

Dynamically induced spin-2 mass in a Weyl-invariant framework

Ioannis D. Gialamas^{*}

*Laboratory of High Energy and Computational Physics,
National Institute of Chemical Physics and Biophysics,
Rävala pst. 10, Tallinn, 10143, Estonia*

Kyriakos Tamvakis[†]

Physics Department, University of Ioannina, 45110, Ioannina, Greece



(Received 29 July 2025; accepted 1 October 2025; published 17 October 2025)

We combine the ghost-free bimetric theory of gravity with the concept of local Weyl invariance, realized in the framework of Einstein-Cartan gravity. The gravitational sector, characterized by two independent metrics and two independent connections, is coupled to a scalar field that can in principle develop a nonvanishing expectation value through radiative corrections. The spectrum of the model, apart from the massless standard graviton and a pair of axionlike pseudoscalars, associated with the presence of the Holst invariants in the action, includes an additional spin-2 state of a nonvanishing Fierz-Pauli mass proportional to the scalar field vacuum expectation value. We analyze the phenomenology of the model and specify the conditions under which the massive spin-2 state could be a primary dark matter candidate.

DOI: [10.1103/hwht-srsz](https://doi.org/10.1103/hwht-srsz)

I. INTRODUCTION

Efforts to formulate a theory of a massive spin-2 field have a rather long history, starting with the linear theory of Fierz and Pauli [1]. Just after the resolution of the van Dam-Veltman-Zakharov discontinuity [2,3] issue in the framework of its nonlinear generalization by Vainshtein [4], the theory came to face the problem of ghosts raised by Boulware and Deser [5]. Bimetric gravity [6], characterized by the existence of two independent metric tensors, came out as a ghost-free answer to this problem. The physical Planck mass as well as the Fierz-Pauli mass of the massive spin-2 excitation arise in terms of two independent gravitational scales, associated with the two dynamical metric tensors of the theory. On the other hand, the problem of multiple energy scales, often vastly different, that arise in physics (hierarchy problem) has often been attributed to approximate or exact scale invariance, broken at the quantum level [7–35]. For instance, the Standard model (SM) is scale invariant with the exception of the Higgs mass term. In gravitational considerations also global scale or Weyl invariance has also been employed to address analogous issues. Weyl invariance can be promoted to a

local gauge symmetry in the framework of the so called Weyl geometry. Gauge symmetries play a very important role in physics, the most obvious example being the SM of particle interactions whose success relies to a large extent on its local gauge symmetries. This philosophy can be carried over to gravitation, promoting gravity to a gauge theory of the Poincaré group [36,37]. This is the Einstein-Cartan theory of gravity [38,39], positing, in addition to the metric, a torsion tensor as a fundamental property of spacetime. The Einstein-Cartan theory can also be combined with local Weyl symmetry with the role of the corresponding Weyl gauge field [40] played by the torsion vector.

In the present article we have combined bimetric gravity (see [41] for a review and [42–48] for recent applications), characterized by an additional gravitational scale that leads to a massive spin-2 field, with the concept of local Weyl symmetry, the latter realized in the framework of Einstein-Cartan gravity. Einstein-Cartan bigravity, in addition to the two independent metrics, posits two independent connections as well. The gravitational Weyl-invariant action (see [49–63] for applications) is restricted to include at most quadratic curvature invariants, so that no unphysical degrees of freedom arise, consisting of quadratic terms of the Ricci scalar curvature and Holst invariants [64] corresponding to both metrics and connections. The interaction potential of the two gravitational sectors is the standard bigravity potential rendered Weyl-invariant in terms of its coupling to a scalar field. The equivalent metric form of the action features the torsion vectors corresponding to the two sectors which are nondynamical and are integrated out.

^{*}Contact author: ioannis.gialamas@kbfi.ee

[†]Contact author: tamvakis@uoi.gr

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

The model, apart from the two interacting spin-2 states, describes a pair of axionlike pseudoscalars, associated with the presence of the Holst terms, and the above scalar that couples to the interaction potential [6]. Considering the linear limit of the theory for proportional backgrounds, we arrive, as in the case of standard bigravity, at one massless graviton state as well as a spin-2 state of nonzero Fierz-Pauli mass. The latter is proportional to the vacuum expectation value (vev) of the scalar field. At this point it is argued that quantum radiative corrections, expected to break the scale invariance in a fashion analogous to dimensional transmutation, lead to a nonvanishing expectation value of the scalar field and as a result to a nonvanishing Fierz-Pauli mass. Although the theory has started with just a set of dimensionless parameters, namely those scaling the curvature invariants and those of the interaction potential, at the end it has exchanged a number of them for the set of pseudoscalar, scalar, and spin-2 masses.

The paper is organized as follows: In Sec. II we lay out the basic elements of Einstein-Cartan gravity and local Weyl invariance. We consider the simplest Weyl-invariant gravitational action that does not lead to unphysical degrees of freedom and examine its coupling to a scalar field ϕ . In Sec. III we combine the above Weyl-Einstein-Cartan framework with the ghost-free theory of bimetric gravity, characterized by two independent interacting metrics and connections. The action, featuring quadratic terms of the Ricci scalars and Holst invariants, is rewritten in terms of pairs of auxiliary and axionlike pseudoscalar fields. We derive the equivalent metric form of the action and integrate out the appearing nondynamical torsion vectors. The resulting Einstein frame action is characterized by two gravitational scales set up by the nondynamical auxiliary fields. In addition to the two gravitons the theory describes the scalar field ϕ and a pair of axionlike pseudoscalars. In Sec. III A we derive the equations of motion and argue that the scalar field ϕ can obtain a nonvanishing expectation value through quantum corrections. In Sec. IV and specifically in Sec. IV A we discuss a special class of solutions in which the two metrics are proportional, derive the corresponding energy-momentum tensors and analyze the conditions for their compatibility imposed by the Einstein's equations. In Sec. IV B we consider perturbations around proportional metric backgrounds and analyze the corresponding linear limit, characterized by a standard massless graviton and a spin-2 state, having a Fierz-Pauli mass. In Sec. V we discuss the phenomenology of the model, examining particular cases where the spin-2 state is ultralight or superheavy and analyzing the possibility of this state to be a primary candidate for dark matter. Finally, in Sec. VI we state briefly our conclusions.

II. FRAMEWORK

In the Einstein-Cartan formulation of gravity [38,39] translations and Lorentz transformations are promoted to

gauge symmetries with the gauge fields being the tetrad e_μ^A and the spin connection ω_μ^{AB} . The corresponding field strengths can be represented by the affine-curvature $\mathcal{R}^\rho_{\sigma\mu\nu}$ and the affine-torsion $T^\mu_{\nu\rho}$

$$\begin{aligned}\mathcal{R}^\rho_{\sigma\mu\nu} &= \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda} \Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda} \Gamma^\lambda_{\mu\sigma}, \\ T^\mu_{\nu\rho} &= \Gamma^\mu_{\nu\rho} - \Gamma^\mu_{\rho\nu},\end{aligned}\quad (1)$$

in terms of the affine-connection $\Gamma^\mu_{\nu\rho} = e_A^\mu (\partial_\nu e_\rho^A + \omega_{\nu B}^A e_\rho^B)$. The torsion tensor can be decomposed as

$$T_{\mu\nu\rho} = -\frac{1}{3}(g_{\mu\nu}T_\rho - g_{\mu\rho}T_\nu) + \frac{1}{6}\epsilon_{\mu\nu\rho\sigma}\hat{T}^\sigma + \tau_{\mu\nu\rho}, \quad (2)$$

in terms of a vector part T^μ , an axial vector part $\hat{T}^\mu$, defined as

$$T^\mu = g_{\nu\rho}T^{\nu\mu\rho}, \quad \hat{T}^\mu = \epsilon^{\mu\nu\rho\sigma}T_{\nu\rho\sigma}, \quad (3)$$

and a tensorial part $\tau_{\nu\rho\sigma}$, defined by $\tau^\alpha_{\mu\alpha} = \tau^\alpha_{\alpha\mu} = \epsilon_\mu^{\nu\rho\sigma}\tau_{\nu\rho\sigma} = 0$.

Global scale symmetry corresponds to invariance under the transformations

$$e_\mu^A \rightarrow q^{-1}e_\mu^A, \quad \omega_\mu^{AB} \rightarrow \omega_\mu^{AB}. \quad (4)$$

This symmetry can be promoted to a local gauge symmetry (*Weyl symmetry*) introducing a gauge field W_μ transforming as

$$W_\mu \rightarrow W_\mu + q^{-1}(x)\partial_\mu q(x). \quad (5)$$

Under this symmetry the torsion vector transforms in an analogous fashion, namely as

$$T_\mu \rightarrow T_\mu - 3q^{-1}(x)\partial_\mu q(x). \quad (6)$$

It is possible to realize this local symmetry without introducing the gauge field as an additional dynamical degree of freedom. Instead, one can employ the torsion vector by setting $W_\mu = \frac{1}{3}T_\mu$, with the corresponding Weyl covariant derivative defined as

$$\nabla_\mu^{(w)} = \partial_\mu + \frac{1}{3}T_\mu. \quad (7)$$

It turns out that, in the specific Einstein-Cartan quadratic models to be considered below, the torsion is nondynamical and, therefore, employing the above identification will not introduce any additional dynamical degrees of freedom.

The set of (local) Weyl transformations are¹

$$\begin{aligned} g_{\mu\nu} &\rightarrow q^{-2}g_{\mu\nu}, & \Gamma^\mu_{\nu\rho} &\rightarrow \Gamma^\mu_{\nu\rho} - q^{-1}\partial_\nu q\delta^\mu_\rho, \\ \mathcal{R} &\rightarrow q^2\mathcal{R}, & \tilde{\mathcal{R}} &\rightarrow q^2\tilde{\mathcal{R}}, \\ T_\mu &\rightarrow T_\mu - 3q^{-1}\partial_\mu q, & \hat{T}_\mu &\rightarrow \hat{T}_\mu, & \tau_{\mu\nu\rho} &\rightarrow q^{-2}\tau_{\mu\nu\rho}, \end{aligned} \quad (8)$$

where $\mathcal{R}$ and $\tilde{\mathcal{R}}$ are the two existing curvature scalars, namely the *Ricci scalar* and the *Holst invariant* [64], defined as

$$\mathcal{R} = g^{\nu\rho}\mathcal{R}^\rho_{\sigma\nu}, \quad \tilde{\mathcal{R}} = \epsilon^{\mu\nu\rho\sigma}\mathcal{R}_{\mu\nu\rho\sigma}. \quad (9)$$

Constructing a theory of pure gravity in the above described Einstein-Cartan framework and subject to the criterion of the absence of any unphysical degrees of freedom, we are restricted to consider at most quadratic terms of the curvature and in particular at most quadratic terms of $\mathcal{R}$ and $\tilde{\mathcal{R}}$. In addition Weyl symmetry excludes the linear terms. Therefore, a general Weyl-Einstein Cartan pure gravity action is²

$$\mathcal{S}_g = \int d^4x \sqrt{-g} \left(\frac{1}{4f^2} \mathcal{R}^2 + \frac{1}{4\tilde{f}^2} \tilde{\mathcal{R}}^2 \right), \quad (10)$$

where g is the metric determinant and $f, \tilde{f}$ are dimensionless parameters. It is also worth mentioning that the above action coincides with the metric-affine case where non-metricity is nonzero. This equivalence arises due to a projective symmetry of the action, generated by a single vector.³ However, if we explicitly include terms involving nonmetricity and/or torsion in the action (as in what follows) [68–70], this equivalence no longer holds.

The action (10) can also be written in terms of two auxiliary fields, namely an *auxiliary scalar* χ and an *axionlike* ζ as

$$\mathcal{S}_g = \int d^4x \sqrt{-g} \left(\frac{\chi}{2f^2} \mathcal{R} + \frac{\zeta}{2\tilde{f}^2} \tilde{\mathcal{R}} - \frac{\chi^2}{4f^2} - \frac{\zeta^2}{4\tilde{f}^2} \right). \quad (11)$$

The coupling of a scalar field ϕ to gravity in this framework will necessarily restrict its self-interaction potential to be quartic. The corresponding action will be

$$\mathcal{S}_m = \int d^4x \sqrt{-g} \left(\frac{1}{2} \xi \phi^2 \mathcal{R} - \frac{1}{2} (\nabla_\mu^{(w)} \phi)^2 - \frac{\lambda}{4} \phi^4 + C \hat{T}^2 \phi^2 \right). \quad (12)$$

¹Scalar fields transform in terms of their canonical dimension as $\phi \rightarrow q^d \phi$.

²We exclude the parity-odd mixed $\mathcal{R}\tilde{\mathcal{R}}$ term. See [65,66] for the inclusion of this term in the context of inflation.

³A new extended projective symmetry, generated by a pair of vectors and leading to interesting phenomenology, has been discussed in [67].

Note also the presence of the Weyl-invariant term $\sqrt{-g} \hat{T}^2 \phi^2$, where $\hat{T}^2 = g^{\mu\nu} \hat{T}_\mu \hat{T}_\nu$, which cannot be excluded. A nonminimal coupling term $\sqrt{-g} \tilde{g} \phi^2 \tilde{\mathcal{R}}$ is also allowed by Weyl invariance, although it is parity odd. Recently, numerous studies have explored the effects of this term in inflation [65,66,71–80].

Closing this section we note that the action can be set in a metric form by substituting in place of the curvature scalars the formulas

$$\mathcal{R} = R(g) + 2\nabla_\mu T^\mu - \frac{2}{3} T_\mu T^\mu + \frac{1}{24} \hat{T}_\mu \hat{T}^\mu + \frac{1}{2} \tau_{\mu\nu\rho} \tau^{\mu\nu\rho}, \quad (13a)$$

$$\tilde{\mathcal{R}} = -\nabla_\mu \hat{T}^\mu + \frac{2}{3} \hat{T}_\mu T^\mu + \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \tau_{\lambda\mu\nu} \tau^\lambda_{\rho\sigma}, \quad (13b)$$

where the metric Ricci scalar $R(g)$ and the covariant derivative are defined in terms of the Levi-Civita connection. In the above formulas, the quadratic terms of the tensorial part $\tau_{\mu\nu\rho}$ can be neglected, as their (algebraic) equation of motion yields $\tau = 0$, allowing us to set them to zero to begin with.

III. THE MODEL

Bimetric gravity as a ghost-free theory of a massive spin-2 particle, positing two independent metric tensors $(g_1)_{\mu\nu}$ and $(g_2)_{\mu\nu}$, formulated in the Einstein-Cartan framework, is also characterized by two independent connections. The Fierz-Pauli mass for one of the spin-2 states is generated by a standard interaction potential of the form⁴ $\sum_{n=0}^4 \beta_n e_n(\sqrt{\Delta})$, where $\Delta^\mu_\nu = g_1^{\mu\rho} g_{2\rho\nu}$, in terms of five parameters β_n . Imposing invariance under common local transformations $g_{a\mu\nu} \rightarrow q^{-2} g_{a\mu\nu}$ and $(\Gamma_a)^\mu_{\nu\rho} \rightarrow (\Gamma_a)^\mu_{\nu\rho} - q^{-1} \partial_\nu q \delta^\mu_\rho$ leads us to consider an action of the form (10) for each metric. Thus, for the noninteracting part of the gravitational action we adopt the action⁵

$$\begin{aligned} \mathcal{S}_g = \int d^4x & \left[\sqrt{-g_1} \left(\frac{1}{4f_1^2} \mathcal{R}_1^2 + \frac{1}{4\tilde{f}_1^2} \tilde{\mathcal{R}}_1^2 \right) \right. \\ & \left. + \sqrt{-g_2} \left(\frac{1}{4f_2^2} \mathcal{R}_2^2 + \frac{1}{4\tilde{f}_2^2} \tilde{\mathcal{R}}_2^2 \right) \right]. \end{aligned} \quad (14)$$

⁴Starting with $e_0(X) = 1$, the polynomials $e_n(X)$ satisfy the recurrence relation

$$e_n(X) = \frac{(-1)^{n+1}}{n} \sum_{k=0}^{n-1} (-1)^k \text{Tr}(X^{n-k}) e_k(X).$$

⁵The impact of a quadratic curvature term in bigravity has been recently discussed in [44,46] using the standard metric formulation and in [42] within the framework of metric-affine gravity. For further applications in the context of $F(R)$ massive gravity and bigravity, refer to [81,82].

Note that the standard interaction potential discussed above is of zero Weyl weight. A Weyl-invariant interaction potential that preserves the results of bimetric gravity can be written in terms of a scalar field ϕ (of canonical dimension $d = 1$) with a quartic interaction, namely

$$V(\sqrt{\Delta}, \phi) = \phi^4 \sum_{n=0}^4 \beta_n e_n(\sqrt{\Delta}), \quad (15)$$

where β_n are five dimensionless parameters. Note that a quartic self-interaction is included in the sum in terms of the $n = 0$ and $n = 4$ terms. Thus, the full action of the model is

$$\mathcal{S} = \mathcal{S}_g + \mathcal{S}_{\text{int},\phi}, \quad (16)$$

where

$$\begin{aligned} \mathcal{S}_{\text{int},\phi} = & \int d^4x \sqrt{-g_1} \left(-\frac{1}{2} (\nabla_\mu^{(w)} \phi)^2 + C \hat{T}_1^2 \phi^2 \right. \\ & \left. + \phi^4 \sum_{n=0}^4 \beta_n e_n(\sqrt{\Delta}) \right). \end{aligned} \quad (17)$$

The specific form of the potential V is determined by the requirement that the theory remains both ghost-free [6,83] and invariant under Weyl transformations. This ensures the internal consistency of the framework, preventing the emergence of unphysical degrees of freedom while preserving its fundamental symmetry properties. Terms of the form $\xi \phi^2 R$ and $\tilde{\xi} \phi^2 \tilde{R}$ also preserve Weyl invariance. However, we omit them at this stage for the sake of simplicity.

We can proceed by introducing auxiliary fields in the fashion of (11), namely a pair of auxiliary scalar fields χ_a and a pair of axionlike pseudoscalars ζ_a , and rewrite the gravitational part of the action, summing up over the two sets of variables in terms of an index a , as

$$\mathcal{S}_g = \int d^4x \sum_{a=1,2} \sqrt{-g_a} \left(\frac{\chi_a}{2f_a^2} \mathcal{R}_a + \frac{\zeta_a}{2\tilde{f}_a^2} \tilde{\mathcal{R}}_a - \frac{\chi_a^2}{4f_a^2} - \frac{\zeta_a^2}{4\tilde{f}_a^2} \right). \quad (18)$$

Weyl invariance allows us to set one of the auxiliary fields to

$$\chi_1 = f_1^2 m_1^2. \quad (19)$$

Next, we can employ the expressions (13) in terms of the torsion vectors and write the full action in metric form as

$$\mathcal{S} = \mathcal{S}_1 + \mathcal{S}_2 + \mathcal{S}_{\text{int}}, \quad (20)$$

with

$$\begin{aligned} \mathcal{S}_1 = & \int d^4x \sqrt{-g_1} \left[\frac{m_1^2}{2} \left(R[g_1] - \frac{2}{3} T_1^2 + \frac{1}{24} \hat{T}_{1\mu} \hat{T}_1^\mu \right) \right. \\ & + \frac{\bar{\xi}_1}{2} \left(\frac{2}{3} T_1^\mu \hat{T}_{1\mu} \right) + \frac{1}{2} \hat{T}_1^\mu \partial_\mu \bar{\xi}_1 - \frac{1}{2} (\partial_\mu \phi)^2 \\ & \left. - \frac{1}{18} \phi^2 T_1^\mu T_{1\mu} - \frac{1}{3} \phi T_1^\mu \partial_\mu \phi + C \phi^2 \hat{T}_1^2 - \frac{f_1^2}{4} m_1^4 - \frac{\tilde{f}_1^2}{4} \bar{\xi}_1^2 \right], \end{aligned} \quad (21a)$$

$$\begin{aligned} \mathcal{S}_2 = & \int d^4x \sqrt{-g_2} \left[\frac{\bar{\chi}_2}{2} \left(R[g_2] - \frac{2}{3} T_2^2 + \frac{1}{24} \hat{T}_{2\mu} \hat{T}_2^\mu \right) \right. \\ & + \frac{\bar{\xi}_2}{2} \left(\frac{2}{3} T_2^\mu \hat{T}_{2\mu} \right) + \frac{1}{2} \hat{T}_2^\mu \partial_\mu \bar{\xi}_2 - T_2^\mu \partial_\mu \bar{\chi}_2 - \frac{f_2^2}{4} \bar{\chi}_2^2 - \frac{\tilde{f}_2^2}{4} \bar{\xi}_2^2 \left. \right], \end{aligned} \quad (21b)$$

$$\mathcal{S}_{\text{int}} = \int d^4x \sqrt{-g_1} \phi^4 \sum_{n=0}^4 \beta_n e_n(\sqrt{\Delta}). \quad (21c)$$

Note that the kinetic and $\hat{T}^2$ terms of ϕ , included in $\mathcal{S}_{\text{int},\phi}$ are not included in $\mathcal{S}_{\text{int}}$, having been incorporated in $\mathcal{S}_1$. Note also that we have omitted the terms $\tau_1^{\mu\nu\rho}, \tau_2^{\mu\nu\rho}$ since they appear quadratically in the action, and their equations of motion yield $\tau_1^{\mu\nu\rho} = \tau_2^{\mu\nu\rho} = 0$ as solutions. We have imposed the gauge $\chi_1 = f_1^2 m_1^2$ and redefined the fields as $\xi_1/\tilde{f}_1^2 = \bar{\xi}_1$, $\xi_2/\tilde{f}_2^2 = \bar{\xi}_2$, and $\chi_2/f_2^2 = \bar{\chi}_2$.

Varying the above action (21) with respect to $T_1, \hat{T}_1, T_2$, and $\hat{T}_2$ we obtain the system of linear equations with solutions

$$\begin{aligned} T_1^\mu &= -\frac{3[12\bar{\xi}_1 \partial^\mu \bar{\xi}_1 + \phi \partial^\mu \phi (m_1^2 + 48C\phi^2)]}{24\bar{\xi}_1^2 + (6m_1^2 + \phi^2)(m_1^2 + 48C\phi^2)}, \\ \hat{T}_1^\mu &= \frac{12[2\bar{\xi}_1 \phi \partial^\mu \phi - \partial^\mu \bar{\xi}_1 (6m_1^2 + \phi^2)]}{24\bar{\xi}_1^2 + (6m_1^2 + \phi^2)(m_1^2 + 48C\phi^2)}, \\ T_2^\mu &= -\frac{3(4\bar{\xi}_2 \partial^\mu \bar{\xi}_2 + \bar{\chi}_2 \partial^\mu \bar{\chi}_2)}{2(\bar{\chi}_2^2 + 4\bar{\xi}_2^2)}, \\ \hat{T}_2^\mu &= \frac{12(\bar{\xi}_2 \partial^\mu \bar{\chi}_2 - \bar{\chi}_2 \partial^\mu \bar{\xi}_2)}{\bar{\chi}_2^2 + 4\bar{\xi}_2^2}. \end{aligned} \quad (22)$$

Substituting the solution back into the action (20) we obtain

$$\mathcal{S}_1 = \int d^4x \sqrt{-g_1} \left[\frac{m_1^2}{2} R[g_1] - \frac{f_1^2}{4} m_1^4 - \frac{\tilde{f}_1^2}{4} \bar{\xi}_1^2 \right. \\ \left. - 3 \frac{(\partial_\mu \bar{\xi}_1)^2 (6m_1^2 + \phi^2) + (\partial_\mu \phi)^2 (m_1^4 + 4\bar{\xi}_1^2 + 48Cm_1^2 \phi^2) - 4\bar{\xi}_1 \phi \partial_\mu \phi \partial^\mu \bar{\xi}_1}{24\bar{\xi}_1^2 + (6m_1^2 + \phi^2)(m_1^2 + 48C\phi^2)} \right], \quad (23a)$$

$$\mathcal{S}_2 = \int d^4x \sqrt{-g_2} \left[\frac{\tilde{\chi}_2}{2} R[g_2] - \frac{f_2^2}{4} \tilde{\chi}_2^2 - \frac{\tilde{f}_2^2}{4} \bar{\xi}_2^2 + \frac{3}{4} \left(\frac{\tilde{\chi}_2 (\partial_\mu \tilde{\chi}_2)^2 - 4\tilde{\chi}_2 (\partial_\mu \bar{\xi}_2)^2 + 8\bar{\xi}_2 \partial_\mu \tilde{\chi}_2 \partial^\mu \bar{\xi}_2}{\tilde{\chi}_2^2 + 4\bar{\xi}_2^2} \right) \right]. \quad (23b)$$

The action $\mathcal{S}_1$ can be diagonalized by performing the field redefinition [60,65]⁶

$$\phi = \sqrt{6}m_1 \sinh\left(\frac{\Phi}{\sqrt{6}m_1}\right), \\ \bar{\xi}_1 = 12m_1^2 e^{-Z/m_1} \cosh^2\left(\frac{\Phi}{\sqrt{6}m_1}\right), \quad (24)$$

which transforms $\mathcal{S}_1$ into

$$\mathcal{S}_1 = \int d^4x \sqrt{-g_1} \left[\frac{m_1^2}{2} R[g_1] - \frac{1}{2} (\partial_\mu \Phi)^2 \right. \\ \left. - \frac{1}{2} K_Z(\Phi, Z) (\partial_\mu Z)^2 - V_1(\Phi, Z) \right], \quad (25)$$

where

$$K_Z(\Phi, Z) = \frac{3}{2} \frac{\cosh^4\left(\frac{\Phi}{\sqrt{6}m_1}\right)}{\left[\cosh^2\left(\frac{\Phi}{\sqrt{6}m_1}\right) + \frac{e^{2Z/m_1}}{(24)^2} (1 + 288C \sinh^2\left(\frac{\Phi}{\sqrt{6}m_1}\right))\right]}, \quad (26a)$$

$$V_1(\Phi, Z) = \frac{m_1^4 f_1^2}{4} + 36m_1^4 \tilde{f}_1^2 e^{-2Z/m_1} \cosh^4\left(\frac{\Phi}{\sqrt{6}m_1}\right). \quad (26b)$$

Before proceeding with the diagonalization of $\mathcal{S}_2$ we first perform a Weyl rescaling of the metric $g_{2\mu\nu}$, given by⁷

$$g_{2\mu\nu} \rightarrow \frac{m_2^2}{\tilde{\chi}_2} g_{2\mu\nu}. \quad (27)$$

⁶The inverse relations are

$$\Phi = \sqrt{6}m_1 \sinh^{-1}(\phi/\sqrt{6}m_1), \quad Z = -m_1 \ln\left(\frac{\tilde{\xi}_1 m_1}{2(\phi^2 + 6m_1^2)}\right).$$

⁷Note that Weyl-invariance has been used to gauge-fix the auxiliary χ_1 , so that no Weyl-rescaling for $g_{1\mu\nu}$ is required. In contrast, the metric $g_{2\mu\nu}$ enters in the Jordan frame and a Weyl-rescaling is appropriate.

The rescaling introduces the standard additional derivative term, $-\frac{3}{4}m_2^2 \frac{(\partial_\mu \tilde{\chi}_2)^2}{\tilde{\chi}_2^2}$. Consequently, the actions $\mathcal{S}_2$ and $\mathcal{S}_{\text{int}}$ are modified accordingly

$$\mathcal{S}_2 = \int d^4x \sqrt{-g_2} \left[\frac{m_2^2}{2} R[g_2] - \frac{3}{4} m_2^2 \frac{(\partial_\mu \tilde{\chi}_2)^2}{\tilde{\chi}_2^2} \right. \\ \left. + \frac{3}{4} m_2^2 \frac{(\partial_\mu \tilde{\chi}_2)^2 - 4(\partial_\mu \bar{\xi}_2)^2 + 8\bar{\xi}_2 \partial_\mu \tilde{\chi}_2 \partial^\mu \bar{\xi}_2}{\tilde{\chi}_2^2 + 4\bar{\xi}_2^2} \right. \\ \left. - \frac{1}{4} m_2^4 f_2^2 - \frac{1}{4} m_2^4 \tilde{f}_2^2 \frac{\bar{\xi}_2^2}{\tilde{\chi}_2^2} \right], \quad (28)$$

$$\mathcal{S}_{\text{int}} = 36m_1^4 \int d^4x \sqrt{-g_1} \sinh^4\left(\frac{\Phi}{\sqrt{6}m_1}\right) \\ \times \sum_{n=0}^4 \left(\frac{m_2}{\sqrt{\tilde{\chi}_2}}\right)^n \beta_n e_n(\sqrt{\Delta}), \quad (29)$$

where we made use of the property $e_n(\sqrt{\lambda x}) = \lambda^{n/2} e_n(\sqrt{x})$. Introducing the field

$$\sigma = m_2 \frac{\bar{\xi}_2}{\tilde{\chi}_2}, \quad (30)$$

$\mathcal{S}_2$ reduces to a single-field action

$$\mathcal{S}_2 = \int d^4x \sqrt{-g_2} \left[\frac{m_2^2}{2} R[g_2] - \frac{1}{2} K_\sigma(\sigma) (\partial_\mu \sigma)^2 - V_2(\sigma) \right], \quad (31)$$

where

$$K_\sigma(\sigma) = \frac{6}{1 + 4\sigma^2/m_2^2} \quad \text{and} \quad V_2(\sigma) = \frac{1}{4} m_2^4 f_2^2 + \frac{1}{4} m_2^2 \tilde{f}_2^2 \sigma^2. \quad (32)$$

Our complete final action is given by

$$\begin{aligned} \mathcal{S}_1 + \mathcal{S}_2 + \mathcal{S}_{\text{int}} = & \int d^4x \sqrt{-g_1} \left[\frac{m_1^2}{2} R[g_1] - \frac{1}{2} (\partial_\mu \Phi)^2 - \frac{1}{2} K_Z(\Phi, Z) (\partial_\mu Z)^2 - V_1(\Phi, Z) \right] \\ & + \int d^4x \sqrt{-g_2} \left[\frac{m_2^2}{2} R[g_2] - \frac{1}{2} K_\sigma(\sigma) (\partial_\mu \sigma)^2 - V_2(\sigma) \right] \\ & + 36m_1^4 \int d^4x \sqrt{-g_1} \sinh^4 \left(\frac{\Phi}{\sqrt{6}m_1} \right) \sum_{n=0}^4 \left(\frac{m_2}{\sqrt{\tilde{\chi}_2}} \right)^n \beta_n e_n(\sqrt{\Delta}), \end{aligned} \quad (33)$$

where the involved functions are given by (26) and (32). The absence of ghost degrees of freedom in standard massive gravity [84] and bigravity [6] was explicitly demonstrated in [83,85]. For actions similar to (33), where the interaction potential is field-dependent, this was proven in [46] and was also adopted in the context of multi-gravity in [86].

Note that this action, apart from the two spin-2 particles, describes one dynamical scalar field and two dynamical pseudoscalars, namely Z and σ , associated with the presence of the Holst terms. The appearing auxiliary χ_2 is clearly nondynamical, not possessing a kinetic term. As it has been pointed out, the coupling of the pseudoscalar sector to the axial vector current of SM fermions can lead to a possible solution of the strong CP problem [60].

A. Equations of motion

Varying the action (29) with respect to the nondynamical scalar $\tilde{\chi}_2$ we get

$$\frac{\delta \mathcal{S}_{\text{int}}}{\delta \tilde{\chi}_2} = 0 \Rightarrow \sum_{n=0}^4 (\tilde{\chi}_2)^{-n/2-1} n \dot{\beta}_n e_n(\sqrt{\Delta}) = 0, \quad (34)$$

where $\dot{\beta}_n = m_2^n \beta_n$.

This equation amounts

$$\begin{aligned} & \dot{\beta}_1 e_1(\sqrt{\Delta}) \tilde{\chi}_2^{3/2} + 2\dot{\beta}_2 e_2(\sqrt{\Delta}) \tilde{\chi}_2 + 3\dot{\beta}_3 e_3(\sqrt{\Delta}) \tilde{\chi}_2^{1/2} \\ & + 4\dot{\beta}_4 e_4(\sqrt{\Delta}) = 0, \end{aligned} \quad (35)$$

with the $\tilde{\chi}_2$ coefficients becoming constant for proportional backgrounds. Note that for the so-called *minimal choice* [6] ($\beta_2 = \beta_3 = 0$), (35) yields

$$\tilde{\chi}_2^{1/2} = m_2 \left(\frac{-4\beta_4 e_4(\sqrt{\Delta})}{\beta_1 e_1(\sqrt{\Delta})} \right)^{1/3}. \quad (36)$$

By varying the action (33) with respect to Φ , Z , and σ , we obtain the scalar field equations of motion

$$\begin{aligned} \square \Phi - \frac{1}{2} \frac{\partial K_Z}{\partial \Phi} (\partial_\mu Z)^2 + 12\sqrt{6}m_1^3 \sinh \left(\frac{2\Phi}{\sqrt{6}m_1} \right) \\ \times \left[\sinh^2 \left(\frac{\Phi}{\sqrt{6}m_1} \right) \sum_{n=0}^4 \tilde{\chi}_2^{-n/2} \dot{\beta}_n e_n(\sqrt{\Delta}) \right. \\ \left. - \tilde{f}_1^2 \cosh^2 \left(\frac{\Phi}{\sqrt{6}m_1} \right) e^{-2Z/m_1} \right] = 0, \end{aligned} \quad (37a)$$

$$\square Z - \frac{1}{2} \frac{\partial K_Z}{\partial Z} (\partial_\mu Z)^2 + 72m_1^3 \tilde{f}_1^2 e^{-2Z/m_1} \cosh^4 \left(\frac{\Phi}{\sqrt{6}m_1} \right) = 0, \quad (37b)$$

$$\square \sigma - \frac{1}{2} \frac{\partial K_\sigma}{\partial \sigma} (\partial_\mu \sigma)^2 - \frac{1}{2} m_2^2 \tilde{f}_2^2 \sigma = 0. \quad (37c)$$

Variations with respect to the metrics $g_{1\mu\nu}$ and $g_{2\mu\nu}$ give the Einstein equations

$$R_{\mu\nu}(g_1) - \frac{1}{2} g_{1\mu\nu} R(g_1) = \frac{1}{m_1^2} T_{\mu\nu}^{(1)}, \quad (38a)$$

$$R_{\mu\nu}(g_2) - \frac{1}{2} g_{2\mu\nu} R(g_2) = \frac{1}{m_2^2} T_{\mu\nu}^{(2)}, \quad (38b)$$

where

$$\begin{aligned} T_{\mu\nu}^{(1)} = & \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} g_{1\mu\nu} (\partial_\rho \Phi)^2 + K_Z \partial_\mu Z \partial_\nu Z \\ & - \frac{K_Z}{2} g_{1\mu\nu} (\partial_\rho Z)^2 - g_{1\mu\nu} V_1(\Phi, Z) \\ & - 72m_1^4 \sinh^4 \left(\frac{\Phi}{\sqrt{6}m_1} \right) \sum_{n=0}^4 \dot{\beta}_n \frac{1}{\sqrt{-g_1}} \frac{\delta}{\delta g_1^{\mu\nu}} \\ & (\sqrt{-g_1} \tilde{\chi}_2^{-n/2} e_n(\sqrt{\Delta})), \end{aligned} \quad (39a)$$

$$\begin{aligned}
T_{\mu\nu}^{(2)} = & K_\sigma \partial_\mu \sigma \partial_\nu \sigma - \frac{K_s}{2} g_{2\mu\nu} (\partial_\rho \sigma)^2 - g_{2\mu\nu} V_2(\sigma) \\
& - 72m_1^4 \sinh^4 \left(\frac{\Phi}{\sqrt{6}m_1} \right) \sum_{n=0}^4 \dot{\beta}_n \frac{1}{\sqrt{-g_2}} \frac{\delta}{\delta g_2^{\mu\nu}} \\
& (\sqrt{-g_1} \tilde{\chi}_2^{-n/2} e_n(\sqrt{\Delta})). \quad (39b)
\end{aligned}$$

The remaining variations in the equations above, which have not yet been computed, can be expressed as

$$\begin{aligned}
& \sum_{n=0}^4 \dot{\beta}_n \frac{\delta(\sqrt{-g_1} \tilde{\chi}_2^{-n/2} e_n(\sqrt{\Delta}))}{\delta g_a^{\mu\nu}} \\
& = \sum_{n=0}^4 \dot{\beta}_n \left[\tilde{\chi}_2^{-n/2} \frac{\delta(\sqrt{-g_1} e_n(\sqrt{\Delta}))}{\delta g_a^{\mu\nu}} \right. \\
& \quad \left. - \frac{n}{2} \sqrt{-g_1} e_n(\sqrt{\Delta}) \tilde{\chi}_2^{-n/2-1} \frac{\delta \tilde{\chi}_2}{\delta g_a^{\mu\nu}} \right] \\
& = \sum_{n=0}^4 \dot{\beta}_n \tilde{\chi}_2^{-n/2} \frac{\delta(\sqrt{-g_1} e_n(\sqrt{\Delta}))}{\delta g_a^{\mu\nu}}, \quad (40)
\end{aligned}$$

where the second term on the right-hand side of the first line vanishes on-shell due to the equation of motion for $\tilde{\chi}_2$ (34). At this stage, Eq. (40) contains only the standard variations in the bigravity framework, given by

$$\begin{aligned}
\frac{-2}{\sqrt{-g_1}} \frac{\delta(\sqrt{-g_1} e_n(\sqrt{\Delta}))}{\delta g_1^{\mu\nu}} &= V_{\mu\nu}^{(n)} \quad \text{and} \\
\frac{-2}{\sqrt{-g_2}} \frac{\delta(\sqrt{-g_1} e_n(\sqrt{\Delta}))}{\delta g_2^{\mu\nu}} &= \tilde{V}_{\mu\nu}^{(4-n)}, \quad (41)
\end{aligned}$$

where

$$\begin{aligned}
V_{\mu\nu}^{(n)} &= g_{1\mu\lambda} Y_{(n)\nu}^\lambda \left(\sqrt{g_1^{-1} g_2} \right) \quad \text{and} \\
\tilde{V}_{\mu\nu}^{(n)} &= g_{2\mu\lambda} Y_{(n)\nu}^\lambda \left(\sqrt{g_2^{-1} g_1} \right), \quad (42)
\end{aligned}$$

with

$$Y_{(n)}(X) = \sum_{r=0}^n (-X)^{n-r} e_r(X). \quad (43)$$

As a result the energy momentum tensors (39) can be written as

$$\begin{aligned}
T_{\mu\nu}^{(1)} = & \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} g_{1\mu\nu} (\partial_\rho \Phi)^2 + K_Z \partial_\mu Z \partial_\nu Z \\
& - \frac{K_Z}{2} g_{1\mu\nu} (\partial_\rho Z)^2 - g_{1\mu\nu} V_1(\Phi, Z) \\
& + 36m_1^4 \sinh^4 \left(\frac{\Phi}{\sqrt{6}m_1} \right) \sum_{n=0}^3 \dot{\beta}_n \tilde{\chi}_2^{-n/2} V_{\mu\nu}^{(n)}, \quad (44a)
\end{aligned}$$

$$\begin{aligned}
T_{\mu\nu}^{(2)} = & K_\sigma \partial_\mu \sigma \partial_\nu \sigma - \frac{K_s}{2} g_{2\mu\nu} (\partial_\rho \sigma)^2 - g_{2\mu\nu} V_2(\sigma) \\
& + 36m_1^4 \sinh^4 \left(\frac{\Phi}{\sqrt{6}m_1} \right) \sum_{n=1}^4 \dot{\beta}_n \tilde{\chi}_2^{-n/2} \tilde{V}_{\mu\nu}^{(4-n)}. \quad (44b)
\end{aligned}$$

Closing this section we see that the equations of motion for the two gravity related pseudoscalars σ and ζ_1 are satisfied with $\sigma = 0$ and $\zeta_1 = 0$. The latter corresponds to $Z = +\infty$ through the transformation relation $Z = -m_1 \ln(\tilde{\zeta}_1 m_1 / 2(\phi^2 + m_1^2))$. A vanishing classical vacuum is also a solution for ϕ . Nevertheless, in our general framework of consideration, where the effective theory of gravity is treated classically, while matter fields are sensitive to quantum corrections, it would be reasonable to expect that quantum corrections involving ϕ could be important in the flat limit. The potential expressed in terms of ϕ is just a quartic potential with the role of the bare coupling played by β_0 , while the derivative interactions are suppressed for $\phi \ll m_1$. Note that ϕ could very well be a SM field or even the Higgs itself, interacting with the rest of matter. Dimensional transmutation via logarithmic corrections to the potential $\Delta V \sim \phi^4 \ln(\phi^2/\mu^2)$ could take place, thus, generating dynamically a scale $\langle \phi \rangle = v \neq 0$. We are going to make this assumption in the rest our considerations of the above model.

IV. BACKGROUND SOLUTIONS AND PARTICLE SPECTRUM

In this section, we shall consider a special class of solutions for the metric tensors, known as proportional solutions. Perturbing around these solutions we derive the corresponding particle spectrum which includes the standard graviton and a massive spin-2 particle.

A. Proportional solutions

Proportional solutions [87] by definition are those for which the two metrics are related by a constant proportionality factor

$$g_{2\mu\nu} = c^2 g_{1\mu\nu} = c^2 g_{\mu\nu}. \quad (45)$$

These solutions play a crucial role in simplifying the structure of the field equations⁸ and allow for analytic progress in understanding the vacuum structure and perturbative stability of the theory. Note that the requirement that both metrics should have a flat limit is easier to satisfy in the case of proportional metrics, in contrast to a case of independent metrics, which poses a more difficult issue to

⁸Note that for proportional backgrounds

$$e_n(\sqrt{\Delta}) = \binom{4}{n} c^n. \quad (46)$$

be studied. Given this relationship, the tensors $V_{\mu\nu}^{(n)}$ and $\tilde{V}_{\mu\nu}^{(n)}$, defined in (42), take the form

$$V_{\mu\nu}^{(n)} = g_{\mu\nu} \binom{3}{n} c^n \quad \text{and} \\ \tilde{V}_{\mu\nu}^{(n)} = g_{\mu\nu} \binom{3}{n} c^{2-n}, \quad \text{with } n = 0, 1, 2, 3. \quad (47)$$

These expressions show that, under proportional solutions, the interaction tensors retain a simple dependence on c , greatly simplifying the field equations.

These solutions can be valid in vacuum states, where the fields acquire their vacuum expectation values, or along specific directions in field space [44]. In the following, we analyze the behavior of these solutions in the vacuum. The equations of motion for the pseudoscalars are satisfied for $\sigma = 0$ and $\zeta_1 = 0$, while, as we have argued in the previous section, the scalar ϕ can in principle acquire a nonvanishing expectation value. The potential for Φ , expressed in terms of the original variable ϕ , arising only from the interaction term, for proportional backgrounds $g_{2\mu\nu} = c^2 g_{1\mu\nu}$ corresponds to a quartic potential

$$V_{\text{int}} = \phi^4 \sum_{n=0}^3 \dot{\beta}_n \bar{\chi}_2^{-n/2} e_n(\sqrt{\Delta}) \stackrel{(45)}{\Rightarrow} \\ V_{\text{int}} = \phi^4 \sum_{n=0}^4 \binom{4}{n} \dot{\beta}_n \bar{\chi}_2^{-n/2} c^n, \quad (48)$$

where $\bar{\chi}_2$ is a constant solution of the equation $\sum_{n=1}^4 \binom{4}{n} n \dot{\beta}_n c^n \bar{\chi}_2^{-n/2} = 0$. Quantum corrections due to the ϕ self-interaction and to interactions with matter fields, will necessarily generate logarithmic corrections $\Delta V \propto \phi^4 \ln(\phi^2/\mu^2)$, which are expected to break its classical scale invariance and lead to a nonvanishing expectation value $\langle \Phi \rangle = v_\Phi \neq 0$.

Assuming proportional backgrounds (45) the energy momentum tensors at the vacuum read

$$T_{\mu\nu}^{(1)} = -g_{\mu\nu} \frac{1}{4} m_1^4 f_1^2 + 36 m_1^4 \sinh^4 \left(\frac{v_\Phi}{\sqrt{6} m_1} \right) \\ \times \sum_{n=0}^3 \dot{\beta}_n \bar{\chi}_2^{-n/2} \binom{3}{n} c^n g_{\mu\nu} = -m_1^2 \Lambda_1 g_{\mu\nu}, \quad (49a)$$

$$T_{\mu\nu}^{(2)} = -g_{\mu\nu} \frac{1}{4} c^2 m_2^4 f_2^2 + 36 m_1^4 \sinh^4 \left(\frac{v_\Phi}{\sqrt{6} m_1} \right) \\ \times \sum_{n=1}^4 \dot{\beta}_n \bar{\chi}_2^{-n/2} \binom{3}{4-n} c^{n-2} g_{\mu\nu} \\ = -g_{\mu\nu} \frac{1}{4} c^2 m_2^4 f_2^2 = -\alpha^2 m_1^2 \Lambda_2 g_{\mu\nu}, \quad (49b)$$

where $\alpha = m_2/m_1$ represents the ratio of the gravitational masses in the two sectors. Note that the $\dot{\beta}_n$ -dependent term of $T_{\mu\nu}^{(2)}$ vanishes on-shell due to the equation of motion (34) for $\bar{\chi}_2$. This follows from the combinatorial identity $\binom{3}{4-n} = \frac{n}{4} \binom{4}{n}$.

A crucial property of the Einstein tensor, $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$, is that it remains invariant under uniform scaling of the metric as in (45). Consequently, the left-hand sides of the Einstein equations (38a) and (38b) are identical, leading to the proportionality of their corresponding energy-momentum tensors, that is $\alpha^2 T_{\mu\nu}^{(1)} = T_{\mu\nu}^{(2)}$. By redefining the parameters $\dot{\beta}_n$ as $\bar{\beta}_n = \dot{\beta}_n c^n \bar{\chi}_2^{-n/2}$ and using the proportionality of the energy-momentum tensors, we obtain the following parameter relation

$$\frac{1}{4} f_1^2 - 36 \sinh^4 \left(\frac{v_\Phi}{\sqrt{6} m_1} \right) (\bar{\beta}_0 + 3\bar{\beta}_1 + 3\bar{\beta}_2 + \bar{\beta}_3) = \frac{1}{4} \bar{\alpha}^2 f_2^2, \quad (50)$$

where $\bar{\alpha} = \alpha c$. Furthermore, the proportionality condition affects the equation of motion for $\bar{\chi}_2$, given by (35), which takes the form $\dot{\beta}_1 \bar{\chi}_2^{3/2} + 3c \dot{\beta}_2 \bar{\chi}_2 + 3c^2 \dot{\beta}_3 \bar{\chi}_2^{1/2} + c^3 \dot{\beta}_4 = 0$. This is a cubic equation for $\bar{\chi}_2^{1/2}$ that can be solved to express $\bar{\chi}_2$ as a function of the parameters, that is, $\bar{\chi}_2 = \bar{\chi}_2(\bar{\alpha}, \beta_1, \beta_2, \beta_3, \beta_4)$.

B. Linearization

Perturbation theory in bimetric gravity faces significant challenges due to the presence of the square root matrix in the interaction potential. This complication generally makes the study of fluctuations more intricate. Nevertheless, for proportional solutions, the structure of the equations simplifies considerably, allowing for a more tractable analysis and providing a simplified setting for analyzing fluctuations.

Linear perturbations around proportional backgrounds are studied by introducing the metric fluctuations $h_{\mu\nu}$ and $l_{\mu\nu}$ as deviations from the background metrics $g_{1\mu\nu}$ and $g_{2\mu\nu}$ respectively. Specifically, we define

$$g_{1\mu\nu} = g_{\mu\nu} + h_{\mu\nu} \quad \text{and} \quad g_{2\mu\nu} = c^2 g_{\mu\nu} + l_{\mu\nu}. \quad (51)$$

Since the theory generically leads to a massive and a massless spin-2 excitation, it is useful to construct linear combinations of $h_{\mu\nu}$ and $l_{\mu\nu}$ that diagonalize the mass spectrum. The canonically normalized mass eigenstates are then given by

$$G_{\mu\nu} = \frac{M_P}{(1 + \alpha^2 c^2)} (h_{\mu\nu} + \alpha^2 l_{\mu\nu}), \\ M_{\mu\nu} = \frac{\alpha M_P}{c(1 + \alpha^2 c^2)} (l_{\mu\nu} - c^2 h_{\mu\nu}), \quad (52)$$

where $G_{\mu\nu}$ corresponds to the massless spin-2 mode and $M_{\mu\nu}$ represents the massive eigenstate. The physical Planck mass, which governs gravitational interactions, is defined as⁹ $M_P = \sqrt{m_1^2 + c^2 m_2^2}$. These redefinitions ensure that the kinetic terms for the mass eigenstates are properly normalized, simplifying the analysis of their interactions. The linearized action for these perturbations takes the form

$$\begin{aligned} \mathcal{S} = \frac{1}{2} \int d^4x \sqrt{-g} & \left[\frac{1}{4} G_{\mu\nu} \mathcal{E}^{\mu\nu\rho\sigma} G_{\rho\sigma} + \frac{1}{4} M_{\mu\nu} \mathcal{E}^{\mu\nu\rho\sigma} M_{\rho\sigma} \right. \\ & + \frac{\Lambda_1}{2} \left(G_{\mu\nu} G^{\mu\nu} - \frac{1}{2} G^2 \right) + \frac{\Lambda_1}{2} \left(M_{\mu\nu} M^{\mu\nu} - \frac{1}{2} M^2 \right) \\ & \left. - \frac{m_{\text{FP}}^2}{4} (M_{\mu\nu} M^{\mu\nu} - M^2) - M_P \Lambda_1 G - 2M_P^2 \Lambda_1 \right], \quad (53) \end{aligned}$$

where we have used the equality of cosmological constants¹⁰ $\Lambda_1 = \Lambda_2$. The structure of this action highlights the presence of two independent sectors: one corresponding to a massless graviton and another to a massive spin-2 field. The massive sector exhibits a standard Fierz-Pauli structure with mass term m_{FP}^2 , which ensures that the theory propagates the correct number of degrees of freedom without introducing ghostlike instabilities. The Fierz-Pauli mass of the massive spin-2 field is explicitly given by

$$m_{\text{FP}}^2 = -\frac{36M_P^2}{\bar{\alpha}^2} \sinh^4 \left(\frac{v_\Phi \sqrt{1 + \bar{\alpha}^2}}{\sqrt{6}M_P} \right) (\bar{\beta}_1 + 2\bar{\beta}_2 + \bar{\beta}_3). \quad (54)$$

This mass term arises from the interaction potential between the two metrics and plays a crucial role in determining the phenomenology of the massive spin-2 mode.

V. MASSIVE SPIN-2 PHENOMENOLOGY

In this section, we shall examine the Fierz-Pauli mass (54) and see how it relates to the model parameters, specifically v_Φ , the ratio of the gravitational scales $\bar{\alpha}$, and the β parameters. We shall also investigate whether the massive spin-2 particle can constitute DM and, using previously established constraints, derive limits on the vacuum expectation value v_Φ . We shall focus on the so-called minimal choice of parameters, where the parameters β_2 and β_3 are set to zero. This choice simplifies the analysis while preserving the essential features of the theory. In this scenario, solving the equation of motion for $\bar{\chi}_2$, we obtain

⁹We shall assume that the coupling to matter is realized exclusively through $g_{1\mu\nu}$. In this case

$$T_{\mu\nu}^{(m)} g_1^{\mu\nu} = T_{\mu\nu}^{(m)} g^{\mu\nu} + \frac{1}{M_P} G^{\mu\nu} T_{\mu\nu}^{(m)} - \frac{\bar{\alpha}}{M_P} M^{\mu\nu} T_{\mu\nu}^{(m)}.$$

¹⁰Note that $\Lambda_1 = m_1^2 (\frac{1}{4} f_1^2 - 36 \sinh^4 (\frac{v_\Phi}{\sqrt{6}m_1})) (\bar{\beta}_0 + 3\bar{\beta}_1 + 3\bar{\beta}_2 + \bar{\beta}_3)$.

$$\bar{\chi}_2^{1/2} = \frac{\bar{\alpha} M_P}{\sqrt{1 + \bar{\alpha}^2}} (-\beta_4/\beta_1)^{1/3}, \quad (55)$$

which leads to the rescaled $\bar{\beta}$ parameters

$$\bar{\beta}_1 = \frac{\beta_1}{(-\beta_4/\beta_1)^{1/3}} \quad \text{and} \quad \bar{\beta}_4 = \frac{\beta_4}{(-\beta_4/\beta_1)^{4/3}}. \quad (56)$$

Note that even in the most complex case, where none of the β parameters are zero, the $\bar{\beta}$ parameters always depend solely on the β parameters and not on any other parameters of our model. To explore the behavior of the Fierz-Pauli mass, we take the β parameters to be of order $\mathcal{O}(1)$, and analyze how the mass evolves for different values of the vev v_Φ , ranging between $\mathcal{O}(M_W)$ and $\mathcal{O}(M_P)$. Taking the specific values $\beta_1 = -1 = -\beta_4$, the Fierz-Pauli mass becomes

$$m_{\text{FP}} = \frac{6M_P}{\bar{\alpha}} \sinh^2 \left(\frac{v_\Phi \sqrt{1 + \bar{\alpha}^2}}{\sqrt{6}M_P} \right). \quad (57)$$

In Fig. 1 we show the Fierz-Pauli mass as function of v_Φ (left) and $\bar{\alpha}$ (right), computed using Eq. (57). It is clear that the Fierz-Pauli mass can vary significantly depending on the values of v_Φ and $\bar{\alpha}$. In the following, we discuss the limiting cases of ultralight and superheavy massive spin-2 particles as well as the case of the spin-2 particle being dark matter [88–98].

A. Ultralight massive spin-2

For small vevs, i.e., $v_\Phi \ll M_P$, assuming $\bar{\alpha} < 1$, the Fierz-Pauli mass formula simplifies to the approximate form $m_{\text{FP}} \simeq v_\Phi^2 / (\bar{\alpha} M_P)$, which shows that it scales quadratically with v_Φ , suppressed by $\bar{\alpha} M_P$. If we take v_Φ to be of the order of the electroweak scale, $v_\Phi \sim M_W$, the corresponding mass is estimated to $m_{\text{FP}} \simeq 3 \times 10^{-6} \bar{\alpha}^{-1} \text{ eV}$, indicating an ultralight massive spin-2 particle in the limiting case where $\bar{\alpha} \sim \mathcal{O}(1)$. As $\bar{\alpha}$ decreases, the Fierz-Pauli mass correspondingly increases, highlighting its inverse dependence on $\bar{\alpha}$.

B. Superheavy massive spin-2

Restricting our analysis to the case where $\bar{\alpha} < 1$ and considering a vev near the Planck scale, we find a Fierz-Pauli mass $m_{\text{FP}} \sim M_P / \bar{\alpha}$, which is always super-Planckian.¹¹ This excludes such high values for the vev v_Φ . However, if we consider lower vev values $v_\Phi \sim 10^{-\nu} M_P$, as for example at the grand unified theory (GUT) scale ($\nu \sim 2$ or 3), we obtain $m_{\text{FP}} \sim 10^{-2\nu} M_P / \bar{\alpha}$. For m_{FP} to remain sub-Planckian, we have to require $\bar{\alpha} > 10^{-2\nu}$. More generally, since we cannot have masses exceeding the Planck scale, we must have $v_\Phi \lesssim M_P \sqrt{\bar{\alpha}}$.

¹¹See also the orange line in the right panel of Fig. 1.

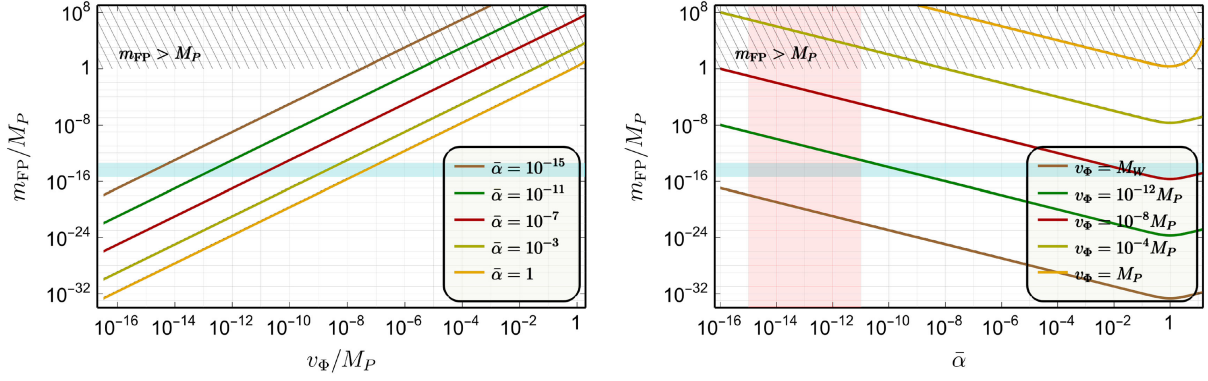


FIG. 1. The Fierz-Pauli mass, m_{FP} , as function of the v_ϕ (left) and $\tilde{\alpha}$ (right), with parameters $\beta_1 = -1 = -\beta_4$. The cyan and red shaded regions indicate the constraints on m_{FP} and $\tilde{\alpha}$ derived in [88], under the assumption that the massive spin-2 particle constitutes DM.

The above distinction between the ultralight and super-heavy regimes is based on our assumption that the β parameters are of order $\mathcal{O}(1)$. Since these parameters act as multiplicative factors in the Fierz-Pauli mass, they can significantly alter its value if we assume that β is either very large or very small. This would allow for a smooth transition between the ultralight and superheavy regimes by appropriately tuning the β parameters. In general, there is considerable flexibility in setting the mass of the spin-2 particle, allowing us to satisfy various constraints, such as those arising when the massive spin-2 particle is identified as a candidate for dark matter [88,91].

C. Massive spin-2 as dark matter

It has been well established that the massive spin-2 particles predicted by bimetric gravity could serve as viable DM candidates due to their extremely weak interactions with SM particles. The parameter space for massive spin-2 DM can be constrained by considering the most stringent lifetime bounds for decaying DM, limits imposed by DM overproduction, and the exclusion of regions where perturbative analysis is not applicable. By incorporating these constraints, the authors of [88] found that the allowed range for the Fierz-Pauli mass is significantly restricted to $1 \text{ TeV} \lesssim m_{\text{FP}} \lesssim 100 \text{ TeV}$, while the gravitational mass associated with the second metric lies within the range is $1 \text{ TeV} \lesssim m_2 \lesssim 10^4 \text{ TeV}$. These values, in turn, correspond to the parameter range $10^{-15} \lesssim \tilde{\alpha} \lesssim 10^{-11}$. The regions satisfying these constraints are indicated by the shaded areas in Fig. 1. Applying these bounds, the vacuum expectation value of the scalar field is determined to be

$$1.5 \times 10^3 \text{ GeV} \lesssim v_\phi \lesssim 1.5 \times 10^6 \text{ GeV}. \quad (58)$$

As mentioned earlier, this result holds under the assumption that the parameters β are of order $\mathcal{O}(1)$. If the β parameters take on smaller (larger) values, they lead to larger (smaller) vevs. Note that if we relax the assumption that the β 's are of $\mathcal{O}(1)$, the massive spin-2 particles can constitute DM with

the characteristic vev of the SM Higgs boson $v_H \sim 246 \text{ GeV}$ for a wide range $10^3 \lesssim \beta \lesssim 10^{15}$.

To conclude this section, let us have a closer look at the arising cosmological constants. They are given by

$$\Lambda_1 = \frac{f_1^2 M_P^2}{4(1 + \tilde{\alpha}^2)} - \frac{36 M_P^2}{1 + \tilde{\alpha}^2} \sinh^4 \left(\frac{v_\phi \sqrt{1 + \tilde{\alpha}^2}}{\sqrt{6} M_P} \right) (\beta_0 - 3), \quad (59a)$$

$$\Lambda_2 = \frac{\tilde{\alpha}^2 f_2^2 M_P^2}{4(1 + \tilde{\alpha}^2)}, \quad (59b)$$

where we have used the relation $\tilde{\beta}_0 + 3\tilde{\beta}_1 + 3\tilde{\beta}_2 + \tilde{\beta}_3 = \beta_0 - 3$ for $\beta_1 = -1$. In order to satisfy the observational constraint $\Lambda_1 = \Lambda_2 \simeq 10^{-120} M_P^2$ we can set $\beta_0 = 3$ to cancel the dominant contribution to the cosmological constant, arising from the nonzero vev. Then, the gravitational contributions, which have to satisfy the constraint $f_1 = \tilde{\alpha} f_2$, can meet the observational constraint for exceedingly small values of the parameters $f_1 = \tilde{\alpha} f_2 \sim 10^{-60}$. Nevertheless, additional perturbative and nonperturbative contributions should be expected that invalidate these arrangements. For instance, the contribution from the SM, mostly affecting Λ_1 , is expected to be of $\mathcal{O}(M_W^4)$. Thus, a milder, equally agnostic, assumption could be made that the above gravitational contributions are at least of that order, leading to $f \sim \mathcal{O}(M_W^2/M_P^2) \approx 10^{-32}$. Note that, assuming that the couplings $\tilde{f}$ are of the same order as the f -couplings, the smallness of the latter reflects directly on the smallness of the axion Z , σ masses, being $M_1 \sim$

$$\frac{\tilde{f}_1 M_P}{\sqrt{1 + \tilde{\alpha}^2}} \text{ and } M_2 \sim \frac{\tilde{\alpha} \tilde{f}_2 M_P}{\sqrt{1 + \tilde{\alpha}^2}}.$$

VI. SUMMARY AND CONCLUSIONS

We started by considering the ghost-free bimetric theory of gravity, characterized by two independent metric tensors accompanied by two independent gravitational mass scales.

Motivated by the use of classical scale invariance to face large scale hierarchies in particle physics, we imposed on bimetric gravity local Weyl invariance. We restricted the gravitational action to include at most quadratic terms, avoiding any unphysical degrees of freedom. The theory was coupled to a scalar field that modified the standard bigravity interaction potential by a quartic scalar field factor, thus, rendering it Weyl invariant.

Furthermore, local Weyl invariance was realized in the framework of Einstein-Cartan gravity, with the role of a gauge vector taken up by a corresponding torsion vector and, thus, not introducing any additional dynamical degrees of freedom. The action, depending only on the dimensionless gravitational couplings and on the dimensionless parameters of the potential and consisting of quadratic terms of the Ricci scalars and the Holst invariants, was written in terms of axionlike pseudoscalars and two auxiliary scalar fields, the latter being exchanged for the two independent gravitational mass scales m_1 and m_2 .

The scalar field, being massless at the classical level, is expected to receive radiative corrections either from its self-interaction or from possible interactions with the rest of matter. These corrections, unavoidably leading to the breaking of scale invariance at the quantum level, are likely to generate a nonvanishing expectation value through logarithms $\sim \phi^4 \ln(\phi^2/\mu^2)$ in analogy to standard dimensional transmutation. Assuming this, we have considered metric perturbations around proportional backgrounds $g_{2\mu\nu} = c^2 g_{1\mu\nu}$ and analyzed the resulting linearized theory. Consistency leads to a proportionality relation between the corresponding energy-momentum tensors that results in a relation among parameters. The spectrum of the theory, apart from the standard massless graviton, includes a pair of axionlike pseudoscalars, the scalar and the additional spin-2 state having the Fierz-Pauli mass. The latter depends on the vacuum expectation value of the scalar field, on the ratio

$\bar{\alpha} = c(m_2/m_1)$ and on the dimensionless β parameters of the interaction potential.

Having established the general structure of the spectrum and the parametric constraints of the model, we proceeded to analyze the phenomenology of the model focusing on the massive spin-2 state. Assuming that the initial β parameters of the standard bigravity potential are of order $\mathcal{O}(1)$, we found that the mass of the spin-2 state could range from ultralight values, $\mathcal{O}(10^{-6} \text{ eV})$, to superheavy approaching the Planck scale. Finally, the massive spin-2 particle can be identified as dark matter, provided that the vacuum expectation value of the scalar field lies within the range $\mathcal{O}(10^3 \text{ GeV}) \lesssim v_\phi \lesssim \mathcal{O}(10^6 \text{ GeV})$, although this range could shift for larger or smaller values that include the SM Higgs vev if we relax the assumption that the β parameters are of $\mathcal{O}(1)$.

As for future work, it would be interesting to perform a thorough analysis of cosmological backgrounds in our setup. In particular, it would be important to investigate the behavior of perturbations around cosmological solutions and assess their stability conditions [99–101], as well as to study the impact of the Fierz–Pauli mass on late-time cosmology, including possible implications for dark energy [48,102,103].

ACKNOWLEDGMENTS

I. D. G. thanks Hardi Veermäe for useful discussions. The work of I. D. G. was supported by the Estonian Research Council Grants MOB3JD1202, RVTT3, RVTT7, and by the CoE program TK202 “Fundamental Universe.”

DATA AVAILABILITY

No data were created or analyzed in this study.

-
- [1] M. Fierz and W. Pauli, On relativistic wave equations for particles of arbitrary spin in an electromagnetic field, *Proc. R. Soc. A* **173**, 211 (1939).
 - [2] H. van Dam and M. J. G. Veltman, Massive and massless Yang-Mills and gravitational fields, *Nucl. Phys.* **B22**, 397 (1970).
 - [3] V. I. Zakharov, Linearized gravitation theory and the graviton mass, *JETP Lett.* **12**, 312 (1970).
 - [4] A. I. Vainshtein, To the problem of nonvanishing gravitation mass, *Phys. Lett.* **39B**, 393 (1972).
 - [5] D. G. Boulware and S. Deser, Can gravitation have a finite range?, *Phys. Rev. D* **6**, 3368 (1972).
 - [6] S. F. Hassan and R. A. Rosen, Bimetric gravity from ghost-free massive gravity, *J. High Energy Phys.* **02** (2012) 126.
 - [7] M. Shaposhnikov and D. Zenhausern, Scale invariance, unimodular gravity and dark energy, *Phys. Lett. B* **671**, 187 (2009).
 - [8] M. Shaposhnikov and D. Zenhausern, Quantum scale invariance, cosmological constant and hierarchy problem, *Phys. Lett. B* **671**, 162 (2009).
 - [9] D. Blas, M. Shaposhnikov, and D. Zenhausern, Scale-invariant alternatives to general relativity, *Phys. Rev. D* **84**, 044001 (2011).
 - [10] J. Garcia-Bellido, J. Rubio, M. Shaposhnikov, and D. Zenhausern, Higgs-Dilaton cosmology: From the early to the late Universe, *Phys. Rev. D* **84**, 123504 (2011).
 - [11] F. Bezrukov, G. K. Karananas, J. Rubio, and M. Shaposhnikov, Higgs-Dilaton cosmology: An

- effective field theory approach, *Phys. Rev. D* **87**, 096001 (2013).
- [12] V. V. Khoze, Inflation and dark matter in the Higgs portal of classically scale invariant standard model, *J. High Energy Phys.* **11** (2013) 215.
- [13] A. Karam and K. Tamvakis, Dark matter and neutrino masses from a scale-invariant multi-Higgs portal, *Phys. Rev. D* **92**, 075010 (2015).
- [14] K. Kannike, A. Racioppi, and M. Raidal, Embedding inflation into the standard model—more evidence for classical scale invariance, *J. High Energy Phys.* **06** (2014) 154.
- [15] C. Csaki, N. Kaloper, J. Serra, and J. Terning, Inflation from broken scale invariance, *Phys. Rev. Lett.* **113**, 161302 (2014).
- [16] K. Kannike, G. Hütsi, L. Pizza, A. Racioppi, M. Raidal, A. Salvio, and A. Strumia, Dynamically induced Planck scale and inflation, *J. High Energy Phys.* **05** (2015) 065.
- [17] M. Rinaldi and L. Vanzo, Inflation and reheating in theories with spontaneous scale invariance symmetry breaking, *Phys. Rev. D* **94**, 024009 (2016).
- [18] P. G. Ferreira, C. T. Hill, and G. G. Ross, Scale-independent inflation and hierarchy generation, *Phys. Lett. B* **763**, 174 (2016).
- [19] L. Marzola and A. Racioppi, Minimal but non-minimal inflation and electroweak symmetry breaking, *J. Cosmol. Astropart. Phys.* **10** (2016) 010.
- [20] G. K. Karananas and J. Rubio, On the geometrical interpretation of scale-invariant models of inflation, *Phys. Lett. B* **761**, 223 (2016).
- [21] K. Kannike, M. Raidal, C. Spethmann, and H. Veermäe, The evolving Planck mass in classically scale-invariant theories, *J. High Energy Phys.* **04** (2017) 026.
- [22] P. G. Ferreira, C. T. Hill, and G. G. Ross, Weyl current, scale-invariant inflation and Planck scale generation, *Phys. Rev. D* **95**, 043507 (2017).
- [23] D. M. Ghilencea, Z. Lalak, and P. Olszewski, Standard model with spontaneously broken quantum scale invariance, *Phys. Rev. D* **96**, 055034 (2017).
- [24] J. Rubio and C. Wetterich, Emergent scale symmetry: Connecting inflation and dark energy, *Phys. Rev. D* **96**, 063509 (2017).
- [25] D. Benisty and E. I. Guendelman, Two scalar fields inflation from scale-invariant gravity with modified measure, *Classical Quantum Gravity* **36**, 095001 (2019).
- [26] P. G. Ferreira, C. T. Hill, J. Noller, and G. G. Ross, Inflation in a scale invariant universe, *Phys. Rev. D* **97**, 123516 (2018).
- [27] A. Karam, T. Pappas, and K. Tamvakis, Nonminimal Coleman–Weinberg inflation with an R^2 term, *J. Cosmol. Astropart. Phys.* **02** (2019) 006.
- [28] P. G. Ferreira, C. T. Hill, and G. G. Ross, Inertial spontaneous symmetry breaking and quantum scale invariance, *Phys. Rev. D* **98**, 116012 (2018).
- [29] I. D. Gialamas, A. Karam, and A. Racioppi, Dynamically induced Planck scale and inflation in the Palatini formulation, *J. Cosmol. Astropart. Phys.* **11** (2020) 014.
- [30] M. Shaposhnikov and A. Shkerin, Gravity, scale invariance and the hierarchy problem, *J. High Energy Phys.* **10** (2018) 024.
- [31] D. Iosifidis and T. Koivisto, Scale transformations in metric-affine geometry, *Universe* **5**, 82 (2019).
- [32] S. Casas, G. K. Karananas, M. Pauly, and J. Rubio, Scale-invariant alternatives to general relativity. III. The inflation-dark energy connection, *Phys. Rev. D* **99**, 063512 (2019).
- [33] G. K. Karananas, M. Shaposhnikov, A. Shkerin, and S. Zell, Scale and Weyl invariance in Einstein–Cartan gravity, *Phys. Rev. D* **104**, 124014 (2021).
- [34] I. D. Gialamas, A. Karam, T. D. Pappas, and V. C. Spanos, Scale-invariant quadratic gravity and inflation in the Palatini formalism, *Phys. Rev. D* **104**, 023521 (2021).
- [35] C. Cecchini, M. De Angelis, W. Giarè, M. Rinaldi, and S. Vagnozzi, Testing scale-invariant inflation against cosmological data, *J. Cosmol. Astropart. Phys.* **07** (2024) 058.
- [36] R. Utiyama, Invariant theoretical interpretation of interaction, *Phys. Rev.* **101**, 1597 (1956).
- [37] T. W. B. Kibble, Lorentz invariance and the gravitational field, *J. Math. Phys. (N.Y.)* **2**, 212 (1961).
- [38] E. Cartan, Sur les variétés à connexion affine et la théorie de la relativité généralisée. (première partie), *Ann. Sci. Ecole Norm. Sup.* **40**, 325 (1923).
- [39] E. Cartan, Sur les variétés à connexion affine et la théorie de la relativité généralisée. (première partie) (Suite), *Ann. Sci. Ecole Norm. Sup.* **41**, 1 (1924).
- [40] F. W. Hehl, P. Von Der Heyde, G. D. Kerlick, and J. M. Nester, General relativity with spin and torsion: Foundations and prospects, *Rev. Mod. Phys.* **48**, 393 (1976).
- [41] A. Schmidt-May and M. von Strauss, Recent developments in bimetric theory, *J. Phys. A* **49**, 183001 (2016).
- [42] I. D. Gialamas and K. Tamvakis, Bimetric-affine quadratic gravity, *Phys. Rev. D* **107**, 104012 (2023).
- [43] M. Rahman, A. A. Sen, and S. S. Bohra, Traversable wormholes in bimetric gravity, *Phys. Rev. D* **108**, 104008 (2023).
- [44] I. D. Gialamas and K. Tamvakis, Bimetric Starobinsky model, *Phys. Rev. D* **108**, 104023 (2023).
- [45] W.-Y. Hu, Q.-Y. Wang, Y.-Q. Ma, and Y. Tang, Gravitational waves from preheating in inflation with Weyl symmetry, *Phys. Rev. D* **109**, 083542 (2024).
- [46] I. D. Gialamas and K. Tamvakis, On the absence of ghosts in quadratic bigravity, *J. Cosmol. Astropart. Phys.* **03** (2024) 016.
- [47] T. Inagaki and N. Yoshioka, Nonthermal particle production in Einstein–Cartan gravity with modified Holst term and nonminimal couplings, *Phys. Rev. D* **110**, 103537 (2024).
- [48] S. Dwivedi and M. Högas, 2D BAO vs. 3D BAO: Solving the Hubble tension with bimetric cosmology, *Universe* **10**, 406 (2024).
- [49] A. Edery and Y. Nakayama, Restricted Weyl invariance in four-dimensional curved spacetime, *Phys. Rev. D* **90**, 043007 (2014).
- [50] A. Edery and Y. Nakayama, Generating Einstein gravity, cosmological constant and Higgs mass from restricted Weyl invariance, *Mod. Phys. Lett. A* **30**, 1550152 (2015).
- [51] D. M. Ghilencea, Spontaneous breaking of Weyl quadratic gravity to Einstein action and Higgs potential, *J. High Energy Phys.* **03** (2019) 049.

- [52] D. M. Ghilencea, Palatini quadratic gravity: Spontaneous breaking of gauged scale symmetry and inflation, *Eur. Phys. J. C* **80** (4, 2020) 1147.
- [53] D. M. Ghilencea and T. Harko, Cosmological evolution in Weyl conformal geometry, [arXiv:2110.07056](#).
- [54] D. M. Ghilencea, Standard model in Weyl conformal geometry, *Eur. Phys. J. C* **82**, 23 (2022).
- [55] Y. Shtanov, Electroweak symmetry breaking by gravity, *J. High Energy Phys.* **02** (2024) 221.
- [56] Q.-Y. Wang, Y. Tang, and Y.-L. Wu, Dark matter production in Weyl R2 inflation, *Phys. Rev. D* **106**, 023502 (2022).
- [57] J.-Z. Yang, S. Shahidi, and T. Harko, Black hole solutions in the quadratic Weyl conformal geometric theory of gravity, *Eur. Phys. J. C* **82**, 1171 (2022).
- [58] P. Burikham, T. Harko, K. Pimsamarn, and S. Shahidi, Dark matter as a Weyl geometric effect, *Phys. Rev. D* **107**, 064008 (2023).
- [59] C. Condeescu, D. M. Ghilencea, and A. Micu, Weyl quadratic gravity as a gauge theory and non-metricity vs torsion duality, *Eur. Phys. J. C* **84**, 292 (2024).
- [60] G. K. Karananas, M. Shaposhnikov, and S. Zell, Weyl-invariant Einstein-Cartan gravity: Unifying the strong *CP* and hierarchy puzzles, *J. High Energy Phys.* **11** (2024) 146.
- [61] T. Harko and S. Shahidi, Cosmological implications of the Weyl geometric gravity theory, *Eur. Phys. J. C* **84**, 509 (2024).
- [62] C. Condeescu and A. Micu, The gauge theory of Weyl group and its interpretation as Weyl quadratic gravity, *Classical Quantum Gravity* **42**, 065011 (2025).
- [63] D. Glavan, R. Noris, and T. Zlosnik, Critical reassessment of the restricted Weyl symmetry, *Phys. Rev. D* **110**, 125011 (2024).
- [64] S. Holst, Barbero's Hamiltonian derived from a generalized Hilbert-Palatini action, *Phys. Rev. D* **53**, 5966 (1996).
- [65] I. D. Gialamas and K. Tamvakis, Inflation in Weyl-invariant Einstein-Cartan gravity, *Phys. Rev. D* **111**, 044007 (2025).
- [66] G. K. Karananas, Geometrical origin of inflation in Weyl-invariant Einstein-Cartan gravity, *Phys. Lett. B* **862**, 139343 (2025).
- [67] W. Barker and S. Zell, Consistent particle physics in metric-affine gravity from extended projective symmetry, [arXiv:2402.14917](#).
- [68] D. Iosifidis, The full quadratic metric-affine gravity (including parity odd terms): Exact solutions for the affine-connection, *Classical Quantum Gravity* **39**, 095002 (2022).
- [69] C. Rigouzzo and S. Zell, Coupling metric-affine gravity to a Higgs-like scalar field, *Phys. Rev. D* **106**, 024015 (2022).
- [70] C. Rigouzzo and S. Zell, Coupling metric-affine gravity to the standard model and dark matter fermions, *Phys. Rev. D* **108**, 124067 (2023).
- [71] M. Langvik, J.-M. Ojanperä, S. Raatikainen, and S. Rasanen, Higgs inflation with the Holst and the Nieh-Yan term, *Phys. Rev. D* **103**, 083514 (2021).
- [72] M. Shaposhnikov, A. Shkerin, I. Timiryasov, and S. Zell, Higgs inflation in Einstein-Cartan gravity, *J. Cosmol. Astropart. Phys.* **02** (2021) 008; **10** (2021) E01(E).
- [73] A. Salvio, Inflating and reheating the Universe with an independent affine connection, *Phys. Rev. D* **106**, 103510 (2022).
- [74] I. D. Gialamas and K. Tamvakis, Inflation in metric-affine quadratic gravity, *J. Cosmol. Astropart. Phys.* **03** (2023) 042.
- [75] A. Di Marco, E. Orazi, and G. Pradisi, Einstein-Cartan pseudoscalar inflation, *Eur. Phys. J. C* **84**, 146 (2024).
- [76] M. He, M. Hong, and K. Mukaida, Starobinsky inflation and beyond in Einstein-Cartan gravity, *J. Cosmol. Astropart. Phys.* **05** (2024) 107.
- [77] A. Racioppi and A. Salvio, Natural metric-affine inflation, *J. Cosmol. Astropart. Phys.* **06** (2024) 033.
- [78] A. Racioppi, ξ -attractors in metric-affine gravity, *J. Cosmol. Astropart. Phys.* **04** (2025) 084.
- [79] I. D. Gialamas and A. Racioppi, Symmetry-breaking inflation in non-minimal metric-affine gravity, *J. Cosmol. Astropart. Phys.* **05** (2025) 072.
- [80] T. Katsoulas and K. Tamvakis, General Einstein-Cartan quadratic gravity with derivative couplings, *J. Cosmol. Astropart. Phys.* **06** (2025) 022.
- [81] J. Klusoň, S. Nojiri, and S. D. Odintsov, New proposal for non-linear ghost-free massive $F(R)$ gravity: Cosmic acceleration and Hamiltonian analysis, *Phys. Lett. B* **726**, 918 (2013).
- [82] S. Nojiri, S. D. Odintsov, and N. Shirai, Variety of cosmic acceleration models from massive $F(R)$ bigravity, *J. Cosmol. Astropart. Phys.* **05** (2013) 020.
- [83] S. F. Hassan and R. A. Rosen, Confirmation of the secondary constraint and absence of ghost in massive gravity and bimetric gravity, *J. High Energy Phys.* **04** (2012) 123.
- [84] C. de Rham, G. Gabadadze, and A. J. Tolley, Resummation of massive gravity, *Phys. Rev. Lett.* **106**, 231101 (2011).
- [85] S. F. Hassan and R. A. Rosen, Resolving the ghost problem in non-linear massive gravity, *Phys. Rev. Lett.* **108**, 041101 (2012).
- [86] K. Wood, A new look at multi-gravity and dimensional deconstruction, [arXiv:2501.16442](#).
- [87] S. F. Hassan, A. Schmidt-May, and M. von Strauss, On consistent theories of massive spin-2 fields coupled to gravity, *J. High Energy Phys.* **05** (2013) 086.
- [88] E. Babichev, L. Marzola, M. Raidal, A. Schmidt-May, F. Urban, H. Veermäe, and M. von Strauss, Heavy spin-2 dark matter, *J. Cosmol. Astropart. Phys.* **09** (2016) 016.
- [89] K. Aoki and K.-i. Maeda, Dark matter in ghost-free bigravity theory: From a galaxy scale to the universe, *Phys. Rev. D* **90**, 124089 (2014).
- [90] K. Aoki and S. Mukohyama, Massive gravitons as dark matter and gravitational waves, *Phys. Rev. D* **94**, 024001 (2016).
- [91] E. Babichev, L. Marzola, M. Raidal, A. Schmidt-May, F. Urban, H. Veermäe, and M. von Strauss, Bigravitational origin of dark matter, *Phys. Rev. D* **94**, 084055 (2016).
- [92] L. Marzola, M. Raidal, and F. R. Urban, Oscillating spin-2 dark matter, *Phys. Rev. D* **97**, 024010 (2018).

- [93] Y. Manita, K. Aoki, T. Fujita, and S. Mukohyama, Spin-2 dark matter from anisotropic Universe in bigravity, [Phys. Rev. D **107**, 104007 \(2023\)](#).
- [94] E. W. Kolb, S. Ling, A. J. Long, and R. A. Rosen, Cosmological gravitational particle production of massive spin-2 particles, [J. High Energy Phys. **05** \(2023\) 181](#).
- [95] Q. Chen and S.-Y. Zhou, Bigravity portal dark matter, [Phys. Rev. D **109**, 095035 \(2024\)](#).
- [96] R.-G. Cai, J.-R. Zhang, and Y.-L. Zhang, Angular correlation and deformed Hellings-Downs curve from spin-2 ultralight dark matter, [Phys. Rev. D **110**, 044052 \(2024\)](#).
- [97] D. Blas, J. Carlton, and C. McCabe, Massive graviton dark matter searches with long-baseline atom interferometers, [Phys. Rev. D **111**, 115020 \(2025\)](#).
- [98] J.-R. Zhang, J. Chen, H.-S. Jiao, R.-G. Cai, and Y.-L. Zhang, Probing spin-2 ultralight dark matter with space-based gravitational wave detectors in millihertz, [Phys. Rev. D **112**, 064030 \(2025\)](#).
- [99] M. S. Volkov, Cosmological solutions with massive gravitons in the bigravity theory, [J. High Energy Phys. **01** \(2012\) 035](#).
- [100] F. Koennig, Y. Akrami, L. Amendola, M. Motta, and A. R. Solomon, Stable and unstable cosmological models in bimetric massive gravity, [Phys. Rev. D **90**, 124014 \(2014\)](#).
- [101] M. Lagos and P. G. Ferreira, Cosmological perturbations in massive bigravity, [J. Cosmol. Astropart. Phys. **12** \(2014\) 026](#).
- [102] J. Smirnov, Dynamical dark energy emerges from massive gravity, [arXiv:2505.03870](#).
- [103] M. Högas and E. Mörtzell, Bimetric gravity improves the fit to DESI BAO and eases the Hubble tension, [arXiv: 2507.03743](#).