Eq. (7), one has $\text{Im} \Gamma_{\sigma\sigma, \text{R}}^{(2)} \sim 1/|\mathbf{p}|$ in the limit of $|\mathbf{p}| \rightarrow 0$, which is discussed in detail in the Supplemental Material [47]. It should be noted that a finite value of the imaginary part of $\Gamma_{\sigma\sigma, \text{R}}^{(2)}$ arises from the Landau damping effect, i.e., the particle-hole process of quarks near the Fermi surface, which has been discussed in detail in [35]. The critical mode exchanges the energy of slow modes with the heat bath through Landau damping.

We close this section by briefly discussing the systematics of QCD calculations at finite temperature and densities within the fRG approach [11]. In the fRG approach to QCD, quantum fluctuations of different scales are integrated in successively with the evolution of the RG scale from the high-energy perturbative to low-energy nonperturbative regimes, such that a full quantum effective action, as well as a variety of correlation functions, is obtained at every fixed temperature T and baryon chemical potential μ_B . This allows us to map out the QCD phase diagram. The obtained phase boundary is quantitatively consistent with the lattice results in the region of $\mu_B/T \lesssim 4$, while the systematic errors increase sizably out of this region. The follow-up studies, e.g., Refs. [34,35,66], indicate that the estimate of systematic uncertainty could be $\lesssim 10\%$ in the regime of $\mu_B/T \gtrsim 4$.

Simulations of QCD-assisted Langevin transport in 3 + 1 dimensions. With the quantities in (3), (5), and (7) computed within the fRG approach to QCD at finite temperature T and baryon chemical potential μ_B [11], one is able to simulate the Langevin equation (2) in 3 + 1 dimensions; see also related classical-statistical simulations [67–70]. We choose a three-dimensional spatial lattice of periodic boundary conditions with lattice constant a and size N_L^3 . Dependence of simulation results on the lattice spacing a and extent N_L are investigated in detail, and the relevant results are presented in the Supplemental Material [47]. We find that

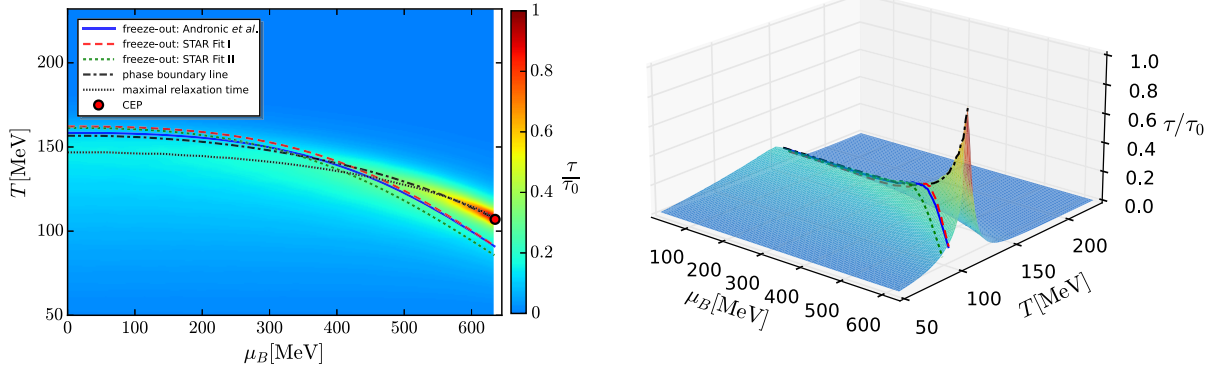


FIG. 1. Left panel: heatmap of the reduced relaxation time τ/τ_0 in the QCD phase diagram. The relaxation time is normalized by the maximal relaxation time τ_0 in this plane, which is located at a point near the CEP, denoted by the red dot. Three typical chemical freeze-out curves, which are also used in [25], are shown. The black dot-dashed line denotes the phase boundary of the crossover obtained in QCD within the fRG [11]. The black dotted line stands for the line of the maximal relaxation time along the temperature direction at a fixed baryon chemical potential. Right panel: three-dimensional plot of the left panel.

convergent results are obtained with $a = 0.1$ fm and $N_L = 32$, where the continuum limit is accessible and the effects of finite volume are negligible. The Langevin equation (2) is implemented as a stochastic partial differential equation in Julia on GPUs [71,72], where the time step is chosen adaptively with the control of relative error. It is found that a relative error of around 10^{-2} is sufficient to resolve the temporal evolution. The Gaussian white noises are generated by N_L^3 independent generators of random numbers with normal distribution.

In the simulations, it is more convenient to work with the renormalized quantities, e.g., the renormalized sigma field $\tilde{\sigma} = \sqrt{Z_\phi^{(i)}} \sigma$; cf. more details in the Supplemental Material [47]. The average of the sigma field on the lattice for one sample in the ensemble reads

$$\tilde{\sigma}(t) = \frac{1}{N_L^3} \sum_{\mathbf{x}} \tilde{\sigma}(t, \mathbf{x}). \quad (8)$$

Then, the ensemble average $\langle \tilde{\sigma}(t) \rangle$ is obtained by averaging $\tilde{\sigma}(t)$ over N_{ens} samples in the ensemble. The error arising from a finite sample number N_{ens} is investigated in the Supplemental Material [47], and it is found that the errors are very small when the sample number is increased up to, e.g., $N_{\text{ens}} = 256$. Different schemes for the initial conditions of the sigma field are adopted for the studies of critical mode relaxation from nonequilibrium to equilibrium, e.g., spatially homogeneous initial field configurations, field configurations of the Gaussian distribution, quenching from another equilibrium state, etc. It is found that the relaxation time is independent of the choice of initial conditions; see the Supplemental Material [47] for more detailed discussions. The relaxation time τ can be extracted by fitting the ensemble-averaged sigma field with an exponential decay, as follows:

$$\langle \tilde{\sigma}(t) \rangle = C e^{-t/\tau} + \langle \tilde{\sigma} \rangle_{\text{eq}}, \quad (9)$$

where $\langle \tilde{\sigma} \rangle_{\text{eq}}$ is the value in equilibrium and C is a constant.

Relaxation time of the critical mode in the QCD phase diagram. In the left panel of Fig. 1, we show the reduced relaxation time τ/τ_0 in the QCD phase diagram. The relaxation time is normalized by some reference relaxation time τ_0 , which is chosen to be the maximal value in the part of the phase diagram shown in Fig. 1. Apparently, this is a point near the CEP. The red dot stands for the location of the CEP as shown in Eq. (10). In the phase diagram, we also show the phase boundary obtained in [11], which is depicted through the pseudocritical temperature of the renormalized light quark condensate as a function of the baryon chemical potential; see [11] for more details. Moreover, we also show three typical freeze-out curves, which are also used in [25,26]. The Andronic *et al.* freeze-out is obtained from the freeze-out data in [4]. The STAR Fit I freeze-out and STAR Fit II freeze-out are obtained from the STAR freeze-out data [73], but for the latter some flawed data from general considerations are dropped; see [25] for more details. The 3D plot of the relaxation time in the phase diagram is also shown in the right panel of Fig. 1.

The location of the CEP is found to be

$$(T_{\text{CEP}}, \mu_{B_{\text{CEP}}}) = (107, 635) \text{ MeV} \quad (10)$$

in [11]. Simulations of Langevin transport become difficult as we approach the CEP. Therefore, the computations are constrained in the regime of up to the baryon chemical potential $\mu_B = 630$ MeV in this work. In this regime, the maximal relaxation time τ_0 is found to take place at $(T, \mu_B) \sim (108, 630)$ MeV, which is also used to normalize the relaxation time; thus, we are left with the reduced relaxation time τ/τ_0 . In the left panel of Fig. 1, we also

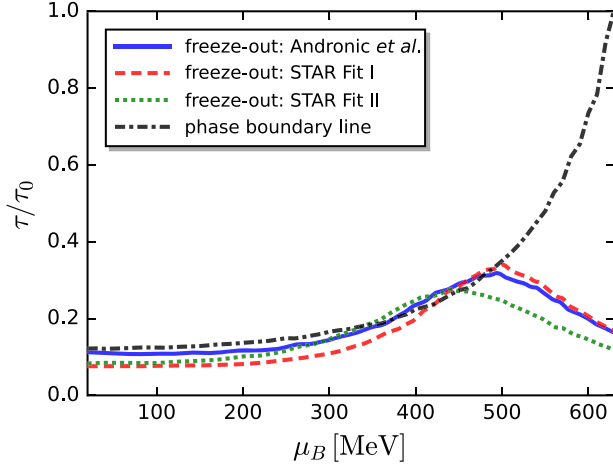


FIG. 2. Relaxation time as a function of the baryon chemical potential along the phase boundary and the three chemical freeze-out curves, respectively, as shown in Fig. 1.

show the line of maximal relaxation time, which is determined by the temperature related to the maximal relaxation time for each value of μ_B . One can see that the line of maximal relaxation time is below the line of the phase boundary in the regime of small μ_B , which is consistent with the nature of continuous crossover in this regime. The phase boundary and the line of maximal relaxation approach each other with increasing μ_B and converge near the CEP.

Another interesting finding is that as the baryon chemical potential is increased up to, e.g., $\mu_B \gtrsim 400$ MeV, the freeze-out curves bend downwards and deviate from the phase boundary line. Consequently, the relaxation time decreases with μ_B rather than increasing along the freeze-out curves in the region of large μ_B , which is shown clearly in Fig. 2. We also show the relaxation time along the phase boundary line in Fig. 2 for comparison, which increases with μ_B monotonically.

The observation that the relaxation time on the chemical freeze-out curves is not increased significantly with the increase of μ_B is also attributed to the fact found from our calculations, that is, a small dynamical critical region. As shown in the left panel of Fig. 1, the red region in the proximity of the CEP is very small. This is even clearer in the 3D plot in the right panel of Fig. 1, where one can see the relaxation time drops immediately once it deviates from the CEP. The small size of the dynamical critical region is consistent with the small size of the static critical region in QCD [34,43]. Since the chemical freeze-out curves lie outside the dynamical critical region, the relaxation time along them is not strongly enhanced in our simulations, suggesting that the effects of critical slowing down may be limited there. The present calculation is, however, performed at fixed T and μ_B , while in heavy-ion collisions the fireball is rapidly expanding, finite-lived, and inhomogeneous. The importance of nonequilibrium effects

therefore also depends on the expansion rate and on the trajectory through the phase diagram. Bearing in mind this caveat, together with the Model A limitation noted earlier, our results indicate that observables determined mainly at the chemical freeze-out, such as the baryon number fluctuations, may be reasonably described by hydrodynamic simulations combined with local equilibrium critical fluctuations [25,26].

Conclusions and outlook. In this letter, we have developed a QCD-assisted relaxation dynamic model for the critical mode of the CEP in the QCD phase diagram at finite temperature and baryon chemical potential. Both the static effective potential and the dynamic transport coefficient in the relaxation dynamic model are computed directly from the flow equations of Euclidean and Minkowski correlation functions in QCD within the fRG approach. This allows us to investigate the critical slowing-down effect quantitatively in the QCD phase diagram, especially in the proximity of the CEP, without any phenomenological parameters.

The relaxation time from nonequilibrium to equilibrium in the QCD phase diagram is extracted from the Langevin simulations of the QCD-assisted relaxation dynamic model. It is found that in a narrow region along the phase boundary radiated from the CEP, the relaxation time is enhanced significantly. Outside this narrow region, the relaxation time drops drastically, which implies that the dynamic critical region is small in the QCD phase diagram. Moreover, we also compute the relaxation time on the chemical freeze-out curves. It is found that with the increase of the baryon chemical potential, the relaxation time is not increasing monotonically but instead develops a peak structure and decreases in the regime of high baryon chemical potential. This is attributed to the fact that the freeze-out curves bend down and deviate from the phase boundary in the regime of large baryon chemical potential. In short, this is due to the following: a small dynamical critical region and the chemical freeze-out curves deviating from the phase boundary in the regime of large baryon chemical potential. The relaxation time is not strongly enhanced along the considered freeze-out curves within the present static-background, Model A-like setup.

Relativistic heavy-ion collisions are the primary terrestrial laboratory tool for studying hot and dense QCD matter. Our work directly connects to the core scientific goal of ongoing and planned major experiments worldwide (e.g., the RHIC-STAR beam energy scan, future FAIR, NICA)—the search for the QCD critical point. Our clear conclusion regarding the mildness of critical slowing down, backed by a first-principles approach, will provide key input for a broad community of experimentalists and theorists working on heavy-ion collisions, influencing how they interpret data and design future analyses.

Note that in this letter we develop the technique of coupling first-principles calculations of nonequilibrium

real-time correlation functions in a microscopic quantum field theory with stochastic differential equation simulations, implemented for the first time in the context of QCD phase transition dynamics. It is not only applicable to the critical point problem studied here but can be directly applied to study the dynamics of chiral symmetry restoration, spinodal decomposition, and more general far-from-equilibrium evolution of QCD matter. Moreover, the developed methodology integrating nonequilibrium field theory with stochastic dynamics is of instructive and transferable value to researchers studying phase transition dynamics in statistical and condensed matter physics. Furthermore, it would be very interesting to extend the QCD-assisted relaxation dynamic model discussed here to include, e.g., the conservation of energy and momentum, nontrivial Poisson bracket, etc., as in the dynamic Model G or Model H. We will report the relevant progress in the future.

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Data availability. The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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- [1] M. A. Stephanov, QCD phase diagram: An overview, *Proc. Sci.* 024 (**2006**) LAT2006, [[arXiv:hep-lat/0701002](#)].
 - [2] M. M. Aggarwal *et al.* (STAR Collaboration), An experimental exploration of the QCD phase diagram: The search for the critical point and the onset of de-confinement, [arXiv:1007.2613](#).
 - [3] X. Luo and N. Xu, Search for the QCD critical point with fluctuations of conserved quantities in relativistic heavy-ion collisions at RHIC: An overview, *Nucl. Sci. Tech.* **28**, 112 (2017).
 - [4] A. Andronic, P. Braun-Munzinger, K. Redlich, and J. Stachel, Decoding the phase structure of QCD via particle production at high energy, *Nature (London)* **561**, 321 (2018).
 - [5] A. Bzdak, S. Esumi, V. Koch, J. Liao, M. Stephanov, and N. Xu, Mapping the phases of quantum chromodynamics with beam energy scan, *Phys. Rep.* **853**, 1 (2020).
 - [6] J. Chen *et al.*, Properties of the QCD matter: Review of selected results from the relativistic heavy ion collider beam energy scan (RHIC BES) program, *Nucl. Sci. Tech.* **35**, 214 (2024).
 - [7] N. Dupuis, L. Canet, A. Eichhorn, W. Metzner, J. M. Pawłowski, M. Tissier, and N. Wschebor, The nonperturbative functional renormalization group and its applications, *Phys. Rep.* **910**, 1 (2021).
 - [8] W.-j. Fu, QCD at finite temperature and density within the fRG approach: An overview, *Commun. Theor. Phys.* **74**, 097304 (2022).
 - [9] M. A. Stephanov, K. Rajagopal, and E. V. Shuryak, Signatures of the tricritical point in QCD, *Phys. Rev. Lett.* **81**, 4816 (1998).
 - [10] M. A. Stephanov, K. Rajagopal, and E. V. Shuryak, Event-by-event fluctuations in heavy ion collisions and the QCD critical point, *Phys. Rev. D* **60**, 114028 (1999).
 - [11] W.-j. Fu, J. M. Pawłowski, and F. Rennecke, QCD phase structure at finite temperature and density, *Phys. Rev. D* **101**, 054032 (2020).
 - [12] F. Gao and J. M. Pawłowski, Chiral phase structure and critical end point in QCD, *Phys. Lett. B* **820**, 136584 (2021).
 - [13] P. J. Gunkel and C. S. Fischer, Locating the critical endpoint of QCD: Mesonic backcoupling effects, *Phys. Rev. D* **104**, 054022 (2021).
 - [14] D. A. Clarke, P. Dimopoulos, F. Di Renzo, J. Goswami, C. Schmidt, S. Singh, and K. Zambello, Searching for the QCD critical end point using multipoint Padé approximations, *Phys. Rev. D* **112**, L091504 (2025).
 - [15] S. Borsanyi, Z. Fodor, J. N. Guenther, P. Parotto, A. Pasztor, C. Ratti, V. Vovchenko, and C. H. Wong, Lattice QCD constraints on the critical point from an improved precision equation of state, *Phys. Rev. D* **112**, L111505 (2025).
 - [16] J. Adam *et al.* (STAR Collaboration), Nonmonotonic energy dependence of net-proton number fluctuations, *Phys. Rev. Lett.* **126**, 092301 (2021).
 - [17] M. S. Abdallah *et al.* (STAR Collaboration), Measurements of proton high order cumulants in $\sqrt{s_{NN}} = 3$ GeV Au + Au collisions and implications for the QCD critical point, *Phys. Rev. Lett.* **128**, 202303 (2022).
 - [18] B. Aboona *et al.* (STAR Collaboration), Beam energy dependence of fifth and sixth-order net-proton number fluctuations in Au + Au collisions at RHIC, *Phys. Rev. Lett.* **130**, 082301 (2023).
 - [19] M. Abdallah *et al.* (STAR Collaboration), Higher-order cumulants and correlation functions of proton multiplicity distributions in $\sqrt{s_{NN}} = 3$ GeV Au + Au collisions at the RHIC STAR experiment, *Phys. Rev. C* **107**, 024908 (2023).
 - [20] B. E. Aboona *et al.* (STAR Collaboration), Precision measurement of net-proton-number fluctuations in Au + Au collisions at RHIC, *Phys. Rev. Lett.* **135**, 142301 (2025).

- [21] M. A. Stephanov, On the sign of kurtosis near the QCD critical point, *Phys. Rev. Lett.* **107**, 052301 (2011).
- [22] W.-j. Fu and J. M. Pawłowski, Relevance of matter and glue dynamics for baryon number fluctuations, *Phys. Rev. D* **92**, 116006 (2015).
- [23] W.-j. Fu and J. M. Pawłowski, Correlating the skewness and kurtosis of baryon number distributions, *Phys. Rev. D* **93**, 091501 (2016).
- [24] W.-j. Fu, J. M. Pawłowski, F. Rennecke, and B.-J. Schaefer, Baryon number fluctuations at finite temperature and density, *Phys. Rev. D* **94**, 116020 (2016).
- [25] W.-j. Fu, X. Luo, J. M. Pawłowski, F. Rennecke, R. Wen, and S. Yin, Hyper-order baryon number fluctuations at finite temperature and density, *Phys. Rev. D* **104**, 094047 (2021).
- [26] W.-j. Fu, X. Luo, J. M. Pawłowski, F. Rennecke, and S. Yin, Ripples of the QCD critical point, *Phys. Rev. D* **111**, L031502 (2025).
- [27] Y. Lu, F. Gao, Y.-x. Liu, and J. M. Pawłowski, Finite density signatures of confining and chiral dynamics in QCD thermodynamics and fluctuations of conserved charges, *Phys. Rev. D* **113**, 054019 (2026).
- [28] B. Berdnikov and K. Rajagopal, Slowing out-of-equilibrium near the QCD critical point, *Phys. Rev. D* **61**, 105017 (2000).
- [29] S. Mukherjee, R. Venugopalan, and Y. Yin, Real time evolution of non-Gaussian cumulants in the QCD critical regime, *Phys. Rev. C* **92**, 034912 (2015).
- [30] P. Hohenberg and B. Halperin, Theory of dynamic critical phenomena, *Rev. Mod. Phys.* **49**, 435 (1977).
- [31] K. Rajagopal and F. Wilczek, Static and dynamic critical phenomena at a second order QCD phase transition, *Nucl. Phys. B* **399**, 395 (1993).
- [32] D. T. Son and M. A. Stephanov, Dynamic universality class of the QCD critical point, *Phys. Rev. D* **70**, 056001 (2004).
- [33] J. Braun, W.-j. Fu, J. M. Pawłowski, F. Rennecke, D. Rosenblüh, and S. Yin, Chiral susceptibility in $(2+1)$ -flavor QCD, *Phys. Rev. D* **102**, 056010 (2020).
- [34] J. Braun *et al.*, Soft modes in hot QCD matter, *Phys. Rev. D* **111**, 094010 (2025).
- [35] W.-j. Fu, J. M. Pawłowski, R. D. Pisarski, F. Rennecke, R. Wen, and S. Yin, QCD moat regime and its real-time properties, *Phys. Rev. D* **111**, 094026 (2025).
- [36] L. Canet and H. Chate, Non-perturbative approach to critical dynamics, *J. Phys.* **40**, 1937 (2007).
- [37] L. Canet, H. Chate, and B. Delamotte, General framework of the non-perturbative renormalization group for non-equilibrium steady states, *J. Phys. A* **44**, 495001 (2011).
- [38] D. Mesterházy, J. H. Stockemer, L. F. Palhares, and J. Berges, Dynamic universality class of Model C from the functional renormalization group, *Phys. Rev. B* **88**, 174301 (2013).
- [39] J. M. Pawłowski, I.-O. Stamatescu, and F. P. G. Ziegler, Cooling stochastic quantization with colored noise, *Phys. Rev. D* **96**, 114505 (2017).
- [40] M. Bluhm, Y. Jiang, M. Nahrgang, J. M. Pawłowski, F. Rennecke, and N. Wink, Time-evolution of fluctuations as signal of the phase transition dynamics in a QCD-assisted transport approach, *Nucl. Phys. A* **982**, 871 (2019).
- [41] Y.-r. Chen, Y.-y. Tan, and W.-j. Fu, Critical dynamics within the real-time FRG approach, *Phys. Rev. D* **109**, 094044 (2024).
- [42] J. V. Roth and L. von Smekal, Critical dynamics in a real-time formulation of the functional renormalization group, *J. High Energy Phys.* **10** (2023) 065.
- [43] Y.-y. Tan, Y.-r. Chen, W.-j. Fu, and W.-J. Li, Universality of pseudo-Goldstone damping near critical points, *Nat. Commun.* **16**, 2916 (2025).
- [44] Y.-r. Chen, Y.-y. Tan, and W.-j. Fu, Critical dynamics of model H within the real-time FRG approach, *Phys. Rev. D* **111**, 094025 (2025).
- [45] J. V. Roth, Y. Ye, S. Schlichting, and L. von Smekal, Dynamic critical behavior of the chiral phase transition from the real-time functional renormalization group, *J. High Energy Phys.* **01** (2025) 118.
- [46] J. V. Roth, Y. Ye, S. Schlichting, and L. von Smekal, Universal critical dynamics near the chiral phase transition and the QCD critical point, *Phys. Rev. D* **111**, L111901 (2025).
- [47] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/hy7v-z3vz> for details of inputs of the Langevin transport model from fRG calculations, simulation setup, threshold functions, and the behavior of the retarded two-point correlation function, which includes Refs. [7,8,11,33–35,43,48–57].
- [48] J. Braun, L. Fister, J. M. Pawłowski, and F. Rennecke, From quarks and gluons to hadrons: Chiral symmetry breaking in dynamical QCD, *Phys. Rev. D* **94**, 034016 (2016).
- [49] M. Mitter, J. M. Pawłowski, and N. Strodthoff, Chiral symmetry breaking in continuum QCD, *Phys. Rev. D* **91**, 054035 (2015).
- [50] F. Rennecke, Vacuum structure of vector mesons in QCD, *Phys. Rev. D* **92**, 076012 (2015).
- [51] A. K. Cyrol, L. Fister, M. Mitter, J. M. Pawłowski, and N. Strodthoff, Landau gauge Yang-Mills correlation functions, *Phys. Rev. D* **94**, 054005 (2016).
- [52] A. K. Cyrol, M. Mitter, J. M. Pawłowski, and N. Strodthoff, Nonperturbative quark, gluon, and meson correlators of unquenched QCD, *Phys. Rev. D* **97**, 054006 (2018).
- [53] L. Corell, A. K. Cyrol, M. Mitter, J. M. Pawłowski, and N. Strodthoff, Correlation functions of three-dimensional Yang-Mills theory from the FRG, *SciPost Phys.* **5**, 066 (2018).
- [54] F. Ihssen, J. M. Pawłowski, F. R. Sattler, and N. Wink, Towards quantitative precision in functional QCD I, *Phys. Rev. D* **113**, 094038 (2026).
- [55] W.-j. Fu, C. Huang, J. M. Pawłowski, Y.-y. Tan, and L.-j. Zhou, Four-quark scatterings in QCD III, *Phys. Rev. D* **112**, 054047 (2025).
- [56] D. F. Litim, Optimization of the exact renormalization group, *Phys. Lett. B* **486**, 92 (2000).
- [57] D. F. Litim, Optimized renormalization group flows, *Phys. Rev. D* **64**, 105007 (2001).
- [58] Y.-y. Tan, Y.-r. Chen, and W.-j. Fu, Real-time dynamics of the $O(4)$ scalar theory within the fRG approach, *SciPost Phys.* **12**, 026 (2022).
- [59] J. Berges and G. Hoffmeister, Nonthermal fixed points and the functional renormalization group, *Nucl. Phys. B* **813**, 383 (2009).

- [60] J. M. Pawłowski and N. Strodthoff, Real time correlation functions and the functional renormalization group, *Phys. Rev. D* **92**, 094009 (2015).
- [61] R. Alkofer and L. von Smekal, The infrared behavior of QCD Green's functions: Confinement dynamical symmetry breaking, and hadrons as relativistic bound states, *Phys. Rep.* **353**, 281 (2001).
- [62] C. S. Fischer, A. Maas, and J. M. Pawłowski, On the infrared behavior of Landau gauge Yang-Mills theory, *Ann. Phys. (N.Y.)* **324**, 2408 (2009).
- [63] H. Gies and C. Wetterich, Renormalization flow of bound states, *Phys. Rev. D* **65**, 065001 (2002).
- [64] H. Gies and C. Wetterich, Universality of spontaneous chiral symmetry breaking in gauge theories, *Phys. Rev. D* **69**, 025001 (2004).
- [65] J. M. Pawłowski, Aspects of the functional renormalisation group, *Ann. Phys. (N.Y.)* **322**, 2831 (2007).
- [66] W.-j. Fu, C. Huang, J. M. Pawłowski, F. Rennecke, R. Wen, and S. Yin, Strangeness neutrality and the QCD phase diagram, [arXiv:2603.13455](https://arxiv.org/abs/2603.13455).
- [67] G. Aarts, Spectral function at high temperature in the classical approximation, *Phys. Lett. B* **518**, 315 (2001).
- [68] L. Batini, E. Grossi, and N. Wink, Dissipation dynamics of a scalar field, *Phys. Rev. D* **108**, 125021 (2023).
- [69] D. Schweitzer, S. Schlichting, and L. von Smekal, Critical dynamics of relativistic diffusion, *Nucl. Phys.* **B984**, 115944 (2022).
- [70] C. Chattopadhyay, J. Ott, T. Schaefer, and V. V. Skokov, Simulations of stochastic fluid dynamics near a critical point in the phase diagram, *Phys. Rev. Lett.* **133**, 032301 (2024).
- [71] J. Bezanson, A. Edelman, S. Karpinski, and V. B. Shah, Julia: A fresh approach to numerical computing, *SIAM Rev.* **59**, 65 (2017).
- [72] T. Besard, C. Foket, and B. De Sutter, Effective extensible programming: Unleashing Julia on GPUs, *IEEE Trans. Parallel Distrib. Syst.*, **30**, 827 (2019).
- [73] L. Adamczyk *et al.* (STAR Collaboration), Bulk properties of the medium produced in relativistic heavy-ion collisions from the beam energy scan program, *Phys. Rev. C* **96**, 044904 (2017).
- [74] fQCD Collaboration, <https://fqcd-collaboration.github.io>.