

Antiflatness and nonlocal nonstabilizerness in two-particle scattering processes

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Nonlocal nonstabilizerness (NL NS) and antiflatness provide a measure of the quantum complexity in the wave function of a physical system. Supported by entanglement, they cannot be removed by local unitary operations, thus providing basis-independent measures, and sufficiently large values underpin the need for quantum computers in order to perform precise simulations of the system at scale. Towards a better understanding of the quantum-complexity generation by fundamental interactions, the building blocks of many-body systems, we consider nonlocal nonstabilizerness and antiflatness in two-particle scattering processes, specifically focusing on low-energy nucleon-nucleon scattering and high-energy Møller scattering. We find that the nonlocal nonstabilizerness induced in both interactions is four times the antiflatness (which is found to be true for any two-qubit wave function), and verify the relation between the Clifford-averaged antiflatness and total nonstabilizerness. For these processes, the antiflatness is a more experimentally accessible quantity as it can be determined from one of the final-state particles, and does not require spin correlations. While the Møller experiment at the Thomas Jefferson National Accelerator Facility does not include final-state spin measurements, the results presented here may add motivation to consider their future inclusion.

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I. INTRODUCTION

Understanding the complexity of quantum many-body systems is key to progress in an array of research and technology domains, including in accelerating future simulations using quantum computers. To that aim, techniques quantifying the quantum complexity, and hence classical computational hardness of simulating such systems, both modest-sized and at scale, continue to improve, driven in part by challenges facing classical computation and the development of quantum computers. One challenging area for classical simulations, in general, is predicting the properties and dynamics of matter under extreme

conditions. They are plagued by sign problems, both in finding low-lying states at finite density and in their real-time evolution. Understanding how various aspects of complexity are generated by fundamental forces, and how they evolve in few-body and many-body environments is key towards better understanding the role of quantum information in these physical phenomena and designing resource-efficient simulation strategies.

The role of entanglement—which characterizes one aspect of quantum complexity—has been investigated in various areas of high-energy physics (HEP) and nuclear physics (NP). For example, building on the work of Refs. [1,2] which identified maximum entanglement in Standard Model scattering processes, the suppression of fluctuations in entanglement from the S-matrix has been connected to the emergence of global symmetries beyond those explicit in the Lagrange density [3], and further explored in Refs. [4–8]. These are consistent with the results obtained from the large- N_c limit of QCD [9], and extend to larger SU(N) spin-flavor symmetry groups [3,10], consistent with the results of lattice QCD calculations [11]. This concept has also been applied in other settings, such as the Higgs sector of the Standard Model [12,13], bosonic

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field theories [14], and higher-spin systems [15]. The maximum entanglement discussion of Refs. [1,2] has been recently extended to perturbative gluon-gluon scattering in QCD [16]. Various studies of entanglement in few-nucleon scatterings [17–24], and in many-body nuclear systems [25–41] have also followed.

Quantum complexity beyond entanglement, in particular nonstabilizerness (NS) [42,43], or “quantum magic” [44], is now being considered in an array of physical systems. The nonstabilizerness in a wave function is a measure of how much it differs from one that can be prepared efficiently classically, either as a measure of the distance to the nearest stabilizer state, or the number of stabilizer states needed to represent it (stabilizer rank). As such, nonstabilizerness can provide guidance for partitioning simulations between classical and quantum hardware, both in the Noisy intermediate scale quantum (NISQ) [45] and fault-tolerant regimes. The recent development of practically computable measures of nonstabilizerness based on stabilizer Rényi entropies (SREs) [46–50] have triggered a wide range of works investigating nonstabilizerness in various aspects of few-body and many-body structure and dynamics. In HEP and NP, this includes studies of nonstabilizerness in the dynamics of neutrino systems [51], in thermalization in gauge theories [52], in simulations of lattice gauge theories [53], in high-energy particle production [54,55] and scattering [56–58], in nuclear and hypernuclear forces [59], in many-body nuclear models [60] and atomic nuclei [41]. These studies constitute efforts towards better understanding the role of quantum complexity in physical behaviors and designing optimal simulation strategies.¹

Entanglement or nonstabilizerness alone, however, do not reflect the full complexity of a quantum state. This is because states with both large nonstabilizerness and low entanglement, and those with large entanglement and low nonstabilizerness can be prepared efficiently with classical resources. However, this becomes increasingly difficult as the entanglement or nonstabilizerness increases, respectively. Therefore it is the interplay between these two aspects that drive quantum complexity, and the need for quantum computers [62,63]. Recently it has been put forward that this interplay is captured by the nonlocal nonstabilizerness (NL NS) [62]—the nonstabilizerness that cannot be removed from the system via local operations—and thus large nonlocal nonstabilizerness will be the key feature of quantum systems that drive future quantum advantages [44,61,64–84]. In holography theory, nonlocal nonstabilizerness has been shown to be responsible for gravitational backreaction and necessary for creating patterns of multipartite entanglement [62]. Nonlocal nonstabilizerness has also been studied in the

transverse-field Ising model [82], and a similar concept has been studied in relation to the phases of matter [85].

One issue in accessing nonlocal nonstabilizerness is that it requires minimization under local operations, which quickly becomes computationally demanding. On the other hand, it has recently been established that the antiflatness of the bipartite entanglement spectrum of a state, which is related to the total nonstabilizerness [86,87], can also partly capture the entanglement-nonstabilizerness interplay [63] and provide a lower bound to the nonlocal nonstabilizerness [62]. Antiflatness has been explored in a number of systems [88], including the detection of phase transitions [89], in scrambling [90], in the thermalization of non-Abelian gauge field theories [52,91], as well as in relation with quantum learnability [92].

In this work, we consider the role of antiflatness and nonlocal nonstabilizerness in two-body scattering processes, specifically low-energy nucleon-nucleon scattering and high-energy Møller scattering. While these systems do not present challenges from a computational standpoint, they probe the potential capabilities of these fundamental processes to drive the quantum complexity of larger many-body systems. Previously, it has been shown that the Clauser-Horne-Shimony-Holt (CHSH) inequality can be violated in polarized Møller scattering [93], building on some of their previous works, e.g., Ref. [94]. Given the simplicity of two-particle systems, along with the fact that antiflatness of the system can be recovered from measurements performed on one of the particles, we propose that polarized measurements performed on one of the final-state particles from a set of specifically prepared polarized initial states are sufficient to reveal the changes in quantum complexity imparted by fundamental forces. Interestingly, it has recently been shown [88] that violations of the CHSH inequality depend upon the presence of local nonstabilizerness, but are suppressed by the presence of nonlocal nonstabilizerness.

II. ANTIFLATNESS AND NONLOCAL NONSTABILIZERNESS

The most general wave function of a system of two spin- $\frac{1}{2}$ particles (mapped to qubits), $|\psi\rangle$, requires the universal quantum gate set to prepare it from an arbitrary (classical) tensor-product state. The vector of matrix elements of the 16 generalized Pauli operators formed from $\bar{\sigma} \in \{\hat{X}, \hat{Y}, \hat{Z}, \hat{I}\}$, $\hat{P}_{ij} = \bar{\sigma}_i \otimes \bar{\sigma}_j$, defines the Ξ_P values, as follows:

$$\Xi_P = c_P^2/d, \quad c_P = \langle \psi | \hat{P} | \psi \rangle, \quad (1)$$

where $d = 2^2$ and $\sum_P \Xi_P = 1$. If $|\psi\rangle$ is a stabilizer state, it can be prepared efficiently using classical resources (using Clifford gates), and only d of the d^2 quantities Ξ_P are nonzero, with values $\Xi_P = 1/d$, such that $\xi = d \sum_P \Xi_P^2 = 1$, along with other analogous relations.

¹The number of non-Clifford gates required to prepare a quantum state has been shown to be lower bounded by the nonstabilizerness in that state. However, a strict relation remains to be established [46,61].

For a nonstabilizer state, while $\sum_P \Xi_P = 1$ remains valid, $1/d \leq \xi < 1$. A family of SREs, $\mathcal{M}_\alpha$ are defined by [46]

$$\mathcal{M}_\alpha(|\psi\rangle) = \frac{1}{1-\alpha} \log_2 \sum_P \Xi_P^\alpha - \log_2 d. \quad (2)$$

Out of these measures the stabilizer two-Rényi entropy $\mathcal{M}_2(|\psi\rangle)$ has been shown to satisfy property of monotonicity [48,49]. Here we will pursue $\mathcal{M}_{\text{lin}}(|\psi\rangle)$, the linear version of $\mathcal{M}_2$, as follows:

$$\mathcal{M}_{\text{lin}}(|\psi\rangle) = 1 - \xi. \quad (3)$$

The main reason behind this choice is that $\mathcal{M}_{\text{lin}}(|\psi\rangle)$ has been shown to be directly related to some aspect of interplay between entanglement and nonstabilizer [86], which will become clearer below. Moreover, $\mathcal{M}_{\text{lin}}(|\psi\rangle)$ has been shown to be a strong monotone [49] and directly yields the stabilizer two-Rényi entropy through

$$\mathcal{M}_2(|\psi\rangle) = -\log_2 \xi = -\log_2(1 - \mathcal{M}_{\text{lin}}(|\psi\rangle)). \quad (4)$$

As defined, these measures of nonstabilizer contain contributions from both local and nonlocal nonstabilizer-ness. Given that the local nonstabilizer-ness can be eliminated, by definition, by applications of local unitary operators, the bipartite nonlocal nonstabilizer-ness² is defined by [62,95]

$$\mathcal{M}_i^{(NL)}(|\psi\rangle) = \min_{\hat{U}_A \otimes \hat{U}_B} [\mathcal{M}_i^{(NL)}(\hat{U}_A \otimes \hat{U}_B |\psi\rangle)], \quad (5)$$

$i \equiv \alpha, \text{lin},$

where A and B denote the two chosen partitions (here the two qubits) and where the minimization is over all possible local unitary transformations on A and B , providing a basis-independent quantity. For the two-qubit system, the minimization corresponds to the simple task of searching over the six angles defining transformations over each Bloch sphere. In some instances, this can be accomplished analytically, while in many others it requires a numerical procedure.

It has been recently shown that measures of total nonstabilizer-ness and nonlocal nonstabilizer-ness in a pure state $|\psi\rangle$ can be related to the antiflatness of the bipartite entanglement spectrum of that state. Such antiflatness is defined by a polynomial function of the reduced density matrix $\hat{\rho}_A = \text{Tr}_B \hat{\rho}_{AB}$ of subsystem A [86] as

$$\mathcal{F}_A(|\psi\rangle) = \langle \rho_A^2 \rangle - \langle \rho_A \rangle^2 = \text{Tr}(\rho_A^3) - (\text{Tr} \rho_A^2)^2, \quad (6)$$

and corresponds to the variance of the reduced density matrix. The two contributions to the antiflatness cancel

²The definition of nonlocal nonstabilizer-ness can be straightforwardly generalized to systems with more than two qubits, both for bipartitions and multipartitions [62,95].

when the nonzero eigenvalues of $\hat{\rho}_A$ are equal (flat entanglement spectrum), which occurs if $|\psi\rangle$ is unentangled or has no nonstabilizer-ness [86]. It is important to note that, similarly to the nonlocal nonstabilizer-ness, the antiflatness is independent on the local basis, but is not invariant under global Clifford operations.

In Ref. [86] it has been shown that the total linear nonstabilizer-ness is directly proportional to the average antiflatness associated with applications of all possible combinations of Clifford gates to the complete state, as follows:

$$\langle \mathcal{F}_A(\hat{\Gamma}|\psi\rangle) \rangle_C = c(d, d_A) \mathcal{M}_{\text{lin}}(|\psi\rangle), \quad (7)$$

where the lhs is the antiflatness of $\hat{\Gamma}|\psi\rangle$ averaged over Clifford unitaries $\hat{\Gamma} \in \mathcal{C}$, and the proportionality constant $c(d, d_A)$ is given by

$$c(d, d_A) = \frac{(d^2 - d_A^2)(d_A^2 - 1)}{(d^2 - 1)(d + 2)d_A^2}, \quad (8)$$

where d_A is the dimensionality of system- A Hilbert space. The averaging over Clifford operators provides a quantity that is Clifford invariant. At the same time, it redistributes the nonstabilizer-ness among local and nonlocal components, and thus can reintroduce local nonstabilizer-ness.

Further, Ref. [62] showed that the presence of nonlocal nonstabilizer-ness is a necessary and sufficient condition for antiflatness of the entanglement spectrum,³ and is lower bounded by the latter. In the present study, we find that the antiflatness in two-particle scattering (mapped onto two qubits) is exactly proportional to the nonlocal linear nonstabilizer-ness.

III. TWO-PARTICLE SCATTERING

As the dominant force between particles in nuclei and many processes involving fundamental particles, two-particle processes encapsulated by the S-matrix and T-matrix provide the basic building blocks of quantum complexity in many-body systems. For demonstrative purposes, motivated by what might be experimentally possible, we examine the antiflatness and nonlocal nonstabilizer-ness in the spin-sector of low-energy (nonrelativistic) S-wave nucleon-nucleon scattering, and high-energy (relativistic) Møller scattering, $e^- e^- \rightarrow e^- e^-$. These processes probe different sectors of the fundamental forces of nature. While the simple frameworks adopted in this work yield similar analytic structures for both processes, they already reveal notable differences in the ability of the underlying forces to generate and remove nonlocal nonstabilizer-ness and entanglement.

³The sufficiency condition is verified for most nonstabilizer-ness measures, except for some particular noninteger SREs [62].

A. Nucleon-nucleon scattering

Low-energy S -wave neutron-proton (np) scattering is described by an S -matrix of the form

$$\hat{S} = \begin{pmatrix} e^{2i\delta_1} & 0 & 0 & 0 \\ 0 & \frac{1}{2}(e^{2i\delta_1} + e^{2i\delta_0}) & \frac{1}{2}(e^{2i\delta_1} - e^{2i\delta_0}) & 0 \\ 0 & \frac{1}{2}(e^{2i\delta_1} - e^{2i\delta_0}) & \frac{1}{2}(e^{2i\delta_1} + e^{2i\delta_0}) & 0 \\ 0 & 0 & 0 & e^{2i\delta_1} \end{pmatrix}, \quad (9)$$

where $\delta_{0,1}$ are the phase shifts in the $S = 0$ and $S = 1$ spin channels. These are the dominant, but not only, channels contributing at low energies, and a more complete analysis would include contributions from P-wave, SD-wave mixing (induced by the tensor force) and higher.

When applied to some initial incoming state, the S -matrix in Eq. (9) defines the asymptotic final state of the scattering system. In Ref. [59], we studied the non-stabilizing power of the scattering S -matrix as the average SRE induced by the S -matrix acting on the set of the 60 two-qubit stabilizer states (listed in Appendix B), as follows:

$$\overline{\mathcal{M}}_{\text{lin}}(\hat{S}) \equiv \frac{1}{60} \sum_{i=1}^{60} \mathcal{M}_{\text{lin}}(\hat{S}|\psi_i\rangle). \quad (10)$$

This quantity included both the local and nonlocal non-stabilizing powers, which were not separated. In order to separate them, we define here the nonlocal nonstabilizing power and antiflatness power in a similar manner. To further isolate the induced changes of entanglement, we however choose to restrict the average to the 36 tensor-product states (formed from the six single-qubit stabilizer states) only. That is,

$$\overline{\mathcal{M}}_{\text{lin}}^{(NL)}(\hat{S}) \equiv \frac{1}{36} \sum_{i=1}^{36} \mathcal{M}_{\text{lin}}^{(NL)}(\hat{S}|\psi_i\rangle), \quad (11)$$

$$\overline{\mathcal{F}}_A(\hat{S}) \equiv \frac{1}{36} \sum_{i=1}^{36} \mathcal{F}_A(\hat{S}|\psi_i\rangle). \quad (12)$$

Quantities averaged over the full set of stabilizer states are denoted with a single bar $\bar{\mathcal{O}}$, while those averaged over the tensor-product stabilizer states are denoted with a double bar $\overline{\mathcal{O}}$.

Direct optimization over the six Euler angles defining the two Bloch spheres yields the following expression for the nonlocal nonstabilizing power:

$$\overline{\mathcal{M}}_{\text{lin}}^{(NL)}(\hat{S}) = \frac{1}{48} (11 + 5 \cos(4\Delta\delta)) \sin^2(2\Delta\delta), \quad (13)$$

where $\Delta\delta = \delta_1 - \delta_0$. This is to be compared to the total nonstabilizing power, which is given by [59]

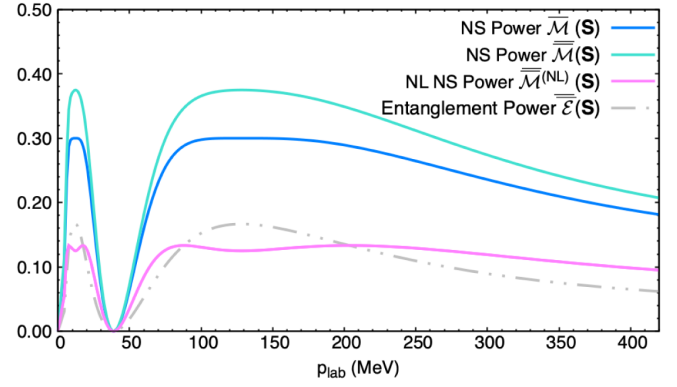


FIG. 1. Total NS powers, $\overline{\mathcal{M}}_{\text{lin}}(\hat{S})$ and $\overline{\mathcal{M}}_{\text{lin}}^{(NL)}(\hat{S})$, and the NL NS power, $\mathcal{M}_{\text{lin}}^{(NL)}(\hat{S})$, for low-energy S -wave neutron-proton scattering as a function of momentum in the laboratory. The difference in phase shifts are determined from the Nijmegen phase-shift analysis [96,97]. The entanglement power is also shown, as a dashed line.

$$\overline{\mathcal{M}}_{\text{lin}}(\hat{S}) = \frac{3}{20} (3 + \cos(4\Delta\delta)) \sin^2(2\Delta\delta). \quad (14)$$

The results obtained using Nijmegen Nijm93 phase shifts [96,97] (determined from fitting NN experimental data) are shown in Fig. 1. For comparison, we also show the linear nonstabilizing power $\overline{\mathcal{M}}_{\text{lin}}(\hat{S})$ averaged over tensor-product stabilizer states only, along with the entanglement power for information.⁴ It is seen that in the regime $p_{\text{lab}} \lesssim 150$ MeV, only about one third of the nonstabilizerness generated is nonlocal, while the rest can be eliminated via local basis transformation.

Further, we find that the nonlocal nonstabilizing power is exactly equal to the antiflatness power, i.e. $\overline{4\mathcal{F}}_A(\hat{S}) = \overline{\mathcal{M}}_{\text{lin}}^{(NL)}(\hat{S})$. In fact, we find this exact equality to be general for two-qubit states $|\psi\rangle$, as

$$4\mathcal{F}_A(|\psi\rangle) = \mathcal{M}_{\text{lin}}^{(NL)}(|\psi\rangle). \quad (16)$$

We have verified this result numerically for a large set (10^5) of random two-qubit states, with precision $\sim 10^{-28}$. This equality between nonlocal linear nonstabilizerness and antiflatness is however only valid for two-qubit systems. For n -qubit systems with $n > 2$, the antiflatness only provides a lower bound on the nonlocal nonstabilizerness, which contains higher-order effects. It was also recently shown (after submission of the present paper) that the equality in Eq. (16) does not hold for higher-dimensional two-qudit systems, such as two-qudrits [98].

⁴The corresponding entanglement power is given by [3,59]

$$\bar{\mathcal{E}}(\hat{S}) = \frac{1}{6} \sin^2(2\Delta\delta). \quad (15)$$

We have also numerically verified the relation in Eq. (7), $\langle \mathcal{F}_A(\Gamma|\psi) \rangle_C = c(d, d_A) \mathcal{M}_{\text{lin}}(|\psi\rangle)$.

We analyze in more detail the individual contribution of outgoing states produced from different groups of initial stabilizer states. As we identified previously [59], these 36 states divide into three groups, Groups 1, 2, and 3, in which there are six, six, and 24 tensor-product initial states, respectively. These groups are chosen so that the states in each group yield the same total outgoing nonstabilizer-ness, and are listed in Appendix C. The antiflatness is again equal to four times the nonlocal linear nonstabilizer-ness, and interestingly, we find that, in each group, the nonlocal linear nonstabilizer-ness is also proportional to the total linear nonstabilizer-ness, as follows:

$$\text{Group 1: } \mathcal{F}_A = \mathcal{M}_{\text{lin}}^{(NL)} = \mathcal{M}_{\text{lin}} = 0, \quad (17)$$

$$\text{Group 2: } 4\mathcal{F}_A = \mathcal{M}_{\text{lin}}^{(NL)} = \mathcal{M}_{\text{lin}} = \frac{1}{4} \sin^2(4\Delta\delta), \quad (18)$$

$$\begin{aligned} \text{Group 3: } 4\mathcal{F}_A = \mathcal{M}_{\text{lin}}^{(NL)} &= \frac{1}{3} \mathcal{M}_{\text{lin}} \\ &= \frac{1}{32} (7 + \cos(4\Delta\delta)) \sin^2(2\Delta\delta). \end{aligned} \quad (19)$$

The proportionality constant is however different for each of the groups. In contrast to Groups 2 and 3, scattering of Group 1 states does not create nonstabilizer-ness or antiflatness. For scattering of states in Group 2, the total nonstabilizer-ness of the final states is saturated by the nonlocal nonstabilizer-ness. That is to say that all the nonstabilizer-ness generated is bound to the outgoing system and there is no nonstabilizer-ness in the final state that can be removed by changes of local basis. In Group 3, the nonlocal nonstabilizer-ness constitutes only a third of the total nonstabilizer-ness. In Appendix C, we show the nonstabilizer-ness, nonlocal nonstabilizer-ness (here equivalent to antiflatness) and entanglement for scattering of states in each group, and also show results when considering entangled stabilizer states.

Tensor-product stabilizer states representative of each group, denoted by $|\psi_{1,2,3}\rangle$, that are candidates for experimental initial-state preparation are

$$\begin{aligned} |\psi_1\rangle &= |\uparrow\rangle \otimes |\uparrow\rangle, & |\psi_2\rangle &= |\uparrow\rangle \otimes |\downarrow\rangle, \\ |\psi_3\rangle &= \frac{1}{\sqrt{2}} [|\uparrow\rangle + |\downarrow\rangle] \otimes |\uparrow\rangle, \end{aligned} \quad (20)$$

corresponding to states 33, 34 and 25 in Table I. Experimentally, the wave function associated with the selected Group 3 state can be prepared by a single-spin $\pi/2$ rotation about the y-axis.

We have previously found that the entanglement and (total) nonstabilizing power in Σ^-n scattering is large and energy independent over a large range, after a rapid rise at low energies [59]. Extending the above analysis to the Σ^-n

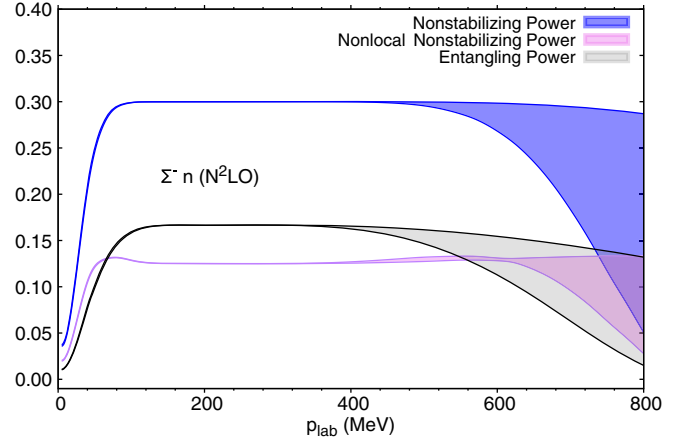


FIG. 2. Total nonstabilizing power, $\bar{\mathcal{M}}_{\text{lin}}(\hat{S})$, nonlocal nonstabilizing power, $\bar{\mathcal{M}}_{\text{lin}}^{(NL)}(\hat{S})$, and entanglement power, $\bar{\mathcal{E}}_{\text{lin}}(\hat{S})$, for low-energy S-wave Σ^-n scattering as a function of momentum in the laboratory. The results are obtained using next-to-next-to leading order ($N^2\text{LO}$) chiral effective field theory phase shifts from Ref. [99]. We have assumed isospin symmetry between Σ^+p and Σ^-n , and neglected Coulomb interactions. The uncertainty bands represent the maximum and minimum values in nonstabilizer-ness and entanglement derived from the $N^2\text{LO}$ phase-shift uncertainty bands [99].

sector, shows that the nonlocal nonstabilizer-ness (or antiflatness) exhibit similar behaviors. They rise rapidly and remain constant, with a small amplitude modulation over an extended energy interval. Nonlocal nonstabilizer-ness is found to capture less than half ($\simeq 0.4$) of the total nonstabilizer-ness, and this fraction remains constant even at large momenta. This is shown in Fig. 2.

B. High-energy Møller scattering

High-energy Møller scattering provides a sensitive probe for new physics, and establishes increasingly precise constraints on electroweak interactions in the Standard Model. For example, the Measurement of a Lepton-Lepton Electroweak Reaction (MOLLER) experiment at the Thomas Jefferson National Accelerator Facility (JLab) [100,101] is designed to measure the parity-violating asymmetry, A_{PV} , to a precision of 0.7 ppb, thereby probing the interference of amplitude from the exchange of a neutral weak-gauge boson, Z^0 , and the electromagnetic amplitude with remarkable sensitivity. This process also provides somewhat of a complementary process to NN scattering for exposing nonlocal nonstabilizer-ness and antiflatness in two-body scattering. It has been considered in previous works exploring quantum complexity in fundamental processes, including (the first) work that highlighted that entanglement seems to be maximal [1,2], and a recent consideration of total nonstabilizer-ness [56]. Following Ref. [56], instead of considering the structure of the complete S-matrix, we consider the final state wave

function generated at leading order in perturbation theory (in the electromagnetic coupling constant, e) by a single insertion of the T-matrix. This preserves unitarity up to $\mathcal{O}(e^4)$, and necessarily requires a projective measurement on the two-electron final state.

The postscattering wave function can be determined from the expressions in Appendix D, and using the helicity-amplitude basis, the scattering amplitude matrix in the high-energy limit ($m_e \rightarrow 0$) becomes [1,2,102]

$$\mathcal{A} \propto \begin{pmatrix} -8\csc^2(\theta) & 0 & 0 & 0 \\ 0 & -2\cot^2(\frac{\theta}{2}) & 2\tan^2(\frac{\theta}{2}) & 0 \\ 0 & 2\tan^2(\frac{\theta}{2}) & -2\cot^2(\frac{\theta}{2}) & 0 \\ 0 & 0 & 0 & -8\csc^2(\theta) \end{pmatrix}, \quad (21)$$

where θ is the center-of-momentum scattering angle. The proportionality constant, including factors of e^2 are inconsequential upon renormalizing the final-state wave function. For any initial-state wave function, $|\psi\rangle$, the final-state wave function is given by $|\chi\rangle = \mathcal{N}\mathcal{A}|\psi\rangle$. The normalization constant, $\mathcal{N}$, is determined (up to an overall phase) by $\langle\chi|\chi\rangle = 1$. In contrast to Ref. [56] with amplitudes given in terms of the computational basis (because of the basis dependence of the total nonstabilizerness), we use the

helicity basis to write $\mathcal{A}$, but any single-particle basis would have served our purposes. The total nonstabilizerness, non-local nonstabilizerness and antiflatness powers take the same expression as in Eqs. (10)–(12), with $\hat{S}|\psi_i\rangle \rightarrow \mathcal{N}\hat{\mathcal{A}}|\psi_i\rangle$.

There are five different groups of stabilizer states with respect to the total nonstabilizerness injected by the scattering amplitude in Eq. (21), which are given in Eq. (D7) in Appendix D. Each state in a given group gives rise to a final state with the same total nonstabilizerness. Representative initial tensor-product states from each group are

$$\begin{aligned} |\psi_1\rangle &= |\uparrow\rangle \otimes |\uparrow\rangle, \\ |\psi_2\rangle &= \frac{1}{\sqrt{2}}[|\uparrow\rangle + |\downarrow\rangle] \otimes \frac{1}{\sqrt{2}}[|\uparrow\rangle + |\downarrow\rangle], \\ |\psi_3\rangle &= \frac{1}{\sqrt{2}}[|\uparrow\rangle + |\downarrow\rangle] \otimes \frac{1}{\sqrt{2}}[|\uparrow\rangle - |\downarrow\rangle], \\ |\psi_4\rangle &= |\uparrow\rangle \otimes |\downarrow\rangle, \\ |\psi_{5a}\rangle &= \frac{1}{\sqrt{2}}[|\uparrow\rangle + i|\downarrow\rangle] \otimes \frac{1}{\sqrt{2}}[|\uparrow\rangle + |\downarrow\rangle], \\ |\psi_{5b}\rangle &= |\uparrow\rangle \otimes \frac{1}{\sqrt{2}}[|\uparrow\rangle + |\downarrow\rangle]. \end{aligned} \quad (22)$$

The values of the total linear nonstabilizerness, $\mathcal{M}_{\text{lin}}(|\chi_i\rangle)$, defined in Eq. (3), of the associated wave function $|\chi_i\rangle = \mathcal{N}\hat{\mathcal{A}}|\psi_i\rangle$ for each of the groups, are

$$\begin{aligned} G1: & 0, \\ G2: & \frac{64 \sin^4(\theta) \cos^2(\theta)}{(\cos(2\theta) + 3)^4}, \\ G3: & \frac{1024 \sin^4(\theta) (20 \cos(2\theta) + \cos(4\theta) + 43)^2}{(12 \cos(2\theta) + \cos(4\theta) + 51)^4}, \\ G4: & \frac{4 \cot^8(\frac{\theta}{2}) (\cot^8(\frac{\theta}{2}) - 1)^2}{(\cot^8(\frac{\theta}{2}) + 1)^4}, \\ G5: & \frac{32 \sin^4(\theta) (799 \cos(2\theta) - 10 \cos(4\theta) + \cos(6\theta) + 1258)}{(\cos(2\theta) + 7)^6}. \end{aligned} \quad (23)$$

The corresponding antiflatness is found to recover the results of numerical minimization of $\mathcal{M}_{\text{lin}}^{(NL)} = 4\mathcal{F}_A$ for each group, which are found to be

$$\begin{aligned} G1: & \mathcal{M}_{\text{lin}} = 0, \\ G2: & \mathcal{M}_{\text{lin}}, \\ G3: & \mathcal{M}_{\text{lin}}, \\ G4: & \mathcal{M}_{\text{lin}}, \\ G5a: & \frac{256 \sin^4(\theta)}{(\cos(2\theta) + 7)^8} (\cos(2\theta) + 15)^2 (28 \cos(2\theta) + \cos(4\theta) + 35) \leq \mathcal{M}_{\text{lin}}, \\ G5b: & \frac{128 \sin^8(\theta)}{(\cos(2\theta) + 7)^8} (175 \cos(2\theta) + 18 \cos(4\theta) + \cos(6\theta) + 318) \ll \mathcal{M}_{\text{lin}}. \end{aligned} \quad (24)$$

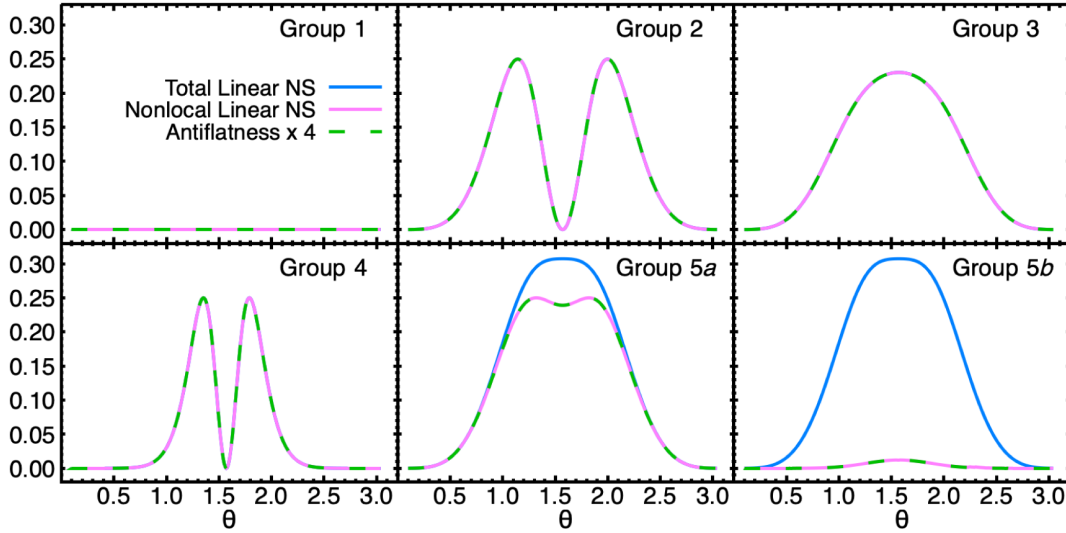


FIG. 3. Linear NS, $\mathcal{M}_{\text{lin}}(|\chi_i\rangle)$, the NL NS, $\mathcal{M}_{\text{lin}}^{(NL)}(|\chi_i\rangle)$ and the antiflatness, $\mathcal{F}_A(|\chi_i\rangle)$ (multiplied by a factor of 4), for outgoing states $|\chi_i\rangle = \mathcal{N}\hat{A}|\psi_i\rangle$, corresponding to the distinct groups of initial-state helicity wave functions $|\psi_i\rangle$ for high-energy Möller scattering. The upper panels from left to right show the results for tensor-product states from Group-1, 2, 3, while the lower panels show the results for Groups 4, 5a, and 5b. The green curves associated with the $\mathcal{M}_{\text{lin}}$ are from Eq. (24), while the dashed blue curves associated with the $\mathcal{F}_A$ are from Eq. (24). The pink curves are from numerical minimization of $\mathcal{M}_{\text{lin}}^{(NL)}(|\chi_i\rangle)$, using Eq. (5).

Notice that the states in Group 5 (G5) produce final states that have two different values of antiflatness. Therefore, we further subdivide these states into Group 5a and Group 5b, as given in Eq. (D8) in Appendix D. For all states, the relation $\mathcal{M}_{\text{lin}}^{(NL)}(|\chi_i\rangle) = 4\mathcal{F}_A(|\chi_i\rangle)$ is verified. For Groups 1–4, the outgoing states only exhibit nonlocal nonstabilizerlessness as $\mathcal{M}_{\text{lin}}^{(NL)}(|\chi_i\rangle) = \mathcal{M}_{\text{lin}}(|\chi_i\rangle)$. This is not the case for outgoing states generated from stabilizer states from Group 5a and Group 5b, which also display some local nonstabilizerlessness. In finding the nonlocal nonstabilizerlessness, we are unable to arrive at closed-form results for $\mathcal{M}_{\text{lin}}^{(NL)}$ by analytic minimization of local unitary transformations for some of the groups. However, numerical minimization over the two Bloch spheres is found to be effective. The resulting nonlocal nonstabilizerlessness, as well as the total nonstabilizerlessness and antiflatness, for initial tensor-product states in each of the groups, are shown in Fig. 3. For completeness, the total linear nonstabilizerlessness $\mathcal{M}_{\text{lin}}(|\chi_i\rangle)$, linear entanglement entropy, and nonlocal nonstabilizerlessness $\mathcal{M}_{\text{lin}}^{(NL)}(|\chi_i\rangle)$ of the outgoing states $|\chi_i\rangle = \mathcal{N}|\psi_i\rangle$ for each groups, including for entangled initial states, are shown in Fig. 7 in Appendix D.

The relation between the antiflatness averaged over the Clifford group, given in Eq. (7), has been numerically verified. Figure 4 shows the Clifford-averaged antiflatness and the total linear nonstabilizerlessness divided by 10 for the states in Groups 5a and 5b. The averaged results are consistent with the relation $\langle \mathcal{F}_A(\Gamma|\psi) \rangle_C = c(d, d_A)\mathcal{M}_{\text{lin}}(|\psi\rangle)$ in Eq. (7), where $c(d, d_A)$ in Eq. (8)

is $c(4, 2) = 1/10$. The estimators of $\langle \mathcal{F}_A(\Gamma|\psi) \rangle_C$ in Fig. 4 at each angle were determined from the mean and standard deviation of an ensemble of $\mathcal{F}_A$ generated from 5×10^3 samples of random Clifford gates applied to the final-state wave function.

The current Möller scattering experiment at JLab [100,101] is able to prepare arbitrary polarized initial states with high precision, but currently lacks the capability to measure spin components of final-state particles. This means that the antiflatness cannot be measured with this generation of experiment. However, with sufficient motivation, there could be potential for including such capabilities in a next-generation MOLLER program.

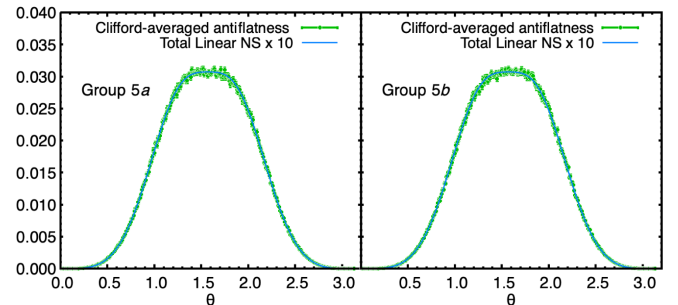


FIG. 4. Clifford-averaged antiflatness, $\langle \mathcal{F}_A(\Gamma|\chi_i) \rangle_C$, (green points) compared with $c(d, d_A)\mathcal{M}_{\text{lin}}(|\chi_i\rangle)$ (blue curve) in Möller scattering from initial states $|\psi_{5a}\rangle$ (left panel) and $|\psi_{5b}\rangle$ (right panel) in Eq. (22). The unaveraged values of antiflatness are shown in Fig. 3.

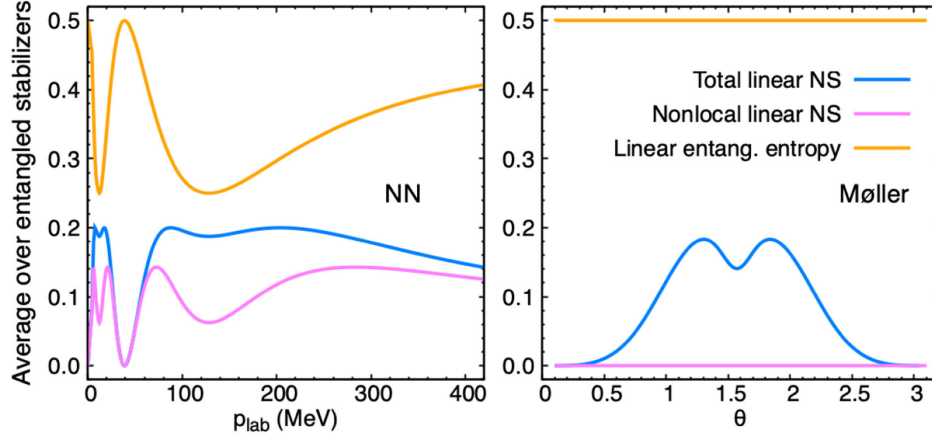


FIG. 5. Total linear NS, NL NS (antiflatness $\times 4$), and linear entanglement entropy in NN scattering and Møller scattering final states, averaged over all 24 entangled stabilizer states.

C. Nonlocal nonstabilizerness generation from entangled initial states:

Nuclear force versus quantum electrodynamics

We have so far focused on the nonlocal nonstabilizerness and antiflatness produced in scattering processes in which the two particles are initially unentangled. Since nonlocal nonstabilizerness requires entanglement, this allowed us to isolate the effectiveness of the underlying interaction to generate both the needed entanglement and the corresponding nonlocal nonstabilizerness. This is also justified from an experimental point of view, as in laboratory scattering experiments the initial incoming particles are typically unentangled.

Nevertheless it is also interesting to investigate the effect of scattering when the two initial particles are prepared in an entangled stabilizer state. These states are those numbered 37 to 60 in Table I in Appendix B. They are maximally entangled but possess, by definition, no nonstabilizerness (local or nonlocal) and no antiflatness. Thus studying scattering from these states can tell us about whether the underlying interaction can build on the initial entanglement to generate antiflatness (complex entanglement patterns), and nonlocal nonstabilizerness. For example, is the nonlocal nonstabilizerness enhanced when the initial states are entangled?

Figure 5 shows the final-state total nonstabilizerness, nonlocal nonstabilizerness (or antiflatness) and linear entanglement entropy averaged over initial entangled stabilizer states for both NN and Møller scattering.⁵ While nonstabilizerness is produced in both processes, Møller scattering does not generate nonlocal nonstabilizerness

when the initial stabilizer states are entangled, and the outgoing particles remain maximally entangled. This means that the outgoing states are themselves stabilizer states in a different local basis. We also observe the same effect in other QED processes such as $e^+e^- \rightarrow \mu^+\mu^-$, and, it was also recently found in gluon, gluino, graviton and gravitino scattering [103]. Since, for two-qubit systems, maximally entangled states are also stabilizers states, the interaction would need to disentangle the system by some amount to introduce nonlocal nonstabilizerness, which does not occur in these processes. More generally, for arbitrary two-qubit initial states, the ability of the interaction to disentangle during those processes is very small, when compared to the entangling power.

In contrast, this feature is not observed in the case of low-energy S-wave np scattering, for which the nuclear force disentangles maximally entangled particles, and produces nonlocal nonstabilizerness (or antiflatness). In fact it is shown in Fig. 6 of Appendix C that the “disentangling power” is about the same or larger than the entangling power in this process, depending on the group of initial states. We also find that the nonlocal nonstabilizerness produced from entangled and unentangled states within Groups 1 and 2 is the same, while initial entanglement in states from Group 3 yield greater nonlocal nonstabilizerness, with a different structure.

To explore this further, we have included the chirality-violating electron mass, and find that the results remain intact. Hence, this does not result from chirality coinciding with helicity in the massless limit. The entangled stabilizer states can be written in terms of sums of eigenstates of angular momentum, with good reflection symmetry in the computational basis. These sums can be shown to be of states of the same $\hat{J}_i$ along one of the Cartesian axes. The fundamental symmetries in Møller scattering, including

⁵The difference between the average final-state linear entanglement entropy and its maximal value of $1/2$ can be identified as the disentanglement power, which is seen to be zero in the Møller scattering process.

identical particles, conservation of total angular momentum and parity, restrict transformation on the maximally entangled states to tensor products of single-spin rotations. These therefore remain maximally entangled. When considering neutron-neutron scattering (as opposed to np scattering of the previous section), this feature also occurs, due to the identical nature of the particles.

IV. SUMMARY

Towards a better understanding of the generation of quantum complexity in few-body and many-body systems from fundamental interactions, we have considered the nonlocal nonstabilizer and antiflatness in two-particle scattering processes. While large-scale nonlocal nonstabilizer and antiflatness in a physical system drive the need for quantum computers in simulations, in two-particle processes they define beyond-classical, basis-independent quantum observables and are fundamental elements of larger systems. We have focused on the spin-space structure of two such processes, low-energy S-wave neutron-proton scattering and high-energy Møller scattering. Starting with each of the tensor-product two-qubit stabilizer states, we have shown that there are a small number of groups of final states exhibiting the same nonstabilizer, and whose nonlinear nonstabilizer and antiflatness are related by a factor of 4. From an experimental standpoint, this relationship is convenient because only the spin of one of the final-state particles is required to be measured from polarized initial states, and not the spin correlations of both particles, as would be required for a CHSH measurement. We have also found substantial differences in the effectiveness of quantum electrodynamics and nuclear forces to disentangle and induce nonlocal nonstabilizer in the scattering processes, when the initial particles are maximally entangled.

However, this is found to result from the fundamental symmetries of the systems, particularly the additional constraint imposed by the scattering of identical particles, as opposed to the structure of the forces themselves.

One may, once again, speculate about the connections between symmetries and the minimal entangling power of the S-matrix in confining theories. In Ref. [3], the vanishing entangling power resulting from identical phase shifts (or phase shifts different by $\pi/2$) was connected to enhanced emergent spin-flavor symmetries in nucleon-nucleon (NN) scattering [Wigner's SU(4) symmetry] and in hyperon-nucleon scattering [SU(16) symmetry]. The latter is a larger group than the SU(6) symmetry group required by the large- N_c limit of QCD, while Wigner's symmetry is the same. Given the results presented in this work, it is not possible to separate vanishing entanglement power from vanishing nonlocal nonstabilizing power. Given that some of the large-scale entangled states can be prepared efficiently with classical resources, while those with large-scale nonlocal nonstabilizer cannot, we highlight the

fact that minimizing fluctuations in entanglement also minimize fluctuations in nonlocal nonstabilizer.

We have made extensive use of Wolfram Mathematica [104].

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: MEASUREMENTS FOR ESTIMATING ANTIPLATNESS

For a two-particle final state, the reduced density matrix corresponds to a 2×2 matrix that, in spin-space, can be reconstructed from expectation values of the spin operator along (any) three Cartesian axes, as follows:

$$\rho_A = \frac{1}{2} \sum_{i=1}^4 \langle \bar{\sigma}_i \rangle_A \bar{\sigma}_i, \quad (\text{A1})$$

where $\langle \bar{\sigma}_i \rangle_A = \text{Tr}(\hat{\rho}_A \bar{\sigma}_i)$. Measuring the spin components of one of the final-state particles, provides an estimator for ρ_A from Eq. (A1), and hence an estimator for the antiflatness using Eq. (6).

APPENDIX B: TWO-QUBIT STABILIZER STATES

For two qubits (with $d = 4$), there are four stabilizer operators for each of the 60 stabilizer states given in Table I. Thirty-six of these states are tensor products formed from one-qubit stabilizers, while the remaining 24 are entangled states.

TABLE I. Complete set of 60 two-qubit stabilizer states. For notational purposes, we identify $|0\rangle = |\uparrow\rangle$ and $|1\rangle = |\downarrow\rangle$. The left set are from the tensor product of one-qubit stabilizer states, while the right set are entangled states. They are (generally) unnormalized, and require coefficients of either 1 or $\frac{1}{\sqrt{2}}$ or $\frac{1}{2}$.

State	$ 00\rangle$	$ 01\rangle$	$ 10\rangle$	$ 11\rangle$	State	$ 00\rangle$	$ 01\rangle$	$ 10\rangle$	$ 11\rangle$
1	1	1	1	1	37	0	1	1	0
2	1	-1	1	-1	38	1	0	0	-1
3	1	1	-1	-1	39	1	0	0	1
4	1	-1	-1	1	40	0	1	-1	0
5	1	1	i	i	41	1	0	0	i
6	1	-1	i	-i	42	0	1	i	0
7	1	1	-i	-i	43	0	1	-i	0
8	1	-1	-i	i	44	1	0	0	-i
9	1	1	0	0	45	1	1	1	-1
10	1	-1	0	0	46	1	1	-1	1
11	0	0	1	1	47	1	-1	1	1
12	0	0	1	-1	48	1	-1	-1	-1
13	1	i	1	i	49	1	i	1	-i
14	1	-i	1	-i	50	1	i	-1	i
15	1	i	-1	-i	51	1	-i	1	i
16	1	-i	-1	i	52	1	-i	-1	-i
17	1	i	i	-1	53	1	1	i	-i
18	1	-i	i	1	54	1	1	-i	i
19	1	i	-i	1	55	1	-1	i	i
20	1	-i	-i	-1	56	1	-1	-i	-i
21	1	i	0	0	57	1	i	i	1
22	1	-i	0	0	58	1	i	-i	-1
23	0	0	1	i	59	1	-i	i	-1
24	0	0	1	-i	60	1	-i	-i	1
25	1	0	1	0					
26	0	1	0	1					
27	1	0	-1	0					
28	0	1	0	-1					
29	1	0	i	0					
30	0	1	0	i					
31	1	0	-i	0					
32	0	1	0	-i					
33	1	0	0	0					
34	0	1	0	0					
35	0	0	1	0					
36	0	0	0	1					

APPENDIX C: MORE DETAILS ON LOW-ENERGY NN SCATTERING

For NN scattering, the stabilizer states separate into three distinct groups, where the states in each group lead to the same total linear nonstabilizerness in the outgoing states:

$$\text{Group 1} = \begin{cases} 1, 4, 17, 20, 33, 36 \text{ (tensor products)} \\ 37 \rightarrow 41, 44, 45, 48, 57, 60 \text{ (entangled)} \end{cases} \quad (\text{C1})$$

$$\text{Group 2} = \begin{cases} 2, 3, 18, 19, 34, 35 \text{ (tensor products)} \\ 42, 43, 46, 47, 58, 59 \text{ (entangled)} \end{cases} \quad (\text{C2})$$

$$\text{Group 3} = \begin{cases} 5 \rightarrow 16 \text{ and } 21 \rightarrow 32 \text{ (tensor products)} \\ 49 \rightarrow 56 \text{ (entangled)}. \end{cases} \quad (\text{C3})$$

Figure 6 shows the full linear nonstabilizerness $\mathcal{M}_{\text{lin}}(\hat{S}|\psi_i\rangle)$, linear entanglement entropy and nonlocal nonstabilizerness (here equivalent to antiflatness) for outgoing states in NN scattering resulting from tensor-product and entangled initial states $|\psi_i\rangle$ from Groups 1–3.

While the total nonstabilizerness produced is the same for tensor-product and entangled initial states within each group, the fluctuations in entanglement differ for Group 3. Specifically it is seen that, in this group, the interaction disentangles more effectively than it entangles. This results in nonlocal nonstabilizerness (or antiflatness) patterns exhibiting more structure when the initial states are entangled. This is not the case for states of Group 2 for which the disentangling power is about the same as the entangling power.

APPENDIX D: MORE DETAILS ON HIGH-ENERGY MØLLER SCATTERING

We choose to use the helicity-amplitude parametrization of the scattering amplitude, with expressions given in Refs. [1,2], and not the computational basis used in Ref. [56]. However, given the basis independence of antiflatness and nonlocal nonstabilizerness, either basis could have been chosen. The helicity amplitudes that are nonzero in the high-energy limit (or $m_e \rightarrow 0$), in terms of Mandelstam variables, are [1,2] (neglecting factors of e^2 , the electromagnetic coupling constant)

$$\begin{aligned} A_{RR:RR} &= A_{LL:LL} = -\frac{2(t+u)^2}{tu}, \\ A_{RL:RL} &= A_{LR:LR} = -\frac{2u}{t}, \\ A_{RL:LR} &= A_{LR:RL} = +\frac{2t}{u}. \end{aligned} \quad (\text{D1})$$

They can be written in terms of the center-of-momentum scattering kinematics,

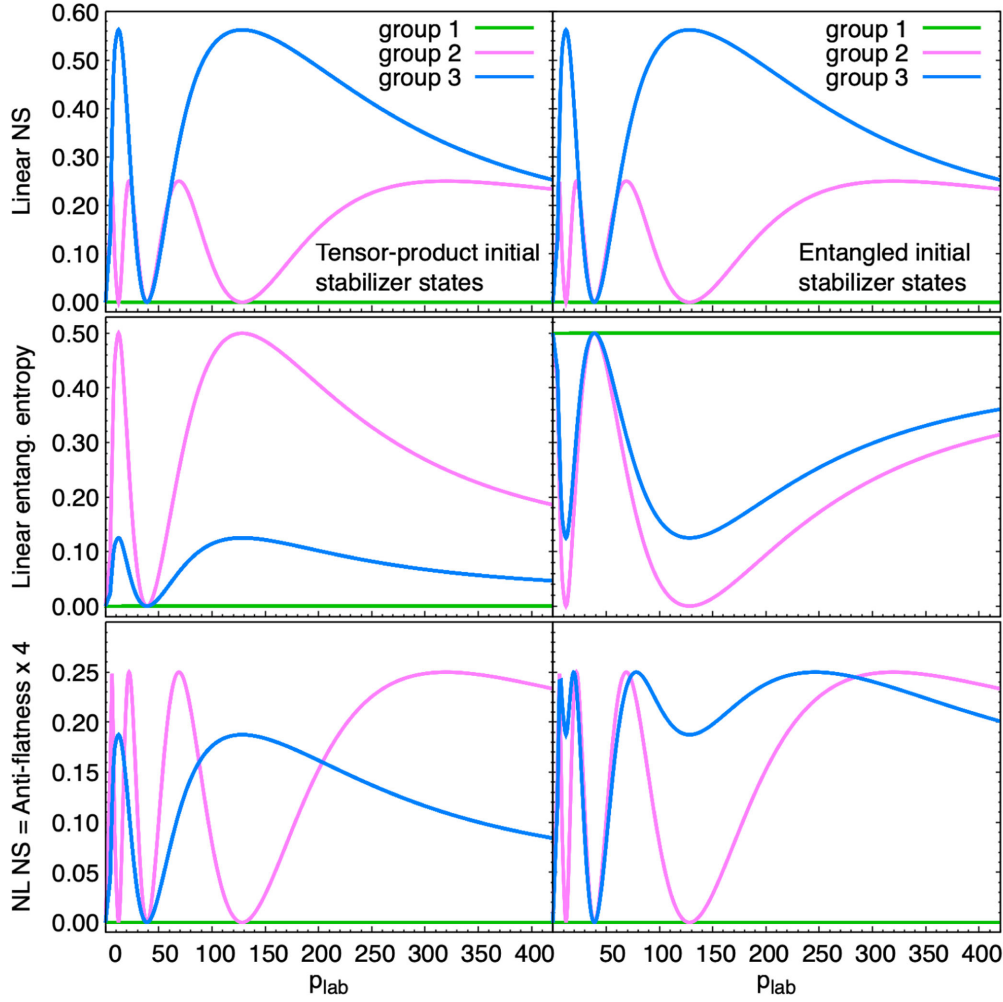


FIG. 6. Total linear NS (top), linear entanglement entropy (middle), and linear NL NS (antiflatness $\times 4$, bottom) in NN scattering. The left (right) panels show the results for outgoing states from initial unentangled (entangled) stabilizer states.

$$\begin{aligned}
 p_1 &= (E/2, 0, 0, |\mathbf{p}_e|), & p_2 &= (E/2, 0, 0, -|\mathbf{p}_e|), \\
 p_3 &= (E/2, 0, |\mathbf{p}_e| \sin \theta, |\mathbf{p}_e| \cos \theta), & p_4 &= (E/2, 0, -|\mathbf{p}_e| \sin \theta, -|\mathbf{p}_e| \cos \theta), \\
 |\mathbf{p}_e|^2 &= (E/2)^2 + m_e^2, \\
 s &= (p_1 + p_2)^2, & t &= (p_1 - p_3)^2, & u &= (p_1 - p_4)^2, \\
 s &= E^2, & t &= -2|\mathbf{p}_e|^2(1 - \cos \theta), & u &= -2|\mathbf{p}_e|^2(1 + \cos \theta),
 \end{aligned} \tag{D2}$$

where θ is the scattering angle in the center of momentum.

For Møller scattering, the stabilizer states contributing to the final-state nonstabilizerness separate into five distinct groups:

$$\text{Group 1} = \begin{cases} 33, 36 \text{ (tensor products)} \\ 37 \rightarrow 41, 44 \text{ (entangled)} \end{cases} \tag{D3}$$

$$\text{Group 2} = \begin{cases} 1, 4, 17, 20 \text{ (tensor products)} \\ 45, 48, 57, 60 \text{ (entangled)} \end{cases} \tag{D4}$$

$$\text{Group 3} = \begin{cases} 2, 3, 18, 19 \text{ (tensor products)} \\ 46, 47, 58, 59 \text{ (entangled)} \end{cases} \tag{D5}$$

$$\text{Group 4} = \begin{cases} 34, 35 \text{ (tensor products)} \\ 42 \text{ (entangled)} \end{cases} \tag{D6}$$

$$\text{Group 5} = \begin{cases} 5 \rightarrow 16 \text{ and } 21 \rightarrow 32 \text{ (tensor products)} \\ 49 \rightarrow 56 \text{ (entangled)}. \end{cases} \tag{D7}$$

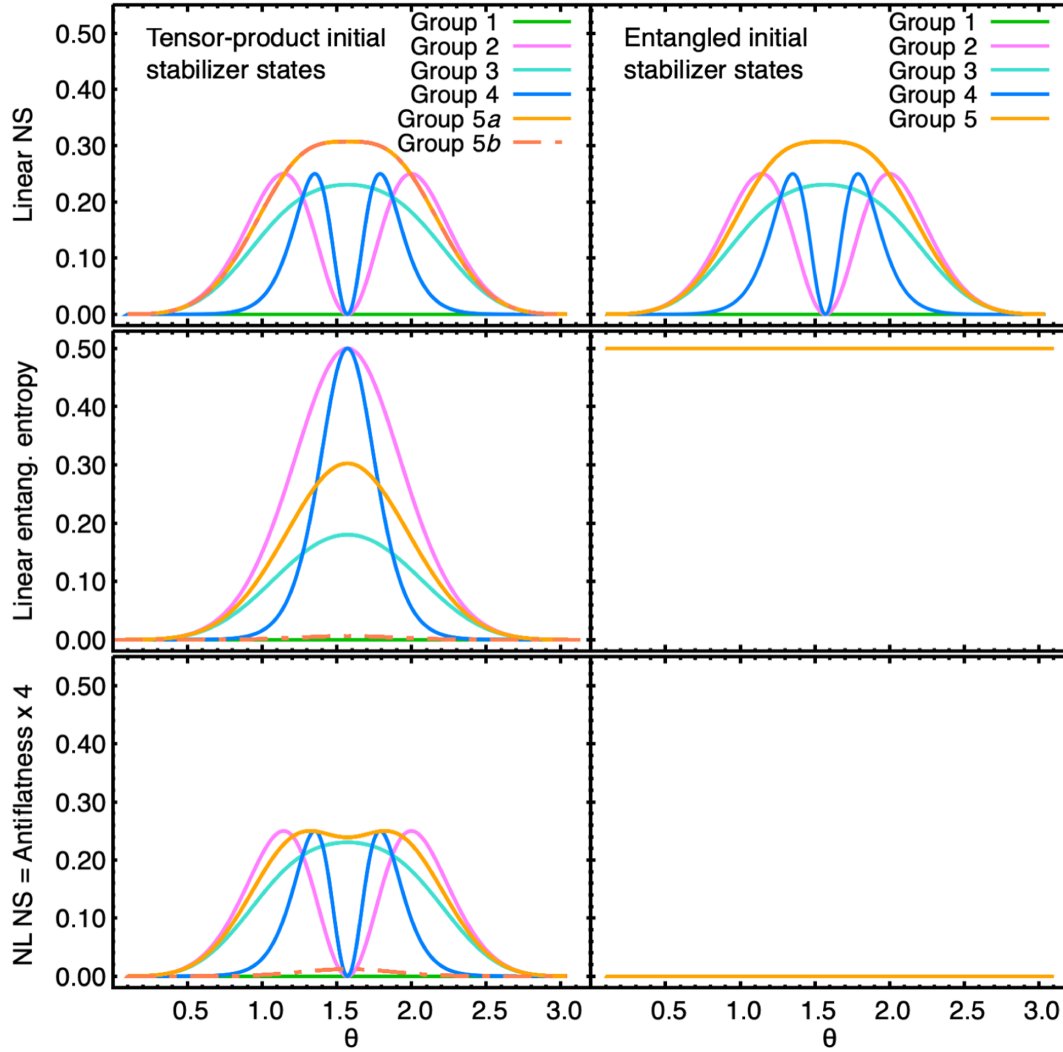


FIG. 7. Total linear NS (top), linear entanglement entropy (middle), and linear NL NS (equal to antiflatness $\times 4$, bottom) in Møller scattering. The left (right) panels show the results for outgoing states from initial unentangled (entangled) stabilizer states.

We find that the tensor-product states within each Group 1–4 display the same antiflatness. However tensor-product states in Group 5 do not. Hence, we further subdivide the tensor-product states in Group 5 into two subgroups:

$$\begin{aligned} \text{Group } 5a &= 5, 6, 7, 8, \quad \text{and} \quad 13, 14, 15, 16, \\ \text{Group } 5b &= 9, 10, 11, 12, \quad \text{and} \quad 21 \rightarrow 32. \end{aligned} \quad (\text{D8})$$

Figure 7 shows the full linear nonstabilizerness, linear entanglement entropy and nonlocal nonstabilizerness (here equivalent to antiflatness) for outgoing states in Møller scattering $|\chi_i\rangle = \mathcal{N}\hat{A}|\psi_i\rangle$ resulting from tensor-product and entangled initial states $|\psi_i\rangle$ from Groups 1–5. It is

interesting to note that the entanglement entropy is maximal at $\theta = \pi/2$ for all groups, and for the average value, consistent with the observations in Refs. [1,2], while this is not the case for the nonstabilizerness.

In contrast to the strong nuclear force in NN scattering, we observe a large asymmetry between the entanglement and disentanglement power in the present QED process. In particular the interaction is not able to decrease the entanglement of maximally entangled initial states, which results in no nonlocal nonstabilizerness (or antiflatness) being produced. Thus in this case the generated nonstabilizerness is entirely local, meaning that the outgoing states remain entangled stabilizer states in a different local basis.

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