

Mass spectrum of the hidden-charm hybrid states via QCD sum rules

Zhi-Gang Wang^{*}

Department of Physics, North China Electric Power University,
Baoding 071003, People's Republic of China



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In this work, we study the mass spectrum of the hidden-charm hybrid states with the $J^{PC} = 0^{-+}, 0^{++}, 0^{--}, 1^{++}, 1^{+-}, 1^{--}, 2^{-+}$ and 2^{++} via the quantum chromodynamics (QCD) sum rules in a consistent way. We calculate the vacuum condensates up to dimensions-6 by taking account of both the leading order and next-to-leading order contributions, and take the energy scale formula $\mu = \sqrt{M_{X/Y/Z}^2 - (2M_c)^2}$ to choose the suitable energy scales of the QCD spectral densities, it is the first time to explore the energy scale dependence of the QCD sum rules for the hidden-charm hybrid states.

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I. INTRODUCTION

In the traditional quark model, the hadrons are classified into mesons and baryons, which are bound states of a quark-antiquark pair or three quarks. However, the quark model does not forbid the possibilities of tetraquark states, pentaquark states, hybrid states, glue-balls, which are the commonly called exotic hadrons or X, Y, Z, P, T states. The quantum chromodynamics (QCD) allows valence color degrees of freedom, such as hybrid states with gluonic excitations or glueballs consist of constituent gluons, which are a major arena for testing our understanding of the strong interactions beyond perturbative region.

In 2003, the Belle collaboration observed the first exotic state $X(3872)$ [1], thereafter, dozens of exotic states have been observed by the ATLAS, BABAR, Belle, BESIII, CDF, CMS, D0, and LHCb collaborations [2]. The theoretical physicists have proposed many interpretations for the nature of those exotic states, such as the tetraquark states, pentaquark states, molecular states, hadron-charmonium states, hybrid states, glueballs, rescattering effects, etc. However, a single theoretical scheme cannot interpret the entire spectroscopy of the exotic states satisfactorily due to shortcomings in one way or another.

It has been argued that the $Y(4260)$, $Y(4360)$, and $Y(4140)$ might be the $\bar{c}cg$ hybrid charmonium states or their essential components [3–7], however, they have the normal quantum numbers $J^{PC} = 1^{--}$ and 1^{++} , respectively,

just like the traditional charmonium states, which make the situations even complex. In 2021, the LHCb collaboration observed the $X(4630)$ in the $J/\psi\phi$ mass spectrum with the favored assignment $J^P = 1^-$ [8]. Although its quantum numbers $J^{PC} = 1^{-+}$ are exotic, it is not necessary to be a hidden-charm hybrid state, the assignment as a tetraquark state or molecular state with the valence quarks $c\bar{c}s\bar{s}$ is also possible [9,10]. We can consult Ref. [11] for detailed analysis of semi-inclusive decays of the hidden-charm (hidden-bottom) hybrid states to charmonium (bottomonium) states based on the Born-Oppenheimer effective field theory to diagnose their nature.

At the light sector, there exist hybrid candidates, such as the $\pi(1400)$ and $\pi(1600)$ with the $J^{PC} = 1^{-+}$ [2]. In 2022, the BESIII collaboration observed the isoscalar resonance $\eta(1855)$ with the exotic quantum numbers $J^{PC} = 1^{-+}$ in the process $J/\psi \rightarrow \gamma\eta(1855) \rightarrow \gamma\eta\eta'$ [12], it might be a possible candidate for the hybrid state [13–15]. Theoretically, the mass spectrum of the hybrid states have been investigated by the MIT bag model [16–18], the confining linear potential model [19], the flux tube model for QCD [20–22], the QCD sum rules [23–42], the lattice QCD [43–47], the Born-Oppenheimer effective field theory [48,49], etc.

The predictions from different theoretical works differ from each other greatly. For instance, the mass of the $\bar{c}cg$ ground state with the $J^{PC} = 1^{-+}$ is 3.70 GeV [33], 3.93 GeV [40], 4.31 GeV [44], 3.96 GeV [49], which makes the relevant problems unresolved until now, new analysis is necessary and interesting.

The QCD sum rules play an important role in studying the hadron masses, decay constants, form-factors, hadronic coupling constants, etc, and have been applied extensively to study the X, Y, Z, P, T states [50]. In Ref. [51], we explore the energy scale dependence of the QCD sum rules for the X, Y and Z states for the first time, and suggest an

^{*}Contact author: zgwang@aliyun.com

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energy scale formula,

$$\mu = \sqrt{M_{X/Y/Z}^2 - (2\mathbb{M}_Q)^2}, \quad (1)$$

with the effective heavy quark masses $\mathbb{M}_Q$ to obtain the ideal energy scales for the QCD sum rules for the hidden-charm and hidden-bottom tetraquark states [52,53], which can enhance the ground state contributions significantly and improve the convergent behavior of the operator product expansion significantly. This is our unique feature.

In our unique scheme of the QCD sum rules, we have performed a systematic analysis of the hidden-charm tetraquark states with the $J^{PC} = 0^{++}, 0^{-+}, 0^{--}, 1^{--}, 1^{-+}, 1^{++}, 1^{+-}, 2^{++}$ [9,54–58], the hidden-bottom tetraquark states with the $J^{PC} = 0^{++}, 1^{+-}, 1^{++}, 2^{++}$ [59], the hidden-charm molecular states with the $J^{PC} = 0^{++}, 1^{+-}, 1^{++}, 2^{++}$ [60], the doubly charm tetraquark (molecular) states with the $J^P = 0^+, 1^+, 2^+$ [61] ([62]), the hidden-charm pentaquark (molecular) states with the $J^P = \frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-$ [63] ([64]). In this work, we extend our previous works to study the hidden-charm hybrid states as there exists the strong fine structure constant $\alpha_s(\mu) = \frac{g_s^2}{4\pi}$ even in the leading order, we should take account of the energy scale dependence in a consistent way, just like what we have done in our previous works.

The article is arranged as follows: we obtain the QCD sum rules for the hidden-charm hybrid states in Sec. II; in Sec. III, we present the numerical results and discussions; Sec. IV is reserved for our conclusion.

II. QCD SUM RULES FOR THE HIDDEN-CHARM HYBRID STATES

First, we write down the two-point correlation functions $\Pi(p)$, $\Pi_{\mu\mu'}(p)$, and $\Pi_{\mu\nu\mu'\nu'}(p)$,

$$\begin{aligned} \Pi(p) &= i \int d^4x e^{ip \cdot x} \langle 0 | T \{ J(x) J^\dagger(0) \} | 0 \rangle, \\ \Pi_{\mu\mu'}(p) &= i \int d^4x e^{ip \cdot x} \langle 0 | T \{ J_\mu(x) J_{\mu'}^\dagger(0) \} | 0 \rangle, \\ \Pi_{\mu\nu\mu'\nu'}(p) &= i \int d^4x e^{ip \cdot x} \langle 0 | T \{ J_{\mu\nu}(x) J_{\mu'\nu'}^\dagger(0) \} | 0 \rangle, \end{aligned} \quad (2)$$

where the T denotes the time-ordering operation, the interpolating currents $J(x) = J^P(x)$, $J^S(x)$, $J_\mu(x) = J_\mu^V(x)$, $J_\mu^A(x)$, $J_{\mu\nu}(x) = J_{\mu\nu}^0(x)$, $J_{\mu\nu}^5(x)$, $J_{\mu\nu}^{\sigma,0}(x)$, $J_{\mu\nu}^{\sigma,5}(x)$, $J_{\mu\nu}^{2,\sigma,0}(x)$, $J_{\mu\nu}^{2,\sigma,5}(x)$,

$$\begin{aligned} J^P(x) &= \bar{c}_i(x) i\gamma_5 \sigma^{\mu\alpha} G_{\alpha\mu}^{ij}(x) c_j(x), \\ J^S(x) &= \bar{c}_i(x) \sigma^{\mu\alpha} G_{\alpha\mu}^{ij}(x) c_j(x), \end{aligned} \quad (3)$$

$$\begin{aligned} J_\mu^V(x) &= \bar{c}_i(x) \gamma^\alpha G_{\alpha\mu}^{ij}(x) c_j(x), \\ J_\mu^A(x) &= \bar{c}_i(x) \gamma^\alpha \gamma_5 G_{\alpha\mu}^{ij}(x) c_j(x), \end{aligned} \quad (4)$$

$$\begin{aligned} J_{\mu\nu}^0(x) &= \bar{c}_i(x) G_{\mu\nu}^{ij}(x) c_j(x), \\ J_{\mu\nu}^5(x) &= \bar{c}_i(x) i\gamma_5 G_{\mu\nu}^{ij}(x) c_j(x), \end{aligned} \quad (5)$$

$$\begin{aligned} J_{\mu\nu}^{\sigma,0}(x) &= \bar{c}_i(x) [\sigma_\mu^\alpha G_{\alpha\nu}^{ij}(x) - \sigma_\nu^\alpha G_{\alpha\mu}^{ij}(x)] c_j(x), \\ J_{\mu\nu}^{\sigma,5}(x) &= \bar{c}_i(x) i\gamma_5 [\sigma_\mu^\alpha G_{\alpha\nu}^{ij}(x) - \sigma_\nu^\alpha G_{\alpha\mu}^{ij}(x)] c_j(x), \end{aligned} \quad (6)$$

$$\begin{aligned} J_{\mu\nu}^{2,\sigma,0}(x) &= \bar{c}_i(x) \left[\sigma_\mu^\alpha G_{\alpha\nu}^{ij}(x) + \sigma_\nu^\alpha G_{\alpha\mu}^{ij}(x) \right. \\ &\quad \left. - \frac{1}{2} g_{\mu\nu} \sigma^{\beta\alpha} G_{\alpha\beta}^{ij}(x) \right] c_j(x), \\ J_{\mu\nu}^{2,\sigma,5}(x) &= \bar{c}_i(x) i\gamma_5 \left[\sigma_\mu^\alpha G_{\alpha\nu}^{ij}(x) + \sigma_\nu^\alpha G_{\alpha\mu}^{ij}(x) \right. \\ &\quad \left. - \frac{1}{2} g_{\mu\nu} \sigma^{\beta\alpha} G_{\alpha\beta}^{ij}(x) \right] c_j(x), \end{aligned} \quad (7)$$

the subscripts and superscripts i and j of the c -quark, $\bar{c}$ -quark and gluon fields are color indexes, the gluon field strength $G_{\alpha\beta}^{ij} = G_{\alpha\beta}^a t_{ij}^a$, $G_{\alpha\beta}^a = \partial_\alpha G_\beta^a - \partial_\beta G_\alpha^a + g_s f^{abc} G_\alpha^b G_\beta^c$, $t^a = \frac{\lambda^a}{2}$, the λ^a is the Gell-Mann matrix. We modify the hybrid currents in Ref. [26] to have definite quantum numbers so as to avoid using complex projectors to obtain the hadronic representations.

The $J^P(x)$ and $J^S(x)$ couple potentially to the hybrid states with the $J^{PC} = 0^{-+}$ and 0^{++} , respectively, the superscripts P and S denote pseudoscalar and scalar, respectively. The $J_\mu^V(x)$ and $J_\mu^A(x)$ couple potentially to the hybrid states with the $J^{PC} = 1^{-+}$ (0^{++}) and 1^{+-} (0^{--}), respectively, the superscripts V and A denote vector and axial-vector, respectively.

The $J_{\mu\nu}^0(x)$ and $J_{\mu\nu}^5(x)$ couple potentially to the hybrid states with the $J^{PC} = 1^{+-}$ and 1^{--} . The $J_{\mu\nu}^{\sigma,0}(x)$ and $J_{\mu\nu}^{\sigma,5}(x)$ couple potentially to the hybrid states with the $J^{PC} = 1^{++}$ and 1^{-+} . The $J_{\mu\nu}^{2,\sigma,0}(x)$ and $J_{\mu\nu}^{2,\sigma,5}(x)$ couple potentially to the hybrid states with the $J^{PC} = 2^{++}$ and 2^{-+} , respectively. The superscripts 0, 5 and σ denote that there exists a Dirac matrix 1, γ_5 and $\sigma_{\alpha\beta}$ in the currents, respectively. The superscript 2 denotes the spin $j = 2$.

At the hadron side, we insert a complete set of intermediate hadronic states with the same quantum numbers as the currents $J(x)$, $J_\mu(x)$ and $J_{\mu\nu}(x)$ into the correlation functions $\Pi(p)$, $\Pi_{\mu\mu'}(p)$ and $\Pi_{\mu\nu\mu'\nu'}(p)$ respectively to obtain the hadronic representation, and isolate the ground state (in other words, pole) contributions [65,66],

$$\Pi(p) = \frac{\lambda_{P/S}^2}{M_{P/S}^2 - p^2} + \dots = \Pi_{P/S}(p^2), \quad (8)$$

$$\begin{aligned}\Pi_{\mu\mu'}(p) &= \frac{\lambda_{A/V}^2}{M_{A/V}^2 - p^2} \tilde{g}_{\mu\mu'} + \frac{\lambda_{P/S}^2}{M_{P/S}^2 - p^2} \tilde{p}_\mu \tilde{p}_{\mu'} + \dots \\ &= \Pi_{A/V}(p^2) \tilde{g}_{\mu\mu'} + \Pi_{P/S}(p^2) \tilde{p}_\mu \tilde{p}_{\mu'},\end{aligned}\quad (9)$$

$$\begin{aligned}\Pi_{\mu\nu\mu'\nu'}^0(p) &= \varepsilon_{\mu\nu\alpha\beta} \varepsilon_{\mu'\nu'\alpha'\beta'} \tilde{g}^{\alpha\alpha'} \tilde{p}^\beta \tilde{p}^{\beta'} \Pi_A(p^2) + S_{\mu\nu\mu'\nu'} \Pi_V(p^2), \\ \Pi_{\mu\nu\mu'\nu'}^5(p) &= \varepsilon_{\mu\nu\alpha\beta} \varepsilon_{\mu'\nu'\alpha'\beta'} \tilde{g}^{\alpha\alpha'} \tilde{p}^\beta \tilde{p}^{\beta'} \Pi_V(p^2) + S_{\mu\nu\mu'\nu'} \Pi_A(p^2),\end{aligned}\quad (10)$$

$$\begin{aligned}\Pi_{\mu\nu\mu'\nu'}^{\sigma,0}(p) &= \varepsilon_{\mu\nu\alpha\beta} \varepsilon_{\mu'\nu'\alpha'\beta'} \tilde{g}^{\alpha\alpha'} \tilde{p}^\beta \tilde{p}^{\beta'} \Pi_A(p^2) + S_{\mu\nu\mu'\nu'} \Pi_V(p^2), \\ \Pi_{\mu\nu\mu'\nu'}^{\sigma,5}(p) &= \varepsilon_{\mu\nu\alpha\beta} \varepsilon_{\mu'\nu'\alpha'\beta'} \tilde{g}^{\alpha\alpha'} \tilde{p}^\beta \tilde{p}^{\beta'} \Pi_V(p^2) + S_{\mu\nu\mu'\nu'} \Pi_A(p^2),\end{aligned}\quad (11)$$

$$\begin{aligned}\Pi_{\mu\nu\mu'\nu'}^{2,\sigma,0/5}(p) &= \frac{\lambda_T^2}{M_T^2 - p^2} \left(\frac{\tilde{g}_{\mu\mu'} \tilde{g}_{\nu\nu'} + \tilde{g}_{\nu\mu'} \tilde{g}_{\mu\nu}}{2} - \frac{\tilde{g}_{\mu\nu} \tilde{g}_{\mu'\nu'}}{3} \right) + \dots, \\ &= \Pi_T(p^2) \left(\frac{\tilde{g}_{\mu\mu'} \tilde{g}_{\nu\nu'} + \tilde{g}_{\nu\mu'} \tilde{g}_{\mu\nu}}{2} - \frac{\tilde{g}_{\mu\nu} \tilde{g}_{\mu'\nu'}}{3} \right) + \dots,\end{aligned}\quad (12)$$

$$S_{\mu\nu\mu'\nu'} = \tilde{g}_{\mu\mu'} \tilde{p}_\nu \tilde{p}_{\nu'} - \tilde{g}_{\nu\mu'} \tilde{p}_\mu \tilde{p}_{\nu'} - \tilde{g}_{\mu\nu'} \tilde{p}_\nu \tilde{p}_{\mu'} + \tilde{g}_{\nu\nu'} \tilde{p}_\mu \tilde{p}_{\mu'},\quad (13)$$

where we have taken the following definitions for the pole residues and polarization vectors,

$$\begin{aligned}\langle 0 | J^P(0) | H_{0^{++}}(p) \rangle &= \lambda_H, \\ \langle 0 | J^S(0) | H_{0^{++}}(p) \rangle &= \lambda_H,\end{aligned}\quad (14)$$

$$\begin{aligned}\langle 0 | J_\mu^V(0) | H_{1^{+-}}(p) \rangle &= \lambda_H \varepsilon_\mu, \\ \langle 0 | J_\mu^A(0) | H_{1^{+-}}(p) \rangle &= \lambda_H \varepsilon_\mu,\end{aligned}\quad (15)$$

$$\begin{aligned}\langle 0 | J_\mu^V(0) | H_{0^{++}}(p) \rangle &= \lambda_H \tilde{p}_\mu, \\ \langle 0 | J_\mu^A(0) | H_{0^{--}}(p) \rangle &= \lambda_H \tilde{p}_\mu,\end{aligned}\quad (16)$$

$$\begin{aligned}\langle 0 | J_{\mu\nu}^0(0) | H_{1^{+-}}(p) \rangle &= \lambda_H \varepsilon_{\mu\nu\alpha\beta} \tilde{p}^\alpha \varepsilon^\beta, \\ \langle 0 | J_{\mu\nu}^0(0) | H_{1^{--}}(p) \rangle &= \lambda_H (\tilde{p}_\mu \varepsilon_\nu - \tilde{p}_\nu \varepsilon_\mu),\end{aligned}\quad (17)$$

$$\begin{aligned}\langle 0 | J_{\mu\nu}^5(0) | H_{1^{+-}}(p) \rangle &= \lambda_H (\tilde{p}_\mu \varepsilon_\nu - \tilde{p}_\nu \varepsilon_\mu), \\ \langle 0 | J_{\mu\nu}^5(0) | H_{1^{--}}(p) \rangle &= \lambda_H \varepsilon_{\mu\nu\alpha\beta} \tilde{p}^\alpha \varepsilon^\beta,\end{aligned}\quad (18)$$

$$\begin{aligned}\langle 0 | J_{\mu\nu}^{\sigma,0}(0) | H_{1^{++}}(p) \rangle &= \lambda_H \varepsilon_{\mu\nu\alpha\beta} \tilde{p}^\alpha \varepsilon^\beta, \\ \langle 0 | J_{\mu\nu}^{\sigma,0}(0) | H_{1^{--}}(p) \rangle &= \lambda_H (\tilde{p}_\mu \varepsilon_\nu - \tilde{p}_\nu \varepsilon_\mu),\end{aligned}\quad (19)$$

$$\begin{aligned}\langle 0 | J_{\mu\nu}^{\sigma,5}(0) | H_{1^{++}}(p) \rangle &= \lambda_H (\tilde{p}_\mu \varepsilon_\nu - \tilde{p}_\nu \varepsilon_\mu), \\ \langle 0 | J_{\mu\nu}^{\sigma,5}(0) | H_{1^{--}}(p) \rangle &= \lambda_H \varepsilon_{\mu\nu\alpha\beta} \tilde{p}^\alpha \varepsilon^\beta,\end{aligned}\quad (20)$$

$$\begin{aligned}\langle 0 | J_{\mu\nu}^{2,\sigma,5}(0) | H_{2^{+-}}(p) \rangle &= \lambda_H \varepsilon_{\mu\nu}, \\ \langle 0 | J_{\mu\nu}^{2,\sigma,0}(0) | H_{2^{++}}(p) \rangle &= \lambda_H \varepsilon_{\mu\nu},\end{aligned}\quad (21)$$

$\tilde{g}_{\mu\mu'} = -g_{\mu\mu'} + \tilde{p}_\mu \tilde{p}_{\mu'}$, $\tilde{p}_\mu \tilde{p}_{\mu'} = \frac{p_\mu p_{\mu'}}{p^2}$, and the symbols $H = P, S, V, A$, and T denote the pseudoscalar, scalar, vector, axialvector, and tensor hybrid states, respectively. We add the subscripts $0^{++}, 0^{-+}, 0^{--}, 1^{++}, 1^{-+}, 1^{+-}, 1^{--}, 2^{-+}$, and 2^{++} to denote the corresponding quantum numbers J^{PC} of the hidden-charm hybrid states.

At the QCD side, we contract the quark and gluon fields in the correlation functions $\Pi(p)$, $\Pi_{\mu\mu'}(p)$, and $\Pi_{\mu\nu\mu'\nu'}(p)$ with the Wick theorem, obtain the results, for example,

$$\Pi_{\mu\mu'}^V(p) = -ig_s^2 t_{ij}^a t_{j'i'}^b \int d^4x e^{ip \cdot x} \text{Tr}[\gamma^\alpha S_c^{ij}(x) \gamma^\beta S_c^{i'i}(-x)] S_{\alpha\mu\beta\mu'}^{ab}(x),\quad (22)$$

where the $S_{\mu\nu\alpha\beta}^{ab}(x)$ and $S_c^{ij}(x)$ are the full gluon and c quark propagators, respectively,

$$\begin{aligned}S_{\mu\nu\alpha\beta}^{ab}(x) &= \frac{\delta_{ab}}{2\pi^2 x^6} [g_{\mu\alpha}(x^2 g_{\nu\beta} - 4x_\nu x_\beta) + g_{\nu\beta}(x^2 g_{\mu\alpha} - 4x_\mu x_\alpha) - g_{\mu\beta}(x^2 g_{\nu\alpha} - 4x_\nu x_\alpha) \\ &\quad - g_{\nu\alpha}(x^2 g_{\mu\beta} - 4x_\mu x_\beta)] - \frac{g_s f^{abc}}{4\pi^2 x^4} [G_{\mu\alpha}^c(x^2 g_{\nu\beta} - 2x_\nu x_\beta) + G_{\nu\beta}^c(x^2 g_{\mu\alpha} - 2x_\mu x_\alpha) \\ &\quad - G_{\mu\beta}^c(x^2 g_{\nu\alpha} - 2x_\nu x_\alpha) - G_{\nu\alpha}^c(x^2 g_{\mu\beta} - 2x_\mu x_\beta)] - \frac{g_s f^{abc}}{8\pi^2 x^4} [g_{\mu\alpha} G_{\lambda\beta}^c(x^2 g_\nu^\lambda - 2x_\nu x^\lambda) \\ &\quad + g_{\nu\beta} G_{\lambda\alpha}^c(x^2 g_\mu^\lambda - 2x_\mu x^\lambda) - g_{\mu\beta} G_{\lambda\alpha}^c(x^2 g_\nu^\lambda - 2x_\nu x^\lambda) - g_{\nu\alpha} G_{\lambda\beta}^c(x^2 g_\mu^\lambda - 2x_\mu x^\lambda)] + \dots,\end{aligned}\quad (23)$$

$$\begin{aligned}
S_c^{ij}(x) = & \frac{i}{(2\pi)^4} \int d^4k e^{-ik \cdot x} \left\{ \frac{\delta_{ij}}{k - m_c} - \frac{g_s G_{\alpha\beta}^n t_{ij}^n \sigma^{\alpha\beta} (k + m_c) + (k + m_c) \sigma^{\alpha\beta}}{4(k^2 - m_c^2)^2} \right. \\
& + \frac{g_s D_\alpha G_{\beta\lambda}^n t_{ij}^n (f^{\lambda\beta\alpha} + f^{\lambda\alpha\beta})}{3(k^2 - m_c^2)^4} - \frac{g_s^2 (t^a t^b)_{ij} G_{\alpha\beta}^a G_{\mu\nu}^b (f^{\alpha\beta\mu\nu} + f^{\alpha\mu\beta\nu} + f^{\alpha\mu\nu\beta})}{4(k^2 - m_c^2)^5} \\
& + \frac{i(D_\alpha D_\beta + D_\beta D_\alpha) g_s G_{\rho\mu}^n t_{ij}^n (f^{\mu\rho\alpha\beta} + f^{\mu\alpha\rho\beta} + f^{\mu\alpha\beta\rho})}{8(k^2 - m_c^2)^5} \\
& \left. + \frac{\langle g_s^3 GGG \rangle (k + m_c) [k(k^2 - 3m_c^2) + 2m_c(2k^2 - m_c^2)] (k + m_c)}{48(k^2 - m_c^2)^6} + \dots \right\}, \quad (24)
\end{aligned}$$

$$f^{\lambda\alpha\beta} = (k + m_c) \gamma^\lambda (k + m_c) \gamma^\alpha (k + m_c) \gamma^\beta (k + m_c),$$

$$f^{\alpha\beta\mu\nu} = (k + m_c) \gamma^\alpha (k + m_c) \gamma^\beta (k + m_c) \gamma^\mu (k + m_c) \gamma^\nu (k + m_c), \quad (25)$$

and $D_\alpha = \partial_\alpha - ig_s G_\alpha^a t^a$, $\langle g_s^3 GGG \rangle = \langle g_s^3 f^{abc} G_\mu^a G_\nu^b G_\rho^c \rangle$ [23,66], we add the superscript V in the correlation function to denote the current $J_\mu^V(x)$. Then we compute the integrals in the coordinate space and momentum space sequentially in the D -dimension, and obtain the QCD spectral densities

$\rho_{\text{QCD}}(s)$ through dispersion relation. For a detailed example, one can consult Ref. [50]. We consider the vacuum condensates up to dimension 6, and compute the vacuum condensates $\langle \frac{\alpha_s}{\pi} GG \rangle$, $\langle g_s^3 GGG \rangle$ and $\langle \bar{q}q \rangle^2$ with $q = u, d$ or s . In calculations, we have used the following formulas,

$$\langle g_s^2 D_\alpha G_{\mu\nu}^a D_\beta G_{\rho\sigma}^a \rangle = \frac{g_s^4 \langle jj \rangle}{36} (g_{\alpha\nu} g_{\beta\sigma} g_{\mu\rho} - g_{\alpha\mu} g_{\beta\sigma} g_{\nu\rho} - g_{\alpha\nu} g_{\beta\rho} g_{\mu\sigma} + g_{\alpha\mu} g_{\beta\rho} g_{\nu\sigma}), \quad (26)$$

$$\begin{aligned}
\langle g_s^2 D_\alpha D_\beta G_{\mu\nu}^a G_{\rho\sigma}^a \rangle = & \left[\frac{5\langle g_s^3 GGG \rangle}{72} - \frac{g_s^4 \langle jj \rangle}{36} \right] g_{\alpha\beta} (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}) \\
& - \left[\frac{\langle g_s^3 GGG \rangle}{144} + \frac{g_s^4 \langle jj \rangle}{72} \right] [g_{\beta\mu} (g_{\alpha\rho} g_{\nu\sigma} - g_{\alpha\sigma} g_{\nu\rho}) - g_{\beta\nu} (g_{\alpha\rho} g_{\mu\sigma} - g_{\alpha\sigma} g_{\mu\rho})] \\
& - \left[\frac{7\langle g_s^3 GGG \rangle}{144} + \frac{g_s^4 \langle jj \rangle}{72} \right] [g_{\alpha\mu} (g_{\beta\rho} g_{\nu\sigma} - g_{\beta\sigma} g_{\nu\rho}) - g_{\alpha\nu} (g_{\beta\rho} g_{\mu\sigma} - g_{\beta\sigma} g_{\mu\rho})], \quad (27)
\end{aligned}$$

$$\langle jj \rangle = \langle \bar{\psi} \gamma_\mu t^a \psi \bar{\psi} \gamma^\mu t^a \psi \rangle = -\frac{4}{9} [\langle \bar{u}u \rangle^2 + \langle \bar{d}d \rangle^2 + \langle \bar{s}s \rangle^2], \quad (28)$$

where $\psi = u, d$ and s . The QCD spectral densities $\rho_{\text{QCD}}(s)$ have the terms $\frac{\alpha_s}{\pi}$, $\langle \frac{\alpha_s}{\pi} GG \rangle$, $\langle g_s^3 GGG \rangle$ and $g_s^4 \langle jj \rangle$ of the leading-order (LO), and the terms $\frac{\alpha_s}{\pi} \langle \frac{\alpha_s}{\pi} GG \rangle$, $\frac{\alpha_s}{\pi} \langle g_s^3 GGG \rangle$ and $\frac{\alpha_s}{\pi} g_s^4 \langle jj \rangle$ of the next-to-leading order (NLO), they are all originated from directly calculating the integrals in Eqs. (22)–(25). Compared with the previous works [31–33,36,40], we perform the operator product expansion in a more comprehensive way by taking account of both the LO and NLO contributions, as the derivatives $D_\alpha G_{\mu\nu}^a$ and $D_\alpha D_\beta G_{\mu\nu}^a$ lead to both the LO and NLO contributions. In Ref. [40], although the higher dimensional condensates $\langle \frac{\alpha_s}{\pi} GG \rangle^2$, $\langle \frac{\alpha_s}{\pi} GG \rangle \langle g_s^3 GGG \rangle$ and $\langle g_s^3 GGG \rangle^2$ are taken into account, they are far from complete, many other vacuum condensates of dimension-8 are neglected [66], furthermore, the important vacuum condensate $\langle jj \rangle$ of dimension-6 is also neglected.

We match the hadronic representation with the QCD representation for the components $\Pi_i(p^2)$ with $i = P, S, V, A$, and T below the continuum thresholds s_0 and accomplish the Borel transformation with respect to the variable $P^2 = -p^2$ to obtain the QCD sum rules

$$\lambda_H^2 \exp\left(-\frac{M_H^2}{T^2}\right) = \int_{4m_c^2}^{s_0} ds \rho_{\text{QCD}}(s) \exp\left(-\frac{s}{T^2}\right), \quad (29)$$

where the T^2 is the Borel parameter.

At last, we differentiate the QCD sum rules in Eq. (29) with respect to the variable $\tau = \frac{1}{T^2}$, and obtain the QCD sum rules for the masses of the hidden-charm hybrid states,

$$M_H^2 = -\frac{\int_{4m_c^2}^{s_0} ds \frac{d}{d\tau} \rho_{\text{QCD}}(s) \exp(-\tau s)}{\int_{4m_c^2}^{s_0} ds \rho_{\text{QCD}}(s) \exp(-\tau s)}. \quad (30)$$

III. NUMERICAL RESULTS AND DISCUSSIONS

We write down the energy-scale dependence of the input parameters,

$$\begin{aligned}\langle\bar{q}q\rangle(\mu) &= \langle\bar{q}q\rangle(1\text{ GeV}) \left[\frac{\alpha_s(1\text{ GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{33-2n_f}}, \\ m_c(\mu) &= m_c(m_c) \left[\frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{12}{33-2n_f}}, \\ \alpha_s(\mu) &= \frac{1}{b_0 t} \left[1 - \frac{b_1 \log t}{b_0^2 t} + \frac{b_1^2 (\log^2 t - \log t - 1) + b_0 b_2}{b_0^4 t^2} \right],\end{aligned}\quad (31)$$

where the quarks $q = u, d$ and $s, t = \log \frac{\mu^2}{\Lambda_{\text{QCD}}^2}$, $b_0 = \frac{33-2n_f}{12\pi}$, $b_1 = \frac{153-19n_f}{24\pi^2}$, $b_2 = \frac{2857-5033n_f+325n_f^2}{128\pi^3}$, $\Lambda_{\text{QCD}} = 210\text{ MeV}$, 292 MeV , and 332 MeV for the flavors $n_f = 5, 4$, and 3 , respectively [2,67]. And we choose $n_f = 4$ in the present analysis.

At the initial points, we take the standard values $\langle\bar{q}q\rangle = -(0.24 \pm 0.01\text{ GeV})^3$, $\langle\bar{s}s\rangle = (0.8 \pm 0.1)\langle\bar{q}q\rangle$, $\pi\langle\frac{\alpha_s GG}{\pi}\rangle = (6.40 \pm 0.30)\text{ GeV}^4$, and $\langle g_s^3 GGG\rangle = (8.2 \pm 1.0)\text{ GeV}^2 \pi\langle\frac{\alpha_s GG}{\pi}\rangle$ at the particular energy scale $\mu = 1\text{ GeV}$ with $q = u$ and d [65,66,68–70], and take the $\overline{\text{MS}}$ mass $m_c(m_c) = (1.275 \pm 0.025)\text{ GeV}$ from the Particle Data Group [2]. The values of the gluon condensate and three-gluon condensate have been updated from time to time, and change considerably, we choose most recent values [69,70]. Thereafter, we would like to refer the c -quark mass, vacuum condensates and continuum threshold parameters as the input parameters.

In our previous works, we take the energy scale formula,

$$\mu = \sqrt{M_{X/Y/Z/P}^2 - (2\mathbb{M}_c)^2}, \quad (32)$$

to choose the optimal energy scales of the QCD spectral densities for the hidden-charm tetraquark (molecular) states and pentaquark (molecular) states [9,54–58,60,63,64], where the effective c -quark mass $\mathbb{M}_c = 1.82\text{ GeV}$ for the diquark type tetraquark and pentaquark states [9,54–58,63]. In this work, we adopt the value $\mathbb{M}_c = 1.82\text{ GeV}$.

As the spectrum of the hidden-charm hybrid states is rather vague, we have no definite knowledge about the energy gaps between the ground states and first radial excitations. In practical calculations, we assume the energy gaps are about $0.6\text{--}0.7\text{ GeV}$, just like in the case of the hidden-charm tetraquark (molecular) states and pentaquark (molecular) states [9,54–58,60,63,64], and change the continuum threshold parameters s_0 and Borel parameters T^2 to satisfy the four criteria:

- (i) Pole dominance at the hadron side;
- (ii) Convergence of the operator product expansion;
- (iii) Appearance of the Borel platforms; and

- (iv) Satisfying the energy scale formula, via trial and error.

At first, we define the pole contributions (PC),

$$\text{PC} = \frac{\int_{4m_c^2}^{s_0} ds \rho_{\text{QCD}}(s) \exp(-\frac{s}{T^2})}{\int_{4m_c^2}^{\infty} ds \rho_{\text{QCD}}(s) \exp(-\frac{s}{T^2})}, \quad (33)$$

and the contributions of the vacuum condensates $D(n)$ of dimension n ,

$$D(n) = \frac{\int_{4m_c^2}^{s_0} ds \rho_{\text{QCD},n}(s) \exp(-\frac{s}{T^2})}{\int_{4m_c^2}^{\infty} ds \rho_{\text{QCD}}(s) \exp(-\frac{s}{T^2})}. \quad (34)$$

After numerous trial and error, we obtain the Borel windows, continuum threshold parameters, optimal energy scales of the spectral densities and pole contributions, which are shown explicitly in Table I. At the Borel windows, the ground state contributions are about (40–60)%, while the central values are slightly larger than 50%, the pole dominance criterion is satisfied, where we have taken the central values of the c -quark mass and vacuum condensates. On the other hand, in the Borel windows shown Table I, the contributions of the vacuum condensates could be classified into five relations $D(0) > D(4) \gg |D(6)|$, $D(0) \gg D(4) \gg |D(6)|$, $D(0) \gg |D(4)| \sim |D(6)|$, $D(0) \gg D(4) > |D(6)|$ and $D(0) \sim D(4) \gg |D(6)|$ for the central values of the input parameters. In all the five cases,

TABLE I. The Borel windows, continuum threshold parameters, energy scales, and pole contributions for the hidden-charm hybrid states.

Currents	J^{PC}	$T^2\text{ (GeV}^2\text{)}$	$\sqrt{s_0}\text{ (GeV)}$	$\mu\text{ (GeV)}$	Pole
$J^P(x)$	0^{-+}	4.0–4.6	4.75 ± 0.10	2.0	(41–60)%
$J^S(x)$	0^{++}	6.3–7.6	5.75 ± 0.10	3.6	(40–61)%
$J_\mu^V(x)$	0^{++}	7.4–9.0	6.05 ± 0.10	3.9	(40–60)%
$J_\mu^A(x)$	0^{--}	7.8–9.5	6.45 ± 0.10	4.5	(40–60)%
$J_\mu^V(x)$	1^{-+}	3.9–4.6	4.60 ± 0.10	1.7	(40–61)%
$J_{\mu\nu}^{\sigma,0/5}(x)$	1^{-+}	3.8–4.5	4.60 ± 0.10	1.7	(40–61)%
$J_{\mu\nu}^{\sigma,0/5}(x)$	1^{++}	6.0–7.3	5.60 ± 0.10	3.4	(40–61)%
$J_{\mu\nu}^0(x)$	1^{+-}	4.8–5.7	5.00 ± 0.10	2.4	(40–61)%
$J_\mu^A(x)$	1^{+-}	5.6–6.7	5.40 ± 0.10	3.1	(40–60)%
$J_{\mu\nu}^5(x)$	1^{+-}	6.6–8.0	5.85 ± 0.10	3.7	(40–60)%
$J_{\mu\nu}^5(x)$	1^{--}	4.3–5.1	4.65 ± 0.10	1.8	(41–61)%
$J_{\mu\nu}^0(x)$	1^{--}	7.3–8.8	6.30 ± 0.10	4.3	(40–60)%
$J_{\mu\nu}^{2,\sigma,5}(x)$	2^{-+}	4.4–5.2	4.90 ± 0.10	2.3	(40–61)%
$J_{\mu\nu}^{2,\sigma,0}(x)$	2^{++}	5.6–6.7	5.45 ± 0.10	3.2	(40–60)%

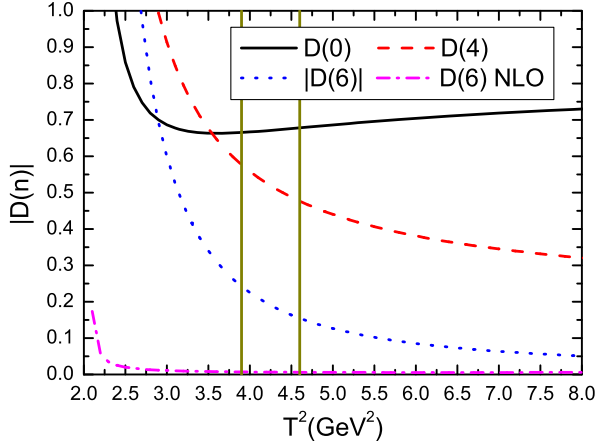


FIG. 1. The contributions of the vacuum condensates for the hybrid state with the $J^{PC} = 1^{-+}$ for the current $J_\mu^V(x)$, where the two vertical lines denote the Borel window.

the operator product expansions converge very well, and we would like to illustrate the first case in Fig. 1.

As an example, in Fig. 1, we plot the contributions of the vacuum condensates for the hidden-charm hybrid state with the $J^{PC} = 1^{-+}$ for the current $J_\mu^V(x)$ with variation of the Borel parameter T^2 for the central values of the input parameters. From the figure, we can see explicitly that the contributions $D(0) > D(4) \gg |D(6)|$ in the Borel window, on the other hand, the contribution of the $D(6)$ of the NLO is about 1%, as the vacuum condensates of dimension-8 are originated from the operators of higher-order expansion of the full heavy quark propagator, see Eq. (24), and accompanied with the powers g_s^4 and g_s^5 , and their contributions are of the NLO [66], and thus they can be neglected safely. All in all, the convergent behaviors of the operator product expansion are very good.

Again, as an example, in Fig. 2, we plot the mass of the hidden-charm hybrid state with the $J^{PC} = 1^{-+}$ with

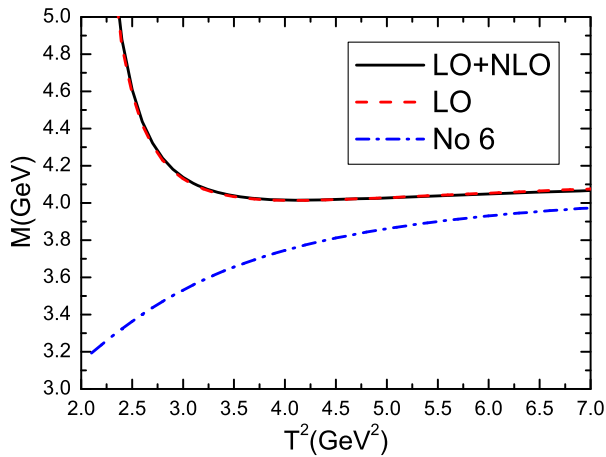


FIG. 2. The mass of the hidden-charm hybrid state with the $J^{PC} = 1^{-+}$ for the current $J_\mu^V(x)$, where the “No 6” denotes the contributions of the vacuum condensates of dimension-6 are not included.

variation of the Borel parameter T^2 for the current $J_\mu^V(x)$ in the cases of different truncations of the operator product expansion for the central values of the input parameters. From the figure, we can see explicitly that the NLO contributions can be absorbed into the pole residue safely, and result in almost degenerated mass, while the LO contributions of dimension-6 play an important role and affect the predicted mass significantly beyond the pole residue.

Finally, we take account of uncertainties of all the parameters and obtain the masses and pole residues of the hidden-charm hybrid states, which are shown explicitly in Table II.

In this work, we calculate the uncertainties δf with the formula,

$$\delta f = \sqrt{\sum_i \left(\frac{\partial f}{\partial x_i} \right)^2 \big|_{x_i=\bar{x}_i} (x_i - \bar{x}_i)^2}, \quad (35)$$

where the f denotes the M_H and λ_H , the x_i denotes the parameters m_c , $\langle \bar{q}q \rangle$, $\langle \bar{s}s \rangle$, $\langle \frac{\alpha_s GG}{\pi} \rangle$, $\langle g_s^2 GGG \rangle$, s_0 , and T^2 . As the partial derivatives $\frac{\partial f}{\partial x_i}$ are difficult to carry out analytically, we take the approximation $(\frac{\partial f}{\partial x_i})^2 (x_i - \bar{x}_i)^2 \approx [f(\bar{x}_i \pm \delta x_i) - f(\bar{x}_i)]^2$ in numerical calculations.

We obtain the masses of the hidden-charm hybrid states from a fraction, see Eq. (30), the uncertainties originate

TABLE II. The masses and pole residues of the hidden-charm hybrid states.

Currents	J^{PC}	M_H (GeV)	λ_H (GeV ⁴)
$J^P(x)$	0^{-+}	4.17 ± 0.08	1.97 ± 0.22
$J^S(x)$	0^{++}	5.10 ± 0.06	5.74 ± 0.44
$J_\mu^V(x)$	0^{++}	5.37 ± 0.06	2.05 ± 0.14
$J_\mu^A(x)$	0^{--}	5.79 ± 0.06	2.01 ± 0.14
$J_\mu^V(x)$	1^{-+}	4.02 ± 0.08	$(6.18 \pm 0.64) \times 10^{-1}$
$J_{\mu\nu}^{\sigma,0/5}(x)$	1^{-+}	4.01 ± 0.08	$(5.80 \pm 0.62) \times 10^{-1}$
$J_{\mu\nu}^{\sigma,0/5}(x)$	1^{++}	4.96 ± 0.06	1.68 ± 0.14
$J_{\mu\nu}^0(x)$	1^{+-}	4.36 ± 0.09	$(4.31 \pm 0.40) \times 10^{-1}$
$J_\mu^A(x)$	1^{+-}	4.76 ± 0.07	1.32 ± 0.11
$J_{\mu\nu}^5(x)$	1^{+-}	5.21 ± 0.07	1.12 ± 0.08
$J_{\mu\nu}^5(x)$	1^{--}	4.07 ± 0.10	$(4.37 \pm 0.40) \times 10^{-1}$
$J_{\mu\nu}^0(x)$	1^{--}	5.61 ± 0.07	1.21 ± 0.09
$J_{\mu\nu}^{2,\sigma,5}(x)$	2^{-+}	4.31 ± 0.08	1.24 ± 0.12
$J_{\mu\nu}^{2,\sigma,0}(x)$	2^{++}	4.85 ± 0.06	2.14 ± 0.18

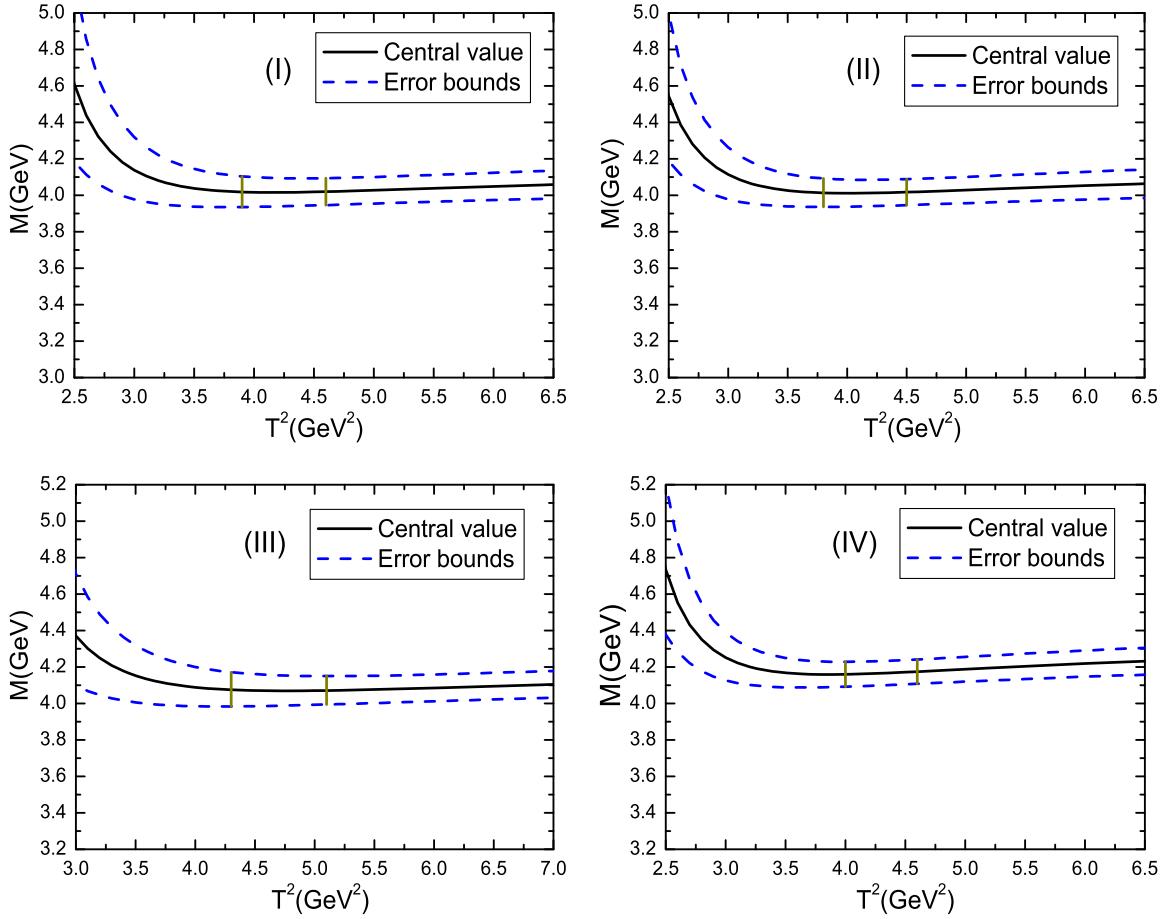


FIG. 3. The masses of the hidden-charm hybrid states, where the (I), (II), (III), and (IV) denote the hybrid states with the $J^{PC} = 1^{-+}$, 1^{-+} , 1^{--} , and 0^{-+} for the currents $J_\mu^V(x)$, $J_{\mu\nu}^{\sigma,0/5}(x)$, $J_{\mu\nu}^5(x)$, and $J^P(x)$, respectively, the two vertical lines denote the Borel windows.

from a parameter in the numerator and denominator are canceled out with each other significantly, the resulting net uncertainties δM_H are very small, about $\delta M_H/M_H \sim (1-3)\%$, while the uncertainties of the pole residues are larger, about $\delta \lambda_H/\lambda_H \sim 10\%$, as there no cancellation occurs. The upper bound $M_H + \delta M_H$ and lower bound $M_H - \delta M_H$ correspond to the continuum threshold parameters $\sqrt{s_0} + \delta\sqrt{s_0}$ and $\sqrt{s_0} - \delta\sqrt{s_0}$, respectively, the relation $\delta M_H \sim \delta\sqrt{s_0} \sim 0.10$ GeV is roughly satisfied, just like in the case of the hidden-charm tetraquark (molecular) states [9,54–58,60]. For example, if we take $\delta\sqrt{s_0} = 0.20$ GeV, then $M_H = 4.02 \pm 0.12$ GeV in stead of 4.02 ± 0.08 GeV for the current $J_\mu^V(x)$, the relation $\delta M_H \sim \delta\sqrt{s_0}$ is deviated significantly, the energy gaps $\sqrt{s_0} - M_H$ and $[\sqrt{s_0} \pm \delta\sqrt{s_0}] - [M_H \pm \delta M_H]$ would not have consistent values.

From Tables I–II, we observe clearly that the energy scale formula, see Eq. (32), is satisfied very good.

In Fig. 3, we plot the masses of the hidden-charm hybrid states with the $J^{PC} = 1^{-+}$, 1^{-+} , 1^{--} , and 0^{-+} for the currents $J_\mu^V(x)$, $J_{\mu\nu}^{\sigma,0/5}(x)$, $J_{\mu\nu}^5(x)$, and $J^P(x)$ respectively with variations of the Borel parameters T^2 , as an example,

where the error bounds originate from uncertainties of the input parameters. From the figure, we can see explicitly that there really appear elegant platforms in the Borel windows, the uncertainties come from the Borel parameters are rather small. In fact, we can choose larger Borel parameters at the cost of smaller pole contributions, thus we obtain more flatter platforms and better convergent behaviors in the operator product expansion, see Figs. 1–3. Compared with Ref. [33], we choose larger pole contributions, in fact, only the representation in Ref. [33] is convenient to compare with.

In Fig. 4, we plot the pole contribution of the hidden-charm hybrid state with the $J^{PC} = 1^{-+}$ for the current $J_\mu^V(x)$ with variation of the Borel parameter T^2 for the central values of the c -quark mass and vacuum condensates. From the figure, we can see explicitly that the pole contribution decreases monotonically with increase of the Borel parameter, at the value larger than 4.5 GeV^2 , the upper bound of the Borel parameter, the pole contribution is smaller than 40%, although the operator product expansion converges better, see Fig. 1. We prefer larger pole contributions, (40–60)%, in an uniform way, and expect to obtain robust predictions.

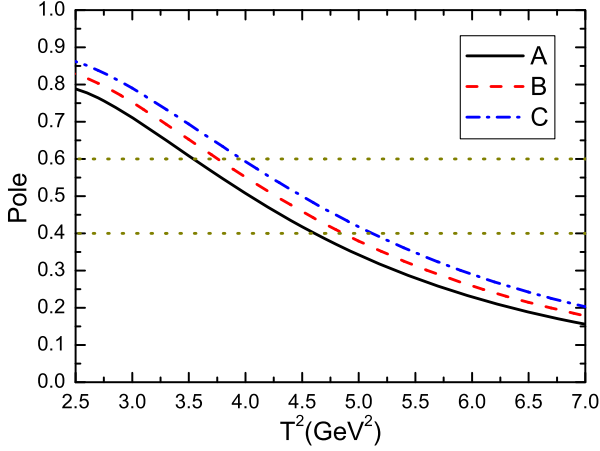


FIG. 4. The pole contribution of the hybrid state with the $J^{PC} = 1^{-+}$ for the current $J_\mu^V(x)$, where the A , B , and C denote the continuum threshold parameters $\sqrt{s_0} = 4.5$ GeV, 4.6 GeV, and 4.7 GeV, respectively.

In Fig. 5, we draw the Feynman diagram for the decays of the hidden-charm hybrid states, where the H denotes the hidden-charm hybrid states, the light quarks $q = u, d, s$, the D denotes the charmed mesons $D, D_s, D^*, D_s^*, D_0, D_{s0}, D_1, D_{s1}, D_2$, and D_{s2} . We can take the pole residues λ_H , see Table II, as input parameters to explore the strong decays of those hidden-charm hybrid states with the three-point QCD sum rules, and obtain ratios among the partial decay widths to diagnose their nature. At the present time, the experimental data on the hidden-charm hybrid states are vague, if there really exist a hybrid state with the $J^{PC} = 1^{-+}$ at about 4.0 GeV, see Table II, then the LHCb's new state $X(4630)$ can be tentatively assigned as the first radial excitation according to the energy gap about 0.6 GeV [8].

In Ref. [71], we construct the color singlet-singlet type four-quark current,

$$J_{\mu\nu}(x) = \frac{1}{\sqrt{2}} [\bar{s}(x)\gamma_\mu c(x)\bar{c}(x)\gamma_\nu s(x) - \bar{s}(x)\gamma_\nu c(x)\bar{c}(x)\gamma_\mu s(x)], \quad (36)$$

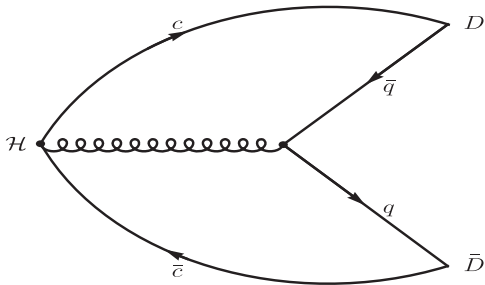


FIG. 5. The Feynman diagram for the decays of the hidden-charm hybrid states.

to study the $D_s^* \bar{D}_{s1} - D_{s1} \bar{D}_s^*$ molecular state with the $J^{PC} = 1^{-+}$, and obtain the prediction, $M_X = 4.67 \pm 0.08$ GeV, which happens to coincide with the mass of the $X(4630)$ from the LHCb collaboration, $4626 \pm 16_{-10}^{+18}$ MeV [8], we tentatively assign the $X(4630)$ as the $D_s^* \bar{D}_{s1} - D_{s1} \bar{D}_s^*$ molecular state with the $J^{PC} = 1^{-+}$ [10]. In Ref. [9], we take the pseudoscalar, scalar, vector, axialvector and tensor diquarks as the basic constituents to construct the four-quark currents, and study the hidden-charm-hidden-strange tetraquark states with the $J^{PC} = 1^{--}$ and 1^{-+} comprehensively. According to the predicted mass 4.68 ± 0.09 GeV of the $[sc]_S[\bar{s}\bar{c}]_{\bar{V}} + [sc]_{\bar{V}}[\bar{s}\bar{c}]_S$ tetraquark state with the $J^{PC} = 1^{-+}$, it is also possible to assign the $X(4630)$ as a tetraquark state. All in all, the $X(4630)$ might have three important Fock components: hybrid state, molecular state and tetraquark state, we have to study its partial decay widths in details to diagnose its substructures, as different substructures could lead to quite different partial decay widths.

IV. CONCLUSION

In this work, we extend our previous works on the hidden-charm tetraquark (molecular) states and pentaquark (molecular) states to study the hidden-charm hybrid states with the quantum numbers $J^{PC} = 0^{-+}, 0^{++}, 0^{--}, 1^{++}, 1^{+-}, 1^{-+}, 1^{--}, 2^{-+}$, and 2^{++} via the QCD sum rules in a systematic way. We calculate the vacuum condensates up to dimensions six in a consistent way by taking account of both the leading-order and next-to-leading order contributions, and take the energy scale formula $\mu =$

$\sqrt{M_{X/Y/Z}^2 - (2M_c)^2}$ to choose the best energy scales of the QCD spectral densities, it is the first time to explore the energy scale dependence of the QCD sum rules for the hidden-charm hybrid states. Finally, we obtain the mass spectrum, which can be confronted to experimental data in the future. While the pole residues can be taken as input parameters to study the two-body strong decays of the hidden-charm hybrid states with the three-point QCD sum rules.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [4].

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