

Worldsheet CFT_2 and celestial CFT_2 : An AdS_3 - CFT_2 perspective

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Celestial CFT_d with symmetry group $\text{ISO}(d+1,1)$ is the putative dual of quantum gravity in asymptotically flat $(d+2)$ dimensional space time. We argue that a class of d dimensional field theories with symmetry group $\text{ISO}(d+1,1)$, refereed to as Poincaré CFT_d , can be engineered via AdS_{d+1} - CFT_d correspondence. Celestial and Poincaré CFT_d may share common features determined by the kinematics because they have the same symmetry group. So understanding Poincaré CFTs may help us better understand some features of celestial CFTs. Our argument is based on the observation that if we zoom in near the boundary of (Euclidean) AdS_{d+1} then the conformal isometry group of EAdS_{d+1} , which is $\text{SO}(d+2,1)$, contracts to the Poincaré group $\text{ISO}(d+1,1)$. This suggests that the near boundary scaling limit of a theory of conformal gravity on EAdS_{d+1} should be dual to a boundary CFT_d with $\text{ISO}(d+1,1)$ symmetry. This dual CFT_d is an example of a Poincaré CFT_d . Similarly, if we have a nonconformal theory of gravity on EAdS_{d+1} then the near boundary scaling limit of such a theory is dual to a Poincaré CFT_d with only $[\text{SO}(d+1,1)]$ Lorentz invariance. This class of less symmetric Poincaré CFT_d is analogous to celestial CFT_d which computes the leaf amplitudes. Now following this logic we discuss, among other things, the near boundary scaling limit of the bosonic string theory on Euclidean AdS_3 in the presence of the NS-NS B field. The AdS_3 part of the world sheet theory is free in this limit and has been studied in the literature in different contexts. This limit describes a “long string” which wraps the (Euclidean) AdS_3 boundary and it has been argued that the space-time CFT_2 which describes the radial fluctuations of a long string is a Liouville CFT. According to our proposal, the dual CFT_2 which describes the long string sector is an example of a Poincaré CFT_2 with only $[\text{SO}(3,1)]$ Lorentz invariance. We do not get a full $\text{ISO}(3,1)$ invariant Poincaré CFT_2 in this way because the string theory does not have target space conformal invariance.

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I. INTRODUCTION

Celestial holography [1–9] posits that the quantum theory of gravity in asymptotically flat $(d+2)$ dimensional space-time is dual to a conformal field theory living on the d dimensional celestial sphere. The Lorentz group $\text{SO}(d+1,1)$ acts on the celestial sphere as the group of conformal transformations. The bulk space-time also has translation symmetry which is realized as internal symmetry in the celestial CFT_d . So the global symmetry group of a celestial CFT_d is the Poincaré group $\text{ISO}(d+1,1)$. However, developments [10,11] in the last few years have taught

us that sometimes it is more convenient to work with celestial CFTs with *only* Lorentz invariance. In such celestial CFTs correlation functions have the standard CFT form and there is specific algorithm following which one can reconstruct the Poincaré invariant S-matrix elements from the celestial CFT correlation functions. In interesting recent works the authors of [12–14] have constructed examples of such celestial CFTs in two dimensions as Liouville theory coupled to conformal matter fields.

Motivated by these developments we study d dimensional field theories with $\text{ISO}(d+1,1)$ global symmetry using a scaling limit of the AdS -CFT correspondence. The subgroup $\text{SO}(d+1,1)$ of the Poincaré group is realized as the conformal symmetry group of the field theory and the Abelian (translation) generators act as internal symmetries. In this paper we refer to such field theories as Poincaré CFT_d . We do not call them celestial CFT_d because their correlation functions have, in general, nothing to do with the scattering amplitudes in asymptotically flat space-time. However, it should be kept in mind that a celestial CFT_d

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and a Poincaré CFT_d may share common features determined by the kinematics because they have the same symmetry group.

In this paper we argue that Poincaré CFT_d can be studied via a scaling limit of the AdS_{d+1} - CFT_d correspondence where we zoom in near the boundary of the AdS. In the near-boundary limit the conformal isometry group $\text{SO}(d+2, 1)$ of the (Euclidean) AdS_{d+1} contracts to the Poincaré group $\text{ISO}(d+1, 1)$. So the near boundary limit of a conformal field theory or conformal gravity on AdS_{d+1} must have the symmetry $\text{ISO}(d+1, 1)$. In this paper we propose and gather some evidence that such a $(d+1)$ dimensional theory obtained in the near-boundary limit is holographically dual to a Poincaré CFT_d which lives on the boundary of the AdS_{d+1} .

Now the Poincaré group $\text{ISO}(d+1, 1)$ is isomorphic to the conformal Carroll group in $(d+1)$ dimensions [15, 16].¹ Therefore the bulk theory that one obtains in the near-boundary limit is nothing but a Carrollian CFT_{d+1} or a Carrollian theory of gravity [19–25]. In fact the metric in the near boundary limit is degenerate and Carrollian theories usually live on degenerate manifolds. Our proposal says that such a $(d+1)$ dimensional Carrollian theory (of gravity) is holographically dual to a Poincaré CFT_d . Carrollian field theories appear as dual theories in flat space holography [20–31]. In this paper they play an equally important but different role. We have more to say on this in the following sections.

The paper is organized as follows: In Sec. II we discuss the near boundary limit of the conformal isometry group of AdS_{d+1} . In Sec. III we state our proposal relating the near boundary limit of a theory on AdS_{d+1} to the boundary Poincaré CFT_d . Section III onward we discuss the near boundary limit of various theories on AdS and the emergent degenerate manifold. This degenerate manifold provides the natural geometric background for Carrollian CFTs [24, 32–35]. In particular, in Sec. VII we discuss the near boundary limit of the bosonic string theory on Euclidean AdS_3 in the presence of the NS-NS flux and discuss the relation between the (long strings or) world sheet instantons which wrap the boundary S^2 and the dual Poincaré CFT_2 which lives on it. We also discuss the near boundary limit of Einstein gravity in the Appendix.

II. ZOOMING IN NEAR THE BOUNDARY OF EUCLIDEAN ADS

For the sake of concreteness let us consider EAdS_3 in Poincaré coordinates. The line element is given by

$$ds^2 = \frac{d\eta^2 + dz d\bar{z}}{\eta^2} \quad (2.1)$$

¹The Carrollian group is obtained by taking the ultrarelativistic limit (i.e., speed of light $(c \rightarrow 0)$) of Poincaré group [17, 18].

The boundary is at $\eta = 0$. The conformal Killing vector fields (CKVs) of AdS_3 (2.1) can be written in the following form²

$$\begin{aligned} \mathcal{L}_m &= -z^{m+1} \partial_z + \frac{1}{2} m(m+1) \eta^2 \partial_{\bar{z}} - \frac{1}{2} (m+1) z^m \eta \partial_{\eta} \\ \bar{\mathcal{L}}_m &= -\bar{z}^{m+1} \partial_{\bar{z}} + \frac{1}{2} m(m+1) \eta^2 \partial_z - \frac{1}{2} (m+1) \bar{z}^m \eta \partial_{\eta} \\ \mathcal{P}_{r,s} &= -z^{r+\frac{1}{2}} \bar{z}^{s+\frac{1}{2}} \partial_{\eta} + 2\eta \left(s + \frac{1}{2} \right) z^{r+s} \partial_z + 2\eta \left(r + \frac{1}{2} \right) \bar{z}^{r+s} \partial_{\bar{z}} \\ &\quad + \left(r + \frac{1}{2} \right) \left(s + \frac{1}{2} \right) \eta^2 \partial_{\eta} \end{aligned} \quad (2.2)$$

where $m \in \{0, \pm 1\}$ and $(r, s) \in \{\pm \frac{1}{2}\}$.

These vector field obey commutation relations

$$\begin{aligned} [\mathcal{L}_m, \mathcal{L}_n] &= (m-n) \mathcal{L}_{m+n}, & [\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] &= (m-n) \bar{\mathcal{L}}_{m+n} \\ [\mathcal{L}_m, \mathcal{P}_{r,s}] &= \frac{1}{2} (m-2r) \mathcal{P}_{m+r,s}, \\ [\mathcal{L}_m, \mathcal{P}_{r,s}] &= \frac{1}{2} (m-2s) \mathcal{P}_{r,m+s} \\ [\mathcal{P}_{r,s}, \mathcal{P}_{r',s'}] &= 2(\epsilon_{rr'} \bar{\mathcal{L}}_{s+s'} + \epsilon_{ss'} \mathcal{L}_{r+r'}) \end{aligned} \quad (2.3)$$

where $\epsilon_{\frac{-1}{2}\frac{-1}{2}} = -\epsilon_{\frac{1}{2}\frac{1}{2}} = 1$. The commutators (2.3) is the Lie algebra of $\text{SO}(4, 1)$ which is the group of conformal transformations of EAdS_3 .

To zoom in near the boundary we define the new coordinate $\tilde{\eta}$ by

$$\eta = \epsilon \tilde{\eta}, \quad \epsilon \rightarrow 0 \quad (2.4)$$

and take $\epsilon \rightarrow 0$ at fixed $\tilde{\eta}$.

In terms of $\tilde{\eta}$ the CKVs become

$$\begin{aligned} \mathcal{L}_m &= -z^{m+1} \partial_z - \frac{\tilde{\eta}}{2} (m+1) z^m \partial_{\tilde{\eta}} + \frac{\epsilon^2 \tilde{\eta}^2}{2} m(m+1) \partial_{\bar{z}} \\ \bar{\mathcal{L}}_m &= -\bar{z}^{m+1} \partial_{\bar{z}} - \frac{\tilde{\eta}}{2} (m+1) \bar{z}^m \partial_{\tilde{\eta}} + \frac{\epsilon^2 \tilde{\eta}^2}{2} m(m+1) \partial_z \\ \mathcal{P}_{r,s} &= -z^{r+\frac{1}{2}} \bar{z}^{s+\frac{1}{2}} \frac{\partial_{\tilde{\eta}}}{\epsilon} + \epsilon \left[2\tilde{\eta} \left(s + \frac{1}{2} \right) z^{r+s} \partial_z \right. \\ &\quad \left. + 2\tilde{\eta} \left(r + \frac{1}{2} \right) \bar{z}^{r+s} \partial_{\bar{z}} + \left(r + \frac{1}{2} \right) \left(s + \frac{1}{2} \right) \tilde{\eta}^2 \partial_{\tilde{\eta}} \right]. \end{aligned} \quad (2.5)$$

Now we take $\epsilon \rightarrow 0$ keeping $\tilde{\eta}$ fixed. In this limit the vector fields $\mathcal{P}_{r,s}$ diverge and to cure this we introduce the rescaled vector fields $P_{r,s}$ defined as

$$P_{r,s} = \epsilon \mathcal{P}_{r,s}, \quad \epsilon \rightarrow 0 \quad (2.6)$$

²See for example [36].

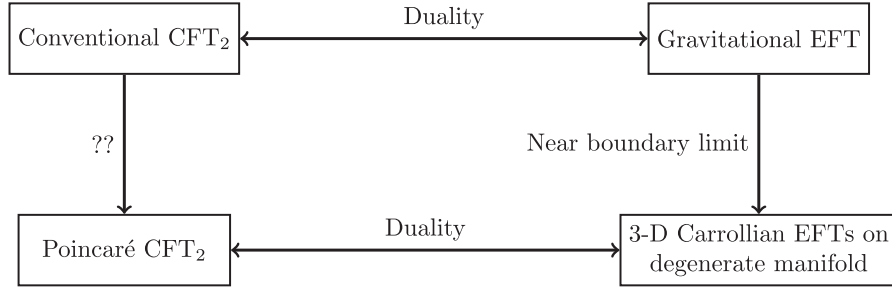


FIG. 1. Diagrammatic representation of our proposal.

So in the limit $\epsilon \rightarrow 0$ of (2.5) we obtain the following vector fields,

$$\begin{aligned}
 L_m &= -z^{m+1} \partial_z - \frac{1}{2}(m+1)z^m \tilde{\eta} \partial_{\tilde{\eta}}, \\
 \bar{L}_m &= -\bar{z}^{m+1} \partial_{\bar{z}} - \frac{1}{2}(m+1)\bar{z}^m \tilde{\eta} \partial_{\tilde{\eta}}, \\
 P_{r,s} &= -z^{r+\frac{1}{2}} \bar{z}^{s+\frac{1}{2}} \partial_{\tilde{\eta}},
 \end{aligned} \tag{2.7}$$

where

$$\mathcal{L}_m \xrightarrow{\epsilon \rightarrow 0} L_m, \quad \bar{\mathcal{L}}_m \xrightarrow{\epsilon \rightarrow 0} \bar{L}_m. \tag{2.8}$$

The vector fields (2.7) satisfy the ISO(3,1) algebra,

$$\begin{aligned}
 [L_m, L_n] &= (m-n)L_{m+n}, \quad [\bar{L}_m, \bar{L}_n] = (m-n)\bar{L}_{m+n} \\
 [L_m, P_{r,s}] &= \frac{1}{2}(m-2r)P_{m+r,s}, \quad [\bar{L}_m, P_{r,s}] = \frac{1}{2}(m-2s)P_{r,m+s} \\
 [P_{r,s}, P_{r',s'}] &= 0.
 \end{aligned} \tag{2.9}$$

So we can see that in the near-boundary limit (2.4) the conformal group of EAdS₃ contracts to the Poincaré group ISO(3,1). The parameter ϵ acts as the Inonu-Wigner contraction parameter.

The Lorentz group $SO(3,1) \subset ISO(3,1)$ acts on the coordinates $(\tilde{\eta}, z, \bar{z})$ as

$$z \rightarrow \frac{az+b}{cz+d}, \quad \bar{z} \rightarrow \frac{\bar{a}\bar{z}+\bar{b}}{\bar{c}\bar{z}+\bar{d}}, \quad \tilde{\eta} \rightarrow \frac{\tilde{\eta}}{|cz+d|^2} \tag{2.10}$$

where we have used the isomorphism $SO(3,1) \rightarrow SL(2, \mathbb{C})/\mathbb{Z}_2$ and

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{C}) \tag{2.11}$$

Similarly the translation $P_{r,s} \in ISO(3,1)$ acts as

$$z \rightarrow z, \quad \bar{z} \rightarrow \bar{z}, \quad \tilde{\eta} \rightarrow \tilde{\eta} + a_{r,s} z^{r+\frac{1}{2}} \bar{z}^{s+\frac{1}{2}} \tag{2.12}$$

where $a_{r,s} = \bar{a}_{s,r}$.

Note that the Lorentz group $SO(3,1)$ is the isometry group of the degenerate metric

$$ds^2 = \frac{dz d\bar{z}}{\tilde{\eta}^2} \tag{2.13}$$

whereas the translations $P_{r,s}$ act as (Weyl) conformal transformations. So if we start with the AdS₃ isometry group then in the near boundary limit we will get only the Lorentz group $SO(3,1)$ acting as the isometries of the degenerate metric (5.10).

Our calculation can be generalized in variety of ways. For example, one could replace Euclidean AdS₃ by Lorentzian AdS₃ and take the same near-boundary limit (2.4). In this limit the conformal group $SO(3,2)$ of Lorentzian AdS₃ contracts to the “Poincaré group” ISO(2,2) of the split signature Minkowski space-time $\mathbb{R}^{2,2}$.

Similarly we can take the near boundary limit (2.4) in any dimension and then the conformal group $SO(d+2,1)$ of EAdS_{d+1} contracts to the Poincaré group $ISO(d+1,1)$.

III. OUR PROPOSAL

A. Conformal gravity on AdS_{d+1}

We have shown that in the near boundary limit (2.4) the conformal group $SO(d+2,1)$ of EAdS_{d+1}³ contracts to the Poincaré group $ISO(d+1,1)$. Now AdS-CFT duality tells us that any theory of quantum gravity on asymptotically EAdS_{d+1} space is holographically dual to a boundary (Euclidean) CFT_d. So if we have a theory of conformal gravity coupled to conformal matter on EAdS_{d+1} then that theory is also dual to a CFT_d with a larger⁴ symmetry group $SO(d+2,1)$ which is the same as the conformal isometry group of EAdS_{d+1}. Now, assuming that the duality survives the near boundary scaling limit, we can say that in this limit the $SO(d+2,1)$ symmetry of the dual CFT_d also reduces to the Poincaré group $ISO(d+1,1)$. Note that in the process the boundary conformal group $SO(d+1,1) \subset ISO(d,1)$ remains untouched.

³The E stands for Euclidean.

⁴The conformal symmetry group of CFT_d is $SO(d+1,1) \subset SO(d+2,1)$.

So in a nutshell, the near boundary limit (2.4) of any conformal gravity theory on EAdS_{d+1} is holographically dual to a boundary CFT_d with symmetry $\text{ISO}(d+1, 1)$. So the dual is a Poincaré CFT_d . The same reasoning is true for conformal gravity theories on Lorentzian AdS_{d+1} except that now in the near boundary limit the Poincaré CFT_d has symmetry group $\text{ISO}(d, 2)$. This identification is based on symmetries alone. See Fig. 1 for a diagrammatic representation of our proposal.

Our proposal gives a way to study a subset of Poincaré CFTs via AdS-CFT duality. In particular, this suggests that Poincaré CFT_2 with symmetry group $\text{ISO}(3, 1)$ arises in a certain scaling limit of the AdS_3 - CFT_2 correspondence.

Now the bulk theory obtained in the near-boundary limit is naturally a Carrollian conformal field theory or a Carrollian theory of gravity [19–25] because the near-boundary metric (2.13) is degenerate. So another statement of our proposed duality is:

A Carrollian conformal field theory or a (conformal) Carrollian gravity theory living on a $(d+1)$ dimensional degenerate metric of the form (2.13) is holographically dual to a Poincaré CFT_d living on the boundary.

Now in the context of flat space holography there is a proposed relation [7, 27–29] between the Carrollian and the celestial approach. This proposed relation is an algebraic way to turn correlators of a celestial CFT into the correlators of a Carrollian CFT and vice-versa. However, in the present case the relation between a Carrollian CFT and a Poincaré CFT is one of holographic duality. For example, in Sec. V we compute the boundary two point function of a Poincaré CFT_2 by extrapolating the bulk two point function of a Carrollian CFT_3 . Now, in the context of flat space holography both the Carrollian and the celestial descriptions do not involve dynamical gravity because both are proposed to be the dual descriptions of bulk gravity theory. In the present case the bulk theory is a Carrollian theory of (conformal) gravity and the dual boundary theory is a Poincaré CFT without any dynamical gravity. So our proposal points to a relation between the Carrollian theories of gravity and Poincaré CFTs which is fundamentally different from what appears in the context of flat space holography.

B. Einstein gravity on AdS_{d+1}

Now suppose that the bulk EAdS_{d+1} theory is matter coupled to Einstein gravity. Then the dual theory is a CFT_d with symmetry $\text{SO}(d+1, 1)$. We can take the near boundary scaling limit (2.4) even in this case. Scaling limit does not change the bulk isometry algebra $\text{SO}(d+1, 1)$ but it acts nontrivially on the correlation functions of bulk fields. Therefore in this limit the bulk theory changes. So again if we assume that the duality survives the scaling limit then we can say that the boundary CFT_d changes to a Poincaré CFT_d but now with only (Lorentz) $\text{SO}(d+1, 1)$

invariance. We do not get the full Poincaré invariance because the EAdS_{d+1} bulk theory we start with does not have the conformal group $\text{SO}(d+2, 1)$ as (asymptotic) symmetry.

Therefore we can say that the near boundary limit (2.4) of Einstein gravity coupled to matter on EAdS_{d+1} is holographically dual to a Poincaré CFT_d with only (Lorentz) $\text{SO}(d+1, 1)$ invariance. Similarly, if we start with theories on Lorentzian AdS_{d+1} then the dual Poincaré CFT_{d+1} has symmetry group $\text{SO}(d, 2)$ which is the Lorentz group of the $(d+2)$ dimensional split signature Minkowski space.

Our proposal is that in the near boundary limit bulk theories of gravity on AdS should be dual to Poincaré CFTs. In the following sections we discuss the near boundary limit of various theories on AdS including the world sheet theory of bosonic strings on AdS_3 in the presence of NS-NS flux. In the Appendix, we also discuss the near boundary limit of Einstein gravity on EAdS_{d+1} .

C. FAQ

Is any (conventional) CFT_d also a Poincaré CFT_d without bulk translation invariance, i.e., without the full $\text{ISO}(d+1, 1)$ symmetry?

The answer is no but, we do not yet have the criteria to decide whether a given CFT is Poincaré or not which depends only on the intrinsic properties of a CFT. Our proposal sheds light on why the answer is no in the cases of holographic CFTs. We say a few words on this very important point now.

Given a CFT_d , which has a dual (semiclassical) gravity description on AdS_{d+1} , we can proceed in the following way. We take the near boundary limit of the dual AdS_{d+1} theory and, according to our proposal, the resulting theory is dual to a Poincaré CFT_d living on the boundary. So the Poincaré CFT_d we end up with cannot be the same as the conventional (holographic) CFT_d we started with. Currently we do not understand this limit in the field theory side. If we understand this then we will be able to take the “Poincaré limit” of any (conventional) CFT without using the AdS-CFT duality. We leave this question for future research.

IV. NEAR-BOUNDARY LIMIT OF A CFT TWO-POINT FUNCTION AND EMERGENT CARROLLIAN STRUCTURE

Let us consider an Euclidean CFT_{d+1} on AdS_{d+1} with metric

$$ds^2 = \frac{d\eta^2 + d\vec{x}^2}{\eta^2}, \quad \vec{x} \in \mathbb{R}^d \quad (4.1)$$

Since (4.1) is conformally flat the two-point function of a scalar conformal primary Φ_Δ of dimension Δ is given by

$$\begin{aligned} & \langle \Phi_{\Delta}(\eta_1, \vec{x}_1) \Phi_{\Delta}(\eta_2, \vec{x}_2) \rangle_{\text{AdS}_{d+1}} \\ &= \eta_1^{\Delta} \eta_2^{\Delta} \langle \Phi_{\Delta}(\eta_1, \vec{x}_1) \Phi_{\Delta}(\eta_2, \vec{x}_2) \rangle_{R^{d+1}} \end{aligned} \quad (4.2)$$

where $\langle \Phi_{\Delta}(\eta_1, \vec{x}_1) \Phi_{\Delta}(\eta_2, \vec{x}_2) \rangle_{R^{d+1}}$ is the two-point function of the scalar primary Φ_{Δ} on R^{d+1} . Therefore,

$$\langle \Phi_{\Delta}(\eta_1, z_1, \bar{z}_1) \Phi_{\Delta}(\eta_2, z_2, g\bar{z}_2) \rangle_{\text{AdS}_{d+1}} = \frac{\eta_1^{\Delta} \eta_2^{\Delta}}{((\eta_{12})^2 + \vec{x}_{12}^2)^{\Delta}} \quad (4.3)$$

where $\vec{x}_{ij} = \vec{x}_i - \vec{x}_j$.

To take the near-boundary limit of (4.3) we change coordinate $\eta \rightarrow \tilde{\eta} = \frac{\eta}{\epsilon}$ and the expression for the two-point function becomes

$$\langle \Phi_{\Delta}(\epsilon \tilde{\eta}_1, \vec{x}_1) \Phi_{\Delta}(\epsilon \tilde{\eta}_2, \vec{x}_2) \rangle_{\text{AdS}_{d+1}} = \frac{\epsilon^{2\Delta} \tilde{\eta}_1^{\Delta} \tilde{\eta}_2^{\Delta}}{(\epsilon^2 \tilde{\eta}_{12}^2 + \vec{x}_{12}^2)^{\Delta}}. \quad (4.4)$$

Now we let $\epsilon \rightarrow 0$ keeping $\tilde{\eta}$ fixed. In this limit the correlation function can be expanded as⁵

$$\begin{aligned} & \langle \Phi_{\Delta}(\epsilon \tilde{\eta}_1, \vec{x}_1) \Phi_{\Delta}(\epsilon \tilde{\eta}_2, \vec{x}_2) \rangle_{\text{AdS}_{d+1}} \\ & \rightarrow \epsilon^d \left(\pi^{\frac{d}{2}} \frac{\Gamma(\Delta - \frac{d}{2})}{\Gamma(\Delta)} \frac{\tilde{\eta}_1^{\Delta} \tilde{\eta}_2^{\Delta}}{|\tilde{\eta}_{12}|^{2\Delta-d}} \delta^d(\vec{x}_1 - \vec{x}_2) \right) \\ & + \epsilon^{2\Delta} \left(\frac{\tilde{\eta}_1^{\Delta} \tilde{\eta}_2^{\Delta}}{|\vec{x}_1 - \vec{x}_2|^{2\Delta}} \right) \end{aligned} \quad (4.5)$$

So we consider two different cases depending on the dimension Δ :

A. Electric sector: $\Delta > \frac{d}{2}$

In this case the first term in (4.5) is dominant as $\epsilon \rightarrow 0$. So if $\Delta > \frac{d}{2}$ then the near boundary limit of the two point function is

$$\begin{aligned} & \langle \Phi_{\Delta}(\epsilon \tilde{\eta}_1, \vec{x}_1) \Phi_{\Delta}(\epsilon \tilde{\eta}_2, \vec{x}_2) \rangle_{\text{AdS}_{d+1}} \\ & \rightarrow \epsilon^d \left(\pi^{\frac{d}{2}} \frac{\Gamma(\Delta - \frac{d}{2})}{\Gamma(\Delta)} \frac{\tilde{\eta}_1^{\Delta} \tilde{\eta}_2^{\Delta}}{|\tilde{\eta}_{12}|^{2\Delta-d}} \delta^d(\vec{x}_1 - \vec{x}_2) \right), \quad \Delta > \frac{d}{2} \end{aligned} \quad (4.6)$$

We can define rescaled fields as

$$\Phi_{\Delta}(\epsilon \tilde{\eta}, \vec{x}) = \epsilon^{\frac{d}{2}} \tilde{\Phi}_{\Delta}(\tilde{\eta}, \vec{x}) \quad (4.7)$$

in terms of which the correlation function becomes

$$\begin{aligned} & \langle \tilde{\Phi}_{\Delta}(\tilde{\eta}_1, \vec{x}_1) \tilde{\Phi}_{\Delta}(\tilde{\eta}_2, \vec{x}_2) \rangle_{\text{Near-Boundary}} \\ &= \pi^{\frac{d}{2}} \frac{\Gamma(\Delta - \frac{d}{2})}{\Gamma(\Delta)} \frac{\tilde{\eta}_1^{\Delta} \tilde{\eta}_2^{\Delta}}{|\tilde{\eta}_{12}|^{2\Delta-d}} \delta^d(\vec{x}_1 - \vec{x}_2), \quad \Delta > \frac{d}{2} \end{aligned} \quad (4.8)$$

⁵This type of formula appears for example in [6].

This has precisely the structure of the two point function of Carrollian [17,18] conformal primaries [7,27,29–31] in the electric sector a Carrollian CFT_{d+1} living on the degenerate metric

$$ds^2 = \frac{d\vec{x}^2}{\tilde{\eta}^2}, \quad \vec{x} \in R^d \quad (4.9)$$

Note that as mentioned in the Introduction, the conformal group of this metric which is ISO($d+1, 1$) is isomorphic to the conformal Carroll group [15] in ($d+1$) dimensions. So this result is not unexpected. We can get the more familiar form of the electric sector two point function if we do a conformal rescaling and define

$$\Phi_{\Delta}(\tilde{\eta}, \vec{x}) = \epsilon^{\frac{d}{2}} \tilde{\Phi}_{\Delta}(\tilde{\eta}, \vec{x}) = \epsilon^{\frac{d}{2}} \tilde{\eta}^{\Delta} \tilde{\Phi}'_{\Delta}(\tilde{\eta}, \vec{x}) \quad (4.10)$$

In terms of the primed fields the two point function becomes

$$\begin{aligned} & \langle \tilde{\Phi}'_{\Delta}(\tilde{\eta}_1, \vec{x}_1) \tilde{\Phi}'_{\Delta}(\tilde{\eta}_2, \vec{x}_2) \rangle \\ &= \pi^{\frac{d}{2}} \frac{\Gamma(\Delta - \frac{d}{2})}{\Gamma(\Delta)} \frac{\delta^d(\vec{x}_1 - \vec{x}_2)}{|\tilde{\eta}_{12}|^{2\Delta-d}}, \quad \Delta > \frac{d}{2} \end{aligned} \quad (4.11)$$

The Carrollian CFT_{d+1} in which the two point function (4.11) is now defined lives on the “flat” manifold

$$ds^2 = 0 \cdot d\tilde{\eta}^2 + d\vec{x}^2 \quad (4.12)$$

We now discuss the possible physical interpretation of the bound on the scaling dimension Δ .

1. $\Delta > \frac{d}{2}$ bound in the electric sector

As $\Delta \rightarrow \frac{d}{2} +$ the finite part of the correlation function (4.8) can be written as

$$\begin{aligned} & \langle \tilde{\Phi}_{\Delta}(\tilde{\eta}_1, \vec{x}_1) \tilde{\Phi}_{\Delta}(\tilde{\eta}_2, \vec{x}_2) \rangle \\ &= -\frac{\pi^{\frac{d}{2}}}{\Gamma(\frac{d}{2})} \tilde{\eta}_1^{\frac{d}{2}} \tilde{\eta}_2^{\frac{d}{2}} \ln |\tilde{\eta}_1 - \tilde{\eta}_2|^2 \delta^d(\vec{x}_1 - \vec{x}_2) \end{aligned} \quad (4.13)$$

So in the electric sector of a Carrollian CFT_{d+1}, the scalar operator with dimension $\Delta = \frac{d}{2}$ is not a conformal primary. This is more obvious in terms of the rescaled correlation function which becomes

$$\begin{aligned} & \langle \tilde{\Phi}'_{\Delta}(\tilde{\eta}_1, \vec{x}_1) \tilde{\Phi}'_{\Delta}(\tilde{\eta}_2, \vec{x}_2) \rangle \\ &= -\frac{\pi^{\frac{d}{2}}}{\Gamma(\frac{d}{2})} \ln |\tilde{\eta}_1 - \tilde{\eta}_2|^2 \delta^d(\vec{x}_1 - \vec{x}_2) \end{aligned} \quad (4.14)$$

The situation is similar to that in two dimensions where a free scalar field with dimension zero is not a conformal field because it has logarithmic two point function.

However, the derivative of the scalar field is a conformal primary operator. This was also observed in [30].

In the present case the $\tilde{\eta}$ dependent part of the two point function (4.11) transforms like the correlation function of conformal primary operators of effective dimension $\Delta_{\text{eff}} = \Delta - \frac{d}{2}$. Δ_{eff} vanishes at $\Delta = \frac{d}{2}$ and this leads to the logarithmic two point function. It is also easy to see that the derivatives $\partial_{\tilde{\eta}}^p \tilde{\Phi}'_{d/2}$ transform like Carrollian conformal fields of dimension $\Delta = \frac{d}{2} + p$ and has power-law correlation functions. All of these suggest that the $\Delta > \frac{d}{2}$ bound should be thought of as the unitarity bound. To be more precise, we suggest that,

In a unitary Carrollian CFT_{d+1} the dimension Δ of a scalar primary in the electric sector is bounded from below by $\frac{d}{2}$, i.e., $\Delta > \frac{d}{2}$.

In a relativistic CFT_{d+1} the corresponding bound $\Delta \geq \frac{d-1}{2}$ is less stringent.

We emphasize that our “derivation” of the bound is purely algebraic. It will be very interesting to prove the proposed bound from first principle and in the absence of such a proof we cannot be sure that this bound is indeed a unitarity bound.

2. Transformation of $\tilde{\Phi}'_{\Delta}$ under the Poincaré group ISO(3,1)

In this section we show that $\tilde{\Phi}'_{\Delta}$ transforms like a Carrollian conformal primary under the near boundary (conformal) symmetry group ISO(3,1) of EAdS₃.

In order to find the transformation law of $\tilde{\Phi}'_{\Delta}(\tilde{\eta}, z, \bar{z})$ we start with the transformation law

$$\delta\Phi_{\Delta}(\eta, z, \bar{z}) = -\xi^{\mu}\partial_{\mu}\Phi_{\Delta}(\eta, z, \bar{z}) - \frac{\Delta}{3}\nabla_{\mu}\xi^{\mu}\Phi_{\Delta}(\eta, z, \bar{z}) \quad (4.15)$$

of the (relativistic) scalar conformal primary $\Phi_{\Delta}(\eta, z, \bar{z})$ under the Euclidean conformal group SO(4,1). The vector field ξ is a linear combination of the CKVs (2.2),

$$\xi = a_m\mathcal{L}_m + \bar{a}_m\bar{\mathcal{L}}_m + p_{r,s}\mathcal{P}_{r,s} \quad (4.16)$$

If the conformal symmetry is generated by the charge Q_{ξ} , then the commutator of the charge with the scalar primary $\Phi_{\Delta}(\eta, z, \bar{z})$ should reproduce the transformation law (4.15) of the field, i.e.,

$$[Q_{\xi}, \Phi_{\Delta}(\eta, z, \bar{z})] = -\xi^{\mu}\partial_{\mu}\Phi_{\Delta}(\eta, z, \bar{z}) - \frac{\Delta}{3}\nabla_{\mu}\xi^{\mu}\Phi_{\Delta}(\eta, z, \bar{z}). \quad (4.17)$$

This leads to the following commutation relation between the generators of SO(4,1) and $\Phi_{\Delta}(\eta, z, \bar{z})$,

$$\begin{aligned} [\mathcal{L}_m, \Phi_{\Delta}] &= z^{m+1}\partial_z\Phi_{\Delta} + \frac{1}{2}(m+1)z^m\eta\partial_{\eta}\Phi_{\Delta} - \frac{1}{2}m(m+1)\eta^2\partial_{\bar{z}}\Phi_{\Delta} \\ [\bar{\mathcal{L}}_m, \Phi_{\Delta}] &= \bar{z}^{m+1}\partial_{\bar{z}}\Phi_{\Delta} + \frac{1}{2}(m+1)\bar{z}^m\eta\partial_{\eta}\Phi_{\Delta} - \frac{1}{2}m(m+1)\eta^2\partial_z\Phi_{\Delta} \\ [\mathcal{P}_{r,s}, \Phi_{\Delta}] &= z^{r+\frac{1}{2}}\bar{z}^{s+\frac{1}{2}}\left(\partial_{\eta}\Phi_{\Delta} - \frac{\Delta\Phi_{\Delta}}{\eta}\right) - \left(2\eta\left(s+\frac{1}{2}\right)z^{r+s}\partial_z\Phi_{\Delta} + 2\eta\left(r+\frac{1}{2}\right)\bar{z}^{r+s}\partial_{\bar{z}}\Phi_{\Delta}\right. \\ &\quad \left.+ \left(\left(r+\frac{1}{2}\right)\left(s+\frac{1}{2}\right)\right)(\eta^2\partial_{\eta}\Phi_{\Delta} + \Delta\eta\Phi_{\Delta})\right). \end{aligned} \quad (4.18)$$

Now obtaining the (conformal Carrollian or) near-boundary limit of the transformation laws (4.18) is straightforward. We substitute (2.4) and (4.10) in (4.18) and take $\epsilon \rightarrow 0$. In this limit we get

$$\begin{aligned} [L_m, \tilde{\Phi}'_{\Delta}(\tilde{\eta}, z, \bar{z})] &= z^{m+1}\partial_z\tilde{\Phi}'_{\Delta}(\tilde{\eta}, z, \bar{z}) + (m+1)z^m\left(\frac{\tilde{\eta}}{2}\partial_{\tilde{\eta}} + \frac{\Delta}{2}\right)\tilde{\Phi}'_{\Delta}(\tilde{\eta}, z, \bar{z}) \\ [\bar{L}_m, \tilde{\Phi}'_{\Delta}(\tilde{\eta}, z, \bar{z})] &= \bar{z}^{m+1}\partial_{\bar{z}}\tilde{\Phi}'_{\Delta}(\tilde{\eta}, z, \bar{z}) + (m+1)\bar{z}^m\left(\frac{\tilde{\eta}}{2}\partial_{\tilde{\eta}} + \frac{\Delta}{2}\right)\tilde{\Phi}'_{\Delta}(\tilde{\eta}, z, \bar{z}) \\ [P_{r,s}, \tilde{\Phi}'_{\Delta}(\tilde{\eta}, z, \bar{z})] &= z^{r+\frac{1}{2}}\bar{z}^{s+\frac{1}{2}}\partial_{\tilde{\eta}}\tilde{\Phi}'_{\Delta}(\tilde{\eta}, z, \bar{z}) \end{aligned} \quad (4.19)$$

where we have taken into account the scaling (2.6) and also the relations (2.8). Equation (4.19) is precisely the transformation law of a scalar Carrollian primary of dimension Δ [19,37] in a Carrollian CFT₃ which lives on the manifold with degenerate metric (4.12). Therefore near-boundary limit indeed contracts representations.

B. Magnetic sector: $\frac{d-1}{2} \leq \Delta \leq \frac{d}{2}$

When the conformal dimension Δ of the (relativistic) conformal field is $< \frac{d}{2}$, the second term in the expansion (4.5) dominates as we take $\epsilon \rightarrow 0$. So in the near boundary limit we get

$$\langle \Phi_{\Delta}(\epsilon \tilde{\eta}_1, \vec{x}_1) \Phi_{\Delta}(\epsilon \tilde{\eta}_2, \vec{x}_2) \rangle_{\text{Near Boundary}} \rightarrow \epsilon^{2\Delta} \left(\frac{\tilde{\eta}_1^{\Delta} \tilde{\eta}_2^{\Delta}}{|\vec{x}_1 - \vec{x}_2|^{2\Delta}} \right), \quad \frac{d-1}{2} \leq \Delta \leq \frac{d}{2} \quad (4.20)$$

The lower bound on Δ is nothing but the unitarity bound of the parent relativistic CFT _{$d+1$} . Now we can define the rescaled field

$$\Phi_{\Delta}(\epsilon \tilde{\eta}, \vec{x}) = \epsilon^{\Delta} \tilde{\Phi}_{\Delta}(\tilde{\eta}, \vec{x}) \quad (4.21)$$

in terms of which the two point function becomes

$$\langle \tilde{\Phi}_{\Delta}(\tilde{\eta}_1, \vec{x}_1) \tilde{\Phi}_{\Delta}(\tilde{\eta}_2, \vec{x}_2) \rangle_{\text{Near Boundary}} = \frac{\tilde{\eta}_1^{\Delta} \tilde{\eta}_2^{\Delta}}{|\vec{x}_1 - \vec{x}_2|^{2\Delta}}, \quad \frac{d-1}{2} \leq \Delta \leq \frac{d}{2} \quad (4.22)$$

This is the two point function of Carrollian conformal primaries in the magnetic sector. Equation (4.22) suggests the following bound

In a unitary Carrollian CFT _{$d+1$} the dimension Δ of a scalar primary in the magnetic sector is both bounded from below and above by $\frac{d-1}{2} \leq \Delta \leq \frac{d}{2}$.

The lower limit is simply the unitarity bound of the parent relativistic CFT. The upper bound is more mysterious. It will be very interesting to see how this can be derived from the first principle.

To summarize, in the near boundary limit a scalar conformal primary of dimension Δ in a relativistic CFT _{$d+1$} becomes electric Carrollian primary if $\Delta > \frac{d}{2}$ and becomes magnetic Carrollian primary if $\frac{d-1}{2} \leq \Delta \leq \frac{d}{2}$.

V. NEAR BOUNDARY LIMIT OF (NONCONFORMAL) FIELD THEORY EAdS₃

Let us consider a scalar field of mass m on Euclidean AdS₃. We take the AdS₃ metric to be

$$ds^2 = d\phi^2 + e^{2\phi} d\gamma d\bar{\gamma} \quad (5.1)$$

where $-\infty < \phi < \infty$ and γ and $\bar{\gamma}$ are complex conjugates. In these coordinates the AdS boundary is at $\phi = \infty$.

The relation between the coordinates $(\phi, \gamma, \bar{\gamma})$ and the ones used in (2.1) is

$$\phi = -\ln \eta, \quad \gamma = z, \quad \bar{\gamma} = \bar{z} \quad (5.2)$$

In terms of ϕ the near boundary limit (2.4) becomes

$$\phi = -\ln \epsilon + \tilde{\phi}, \quad \epsilon \rightarrow 0 \quad (5.3)$$

with $\tilde{\phi}$ held fixed.

The action of the scalar field is given by

$$S = \frac{1}{2} \int d\phi d\gamma d\bar{\gamma} e^{2\phi} ((\partial_{\phi} \Psi)^2 + e^{-2\phi} \partial_i \Psi \partial_i \Psi + m^2 \Psi^2) \quad (5.4)$$

We can see that in the near boundary limit $\phi \rightarrow \infty$, we can neglect the spatial derivative term and the action becomes

$$S_{\text{NB}} = \frac{1}{2} \int d\phi d\gamma d\bar{\gamma} e^{2\phi} ((\partial_{\phi} \Psi)^2 + m^2 \Psi^2) \quad (5.5)$$

The procedure can be formalized as follows. First we make the change of coordinate (5.3) and do the field rescaling (4.7) $\Psi \rightarrow \epsilon \tilde{\Psi}$. This gives us

$$S = \frac{1}{2} \int d\tilde{\phi} d\gamma d\bar{\gamma} e^{2\tilde{\phi}} ((\partial_{\tilde{\phi}} \tilde{\Psi})^2 + \epsilon^2 e^{-2\tilde{\phi}} \partial_i \tilde{\Psi} \partial_i \tilde{\Psi} + m^2 \tilde{\Psi}^2) \quad (5.6)$$

Now we take $\epsilon \rightarrow 0$ keeping all other quantities fixed and we get

$$S_{\text{NB}} = \frac{1}{2} \int d\tilde{\phi} d\gamma d\bar{\gamma} e^{2\tilde{\phi}} ((\partial_{\tilde{\phi}} \tilde{\Psi})^2 + m^2 \tilde{\Psi}^2) \quad (5.7)$$

Note that here the range of $\tilde{\phi}$ is again $(-\infty, \infty)$. So as expected (5.7) and (5.5) have identical structure except the replacement $(\phi, \Psi) \rightarrow (\tilde{\phi}, \tilde{\Psi})$.

So for notational simplicity we will work with the action (5.5) with the understanding that the range of ϕ is the whole real line $(-\infty, \infty)$.

A. (Infinite) symmetries of the near-boundary action

The near boundary action

$$S_{\text{NB}} = \frac{1}{2} \int d\phi d\gamma d\bar{\gamma} e^{2\phi} ((\partial_{\phi} \Psi)^2 + m^2 \Psi^2) \quad (5.8)$$

is invariant under SO(3,1) which acts on the field and coordinates as follows

$$\begin{aligned} \Psi'(\phi', \gamma', \bar{\gamma}') &= \Psi(\phi, \gamma, \bar{\gamma}) \\ \phi' &= \phi + \ln |c\gamma + d|^2, \quad \gamma' = \frac{a\gamma + b}{c\gamma + d}, \\ \bar{\gamma}' &= \frac{\bar{a}\bar{\gamma} + \bar{b}}{\bar{c}\bar{\gamma} + \bar{d}}, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{C}) \end{aligned} \quad (5.9)$$

The coordinate transformation is the isometry of the degenerate metric

$$ds^2 = e^{2\phi} d\gamma d\bar{\gamma} \quad (5.10)$$

on which the scalar field lives. However, the metric (5.10) and the action (5.8) has the infinite dimensional local symmetry of arbitrary holomorphic reparametrization given by

$$\begin{aligned} \Psi'(\phi', \gamma', \bar{\gamma}') &= \Psi(\phi, \gamma, \bar{\gamma}) \\ \phi' &= \phi - \frac{1}{2} \ln \left| \frac{d\gamma'}{d\gamma} \right|^2, \quad \gamma' = \gamma'(\gamma), \quad \bar{\gamma}' = \bar{\gamma}'(\bar{\gamma}) \end{aligned} \quad (5.11)$$

Therefore in the near boundary limit in AdS₃ the symmetry of the bulk action is enhanced to the infinite dimensional Virasoro algebra.

B. Boundary correlation function

We want to compute the boundary two point correlation function when the bulk action is given by

$$S_{\text{NB}} = \frac{1}{2} \int d\phi d\gamma d\bar{\gamma} e^{2\phi} ((\partial_\phi \Psi)^2 + m^2 \Psi^2) \quad (5.12)$$

The boundary is at $\phi = \infty$. To compute the boundary correlation function we first compute the bulk Green's function defined as

$$\begin{aligned} & \left(-e^{-2\phi} \frac{\partial}{\partial \phi} \left(e^{2\phi} \frac{\partial}{\partial \phi} \right) + m^2 \right) G(\phi, \gamma, \bar{\gamma}; \phi', \gamma', \bar{\gamma}') \\ &= \frac{\delta(\phi - \phi') \delta^2(\gamma - \gamma')}{e^{2\phi}} \end{aligned} \quad (5.13)$$

The Green's function G can be easily computed to be

$$\begin{aligned} G(\phi, \gamma, \bar{\gamma}; \phi', \gamma', \bar{\gamma}') &= \frac{e^{-\Delta_+ \phi} e^{-\Delta_- \phi'}}{2\sqrt{1+m^2}} \theta(\phi - \phi') \delta^2(\gamma - \gamma') \\ &+ \frac{e^{-\Delta_- \phi} e^{-\Delta_+ \phi'}}{2\sqrt{1+m^2}} \theta(\phi' - \phi) \delta^2(\gamma - \gamma') \end{aligned} \quad (5.14)$$

where

$$\Delta_{\pm} = 1 \pm \sqrt{1+m^2} \quad (5.15)$$

The form of the Green's function becomes more familiar if we write it in terms of the coordinate $\eta = e^{-\phi}$. The boundary is now located at $\eta = 0$. In term of η we get

$$\begin{aligned} G(\eta, \gamma, \bar{\gamma}; \eta', \gamma', \bar{\gamma}') &= \frac{\eta^{\Delta_+} \eta'^{\Delta_-}}{2\sqrt{1+m^2}} \theta(\eta' - \eta) \delta^2(\gamma - \gamma') \\ &+ \frac{\eta^{\Delta_-} \eta'^{\Delta_+}}{2\sqrt{1+m^2}} \theta(\eta - \eta') \delta^2(\gamma - \gamma') \end{aligned} \quad (5.16)$$

Now we can define the boundary two point function using the extrapolate dictionary. So we take (η, η') to zero and the boundary correlator we get is

$$\begin{aligned} \langle O_{\Delta_+}(\gamma, \bar{\gamma}) O_{\Delta_-}(\gamma', \bar{\gamma}') \rangle &= \delta^2(\gamma - \gamma'), \quad \Delta_+ + \Delta_- = 2 \\ \langle O_{\Delta_+}(\gamma, \bar{\gamma}) O_{\Delta_+}(\gamma', \bar{\gamma}') \rangle &= \langle O_{\Delta_-}(\gamma, \bar{\gamma}) O_{\Delta_-}(\gamma', \bar{\gamma}') \rangle = 0 \end{aligned} \quad (5.17)$$

So the boundary two point function is a pure contact term because the theory is ultralocal in the boundary direction. It is easy to check that (5.17) is conformally invariant. The dimensions of the boundary operators is fixed by the exponents of the (η, η') . So we conclude that

In the near boundary limit in AdS_{d+1} a scalar field of mass m corresponds to two boundary operators of dimensions

$$\Delta_{\pm} = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + m^2} \quad (5.18)$$

We would like to point out that the two point function of a celestial CFT₂ is also a contact term and is nonzero precisely when $\Delta_+ + \Delta_- = 2$.⁶ This is not surprising because this feature is determined by symmetry. However, in celestial CFT₂ the primary operators transform in the unitary principal series representation and the operator dimension is continuous. But for a Poincaré CFT₂ considered here the operator dimension is discrete and is given by (5.18). This difference is due to different dynamics of the two theories.

⁶To be more precise, the celestial primary operators transform in the unitary principal series representation of the (Lorentz) conformal group. For a celestial CFT₂ the operator dimension $\Delta = 1 + i\lambda, \lambda \in \mathbb{R}$. Celestial two point function of two massless primaries is given by the Mellin transformation

$$\begin{aligned} & \langle O_{\Delta_+}(z, \bar{z}) O_{\Delta_-}(z', \bar{z}') \rangle \\ &= \int_0^\infty \int_0^\infty d\omega d\omega' \omega^{\Delta_+ - 1} \omega'^{\Delta_- - 1} \omega(1 + z\bar{z}) \delta^3(\vec{p} - \vec{p}') \\ &\propto \delta(\lambda_+ + \lambda_-) \delta^2(z - z') \\ &= \delta(\Delta_+ + \Delta_- - 2) \delta^2(z - z') \end{aligned} \quad (5.19)$$

where $p^\mu = \omega(1 + z\bar{z}, z + \bar{z}, -i(z - \bar{z}), 1 - z\bar{z})$.

VI. NEAR BOUNDARY LIMIT OF THE WORLDLINE ACTION

In this section we study the worldline action of a particle of mass m^2 moving on the Euclidean AdS₃. Our goal is to take the near boundary limit in the worldline action.

With usual notation the worldline action can be written as

$$S = \frac{1}{2} \int d\lambda \left(\frac{1}{e(\lambda)} g_{ab} \frac{dx^a}{d\lambda} \frac{dx^b}{d\lambda} - m^2 e(\lambda) \right) \quad (6.1)$$

where g_{ab} is the AdS₃ metric (5.1).

We now choose the standard gauge

$$e(\lambda) = 1 \quad (6.2)$$

In this gauge the action becomes

$$\begin{aligned} S &= \frac{1}{2} \int d\lambda \left(g_{ab} \frac{dx^a}{d\lambda} \frac{dx^b}{d\lambda} - m^2 \right) \\ &= \frac{1}{2} \int d\lambda \left(\left(\frac{d\phi}{d\lambda} \right)^2 + 2e^{2\phi} \frac{d\gamma}{d\lambda} \frac{d\bar{\gamma}}{d\lambda} - m^2 \right) \end{aligned} \quad (6.3)$$

To take the near boundary limit we have to take $\phi \rightarrow \infty$. In order to take this limit we introduce two Lagrange multiplier fields $[\beta(\lambda), \bar{\beta}(\lambda)]$ and write the action as,

$$S = \int d\lambda \left(\frac{1}{2} \left(\frac{d\phi}{d\lambda} \right)^2 + \beta \frac{d\gamma}{d\lambda} + \bar{\beta} \frac{d\bar{\gamma}}{d\lambda} - \frac{m^2}{2} - \beta \bar{\beta} e^{-2\phi} \right) \quad (6.4)$$

In the $\phi \rightarrow \infty$ limit we can neglect the last term in (6.4) and get

$$S_{\text{NB}} = \int d\lambda \left(\frac{1}{2} \left(\frac{d\phi}{d\lambda} \right)^2 + \beta \frac{d\gamma}{d\lambda} + \bar{\beta} \frac{d\bar{\gamma}}{d\lambda} - \frac{m^2}{2} \right) \quad (6.5)$$

Again we take $-\infty < \phi < \infty$ in (6.5).

The Hamiltonian derived from the near-boundary action (6.5) is given by

$$H = \frac{1}{2} (p_\phi^2 + m^2) \quad (6.6)$$

where p_ϕ is the canonical momentum conjugate to ϕ . Now if we quantize the worldline theory then the physical states satisfy the constraint [or the missing equation of motion for $e(\lambda)$]

$$H|\text{phys}\rangle = 0 \quad (6.7)$$

So the wave functions $\tilde{\Psi}(\phi, \gamma, \bar{\gamma})$ satisfy the equation

$$\frac{1}{2} \left(-\frac{\partial^2}{\partial \phi^2} + m^2 \right) \tilde{\Psi}(\phi, \gamma, \bar{\gamma}) = 0 \quad (6.8)$$

where we have substituted, $p_\phi = -i \frac{\partial}{\partial \phi}$. The Schrodinger equation (6.8) can be derived from the space-time action

$$\tilde{S} = \frac{1}{2} \int d\phi d\gamma d\bar{\gamma} ((\partial_\phi \tilde{\Psi})^2 + m^2 \tilde{\Psi}^2) \quad (6.9)$$

To bring it to the more familiar form we define

$$\Psi = e^{-\phi} \tilde{\Psi} \quad (6.10)$$

in terms of which the action becomes

$$S = \frac{1}{2} \int d\phi d\gamma d\bar{\gamma} e^{2\phi} ((\partial_\phi \Psi)^2 + m^2 \Psi^2) \quad (6.11)$$

The action (6.11) matches precisely with the action (5.5) which is the near boundary limit of the action of a massive scalar field theory on Euclidean AdS₃.

So if we take the near boundary limit of the worldline action, written in the first order form, then the space-time theory we get (after first quantization) is an electric Carroll theory of a massive scalar field.

We will use this result to understand the near-boundary limit of the string sigma model of Euclidean AdS₃. Before we end this section let us discuss the physical picture.

The equations of motion of the fields $(\beta, \bar{\beta})$ following from the action (6.5) are

$$\gamma = \text{const}, \quad \bar{\gamma} = \text{const} \quad (6.12)$$

So the particle described by (6.5) is pinned at a point of the boundary and moves only in the radial direction in the bulk. This is why the near boundary limit of the action of a massive scalar field (5.5) does not have any spatial derivatives and is described by an ultralocal (electric) Carrollian theory.

This worldline picture will be useful when we study the near boundary limit of string theory on EAdS₃ in the presence of NS-NS two form flux.

VII. NEAR BOUNDARY LIMIT OF STRING THEORY ON AdS₃ AND THE DUAL POINCARÉ CFT₂

The conventional string theories do not have target space conformal invariance and in the low energy limit give Einstein gravity rather than conformal gravity. Therefore in the near boundary limit string theories on AdS should be dual to Poincaré CFTs with only conformal invariance but not Poincaré invariance.⁷

In this section we consider bosonic string theory on Euclidean AdS₃ $\times \mathcal{N}$ in the presence of Neveu-Schwarz

⁷To study Poincaré CFT₂ with ISO(3,1) invariance one needs to study the near boundary limit of twistor string theory on AdS₃. This is beyond the scope of this paper.

$B_{\mu\nu}$ field and $\mathcal{N}$ is some compact space. The AdS_3 metric is

$$ds^2 = l^2(d\phi^2 + e^{2\phi}d\gamma d\bar{\gamma}) \quad (7.1)$$

where l is the AdS_3 radius.

Now the bosonic part of the world sheet Lagrangian on AdS_3 is given by the Wess-Zumino-Witten model for the coset $SL(2, \mathbb{C})/SU(2)$ and can be written as [38–40]

$$S = \frac{2l^2}{l_s^2} \int d^2w (\partial\phi\bar{\partial}\phi + e^{2\phi}\bar{\partial}\gamma\partial\bar{\gamma}) + S_{\text{int}} \quad (7.2)$$

where S_{int} is the action describing string propagation on compact internal space.

Now to take the near boundary limit we write the action in the first order form as [38–40]

$$S = \int d^2w \left(\partial\phi\bar{\partial}\phi - \frac{2}{\alpha_+} \sqrt{g}R\phi + \beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} - \beta\bar{\beta}e^{-\frac{2}{\alpha_+}\phi} \right) + S_{\text{int}} \quad (7.3)$$

where (β, γ) are bosonic holomorphic fields with weights $(1,0)$ and $(0,0)$. $(\bar{\beta}, \bar{\gamma})$ are the corresponding antiholomorphic fields. The parameter α_+ is related to the string length l_s and the AdS radius l by

$$\alpha_+^2 = 2k - 4 = 2\frac{l^2}{l_s^2} - 4 \quad (7.4)$$

R is the world sheet curvature.

In the near boundary limit $\phi \rightarrow \infty$ the last term in the action (7.3) can be neglected and we get the free world sheet action,

$$S_{\text{NB}} = \int d^2w \left(\partial\phi\bar{\partial}\phi - \frac{2}{\alpha_+} \sqrt{g}R\phi + \beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} \right) + S_{\text{int}} \quad (7.5)$$

That this is the correct near boundary limit follows from our analysis of the near boundary limit of the worldline action in Sec. VI.

A. Properties of the near boundary world sheet action

The free world sheet action

$$S_{\text{NB}} = \int d^2w \left(\partial\phi\bar{\partial}\phi - \frac{2}{\alpha_+} \sqrt{g}R\phi + \beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} \right) + S_{\text{int}} \quad (7.6)$$

has been studied in the context of the AdS_3 - CFT_2 correspondence and below we summarize some of the key points:

- (1) The equation of motion of $\beta(\bar{\beta})$

$$\bar{\partial}\gamma = 0, \quad \partial\bar{\gamma} = 0 \quad (7.7)$$

require γ ($\bar{\gamma}$) to be a holomorphic (antiholomorphic) map from the world sheet to the boundary sphere⁸ (S^2) in the target space. These world sheet instantons or wrapped strings are known as the “long-strings” [38–41].

- (2) The dual or space-time CFT_2 describing the long-strings has central charge [38] $c = 6kp$ where p is the number of times the world sheet wraps the boundary sphere. Seiberg and Witten [41] argued that the space-time CFT_2 which describes the long strings has a Liouville sector with background charge (for $p = 1$)

$$Q = (k-3)\sqrt{\frac{2}{k-2}} \quad (7.8)$$

and central charge⁹

$$c_L = 1 + 3Q^2 = 1 + 6\frac{(k-3)^2}{k-2} \quad (7.9)$$

The Liouville field describes the radial (ϕ) fluctuations of the long-string world sheet.

- (3) It was shown in [38] that the affine symmetries of the free action (7.6) can be “lifted” to the space-time and become affine symmetries of the dual CFT_2 . To be more precise, if the world sheet theory has an affine $\hat{G}$ symmetry with level k' then after lifting to the space time it becomes an affine $\hat{G}$ symmetry of the dual CFT_2 with level $\tilde{k}$ given by $\tilde{k} = pk$. Here p is the number of times the string world sheet wraps the boundary sphere.

Similarly the affine $SL_2 \times SL_2$ symmetry of the action (7.6), after lifting to the space time, becomes the Virasoro algebra of the dual CFT_2 with central charge $c = 6kp$.

B. Dual Poincaré CFT_2

We have seen that in the near boundary limit the world sheet action reduces to

⁸We are calling it boundary sphere because we have already taken the near boundary limit.

⁹ $c_L = 6k - c_{\text{int}}$ where c_{int} is the central charge of S_{int} which describes the motion of the string in the 23 dimensional compact space. c_{int} can be determined in terms of k using the condition that the total matter central charge of the world sheet CFT is 26.

$$S_{\text{NB}} = \int d^2w \left(\partial\phi\bar{\partial}\phi - \frac{2}{\alpha_+} \sqrt{g} R\phi + \beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} \right) + S_{\text{int}} \quad (7.10)$$

According to our proposal.

The space-time CFT₂ dual to the string theory described by the world sheet action S_{NB} is an example of a Poincaré CFT₂ (with only Lorentz invariance). The Poincaré CFT₂ contains a *Liouville* sector which describes the radial fluctuation of the long string world sheet.¹⁰ Other fields which live on the long string world sheet are the conformal fields which describe the position of the string in the compact space. Moreover, the infinite dimensional symmetries of the Poincaré CFT₂ can be thought of as lifts of the infinite dimensional symmetries on the world sheet.

In very interesting recent works¹¹ [12–14] examples of celestial CFT₂ were constructed which are Liouville theory coupled to various matter fields. These theories are Lorentz invariant but not Poincaré invariant. The existence of the Liouville sector in these examples and in the long-string CFT₂ suggests that Liouville sector could be a universal feature shared by both celestial and Poincaré CFT₂.

We would also like to point out that the relation between the world sheet CFT and the Poincaré CFT, that we find in this paper, seems to resonate with the observations of [48,49] that in the high energy limit in asymptotically flat space-time one can identify the string world sheet with the celestial sphere. It will be interesting to clarify the connection between our work and the works of [48,49].

Last but not the least, the techniques developed in [50–52] and the related papers, may help us to understand the nature of Poincaré CFT₂ better. We hope to return to this in the near future.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

¹⁰A more precise statement is contained in [42]. See also [43] for recent work on long string CFT.

¹¹See also [44–47] for related work.

APPENDIX: NEAR BOUNDARY LIMIT OF EINSTEIN GRAVITY ON EAdS₃

In this Appendix, we derive the near boundary limit of the Einstein-Hilbert action and the equations of motion. For this let us consider the metric of locally EAdS₃ in Fefferman-Graham gauge with the following line element

$$ds^2 = d\phi^2 + e^{2\phi} h_{ij}(\phi, x^i) dx^i dx^j. \quad (A1)$$

The boundary coordinates are labeled by $x^i = \{\gamma, \bar{\gamma}\}$. On a constant ϕ hypersurface, the two dimensional induced boundary metric a_{ij} is

$$a_{ij} = e^{2\phi} h_{ij} \quad (A2)$$

The Einstein-Hilbert action for EAdS₃ gravity is given by

$$S_{\text{EH}} = \int d\phi d^2\gamma \sqrt{g} (R + 2), \quad (A3)$$

where we have set AdS radius $l = 1$. For the metric in (A1), the action becomes

$$S_{\text{EH}} = \int d\phi d^2\gamma e^{2\phi} \sqrt{h} (K^2 - K_{ij} K^{ij} + 2) + \int d\phi d^2\gamma \sqrt{h} \mathcal{R}^{(2)}, \quad (A4)$$

where the extrinsic curvature K_{ij} is defined as

$$K_{ij} = \frac{1}{2} \partial_\phi a_{ij}, \quad K = a^{ij} K_{ij}. \quad (A5)$$

and $\mathcal{R}^{(2)}$ is the Ricci scalar of metric h_{ij} . One can see that in the near boundary limit $\phi \rightarrow \infty$, the first term in (A4) dominates and therefore the near boundary action is given as¹²

$$S_{\text{NB}} = \int d\phi d^2\gamma e^{2\phi} \sqrt{h} (K^2 - K_{ij} K^{ij} + 2), \quad -\infty < \phi < \infty. \quad (A6)$$

This matches with the Carrollian limit of the Einstein-Hilbert action derived in [25].

The action (A6) is defined on the degenerate manifold with line element

$$ds^2 = 0.d\phi^2 + e^{2\phi} h_{ij}(\phi, x^i) dx^i dx^j. \quad (A7)$$

The Einstein equation is

¹²Note that the procedure to obtain near boundary limit that was formalized in the case of scalar field theory will lead to the same expression in terms of $\tilde{\phi}$ but for notational simplicity we work with the coordinate ϕ .

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} - g_{\mu\nu} = 0 \quad (\text{A8})$$

Since we already fixed the gauge to be (A1), $G_{\phi\phi}$ and $G_{\phi i}$ component of the equations of motion (A8) are constraints given as

$$G_{\phi\phi} = \frac{1}{2}(K^2 - K_{ij}K^{ij} - 2) - \frac{1}{2}e^{-2\phi}\mathcal{R}^{(2)} = 0 \quad (\text{A9})$$

$$G_{\phi i} = (a^{kl}\nabla_k K_{il} - a^{kl}\nabla_i K_{kl}) = 0 \quad (\text{A10})$$

The covariant derivative ∇ is with respect to a_{ij} . The G_{ij} component is

$$G_{ij} = (a_{ij}\partial_\phi K + K_{mn}K^{mn}a_{ij} - a_{ij}) - \partial_\phi K_{ij} + a^{nl}K_{il}K_{nj} = 0 \quad (\text{A11})$$

In the limit $\phi \rightarrow \infty$, the Eq. (A9) becomes,

$$K^2 - K_{ij}K^{ij} - 2 = 0 \quad (\text{A12})$$

whereas the equations for component $G_{\phi i}$ and G_{ij} remain unchanged as all the terms appear at the same order in $e^{2\phi}$.

To write the equations in term of h_{ij} , we introduce the following quantities:

$$\zeta_{ij} = \frac{1}{2}\partial_\phi h_{ij}, \quad \Theta = h^{ij}\zeta_{ij} \quad (\text{A13})$$

then the equation (A12), (A10), (A11) are given by

$$\Theta - \frac{1}{2}\zeta_{ij}\zeta^{ij} + \frac{1}{2}\Theta^2 = 0 \quad (\text{A14})$$

$$h^{kl}\nabla_k \zeta_{il} - h^{kl}\nabla_i \zeta_{kl} = 0 \quad (\text{A15})$$

$$(\partial_\phi \Theta h_{ij} - \partial_\phi \zeta_{ij} + \zeta_{kl}\zeta^{kl}h_{ij} + \zeta_i^k \zeta_{kj} + 2\Theta h_{ij} - 2\zeta_{ij}) = 0 \quad (\text{A16})$$

All indices are raised and lowered by h_{ij} .

1. Solutions to the equations

Taking trace of (A16) we have

$$2\Theta + \partial_\phi \Theta + \zeta_{ij}\zeta^{ij} = 0. \quad (\text{A17})$$

One can use (A17) to simplify (A16) as follows:

$$\partial_\phi \zeta_{ij} + 2\zeta_{ij} - \zeta_i^k \zeta_{kj} = 0 \quad (\text{A18})$$

The solution of above equation is exact and can be written as

$$h_{ij}(\phi, x^i) = h_{ij}^{(0)}(x^i) + e^{-2\phi}h_{ij}^{(1)}(x^i) + e^{-4\phi}h_{ij}^{(2)}(x^i) \quad (\text{A19})$$

with $h_{ij}^{(2)}(x^i)$ given by

$$h_{ij}^{(2)} = \frac{1}{4}h_{ik}^{(1)}h_{(0)}^{kl}h_{lj}^{(1)} \quad (\text{A20})$$

Substituting (A19) in (A14), $\mathcal{O}(e^{-2\phi})$ component implies

$$\boxed{h_{(0)}^{ij}h_{ij}^{(1)} = 0.} \quad (\text{A21})$$

The equation (A15) to the lowest order gives

$$-h_{(0)}^{kl}D_k h_{il}^{(1)} + h_{(0)}^{kl}D_i h_{kl}^{(1)} = 0 \quad (\text{A22})$$

where D_i is covariant derivative with respect to $h_{ij}^{(0)}$. Due to the trace condition second term vanishes and therefore, one gets the conservation law

$$\boxed{D^k h_{ki}^{(1)} = 0.} \quad (\text{A23})$$

To summarize, the solution space is parametrized by $h_{ij}^{(0)}$ and $h_{ij}^{(1)}$ with the latter being a conserved and traceless tensor.

Comments: The tracelessness of $h_{ij}^{(1)}$ in (A21) implies that the Poincaré CFT₂ dual to the near boundary limit of Einstein gravity has zero central charge.

From (A23), the boundary stress tensor of this dual Poincaré CFT₂ will have holomorphic and antiholomorphic components that will generate two commuting copies of Virasoro algebra. One can contrast this with conventional AdS₃/CFT₂ setup where the symmetry algebra of boundary CFT is two commuting copies of Virasoro algebra with nonzero central charge [53].

2. Extension to higher dimension EAdS_{d+1}

Here we extend the results of previous section to general $d+1$ dimensions. The Einstein-Hilbert action in the Fefferman-Graham gauge becomes

$$S_{\text{EH}} = \int d\phi d^d x e^{d\phi} \sqrt{h} (K^2 - K_{ij}K^{ij} - 2\Lambda) + \int d\phi d^d x e^{(d-2)\phi} \sqrt{h} \mathcal{R}^{(d)} \quad (\text{A24})$$

As $\phi \rightarrow \infty$, the action is given by

$$S_{\text{NB}} = \int d\phi d^d x e^{d\phi} \sqrt{h} (K^2 - K_{ij}K^{ij} - 2\Lambda) \quad (\text{A25})$$

The constraint equations in the near boundary limit is given by

$$\frac{1}{2}(K^2 - K_{ij}K^{ij} + 2\Lambda) = 0 \quad (\text{A26})$$

$$(a^{kl}\nabla_k K_{il} - a^{kl}\nabla_i K_{kl}) = 0 \quad (\text{A27})$$

The equation of motion is given by

$$\begin{aligned} -KK_{ij} + \frac{1}{2}K_{mn}K^{mn}a_{ij} + \Lambda a_{ij} - a_{\text{in}}\partial_\phi K^n_j \\ + \partial_\phi K a_{ij} + \frac{1}{2}K^2 a_{ij} = 0 \end{aligned} \quad (\text{A28})$$

The constraint equations in terms of the metric h_{ij} are

$$\begin{aligned} \Theta^2 - \zeta_{ij}\zeta^{ij} + (d-1)(d+2\Theta) + 2\Lambda = 0 \\ h^{kl}\nabla_k \zeta_{il} - h^{kl}\nabla_i \zeta_{kl} = 0 \end{aligned} \quad (\text{A29})$$

The equation for h_{ij} can be written as

$$\begin{aligned} \frac{h_{ij}}{2}(\zeta_{mn}\zeta^{mn} + 2\partial_\phi \Theta + \Theta^2 + 2\Theta d + d(d-1)) \\ - (\Theta + d)\zeta_{ij} - a_{\text{in}}\partial_\phi \zeta^n_j = 0 \end{aligned} \quad (\text{A30})$$

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